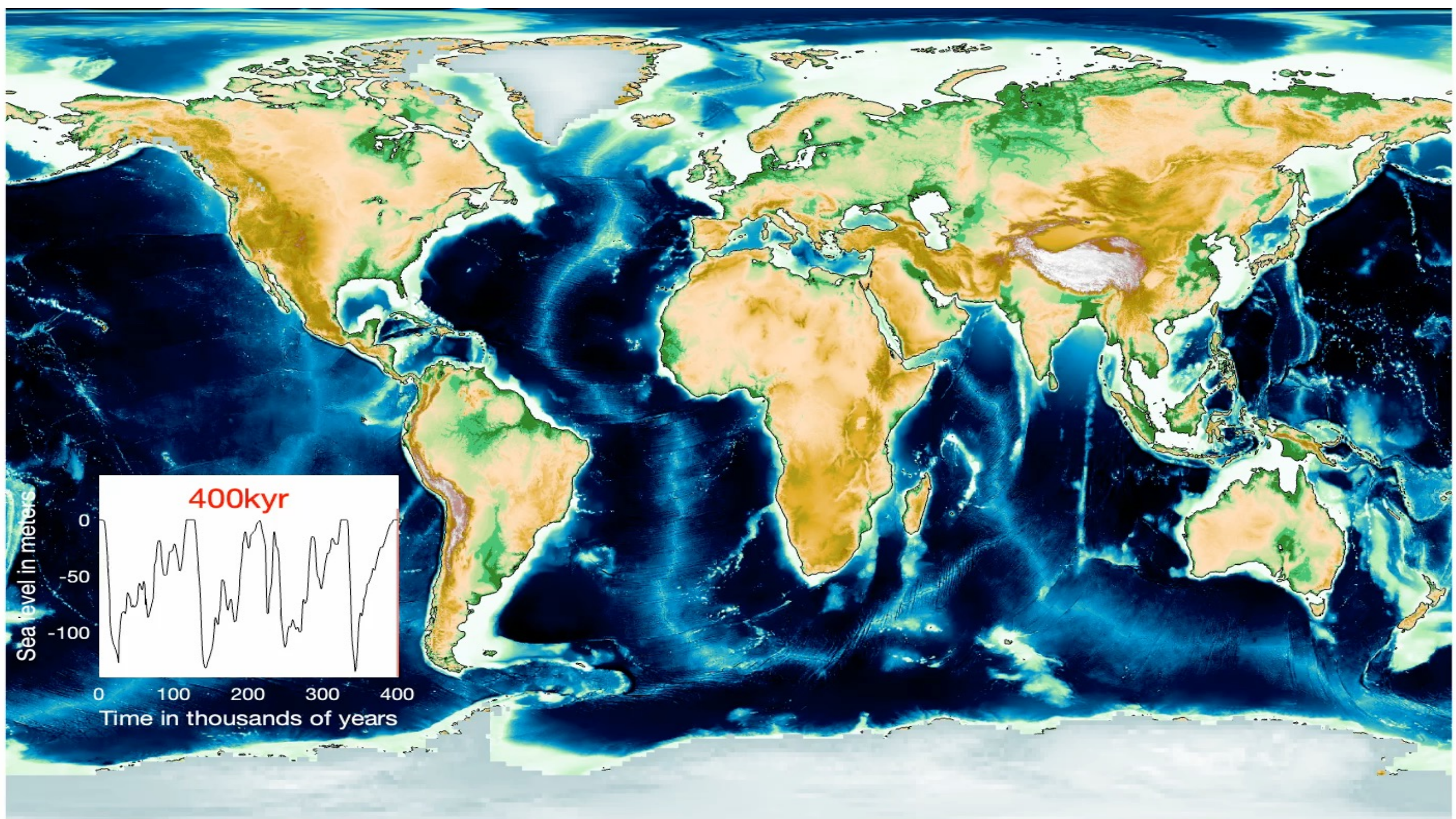
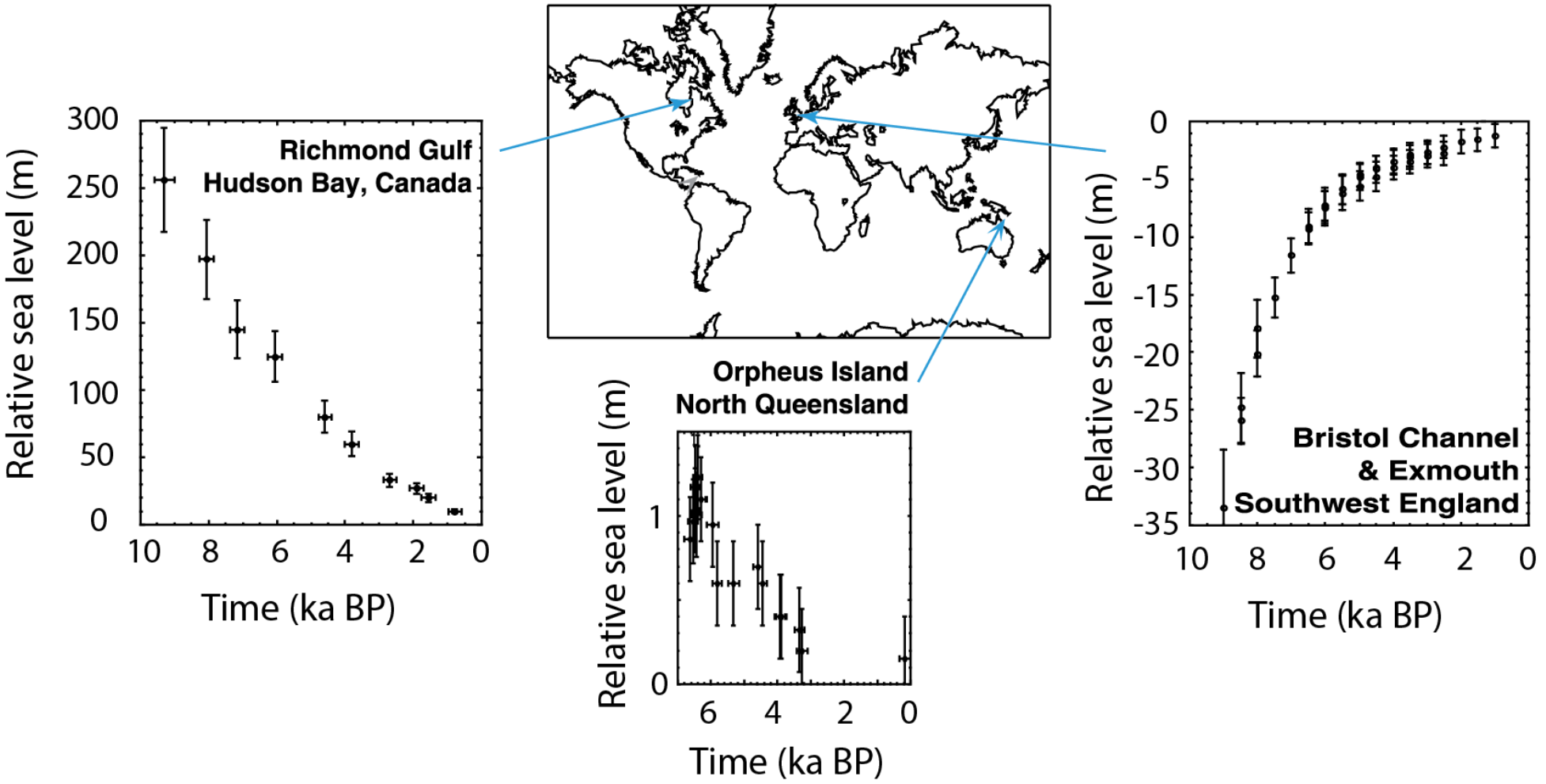
Solving the sea level equation: Earth's response to ice and ocean load changes

Jacky Austermann & Sam Chester


400kyr
0
-50
-100
Sea level in meters
0
100
200
300
400
Time in thousands of years

Sea level change is not uniform across the globe



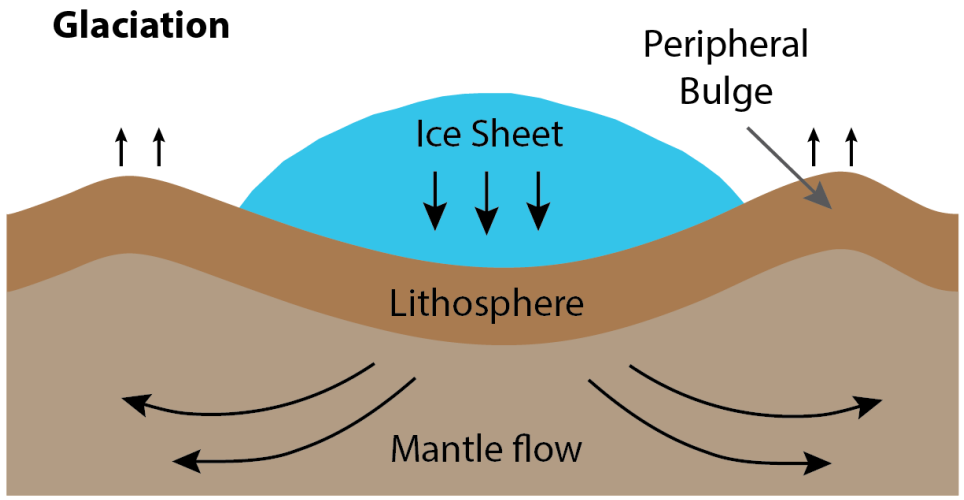
Lambeck et al., 2001

Glacial isostatic adjustment (GIA)

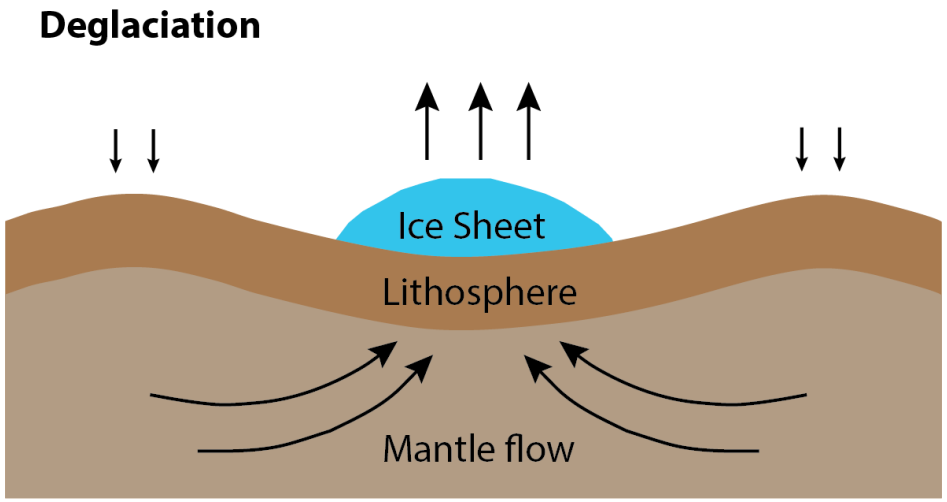
Postglacial rebound: Viscoelastic rebound of the solid Earth after ice melt.

Glacial isostatic adjustment: the viscoelastic response of Earth's solid surface, its gravity field and rotation axis to changes in ice and ocean load.

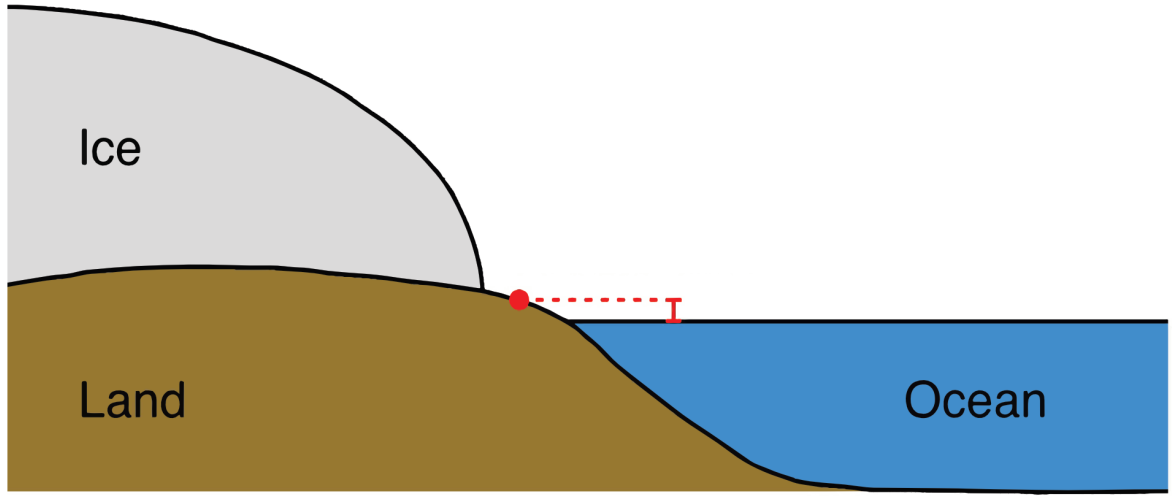
During / after ice growth



During / after ice melt



Ice melt causes changes in Earth's gravity field



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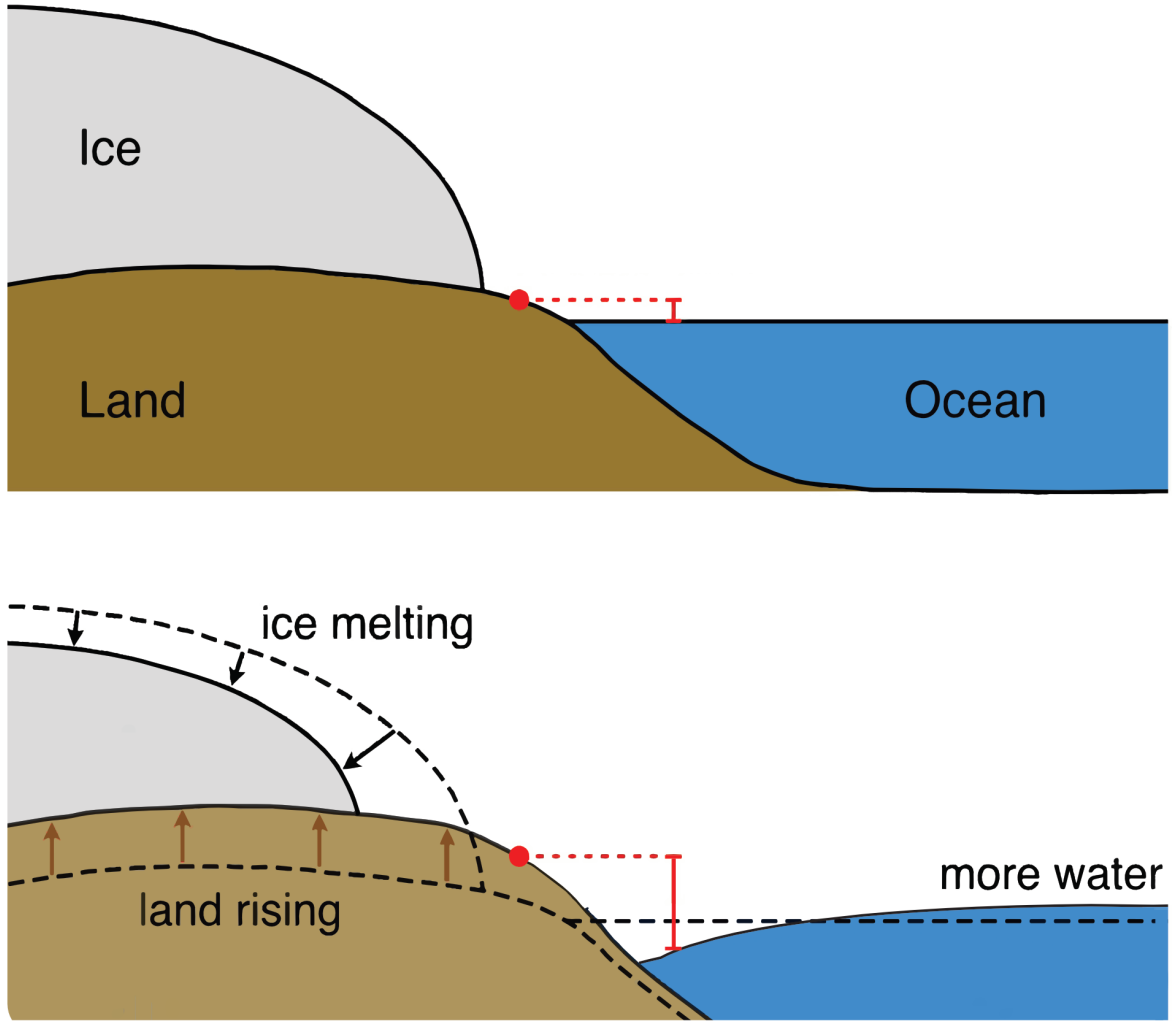

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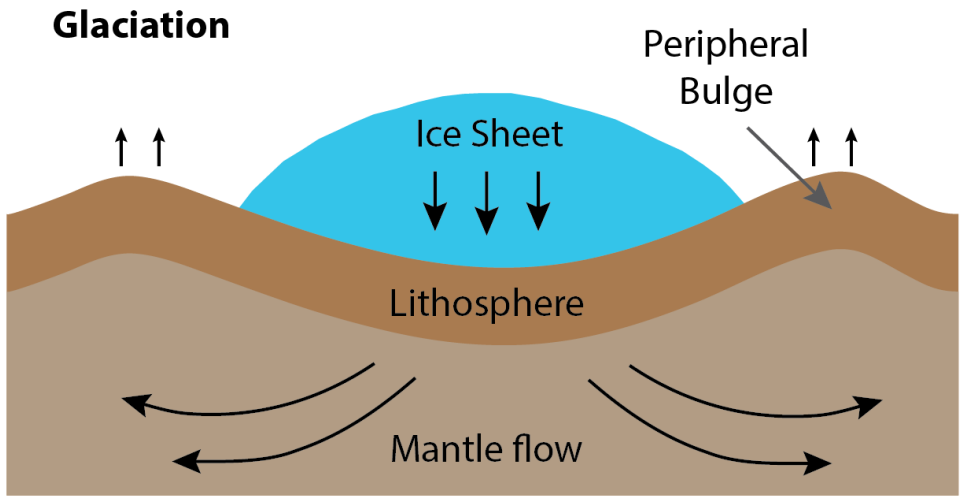

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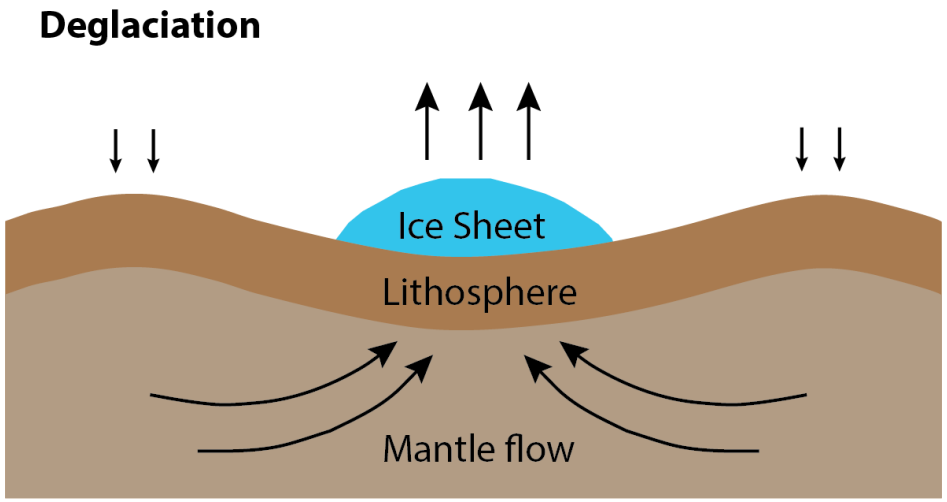
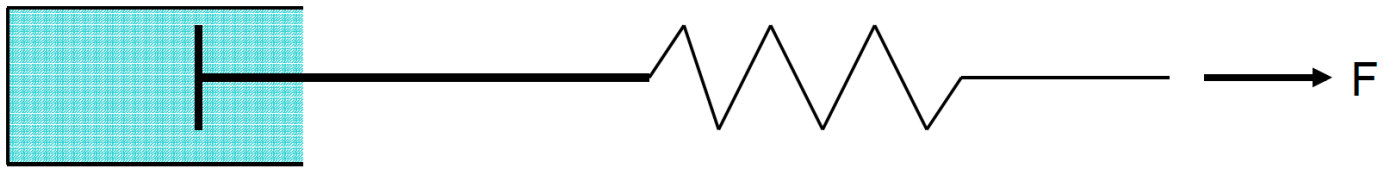


What does viscoelastic mean?

We need to make choices about the rheology of the mantle, i.e. the relationship between stress, i.e. force that is applied, and strain, i.e. deformation that occurs.

Viscoelastic: Viscous and elastic components to the deformation

Maxwell viscoelastic: A conceptual spring and dashpot in series



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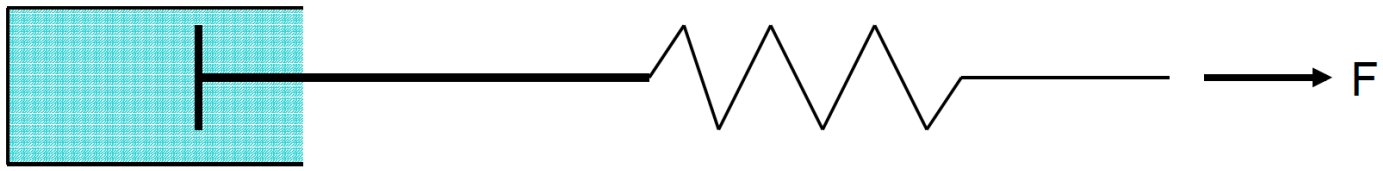

$$\dot{e} = \frac{1}{k}\dot{\sigma} + \frac{1}{\eta}\sigma$$

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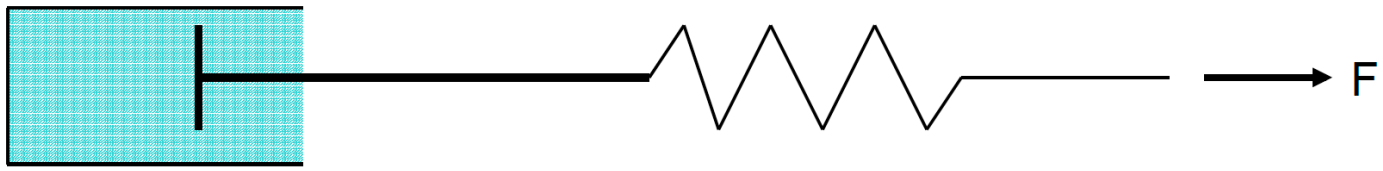

stress

$$\dot{e} = \frac{1}{k}\dot{\sigma} + \frac{1}{\eta}\sigma$$

strain

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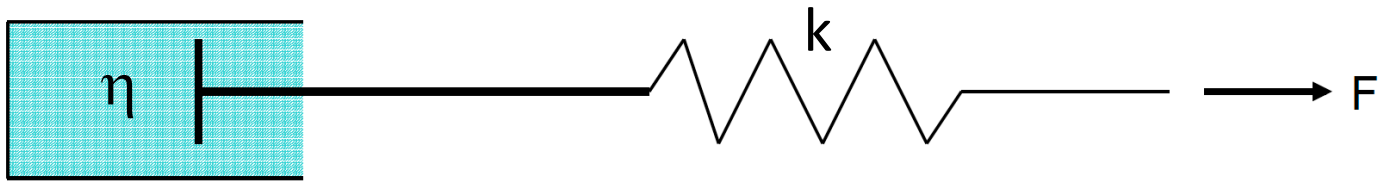


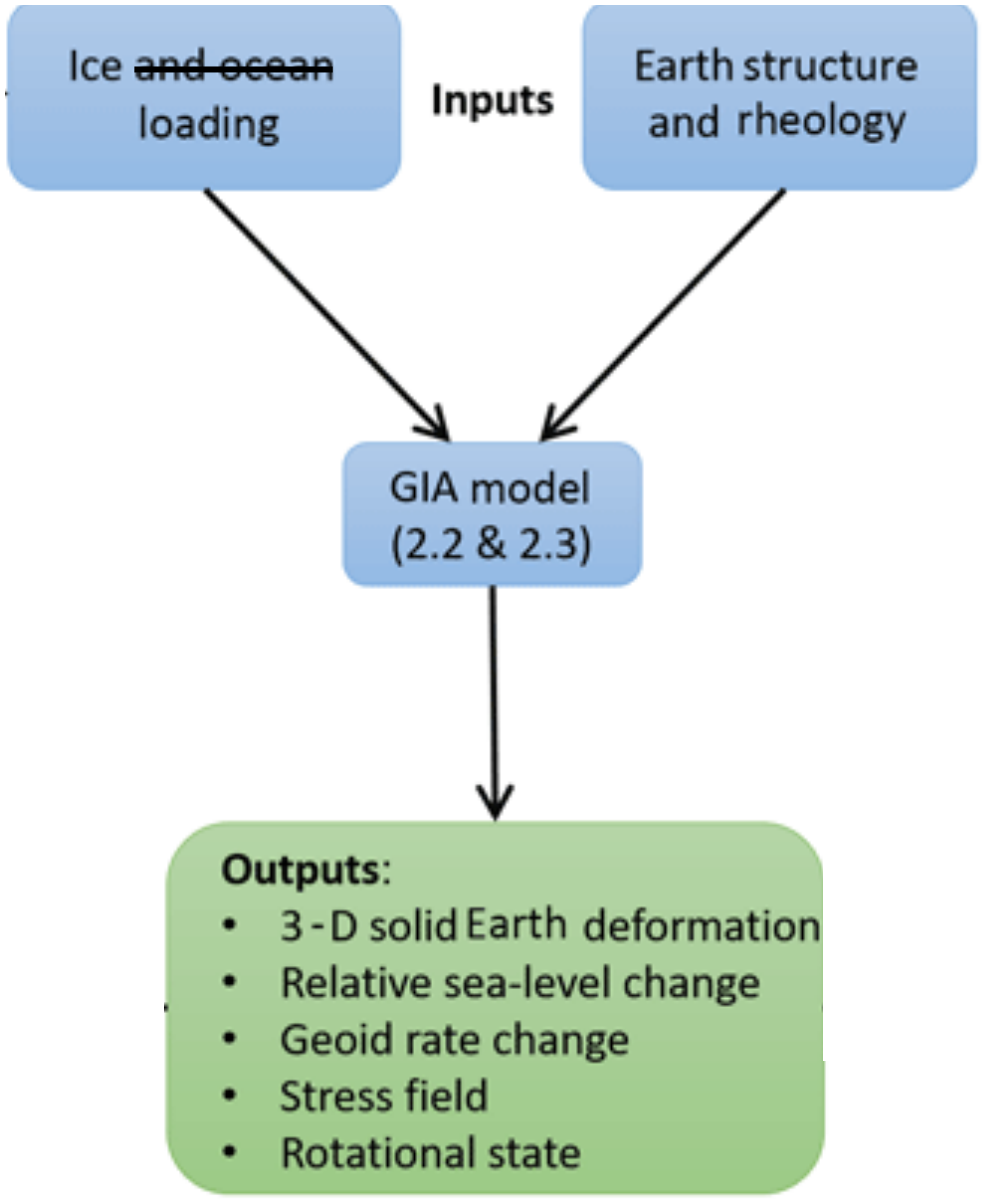
stress

$$\dot{e} = \frac{1}{k}\dot{\sigma} + \frac{1}{\eta}\sigma$$

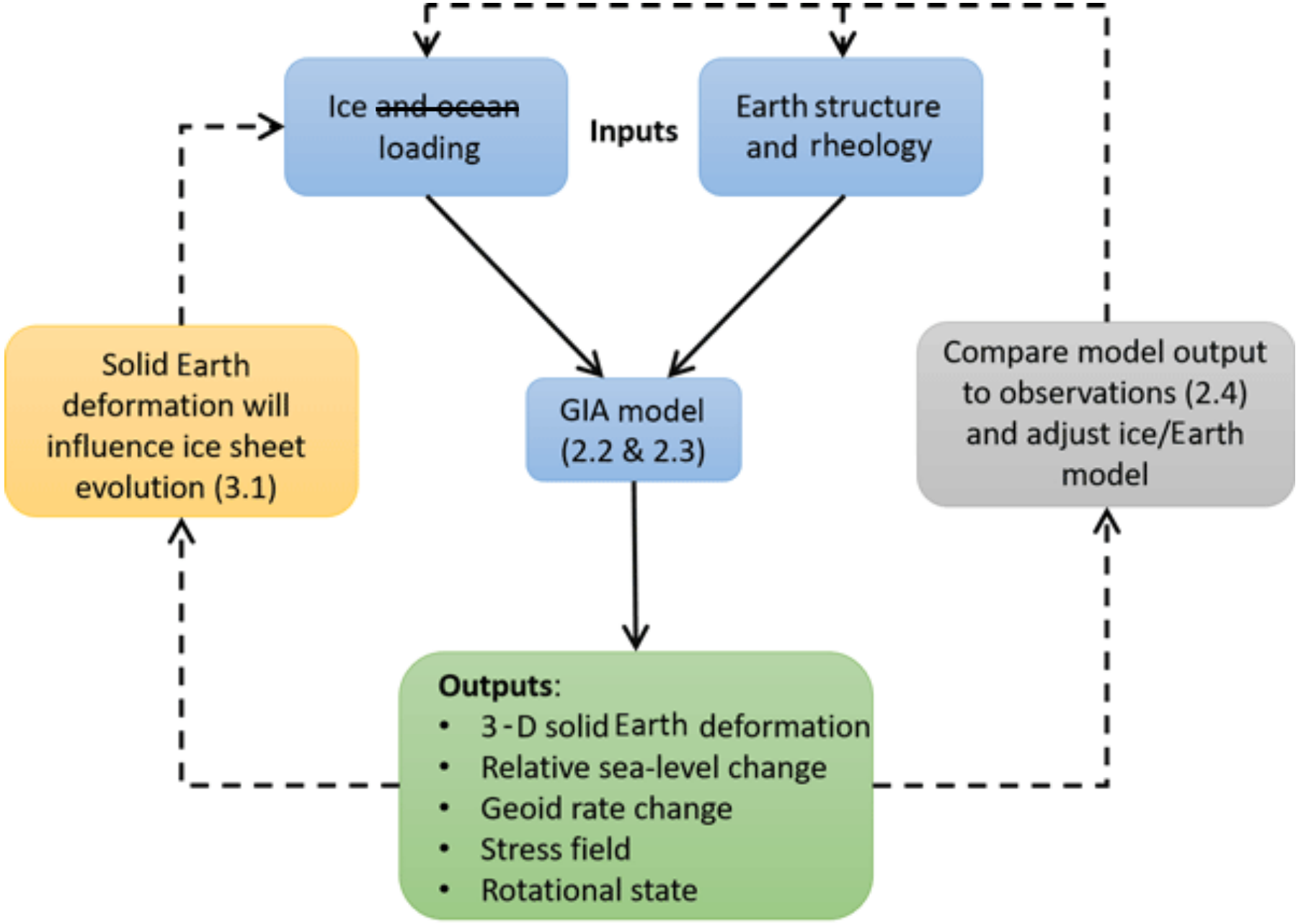
strain

viscosity

elastic constant



Whitehouse, 2018



Whitehouse, 2018

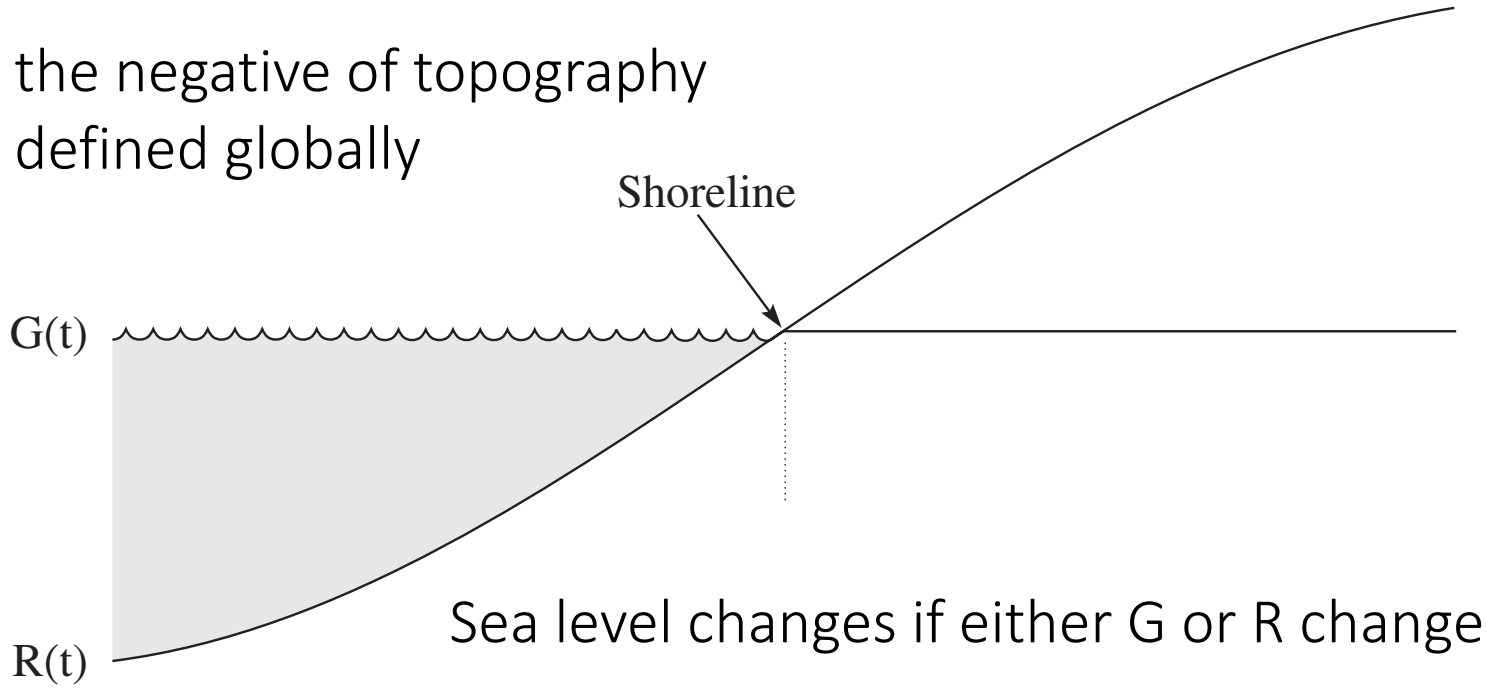
Some basic concepts

SL: Sea level
G: Gravitational equipotential surface that coincides with mean sea level
R: Position of the solid Earth
T: Topography

$$SL(\theta, \psi, t) \equiv G(\theta, \psi, t) - R(\theta, \psi, t).$$

$$T(\theta, \psi, t) \equiv R(\theta, \psi, t) - G(\theta, \psi, t) = -SL(\theta, \psi, t)$$

- Sea level is the negative of topography
- Sea level is defined globally



Sea level changes if either G or R changes

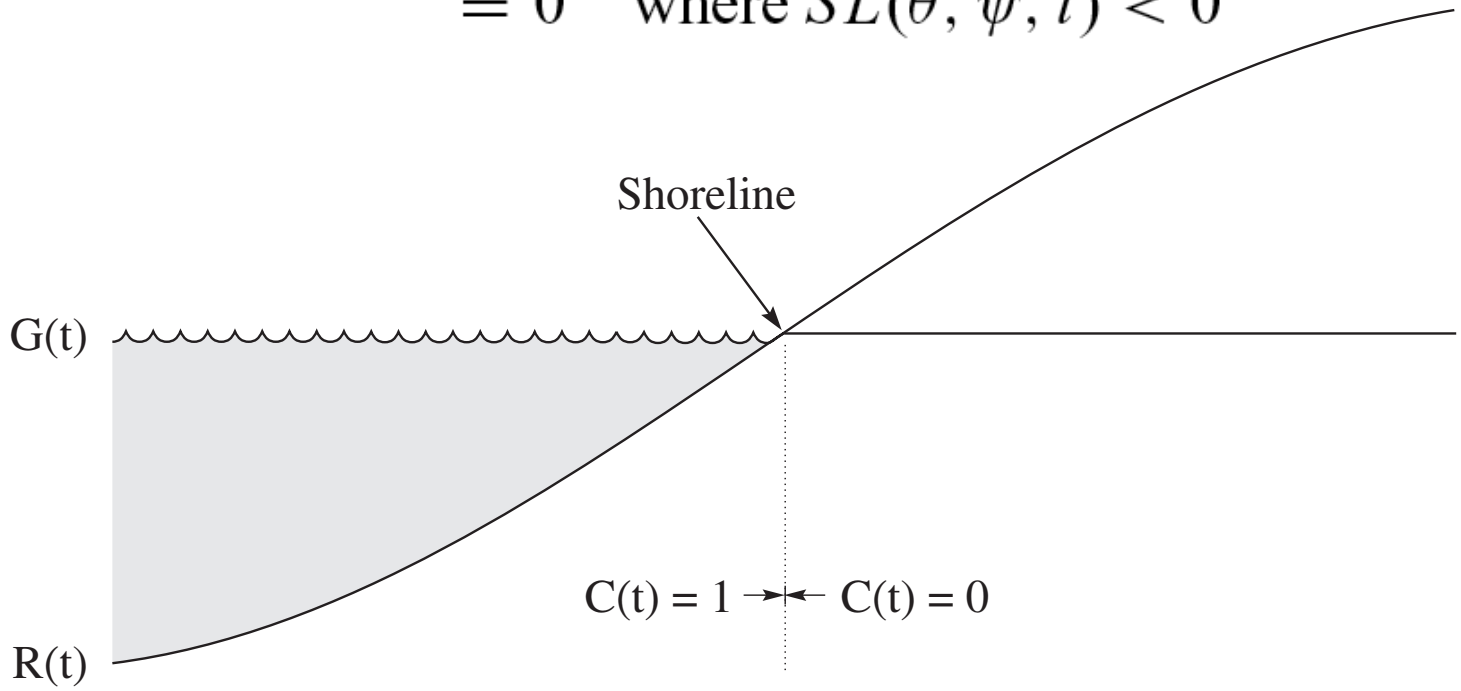
Some basic concepts

SL: Sea level
G: Gravitational equipotential surface that coincides with mean sea level
R: Position of the solid Earth
T: Topography

Sea surface height S: $S(\theta, \psi, t) = SL(\theta, \psi, t)C(\theta, \psi, t),$

Ocean function C: $C(\theta, \psi, t) = 1 \quad \text{where } SL(\theta, \psi, t) > 0$

$= 0 \quad \text{where } SL(\theta, \psi, t) < 0$



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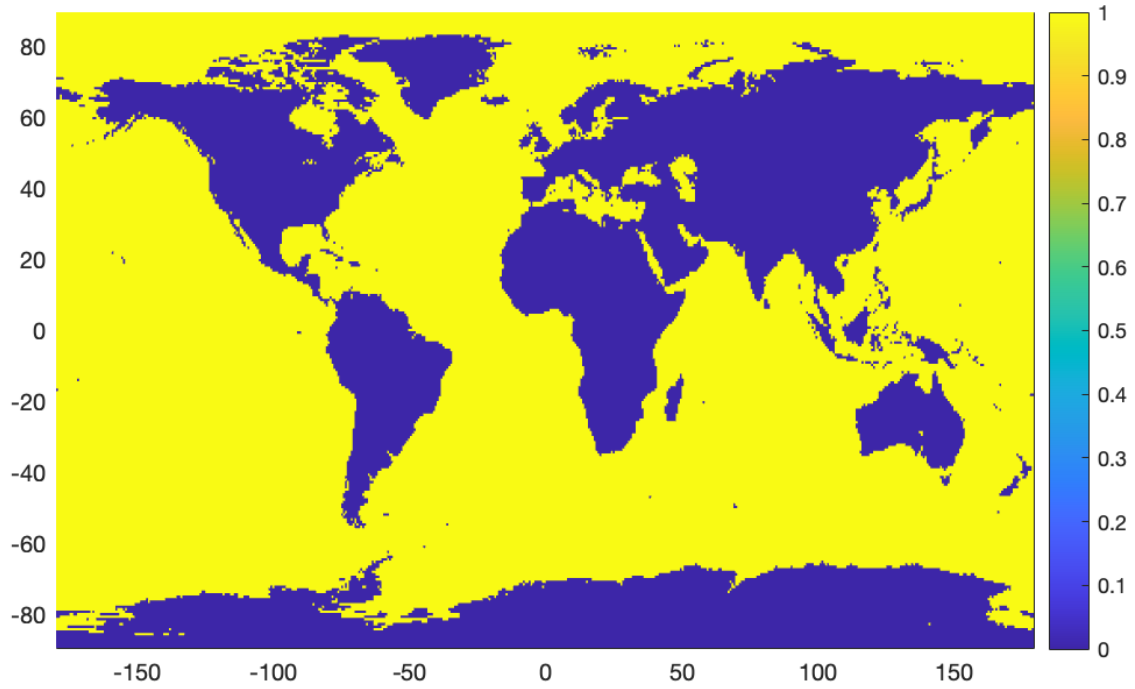

Some basic concepts

SL: Sea level
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Changes from the initial state:

$$\Delta SL(\theta, \psi, t_j) = SL(\theta, \psi, t_j) - SL(\theta, \psi, t_0)$$

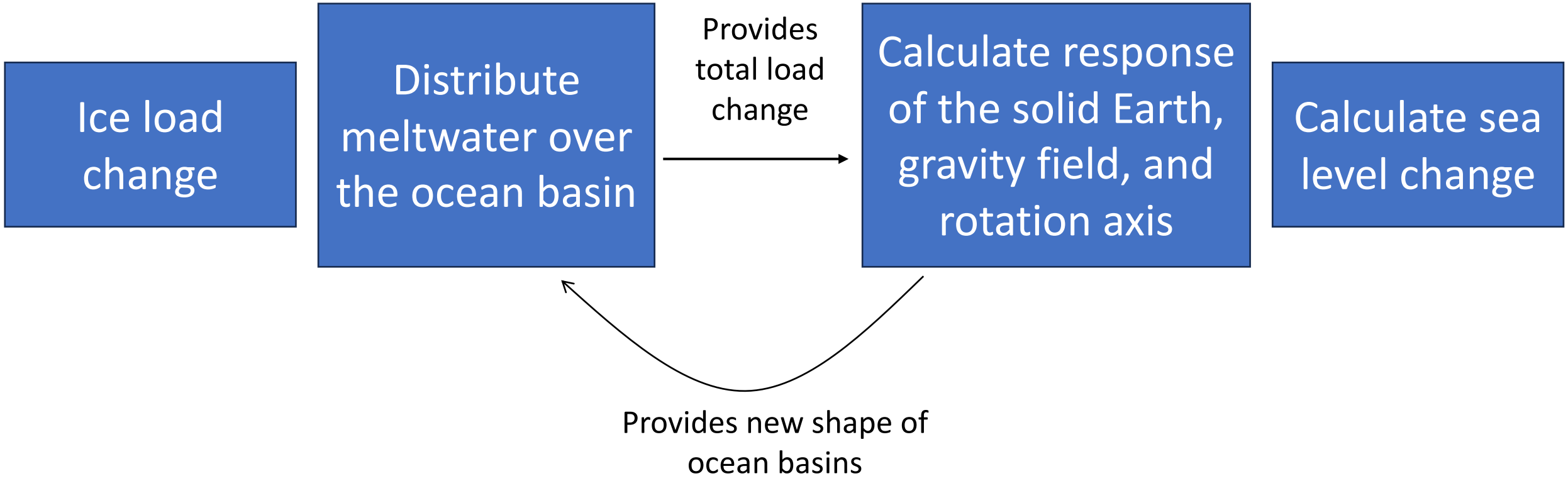
$$\Delta SL(\theta, \psi, t_j) = \Delta G(\theta, \psi, t_j) - \Delta R(\theta, \psi, t_j),$$

$$\begin{aligned}\Delta T(\theta, \psi, t_j) &= \Delta R(\theta, \psi, t_j) - \Delta G(\theta, \psi, t_j) \\ &= -\Delta SL(\theta, \psi, t_j).\end{aligned}$$

We aim to calculate ΔSL.

$$RSL(t_j) = SL(t_j) - SL(present)$$

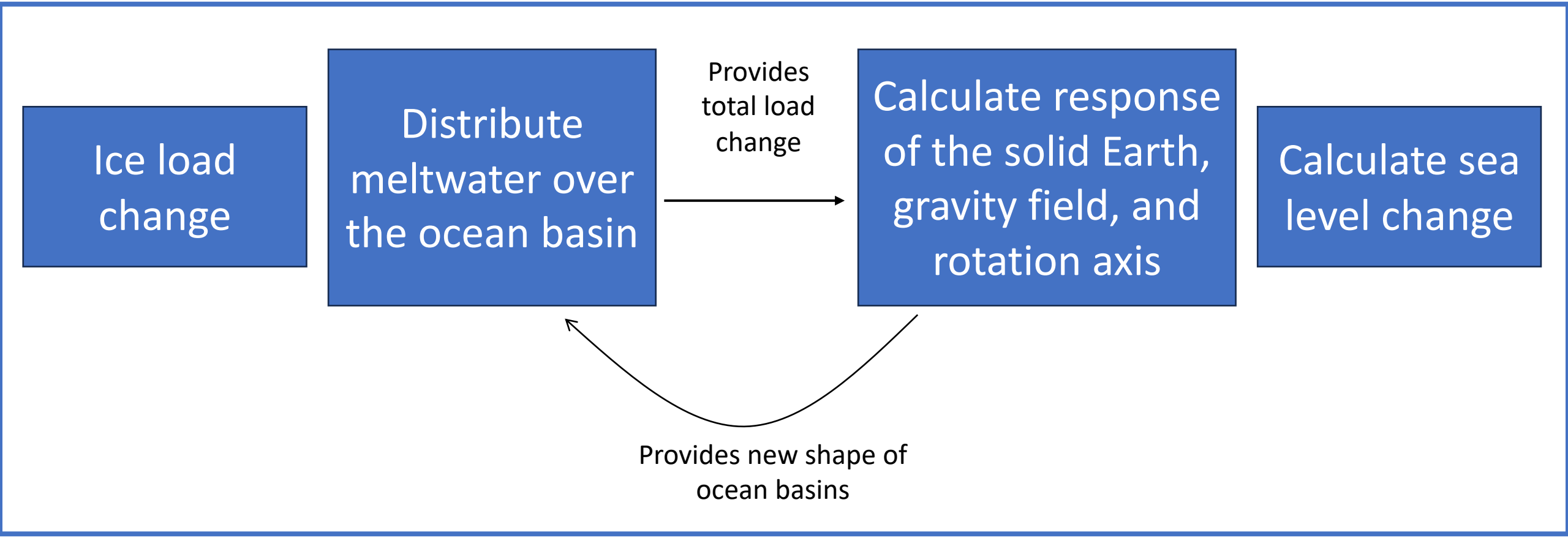
The sea level algorithm



Calculate response of the solid Earth, gravity field, and rotation axis

- This step is done using Green's functions.
- This assumes Earth is radially symmetric.
- For the viscoelastic loading problem, the parameters in the Green's functions are called Love numbers.
- Love numbers are calculated separately for specific Earth viscosity profiles.
- This step is done in the spectral domain, therefore there are several places in the code that take you in and out of the spectral space (spherical harmonic basis functions)

The sea level algorithm



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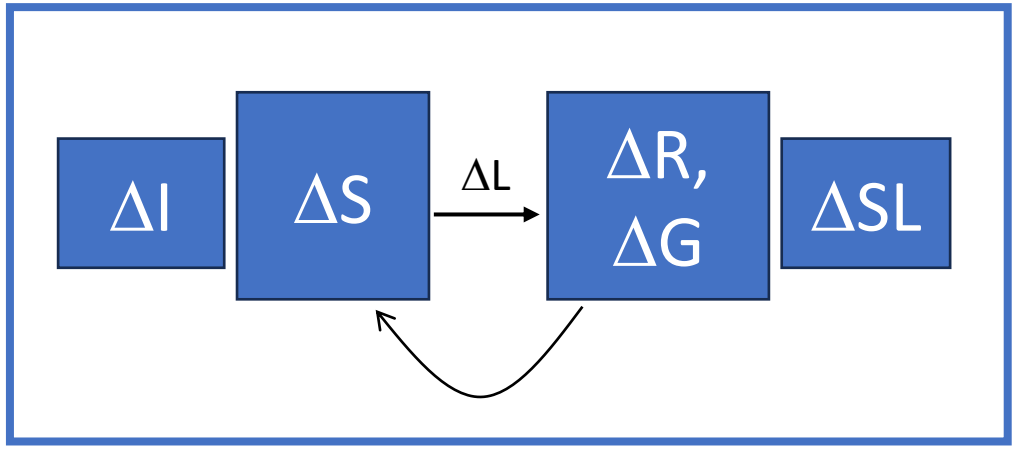

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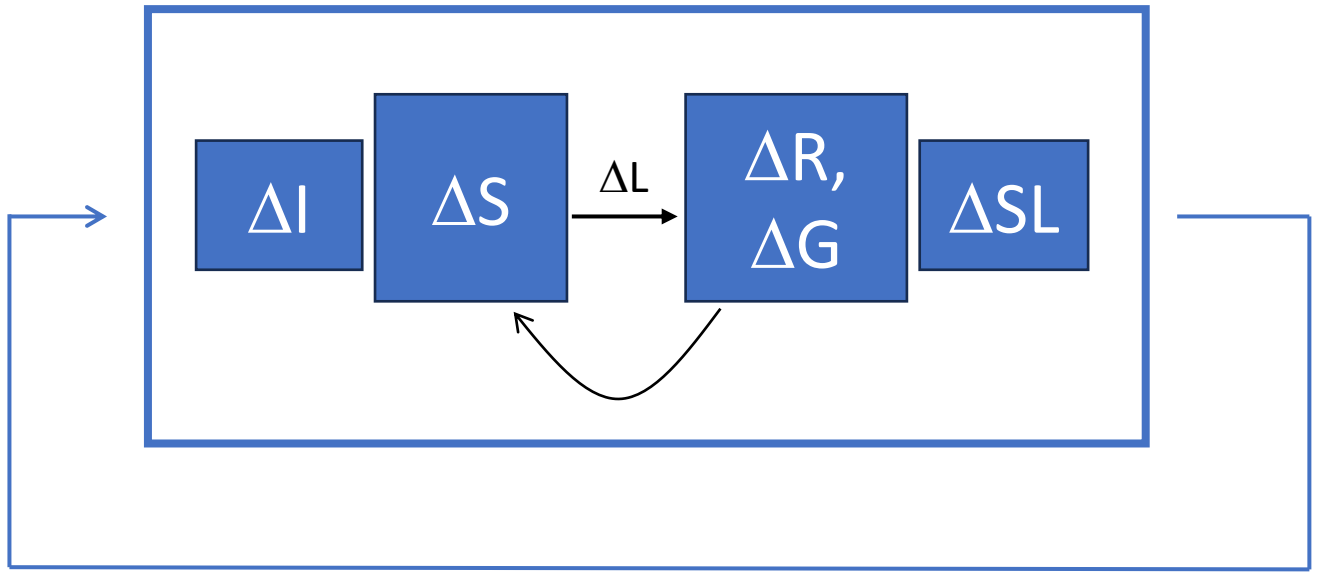
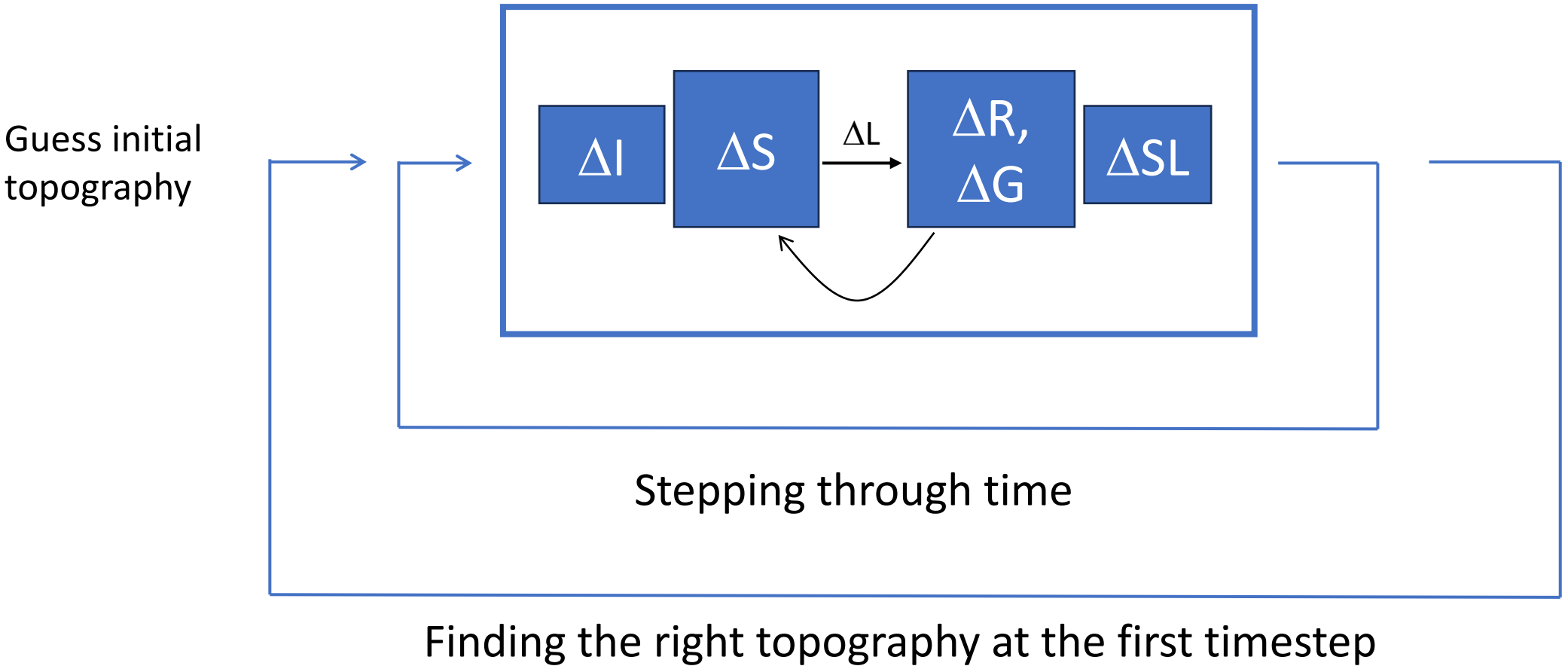


Stepping through time

The sea level algorithm



Geophys. J. Int. (2005) **161,** 679–706 doi: 10.1111/j.1365-246X.2005.02553.x

On post-glacial sea level – II. Numerical formulation and comparative results on spherically symmetric models

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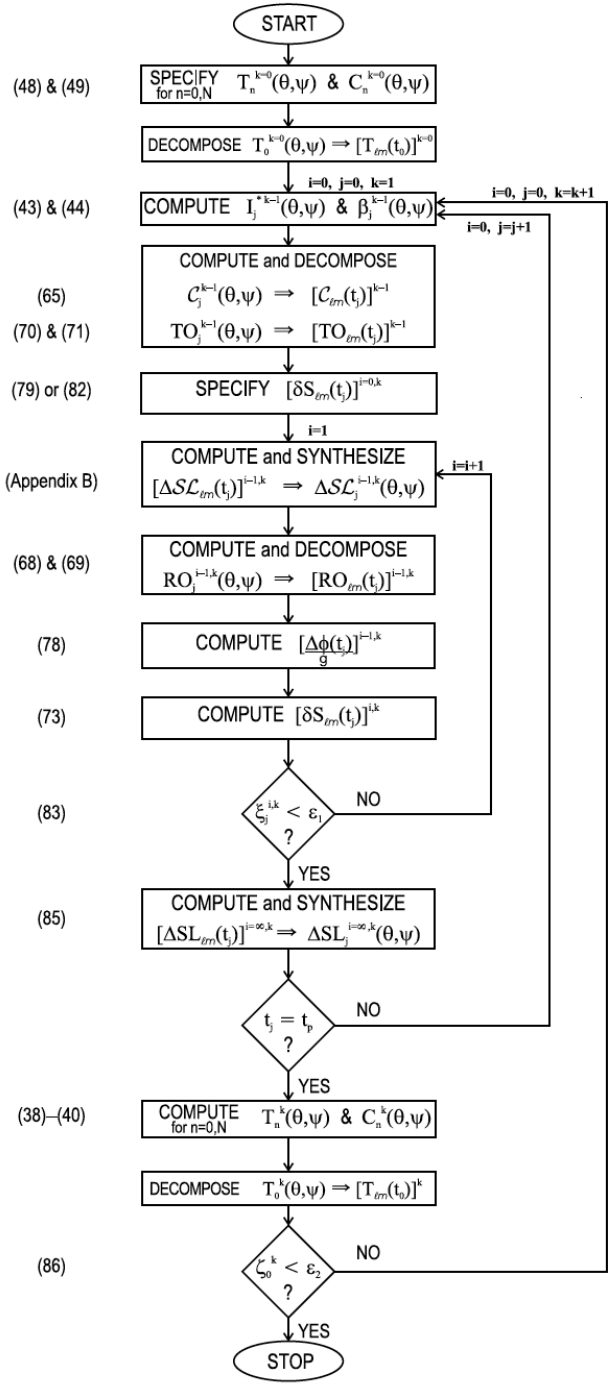
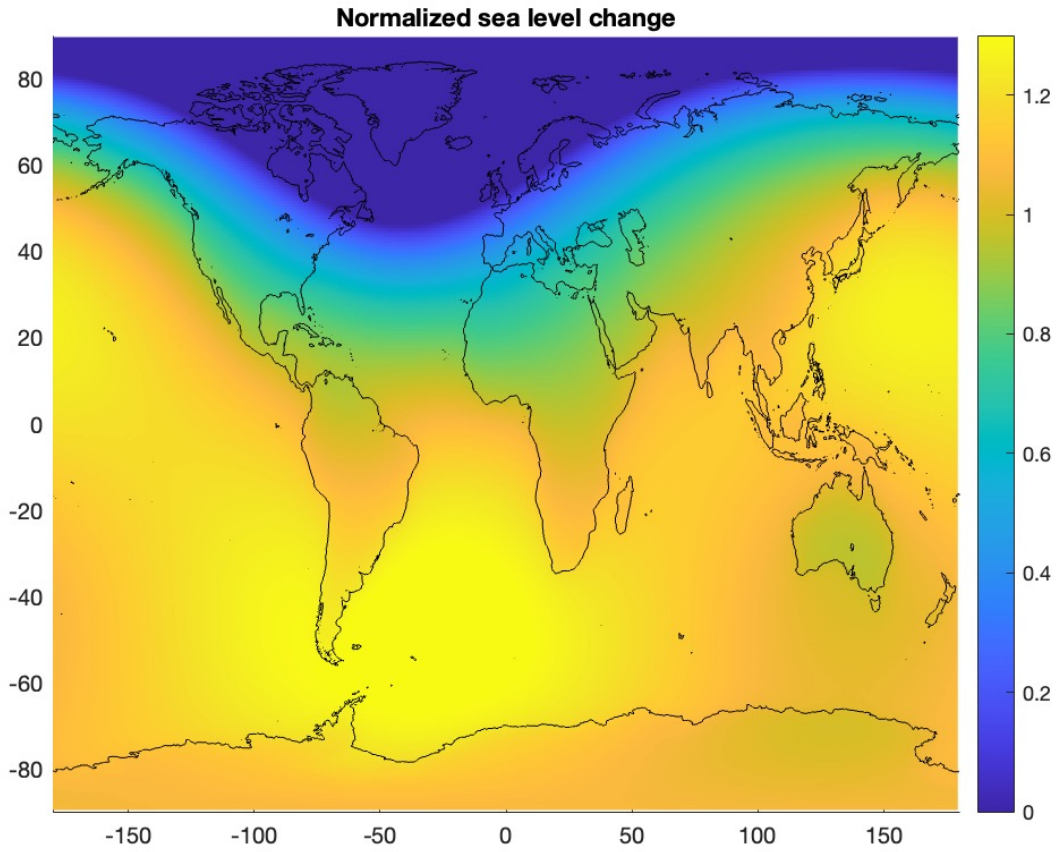
[2]*Department of Geological Sciences, University of Durham, Science Labs, South Road, Durham* DH1 3LE, *UK*

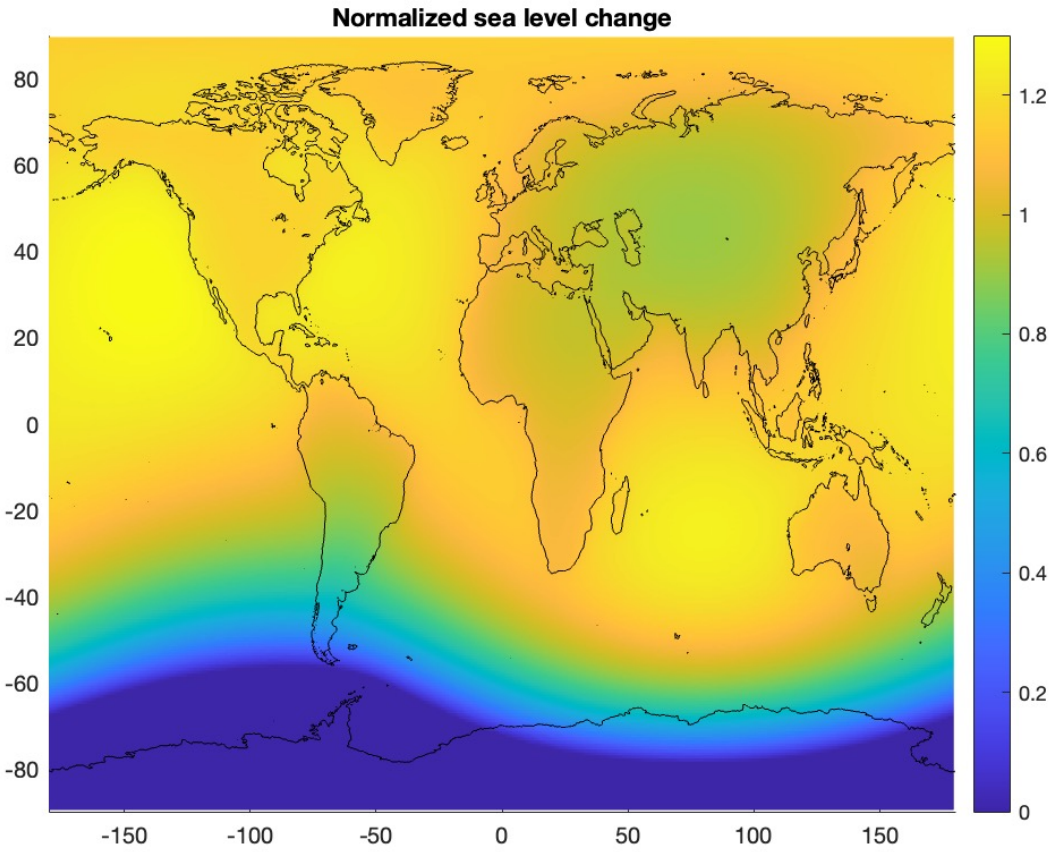


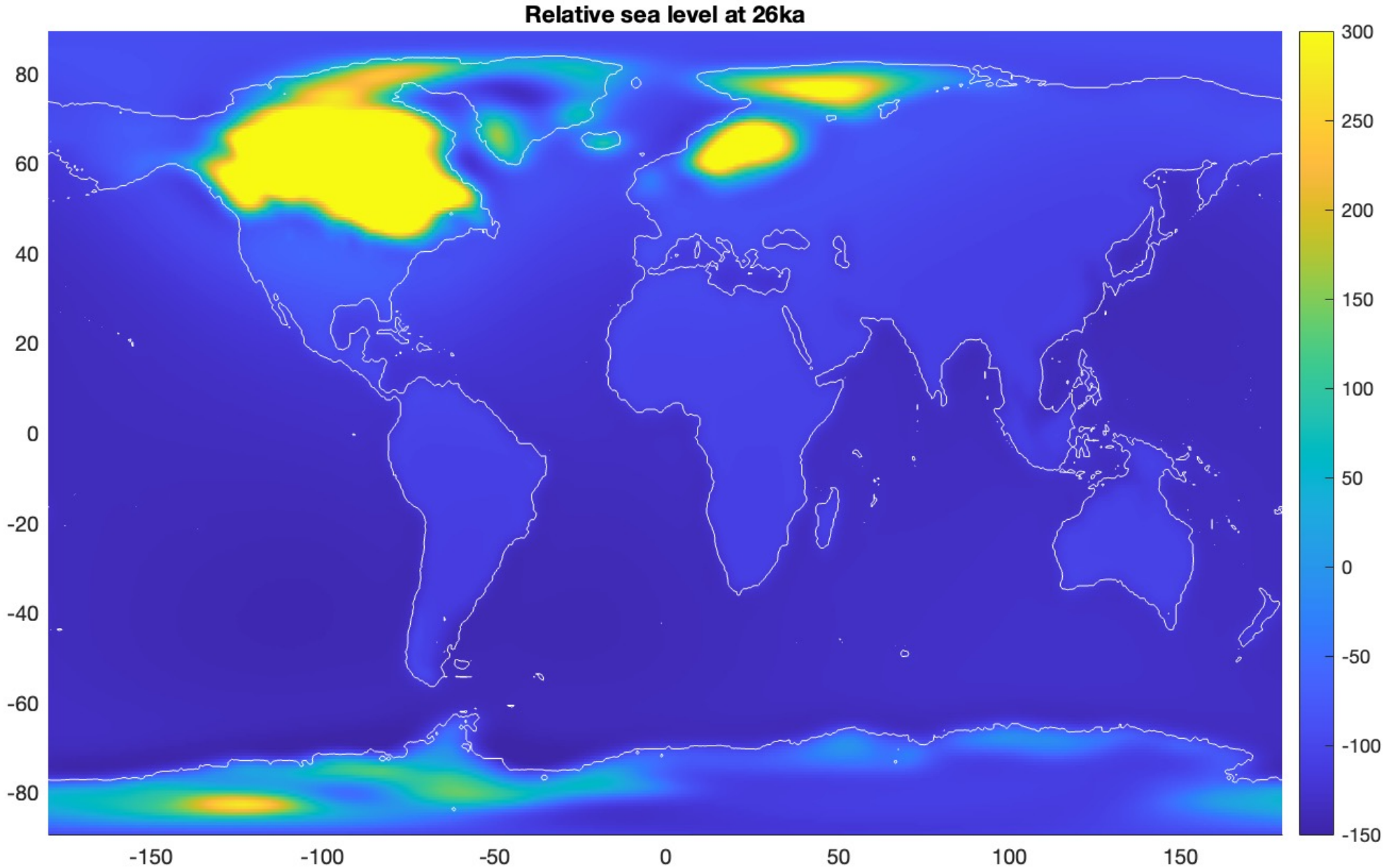
Kendall et al., 2005

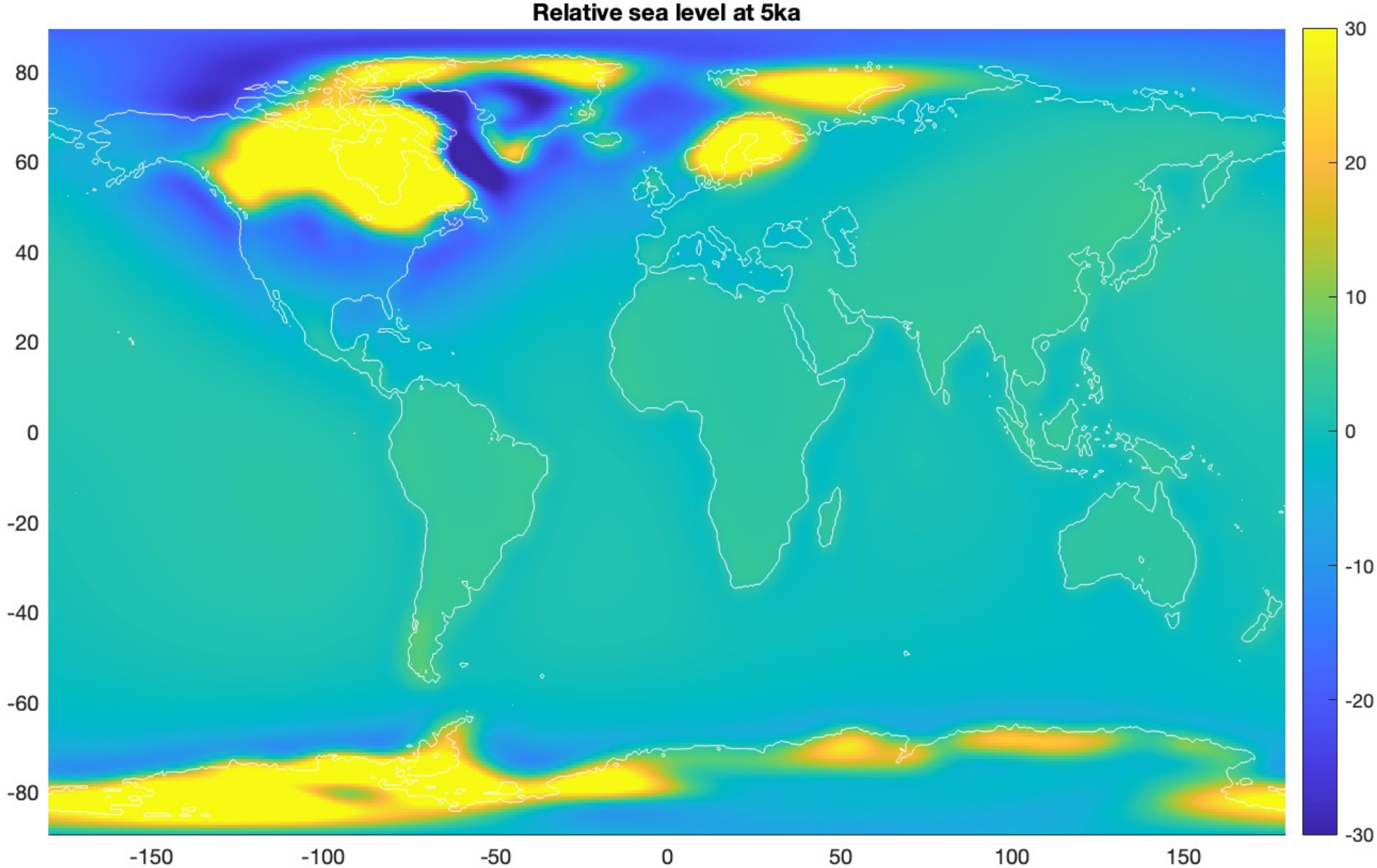
Greenland



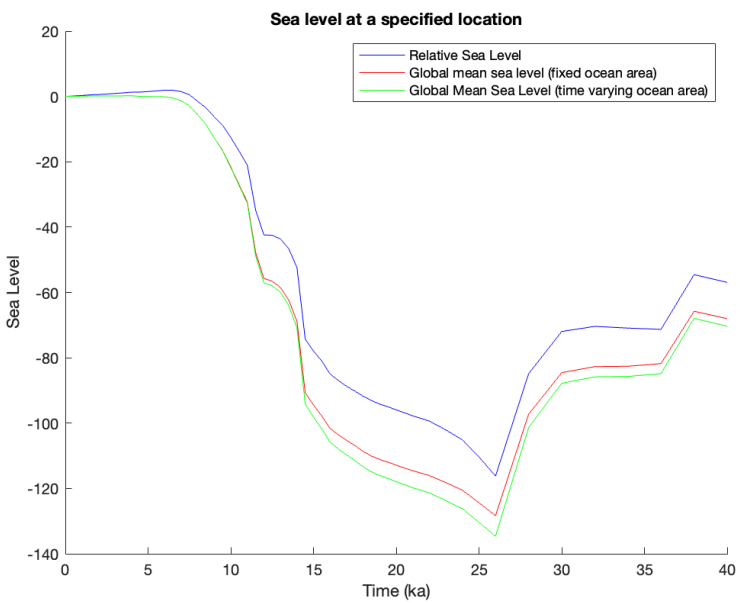
West Antarctica



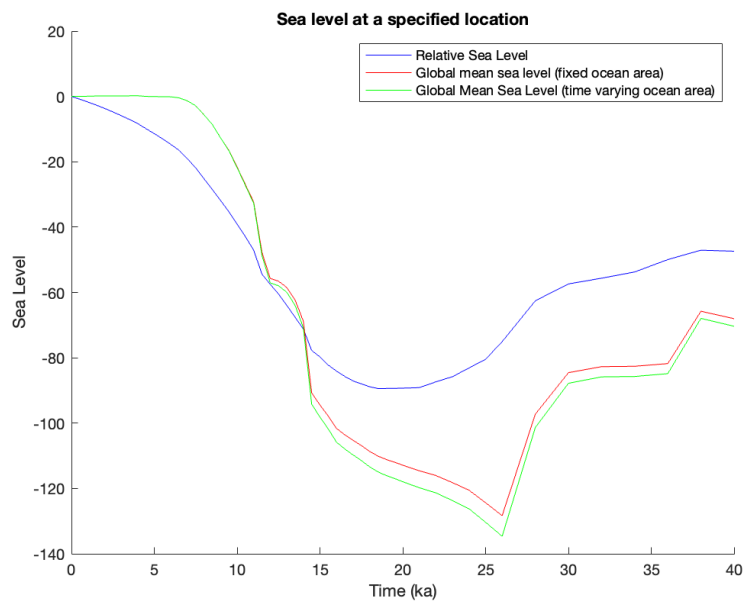

Relative sea level at 26ka
80
60
40
20
0
-20
-40
-60
-80
-150
-100
-50
0
50
100
150
300
250
200
150
100
50
0
-50
-100
-150


Relative sea level at 5ka
80
60
40
20
0
-20
-40
-60
-80
-150
-100
-50
0
50
100
150
30
20
10
0
-10
-20
-30

Sydney



North Carolina



Hudson Bay