## rnn: a Recurrent Neural Network in R\*

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#### Abstract

The rnn package implements the first Recurrent Neural Network (RNN) in the R language. RNN algorithms expand on traditional feed-forward neural networks, allowing for greater complexity and dynamics, by implementing a memory state. This temporal nature of the algorithm makes it explicitly well suited for dynamic problems such as time series prediction. Additionally, this also allows for inputs of undefined or changing length, allowing models to be updated as new data comes in. The rnn package is the first implementation of a Recurrent Neural Network in the R language, making it both operable and understandable to R users. Here I apply the package to two problems, the classic complex problem of carrying a 1, in bit by bit (column by column) binary addition, as well as foreign exchange rate prediction. Interactive live versions of these examples are available on my website http://qua.st/rnn.

#### 1 Introduction

This package implements the first Recurrent Neural Network in the R language. At this point it is useful to clarify the nomenclature in this article. I use the term RNN (capitalised) for the general concept of a Recurrent Neural Network and I use rnn (in minuscules and using a monospace font) to refer to the R package.

 $<sup>*</sup> https://cran.r-project.org/package=rnn \ | \ https://github.com/bquast/rnn$ 

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Table 1: rnn Package

```
# load library
library(rnn)

# list included functions
ls('package:rnn')

## [1] "bin2int" "int2bin" "predictr"
## [4] "run.finance_demo" "trainr"
```

As is listed above, the package contains the following functions:

- bin2int(): conversion of a matrix of numbers in binary representation to decimal representation;
- int2bin(): conversion of a vector numbers in decimal representation to binary representation;
- predictr(): predicts response variable based on a trainr() model and input data;
- trainr(): primary function, trains a model based on training data and hyperparameters.

In addition to these functions there are also two internal functions i2b() and b2i(), these functions are used by int2bin() and bin2int() internally to change a single number from decimal to binary or visa versa.

The main trainr() function takes two arrays as inputs, the response variable Y and the input variable X, it returns a model that can be used with by the predictr() function together with a testing data input array X.

Internally, the functions make use of one or more sigmoid functions. In order to make the Sigmoid functions more generally available, these were moved to a separate package sigmoid.

The sigmoid() function is a wrapper, that defaults to the logistic() function, which maps the inputs to (0,1) using the logistic function, when using the default parameters, this is the standard logistic function.

### 2 Data

As mentioned above, an explicit element of Recurrent Neural Networks in the temporal aspect. As a result, both the input and the output can have up to three dimensions:

- 1. variables
- 2. observations

Table 2: sigmoid

#### 3. time periods

As a result of this, the functions take inputs of the type array, is a matrix is used as an input, the matrix is converted to an array. Conversely, if the output is a 2 dimensional, it will be simplified to a matrix.

### 2.1 Binary Addition

The vectors X1 and X2 are independent variables, the Y vector is the sum of X1 and X2 and acts as the response variable (for more info see help('trainr')).

Training data can be generated using base package's sample() function. For reproducibility, we also set the seed value of the pseudo-random number generator that R uses internally to 1. After generating X1 and X2, I add the two pairwise and store the result in Y. Finally, I convert both the input variables and the response variable to binary representation using the int2bin() included with the package.

Table 3: Binary Numbers

```
# use the same random numbers
set.seed(1)

# create training inputs
X1 = sample(0:127, 7000, replace=TRUE)
X2 = sample(0:127, 7000, replace=TRUE)

# create training output
Y <- X1 + X2

# create 3d array: dim 1: samples; dim 2: time; dim 3: variables
X <- array( c(X1,X2), dim=c(dim(X1),2) )</pre>
```

Internally the int2bin() function converts these characters into binary format using the intToBits() function, the bin2int() function converts it back into decimal format for printing using the packBits() function, both functions are included in the base package.

We can for instance take the first value of X1 and convert it to a binary representation, whereby the binary\_dim argument to the trainr() function determines the length of the binary representation, throughout this paper we will use 8 bit representations (which limits numbers to the range 0-255), but the theoretical limit is 32 bits.

Table 4: Binary Representation

```
int2bin( X1[1] )
## [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,] 0 0 1 0 0 0 1
```

Lets check look at the first sum in decimal representation.

Table 5: Decimal Summation

```
X1[1]

## [1] 33

X2[1]

## [1] 89

X1[1] + X2[1]

## [1] 122

Y[1]

## [1] 122
```

and now in binary representation.

Table 6: Binary Summation

```
int2bin( X1[1] )
int2bin( X2[1] )
print('-----
int2bin( Y[1] )
##
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
##
  [1,]
      0 0 1 0 0
                              0
##
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,]
         0
           1
               0 1
                                  0 1
                        1
                              0
  [1] "-----
##
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
         0 1 1 1 1
```

As can be seen from the above output, the first values of X1 and X2, 33 and 89 respectively, are both in the range 0-127, which can be represented with only 7 bits. Yet the sum of the two - 122 - is almost outside of the range 0-127, . If we sampled numbers great than 127 for X1 and X2 then the sum of the two could be greater than 255, which requires a ninth bit (or length=9).

We can now convert the entire vectors to binary matrices.

```
# convert to binaries of 8 bit (default)
X1 <- int2bin(X1)
X2 <- int2bin(X2)
Y <- int2bin(Y)</pre>
```

#### 2.2 Foreign Exchange Prediction

In this second example I train a RNN on more real-life data. I use the exchange rates of four major international currencies against the US Dollar.

Table 7: Foreign Exchange Data

```
library(quantmod) # for downloading FX data
start = '1998-12-14'
end = '2001-09-01'

# download values
# output is automatically returned to
# the global environment (.GlobalEnv)
getFX('CHF/USD', from = start, to = end)
getFX('GBP/USD', from = start, to = end)
getFX('JPY/USD', from = start, to = end)
getFX('EUR/USD', from = start, to = end)
```

Input data should be on the domain [0,1]. Exchange rates are well suited for this since either A/B or B/A has to be in this domain. However, it is of course possible that within the time period studied, currency A, initially being worth less than currency B, becomes worth more. This is exactly what happened with the EUR/USD exchange rate. This means that neither EUR/USD nor USD/EUR is within the [0,1] domain for the entire period.

It is for this reason that sigmoid functions are used to map any real number to the domain [0,1]. The most function for this is the logistic function. At the end of the process, the outputs are mapped again to the original domain, using the inverse of the sigmoid function, in the case of the logistic, this is the logit function.

By specifying the x0.

# 3 Methodology

The workhorse of the rnn package is the trainr() function.

For example, if we add the binary numbers 0 0 1 (decimal system: 1) and 1 0 1 (decimal system: 5), we start by adding the right column, 1 and 1 make

1 0 (similar to when 5 and 5 make 1 0 in the decimal system), the 0 is stored in the right column, the 1 is carried over to the middle column and added with the two existing bits 0 and 0, to form 1, which is stored in the middle column. This time nothing is carried over and the left column sums 0 and 1 to make 1, which gives the outcome 1 1 0 (decimal system: 6).

If we go back to the output of the int2bin() function for X1, X2, and Y, we see that in the 4th column (from right to left), a 0 and a 0 are added, resulting in an output of 1. This is because in the previous iteration 3rd column (from right to left) a 1 and a 1 are added, which becomes 1 0, so the 0 goes into column 3 and the 1 is carried over to column 4. Since the summation is done bit by bit (or column by column), the neural network need to remember from the 3rd iteration until the 4th iteration that it is carrying a 1 over. It is this remembering that a feed-forward neural network cannot teach itself.

#### 3.1 Binary Addition

The trainr() function will run until it has evaluated all rows in the matrices that it is fed and repeat this according to the number of epochs specified in the numepochs argument. Since the training of the network, particularly the carrying part, takes many iterations to learn (the exact number of iterations varies but depends on the hyperparameters, more on this in the next section), it is therefore advisable to sample several thousand values (I use 7000).

After each iteration the difference between the predicted value and the actual value is fed back into the neural network using a method called back-propagation (an application the chain rule of differential calculus).

Table 8: trainr() Binary Addition

```
# train the network
m1 <- trainr(Y,</pre>
            Χ,
            hidden_dim = 5,
            numepochs = 10,
            learningrate = 0.1,
            start_from_end = TRUE)
## Training epoch: 1 - Learning rate:
## Epoch error: 3.86763515257091
## Training epoch: 2 - Learning rate:
## Epoch error: 1.60548908339123
## Training epoch: 3 - Learning rate:
## Epoch error: 0.395638289496469
## Training epoch: 4 - Learning rate:
## Epoch error: 0.238016297019233
## Training epoch: 5 - Learning rate:
## Epoch error: 0.185323281721302
## Training epoch: 6 - Learning rate:
## Epoch error: 0.157134942538396
## Training epoch: 7 - Learning rate:
## Epoch error: 0.138868121568199
## Training epoch: 8 - Learning rate:
## Epoch error: 0.125779706036699
## Training epoch: 9 - Learning rate: 0.1
## Epoch error: 0.115801813496506
## Training epoch: 10 - Learning rate: 0.1
## Epoch error: 0.107867174922182
```

#### 3.2 Foreign Exchange Prediction

The trainr() and predictr() functions internally make use of the logistic() function, specifically the standard logistic function, which takes the range (Infinity, Infinity) and maps it to the range (0, 1).

Table 9: Logistic Source Code

```
# print arguments to the function
args(logistic)

## function (x, k = 1, x0 = 0)

## NULL

# print source code of the logistic function
logistic

## function (x, k = 1, x0 = 0)

## 1/(1 + exp(-k * (x - x0)))

## <environment: namespace:sigmoid>
```

For instance:

Table 10: Logistic Examples

```
logistic(-137)
logistic(5.3)

## [1] 3.174359e-60
## [1] 0.9950332
```

The rough shape of the sigmoid function is shown below.

Figure 1: Standard Logistic Shape

```
library(ggplot2) # load plotting package

# sequence of -10 through 10
x = seq(-10, 10)

# plot sigmoid shape
qplot(x = x, y = logistic(x), geom='line')
```

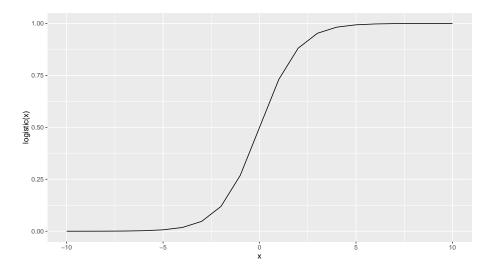


Figure 2: Logistic Mapping FX

```
# logistic map, write to new objects
chfusd <- logistic(CHFUSD, k=sd(CHFUSD)^-1, x0=mean(CHFUSD))
gbpusd <- logistic(GBPUSD, k=sd(GBPUSD)^-1, x0=mean(GBPUSD))
jpyusd <- logistic(JPYUSD, k=sd(JPYUSD)^-1, x0=mean(JPYUSD))
eurusd <- logistic(EURUSD, k=sd(EURUSD)^-1, x0=mean(EURUSD))</pre>
```

# 4 Results

The eventual purpose is to use the model generated by the trainr() function as an input to the predictr() function, in order to predict the values of new inputs.

### 4.1 Binary Addition Sums

Table 11: Binary Addition Test Data

```
# create test inputs
C1 <- int2bin( sample(0:127, 7000, replace=TRUE) )
C2 <- int2bin( sample(0:127, 7000, replace=TRUE) )

# stack matrices in array
C <- array( c(C1,C2), dim=c(dim(C1),2) )</pre>
```

Now predict using the predictr() function.

Table 12: predictr() Binary Addition

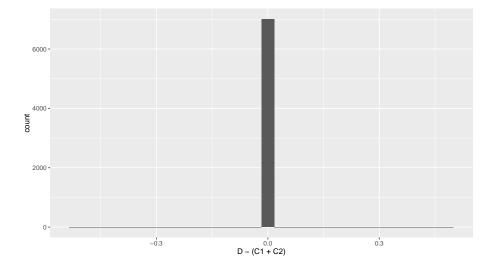
```
# predict
D <- predictr(model = m1, X = C)</pre>
```

We can now convert the predictions and the inputs back to decimals and plot them.

Figure 3: Binary Addition Sums

```
# convert back to decimal
C1 <- bin2int(C[,,1])
C2 <- bin2int(C[,,2])
D <- bin2int(D)

# inspect the differences
qplot( D - (C1+C2) )</pre>
```



As can be seen from the results, the difference is almost always 0.

### 4.2 Foreign Exchange Rate Predictions

Table 13: Foreign Exchange Rate Predictions

### 5 Conclusion

CRAN and the rest of the R ecosystem show that there is a strong interest in using the R language for neural network analysis. Existing package such as the built in nnet package and the caret package make available very powerful neural network tools to R users. The RSNNS package acts as an R wrapper for the Stuttgart Neural Network Simulator library, which is written in C, and thereby makes available to partial RNNs such as Elman and Jordan networks.

The enormous popularity of full Recurrent Neural Networks in other languages, primarily Python and C, show that there is a great amount of interest

for using this methodology, including interest from Economist, Data Scientists, and other non-professional programmers. Although Python is a relatively accessible programming language for laymen, it has a smaller user base in terms of data analysts. The rnn package attempts to address this need by showing that Recurrent Neural Networks can be made available and perhaps more importantly, made available in native R, which allows user to delve into the code and understand the method and developer a more thorough understanding of how to use it.

### A Source code of trainr() function

#### 0 Users quast rnn R trainr.R

```
@name \ trainr
 1
       @export
       @importFrom stats runif
   #' @importFrom sigmoid logistic sigmoid output to derivative
   #' @title Recurrent Neural Network
   #' @description Trains a Recurrent Neural Network.
   \#' @param Y array of output values, dim 1: samples (must be equal
        to \dim 1 of X), \dim 2: time (must be equal to \dim 2 of X), \dim
        3: variables (could be 1 or more, if a matrix, will be coerce
        to array)
   #' @param X array of input values, dim 1: samples, dim 2: time, dim
 8
         3: variables (could be 1 or more, if a matrix, will be coerce
        to array)
   #' @param learningrate learning rate to be applied for weight
        iteration
   \#' @param numepochs number of iteration, i.e. number of time the
10
        whole\ dataset\ is\ presented\ to\ the\ network
   #' @param hidden dim dimension of hidden layer
11
   #' @param start from end should the sequence start from the end
   #' @param learningrate decay coefficient to apply to the learning
13
        rate at each weight iteration
   \#' @param momentum coefficient of the last weight iteration to keep
14
         for faster learning
   \#' @return a model to be used by the predictr function
15
   #' @examples
16
   \#' \# create training numbers
   \#'X1 = sample(0.127, 7000, replace=TRUE) \ \#'X2 = sample(0.127, 7000, replace=TRUE)
18
19
   #'
20
   \#' \# create training response numbers
21
   #' Y <- X1 + X2
23
   #' # convert to binary
   \#'X1 \leftarrow int2bin(X1, length=8)
   \#' X2 < -int2bin(X2, length=8)
26
   \#'Y \leftarrow int2bin(Y, length=8)
27
28
   \#'\ \#\ create\ 3d\ array:\ dim\ 1:\ samples;\ dim\ 2:\ time;\ dim\ 3:\ variables
   \#' \ X < - \ array \left( \ c \left( X1, X2 \right), \ dim = c \left( dim \left( X1 \right), 2 \right) \ \right)
30
31
   \#' \# train the model
32
       model \leftarrow trainr(Y=Y)
33
   #'
34
                         X=X
   #'
                         learning rate
                                          = 0.1.
35
   #'
                         hidden dim
                                          = 10,
36
   #'
                         start \overline{f}rom end = TRUE)
37
38
39
    trainr <- function(Y, X, learningrate, learningrate decay = 1,
40
        momentum = 0, hidden dim, numepochs = 1, start from end=FALSE)
41
      \# check the consistency
42
```

```
if(dim(X)[2] != dim(Y)[2]) {
43
         stop ("The_time_dimension_of_X_is_different_from_the_time_
44
              dimension_of_Y._Only_sequences_to_sequences_is_supported")
45
       if(dim(X)[1] != dim(Y)[1])
46
         stop ("The_sample_dimension_of_X_is_different_from_the_sample_
47
              dimension_of_Y.")
48
49
      # coerce to array if matrix
50
       if(length(dim(X)) = 2){
51
         X \leftarrow \operatorname{array}(X, \operatorname{dim}=\operatorname{c}(\operatorname{dim}(X), 1))
52
53
       if(length(dim(Y)) == 2){
54
         Y \leftarrow \operatorname{array}(Y, \operatorname{dim}=\operatorname{c}(\operatorname{dim}(Y), 1))
55
56
57
      # extract the network dimensions
58
      input dim = dim(X)[3]
59
      \operatorname{output}_{\operatorname{\mathbf{dim}}} = \operatorname{\mathbf{dim}}(Y)[3]
60
      \operatorname{binary} \operatorname{\mathbf{dim}} = \operatorname{\mathbf{dim}}(X)[2]
61
62
      # initialize neural network weights
63
      synapse 0 = matrix(stats::runif(n = input dim*hidden dim, min=-1,
            max=1), nrow=input dim)
      synapse 1 = matrix(stats::runif(n = hidden dim*output dim, min
65
           =-1, max=1), nrow=hidden dim)
      synapse h = matrix(stats::runif(n = hidden dim*hidden dim, min
66
           =-\overline{1}, max=1), nrow=hidden dim)
67
      # initialize the update
68
      synapse 0 update = matrix(0, nrow = input dim, ncol = hidden dim)
69
      synapse 1 update = matrix (0, nrow = hidden dim, ncol = output dim
70
      synapse\_h\_update = matrix(0, nrow = hidden dim, ncol = hidden dim
71
72
73
      # initialize the old update for the momentum
      synapse 0 old update = matrix(0, nrow = input dim, ncol = hidden
74
      synapse_1_old_update = matrix(0, nrow = hidden_dim, ncol = output
75
            \overline{\dim}
      synapse h old update = matrix(0, nrow = hidden dim, ncol = hidden
           _dim)
77
78
      \# Storing layers states
79
      store output \leftarrow \mathbf{array}(0, \mathbf{dim} = \mathbf{dim}(Y))
80
      store hidden \leftarrow array (0, dim = c(dim(Y)[1:2], hidden dim))
81
      \# Storing errors, dim 1: samples, dim 2 is epochs, we could store
83
             also the time and variable dimension
84
       error \leftarrow array (0, dim = c(dim(Y)[1], numepochs))
85
      \# training logic
      for(epoch in seq(numepochs)){
87
```

```
message \, (\, paste \, 0 \, (\, "\, Training \, \_epoch \, : \, \_" \, , epoch \, , \, " \, \_- \, \_Learning \, \_rate \, : \, \_" \, ,
88
               learningrate))
          for (j \text{ in } 1: dim(Y)[1]) {
89
90
            \# generate a simple addition problem (a + b = c)
91
            a = array(X[j, ], dim=c(dim(X)[2], input dim))
92
93
            # true answer
94
            \mathbf{c} = \mathbf{array}(Y[j, ], \mathbf{dim} = \mathbf{c}(\mathbf{dim}(Y)[2], \mathbf{output} \mathbf{dim}))
95
96
            overallError = 0
97
            layer 2 deltas = matrix(0,nrow=1, ncol = output dim)
99
            layer 1 values = matrix(0, nrow=1, ncol = hidden dim)
100
            \# layer_1\_values = rbind(layer_1\_values, matrix(0, nrow=1,
101
                 n c o \overline{l} = \overline{h} i d d e n d i m))
102
            # time index vector, needed because we predict in one
103
                 direction but update the weight in an other
            if(start\_from\_end == TRUE) \ \{
104
                              <- binary dim:1
105
               pos vec back <- 1: binary dim
106
            } else {
107
              \mathbf{pos} vec
                              <- 1: binary dim
108
              pos_vec_back <- binary_dim:1
109
111
            # moving along the time
112
113
            for (position in pos vec) {
114
              \# generate input and output
115
116
              x = a[position,]
              y = c[position,]
117
118
               # hidden layer (input ~+ prev hidden)
119
               layer 1 = sigmoid :: logistic((\overline{x}\% - x) - y) + (layer_1_
120
                    values [dim(layer_1_values)[1],] %*% synapse_h))
121
               \# output layer (new binary representation)
122
               layer 2 = sigmoid::logistic(layer 1 %*% synapse 1)
123
124
               \# did we miss?... if so, by how much?
125
              layer 2 error = y - layer 2
              layer_2_deltas = rbind(layer_2_deltas, layer_2_error *
    sigmoid :: sigmoid_output_to_derivative(layer_2))
127
128
               overallError = overallError + sum(abs(layer_2_error))
129
              \# storing
130
               store_output[j,position,] = layer_2
131
               store hidden [j, position,] = layer
132
133
              # store hidden layer so we can print it out. Needed for
134
                    error calculation and weight iteration
               layer_1_values = rbind(layer_1_values, layer_1)
135
            }
137
138
```

```
# store errors
139
           error[j,epoch] <- overallError
140
141
           future layer 1 delta = matrix(0, nrow = 1, ncol = hidden dim)
142
143
           # Weight iteration,
144
           for (position in 0:(binary dim-1)) {
146
                             = a[pos vec back[position+1],]
147
              layer 1
                            = layer_1_values[dim(layer_1_values)[1] -
148
                  position,]
              \mathbf{prev}\_\mathtt{layer}\_\mathtt{1} = \mathtt{layer}\_\mathtt{1}\_\mathtt{values} \left[ \mathbf{dim} (\mathtt{layer}\_\mathtt{1}\_\mathtt{values}) \left[ \mathtt{1} \right] - (\mathtt{normal}) \right]
149
                  \overline{position} + 1),
150
              \# error at output layer
151
              layer 2 delta = layer 2 deltas [dim(layer 2 deltas)[1] -
152
                  position,]
                error at hidden layer
153
              layer 1 delta = (future layer 1 delta %*% t(synapse h) +
                  layer 2 delta %*% t(synapse_1)) *
                sigmoid::sigmoid output to derivative(layer 1)
156
              # let's update all our weights so we can try again
157
              synapse 1 update = synapse 1 update + matrix(layer 1) %*%
                  layer_2_delta
              synapse h update = synapse h update + matrix(prev layer 1)
                  %*% layer 1 delta
              synapse 0 update = synapse 0 update + c(x) %*% layer 1
160
                   delta \# I \ had \ to \ change \ \overline{X} \ as \ a \ vector \ as \ it \ is \ not \ a
                   matrix anymore, other option, define it as a matrix of
                  dim()=c(1, input dim)
161
              future layer 1 delta = layer 1 delta
162
163
           }
164
           # Calculate the real update including learning rate and
                momentum
166
           synapse 0 update = synapse 0 update * learningrate + synapse
                0 old update * momentum
           synapse 1 update = synapse 1 update * learningrate + synapse
167
                1 old update * momentum
           synapse h update = synapse h update * learningrate + synapse
168
                h old update * momentum
169
           \# Applying the update
170
           synapse 0 = synapse 0 + synapse 0 update
171
           synapse 1 = synapse 1 + synapse 1 update
172
           synapse h = synapse h + synapse h update
174
           # Update the learning rate
           learningrate <- learningrate * learningrate_decay</pre>
176
177
178
           # Storing the old update for next momentum
           synapse 0 old update = synapse 0 update
179
           synapse 1 old update = synapse 1 update
           synapse_h_old_update = synapse_h_update
181
182
```

```
# Initializing the update
183
            synapse 0 update = synapse 0 update * 0
            synapse 1 update = synapse 1 update * 0
185
            synapse h update = synapse h update * 0
186
187
          # update best guess if error is minimal
188
189
          if(colMeans(error)[epoch] <= min(colMeans(error)[1:epoch])){</pre>
            store output best <- store output
190
            store hidden best <- store hidden
191
192
          message(paste0("Epoch_error: _ ", colMeans(error)[epoch]))
193
194
195
       # create utput object
196
       output=list (synapse 0
                                            = synapse_0,
197
                      synapse
                                            = synapse^{-1},
198
199
                      synapse h
                                            = synapse_h,
                      error
                                            = error,
200
                      store output
                                            = store output,
201
                      store_hidden
                                            = store_hidden,
202
                      \begin{array}{lll} store\_hidden\_best = store\_hidden\_best \;, \\ store\_output\_best = store\_output\_best \;, \end{array}
203
204
                      start from end
                                            = start from end)
205
206
       attr(output, 'error') <- colMeans(error)</pre>
207
208
209
       \# return output
210
       return (output)
211
212
```

## B Source code of predictr() function

#### 1 Users quast rnn R predictr.R

```
@name predictr\\
      @export
   #'
      @importFrom stats runif
      @importFrom\ sigmoid\ sigmoid
      @title Recurrent Neural Network
   \#' @description predict the output of a RNN model
      @param\ model\ output\ of\ the\ trainr\ function
      @param\ X\ array\ of\ input\ values\ ,\ dim\ 1:\ samples\ ,\ dim\ 2:\ time\ ,\ dim
         3: variables (could be 1 or more, if a matrix, will be coerce
        to array)
   #' @param hidden should the function output the hidden units states
9
   \#' @param ... arguments to pass on to sigmoid function
   \#' @return array or matrix of predicted values
11
   \#' @examples
12
   \#' \# create training numbers
   \#'X1 = sample(0:127, 7000, replace=TRUE)
  ig| \#' X2 = sample(0:127, 7000, replace = TRUE)
   #'
16
   \#' \# create training response numbers
```

```
| \#' Y < - X1 + X2
18
    #'
19
   #' # convert to binary
20
   \#' X1 \leftarrow int2bin(X1)
   \#' X2 \leftarrow int2bin(X2)
22
    \#'Y \leftarrow int2bin(Y)
23
24
    #' # Create 3d array: dim 1: samples; dim 2: time; dim 3: variables
25
    \#' \ X < - \ array\left( \ c\left(X1,X2\right), \ dim = c\left(dim\left(X1\right),2\right) \ \right)
26
27
    \#' \# train the model
28
   \#' \mod el \leftarrow trainr(Y=Y,
29
   #'
                             X=X
    #'
                             learningrate = 0.1,
31
    #'
                             hidden dim
                                                = 10,
32
    #'
                             start \overline{f}rom end = TRUE )
33
34
   \#' \# create test inputs
    \#'\ A1 = int2bin(\ sample(0:127,\ 7000,\ replace=TRUE)\ )\ \#'\ A2 = int2bin(\ sample(0:127,\ 7000,\ replace=TRUE)\ )
36
37
38
   \#' \# create 3d array: dim 1: samples; dim 2: time; dim 3: variables
39
    \#' A \leftarrow array(c(A1, A2), dim=c(dim(A1), 2))
40
41
    #' # predict
42
    \#'B \leftarrow predictr(model,
43
44
45
    #' # convert back to integers
46
    \#' A1 \leftarrow bin2int(A1)
47
    \parallel \#' A2 < -bin2int(A2)
48
   \#'B \leftarrow bin2int(B)
50
    # ' # inspect the differences
51
    \#' table (B-(A1+A2))
52
53
    \#' # plot the difference
    \#' \ hist( B-(A1+A2) )
55
56
57
    predictr <- function(model, X, hidden = FALSE, ...) {
58
       \# coerce to array if matrix
60
       if(length(dim(X)) = 2){
61
         X \leftarrow \operatorname{array}(X, \operatorname{dim} = \operatorname{c}(\operatorname{dim}(X), 1))
62
63
      # load neural network weights
65
       synapse 0
                         = model$synapse 0
66
       synapse_1
                         = model$synapse_1
67
       synapse h
                        = model$synapse h
68
69
       start from end = model$start from end
70
       \# extract the network dimensions, only the binary dim
71
       input dim = dim(synapse 0)[1]
72
       output \dim = \dim(\operatorname{synapse} 1)[2]
```

```
hidden dim = dim(synapse 0)[2]
74
       \operatorname{binary} \operatorname{\mathbf{dim}} = \operatorname{\mathbf{dim}}(X)[2]
 75
76
       \# Storing layers states
77
       store output \leftarrow array(0, dim = c(dim(X)[1:2], output dim))
78
       store hidden \leftarrow array (0, \dim = c(\dim(X)[1:2], \operatorname{hidden \overline{dim}}))
79
80
       for (j in 1:dim(X)[1]) {
81
82
         \# generate a simple addition problem (a + b = c)
83
         a = array(X[j,,], dim=c(dim(X)[2], input dim))
84
 85
86
         layer 1 values = matrix(0, nrow=1, ncol = hidden dim)
88
         # time index vector, needed because we predict in one direction
89
                but \ update \ the \ weight \ in \ an \ other
          if(start from end == T){
90
            pos vec <- binary dim:1
            pos_vec_back <- 1:binary_dim
92
93
            pos vec <- 1: binary dim
94
            pos_vec_back <- binary_dim:1
95
96
97
         \# moving along the time
         for (position in pos vec) {
99
100
            \# generate input and output
101
            x = a[position,]
102
103
            \# hidden layer (input \tilde{\ }+ prev hidden)
104
            layer 1 = \text{sigmoid} :: \text{sigmoid} ((x \sqrt{x}) \text{synapse } 0) + (\text{layer } 1 \text{ values})
105
                 dim(layer_1_values)[1],] %*% synapse_h), ...)
106
            # output layer (new binary representation)
107
            layer\_2 = sigmoid :: sigmoid (layer\_1 \% \% synapse\_1, ...)
108
109
            \# storing
110
            store output[j,position,] = layer 2
111
            store_hidden[j, position,] = layer_1
112
113
            \# store hidden layer so we can print it out. Needed for error
114
                  calculation \ and \ weight \ iteration
            layer 1 values = rbind(layer 1 values, layer 1)
115
116
         }
117
       }
118
119
120
       \# return output vector
121
       if(hidden == FALSE){
122
123
         # convert to matrix if 2 dimensional
          if(dim(store output)[3]==1) {
124
125
            store output <- matrix(store output,
                                 nrow = dim(\overline{store}\_output)[1],
126
127
                                 ncol = dim(store output)[2])
```

```
128  # return output

129  return(store_output)

130  }else{

131  return(store_hidden)

132  }

133  }
```