Simulation of Everting Tube Experiments
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1 Introduction

1.1 Literature Review

Prior studies of eversion mechanics have relied on the assumption of constant pressure over time and throughout the extent of the device. Blumenschein et al.[1] applied biomechanical models for plant shoot growth to analogous physical processes in everting tubes and compared simulations with experimental measurements. Notably, for the speeds and design parameters they studied, they did not find an effect indicating air flow resistance between the housing and the everting tube. [**?**] Performed mostly quasi-static modeling of ETs but studied the important buckling phenomena in complex geometric environments which we do not consider here.

[2] created a dynamic, lumped parameter mechanical model of eversion with special emphasis on friction properties between an everting tube and its environment as well as a second contact between the tube and a rod (catheter) in contact with the everted material. This work is close in spirit to ours, but their eversion system was controlled by an “active channel” to which the inner part of the everting tube was connected. Thus eversion was only possible as allowed by active channel motion. This work studied slower rates of eversion (0-6$mmsec^{-1}$) and did not compare time responses, instead comparing velocity which was apparently constant under measured conditions. They measured a “breakway” pressure of 115kPa which was comparable to our experiments. This model included a sophisticated friction model having static, Coulomb, and viscous components.

1.2 Observed Eversion Characteristics

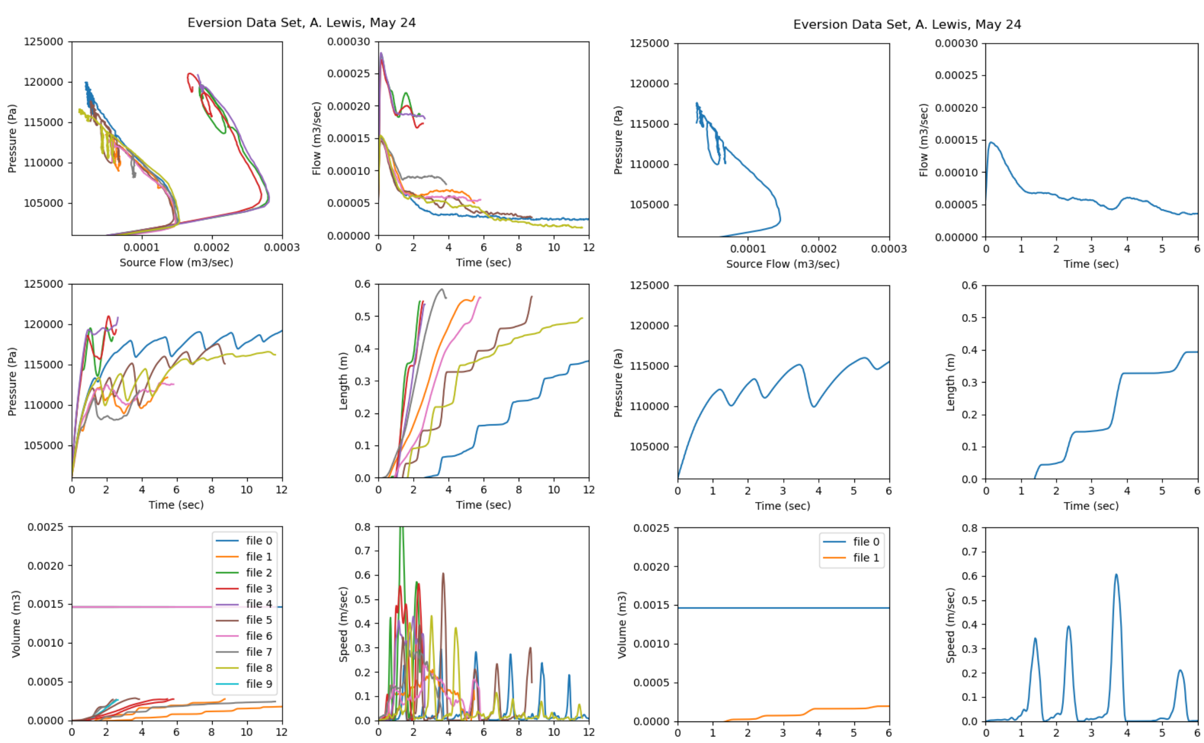

Figure 1: Experimental data displaying dynamic characteristics of loaded eversion. All data (Left), single example (Right) note different time axes for the two plots. (See Lewis, Fig 9. Loads: high inertia, high Friction)

****** Resolve 10x flow rate hack!!! *****

Recently, Lewis [?] measured some complex dynamic behaviors of a tube everting inside a straight support tube of approximately the same diameter (25mm) as the inflated tubing material (Figure 1, Left panel). Data consist of 9 trials on 3 tubes. Tube #1 had 4 trials, tube #2 had 2 trials, and tube #3 had 3 trials. Tubes were supplied by a reel which was free to rotate inside the pressure vessel (housing). Variable loads could be applied to the reel including inertia and coulomb friction. Applied loads are given in Table **??**.

There are two distinct clusters of responses, five slower and three faster (particularly clear in the flow plots (top row). The three datasets having higher flow rates in the top row of Figure 1 are the three trials of tube #3.

One experiment which displays several interesting evertion characteristics is shown in Figure 1, Right Panel. Referring first to the length-vs-time plot (middle right), we see a start-stop or staircase behavior sometimes seen with eversion under relatively constant input pressure. It should be noted that the selected eversion experiment (Right panel) was loaded by "high" inertia and "High" friction applied to the reel. Velocity (lower right) indicates a series of peaks where the eversion "breaks-free" but then stops again a short time later giving length-vs-time a stair-step characteristic.

Pressure (middle left) first grows for about 1 second without corresponding eversion motion, and then oscillates in approximate synchrony with the velocity peaks.

Finally, the pressure-flow phase plot (top left) shows a predominantly diagonal straight line slope. Flow grows rapidly and then the trajectory proceeds from lower right (high initial flow with low pressure) to upper left (lower flow, higher pressure). The trajectory makes loops to lower pressures and back up to the slope at higher pressures. These loops correspond in time to the pressure oscillations (middle left).

These dynamic characteristics (start-stop bursting, pressure oscillations, diagonal phase trajectory, and downward loops) were not all present under all experimental conditions. In summary then, depending on factors to be determined, some or all of these characteristics may or may not be present in a given free eversion run.

1.3 Simulation Goals

The goals of this simulation are:

1. Increase fundamental understanding of the eversion process.
2. Identify key parameters capable of representing the complex dynamic characteristics above.
3. Identify values and value ranges for unknown parameters which fit individual experiments.
4. Clearly segregate the parameters into known, measured quantities vs. free parameters.
5. Codify an efficient manual method for parameter identification.

In any simulation study caution must be exercised when there are a large number of free parameters as there may be several combinations of parameters or manifolds in the parameter space which may fit any dataset. In Section 2.2, below, we review the overall parameter set and classify the parameters into known, independently measureable parameters vs free parameters.

1.4 Dynamic Model

The dynamic model of an eversion drive system includes;

- An everting tube of length L and growth rate $\dot{L}$.
- A pressurized housing
- A reel with rotation $\theta, \dot{\theta}, \ddot{\theta}$, on which tubing is rolled having inertia (assume fixed) of J and radius r.
- A brake which applies a Coulomb friction torque to the reel

$$\tau_c = C\mathrm{sgn}(\dot{\theta})$$

- A "crumple zone" in which eversion material can accumulate between the reel and the everting tube. The length of material in the crumple zone is $L_c >= 0$.

- Eversion happens when the eversion force (pressure $\times$ face area of the tube) exceeds any retarding forces.
- Forces which can oppose eversion include,
  - drag forces and inertial forces required to pull the tubing material inside the deployed tube,
  - reel inertia and reel friction resulting from unspooling material (only when $L_c = 0$).

The everting tube can be in one of two states:

- GROWING (the tube is actively everting, $\dot{L} > 0$)
- STUCK (the tube is not growing due to insufficient everting force, $\dot{L} = 0$)

and the reel/crumple zone can be in one of two additional states:

- TAUGHT (the crumple zone has zero length , $L_c = 0$)
- SLACK (there is material in the crumple zone, $L_c > 0$)

Together the system can be in four states comprising the permutations of these two state variables.

2 Model Structure and System Equations:

We will consider two possible structures:

A *one-compartment* model (Figure 2, Left) assumes that first-order pressure dynamics are present between a source of pressure (P_{source} from a compressor, regulator, or tank accumulator) and a single everting tube system volume (housing plus tube volume, $V_{et} = L \times \pi r^2$.) with uniform pressure, P.

The *two-compartment* model(Figure 2, Right) assumes that additional first-order pressure dynamics are present between the housing volume (pressure, P_1, volume $V_{housing}$ and the tube system volume (V_{et} as above).

For both structures, we can make the tube radius a function of its length, $R_{et}(L)$. This can simulate novel designs with varying radii exploiting new ET fabrication methods such as laser welding or CNC heat sealing.

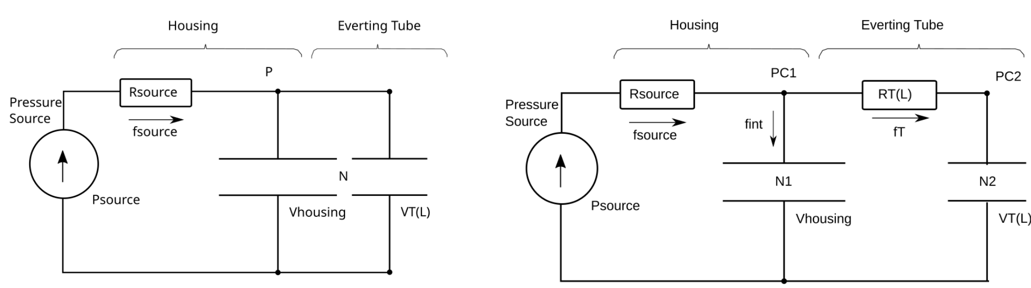

Figure 2: Two model structures applied to eversion experiments of Fig. 1. One compartment model (Left) lumps the housing volume and tube volume together. Two compartment model (Right) introduces length-dependent resistance between the housing and tube, $RT(L)$, to split the pressure state, P into two states, P_{C1}, P_{C2}.

After setting initial conditions (see below), we model eversion dynamics by:

1. Computing pressure, volume, and flow from the source.
2. Computing forces applied to the eversion tip.
3. Accounting for the mechanical advantage (everting material speed is $2 \times \dot{L}$, Pressure applied to everting front develops 1/2 the everting force expected from $P \times A$.)
4. Selecting the dynamic mode from the four combined states above. GROWING or STUCK state is selected by pressure thresholds (related to net eversion force by $F = PA(L)$. TAUGHT or SLACK state is selected by checking length of the crumple zone material.

5. According to the dynamic mode, summing forces, equating to zero, solving for tube and reel accelerations.

6. Eversion does not come to an instant halt. We empirically model a short exponential decay of velocity as tube decelerates.

Specifically for the one-compartment model:

$$V_t = V_{housing} - V_{contents} + LA \tag{1}$$

V_t includes both the reel housing volume (minus the volume of its contents) and the everted tube volume $Vet(L)$ with cross sectional area $A(L)$ where

$$A(L) = \pi Ret(L)^2 \tag{2}$$

where $Ret(L)$ is initially a constant radius, but can also be a function describing a diameter profile manufactured into the everting tube. Alternatively, $Ret(L)$ can describe a constraining tube of radius $R_C(L)$ such that

$$R_C(L) <= Ret(L) \tag{3}$$

When the radius is not constant, tube volume is computed by

$$Vet = \pi \int_0^L Ret(L)^2 dL \tag{4}$$

From the ideal gas equation:

$$P = \frac{NRT}{V_t} \tag{5}$$

where N is the molar mass of gas in the system, R is the gas constant, and T is the temperature in °K. We assume temperature is constant during eversion (isothermal boundary condition).

Computing Forces:

$$F_{ever} = \max(0, PA/2) \tag{6}$$

$$F_c = \tau_{Coulomb}/r \tag{7}$$

Coulomb friction is independent of velocity so does not get scaled by the 2× mechanical advantage. Computing acceleration according to the dynamic state:

GROWING and SLACK:

$$\ddot{L} = \frac{F_{ever} - F_D(FL, \dot{L}) - F_C}{M_T/2} \tag{8}$$

$$\dot{\theta} = \tau_{Coulomb}/J \tag{9}$$

where M_T is the mass of inside tubing, pulled at 1/2 speed by the eversion process, and J is the rotational inertia of the tubing reel inside the housing. We assume J is constant (despite some material being unspooled) and that $M_T = L \cdot \text{ET\_mass\_per\_mm}$

GROWING and TAUGHT:

$$\ddot{L} = (F_{ever} - F_D - F_C)/(M_T/2 + J/r^2) \tag{10}$$

$$\ddot{\theta} = \ddot{L}/r \tag{11}$$

STUCK and (SLACK or TAUGHT):

$$\ddot{L} = -1 * \text{dmax}(0, \alpha * \dot{L}) \tag{12}$$

$$\ddot{\theta} = \tau_{Coulomb}/J \tag{13}$$

where α is an empirical time constant modeling the dynamics of eversion stopping.

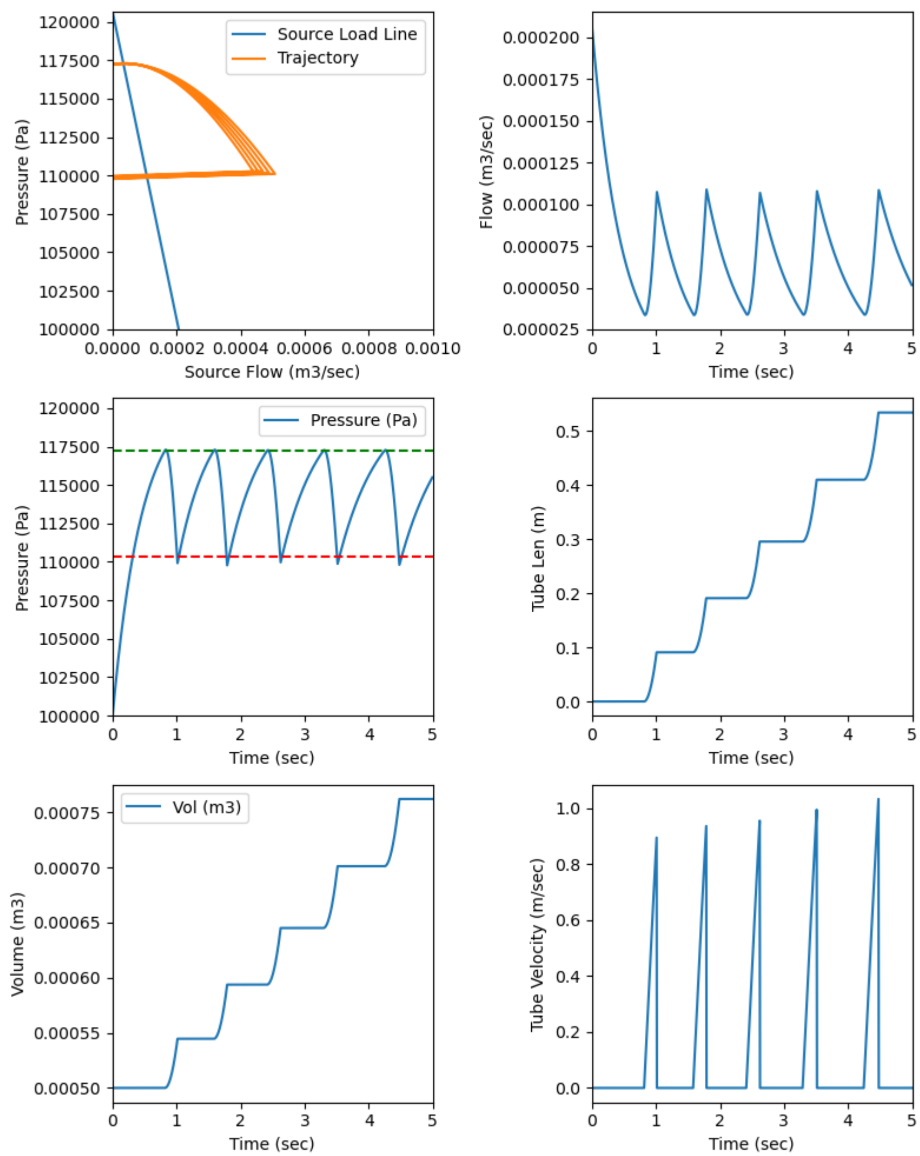

Figure 3: Simulation Run with approximate qualitative match. (See Lewis, Fig 9, hiI, hiTf)

Modeling flow from the pressure source (Thevenin equivalent):

$$Fl_{source} = \frac{P_{source} - P}{R_{source}} \tag{14}$$

Converting airflow (m^3/sec) to rate of molar mass flow:

$$\dot{N} = Fl_{source} \cdot \text{moles\_per\_m3} \tag{15}$$

Model velocity and length dependent eversion force which is resistance to pulling out eversion material:

$$F_D(L, \dot{L}) = \max\Big(2(K_D + LK_{2D})\dot{L}, F_{end}(L)\Big) \tag{16}$$

Where the factor of two accounts for everting material going at twice tube growth rate, $\dot{L}$, and F_{end} is a function to model the everting tube material running into its hard stop:

$$F_{end}(L) = (L/Lmax)^7 P_{source} \pi Ret(L)^2 \tag{17}$$

Update length of crumpled material (if any)

$$L_C = \max(0, r\theta - L) \tag{18}$$

We have a switching model to replicate observed intermittent starting and stopping of eversion which updates the state based on current pressure:

$$\texttt{state} = \begin{cases} \text{GROWING}, & P > P2 \\ \text{unchanged}, & P1 <= P <= P2 \\ \text{STUCK}, & P < P1 \end{cases} \tag{19}$$

We are currently investigating changing this switching model to one based on thresholding net eversion force rather than tip surface pressure:

$$\texttt{state1} = \begin{cases} \text{GROWING}, & F_{ever} > F2 \\ \text{unchanged}, & F1 <= F_{ever} <= F2 \\ \text{STUCK}, & F_{ever} < F1 \end{cases} \tag{20}$$

In either case above, we update the crumple state as

$$\texttt{state2} = \begin{cases} \text{TAUGHT}, & L_C <= 0 \\ \text{SLACK}, & L_C > 0 \end{cases} \tag{21}$$

Finally, we integrate the state variables:

$$\begin{gathered} \dot{L} = \dot{L} + \ddot{L}dt \\ L = L + \dot{L}dt \\ \dot{\theta} = \dot{\theta} + \ddot{\theta}dt \\ \theta = \theta + \dot{\theta}dt \\ N = N + \dot{N}dt \end{gathered} \tag{22}$$

and, if necessary ($Ret(L)$ is not constant),

$$Vet = Vet + \pi Ret(L)^2 \, \dot{(L)} \tag{23}$$

Parameter	Class	Value	Units	Description	source
ET_radius	Known	12.5	mm	Tube radius	tube design
J	Known	5.1E-4	kg/m^2	Reel inertia	design and testing [?]
Lmax	Known	0.5-0.8	m	Tube length	tube design & fitting
Patmosphere	Known	101.325	kPa	Atmospheric pressure	
RT	Known	2.5E+3	m^3Pa/mol	Gas constant x temperature	
T	Known	2.95E+2	$^\circ K$	Room temperature	Thermometer
Tau_coulomb	Known		Nm	Coulomb fric. torque on reel	design and testing [?]
Vhousing_m3	Known	1.256E-3	m^3	Net housing air volume	CAD model
et_MPM	Known	0.1	kg	Tubing mass per meter	measured
rReel	Known		m	Reel radius	CAD model
ET_Res_ratio	Free	0.3—0.6	dimensionless	ET resistance scale	manual tuning
K2drag	Free	0.16—4.0	Nm	Empirical Fit	manual tuning
Kdrag	Free	0.15—6.0	$N/m^2/sec$	Empirical Fit	manual tuning
PBA_static	Free	107—118.5	kPa	Break-away pressure	manual tuning
PHalt_dyn	Free	61—116	kPa	Eversion stopping press.	manual tuning
Psource_SIu	Free	118—145	kPa	Gas source pressure	manual tuning
Rsource_SIu	Free	(8—14)$\times 10^7$	$Pa/m^3/sec$	Slope of P-Fl load-line	manual tuning
Threshold Taper	Free	640—3800	Pa/m	Change in PBA w/ length	manual tuning

Table 1: Parameters used and their sources and values.

2.1 Initial Conditions

For a typical simulation, the initial conditions are

$$\begin{aligned}
P &= 1 \text{ atmosphere} \\
\texttt{state1} &= STUCK \\
\texttt{state2} &= TAUGHT \\
N &= N(V_{housing} - V_{contents})/RT \\
L &= 0 \\
\dot{L} &= 0 \\
\ddot{L} &= 0
\end{aligned} \tag{24}$$

2.2 Parameters

The model parameters are classified in Table 1. The first 10 parameters are independently measured or otherwise known. For example, atmospheric pressure (Patmosphere) and room temperature (T) are known constants. Reel parameters rReel, J, Tau_coulomb, and Vhousing_m3 are known from CAD files and independently confirmed by measurements[?]. The maximum tubing length, Lmax, is obviously known (about 0.6m), but was modified by matching the end length between experiment and simulations.

The modified parameter set is given in Section 4 below. Asterisks (*) denote iteratively changed parameters.

Simulation with the modified parameters (Figure 4) improve the model fit to data by 1) Shrinking loops on the pressure flow plot (upper left) are similar to shrinking loops in experimental pressure-velocity plot' 2) Convergence of the eversion thresholds (center left) causes the shrinking loops above, 3) Peak velocities better match peaks of fig. 9, and match the pattern of the highest velocity appearing near the center of travel. 4) With the modified parmeters there are about 2 bursts per second, similar to the experimental rate.

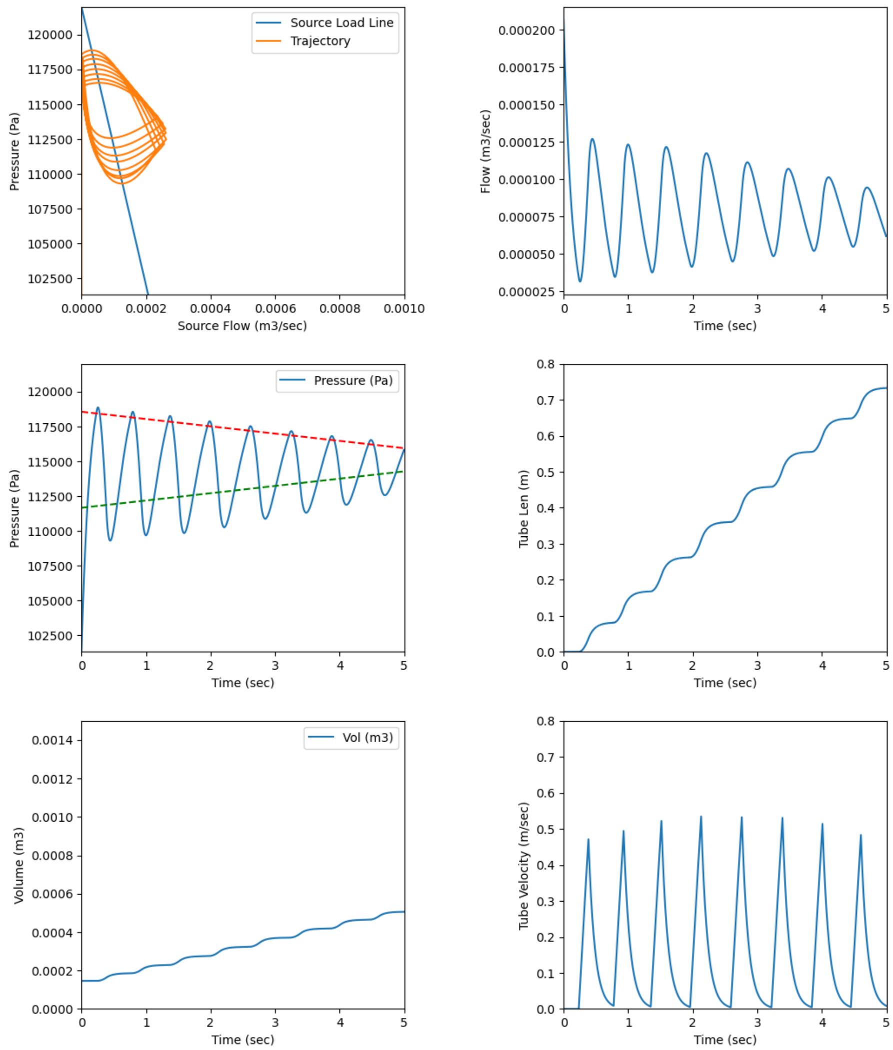

Figure 4: Improved qualitative match with modified parameters. (See Lewis, Fig 9, hiI, hiTf)

3 Baseline Parameter Values

```
moles_per_m3   4.4623E+01  moles / m3
Psource\_SIu    1.2066E+05  Pascals
LLine\_SIu      1.0000E-08  m3/sec / Pascal
Rsource\_SIu    1.0000E+08  Pa/m3/sec
Vhousing_m3    1.4616E-03  m3
Kdrag          3.0000E-01  N / m2 / sec
area_m2        4.9087E-04  m2
Pintercept     1.2066E+05  Pascals
Fintercept     2.0684E-04  m3/sec
Vintercept     4.2137E-01  m/sec
PBA_static     1.1721E+05  Pascals
PHalt_dyn      1.1032E+05  Pascals
Patmosphere    1.0132E+05  Pascals
```

4 Modified Parameter Values

```
moles_per_m3   4.4623E+01  moles / m3 of Air
Patmosphere    1.0132E+05  Pascals
Psource\_SIu    1.2201E+05  Pascals
LLine\_SIu      1.0000E-08  m3/sec / Pascal
Rsource\_SIu    1.0000E+08  Pa/m3/sec
J            * 1.0200E-03  kg/m2
Vhousing_m3  * 1.4616E-04  m3
Kdrag        * 1.2000E+00  N / m2 / sec
area_m2        4.9087E-04  m2
Pintercept     1.2201E+05  Pascals
Fintercept     2.0684E-04  m3/sec
Vintercept     4.2138E-01  m/sec
Threshold Taper *   3.5714E+03  Pa /m
PBA_static     1.1856E+05  Pascals
PHalt_dyn      1.1167E+05  Pascals
```

5 Parameter Estimation from Data

The following process can be used to fit parameters to an eversion data record where the record consists of the following data as a function of time:

- Pressure in the tube storage chamber (Pa)
- Air flow from source into tube storage chamber (m^3/sec)
- Length of everting tube relative to chamber opening (m)
- Velocity of eversion (derived) (m/sec)

These can be plotted multiple ways such as in Fig 3 for example.

We can then use the following procedure to iteratively tune some of the model parameters to match a particular experimental run. Other parameters such as the tube reel radius, inertia, and braking friction are readily measured independently [?] and not fit to the eversion experiments.

1. Adjust load line pressure intercept.

   Data Focus: Pressure/Flow curves (upper left):
   Procedure: Adjust Psource_SIu to move the load line and sim trajectory (they should overlap) up and down. Adjust Rsource_SIu to adjust its slope (higher values slope down more).

2. Adjust stop-start thresholds.

   Data Focus: Pressure-Time curves (middle left):
   Procedure: Adjust PBA_static up or down to match the pressure peaks in experiment (green dashed). Adjust PHalt_dyn up or down to match the pressure valleys.

3. Adjust threshold taper.

   Data Focus: Pressure-Time curves (middle left):
   Procedure: Adjust Threshold Taper up or down to speed or slow down convergence of the thresholds.

4. Adjust Friction or drag

   Data Focus: Length-Time curves (middle right):
   Procedure: Adjust the viscus drag constant of tubing (K_drag) up or down to match the overall slope of the data trace. Adjust max tubing length (Lmax) to match stopping point of data.

5. Experiment with other parameters or go back and repeat the procedure.

6 Model Refinements

6.1 Varying tube profile

While most ET research uses tubing of constant diameter, the effective diameter of an ET can vary with length in two main ways. First, the tube can be fabricated with variable diameter by thermal welding of two sheets. Second, tubing of constant diameter may be everted into a tubular space of changing diameters. While the mechanics of eversion will in general be different in these two cases, we will initially ignore that difference and make tube diameter a function of eversion distance L, $V(L)$.

Tube profiles

Simulations

6.2 Multiple compartment model

So far, pressure and flow dynamics have considered the tubing supply chamber and the everted tubing to be a single compartment for computation of volume (Eqn 1) and pressure (Eqn 5) yielding a single flow (Eqn 14).

In some experimental data [...details...] we noted loops in the pressure-flow plane in which a growing ET deviated significantly from the source load line. This indicates that air pressure does not instantaneously equilibrate along the ET, but instead takes time to flow from the tubing compartment down to new volume at the growing tip. A lumped parameter model of this flow can be formed of two compartments, one is the reel housing from which the tube is everted, and the second is the tubing. A flow resistance, $R_T(L)$, connects the two chambers and this resistance increases with length. The tubing volume increases with length (as it did in Eqn 1).

Such a model is shown in Figure 5.

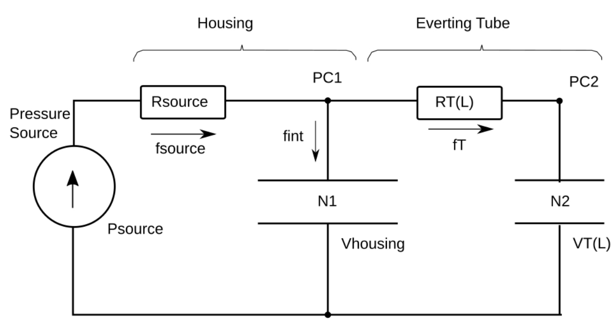

Figure 5: Two compartment, lumped parameter model.

We need to expand our model state variables as follows:

$$
\begin{aligned}
N, \dot{N} &\rightarrow \{N_1, N_2, \dot{N}_1, \dot{N}_2\} \\
V_t &\rightarrow \{V_{housing}, V_T(L)\} \\
f_{source} &\rightarrow \{f_{source}, f_{int}, f_T\} \\
P &\rightarrow \{P_{C1}, P_{C2}\}
\end{aligned} \tag{25}
$$

where N_1 and N_2 are the molar quantity of air in the housing and tube respectively, f_i are the respective flows indicated in Figure 5, and P_{C1} and P_{C2} are the pressures in the housing and tube respectivly.

We then have new equations to replace or expand Equations 5, 14, and 15 as follows:

$$\dot{N}_1 = f_{source} - f_{int} - f_T \tag{26}$$

$$\dot{N}_2 = f_T \tag{27}$$

$$P_{C1} = \frac{N_1 RT}{V_{housing}} \tag{28}$$

$$P_{C2} = \frac{N_2 RT}{V_T(L)} \tag{29}$$

$$f_{source} = (P_{source} - P_{C1})/R_{source} \tag{30}$$

$$f_T = (P_{C1} - P_{C2})/R_T(L) \tag{31}$$

$$f_{int} = f_{source} - f_T \tag{32}$$

7 Results of Two Compartment Model

The data sets were simulated with the two-compartment version of the model. Volume of the ET was a function of length, L. Flow resistance between reel housing (compartment 1) and the ET was approximated by a ratio of Rsource_SIu (typically 0.1).

Results from Tube 1, Trial 2 (hi inertia, hi friction) are given in Figure 6. Notably, the simulated tube pressure deviates below the source loadline during the start-stop oscillations similar to the experimental data.

References

[1] Laura H Blumenschein, Allison M Okamura, and Elliot W Hawkes. Modeling of bioinspired apical extension in a soft robot. In *Conference on Biomimetic and Biohybrid Systems*, pages 522–531. Springer, 2017.

[2] Panagiotis Vartholomeos, Zicong Wu, Hadi Sadati, and Christos Bergeles. Lumped parameter dynamic model of an eversion growing robot: Analysis, simulation and experimental validation. In *IEEE International Conference on Robotics and Automation (ICRA)*, 2024.

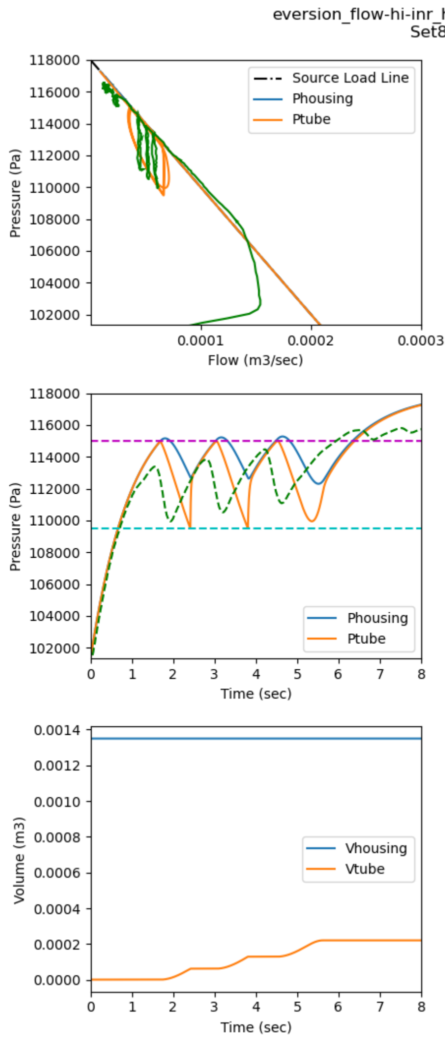

Figure 6: Two compartment model compared with simulation (See Lewis, Fig 9, hiI, hiTf)