

# OI Templates

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# 1 C++ Syntax

## 1.1 Operator Overloading

Operator overloading is a powerful method that allows users to overload operators to perform different operations. We can use this to overload input and output of `std::vector` and `std::pair`.

```
// pair IO
template <class T, class V>
istream& operator>>(istream& is, pair<T, V>& obj) {
    is >> obj.first >> obj.second;
    return is;
}

template <class T, class V>
ostream& operator<<(ostream& os, const pair<T, V>& obj) {
    os << obj.first << ' ' << obj.second;
    return os;
}

// vector IO
template <class T>
istream& operator>>(istream& is, vector<T>& obj) {
    for (int i = 0; i < int(obj.size()); i++) is >> obj[i];
    return is;
}

template <class T>
ostream& operator<<(ostream& os, const vector<T>& obj) {
    for (int i = 0; i < int(obj.size()); i++) {
        os << obj[i]; if (i != int(obj.size()) - 1) os << ' ';
    }
    return os;
}
```

## 1.2 String Functions

To construct a string with a specific character of length n:

```
string str(n, chr);
```

To obtain a substring t from index i to j inclusively:

```
string t = s.substr(i, j - i + 1);
```

To find if a character exists in a string str:

```
if (str.find(chr) != string::npos) {
    cout << "Yes\n";
} else {
    cout << "No\n";
}
```

To access front element / back element of a vector:

```
vec.front();
vec.back();
```

To insert an element x in position idx into a vector:

```
vec.insert(vec.begin() + idx, x);
```

## 1.3 Regular Expression

## 2 Mathematics

### 2.1 Basic Modular Arithmetic

Modular arithmetic is very important especially when dealing with counting problems. Modular arithmetic is essential to solve them.

Here are some common identities.

- If  $a \equiv b \pmod{n}$ , then  $b \equiv a \pmod{n}$
- If  $a \equiv b \pmod{n}$ , and  $b \equiv c \pmod{n}$ , then  $a \equiv c \pmod{n}$
- $(a + b) \bmod m = (a \bmod m + b \bmod m) \bmod m$
- $(a - b) \bmod m = (a \bmod m - b \bmod m) \bmod m$
- $(a \times b) \bmod m = (a \bmod m \times b \bmod m) \bmod m$

### 2.2 Binary Exponentiation

Using this method, we can calculate  $a^b \bmod m$  in  $O(\log n)$

Iterative method:

```
long long bigmd(long long x, long long y, long long z) {
    long long ans = 1;
    while (y){
        if (y & 1) ans = ans * x % z;
        y >>= 1;
        x = x * x % z;
    }
    return ans;
}
```

Recursive method:

```
long long bigmd(int n, int p, int mod){
    if (p == 0) return 1;
    long long tt = bigmd(n, p / 2, mod);
    return (p & 1 ? (n % mod) : 1) * (tt * tt % mod);
}
```

### 2.3 Modular Inverse

Fermat's little theorem states that

$$a^p \equiv a \pmod{p}$$

where  $a$  is an integer and  $p$  is a prime number.

The formula can be also represented as

$$a^{p-1} \equiv 1 \pmod{p}$$

Normally, we call a number  $b$  as an inverse of  $a$  when  $a \times b = 1$ .

e.g. 0.5 is the inverse of 2.

Similarly, we call a number  $b$  as the modular inverse of  $a$  modulo  $n$  when  $a \times b \equiv 1 \pmod{n}$

e.g. 4 is the inverse of 2 modulo 7. ( $4 \times 2 \bmod 7 = 1$ )

If we recall the above formula, we can come into a conclusion that the modular inverse of  $a \bmod p$  is  $a^{p-2} \bmod p$  (for prime number  $p$  only).

Code example:

```
// use the above bigmd function
long long modinv(long long a, long long p){
    return bigmd(a, p - 2, p);
}
```

## 2.4 Matrix

Matrices can be useful in solving linear recurrences.

```
struct matrix {
    int row_size, col_size;
    vector<vector<long long>> container;
    matrix(int n, int m) : row_size(n), col_size(m), container(n, vector<long long>(m))
    {};
    matrix(vector<vector<long long>> mtx) : row_size(mtx.size()), col_size(mtx.size() ==
        0 ? 0 : mtx[0].size()), container(mtx) {};

    matrix& operator *= (const matrix& rhs) {
        if (col_size != rhs.row_size) {
            cerr << "This two matrices cannot be multiplied together.\n";
            exit(EXIT_FAILURE);
        }

        vector<vector<long long>> new_container(row_size, vector<long long>(rhs.col_size
            ));
        int common_side = col_size;
        for (int i = 0; i < row_size; i++) {
            for (int j = 0; j < rhs.col_size; j++) {
                long long result = 0;
                for (int k = 0; k < common_side; k++) {
                    result += container[i][k] * rhs.container[k][j];
                }
                new_container[i][j] = result;
            }
        }

        container = new_container;
        return *this;
    }

    friend matrix operator * (matrix lhs, const matrix& rhs) {
        lhs *= rhs;
        return lhs;
    }

    matrix& operator += (const matrix& rhs) {
        if (col_size != rhs.col_size || row_size != rhs.row_size) {
            cerr << "This two matrices cannot be added together.\n";
            exit(EXIT_FAILURE);
        }

        for (int i = 0; i < row_size; i++) {
            for (int j = 0; j < col_size; i++) {
                container[i][j] += rhs.container[i][j];
            }
        }

        return *this;
    }

    matrix operator + (matrix lhs, const matrix& rhs) {
        return lhs + rhs;
    }

    matrix& operator %= (const int mod) {
```

```

        for (int i = 0; i < row_size; i++) {
            for (int j = 0; j < col_size; j++) {
                container[i][j] %= mod;
            }
        }
        return *this;
    }

    friend matrix operator % (matrix lhs, const int mod) {
        lhs %= mod;
        return lhs;
    }
};

matrix power(matrix n, long long p, long long mod) {
    matrix curr = n;
    matrix result(vector<vector<long long>>({{1, 0}, {0, 1}}));
    while (p > 0) {
        if (p & 1) {
            result *= curr;
            result %= mod;
        }
        p >>= 1;
        curr *= curr;
        curr %= mod;
    }
    return result;
}

```

## 2.5 Extended Euclidean Algorithm

## 3 Data Structures

### 3.1 Disjoint-set Union

This data structure provides the following capabilities. We are given several elements, each of which is a separate set. A DSU will have an operation to combine any two sets, and it will be able to tell in which set a specific element is. The structure is very flexible as you can add different operations to it.

```
class disjoint_set_union {
    vector<int> parent; // ancestor
    vector<int> minn, maxx;
    vector<int> count;

    vector<bool> config;
public:
    disjoint_set_union(int length) {
        parent.resize(length + 1);
        iota(parent.begin(), parent.end(), 0);
    }

    disjoint_set_union(int length) {
        parent.resize(length + 1);
        iota(parent.begin(), parent.end(), 0);
        count.resize(length + 1, 1);
    }

    int find(int node) {
        return parent[node] == node ? node : find(parent[node]);
    }

    void joint(int x, int y) {
        int rootx = find(x), rooty = find(y);

        if (rootx != rooty) {
            parent[rooty] = rootx;
            count[rootx] += count[rooty];
        }
    }
};
```

### 3.2 Binary Indexed Tree / Fenwick Tree

```
class fenwick {
    // must be one-based
    vector<long long> bit;
    int size = 0;

public:
    fenwick(int length) {
        bit.resize(length + 1);
        this->size = length + 1;
    }

    void update(int pos, int val) {
        for (; pos < size; pos += pos & (-pos)) bit[pos] += val;
    }

    long long query(int pos) {
        long long ans = 0;
    }
```

```

        for (;pos > 0; pos -= pos & (-pos)) ans += bit[pos];
        return ans;
    }
};

```

### 3.3 Segment Tree

#### 3.3.1 Basic Segment Tree

```

template <class T>
class segment_tree {
private:
    int n;
    vector<T> tree;
    T operation(T left, T right) {
        return left + right;
    }

    void pushup(int node) {
        tree[node] = operation(tree[node + node], tree[node + node + 1]);
    }

    void build(const vector<T>& v, int curr, int lo, int hi) {
        if (lo == hi) {
            tree[curr] = v[lo];
        } else {
            int mi = lo + (hi - lo) / 2;
            build(v, curr + curr, lo, mi);
            build(v, curr + curr + 1, mi + 1, hi);
            pushup(curr);
        }
    }

    void set(int idx, int val, int curr, int lo, int hi) {
        if (lo == hi) {
            tree[curr] = val;
        } else {
            int mi = lo + (hi - lo) / 2;
            if (idx <= mi) {
                set(idx, val, curr + curr, lo, mi);
            } else {
                set(idx, val, curr + curr + 1, mi + 1, hi);
            }
            pushup(curr);
        }
    }

    T query(int l, int r, int curr, int lo, int hi) { // l, r is the target range!
        if (lo == l && hi == r) {
            return tree[curr];
        } else {
            int mi = lo + (hi - lo) / 2;
            if (r <= mi) {
                return query(l, r, curr + curr, lo, mi);
            } else if (l >= mi + 1) {
                return query(l, r, curr + curr + 1, mi + 1, hi);
            } else {
                return operation(query(l, mi, curr + curr, lo, mi), query(mi + 1, r,
                    curr + curr + 1, mi + 1, hi));
            }
        }
    }
};

```

```

    }
}

public:
    segment_tree(const vector<T>& v) : n(v.size()), tree(4 * v.size()) {
        build(v, 1, 0, n - 1);
    }

    void set(int idx, T val) {
        set(idx, val, 1, 0, n - 1);
    }

    T query(int l, int r) {
        return query(l, r, 1, 0, n - 1);
    }
};

```

### 3.3.2 Online Segment with Maximum Sum

```

class segment_tree {
    int _size = 0;
    int power2size = 1;
    vector<long long> v;
    vector<long long> pref, suf, sum;

public:
    segment_tree(int length) {
        _size = length;
        v.resize(4 * _size, 0);
        pref.resize(4 * _size);
        suf.resize(4 * _size);
        sum.resize(4 * _size);

        length--;
        while (length > 0) power2size <<= 1, length >>= 1;
    }

    void set(int l, int r, int pos, int idx, long long x, int p2s) {
        if (inRange(pos, pos + 1, l, r) == 2) {
            int mid = p2s >> 1;
            if (r - l != 1) {
                set(l, l + mid, pos, 2 * idx + 1, x, mid);
                set(l + mid, r, pos, 2 * idx + 2, x, mid);
                v[idx] = max(max(v[2 * idx + 1], v[2 * idx + 2]), suf[2 * idx + 1] +
                    pref[2 * idx + 2]);
                pref[idx] = max(pref[2 * idx + 1], sum[2 * idx + 1] + pref[2 * idx +
                    2]);
                suf[idx] = max(suf[2 * idx + 2], sum[2 * idx + 2] + suf[2 * idx +
                    1]);
                sum[idx] = sum[2 * idx + 1] + sum[2 * idx + 2];
            } else {
                v[idx] = max(0LL, x);
                pref[idx] = max(0LL, x);
                suf[idx] = max(0LL, x);
                sum[idx] = x;
            }
        }
    }
}

```



```

}

void set(int pos, long long x) {
    set(0, _size, pos, 0, x, power2size);
}

// current left, current right, target left, target right
long long ans(int l, int r, int lt, int rt, int idx, int p2s) {
    int checker = inRange(l, r, lt, rt);
    int mid = p2s >> 1;
    if (checker == 0) return 0;
    else if (checker == 1) return max(ans(l, l + mid, lt, rt, 2 * idx + 1, mid),
        ans(l + mid, r, lt, rt, 2 * idx + 2, mid));
    else return v[idx];
}

// [l, r)
long long ans(int l, int r) {
    return ans(0, _size, l, r, 0, power2size);
}

int size() {
    return _size;
}

friend istream& operator >> (istream& is, segment_tree& obj) {
    for (int i = 0; i < obj.size(); i++) {
        int x;
        is >> x;
        obj.set(i, x);
    }
    return is;
}

private:
    // 0 = completely out of range, 1 = partially in range, 2 = completely in range
    // current segment, target segment
    int inRange(int l, int r, int lt, int rt) {
        if (r <= lt || l >= rt) return 0;
        else if (l >= lt && r <= rt) return 2;
        else return 1;
    }
};

```

### 3.3.3 Lazy Segment Tree

```

template <class T>
class segment_tree {
private:
    int n;
    vector<T> tree, diff; // tree = real value, diff = modification value
    vector<bool> lazy; // lazy = is the current node lazied

    T operation(T left, T right) {
        return min(left, right);
    }

    void pushup(int node) {

```

```

    if (lazy[node]) {
        // un-lazy itself
        lazy[node] = false;
        // lazy its children
        tree[node + node] = tree[node + node + 1] = diff[node];
        diff[node + node] = diff[node + node + 1] = diff[node];
        lazy[node + node] = lazy[node + node + 1] = true;
    }
    tree[node] = operation(tree[node + node], tree[node + node + 1]);
}

void build(const vector<T>& v, int curr, int lo, int hi) {
    if (lo == hi) {
        tree[curr] = v[lo];
    } else {
        int mi = lo + (hi - lo) / 2;
        build(v, curr + curr, lo, mi);
        build(v, curr + curr + 1, mi + 1, hi);
        pushup(curr);
    }
}

void set(int l, int r, T val, int curr, int lo, int hi) {
    if (lo == l && hi == r) {
        tree[curr] = val;
        diff[curr] = val;
        lazy[curr] = true;
    } else {
        pushup(curr);
        int mi = lo + (hi - lo) / 2;
        if (r <= mi) {
            set(l, r, val, curr + curr, lo, mi);
        } else if (l >= mi + 1) {
            set(l, r, val, curr + curr + 1, mi + 1, hi);
        } else {
            set(l, mi, val, curr + curr, lo, mi);
            set(mi + 1, r, val, curr + curr + 1, mi + 1, hi);
        }
        pushup(curr);
    }
}

T query(int l, int r, int curr, int lo, int hi) {
    if (lo == l && hi == r) {
        return tree[curr];
    } else {
        pushup(curr);
        int mi = lo + (hi - lo) / 2;
        if (r <= mi) {
            return query(l, r, curr + curr, lo, mi);
        } else if (l >= mi + 1) {
            return query(l, r, curr + curr + 1, mi + 1, hi);
        } else {
            return operation(query(l, mi, curr + curr, lo, mi), query(mi + 1, r,
                curr + curr + 1, mi + 1, hi));
        }
    }
}

public:

```

```

segment_tree(const vector<T>& v) : n(v.size()), tree(4 * v.size()), diff(4 * v.size()) {
    build(v, 1, 0, n - 1);
}

void set(int l, int r, T val) {
    set(l, r, val, 1, 0, n - 1);
    // cout << '\n';
}

T query(int l, int r) {
    return query(l, r, 1, 0, n - 1);
}
};

```

### 3.4 Sparse Table

```

template <class T>
class sparse_table {
    int n, k;
    vector<vector<T>> table;

    T operation(T left, T right) {
        return left + right;
    }

    int log2_floor(unsigned long long i) {
        return i ? __builtin_clzll(1) - __builtin_clzll(i) : -1;
    }

    void build(const vector<T>& v) {
        // copy(v.begin(), v.end(), table[0]);
        for (int i = 0; i < n; i++) {
            table[0][i] = v[i];
        }
        for (int i = 1; i < k; i++) {
            for (int j = 0; j + (1 << i) <= n; j++) {
                table[i][j] = operation(table[i - 1][j], table[i - 1][j + (1 << (i - 1))]);
            }
        }
    }

public:
    sparse_table(const vector<T>& v) : n(v.size()), k(log2_floor(v.size()) + 1), table(k, vector<T>(v.size())) {
        build(v);
    }

    T query(int l, int r) {
        int size = r - l + 1;
        T ans = 0;
        for (int i = 0; i < k; i++) {
            if (size & 1) {
                ans = operation(ans, table[i][l]);
                l += 1 << i;
            }
            size >>= 1;
        }
    }
}

```

```

        return ans;
    }
};

```

### 3.5 Min Heap / Max Heap

```

template <class T>
using max_heap = priority_queue<T, vector<T>>>;

template <class T>
using min_heap = priority_queue<T, vector<T>, greater<T>>>;

```

### 3.6 Linked List

#### 3.6.1 Singly Linked List

To initiate a linked list:

```
list<int> lst(5, 0)
```

To insert element  $x$  to index  $idx$ :

```

auto it = lst.begin();
advance(it, idx);
lst.insert(it, x);

```

To erase element of index  $idx$ :

```

auto it = lst.begin();
advance(it, idx);
lst.erase(it);

```

#### 3.6.2 Doubly Linked List

## 4 String Algorithms

### 4.1 Hashing

```
struct custom_hash {
    int mod = 31;
    int mod2 = 1000003957;
    // 1000003957, 1000001957, 1000003469, 1000003283, 1000002431
    // 1000010611, 1000009739, 1000009567, 1000012253, 1000011421
    void prefixHash(vector<long long>& dest, string& s) {
        dest.resize(s.size());
        dest[0] = s[0] - 'a';

        for (int i = 1; i < s.size(); i++) {
            dest[i] = dest[i - 1] * mod % mod2 + (s[i] - 'a');
            dest[i] %= mod2;
        }
    }

    void prefixHashSize(vector<long long>& dest, string& s, int size) {
        dest.resize(s.size());
        dest[0] = s[0];

        long long power = bigmd(mod, size, mod2);

        for (int i = 1; i < s.size(); i++) {
            dest[i] = dest[i - 1] * mod % mod2 + (s[i]);

            if (i >= size){
                long long minus = (s[i - size]) * power % mod2;
                dest[i] -= minus;
            }

            if (dest[i] < 0){
                dest[i] += mod2;
            }

            dest[i] %= mod2;
        }
    }

    void suffixHash(vector<long long>& dest, string& s) {
        dest.resize(s.size());
        dest[s.size() - 1] = s[s.size() - 1] - 'a';

        for (int i = s.size() - 2; i >= 0; i--) {
            dest[i] = dest[i + 1] * mod + (s[i] - 'a');
            dest[i] %= mod2;
        }
    }

    long long singlePrefixHash(string& s) {
        long long result = s[0] - 'a';
        for (int i = 1; i < s.size(); i++) {
            result = result * mod + (s[i] - 'a');
            result %= mod2;
        }
        return result;
    }
}
```

```

    long long singleSuffixHash(string& s) {
        long long result = s[s.size() - 1] - 'a';
        for (int i = s.size() - 2; i >= 0; i--) {
            result = result * mod + (s[i] - 'a');
            result %= mod2;
        }
        return result;
    }
} hasher;

```

## 4.2 Trie

```

struct trie_node{
    int children[26]; // index of the next node
    int isWord = -1; // if it is a word
    int cnt = 0;
};

class trie {
    vector<trie_node> v;
    int size = 1;
public:
    trie(){
        v.resize(1);
    }

    void insert(string& s, int idx) {
        int currpos = 0;
        v[currpos].cnt++;
        for (int i = 0; i < s.size(); i++) {
            // create a new node if it doesn't exist
            if (v[currpos].children[s[i] - 'a'] == 0) {
                v[currpos].children[s[i] - 'a'] = size++;
                v.push_back(trie_node());
            }
            currpos = v[currpos].children[s[i] - 'a'];
            v[currpos].cnt++;

            // mark it as a word if it is the end of the loop
            if (i == s.size() - 1) v[currpos].isWord = idx;
        }
    }

    void remove(string& s) {
        int currpos = 0;
        v[currpos].cnt--;
        for (int i = 0; i < s.size(); i++) {
            int next = v[currpos].children[s[i] - 'a'];
            currpos = next;
            v[currpos].cnt--;
        }
    }

    int traverse(string& p) {
        int currpos = 0;
        for (int i = 0; i < p.size(); i++) {
            int next = v[currpos].children[p[i] - 'a'];

```

```
        if (next == 0) return 0;
        currpos = next;
    }
    return v[currpos].cnt;
};
```

### 4.3 KM(P)

## 5 Graph

### 5.1 Bipartite Graph

#### 5.1.1 Bipartite Graph Checking

```
bool isBipartite(vector<bool>& visited, int node, vector<int>& color) {
    visited[node] = true;

    for (auto a : graph[node]) {
        if (!visited[a]) {
            color[a] = 1 - color[node];
            return isBipartite(visited, a, color);
        } else if (color[a] == color[node]) {
            return false;
        }
    }

    return true;
}
```

#### 5.1.2 Maximum Bipartite Matching

Idea: We keep finding valid augmentation to graph until we can find none.

1. We start at a vertex  $A$ , and it connects to vertices  $B_1, B_2, \dots, B_k$ .
2. We try to include edge  $(A, B_1)$  in our maximum matching. If  $B_1$  is not the endpoint of any edge in the current matching configuration, we are done.
3. Otherwise,  $B_1$  has been matched to some other vertex, lets say  $C$ . We have to find if  $C$  can match to some other vertices (not  $B$ ). Only if it is possible, we match  $A$  with  $B_1$ .
4. If  $A$  cannot match with  $B_1$ , try next connected vertex ( $B_2$ ) and repeat from step 2, until you find one augmentation, or you declare that  $A$  cannot be included.

```
bool dfs(vector<bool>& visited, int node) {
    if (visited[node]) return false;

    visited[node] = true;
    for (auto to : graph[node]) {
        if (matching[to] == -1 || dfs(visited, matching[to])) {
            matching[to] = cur;
            return true;
        }
    }

    return false;
}

int maxmatch(vector<vector<int>>& graph) {
    int cnt = 0;
    vector<bool> visited(graph.size());
    for (int i = 1; i < graph.size(); i++) {
        fill(visited.begin(), visited.end(), false);
        cnt += dfs(i);
    }
    return cnt;
}
```



## 5.2 Lowest Common Ancestor

```
class lowest_common_ancestor {
    vector<vector<int>> parent;
    vector<int> depth;
    void dfs(vector<vector<int>>& graph, int node, int steppies) {
        depth[node] = steppies;
        for (auto other : graph[node]) {
            parent[0][other] = node;
            dfs(graph, other, steppies + 1);
        }
    }

public:
    lowest_common_ancestor(int n, int root, vector<vector<int>>& graph) {
        parent.resize(20, vector(n + 1, -1));
        depth.resize(n + 1);
        dfs(graph, root, 1);
        for (int i = 1; i < 20; i++) {
            for (int j = 0; j < n; j++) {
                if (parent[i - 1][j] != -1) parent[i][j] = parent[i - 1][parent[i - 1][j]];
            }
        }

        int lift(int node, int steppies) {
            for (int i = 19; i >= 0; i--) {
                if (steppies & (1 << i)) node = parent[i][node];
            }
            return node;
        }

        int query(int lhs, int rhs) {
            if (depth[lhs] > depth[rhs]) swap(lhs, rhs);
            int required_steppies = depth[rhs] - depth[lhs];
            rhs = lift(rhs, required_steppies);

            if (lhs == rhs) return lhs;

            for (int i = 19; i >= 0; i--) {
                if (parent[i][lhs] != parent[i][rhs]) {
                    lhs = parent[i][lhs], rhs = parent[i][rhs];
                }
            }

            return parent[0][lhs];
        }
    };
};
```

## 5.3 Graph Traversal

### 5.3.1 Dijkstra's Algorithm

A single source weighted shortest path algorithm in  $O(n \log n)$

```
void dijkstra(vector<int>& visited, int node){
    min_heap<pair<int, int>> pq;
    pq.push({0, node});
    visited[node] = 0;
```

```

while (!pq.empty()){
    pair<int, int> curr = pq.top();
    pq.pop();

    if (curr.first > visited[curr.second]) continue;

    for (auto a : graph[curr.second]){
        if (curr.first + a.second < visited[a.first]){
            visited[a.first] = curr.first + a.second;
            pq.push({curr.first + a.second, a.first});
        }
    }
}
}

```

### 5.3.2 Bellman-Ford Algorithm

A single source weighted shortest path algorithm which supports negative edges in  $O(VE)$ .

```

vector<long long> dist(n, inf * inf);
dist[root] = 0;

for (int t = 0; t < n - 1; t++) {
    for (int i = 0; i < n; i++) {
        for (pair<int, int> other : graph[i]) {
            int u = i, v = other.first;
            if (dist[u] != inf * inf && dist[v] > dist[u] + other.second) {
                dist[v] = dist[u] + other.second;
            }
        }
    }
}

// for checking negative cycles
for (int i = 0; i < n; i++) {
    for (pair<int, int> other : graph[i]) {
        int u = i, v = other.first;
        if (dist[u] != inf * inf && dist[v] > dist[u] + other.second) {
            cout << "NEGATIVE CYCLE\n";
            return 0;
        }
    }
}
}

```

### 5.3.3 Floyd-Warshall Algorithm

An all pairs weighted shortest path algorithm which supports negative edges in  $O(V^3)$ .

```

vector dist(n, vector(n, inf + inf));
for (int i = 0; i < n; i++) {
    dist[i][i] = 0;
}

for (int i = 0; i < m; i++) {
    long long u, v, w;
    cin >> u >> v >> w;
    dist[u][v] = w;
}
}

```

```

for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            if (dist[i][k] != inf + inf && dist[k][j] != inf + inf) {
                if (dist[i][k] + dist[k][j] < dist[i][j]) {
                    dist[i][j] = dist[i][k] + dist[k][j];
                }
            }
        }
    }
}

// for checking negative cycles
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        for (int k = 0; k < n; k++) {
            if (dist[i][k] + dist[k][j] + dist[j][i] < 0) {
                cout << "NEGATIVE CYCLE\n";
                return 0;
            }
        }
    }
}

```

#### 5.3.4 0-1 BFS

If the weights of the edges are either 0 or 1, we do not have to use a priority queue. Instead, we can use a deque. When we do insertion, if the newly added edge is 0, place it in front of the deque; otherwise, place it at the back of the deque.

```

vector<int> dist(n, INF);
dist[s] = 0;
deque<int> dq;
dq.push_front(s);
while (!dq.empty()) {
    int node = dq.front();
    dq.pop_front();
    for (auto edge : graph[node]) {
        int other = edge.first;
        int weight = edge.second;
        if (dist[other] + w < dist[node]) {
            dist[other] = dist[node] + w;
            if (w == 1)
                q.push_back(other);
            else
                q.push_front(other);
        }
    }
}

```