Time Value of Money: Compounding

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Last Time

Time Value of Money

- Intuition – time units like different currencies
- Tools – time line and discount factor
- Discounting – Moving CFs back in time
- Lesson: Don't add CFs with different time units…ever!

This Time

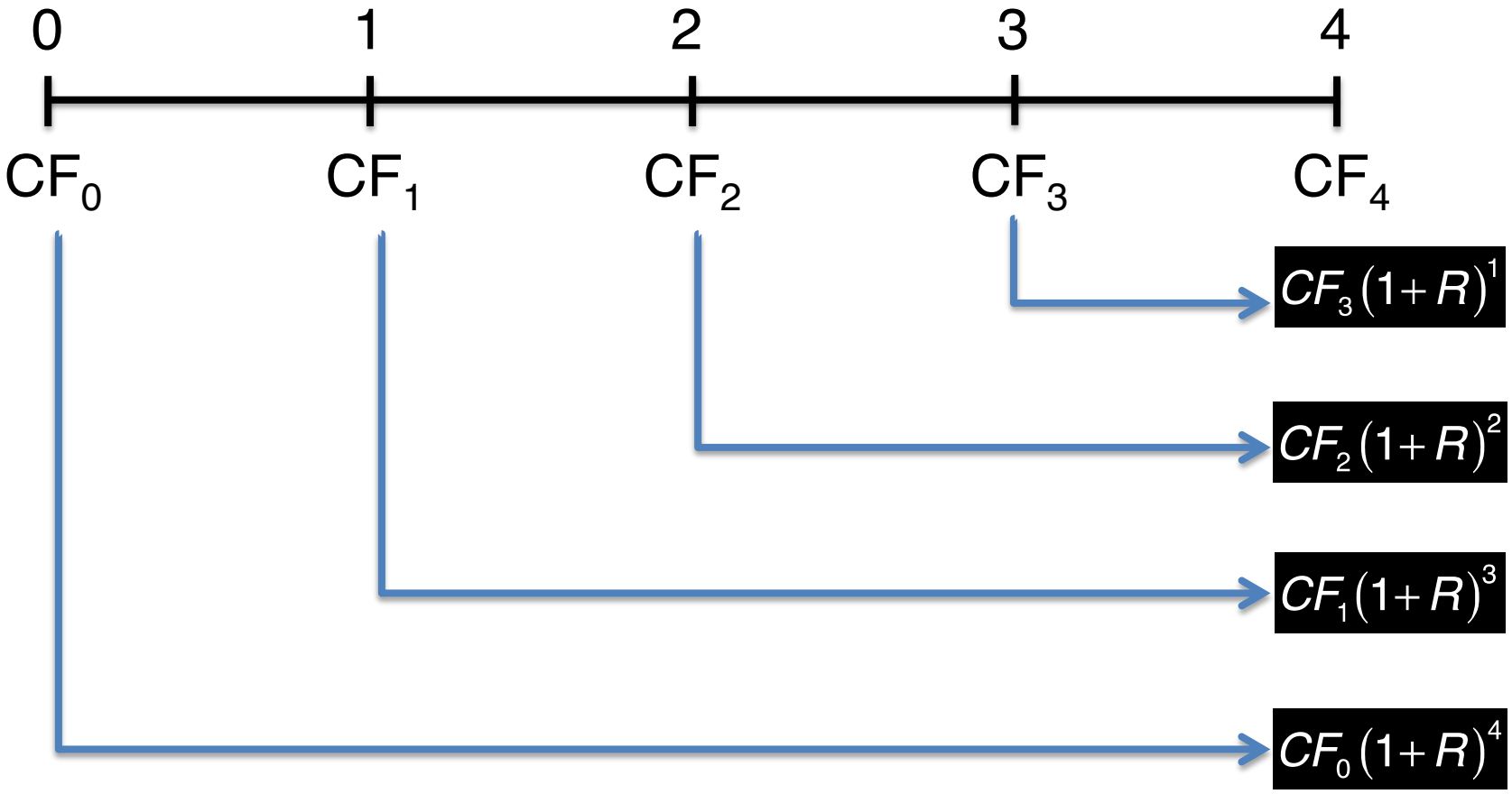
Time Value of Money

- Compounding

USING THE TOOLS: COMPOUNDING

Compounding

Compounding CFs moves them forward in time



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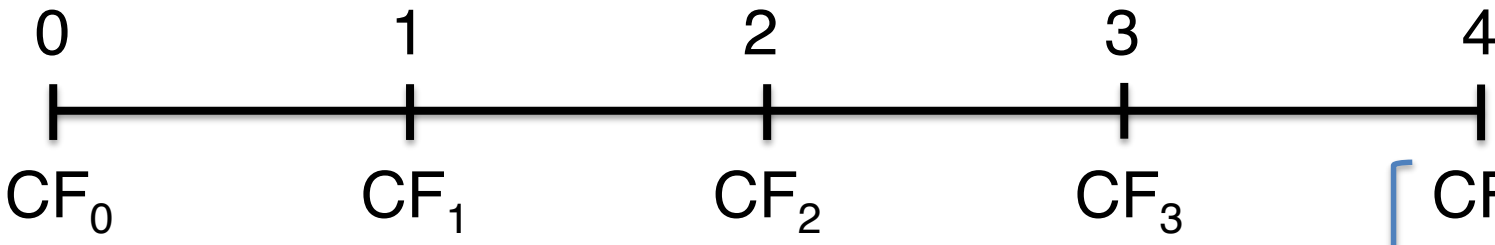
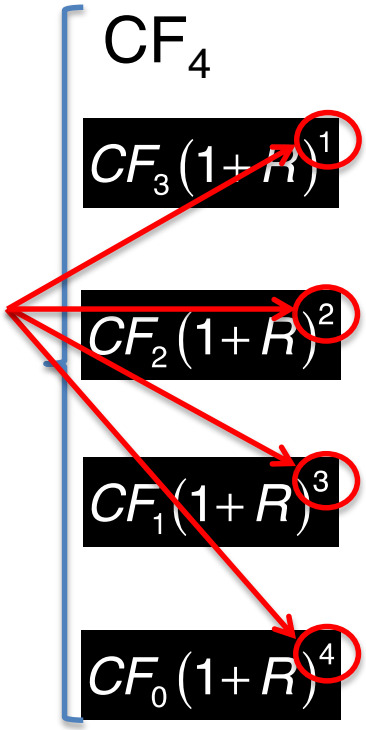


$t > 0$ because we are moving cash flows forward in time



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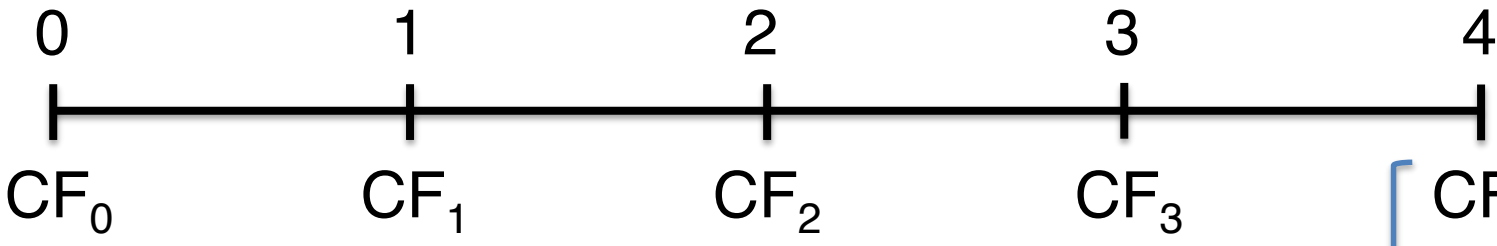


$CF_3(1+R)^1$

$CF_2(1+R)^2$

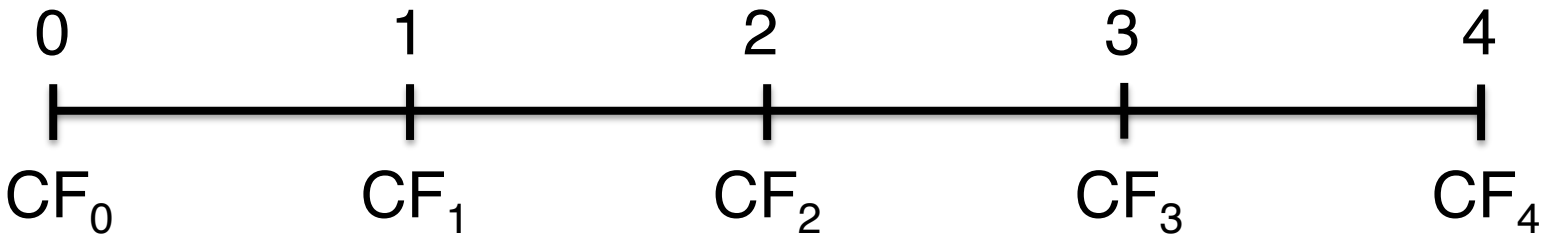
$CF_1(1+R)^3$

$CF_0(1+R)^4$

We can add/subtract these CFs because they are in the same time units (date 4)

Future Value

Future value, $FV_t(\bullet)$ of CFs is compounded value of CFs as of t



These are future values of CFs as of year 4

$$CF_3(1+R)^1 = FV_4(CF_3)$$

$$CF_2(1+R)^2 = FV_4(CF_2)$$

$$CF_1(1+R)^3 = FV_4(CF_1)$$

$$CF_0(1+R)^4 = FV_4(CF_0)$$

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

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This is the future value of the 1,000

Example 2 – Savings

How much money will we have four years from today if we save $100 a year, beginning today, for the next three years, assuming we earn 5% per annum?

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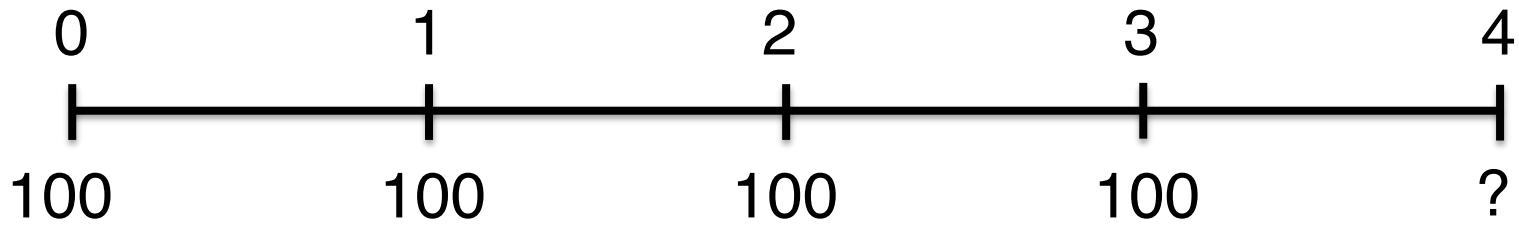
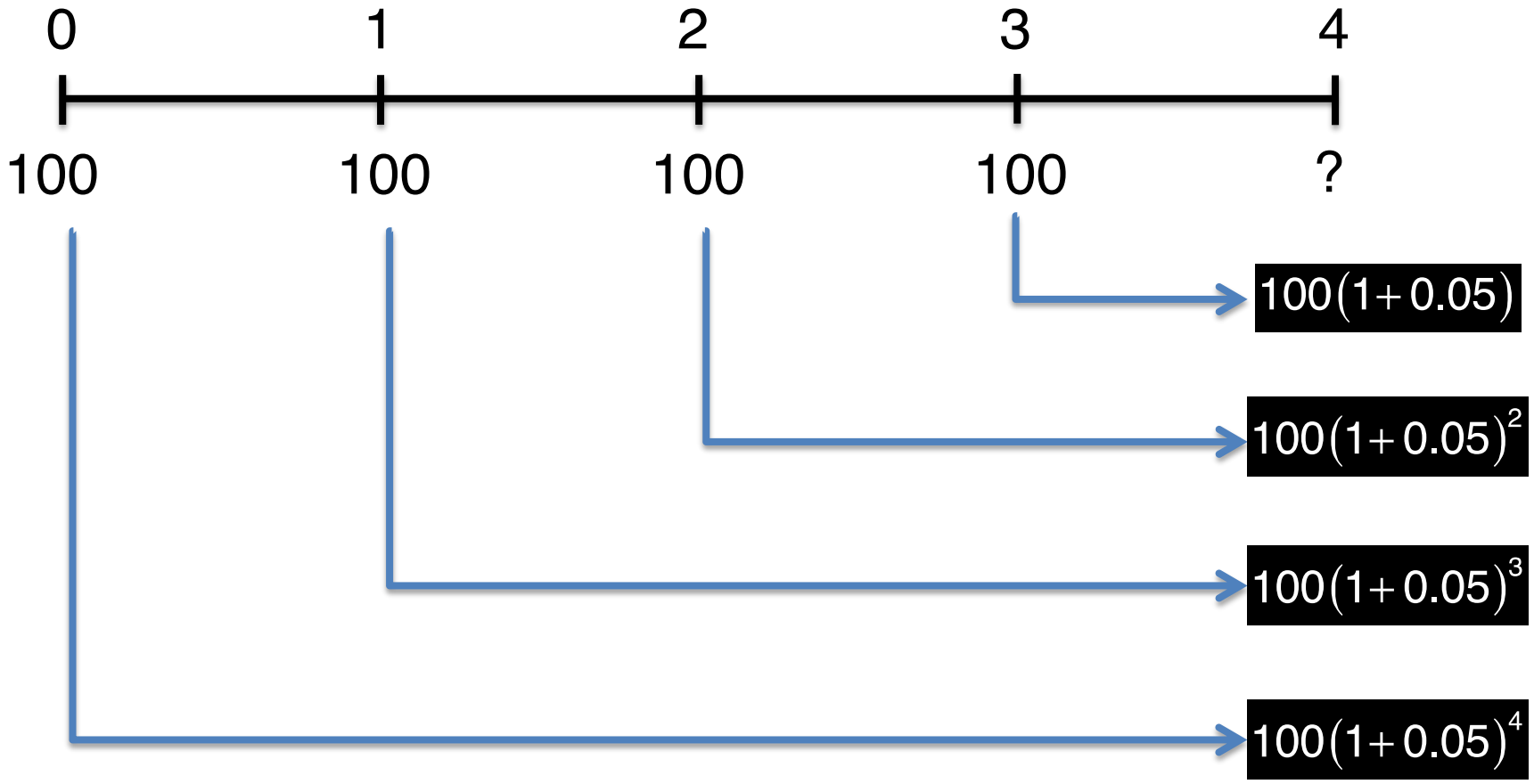


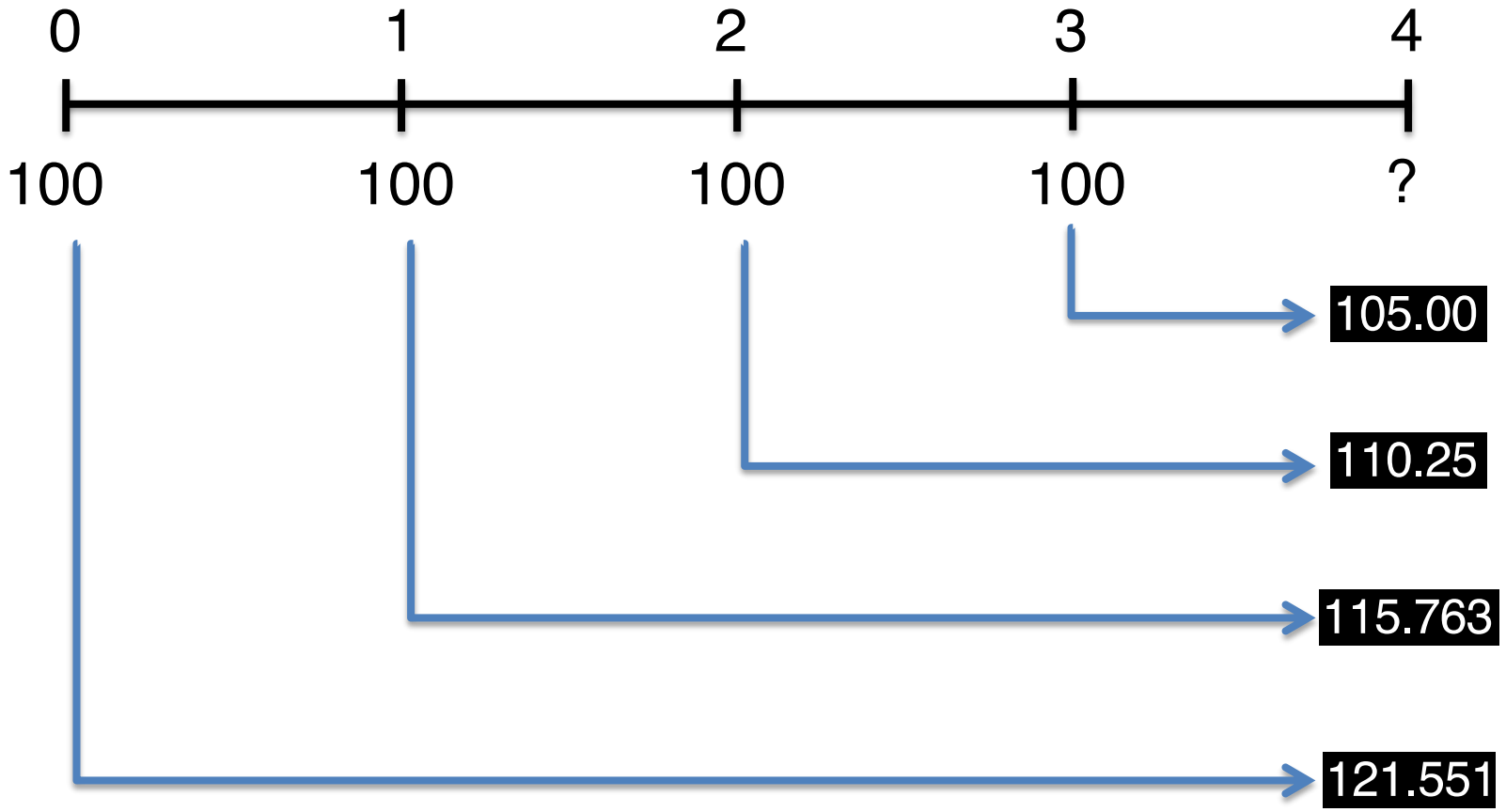
Example 2 – Savings

Step 2: Move CFs forward in time



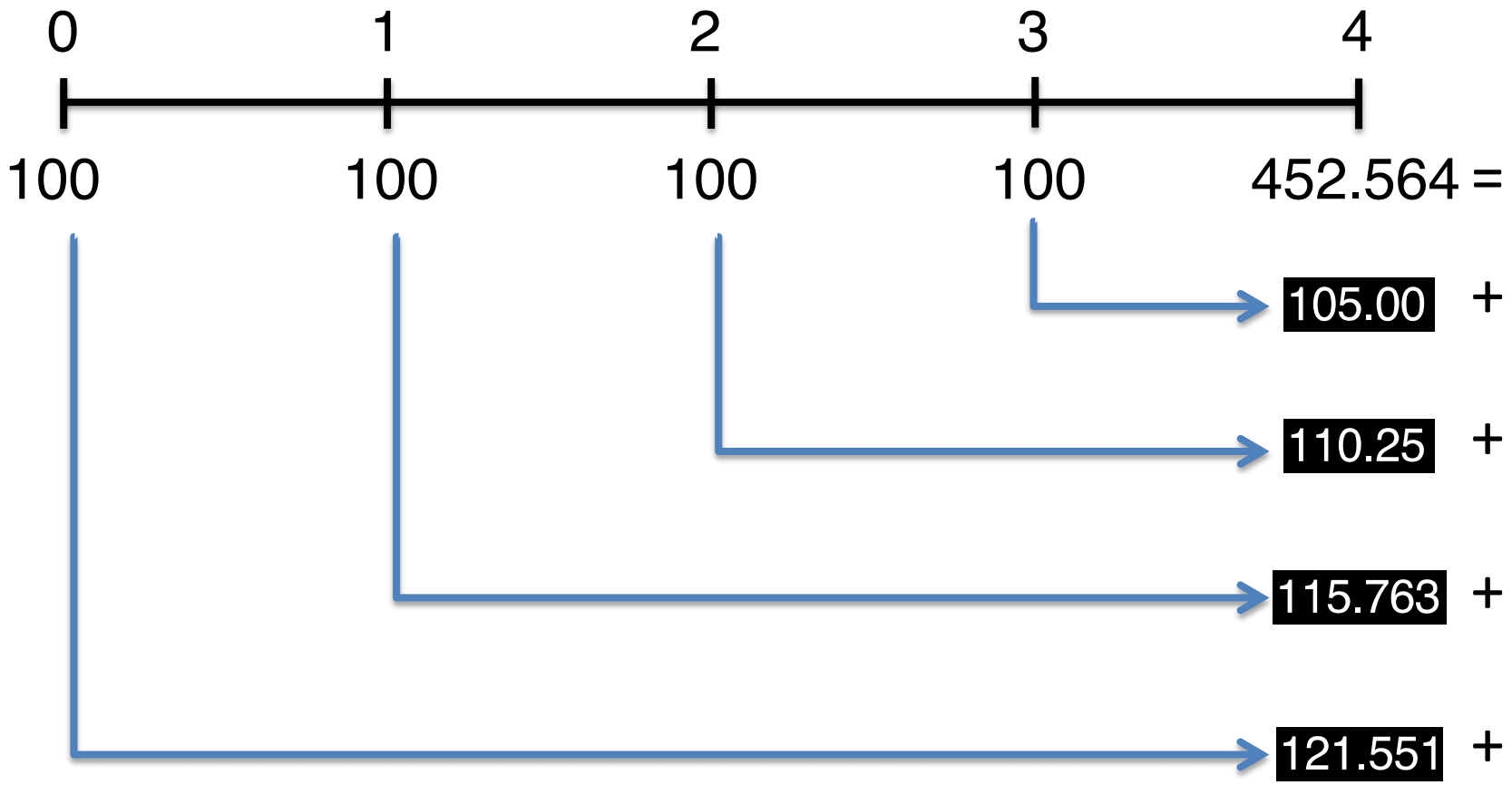
Example 2 – Savings

Step 2: Move CFs forward in time

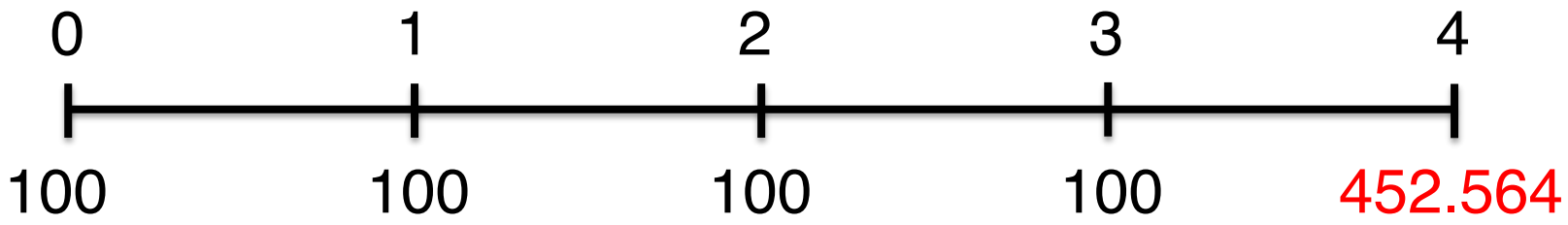


Example 2 – Savings

Step 3: Add up cash flows

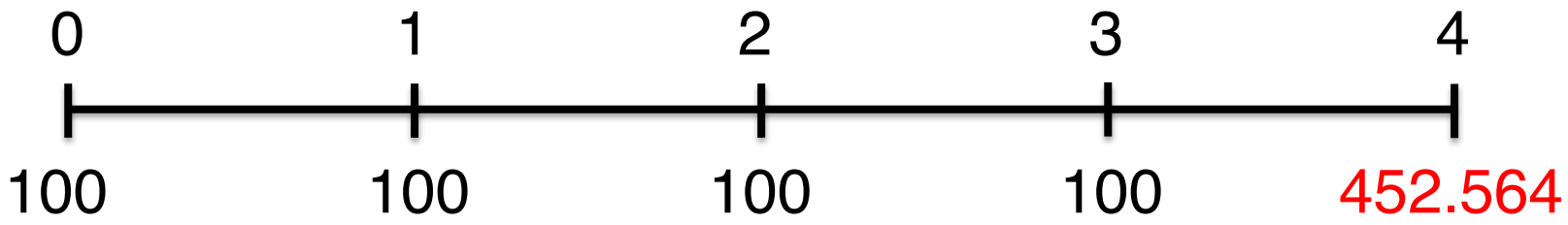


Example 2 – Savings



Interpretation 1: We will have $452.56 at the end of four years if we save $100 starting today for the next three years and our money earns 5% per annum.

Example 2 – Savings



Interpretation 2: The future value four years from today of saving $100 starting today for the next three years at 5% per annum is $452.56.

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00

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Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	[redacted]		

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00		

$$= 100 + 5.00$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00		

$$FV_1(100) = 100 \times (1 + 0.05)^1$$

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Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00

=

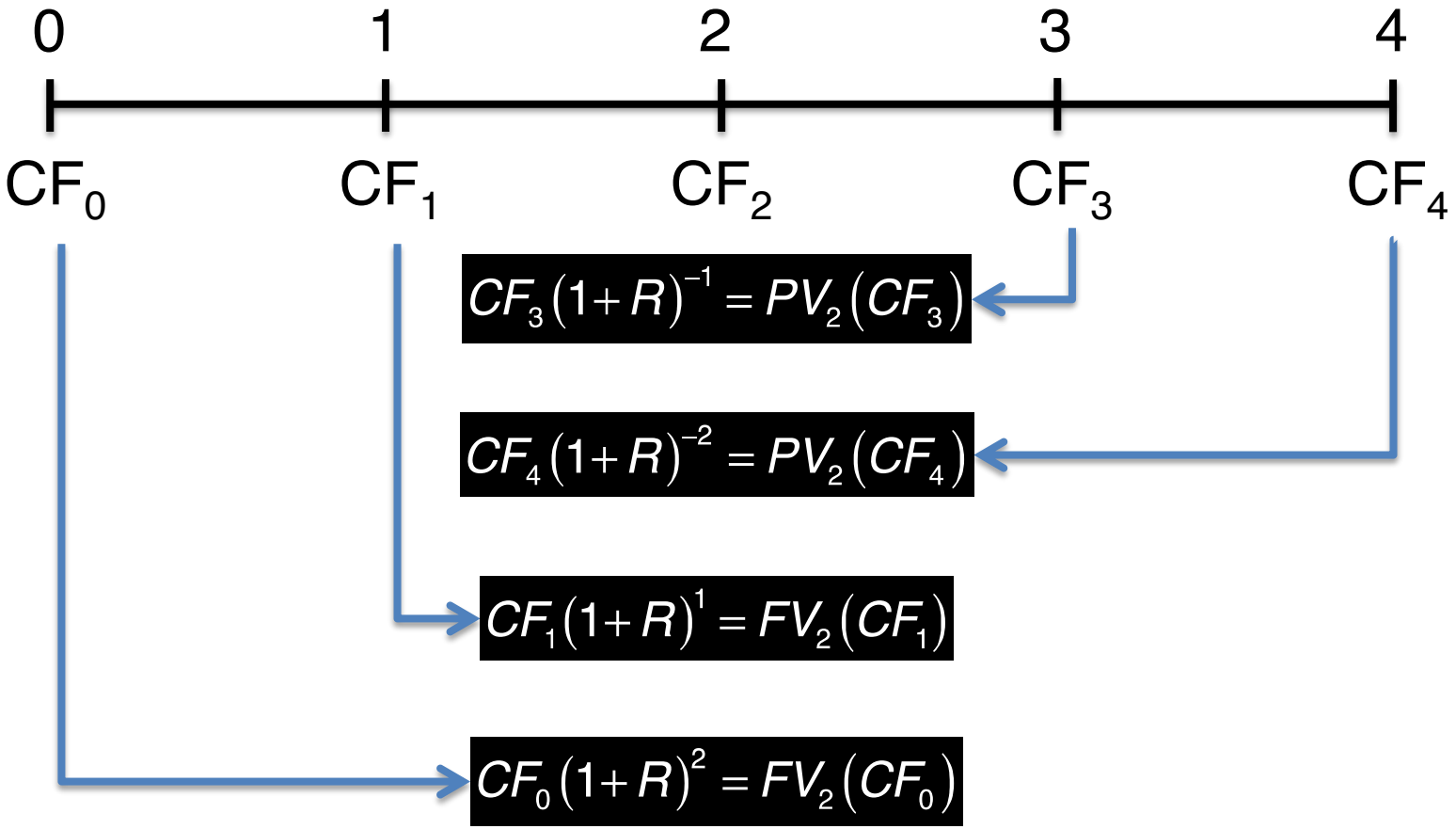
105 + 100

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00
2	$10.25	$215.25	$100.00	$315.25
3	$15.76	$331.01	$100.00	$431.01
4	$21.55	$452.56	$0.00	$452.56

More Generally

Can add CFs at any point in time if same units



Summary

Lessons

- We use compounding to move cash flows forward in time
- Denote the value of cash flows in the future as future value FV_s (CF_t)

$$FV_s(CF_t) = CF_t(1+R)^{s-t} \quad \text{for } t < s$$

Coming up next

- Problem Set

- Useful shortcuts for *PV* and *FV* of common streams of cash flows