Bayesian Statistics, Problem Set 10 (Nov. 1) SOLUTION

The Mudslide Problem

Step 1 — Tabulating the Priors

In[2]:=
```
months = {"Jan", "Feb", "Mar", "Apr",
   "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec"};
priors = {0.005, 0.021, 0.058, 0.264, 0.333, 0.199,
   0.086, 0.03, 0.001, 0.002, 0.001, 0};
TableForm[Table[priors〚i + 1〛, {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Prior P(i)"}}]
```

Out[3]//TableForm=

	Prior P(i)
Jan	0.005
Feb	0.021
Mar	0.058
Apr	0.264
May	0.333
Jun	0.199
Jul	0.086
Aug	0.03
Sep	0.001
Oct	0.002
Nov	0.001
Dec	0

Step 2 — Tabulating the Likelihoods

By eyeball (no sophisticated procedure), I made the following estimates of the fraction of days (in each month) that Ambrosia pollen concentrations were higher than Gramineae / Poaceae pollen concentrations.

In[4]:=
```
likelihoods =
  {0.0, 0.0, 0.0, 1 / 30, 0.0, 0.0, 0.0, 31 / 31, 10 / 30, 31 / 31, 10 / 30, 3 / 31};
TableForm[Table[N[likelihoods[[i + 1]]], {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Likelihood P(O|i)"}}]
```

Out[5]//TableForm=

	Likelihood P(O|i)
Jan	0.
Feb	0.
Mar	0.
Apr	0.0333333
May	0.
Jun	0.
Jul	0.
Aug	1.
Sep	0.333333
Oct	1.
Nov	0.333333
Dec	0.0967742

Whatever you estimated, as long as it was remotely like this, it will work fine in what follows.

Step 3 — Tabulating the Products

In[6]:=
```
TableForm[Table[priors[[i + 1]] * likelihoods[[i + 1]], {i, 0, 11}, {j, 1, 1}],
 TableHeadings → {months, {"Product P(O|i) * P(i)"}}]
```

Out[6]//TableForm=

	Product P(O|i) * P(i)
Jan	0.
Feb	0.
Mar	0.
Apr	0.0088
May	0.
Jun	0.
Jul	0.
Aug	0.03
Sep	0.000333333
Oct	0.002
Nov	0.000333333
Dec	0

Step 4 — Computing the Sum

To finish getting the denominator in $P(i \mid O) = \frac{P(O \mid i)\, P(i)}{\sum_{j=1}^{12} P(O \mid j)\, P(j)}$, compute the sum $\sum_{j=1}^{12} P(O \mid j) P(j)$

In[7]:=
```
denominator = Total[Table[priors[[i + 1]] * likelihoods[[i + 1]], {i, 0, 11}]]
```

Out[7]=
```
0.0414667
```

Step 5 — Computing the Posteriors

In[11]:=
```
posteriors = priors * likelihoods / denominator;
```

In[9]:=
```
TableForm[posteriors]
```

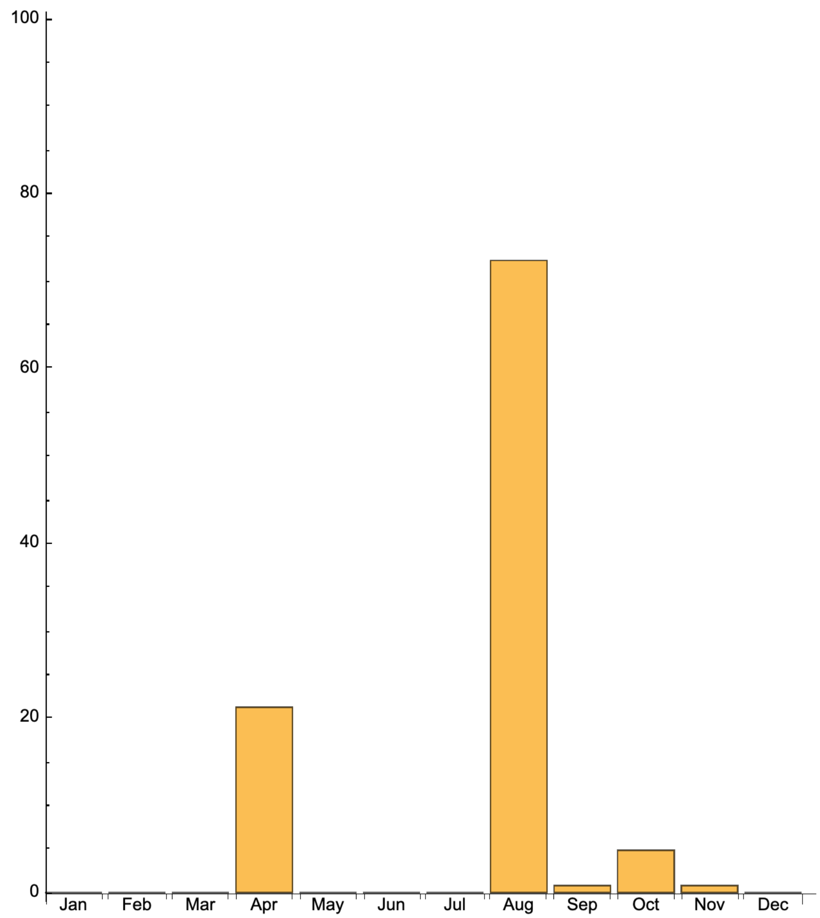
Out[9]//TableForm=
```
0.
0.
0.
0.212219
0.
0.
0.
0.723473
0.00803859
0.0482315
0.00803859
0.
```

Step 6 — Plotting the Posteriors

Make a lovely plot of the posteriors using your table from Step 5. For readability of the table, convert back to percentages if you haven't already.

In[10]:=
```
BarChart[Table[100 * posteriors⟦j⟧, {j, 1, 12}],
 PlotRange → {{0, 12}, {0, 100}}, ChartLabels → months, PlotRangePadding → 1.0,
 FrameLabel → {"Posterior (%)"}, AspectRatio → 1, ImagePadding → 50]
```

Out[10]=



If your answer looks significantly different from this, I am not concerned as long as you did all the steps. The difference between your answer and my answer, if you did the steps correctly, is likely from eyeballing the likelihoods in Step 2. Although the pollen count plot is a beautiful presentation of a lot of data, it is admittedly hard to go from it to an accurate estimate of what fraction of the time in each month Ambrosia concentrations exceeded Gramineae / Poaceae concentrations.