Instrumental Variables using a Neural Network

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```r
library(AER)
data("CigarettesSW")
rprice  <- with(CigarettesSW, price/cpi)
tdiff   <- with(CigarettesSW, (taxs - tax)/cpi)
packs   <- CigarettesSW$packs
```

A manual instrumental variables would look like this (using the built in lm() function.

```r
# first stage
s1 <- lm(rprice ~ tdiff)

# estimate second stage using fitted values (Xhat)
lm(packs ~ s1$fitted.values)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	219.576384	20.8630993	10.524629	0e+00
s1$fitted.values	-1.019485	0.1914682	-5.324565	7e-07

We can verity these results using 'AER''s built in function ivreg().

```r
ivreg(packs ~ rprice | tdiff)

##
## Call:
## ivreg(formula = packs ~ rprice | tdiff)
##
## Coefficients:
## (Intercept)       rprice
##     219.576       -1.019
```

A good way to prove this theoretically is using simulated data.

```r
# library for generation multivariate distributions
library(MASS)
```

```r
# always use the same random numbers
set.seed(123)

# the means and errors for the multivariate distribution
MUs    <- c(10,15)
SIGMAs <- matrix(c(1,   0.5,
                   0.5, 2   ),
                 nrow=2,
                 ncol=2       )

# the multivariate distribution
mdist <- mvrnorm(n     = 1000,
                 mu    = MUs,
                 Sigma = SIGMAs)

# create unobserved covariate
c <- mdist[ , 2]

# create the instrumental variable
z <- rnorm(1000)

# create observed variable
x <- mdist[ , 1] + z

# constuct the dependent variable
y <- 1 + x + c + rnorm(1000, 0, 0.5)
```

Check if the variables behave as expected

```r
cor(x, c)

## [1] 0.1986307

cor(z, c)

## [1] -0.0120011
```

Let's look at the true model.

```r
lm(y ~ x + c)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.9078664	0.1860081	4.880789	1.2e-06
x	1.0155658	0.0118758	85.515517	0.0e+00
c	0.9955182	0.0111828	89.021974	0.0e+00

Now, we assume that we do not have access to c.

```r
lm(y ~ x)
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	13.78678	0.3495719	39.43904	0
x	1.22556	0.0348005	35.21669	0

We now assume to have access to the variable z, and estimate it using two-stage least squares.

```r
# first stage
lms1 <- lm(x ~ z)

# manually obtain fitted values
lmXhat <- lms1$coefficients[2]*z + lms1$coefficients[1]

# estimate second stage using Xhat
(lms2 <- lm(y ~ lmXhat) )
```

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	15.948900	0.6715031	23.75105	0
lmXhat	1.008351	0.0671579	15.01464	0

Or equivalently, using ivreg().

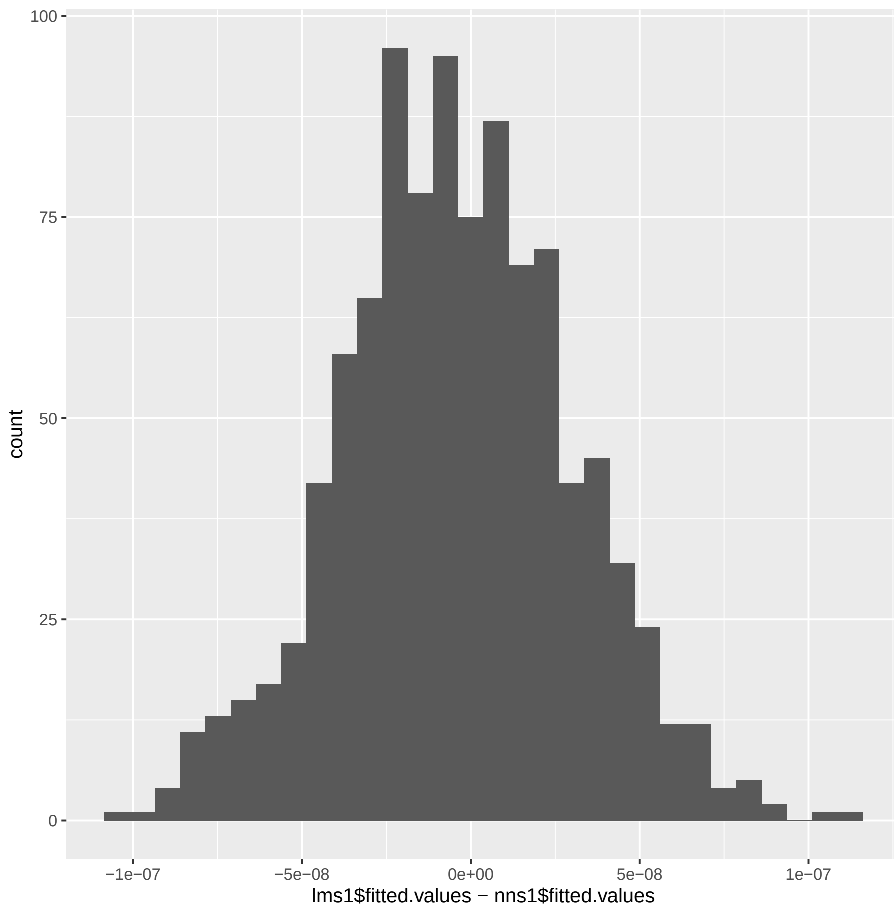
```r
ivreg(y ~ x | z)

## Error in eval(expr, envir, enclos):  could not find function "ivreg"
```

This can also be done using a neural network. Let's start with looking at how a neural network can generally be equivalent to OLS.

```r
library(nnet)
# first stage with neural network
nns1 <- nnet(x ~ z, size=0, skip=TRUE, linout=TRUE)

## # weights:  2
## initial  value 98765.653982
## final  value 924.804075
## converged
```

The results obtained by nns1 are virtually identical to those in lms1.

```r
lms1$coefficients - nns1$wts

##   (Intercept)             z
## -3.365223e-09  3.461435e-08

# graphically
library(ggplot2)
qplot(lms1$fitted.values - nns1$fitted.values)
```



We can now also perform the second stage using neural networks.

```
# manually obtain fitted values
nnXhat <- nns1$fitted.values

# estimate second stage using Xhat
nns2 <- nnet(y ~ nnXhat, size=0, skip=TRUE, linout=TRUE)

## # weights:  2
## initial  value 874286.766246
## final  value 4019.409973
## converged

# evaluate outcome
summary(nns2)

## a 1-0-1 network with 2 weights
```

```
## options were - skip-layer connections  linear output units
##  b->o i1->o
## 15.95  1.01
```

Now compare the final estimates.

```
lms2$coefficients - nns2$wts
```

```
## (Intercept)      lmXhat
##  1.0366e-06 -1.1273e-07
```

```
# graphically
library(ggplot2)
qplot(lms2$fitted.values - nns2$fitted.values)
```