Excercises from Introduction to Algorithms 4th Edition

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1 Strassen's Algorithm

Strassen's algorithm is an efficent algorithm for matrix multiplication. Rather than $O(n^3)$ runtime, it is capable of $O(n^2.81)$ runtime.

Problem 4.2-1

Use Strassen's algorithm to compute the matrix product

$$\begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

1. Partition the input matrices A and B and output matrix C into $n/2 \times n/2$

$$A_{11} = 1, A_{12} = 3, A_{21} = 7, A_{22} = 5$$

$$B_{11} = 6, B_{12} = 8, B_{21} = 4, B_{22} = 2$$

2. Set up the sum or difference in 10 matrices

$$S_1 = B_{12} - B_{22} = -10$$

$$S_2 = A_{11} + A_{12} = 4$$

$$S_3 = A_{21} + A_{22} = 11$$

$$S_4 = B_{21} - B_{11} = -2$$

$$S_5 = A_{11} + A_{22} = 6$$
$$S_6 = B_{11} + B_{22} = 10$$
$$S_7 = A_{12} - A_{22} - 2$$
$$S_8 = B_{21} + B_{22} = 8$$
$$S_9 = A_{11} - A_{21} = -6$$
$$S_10 = B_{11} + B_{12} = 14$$

This step adds $n/2 \times n/2$ matrices, taking $O(n^2)$ time.

3. Step 3 recursively multiplies $n/2 \times n/2$ matrices 7 times.

$$P_1 = A_{11}.S_1 = -10$$
$$P_2 = S_2.B_{22} = 16$$
$$P_3 = S_3.B_{11} = 66$$
$$P_4 = A_{22}.S_4 = -10$$
$$P_5 = S_5.S_6 = 60$$
$$P_6 = S_7.S_8 = -16$$
$$P_7 = S_9.S_10 = -84$$

4. Step 4 adds to and subtracts to create the C product matrix

$$C_{11} = C_{11} + P_5 + P_4 - P_2 + P_6 = 60 + (-10) - 16 + (-16) = 18$$
$$C_{12} = C_{12} + P_1 + P_2 = -10 + 16 = 6$$
$$C_{21} = C_{21} + P_3 + P_4 = 66 - 10 = 56$$
$$C_{22} = C_{22} + P_5 + P_1 - P_3 - P_7 = 60 - 10 - 66 + 84 = 68$$