CS341

Lecture 5

Yaro Gorban

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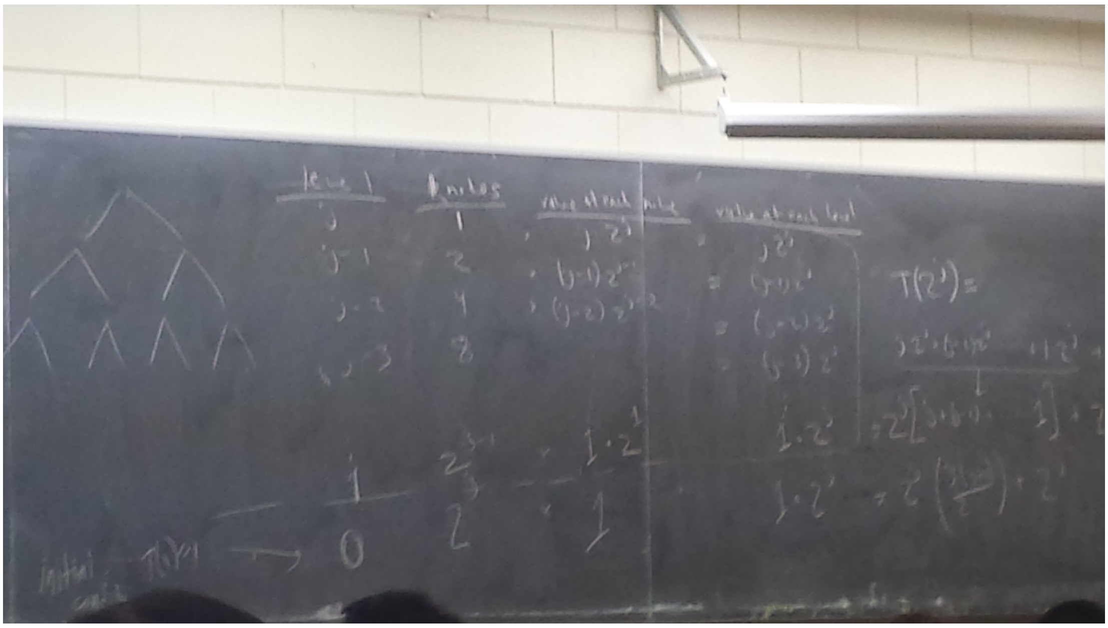
What if we can't solve a recurrence via the master theorem

- We can solve the recurrence directly by using the recursion tree method.
- Start with Root node T(n) and expand
- $T(2^j) = 2^j[\frac{j(j+1)}{2} + 1]$



- $T(n) \in \Theta(n(\log n)^2$

Example:

- $T(n) = T(\lfloor \frac{n}{2} \rfloor) + T(\lfloor \frac{n}{3} \rfloor) + n$
- Base cases: T(1) = 1, T(2) = 2
- Guess - and - Check

- lets guess that $T(n) \in O(n)$
- prove by induction that T(n) $\leq$ cn for all n $\geq$ 1
- $T(1) = 1 \leq c * 1 \Rightarrow c \geq 1$
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- Induction Assumption: $T(n) \leq cn$ for $n<m$
- $T(m) = T(\lfloor \frac{m}{2} \rfloor) + T(\lfloor \frac{m}{3} \rfloor) + m$
- $\leq c\lfloor \frac{m}{2} \rfloor + c\lfloor \frac{m}{3} \rfloor + m$
- $\leq c\frac{m}{2} + c\frac{m}{3} + m$
- $= c(\frac{5m}{6} + m \leq cm$
- So the inequality is true if $\frac{5m}{6} + 1 \leq c$
- $1 \leq c/6$
- $c \geq 6$
- So we can take c = 6 and then:
- $T(n) \leq 6n$ for all $n \geq 1$ by induction