Simulation of Everting Tube Experiments

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1 Introduction

The dynamic model of an eversion drive system includes;

- An everting tube of length L and growth rate $\dot{L}$.
- A pressurized housing
- A reel with rotation $\theta, \dot{\theta}, \ddot{\theta}$, on which tubing is rolled having inertia (assume fixed) of J and radius r.
- A brake which applies a Coulomb friction torque to the reel

$$\tau_c = C\mathrm{sgn}(\dot{\theta})$$

- A "crumple zone" in which eversion material can accumulate between the reel and the everting tube. The length of material in the crumple zone is L_c.
- Eversion happens when the eversion force (pressure $\times$ face area of the tube) exceeds any retarding forces.
- Forces which can oppose eversion include,
  - drag forces inertial forces required to pull the tubing material inside the deployed tube,
  - reel inertia and reel friction resulting from unspooling material,

The everting tube can be in one of two states:

- GROWING (the tube is actively everting, $\dot{L} > 0$)
- STUCK (the tube is not growing due to insufficient everting force, $\dot{L} = 0$)

and the reel/crumple zone can be in one of two additional states:

- TAUGHT (the crumple zone has zero length , $L_c = 0$)
- SLACK (there is material in the crumple zone, $L_c > 0$)

Together the system can be in four states comprising the permutations of these two state variables.

2 System Equations:

After setting initial conditions (see below), we model eversion dynamics by:

1. Computing pressure, volume, and flow from the source.
2. Computing forces applied to the eversion tip.
3. Accounting for the mechanical advantage (everting material speed is $2 \times \dot{L}$, Pressure applied to everting front develops 1/2 the everting force expected from $P \times A$.)
4. Selecting the dynamic mode from the four combined states above. GROWING and STUCK are selected by force thresholds applied to net eversion force. TAUGHT and SLACK are selected by checking length of the crumple zone material.
5. According to the dynamic mode, summing forces, equating to zero, solving for tube and reel accelerations.
6. Eversion does not come to an instant halt. We empirically model exponential decay of velocity as tube decelerates.

Specifically:

$$V_t = V_{housing} - V_{contents} + LA \tag{1}$$

V_t includes both the reel housing minus the volume of its contents, and the everted tube of length L with cross sectional area A.

From the ideal gas equation:

$$P = \frac{NRT}{V_t} \tag{2}$$

where N is the molar mass of gas in the system, R is the gas constant, and T is the temperature in °K, (which we assume is constant).

Computing Forces:

$$F_{ever} = \max(0, P * A/2) \tag{3}$$

$$F_c = \tau_{Coulomb}/r \tag{4}$$

Coulomb friction is independent of velocity so does not get scaled by the 2× mechanical advantage.

Computing acceleration according to the dynamic state:

GROWING and SLACK:

$$\ddot{L} = \frac{1}{M_T}(F_{ever} - F_D(L, \dot{L})) \tag{5}$$

$$\dot{\theta} = \tau_{Coulomb}/J \tag{6}$$

GROWING and TAUGHT:

$$\ddot{L} = (1/M_T + r^2/J) * (F_{ever} - F_D - F_C) \tag{7}$$

$$\ddot{\theta} = \ddot{L}/r \tag{8}$$

STUCK and (SLACK or TAUGHT):

$$\ddot{L} = -1 * (max)(0, \alpha * \dot{L}) \tag{9}$$

$$\ddot{\theta} = \tau_{Coulomb}/J \tag{10}$$

where α is an empiical time constant modeling the dynamics of eversion stopping.

Modeling flow from the pressure source (Thevenin equivalent):

$$Fl_{source} = \frac{P_{source} - P}{R_{source}} \tag{11}$$

Converting airflow (m^3/sec) to rate of molar mass flow:

$$\dot{N} = Fl_{source} \cdot \text{moles\_per\_m3} \tag{12}$$

Model velocity and length dependent eversion force which is resistance to pulling out eversion material:

$$F_D(L, \dot{L} = 2LK_D\dot{L} \tag{13}$$

Where the factor of two accounts for everting material going at twice tube growth rate, $\dot{L}$.

Update length of crumpled material (if any)

$$L_C = \max(0, r\theta - L) \tag{14}$$

We have a switching model to replicate observed intermittent starting and stopping of eversion which updates the state based on current pressure:

$$\texttt{state} = \begin{cases} \text{GROWING}, & P > P2 \\ \text{unchanged}, & P1 <= P <= P2 \\ \text{STUCK}, & P < P1 \end{cases} \tag{15}$$

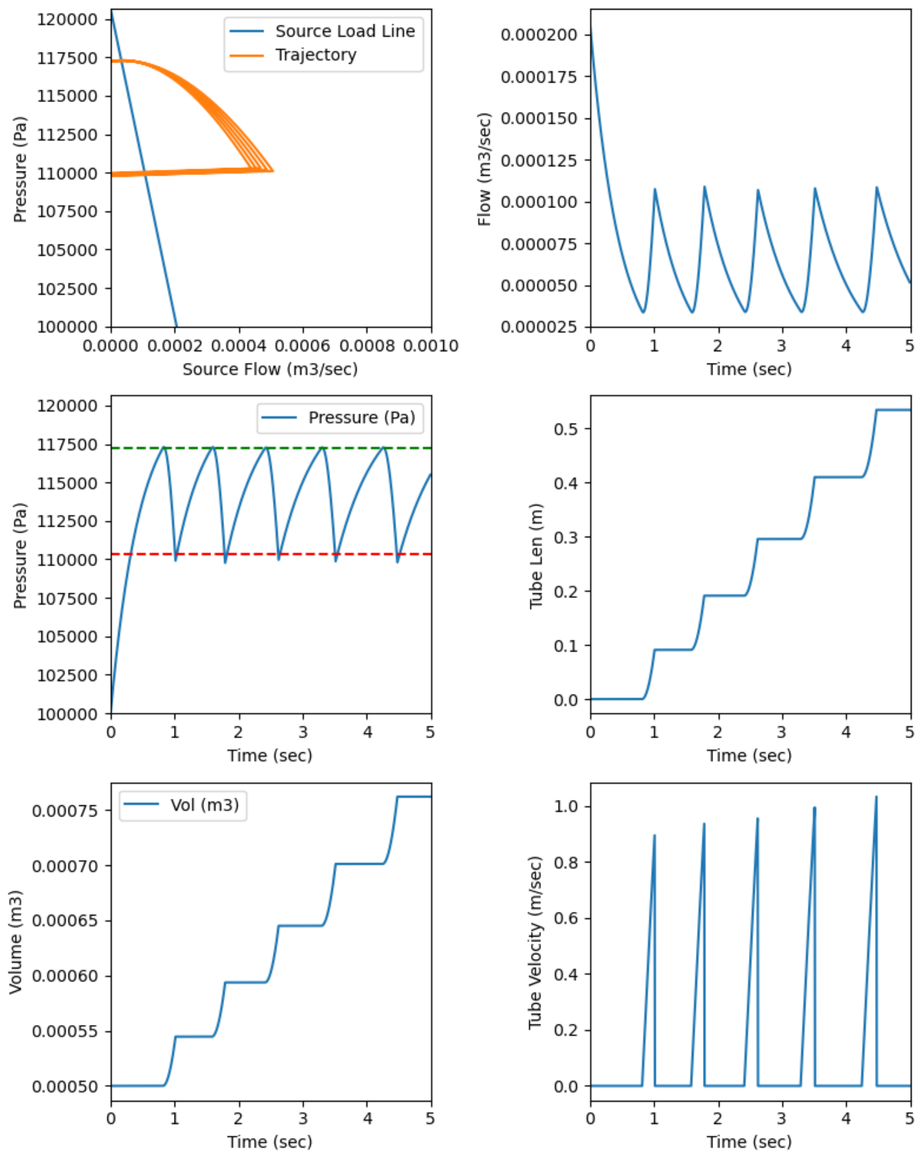

Figure 1: Simulation Run with approximate qualitative match. (See Lewis, Fig 9, hiI, hiTf)

We are currently investigating changing this switching model to one based on thresholding net eversion force rather than tip surface pressure:

$$\texttt{state1} = \begin{cases} \text{GROWING}, & F_{ever} > F2 \\ \text{unchanged}, & F1 <= F_{ever} <= F2 \\ \text{STUCK}, & F_{ever} < F1 \end{cases} \tag{16}$$

In either case above, we update the crumple state as

$$\texttt{state2} = \begin{cases} \text{TAUGHT}, & L_C <= 0 \\ \text{SLACK}, & L_C > 0 \end{cases} \tag{17}$$

Finally, we integrate the state variables:

$$\begin{aligned} \dot{L} &= \dot{L} + \ddot{L}dt \\ L &= L + \dot{L}dt \\ \dot{\theta} &= \dot{\theta} + \ddot{\theta}dt \\ \theta &= \theta + \dot{\theta}dt \\ N &= N + \dot{N}dt \end{aligned} \tag{18}$$

2.1 Initial Conditions

$$\begin{aligned} P &= 1 \text{ atmosphere} \\ \texttt{state1} &= STUCK \\ \texttt{state2} &= TAUGHT \\ N &= N(V_{housing} - V_{contents})/RT \\ L &= 0 \\ \dot{L} &= 0 \\ \ddot{L} &= 0 \end{aligned} \tag{19}$$

3 Simulation Results

4 Simplified Simulation Results

These initial results are from a simplified model assuming the TAUGHT state at all times. All parameters for the above model were computed from measurements of the experimental system with the exception of `PBA_static, PHalt_dyn, Kdrag` which were estimated by a visual match to experimental data. Initial simulation results using the baseline parameters (Sec. 5, below) are given in Figure 1.

Some parameters were iteratively changed based on remaining observed differences between Figure 1 and experiment. Specifically:

Observation	Param	Original Value	Modified Value
Velocities too high	`Kdrag`	0.3	0.6
Pressure thresholds converge with time	`PBA_static`	1.17×10^5Pa	$1.17 \times 10^5 - 500L$ Pa
Pressure thresholds converge with time	`PHalt_dyn`	1.17×10^5Pa	$1.013 \times 10^5 + 500L$ Pa
Pressure rise too slow	$V_{housing}$	0.5×10^{-3} m^3	0.05×10^{-3} m^3
Peak Velocity too high	J	5.10×10^4 kg/m^2	$10.2{\times}10^4$ kg/m^2

The modified parameter set is given in Section 6 below. Asterisks (*) denote iteratively changed parameters.

Simulation with the modified parameters (Figure 2) improve the model fit to data by 1) Shrinking loops on the pressure flow plot (upper left) are similar to shrinking loops in experimental pressure-velocity plot'

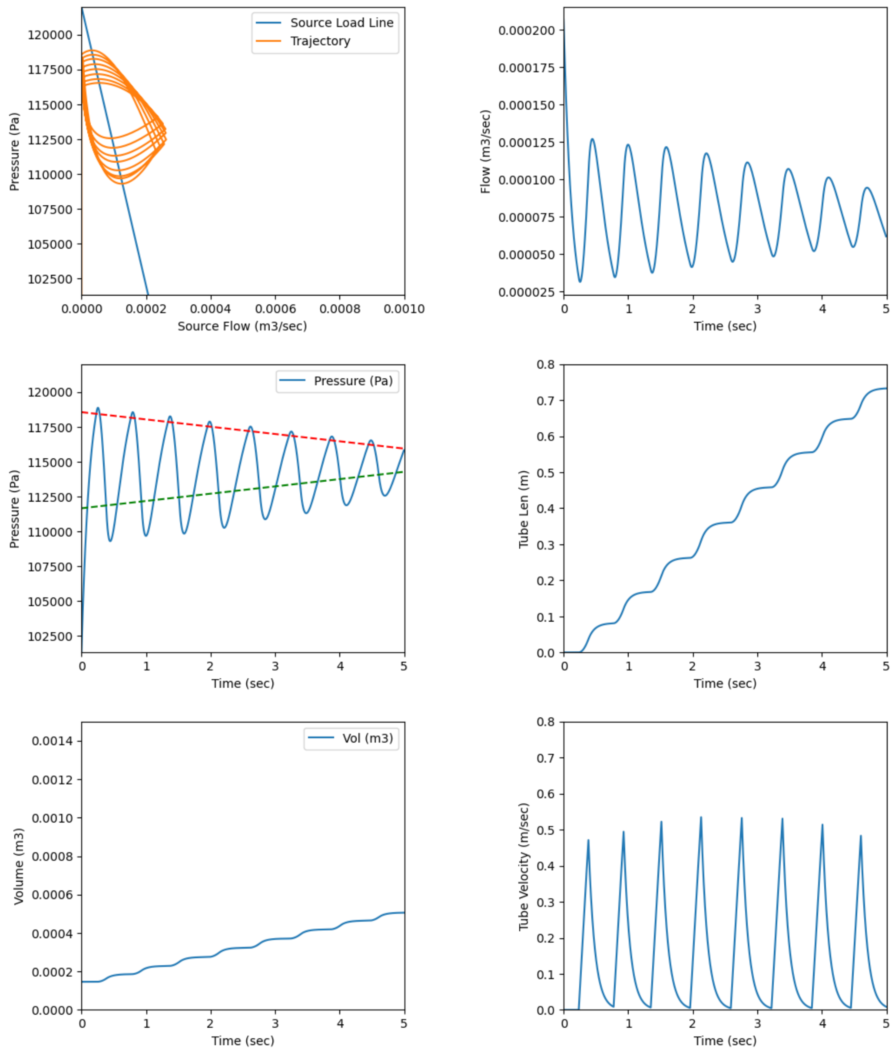

Figure 2: Improved qualitative match with modified parameters. (See Lewis, Fig 9, hiI, hiTf)

2) Convergence of the eversion thresholds (center left) causes the shrinking loops above, 3) Peak velocities better match peaks of fig. 9, and match the pattern of the highest velocity appearing near the center of travel. 4) With the modified parmeters there are about 2 bursts per second, similar to the experimental rate.

5 Baseline Parameter Values

```
moles_per_m3   4.4623E+01  moles / m3
Psource_SIu    1.2066E+05  Pascals
LLine_SIu      1.0000E-08  m3/sec / Pascal
Rsource_SIu    1.0000E+08  Pa/m3/sec
Vhousing_m3    1.4616E-03  m3
Kdrag          3.0000E-01  N / m2 / sec
area_m2        4.9087E-04  m2
Pintercept     1.2066E+05  Pascals
Fintercept     2.0684E-04  m3/sec
Vintercept     4.2137E-01  m/sec
PBA_static     1.1721E+05  Pascals
PHalt_dyn      1.1032E+05  Pascals
Patmosphere    1.0132E+05  Pascals
```

6 Modified Parameter Values

```
moles_per_m3   4.4623E+01  moles / m3 of Air
Patmosphere    1.0132E+05  Pascals
Psource_SIu    1.2201E+05  Pascals
LLine_SIu      1.0000E-08  m3/sec / Pascal
Rsource_SIu    1.0000E+08  Pa/m3/sec
J            * 1.0200E-03  kg/m2
Vhousing_m3  * 1.4616E-04  m3
Kdrag        * 1.2000E+00  N / m2 / sec
area_m2        4.9087E-04  m2
Pintercept     1.2201E+05  Pascals
Fintercept     2.0684E-04  m3/sec
Vintercept     4.2138E-01  m/sec
Threshold Taper *   3.5714E+03  Pa /m
PBA_static     1.1856E+05  Pascals
PHalt_dyn      1.1167E+05  Pascals
```