Exercise 2: Numerical Solutions of the Harmonic Oscillator

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1 Eigenvalues

We expect the first few eigenvalues for the Harmonic Oscillator to be $\lambda_0 = 3$, $\lambda_1 = 7$, and $\lambda_2 = 11$. Included in Table 1 are numerical results for different maximum distances R_{max} and different numbers of integration points, all for $l = 0$. The rate of convergence is similar for different l, but it converges towards different numbers. We expect the eigenvalues to be:

$$\lambda_{n,l} = 4n + 2l + 3 \tag{1}$$

As you can see from Table 1, the choice of R_{max} and the number of integration points are related. A larger R_{max} means a considerably larger amount of integration points necessary to converge to the correct answer. The eigenfunctions for the $R_{max} = 10$ and number of integration points equal to 500 are plotted in Figure 1.

We will now compare results from the Harmonic Oscillator to that of the Woods-Saxon potential in a unit-less form.

$$V_{ws}(r) = \frac{-2}{1 + exp\left((\sqrt{\frac{\hbar^2 c^2}{mc^2 V_0}}\rho - R)/a \right)} \tag{2}$$

The parameters in Equation 2 are $V_0 = 50$ MeV, $a = 0.5$ fm, $R = 1.25$ fm, $\hbar c = 197$ MeV fm, and $mc^2 = 938$ MeV. We made the characteristic length

R_{max}	Integration Points	l	λ_0	λ_1	λ_2
10	10	0	2.7226	5.8946	9.9598
	50		2.9879	6.9394	10.8514
	100		2.9969	6.9846	10.9625
	500		2.9999	6.9994	10.9985
100	10	0	82.6688	330.6027	743.8259
	50		4.3590	15.9012	35.1232
	100		2.6894	6.0798	10.9238
	500		2.9875	6.9372	10.8459
	1000		2.9969	6.9844	10.9618

Table 1: Harmonic Oscillator Eigenvalues.

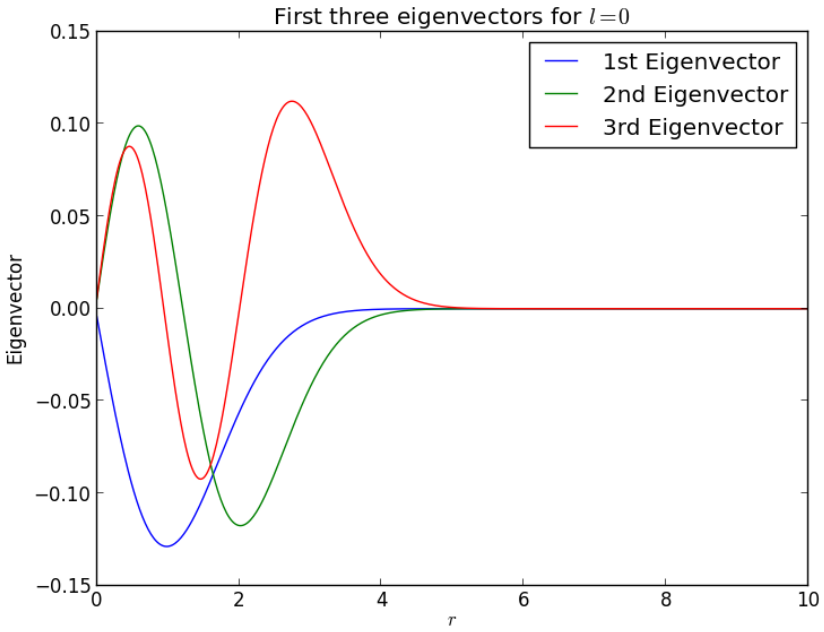

Figure 1: These are the first three eigenvectors for the harmonic oscillators solved numerically for $l = 0$.

scales of the harmonic oscillator and Woods-Saxon the same by making $\hbar\omega = V_0$.

We calculated three eigenvalues for the Woods-Saxon potential for $l = 0$, namely $\lambda_0 = -0.195$, $\lambda_1 = 0.193$, and $\lambda_2 = 0.656$. The three eigenvectors corresponding to these eigenvalues are plotted in Figure 2. The first notable difference between the eigenvectors of the two potentials are the extent of the Woods-Saxon potential eigenvectors, which do not decay to zero nearly as rapidly as the harmonic oscillator eigenfunctions. I think this is because the harmonic oscillator does not take into account the diffuseness of the nuclear surface properly. Otherwise, the two sets of eigenfunctions are very similar in shape if one ignores the difference in sign between the two sets. The sign difference is unimportant because it just suggests an overall phase difference.

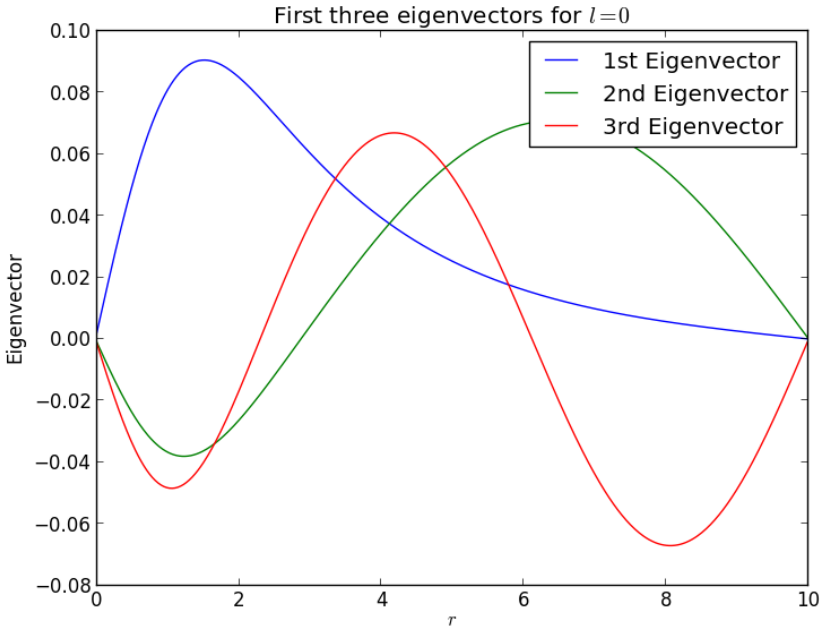

Figure 2: These are the first three eigenvectors for the Woods-Saxon Potential solved numerically for $l = 0$.