Numerical study of the airflow over a high-altitude pseudo-satellite wing

PhD update

Carlo Brunelli

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PhD presentation after 1.5 years

Sections

Numerical Method implemented

The numerical method currently implemented are:

- Variational Multiscale Method (VMS)
- Streamline upwind Petrov–Galerkin (SUPG)

Different solution method are available for all of them

- Non-linear (NLIN)
- Linearized-Coupled (LC-VMS)
- Linearized-Segregated (LS-VMS)

Variational Multiscale Method

- Evolution of the SUPG
- Implict LES
- It does not need calibration
- Residual-based stabilization

Galkerkin formulation

Conservation of mass:

$$\nabla \cdot \vec{u} = 0 \tag{1}$$

Conservation of momentum:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} + \nabla p - \nu \Delta \vec{u} - f = 0 \tag{2}$$

Variational Formulation

$$\begin{aligned} B^G = \int_\Omega \frac{\partial \vec{u}}{\partial t} \cdot \vec{v}\, d\Omega + \int_\Omega (\vec{u} \cdot \nabla)\vec{u} \cdot \vec{v}\, d\Omega + \int_\Omega \nabla(p) \cdot \vec{v}\, d\Omega + \\ \int_\Omega \nu \nabla \vec{u} \cdot \nabla \vec{v}\, d\Omega - \int_\Omega f \cdot \vec{v}\, d\Omega + \int_\Omega q(\nabla \cdot \vec{u})\, d\Omega = 0 \end{aligned} \tag{3}$$

Stabilization equations

$$B^{SUPG}(t,(\vec{u},p),(\vec{v},q)) = \int_{\Omega} (\tau_m(\vec{u}\cdot\nabla\vec{v} + \nabla q)\cdot \vec{R_m}\, d\Omega + \int_{\Omega} \tau_c(\nabla\cdot\vec{v})R_c\, d\Omega \tag{4}$$

$$B^{VMS1}(t,(\vec{u},p),(\vec{v},q)) = \int_{\Omega} (\vec{u}\cdot\nabla\vec{v}') \odot (\tau_m \vec{R_m})\, d\Omega \tag{5}$$

$$B^{VMS2}(t,(\vec{u},p),(\vec{v},q)) = -\int_{\Omega} (\nabla\vec{v} \odot (\tau_m \vec{R_m} \otimes \tau_m \vec{R_m})\, d\Omega \tag{6}$$

Stabilization parameters

$$\tau_m = \left(\frac{4}{\Delta t^2} + \vec{u} \cdot G\vec{u} + C_I \nu^2 G : G \right)^{-1/2} \tag{7}$$

$$\tau_c = \left(\tau_c \vec{g} \cdot \vec{g} \right)^{-1} \tag{8}$$

Where G is the inverse of the gradient of the map cell. For a cubed shaped element, with h the edge length, $G_{ij} = \frac{1}{h^2}\delta_{ij}$, where δ_{ij} is the Kronecker delta.

Linearization

1 Numerical Methods

$$\tilde{\vec{u}} = 2.1875u^{n} - 2.1875u^{n-1} + 1.3125u^{n-2} - 0.3125u^{n-3} \tag{9}$$

$$(\vec{u} \cdot \nabla)\vec{u} \Rightarrow (\tilde{\vec{u}} \cdot \nabla)\vec{u} \tag{10}$$

Boundary Layer Initialization

- Avoid Instabilities (close to the leading edge)
- Avoid velocity ramping
- Higher time-step

Using the function exploited by RANS solvers for the wall distance for computing turbulent parameters but now used to detect the airfoil's contour

$$\begin{cases} \nabla \cdot (|\nabla u_p|^{p-2} \nabla u_p) = -1 & x \in \Omega \\ u_p = 0 & x \in \Omega_D \end{cases} \tag{11}$$

The true-wall distance is provided by solving the equation for $p \to \infty$ Solved using a method that resembles Picard's

The velocity in the x direction in the region identified can be with a simple cubic function (12) where $dn = d/\delta_{99}$, d is the minimum distance to the airfoil, u_∞ is the free-stream flow speed.

$$f(dn) = \begin{cases} u_\infty & dn > 1 \\ (-dn^2 + 2 \cdot dn) \cdot u_\infty & dn < 1 \end{cases} \tag{12}$$

The boundary layer function has been obtained by fixing the following boundary conditions:

- Continuity with the external flow, $f(1) = 1$
- Smooth transition between boundary layer and external flow, $f'(1) = 0$
- Non-slip condition at the wall, $f(0) = 0$

Boundary Layer Initialization

It results in a low-speed zone close to the airfoil, avoiding high speed in really small cells useful for capturing the boundary layer.

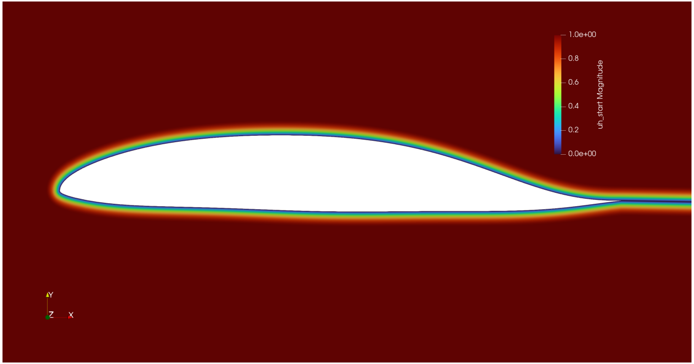


Figure: Boundary Layer Initialization

Sections

2 Performances

Weak Scaling

2 Performances

Weak scalability: the solution time almost does not change with constant problem size per processor

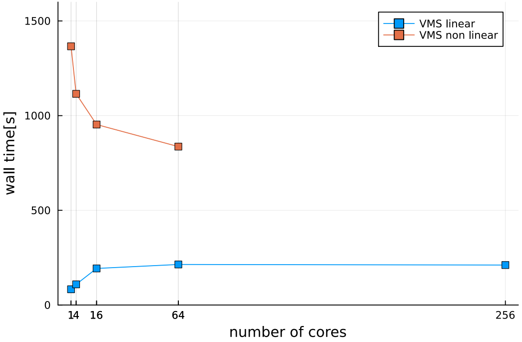


Figure: Taylor Green weak scalability

Strong Scaling

2 Performances

Strong scalability: doubling the number of processors halves the solution time

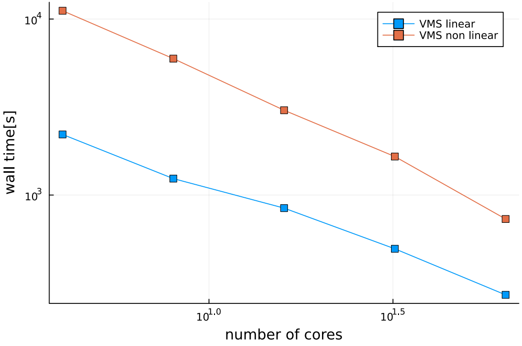


Figure: Taylor Green Strong scalability

Sections

SyntheticEddyMethod.jl

- publication
- presented at *JuliaCon2023* at MIT

Features:

- Create fluctuations that respect the divergence-free condition (DFSEM)
- Create velocity fluctuations for inlet boundary conditions
- Create coherent eddies in 3D domain
- Define custom Reynolds Stress Tensor
- Import from file custom Reynolds Stress Tensor

Synthetic Eddy Method

Reynolds decomposition:

$$\vec{u}(\vec{x},t) = \vec{U}(\vec{x},t) + \vec{u'}(\vec{x},t) \tag{13}$$

Compute velocity fluctuations, using a suitable shape function:

$$u_i(\boldsymbol{x}) = U_i(\boldsymbol{x}) + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} a_{ij} \epsilon_j^k f_{\sigma(\boldsymbol{x})} \left(\boldsymbol{x} - \boldsymbol{x}^k \right) \tag{14}$$

Sections

4 Laminar Separation Bubble

Simulations

- VMS linearized coupled sd7003s - Re $60\,000$ - AoA 4°
- VMS linearized segregated DU89 - Re $250\,000 - 500\,000$ - AoA $1^{\circ} - 5^{\circ}$

Models

4 Laminar Separation Bubble

sd7003s - Re 60 000 - AoA 4°

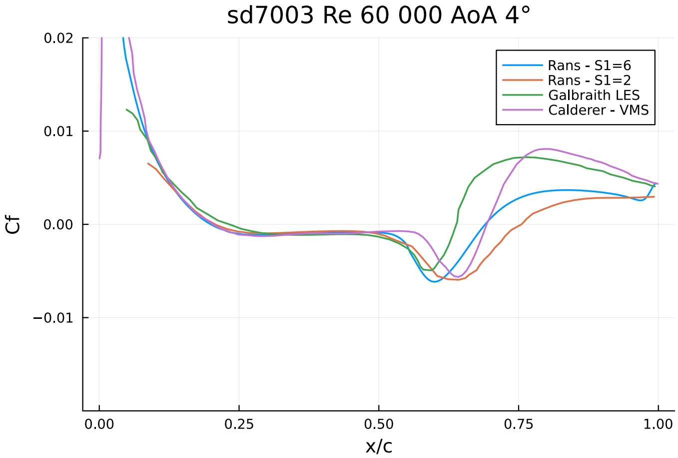


Figure: Different model provides different results. RANs use $\gamma - Re_\theta$ model.

VMS linearized coupled sd7003s

Copuled: velocity and pressure are solved at the same time. Re 60 000 - AoA 4° . Initialization with velocity-ramping

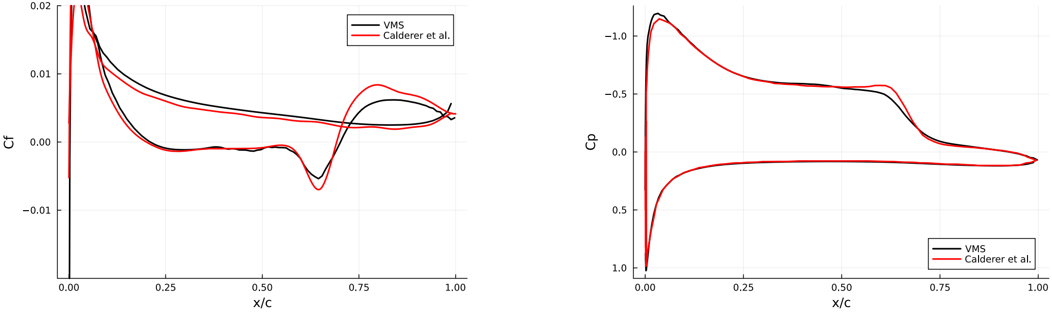


Figure: Comparison with VMS literature results

VMS linearized coupled sd7003s

4 Laminar Separation Bubble

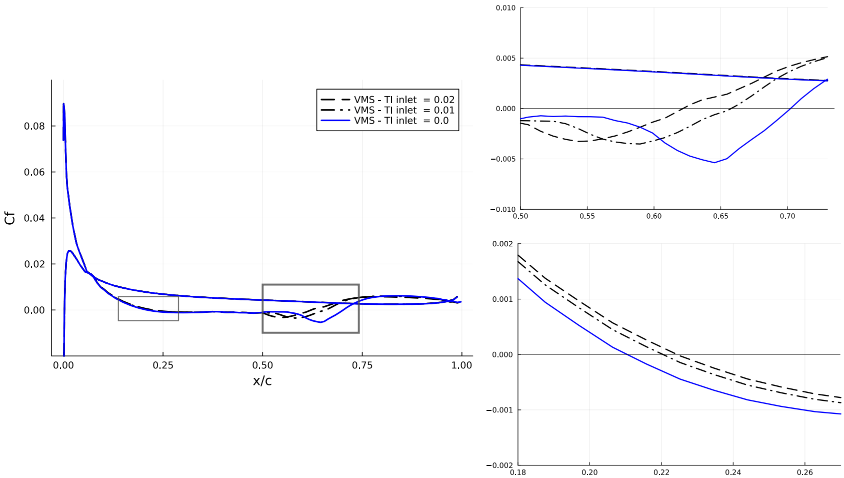
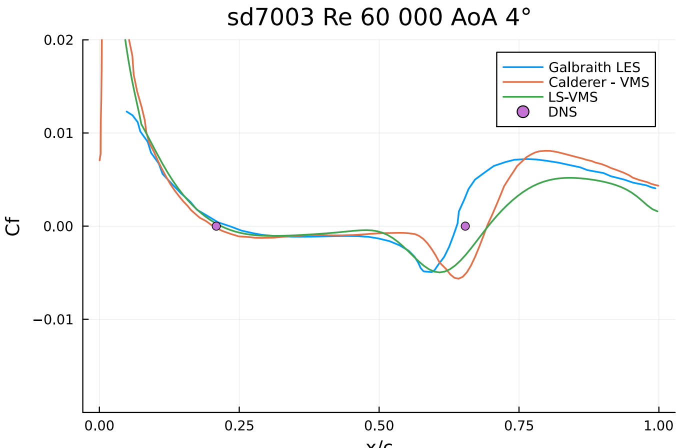


Figure: Bubble position function of freestream turbulence intensity

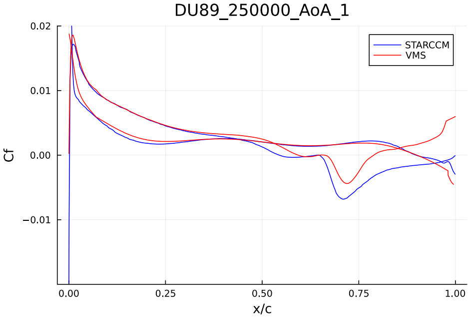
Segregated: each time step pressure and velocity system are solved one after the other multiple times. It is possible to re-use the matrices and preconditioner. It is an iterative method.

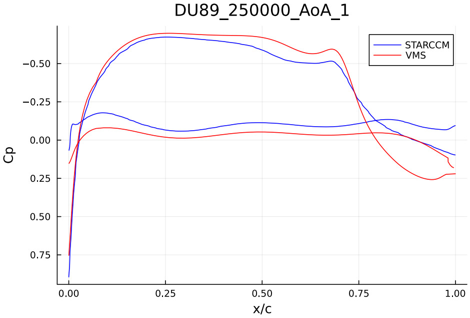


VMS Linearized-Segregated

4 Laminar Separation Bubble

PhD research aims to simulate a new airfoil. Re 250 000 - AoA 1°

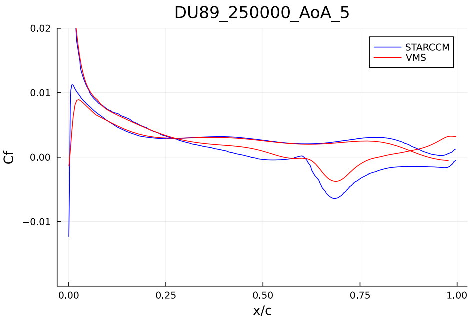


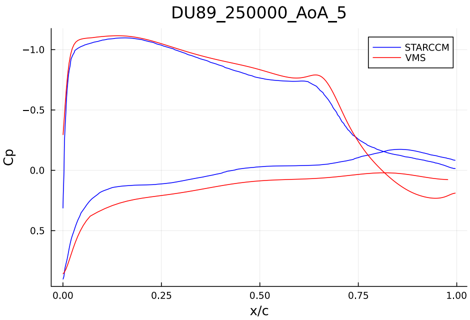


VMS Linearized-Segregated

4 Laminar Separation Bubble

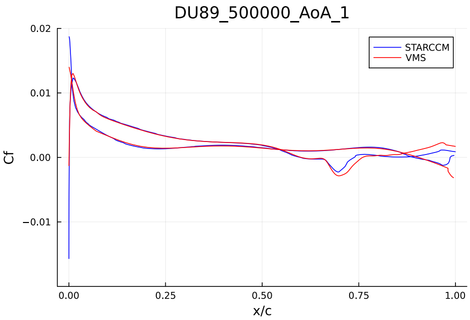
Re 250 000 - AoA 5°

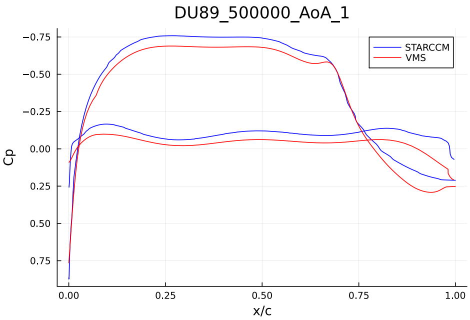




VMS Linearized-Segregated

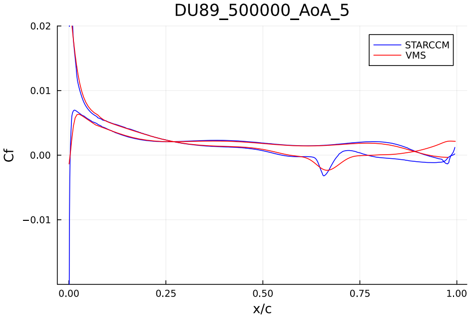
Re 500 000 - AoA 1°

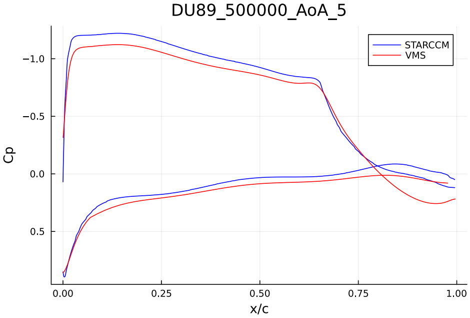




VMS Linearized-Segregated

Re 500 000 - AoA 5°





Mesh Sensitivity

Mesh settings	$\mathcal{C}$	$\mathcal{M}$	$\mathcal{F}$	$\mathcal{SF}$
Airfoil divisions	150	200	200	300
Z divisions	12	16	22	16
First cell height [m]	4.8e-6	2.8e-6	1.6e-6	1.6e-6
Number of Cells	4.1e5	6.6e5	8.5e6	8.9e6
CL	0.3539	0.3514	0.3504	0.3519
CD	0.00915	0.00950	0.00929	0.00947
Separation (x/c)	0.60	0.60	0.60	0.60

Table: Mesh sensitivity analysis DU89, Reynolds 500 000, Aoa 1°

Time Sensitivity

4 Laminar Separation Bubble

Mesh	$\mathcal{C}$	$\mathcal{C}$	$\mathcal{M}$	$\mathcal{M}$
Time average[s]	10	20	10	20
CL	0.3538	0.3539	0.3516	0.3514
CD	0.00910	0.00915	0.00939	0.00950
Separation (x/c)	0.60	0.60	0.60	0.60

Table: Time average sensitivity analysis DU89, Reynolds 500 000, Aoa 1°

dt$[s]$	$2\ 10^{-3}$	$1\ 10^{-3}$	$5\ 10^{-4}$
CL	diverged	0.3514	0.3511
CD	diverged	0.00950	0.00911
Separation (x/c)	diverged	0.60	0.60

Table: Time sensitivity analysis DU89, Reynolds 500 000, Aoa 1°

Royal Military Academy

Sections

5 Uncertainty Quantification

Model Variables

- TI - turbulence intensity
- μr - turbulent viscoity ratio

$k - \omega$ parameters:

- $\sigma\omega 1$
- $\alpha 1$
- $\beta *$

γRe_θ parameters

- $s1$
- $C1$

Sobolov Indexes

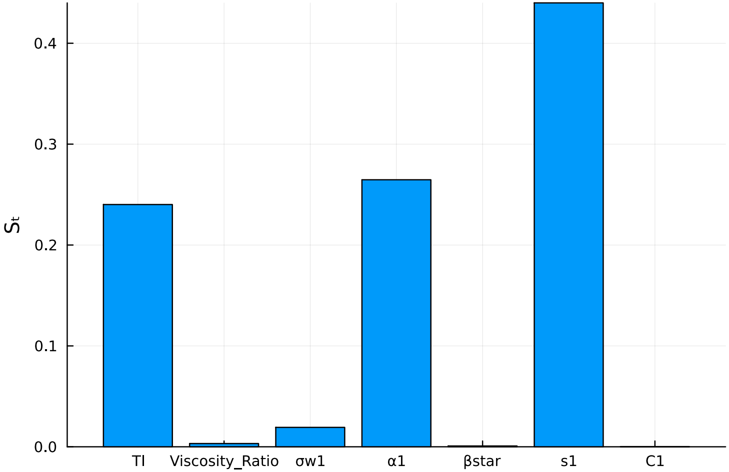


Figure: Sobol' indices for 3D RANS variables

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6 Other activities

Other activities

- Synthetic Eddy Method publication
- JuliaCon 2023 conference
- Co-author AIAA 2023 conference paper
- Testing standard passive flow controls (no improvement in aerodynamic efficiency)

Sections

Elements of novelty

- Systematic usage of Julia in fluid-dynamics
- Usage of VMS for high Reynolds airfoil
- First LES code in Julia - fully parallelized - working with 3D airfoils up to Re 500 000
- Synthetic Eddy Method coded in Julia and coupled with the VMS

It has been challenging, but it seems we are on the right path!

Sections

8 Next steps

Next steps

Expected papers:

- VMS paper (prof. Janssens reading)
- Experimental validation of the LS-VMS - publish paper
- LC-VMS - (publish paper?)

Expected conferences:

- AIAA2024 conferences (LS-VMS, LC-VMS)
- DLES14 (VMS usage, validation test cases)
- ICAS2024 (Passive Flow Controls)

Expected research:

- Uncertainty Quantification using Polynomials Chaos Transformation on $\gamma - Re_\theta$ and VMS
- Start coding the adjoint optimization
- Test a passive flow control with the VMS