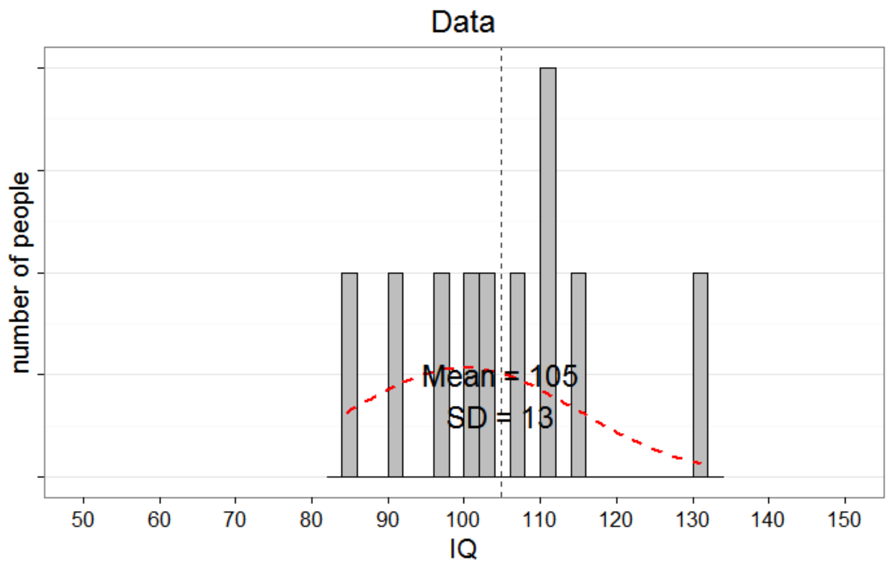
Random Variation and Power Analysis

People find it difficult to think about random variation. Our mind is more strongly geared towards recognizing patterns than randomness. In this assignment, we will learn what random variation looks like, and how to reduce variation by running well-powered studies.

Variation in single samples

Intelligence tests have been designed such that the mean Intelligence Quotient of the entire population of adults is 100, with a standard deviation of 15. This will not be true for every sample we draw from the population. Let's get a feel for what the IQ scores from a sample look like. Which IQ scores will people in our sample have?

We will start by manually calculating the mean and standard deviation of a random sample of 10 individuals. Their IQ scores are: 91.15, 86.52, 75.64, 115.72, 95.83, 105.44, 87.10, 100.81, 82.63, and 106.22. If we sum these 10 scores and divide them by 10, we get the mean of our sample: 94.71. We can also calculate the standard deviation from our sample. First, we subtract the overall mean (94.71) from each individual IQ score. Then, we square these differences and then sum these squared differences (giving 1374.79). We divide this sum of the squared difference by the sample size minus 1 (10-1=9), and finally take the square root of this value, which gives the standard deviation: 12.36. Open Simulate One Group.R, select all the code, and run it, to randomly simulate 10 IQ scores, and plot them.



Above is one example (yours will differ). The grey bars indicate the frequency with which each IQ score was observed. The red dotted line illustrates the hypothetical normal distribution for the population (we will return to this later). Both the mean, as well as the standard deviation, differ from the true mean of the population. Write down the mean and standard deviation in your sample. Simulate 9 more samples of 10 individuals. Write down the mean and standard deviation from each simulated group.

Q1) What is the difference in IQ points between the highest and the lowest mean from your 10 simulations?

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Imagine we did not yet know what the mean IQ was in our population (where M = 100), or the standard deviation (where SD = 15), and that we would only have access to one dataset. Our estimate might be rather far off. This type of variation is perfectly normal in small samples of 10 participants, given the current standard deviation.

Q2) The variability in the means is determined by the standard deviation of the measurement. In real life, the standard deviation can be reduced by for example using multiple and reliable measurements (which is why an IQ test has not just one question, but many different questions). It's not always possible to reduce the standard deviation in the real world, but it is possible in our simulation. Change the *sd* = 15 in line 7 to *sd* = 2. And simulate 10 new samples. What happens?

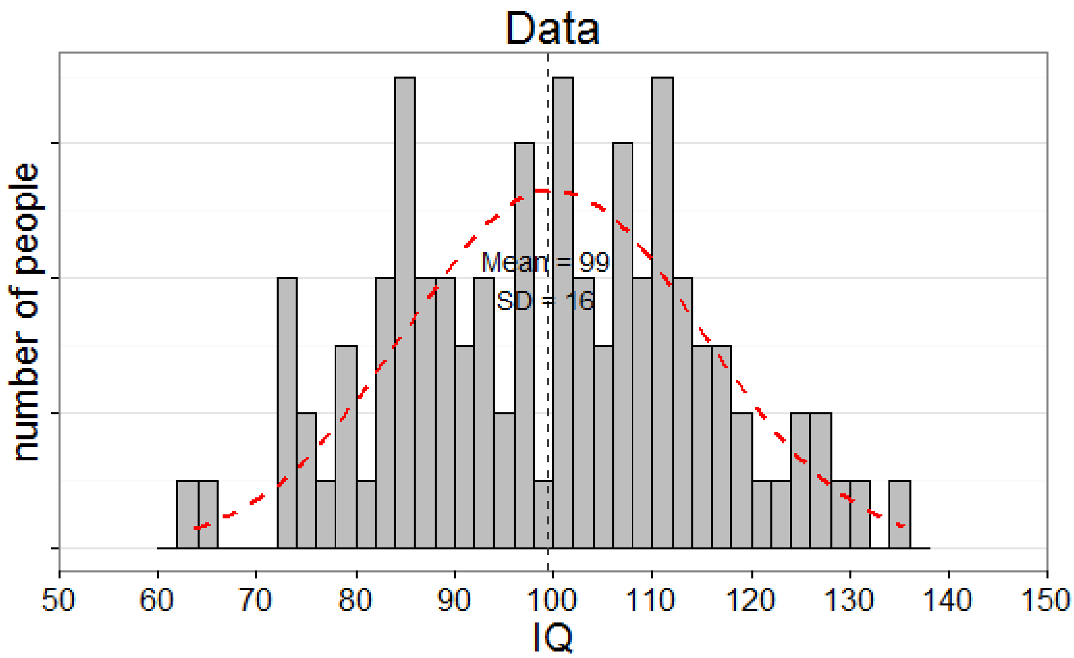
A) There is no difference between the **means** when *sd* = 15 compared to when *sd* = 2.

B) There is no difference between the **standard deviations** when *sd* = 15 compared to when *sd* = 2.

C) With *sd* = 2, the variation in means has **increased** substantially compared to *sd* = 15.

D) With *sd* = 2, the variation in means has **decreased** substantially compared to *sd* = 15.

Change the *sd* = 2 in line 7 back to *sd* = 15. Let's simulate a larger sample, of 100 participants by changing the *n* = 10 in line 6 of the R script to n=100. One sample is plotted below. We are slowly seeing what is known as the **normal distribution** (and the frequency scores start to resemble the red dotted line illustrating the normal distribution of the population). This is the well-known bell shaped curve that represents the distribution of many variables in scientific research (although some other types of distributions are quite common as well). The mean and standard deviation are much closer to the true mean and standard deviation, and this is true for most of the simulated samples.



Q3) Simulate at least 10 samples with n = 10, and 10 samples with n = 100. Look at the distribution of the data you have simulated. Which statement below is true?

A) With small samples, it is very clear the data does not come from a population where IQ scores are normally distributed.

B) With small samples, it is very clear the data comes from a population where IQ scores are normally distributed.

C) The data always come from a population where IQ scores are normally distributed, but this is very difficult to see, especially when n = 10, but sometimes also when n = 100.

D) The data always come from a population where IQ scores are normally distributed. This can easily be seen when we compare the true population normal distribution (the red dotted line) against the data.

Let's simulate really large samples of 1000 people (run the code a number of times, setting n=1000 in line 6). Not every simulated study of 1000 people will yield the true mean and standard deviation, but it will happen quite often. And although the distribution is very close to a normal distribution, even with 1000 people it is not perfect.

Planning for Accuracy

As we have seen above, the accuracy with which you can estimate the IQ of a population depends on the sample size. It is easy to calculate the sample size you need for a given level of accuracy when you know the standard deviation and the percentage of means that should fall within a desired range. Let's assume you want 95% of the results to estimate the mean IQ within an error range of 2 IQ points. You first convert the 95% confidence interval (see the assignment on confidence intervals) to a *z*-score ($z = 1.96$), and use the formula:

$$N = (Z * SD/error)^2$$

In this example, $(1.96*15/2)^2 = 216$ people (rounded down). Feel free to check this number by running the code with n = 216 100 times. In the long run, only 5% of the time will the mean IQ from the sample fall outside the desired accuracy range of 98-102. In your 100 simulations, it might happen fewer or more often, but in thousands of studies, it will end up being 5%!

Variation in *p*-values, statistical power, and a-priori power analysis

Earlier, we learned how *p*-values are a function of the power of the study. You have seen the p-value distribution when simulating large numbers of studies, but here we will take a look at studies at an individual level, to more directly experience their variation. First, take 11 minutes to look at Geoff Cumming's video Dance of the *p*-values. He very clearly explains how *p*-values vary substantially, even when you repeat the experiment in exactly the same manner.

Regardless of the statistics you calculate (effect sizes, likelihoods, confidence intervals, or *p*-values), we always just have the means, standard deviations, and the sample size to calculate with. All the different statistics vary to the same extent, because they are based on the same data. What we often want, is to limit the part of the *p*-value distribution where *p*-values will vary. In power analysis, we calculate the sample size that is required to reach a certain probability of observing a *p*-value smaller than our chosen alpha level. The *p*-values will still vary, but they will remain below our significance threshold, most of the time. Indeed, is we design well-powered experiments, we can turn the dance of the *p*-values into the march of the *p*-values. The goal is to have one after the other *p*-values behave in a consistent and predictable manner, with respect to our significance threshold.

Open MarchOfThePvalues.R. The default setting simulates an independent *t*-test, with two groups of IQ scores that differ by 6 IQ points (lines 14 and 15). Because the standard deviation is 15, the standardized effect size Cohen's $d = 0.4$. With 50 participants in each condition, we have approximately 50% power with an alpha of 0.05. The simulation generates one study at a time, and tell you the *p*-value for each study. It also includes one of 5 emoticons:

:(for p-values larger than 0.10

(._.) for p-values between 0.10 and 0.05

(^.^) for p-values between 0.01 and 0.05

:) for p-values between 0.001 and 0.01, and

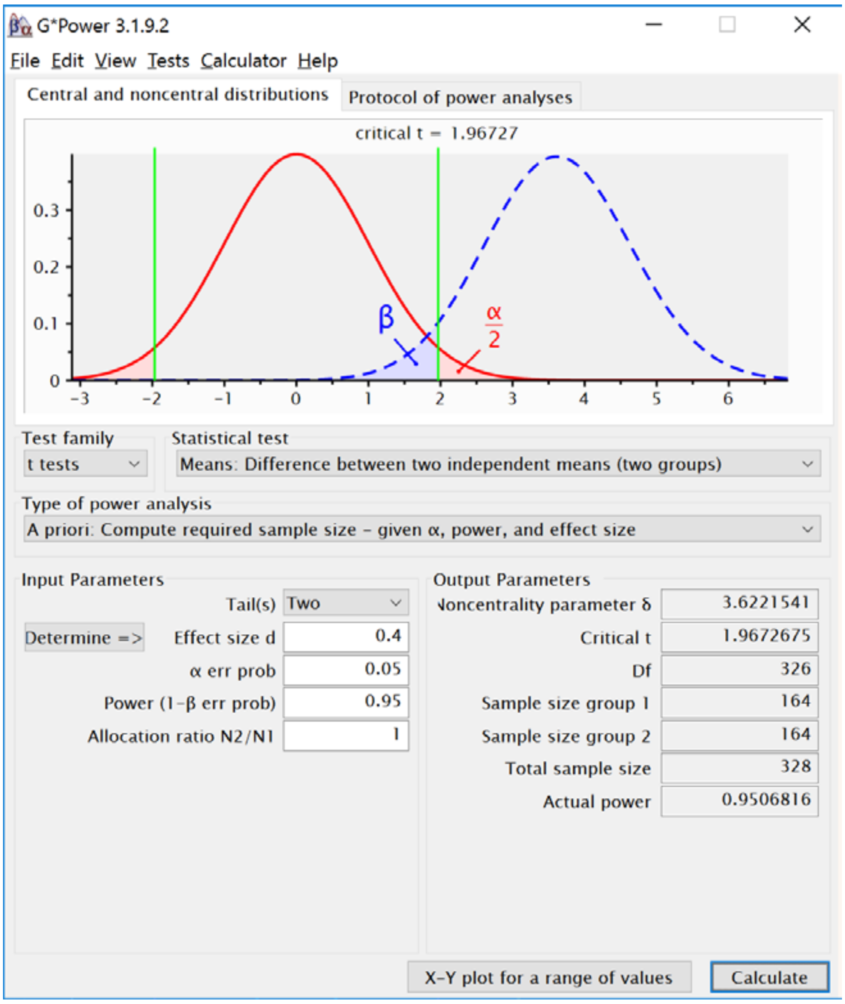
:D for p-values below 0.001.

Select all the lines, and run the script. The *p*-values for 100 simulated studies will appear, one every 0.5 seconds. Look at the range of *p*-values, and get a good feel for the randomness in studies with 50% power. Now let's turn this dance with a lot of variability in a march. To achieve this, we need to determine the sample size that will reduce the likelihood of observing *p*-values > 0.05. The way to choose this sample size is through **power analysis**.

Statistical power is a function of the effect size, the alpha level, and the sample size. The true effect size is what you measure, so you can't determine it yourself. The alpha level is typically set to 0.05. That means you can only directly influence the sample size, if you want to achieve a specific power. Therefore, in what is known as a-priori power analysis, you determine the sample size required to achieve a certain level of statistical power, given an alpha level, and assuming a certain effect size.

In our simulation, we know the effect size (we will discuss different ways to estimate the effect size when it is not known later).

Widely used free software for power analysis is G*Power software, which you can download from here: http://www.gpower.hhu.de/. Install G*Power. Let's calculate the sample size we need to achieve 95% power for our independent *t*-test where the true effect size is $d = 0.4$, and where we use an alpha level of 0.05. In all the drop-down menus (Test family, statistical test, Type of power analysis, tails), follow the selections in the screenshot below. Fill in 0.4 in the 'Effect size d' field, and 0.95 in the 'Power (1-β err prob)' field. Then click 'calculate; to get the same power calculation:



We see we need 164 people in each group (so 328 people in total) to achieve 95% power with an effect size of 0.4. Let's check this in the MarchOfThePvalues.R script. In line 8, change n<-50 into n<-164. Select all lines, and run the simulation.

You should see a lot of **:D**, **:)**, and **(^.^)**. In the 100 simulations, there will be some non-significant results – there will be 5% in the long run, with 95% power. Note that dividing *p*-values in 5 categories makes it look like there is still some variation, but from a Neyman-Pearson perspective, we are really only interested in a distinguishing signal from noise, or happy faces from sad faces, and we want to limit the sad faces to 5%, in the long run, when there is a true effect. If we had programmed the simulation with just a dichotomous output ☺ and ☹, the result would be very repetitive. This is why you want to design high-powered studies: They provide informative results. But they also require a larger sample size.

Q4) We never know the true effect size in real studies. One approach people often use in a-priori power analyses is to set the smallest effect size of interest. For example, using the benchmarks by Cohen, we know $d = 0.2$ is a small effect. We might decide that if we do not observe an effect size of 0.2 or larger, we are not interested in the effect. Use G*power to calculate how many participants we need in each group, in an independent *t*-test, with an alpha level of 0.05, to get 0.95 power to observe an effect size of $d = 0.2$.

A) 51 C) 290

B) 242 D) 651

A second way in which people often determine the effect size, is by calculating the effect sizes observed in a previous study, and using these to determine the effect size in a follow-up experiment. First, download my effect size calculation spreadsheet. You can use it to fill in the test statistics from an article, and it will calculate the effect sizes it can. Go to https://osf.io/wgsi3/ and download the From_R2D2 spreadsheet.

Let's take a look at the 'louder=closer' effect, from [Zhang, Lakens, & IJsselsteijn, 2015](), who showed that participants estimated their conversation partner (who they communicated with over the telephone or over Skype) was closer, when they heard their voice louder (i.e., the volume of the telephone or computer was louder).

1.2. Results and discussion

The data of two participants were excluded based on pre-defined criteria. One participant marked the location of the train station based on the content of the call. The other judged the location as the building of the experimenter's faculty. This left 20 participants in the softer voice condition, and 22 participants in the louder voice condition. The manipulation check confirmed that participants perceived the voice intensity

in the softer condition as softer ($M_{\text{softer}} = 2.75$, $SD_{\text{softer}} = 1.12$) than in the louder condition ($M_{\text{louder}} = 4.00$, $SD_{\text{louder}} = 1.38$), $t(40) = 3.21$, $p = .003$; Cohen's $d = 0.99$. Distance judgments were computed by measuring the Euclidean distance (in centimeters) between participants' marked locations and the library. As expected, participants judged the caller's location to be nearer if they heard a louder compared to a softer voice ($M_{\text{softer}} = 9.93$, $SD_{\text{softer}} = 3.61$; $M_{\text{louder}} = 7.34$, $SD_{\text{louder}} = 3.77$; $t(40) = 2.27$, $p = .029$; Cohen's $d = 0.70$, 95% CI [0.07, 1.32]).[2] On the contrary, the intensity manipulation did not result in significant differences on other measured dimensions (quality, fluency, liking and familiarity). The results provide initial support for the "louder as closer" effect.

Q5) Fill in the statistics for the main effect of caller location as a function of the loudness of the voice over the telephone, $t(40) = 2.27$. Check the effect size calculation using the From_R2D2 spreadsheet if we calculated the reported effect size correctly. You can fill in

N = 42 (for a t-test, N is the degrees of freedom + 2) and t = 2.27. Note that we reported Cohen's d for the sample (in the spreadsheet, indicated by Cohen's d_s). If you want to repeat this study with 90% power, use G*power to calculate how many participants you would you need in each group, assuming the Cohen's d of 0.7 is the true effect size.

A) 24　　C) 44

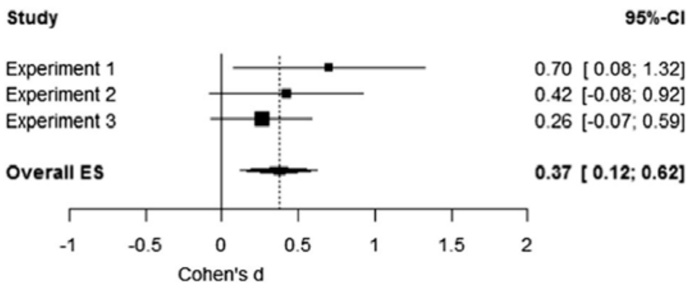
B) 36　　D) 55

Q6: For power analyses, it is best to use unbiased effect size estimates. Redo the power analysis, now using Hedges' g, which you can also get from the From_R2D2 spreadsheet. If you want to repeat this study with 90% power, how many participants would you need in each group?

A) 26　　C) 66

B) 46　　D) 86

Q7: It is very risky to base the power analysis on a single study. Due to publication bias, single studies often overestimate the true effect size, sometimes substantially. A third approach is to use the effect size from a meta-analytic effect size estimate. As it turns out (and as is often the case), the effect size for the loudness = closer effect observed by Zhang, Lakens, & IJsselsteijn in Study 1 was overestimated. After three studies, the authors perform a meta-analysis. Below, we see three lines with the effect sizes and confidence intervals for each study. On the bottom line, we see the overall meta-analytic effect size of 0.37.



Based on an a-priori power analysis, how many participants would you need in each group if you wanted to achieve 90% power? (Note that formally, Hedges' g would be better again, but with very large samples or meta-analytic estimates, the difference is often too small to matter).

A) 59　　C) 126

B) 111　　D) 155

Q8) Let's try a more complex example, where we calculate the power for a repeated measures design. In a repeated measures design, [Lakens (2013)] asked students from the TU Eindhoven to measure their heart rate with their smartphone. Students took three measurements: A baseline measurement, one measurement while they were thinking back to a time they were feeling happy, and once while they thought back to a time when they were sad. The results were as follows (look at the main effect of emotion):

> There was a clear main effect of emotion, $F(2, 132) = 23.89$, $p < .001$, $\eta_p^2 = .27$. HR was higher while recalling the anger ($M = 72.06$, $SD = 12.11$) and happiness ($M = 69.00$, $SD = 12.17$) emotions, compared to the baseline measurement ($M = 64.59$, $SD = 10.33$), $t(67) = 7.11$, $p < .001$, $\eta_p^2 = .43$ and $t(67) = 5.34$, $p < .001$, $\eta_p^2 = .30$, respec-

For ANOVA designs, the effect size needed to perform a power analysis is partial eta squared or η_p^2. This is reported in the result section, but we again should prefer to use the less biased effect size estimates ω_p^2 and ε_p^2. An easy to use spreadsheet to calculate these less biased effect size estimates is available from https://osf.io/sjgv4/. Enter the degrees of freedom for the effect (2) and error (132) and the *F*-value (23.89) in the From_R2D2 spreadsheet.

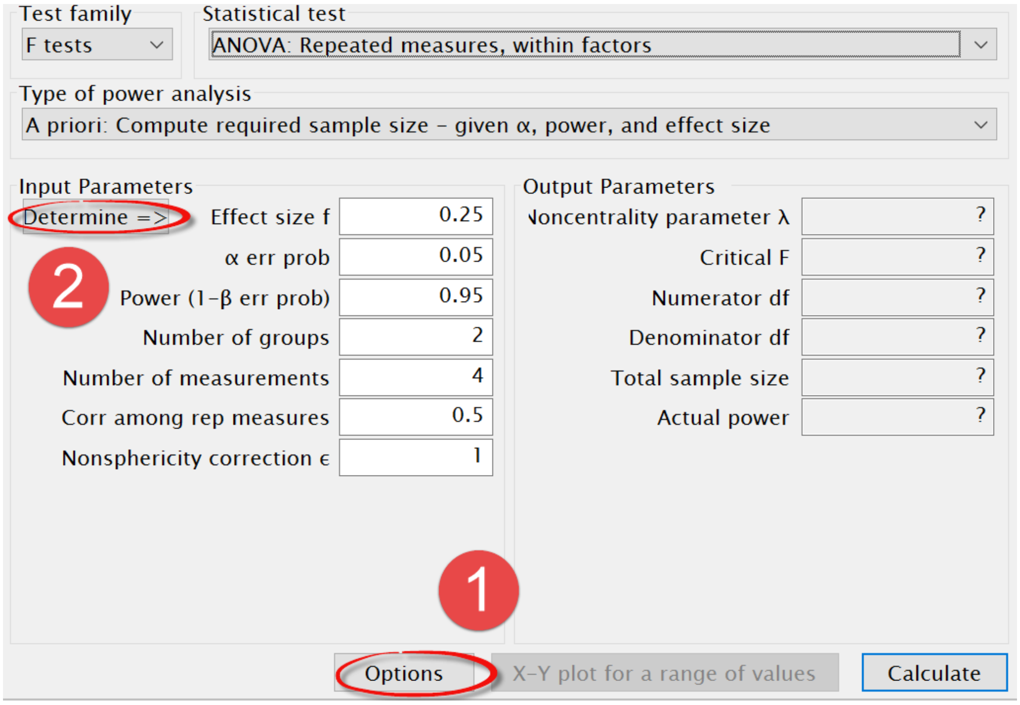
Q8) The η_p^2=0.27. What is the ω_p^2?

A) 0.24 C) 0.26

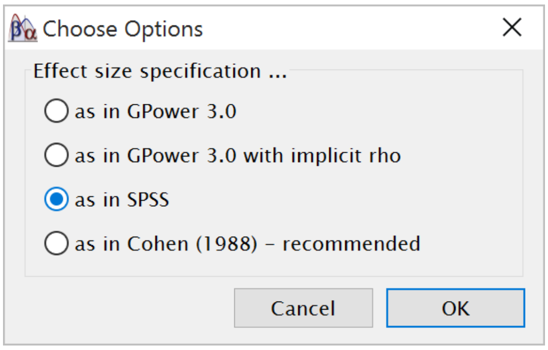
B) 0.25 D) 0.27

It is better to use unbiased effect size estimates in power analyses, so we will use ω_p^2, even though G*Power asks for η_p^2. To perform a power analysis in G*power for the repeated measures interaction, from test family, choose F-tests, from Statistical test choose: ANOVA: Repeated Measures, Within Factors (after all, the three emotion measurements for Baseline, Happy, and Angry were taken within subjects) and set the type of power analysis A priori. First do two things:

G*Power needs the effect size f (which is Cohen's f). We can calculate this from partial eta squared (or ω_p^2), but in a within design, there is a tricky issue: G*power by default expects a different calculated partial eta-squared than software programs such as SPSS and the spreadsheet provide. To tell G*power we will be using the (arguably) default version of partial eta squared (or ω_p^2), first click the 'Options' button (see below)



Check the 'as in SPSS' checkbox. This tricky step is only required when using partial eta squared (or partial omega squared) for **within subject designs**, but then it is very important not to forget it.



Then click 'Determine' (point 2 above). In the window that opens, click the 'Direct' radio button and type in ω_p^2 (rounded to 2 digits after the decimal) in the window. Click 'Calculate' to get the Effect size f(U) of 0.577, and click Calculate and transfer this f to the main window. You can now fill in the remaining numbers. The 'α error prob' is fine at 0.05. Set power to 90%, the number of groups to 1 (we have one group, and everyone does all three measurements), set the number of measurements to 3, and you can ignore the nonsphericity correction. Click 'calculate' to get a total sample size to achieve 90% power with an alpha of 0.05.

Q9) Which sample size is required to achieve 90% power?

A) 14

B) 22

C) 42

D) 58

We've seen there can be a lot of variation in single studies. You can plan the sample size to achieve a certain accuracy, or to achieve a certain probability of observing a $p < 0.05$ in an a-priori power analysis. It is very important that, regardless of the approach you use, you carefully consider the sample size you will collect. Power analysis is one approach, but remember effect size estimates from the published literature are often upwardly biased. Some journals now expect researchers to explicitly justify their sample size when they submit their research for publication, so make sure you know how to justify your sample size!