Bayesian Statistics: The Foundation of Scientific Inference

Unofficial/Short Course Title: Bayesian Statistics

Fall 2024, Deep Springs College, ***Prof. Brian Hill***

Overview

There are two types of inferential statistics that are widely used: frequentist and Bayesian. Frequentist inference is an oversimplification, and it sometimes gives obviously wrong results. However, frequentist inference remains the dominant way that statistics is learned and applied to experimental data to make claims because it is somewhat easier to learn. We will begin with the unassailable basics which underly both frequentist and Bayesian statistics. The basics common to both are known as "descriptive statistics." After covering descriptive statistics, we can spend some time on frequentist inference, including the derivation of the linear regression formulae. Then we will graduate to Bayesian statistics. We will conclude with Monte Carlo methods.

Prerequisite / Joining the Class

I will use mathematics that is commonly known as either Algebra 2 or Algebra 3-4 (depending on whether a school numbers its math classes by the year or the semester). We will need to develop and use some calculus to do statistics, and I will go slowly and assume you have never seen any calculus.

Texts

We are going to use three texts. We are going to use three texts. In order, the three texts we will use are:

- Hugh Young, **Statistical Treatment of Experimental Data** (this is a compact classic that will be of use no matter what additional statistics you do or don't ever use)
- Therese Donovan and Ruth Mickey, **Bayesian Statistics for Beginners** (this is the book we will use the most — it is at a good level for our purposes)
- W.R. Gilks, S. Richardson, David Spiegelhalter, eds., **Markov Chain Monte Carlo in Practice** (the first two chapters of this collection will conclude our course and serve as a bridge from the illustrative but simple Monte Carlo applications in Chapters 13-16 of Donovan and Mickey to realistic and modern epidemiological applications)

Grading

- 45% assignments
- 15% (45% total) for each of three exams, dates to be determined, but coming at about the 5th, 9th, and 14th week of classes
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Being Neat and Organized

There will be at least one problem set every week, and ideally more if I can produce one for every class. There will also be handouts, problem set solutions, exams, and exam solutions. To be organized, locate a three-ring binder and a three-ring hole punch, and file everything chronologically. Reverse-chronological is actually the most convenient, because you always open your binder to what you are currently working on. Problem sets should be *neat* and on standard 8 1/2 x 11 paper. Multi-page problem sets should be stapled. The nicest technical work is facilitated by engineering pads, such as these **Roaring Spring Engineering Pads at Amazon** (which are pretty expensive unless you buy by the case), and done with a mechanical pencil, a ruler, and an eraser at hand.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Deep Springs Handbook.

Daily Schedule Term 2

Week 1 — Uncertainty, Fractional Error, Propagation of Error, and Random Variables — Mean

- Tuesday, Aug. 27 — **Syllabus** — Study pp. 4-15 of Young — Change in volume of a cylinder due to change in radius — The formula for the mean (aka average) — The bar notation for the mean — The formula for a "weighted" mean (in case you have more faith in some of the contributions to the average, you can weight those contributions more) — The Σ summation notation — The formula for σ^2 where σ is the "standard deviation of the mean" — Contrasting standard deviation (which uses squares of differences) with the average of the absolute differences
- Friday, Aug. 30 — Carefully read pp. 1-15 of Young — Do the **Assignment for Friday** — Come prepared with any remaining questions about pp. 1-15 — Let's give some classic examples of and define, what is a "random variable?" — A coin toss (repeated) — A toss of a handful of coins (repeated) — Let's define, what is a "statistic?" — Galton's box (and its relation to coin tosses) — Combinatorics (which is essentially the study of how many ways there are of arriving at the same result) — Pascal's triangle — The choose notation — The factorial notation and its application to combinatoric formulae

Week 2 — Combinatorics and Probability Distributions — Binomial Distribution — Variance

- Tuesday, Sept. 3 — Carefully read to p. 35 of Young (the end of Chapter 2) — Come prepared with any remaining questions about pp. 16-35 — Do the **Assignment for Tuesday**
- Friday, Sept. 6 — Carefully read to p. 57 of Young (stop just before he starts the Poisson distribution) — Come prepared with any remaining questions about pp. 39-57 — Do the **Assignment for Friday** — The relation between small changes, "infinitesimal calculus," and partial derivatives (I finally explain equations 2.7, 2.8, and 2.9 on p. 7) — The relation of the Poisson distribution to the binomial distribution (the tricky derivation on p. 59)

Week 3 — Poisson Distribution — Histograms, Integrals, and Cumulative Distribution Functions

- Tuesday, Sept. 10 — Carefully read to p. 64 of Young (the Poisson distribution) — Do the **Assignment for Tuesday** — Histograms and Integrals — Discussion of Histogram, Probability Distribution Functions (PDFs), and Cumulative Distribution Functions (CDFs), using deaths of males aged 0-109 as our example
- Friday, Sept 13 — To give everyone a break (especially me), there is no assignment due for Friday — Please use this time *to get started* on pp. 64-75 of Young, *but stop in that reading* as soon as you hit something that you really don't yet have the tools to understand, and come prepared with questions about the reading at the point that you decided to stop — Figures like Figure 9.2 on p. 68 should be starting to make sense given Tuesday's discussion

Week 4 — Gaussian Distribution — Start Linear Regression

- Tuesday, Sept. 17 — Our last major topic in Young is the Gaussian distribution, pp. 64-75, which you already started on last week — Do the **Assignment for Tuesday** — Warmup for Linear Regression
- Friday, Sept. 20 — Finish Chapter 3 of Young (to the end of Section 11, p. 86) — Do the **Assignment for Friday**

Week 5 — Exam 1 — Finish Linear Regression — Goodness of Fit

- Tuesday, Sept. 24 — **Exam 1** on Descriptive Statistics (covers Problem Sets 1 to 5 and Young Sections 1 to 9) — Problem Set 6, linear regression and the application of χ-squared will be on Exam 2

- Friday, Sept. 27 — Review Sections 10 and 11 of Young — Read Chapter 1 of **Bayesian Statistics for Beginners** — Do the **Assignment for Friday** (and use **this graph paper** to graph the data and fit for Problem 1) — Finish Discussion of Linear Regression and Goodness of Fit

NOTE: Now that we have covered the essential parts of the compact classic introduction to descriptive statistics by Young, the material for much (but not all!) of the rest of the course will follow an introductory Bayesian statistics text: Therese Donovan and Ruth Mickey, **Bayesian Statistics for Beginners**.

Week 6 — Conditional, Marginal, and Joint Probabilities — Bayes Theorem

- Tuesday, Oct. 1 — Read Chapter 2 of *Bayesian Statistics for Beginners* — Do the **Assignment for Tuesday** — Probabilistic Notation — Venn Diagrams — False Positives — False Negatives — Bayes Theorem
- Friday, Oct. 4 — Read Chapter 3 of *Bayesian Statistics for Beginners* — Do the **Assignment for Friday** — To make the reading more lively and to give us all some comic relief, Tahm has volunteered to open Friday's class with a 5-minute presentation on Morton's toe — Useful rearrangements of Bayes Theorem

Week 7 — Bayesian Inference

- Tuesday, Oct. 8 — Read Chapter 4 of *Bayesian Statistics for Beginners — NB: As preparation for class on Tuesday, Oct. 29, which is the first day of classes after the break, you will be reading Chapter 5 of* Bayesian Statistics for Beginners — Although there is no assignment to hand in for today (Tuesday, Oct. 8), you can get started on or even complete the **Assignment for Tuesday, Oct. 29** — A Prelude to Continuum Versions of Bayes' Theorem
- Friday, Oct. 11 — No class (first day of Term 2-3 break)

Daily Schedule Term 3

Week 8 — Building Your Intuition with More Examples with Mutually-Exclusive Hypotheses: Hamilton vs. Madison and a Mudslide with Pollen

- Tuesday, Oct. 29 — Study Chapter 5 of *Bayesian Statistics for Beginners* — Do the **Assignment for Tuesday** — Preparatory discussion for Chapter 6
- Friday, Nov. 1 — Study Chapter 6 of *Bayesian Statistics for Beginners* — Do the **Assignment for Friday** — Preparatory discussion for Chapters 8 and 9: **A Continuum of Hypotheses** — NOTE: We skipped Chapter 7 because it introduced too little that is new

Week 9 — Bayesian Statistics with Probability Distributions

- Tuesday, Nov. 5 — Dive in at the last half of Chapter 8, pp. 95-107 (if you also try to read pp. 88-94 of Chapter 8, I think you will find it to be a mind-numbing review of what we covered in Young) — Continue in Chapter 9, pp. 108-122 (this is yet more review of Young but in this case the review is useful) — Do the **Assignment for Tuesday** — We reviewed Bernoulli and Binomial Distributions, including the "n choose k" prefactor in the binomial distribution — We looked at the etymology of the terms "probability mass function (PMF)" (an example of which is the Poisson distribution) as Donovan and Mickey use the term in Chapter 8, and "probability density function (PDF)" (an example of which is the Gaussian distribution) as Donovan and Mickey use the term in Chapter 9 — We started the **Assignment for Friday**
- Friday, Nov. 8 — Finish Chapter 9 and the **Assignment for Friday** — We are not starting any new material — Instead, let's consolidate — People asked for **additional practice problems**, which we started in class

Week 10 — Exam 2 — Start Bayesian Conjugates

- Tuesday, Sept. 24 — **Exam 2** on Frequentist and Bayesian Statistics (covers Problem Sets 6 to 12 and Donovan and Mickey Chapters 1 to 9), including linear regression and χ-squared — See also the quick summary of **Problem Sets 10, 10, 11, and 12**
- Friday, Nov. 15 — Study Chapter 10 — Introduce Bayesian Conjugates

Week 11 — Finish Bayesian Conjugates — Start Monte Carlo, Simple Applications

- Tuesday, Nov. 19 — You can study Chapter 11, which introduces gamma functions which are conjugate to Poisson distributions, or alternatively, you can just study my handout on **Bayesian Conjugates** — Please come with questions about Chapters 11 or my summary in the handout of Chapter 11: **Bayesian Conjugates** — Do the **Assignment for Tuesday** — An Advanced Idea from Chapter 12: **A Multi-Dimensional Continuum of Hypotheses** — For completeness, here are **Michael I. Jordan's Stat260 notes**
- Friday, Nov. 22 — While you are doing the **Assignment for Friday** on Chapter 11, I have been working on a **Monte Carlo Methods Introduction** and on a **Monte Carlo Playground and Program** that we will use in class — Rather than you diving in to Chapter 13 already, wait until that for the next class, and instead start by reading **The Beginning of the Monte Carlo Method**

Week 12 — Continue Monte Carlo, Simple Applications — The Metropolis Algorithm

- Tuesday, Nov. 26 — Study Chapter 13 — Do the **Assignment for Tuesday** — Look ahead to the remainder of the course — Describe the three Monte Carlo methods and the Hepatitis B case study

Week 13 — The Metropolis-Hastings Algorithm

- Tuesday, Dec. 3 — See **Monte Carlo Methods - Why Do They Work? - Part I** for my proof that the Metropolis algorithm works — Introducing Metropolis-Hastings
- Friday, Dec. 6 — Study Chapter 15 to p. 229 — More directions for Friday are in the **Assignment for Friday** — See also **Monte Carlo Methods - Why Do They Work? - Part II** for my proof that the Metropolis-Hastings algorithm works

Week 14 — The Gibbs Sampling Algorithm — Exam 3

- Tuesday, Dec. 10 — Our final subject before Exam 3 is Gibbs sampling, and I am *not* going to have you look at Donovan and Mickey Chapter 16, because they have obscured what is almost laughably simple — Instead, in class we'll go through my **Monte Carlo Methods - Why Do They Work? - Part III** — Here is the **In-Class Exercise for Tuesday**
- Friday, Dec. 13 — **Exam 3** covering Chapters 10 and 11 of Donovan and Mickey on Bayesian Conjugates and Chapters 13, 15, and 16 on Markov Chain Monte Carlo — For understanding the theory of chapters 13, 15, and 16, I'd recommend you use my "Monte Carlo Methods - Why do They Work?" write-ups, linked above and my quick summary of **Problem Sets 14 to 18**

Week 15 — A Case Study Using Gibbs Sampling

- Tuesday, Dec. 17 — We will work through this **Monte Carlo Methods Case Study**

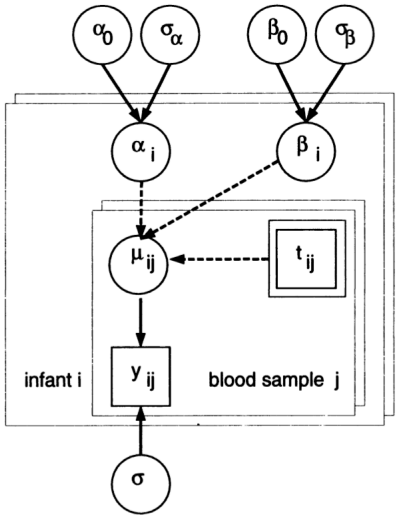

Figure 2.3 *Graphical model for hepatitis B data.*