

CS341  
LECTURE 6  
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September 30, 2015

**Non dominant points problem:**

- Has a worst case of  $O(n^2)$  by comparing each set of points
- not-dominated points for a "staircase"
- all other points are under the stair case
- split into  $S_1$  and  $S_2$
- solve both halves
- to combine we may have to eliminate some of the non-dominated points in  $S_1$
- this runs in  $O(n)$  time

**Complexity**

1. pre-sort  $\Theta(n \log n)$
2.  $T(n) = T(\lfloor \frac{n}{2} \rfloor) + T(\lceil \frac{n}{2} \rceil) + O(n)$
3. sloppy form  $T(n) = 2T(\frac{n}{2}) + O(n)$   $T(n) \in O(n \log n)$
4. since we have 2 halves of  $n \log n$  time we have a time of  $O(n \log n)$

**Closest Pair:**

- Find a Euclidean distance that is minimized
- Trivial Algorithm is comparing each point which is  $O(n^2)$
- Presort with respect to x-coordinates  $\Theta(n \log n)$
- Split the points down the middle
- Find the  $\delta_L$  and the  $\delta_R$

- Combine: determine the smallest distance between a point on the left and the the right. is this distance  $< \min \delta_L, \delta_R$
- Checking every point is not what we want to do
- We create something to create the critical strip
- But what if all the points fall in the critical strip? We didn't cut the run time at all.
- So we sort the points in  $R$ (list of candidates) with respect to the y-coordinates
- **LEMMA:** Suppose the points in the critical strip  $R$  are sorted WRT y-coordinates. Suppose that  $R[j]$  and  $R[k]$  have a distance  $< \delta$  where  $j < k$ . Then  $k \leq j+7$

### Complexity

1. presort  $O(n \log n)$
2.  $T(n) = 2T(\frac{n}{2}) + O(n)(selectcandidates) + O(n \log n)(sort) + O(n)(checkstrip)$
3.  $T(n)$  is  $O(n(\log n)^2)$