CS341
Lecture 3
Yaro Gorban

September 21, 2015

Arithmetic Series: a, a+d, a+2d, ... , a + (n-1)d

Geometric Series:

- a, ar, ar^2, ... ,ar^{n-1}
- if r >1 Sum is: c(r^n - 1)
- if 0 <r <1 Sum is: c(1 - r^n)

Harmonic Series: (reciprocal of the integers)

- $1 + \frac{1}{2} + ... + \frac{1}{n} = H_n(\in \Theta(\log n)$
- more precisely $\lim_{n \to \inf}(H_n - \ln n) =$ Euler's constant

Growth Rate of a Factorial:

- $n! \in \Theta(n^{n+\frac{1}{2}}) = \Theta((\frac{n}{e})^n \sqrt{n})$
- derived from Stirling's formula
- $\ln n! \approx \ln(n^{n+0.5}e^{-n} = (n + 0.5)\ln n - n \in \Theta(n \log n)$

Growth Rates:

- **Polynomial:**
  - $\Theta(1)$
  - $\Theta(\log n)$
  - $\Theta(\sqrt{n})$
  - $\Theta(n)$
- **Exponential:**
  - $\Theta(1.1^n)$
  - $\Theta(2^n)$

- $\Theta(e^n)$
- $\Theta(n!)$

Algorithm Analysis

- We usually strive for a Θ Bound
- Loops have a run time of the sum of each iteration
- separate loops analyse separately and then add them together
- Nested loops start with inner most and work your way out

Analysis of Slide 49:

- everything but the 3 loops is constant time
- The three loops are:
  - i = 1, ... , n
  - j = i, ... , n
  - k = i, ... , j
- Upper Bound:
  - i = 1, ... , n
  - j = 1, ... , n
  - k = 1, ... , n
  - which is $\in O(n^3)$
- Lower Bound:
  - i = 1, ... , $\frac{n}{3}$
  - j = $\frac{2n}{3} + 1, ..., n$
  - k = $\frac{n}{3} + 1, ..., \frac{2n}{3}$
  - $\in \Omega(\frac{n}{3} * \frac{n}{3} * \frac{n}{3})$
  - $\in \Omega(\frac{n^3}{3}$
  - $\in \Omega(n^3)$