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HW #1 Written

a)

- each iteration does $O(1)$
- $i = 2, 4, 16, 256, \ldots \quad i = 2^{(2^m)}$

$i < n$ or $2^{(2^m)} < n \quad = 2^m < \log(n)$

$= m < \log(\log(n))$ so $\boxed{T(n) = \theta(\log(\log(n)))}$

b)

Outer loop: $\sum_{i=1}^{n}$

- if statement is hit when i is a multiple of $m = \sqrt{n}$, $\frac{n}{\sqrt{n}} = \sqrt{n}$
- inner loop runs i^3 times when i is a multiple $m \cdot \sqrt{n}$
  - work: $\theta(i^3) = \theta(m^3 \sqrt{n}^3)$

$\rightarrow \sum_{m=1}^{\sqrt{n}} \theta(m^3 \sqrt{n}^3) \rightarrow \theta(\sqrt{n}^3) \sum_{m=1}^{\sqrt{n}} (m^3)$

ver. of the arithmetic series?

$= \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right)^2 = \theta(\sqrt{n}^4) = \theta(n^2)$

$\rightarrow \theta(\sqrt{n}^3) \cdot \theta(n^2) = \boxed{\theta(n^{\frac{7}{2}})}$

c)

$$\sum_{i=1}^{n} \sum_{k=1}^{n} (\theta(1) + O(\sum_{m=1}^{\log n} \theta(1))$$

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + \sum_{k=1}^{n} \sum_{m=1}^{\log n} \theta(1)$$

$$\sum_{i=1}^{n} \theta(n) + \sum_{k=1}^{n} \theta(\log(n)) \rightarrow \sum_{i=1}^{n} \theta(n) + \theta(n \log n)$$

$$\theta(n^2) + \underbrace{\theta(n * \log n)}$$

so [redacted] $T(n) = \theta(n^2)$

d)

- Outer loop happens n times $\sum_{i=1}^{n} \theta(1) = \theta(n)$
  - when $i == size$, array is resized
    - size grows like (0, 15, 22, 30, ... n) $\log(n)$ times

so outer loop + inner loop = $\underbrace{\theta(n)} + \theta(\log(n))$

So $\theta(n)$

2 a)

call 1 { in1 : 1 -> 2 -> 3 -> 4 -> nullptr
in2 : 5 -> 6 -> nullptr

2 { 1 : 5 -> 6 -> nullptr
2 : 2 -> 3 -> 4 -> nullptr

3 { 1 : 2 -> 3 -> 4 -> nullptr
2 : 6 -> nullptr

4 { 1 : 6 -> nullptr
2 : 3 -> 4 -> nullptr

5 { 1 : 3 -> 4 -> nullptr
2 : nullptr ✱

6 returns [5] : in1 so [3 -> 4 -> nullptr]

Then go back: [3 -> 4 -> nullptr] is after [6] is after [2]
is after [5] is after [1] so...

1 -> 5 -> 2 -> 6 -> 3 -> 4 -> nullptr

b)

call 1 { in1 : nullptr
in2 : 2 -> nullptr

2 { ~~in1~~ :
~~in2~~ :

simply just returns 2 -> nullptr
(base case is reached)