Time Value of Money: Useful Shortcuts

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Last Time

Time Value of Money

- Compounding

This Time

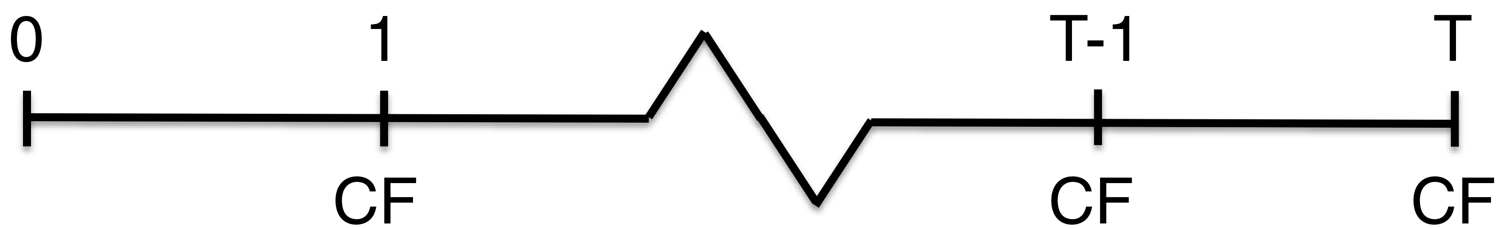
Time Value of Money

- Useful Shortcuts

ANNUITY

Annuity

An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time



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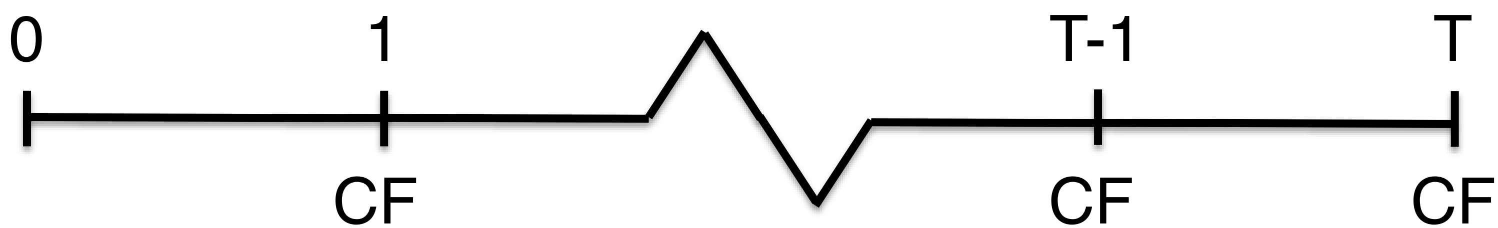

E.g., Savings, vehicle, home mortgage, auto lease, bond payments

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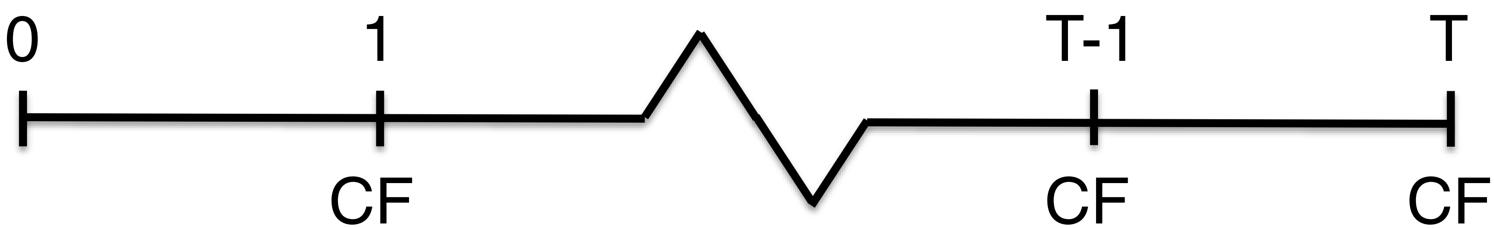

$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

$$= CF \times \underbrace{\frac{1-(1+R)^{-T}}{R}}_{\text{Annuity Factor}}$$

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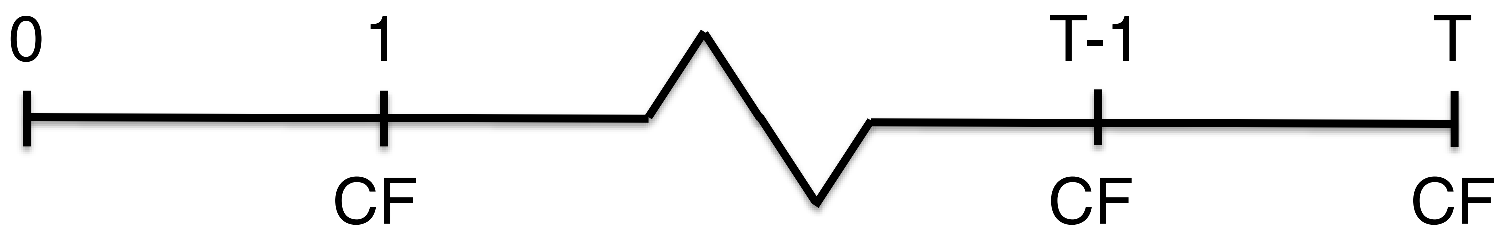

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*The first cash flow arrives one period from today

Example 1 – Savings

How much do you have to save today to withdraw $100 at the end of each of the next 20 years if you can earn 5% per annum?

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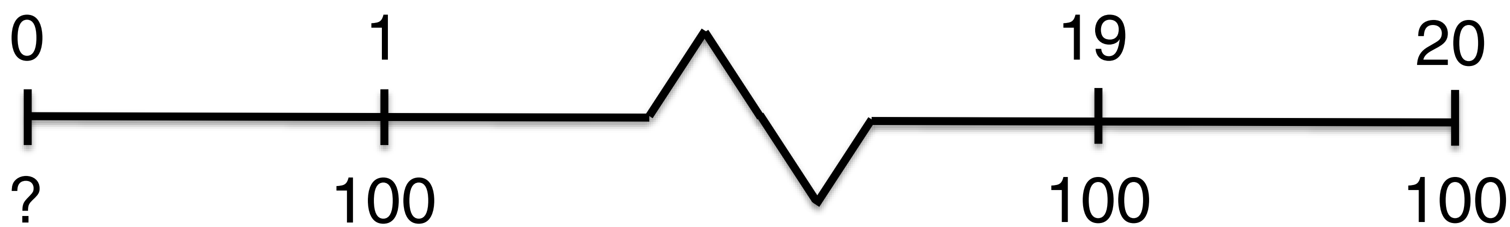

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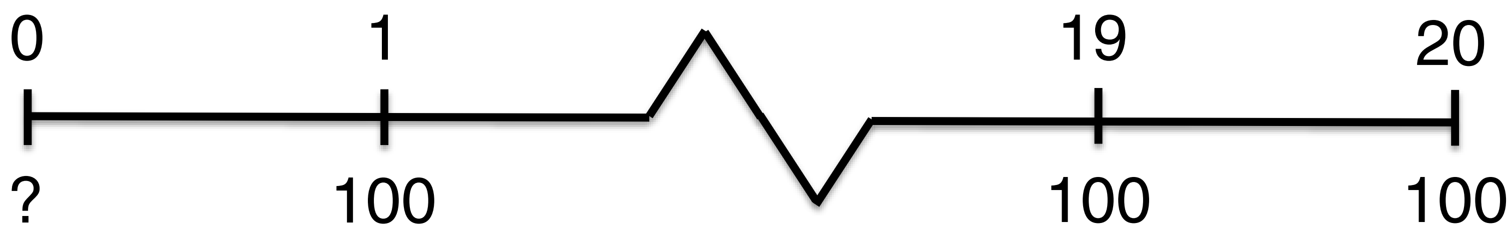
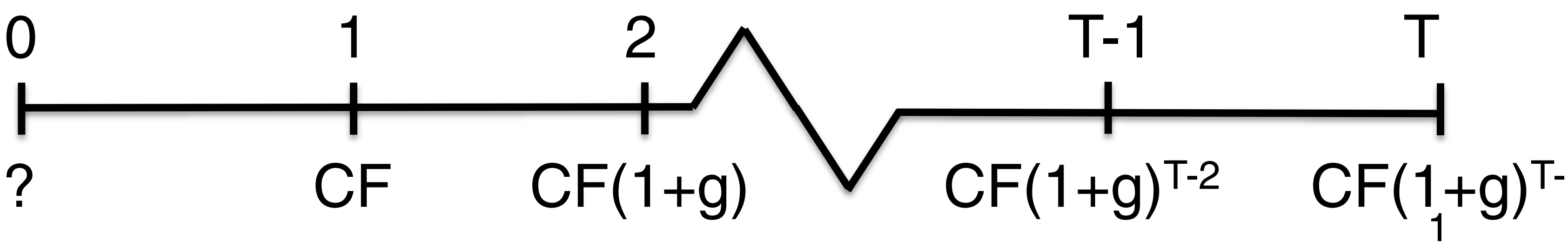


$$\text{PV of Annuity} = \frac{100}{0.05}\left(1-(1+0.05)^{-20}\right) = 1{,}246.22$$

GROWING ANNUITY

Growing Annuity

A growing annuity is a finite stream of cash flows that grow at a constant rate and that are evenly spaced through time



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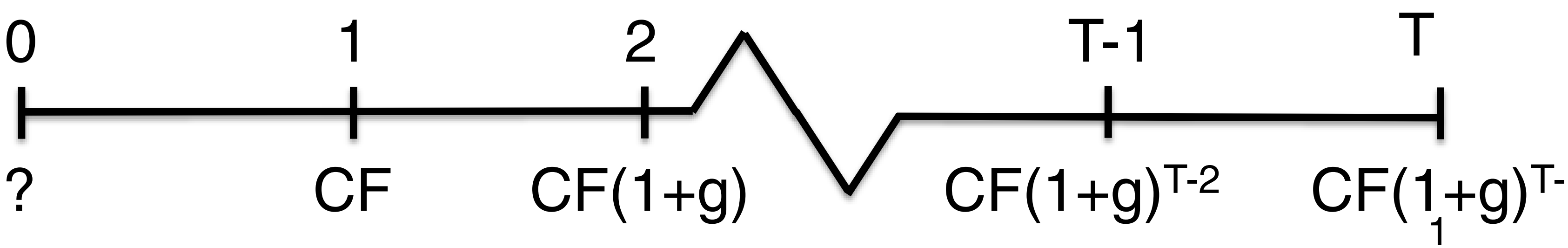

E.g., Income streams, savings strategies, project revenue/expense streams

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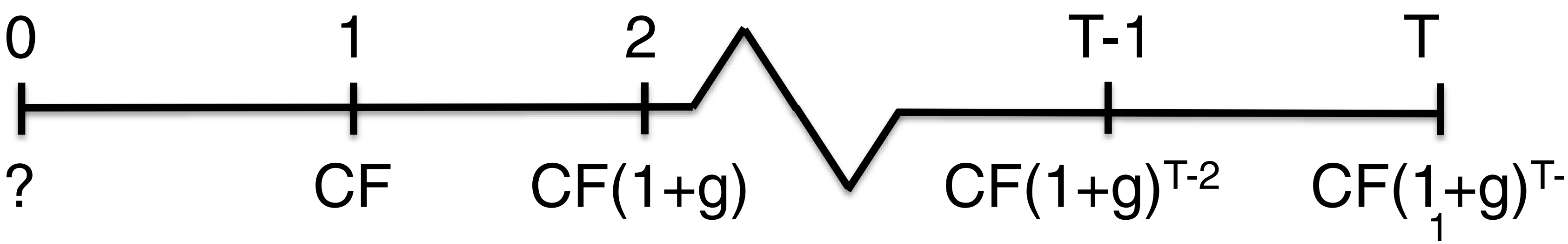

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

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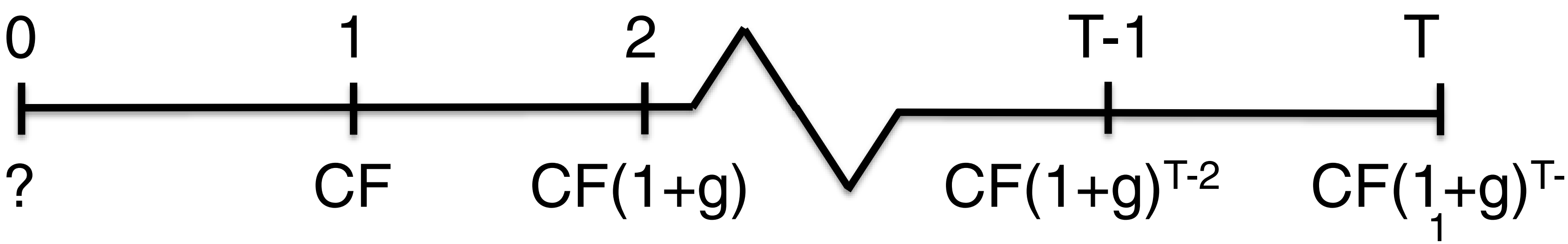

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

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Example 2 – Savings

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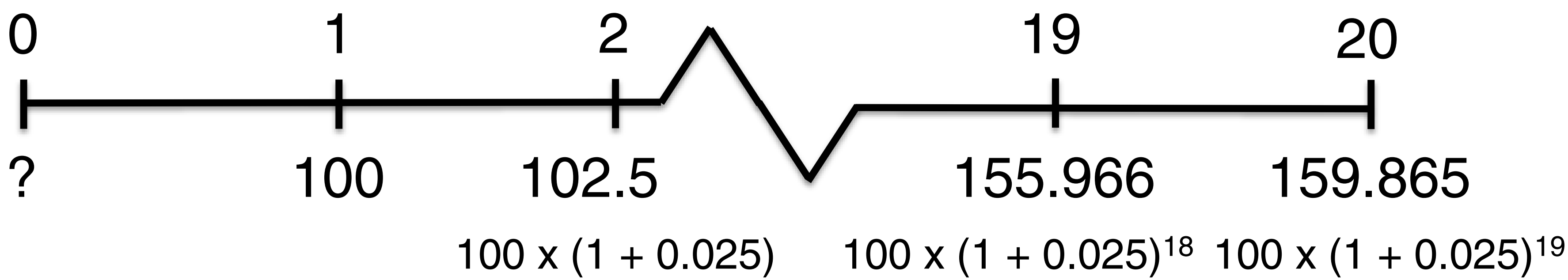

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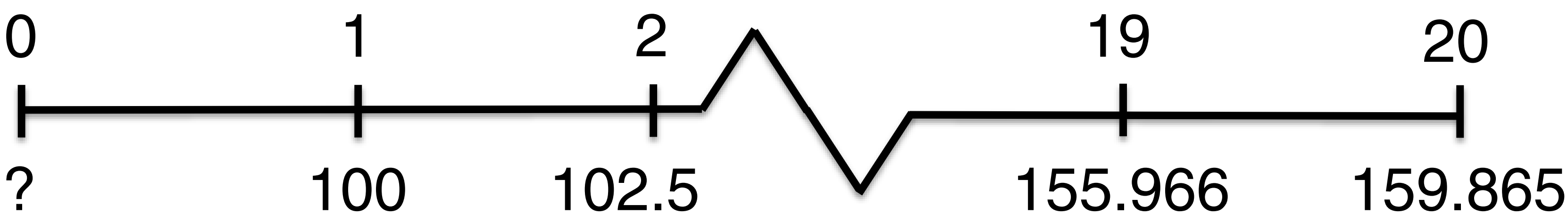


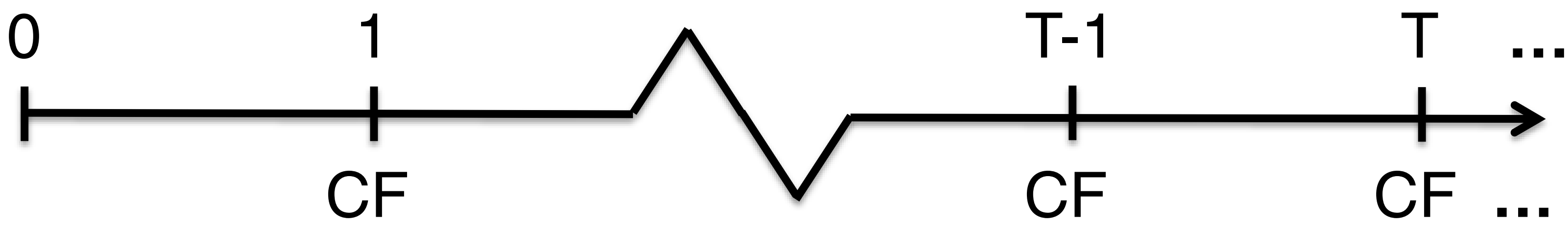
$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

$$= \frac{100}{0.05-0.025}\left(1-\left(\frac{1+0.05}{1+0.025}\right)^{-20}\right) = 1{,}529.69$$

PERPETUITY

Perpetuity

An perpetuity is an infinite stream of cash flows of identical magnitude and equal spacing in time



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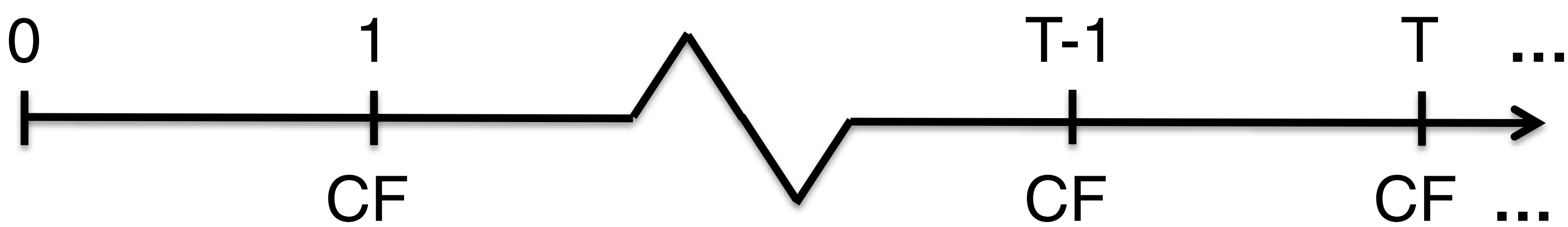

E.g., Perpetuities, consol bonds

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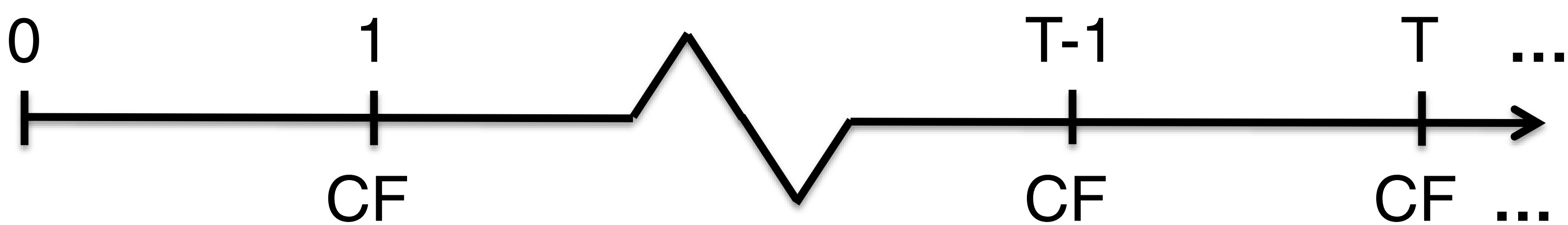

$$\text{PV of Perpetuity} = \frac{CF}{R}$$

Example 3 – Savings

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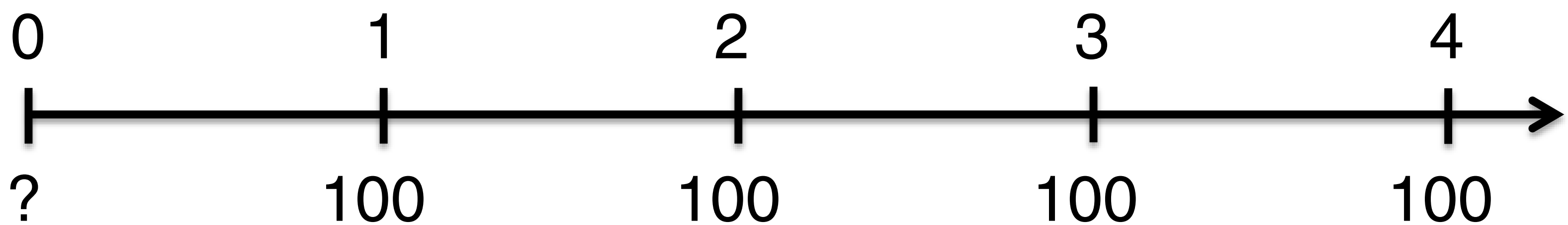

Discount CFs one at a time…impossible!

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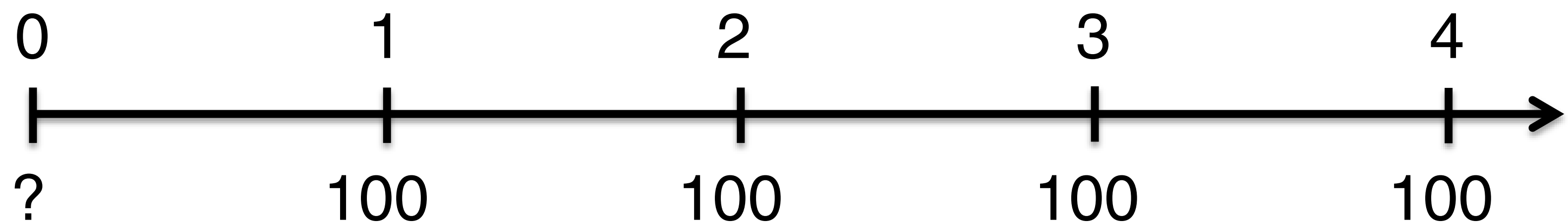

$$\text{PV of Perpetuity} = \frac{100}{0.05} = 2{,}000$$

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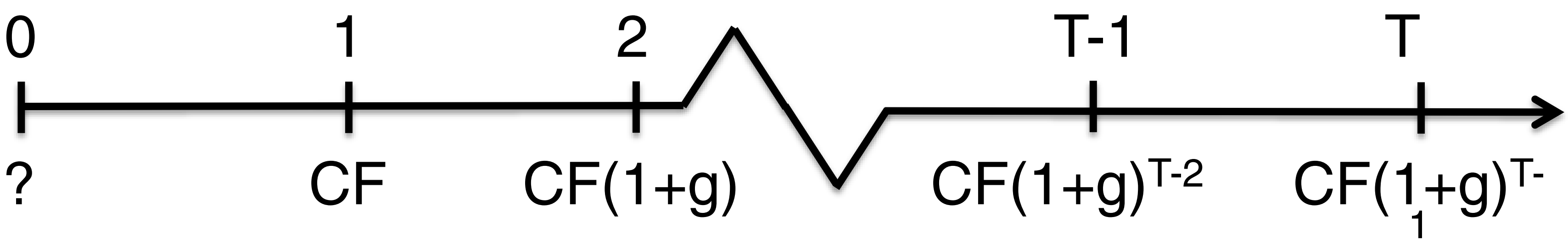

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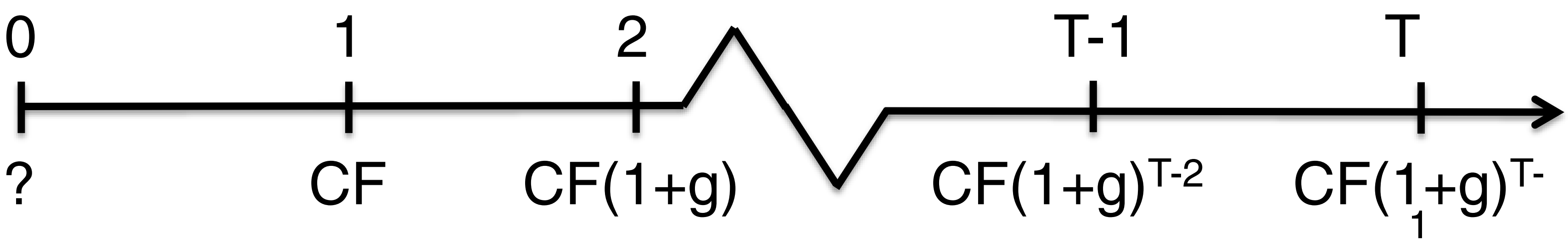

E.g., Dividend streams

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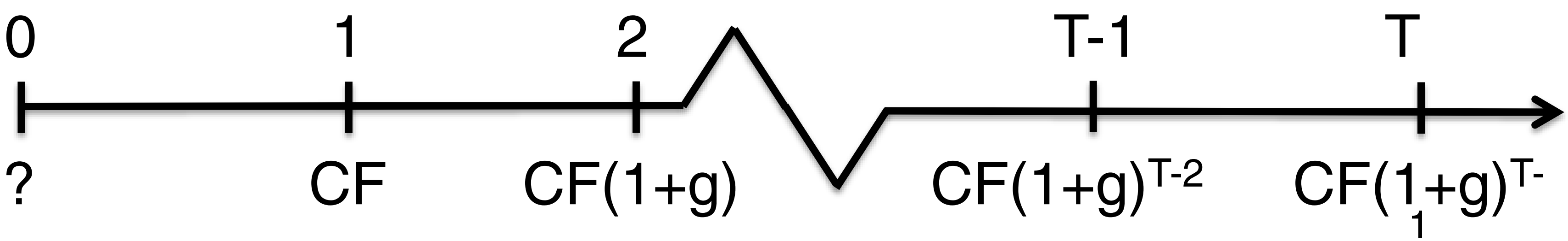

$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g}$$

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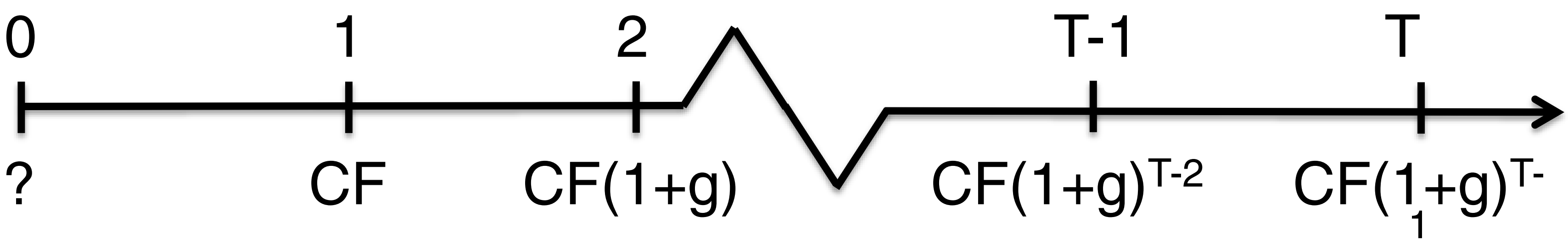

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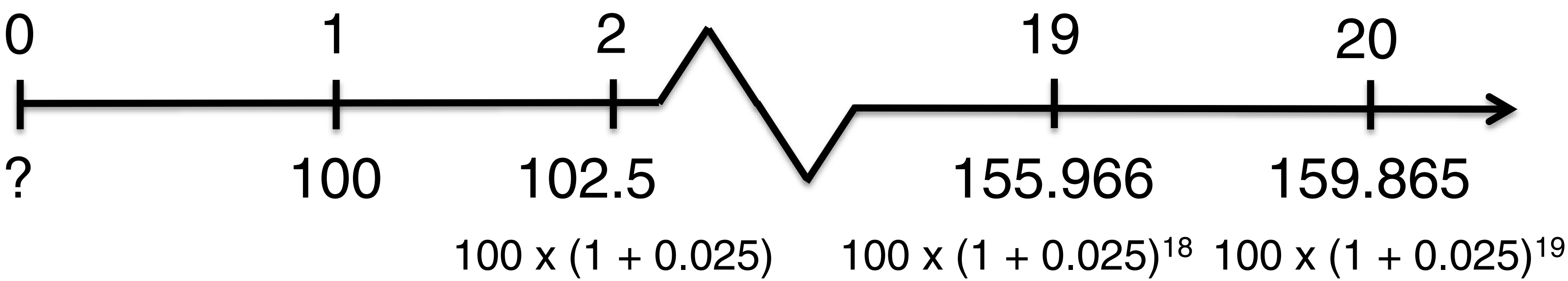

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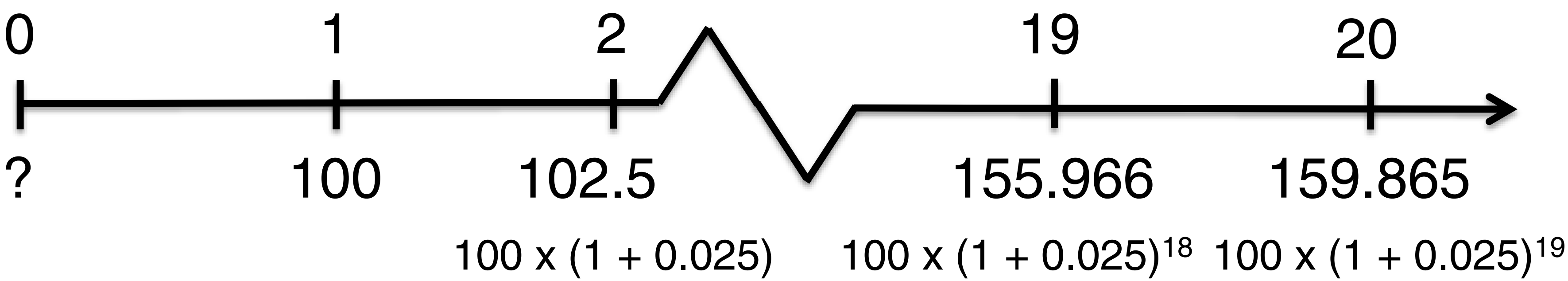

$$\text{PV of Growing Perpetuity} = \frac{CF}{R - g} = \frac{100}{0.05 - 0.025} = 4{,}000$$

Summary

Lessons

- An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time

$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

- A perpetuity is an infinite stream of cash flows of identical magnitude and equal spacing in time

$$\text{PV of Perpetuity} = \frac{CF}{R}$$

Lessons

- A growing annuity is a finite stream of cash flows growing at a constant rate and equally spaced in time

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

- A growing perpetuity is an infinite stream of cash flows growing at a constant rate and equally spaced in time

$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g}$$

Caution

- Annuity and perpetuity formulas assume first cash flow occurs one period from today
- Growth rate, *g,* must be less than the discount rate, *R,* for PV formulas to make sense
- Understand excel functions assumptions

Coming up next

- Problem Set

- Taxes