# Bayesian Statistics, Problem Set 10 (Nov. 1) SOLUTION

## The Mudslide Problem

### Step 1 — Tabulating the Priors

In[47]:=
```
months = {"Jan", "Feb", "Mar", "Apr",
   "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"};
priors = {0.005, 0.021, 0.058, 0.264, 0.333, 0.199,
   0.086, 0.03, 0.001, 0.002, 0.001, 0};
TableForm[Table[priors[[i + 1]], {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Prior P(i)"}}]
```

Out[48]//TableForm=

| | Prior P(i) |
|---|---|
| Jan | 0.005 |
| Feb | 0.021 |
| Mar | 0.058 |
| Apr | 0.264 |
| May | 0.333 |
| Jun | 0.199 |
| Jul | 0.086 |
| Aug | 0.03 |
| Sep | 0.001 |
| Oct | 0.002 |
| Nov | 0.001 |
| Dec | 0 |

### Step 2 — Tabulating the Likelihoods

As the observation, *O*, we will imagine that we looked for and identified two kinds of pollen in the prehistoric mud: Ambrosia pollen and Gramineae / Poaceae pollen. I hastily made the following estimates of the fraction of days (in each month) that Ambrosia pollen concentrations were higher than Gramineae / Poaceae pollen concentrations.

In[49]:=
```
likelihoods =
  {0.0, 0.0, 0.0, 1 / 30, 0.0, 0.0, 0.0, 31 / 31, 10 / 30, 31 / 31, 10 / 30, 3 / 31};
TableForm[Table[N[likelihoods[[i + 1]]], {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Likelihood P(O|i)"}}]
```

Out[50]//TableForm=

| | Likelihood P(O\|i) |
|---|---|
| Jan | 0. |
| Feb | 0. |
| Mar | 0. |
| Apr | 0.0333333 |
| May | 0. |
| Jun | 0. |
| Jul | 0. |
| Aug | 1. |
| Sep | 0.333333 |
| Oct | 1. |
| Nov | 0.333333 |
| Dec | 0.0967742 |

**Whatever you did, as long as it was remotely like this, it will work fine in the remainings steps.**

## Step 3 — Tabulating the Products

In[51]:=
```
TableForm[Table[priors[[i + 1]] * likelihoods[[i + 1]], {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Product P(O|i) * P(i)"}}]
```

Out[51]//TableForm=

| | Product P(O\|i) * P(i) |
|---|---|
| Jan | 0. |
| Feb | 0. |
| Mar | 0. |
| Apr | 0.0088 |
| May | 0. |
| Jun | 0. |
| Jul | 0. |
| Aug | 0.03 |
| Sep | 0.000333333 |
| Oct | 0.002 |
| Nov | 0.000333333 |
| Dec | 0 |

## Step 4 — Computing the Sum

To finish getting the denominator in $P(i \mid O) = \dfrac{P(O \mid i)\, P(i)}{\sum_{j=1}^{12} P(O \mid j)\, P(j)}$, compute the sum $\sum_{j=1}^{12} P(O \mid j)\, P(j)$

In[52]:=
```
denominator = Total[Table[priors[[i + 1]] * likelihoods[[i + 1]], {i, 0, 11}]]
```

Out[52]=

```
0.0414667
```

## Step 5 — Computing the Posteriors

Now that you have done steps 1 to 4 it should be hardly any work at all to compute the posteriors:

In[53]:=
```
posteriors = priors * likelihoods / denominator
```

Out[53]=
```
{0., 0., 0., 0.212219, 0., 0., 0., 0.723473, 0.00803859, 0.0482315, 0.00803859, 0.}
```

In[54]:=
```
TableForm[posteriors]
```

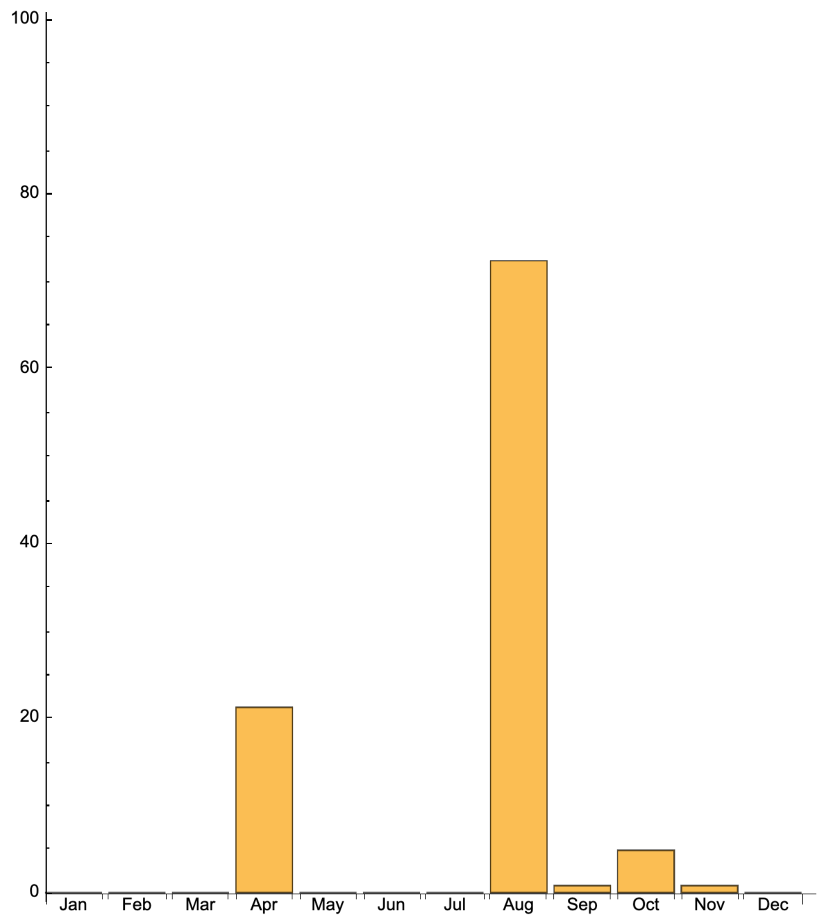
Out[54]//TableForm=
```
0.
0.
0.
0.212219
0.
0.
0.
0.723473
0.00803859
0.0482315
0.00803859
0.
```

## Step 6 — Plotting the Posteriors

Make a lovely plot of the posteriors using your table from Step 5. For readability of the table, convert back to percentages if you haven't already.

In[55]:=
```
BarChart[Table[100 * posteriors[[j]], {j, 1, 12}],
 PlotRange → {{0, 12}, {0, 100}}, ChartLabels → months, PlotRangePadding → 1.0,
 FrameLabel → {"Posterior (%)"}, AspectRatio → 1, ImagePadding → 50]
```

Out[55]=




**If your answer looks significantly different from this, I am not concerned as long as you did all the steps. The difference between your answer an my answer, if you did the steps correctly, is likely from eyeballing the likelihoods in Step 2. Although the pollen count plot was beautiful, it is admittedly hard to go from it to an estimate of what fraction of the time Ambrosia concentrations exceeded Gramineae / Poaceae concentrations.**