Bayesian Statistics, Exam 3

Friday, Dec. 13, 2024 — Bayesian Conjugates and Monte Carlo Methods

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1. A Beta Distribution Prior for Basketball Players Shooting 3-Pointers

An increasingly important shot in basketball is the "3-pointer." In the NBA, this is a shot made from more than 23 feet 9 inches from the basket. There is a stripe on the court marking this distance. An average offensive player that shoots 3-pointers succeeds 30% of the time. (Otherwise you might as well just go for 2-pointers, which offensive players can make about 45% of the time.)

The beta distribution is:

$$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$$

The mean of the beta distribution with parameters α and β is $\mu = \frac{\alpha}{\alpha+\beta}$.

The variance of the distribution is kind of a mess. It is $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2\,(\alpha+\beta+1)}$.

A. Let's assume we have followed a bunch of players and their mean success rate $\mu = 0.3$. What combination of integers α and β will give $\mu = 0.3$? HINT: If you can't just guess the combination of integers that works, try $\alpha = 3$ and see what β has to be.

B. What is the variance with the α and β you chose in Part A? Feel free to leave your answer for σ^2 as a rational fraction.

C. It turns out that the fraction you just calculated in B comes out to about $\sigma^2 = 0.02$ and taking the square root of that, you get a standard deviation, $\sigma = 0.14$. On the plot below, draw three nice vertical lines at μ, at $\mu + \sigma$, and at $\mu - \sigma$. Shade in the region under the curve between these lines.

In[ ]:=
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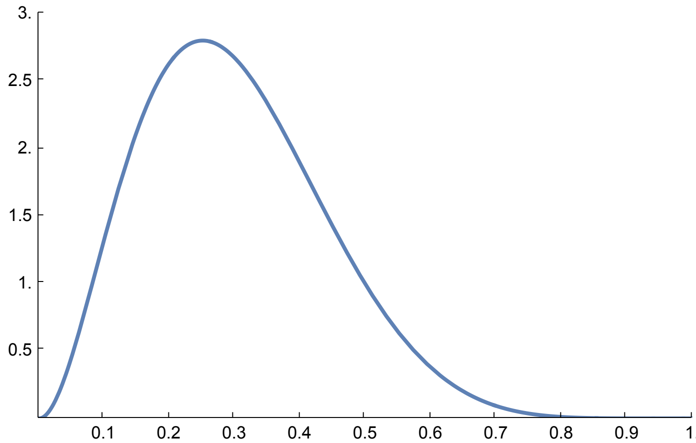
beta[x_] := x^(3-1) (1 - x)^(7-1) / Beta[3, 7]
Plot[beta[x], {x, 0, 1}, PlotRange → {{0.0, 1.0}, {0.0, 3.0}},
 Ticks → {Range[0.1, 1.0, 0.1], Range[0.5, 3.0, 0.5]}]
```

Out[ ]=



D. Make an estimate of the width, the average height, and the area you have shaded in.

E. Convert your answer to D into an estimate of the percentage of players that make 3-pointers between 16% and 34% of the time.

F. To finish off your analysis of the prior, write it out with the α and β you have chosen:

prior(x) = $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ =

2. A Likelihood for Nikola Jokic

On Dec. 8, Nikola Jokic of the Denver Nuggets made 3 out of 6 of the 3-pointers that he attempted against the Atlanta Hawks.

The formula for the binomial distribution is

$$L(x) = \binom{N}{n} x^n (1 - x)^{N-n}$$

You don't know x. It is whatever Nikola Jokic's true average is, and you don't know that, so just leave x as a variable. But simplify $L(x)$ as much as you can:

$L(x) =$

Now you have a likelihood.

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3. A Posterior for Nikola Jokic

A. Take the product of the prior from Problem 1 and the likelihood from Problem 2. Simplify it as much as you can, but note that the denominator is a mess, so just leave it as denominator.

$\text{posterior}(x) = \frac{\text{prior}(x) * L(x)}{\text{denominator}}$

B. Leave all the constants out and the messy denominator out. All that is left is powers of x and 1-x. What is left?

$\text{posterior}(x) \propto$

C. This is another beta distribution. What are the α and β of this distribution?

D. Using your new α and β, you now have a probability distribution for Nikola Jokic's average. What is

$\mu = \frac{\alpha}{\alpha+\beta} =$

for this new distribution? You can leave it as a rational fraction.

E. What is the variance

$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2\,(\alpha+\beta+1)} =$

for this new distribution? You can leave the variance as a rational fraction too.

Name ______________________________________

1. / 3
2. / 3

3. / 4
4. / 5

TOTAL / 15