# Time Value of Money: Intuition and Discounting

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# This Time

## Time Value of Money

- Intuition, tools, and discounting

# Intuition

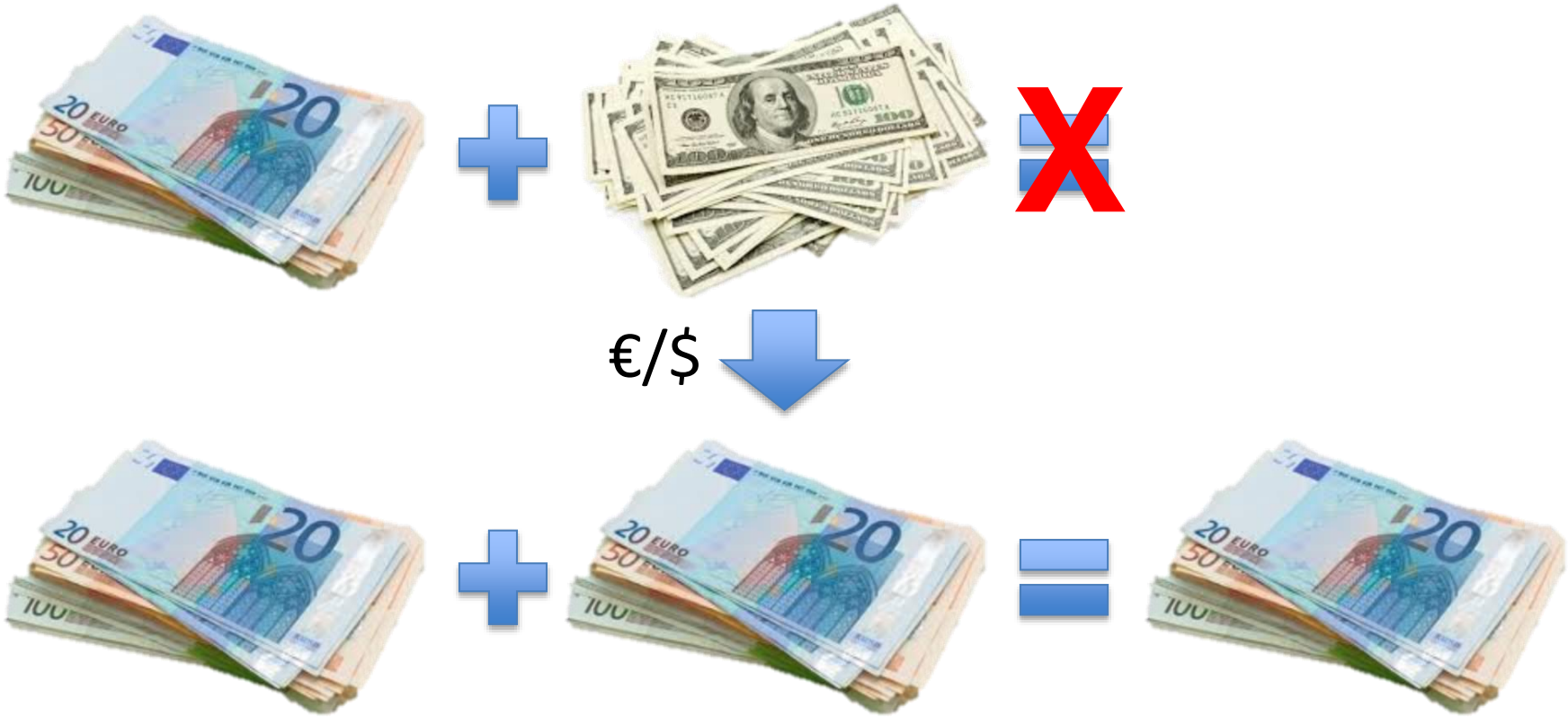
# Currency

# Currency

# Currency

$/€

# Currency



# Currency

# Currency

¥/€ ¥/$

# Currency

$/€ $/¥

# Currency


# Messages (Look up)

1. Can't add/subtract different currencies

2. Must convert currencies to common (base) currency using exchange rate

# Time Value of Money

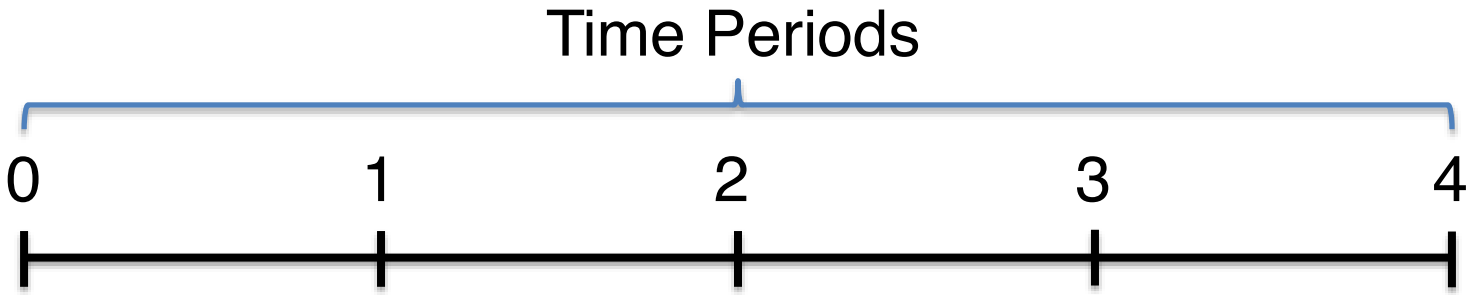
# Time Value of Money

- Money received/paid at different times is like different currencies
  - Money has a time unit

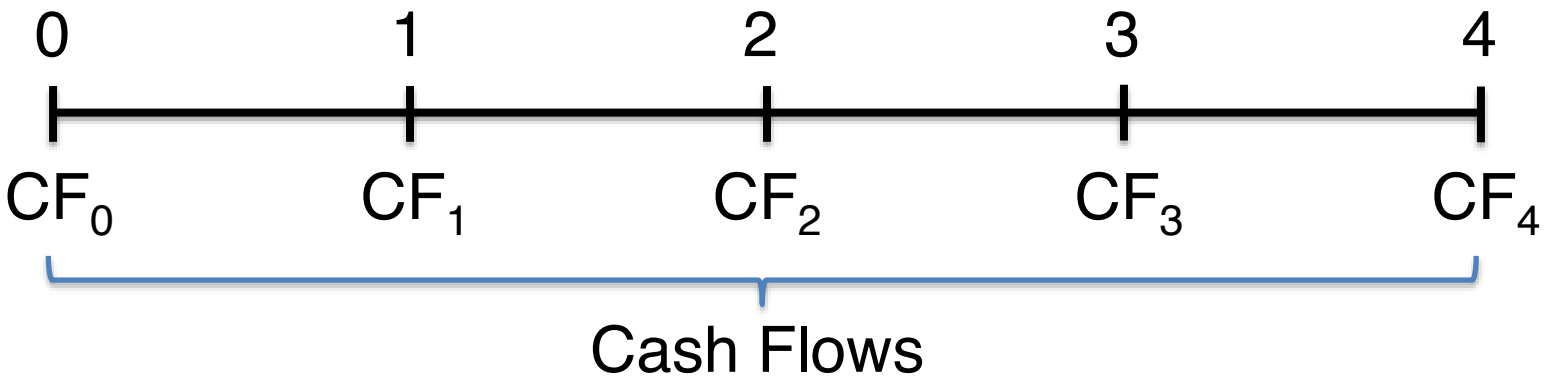
- Must convert to common/base unit to aggregate
  - Need exchange rate for time

# THE TOOLS: TIME LINE & DISCOUNT FACTOR

# Time Line



# Time Line



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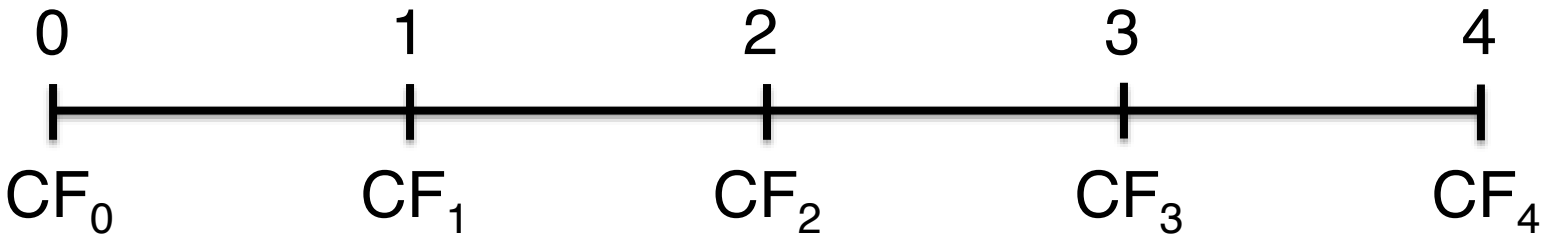
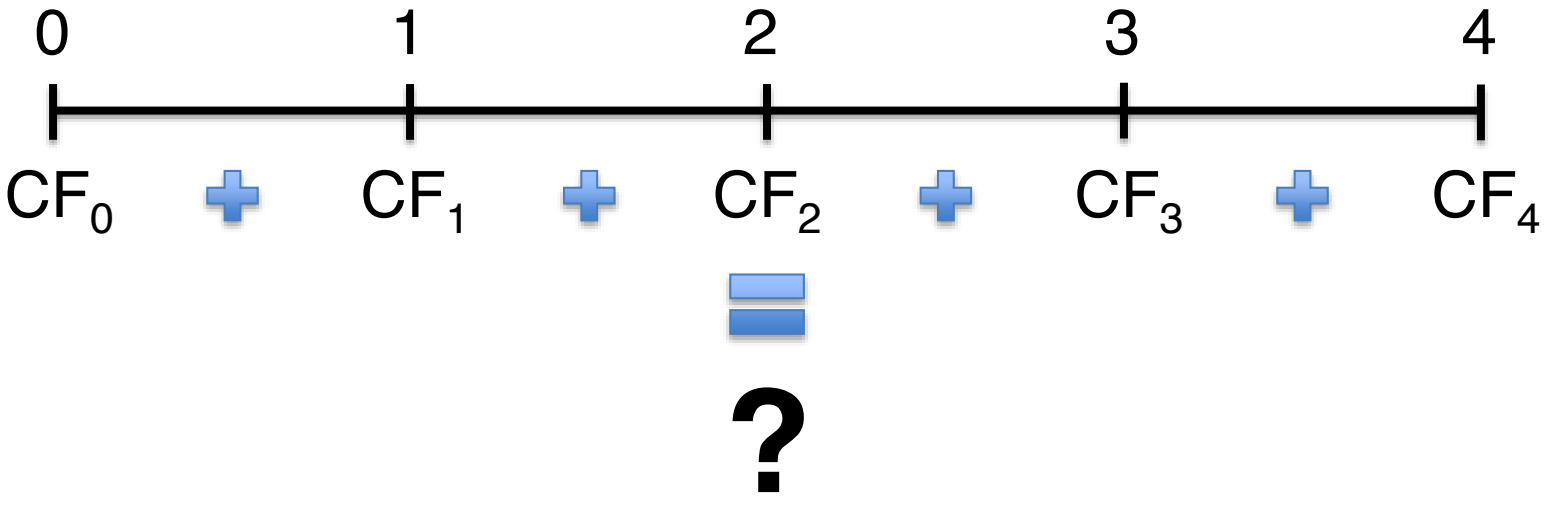


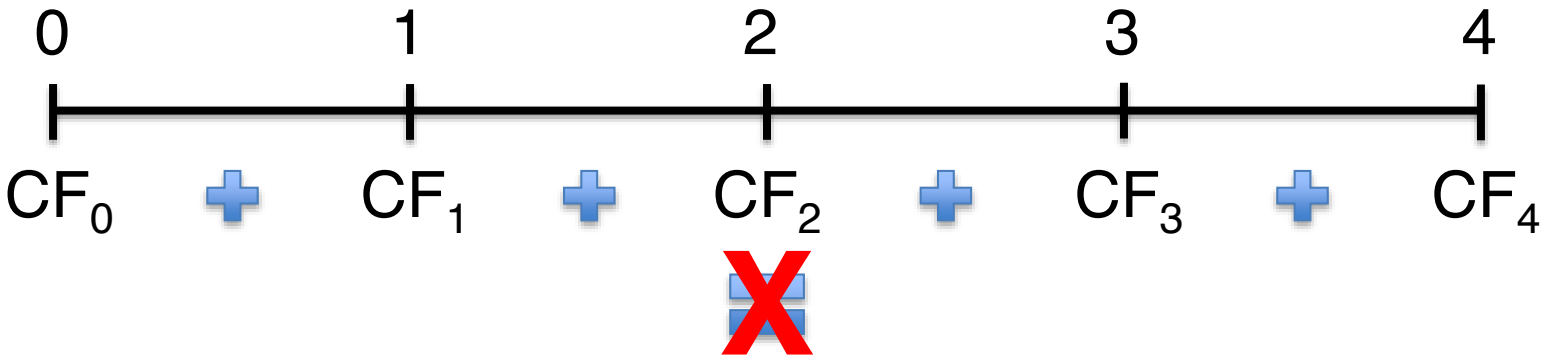

Lesson: Get in the habit of placing cash flows on a time line

# Aggregating Cash Flows



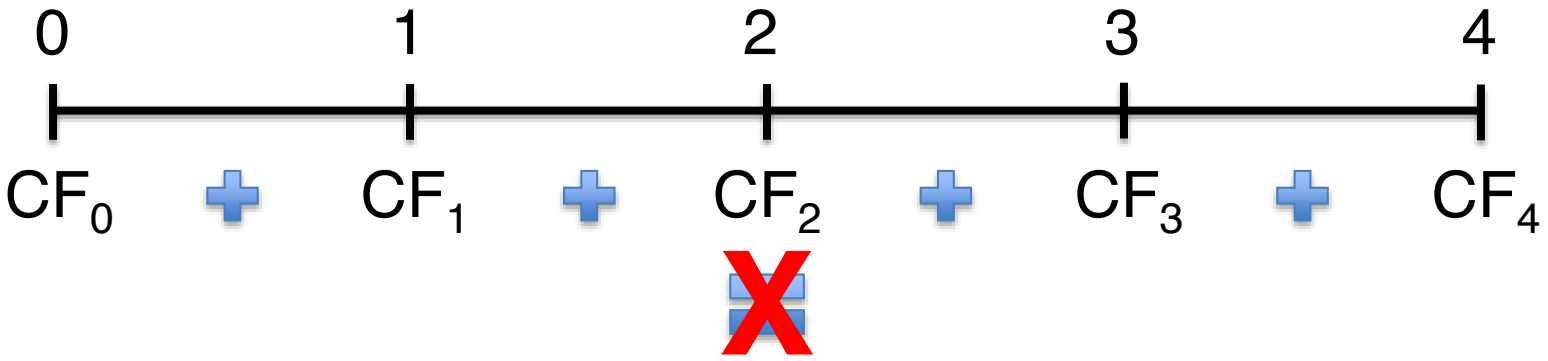

Can we add/subtract cash flows in different time periods

# Aggregating Cash Flows



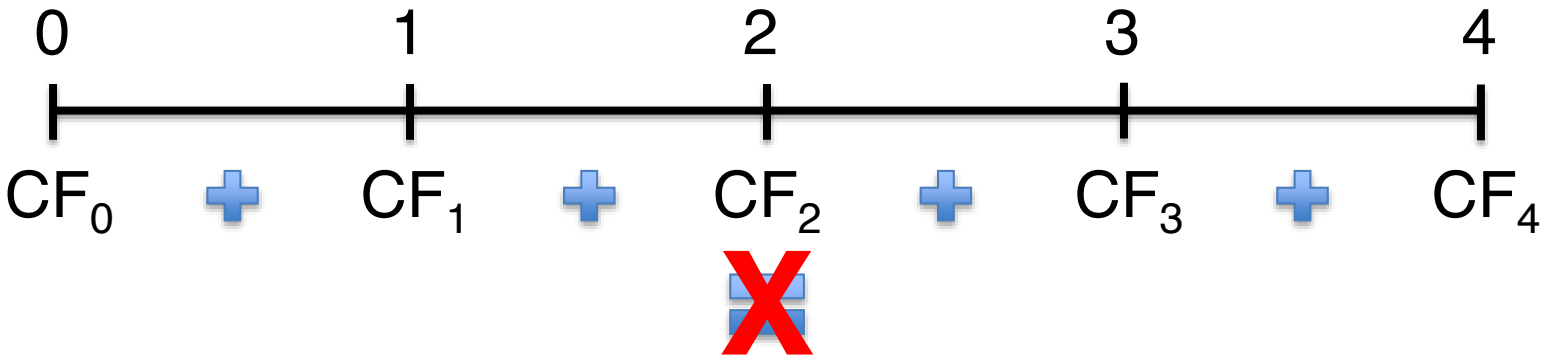

**No!**

# Aggregating Cash Flows




Lesson: Never* add/subtract cash flows received at different times

# Aggregating Cash Flows




Need exchange rate for time to convert to common time unit

# Discount Factor

The discount factor is our exchange rate for time

$$(1+R)^t$$

$t$ = time periods into future ($t > 0$) or past ($t < 0$) to move CFs

$R$ = …

Definition: $R$ is the rate of return offered by investment alternatives in the capital markets of equivalent risk.

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A.k.a., discount rate, hurdle rate, opportunity cost of capital

To determine *R*, consider the risk of the cash flows that you are discounting.

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| Investment | Average Annual Return, *R* |
|---|---|
| Treasury-Bills (30-Day) | 3.49% |
| Treasury-Notes (10-Year) | 5.81% |
| Corporate Bonds (Investment Grade) | 6.60% |
| Large-Cap Stocks | 11.23% |
| Mid-Cap Stocks | 15.15% |
| Small-Cap Stocks | 25.32% |

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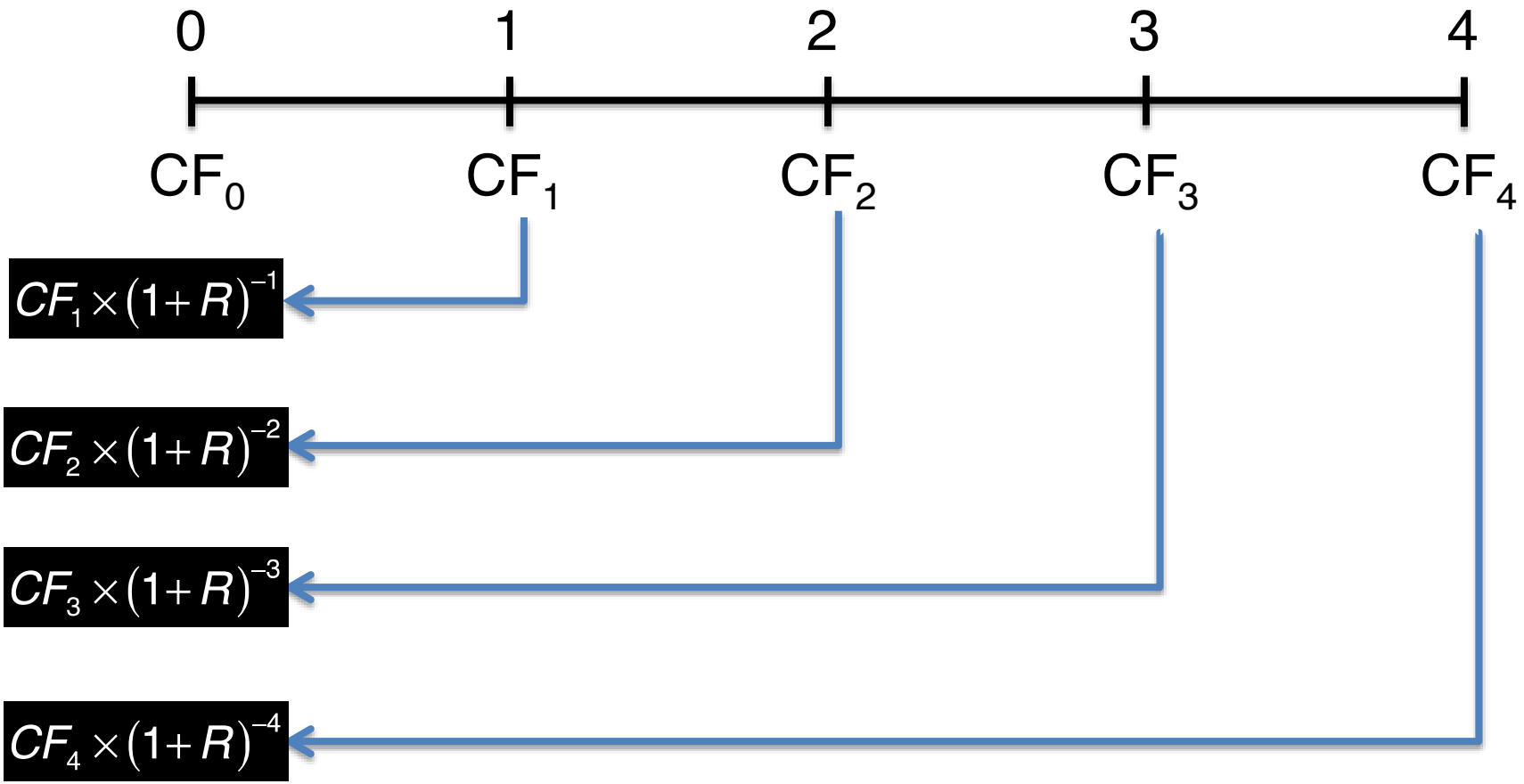
Riskier investment, higher return

# USING THE TOOLS: DISCOUNTING

# Discounting

Discounting CFs moves them back in time



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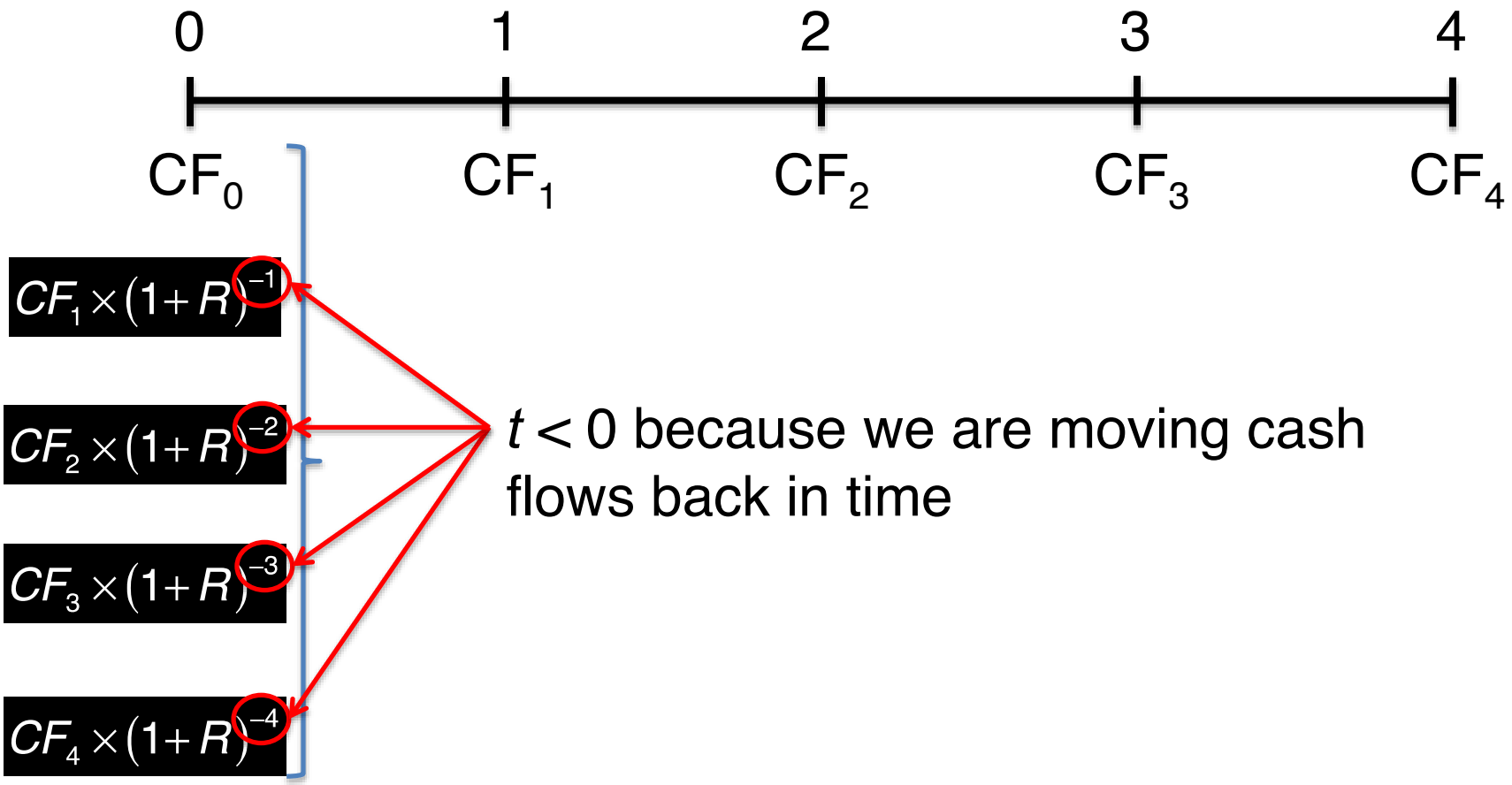

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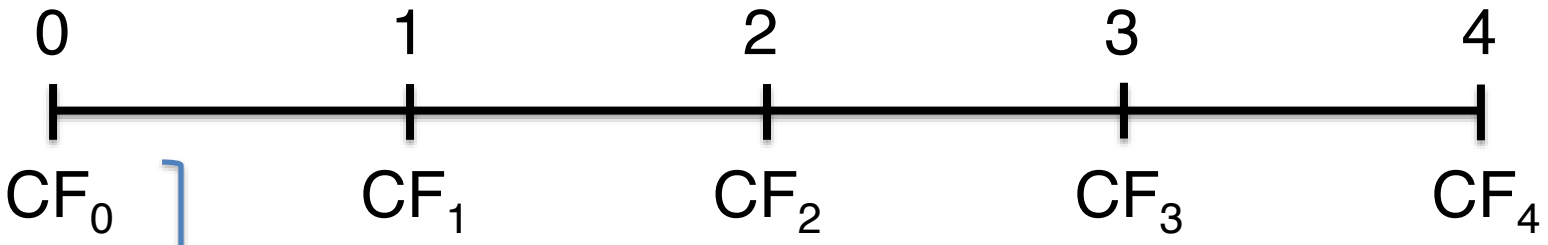



$CF_1 \times (1+R)^{-1}$

$CF_2 \times (1+R)^{-2}$

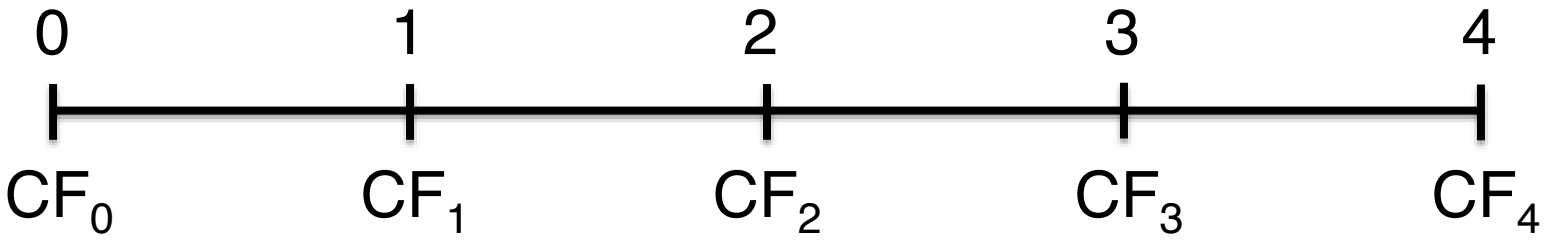
$CF_3 \times (1+R)^{-3}$

$CF_4 \times (1+R)^{-4}$

We can add/subtract these CFs because they are in the same time units (date 0)

# Present Value

Present value, $PV_t(\bullet)$ of CFs is discounted value of CFs as of t




$$CF_1 \times (1+R)^{-1} = PV_0(CF_1)$$

$$CF_2 \times (1+R)^{-2} = PV_0(CF_2)$$

$$CF_3 \times (1+R)^{-3} = PV_0(CF_3)$$

$$CF_4 \times (1+R)^{-4} = PV_0(CF_4)$$

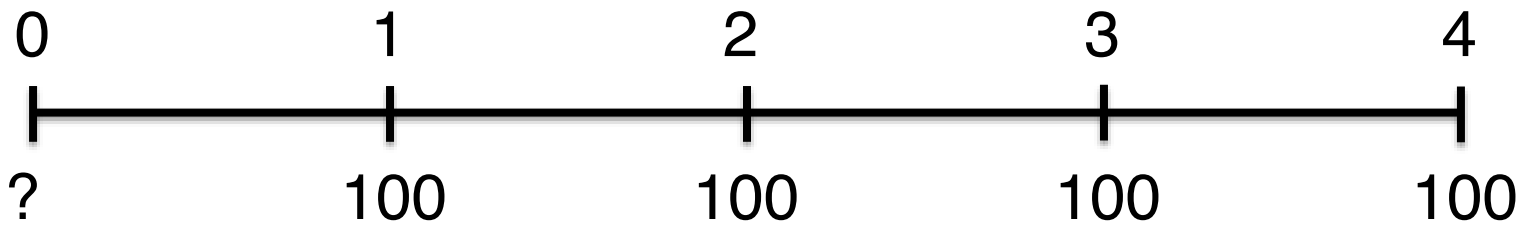
These are present values of future CFs as of today (period 0)

# Example – Savings

How much do you have to save today to withdraw $100 at the end of each of the next four years if you can earn 5% per annum?

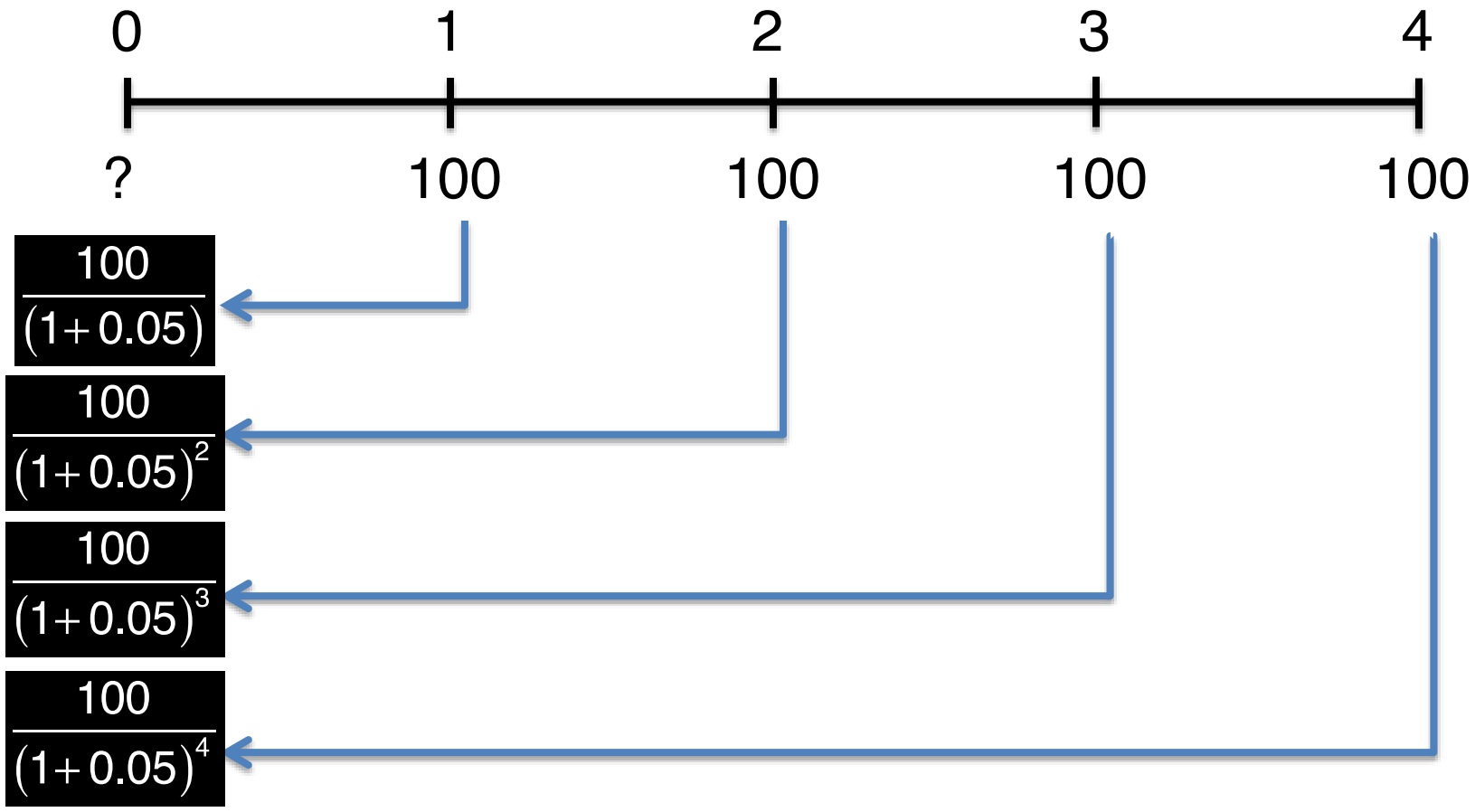
# Example – Savings

Step 1: Put cash flows on a time line



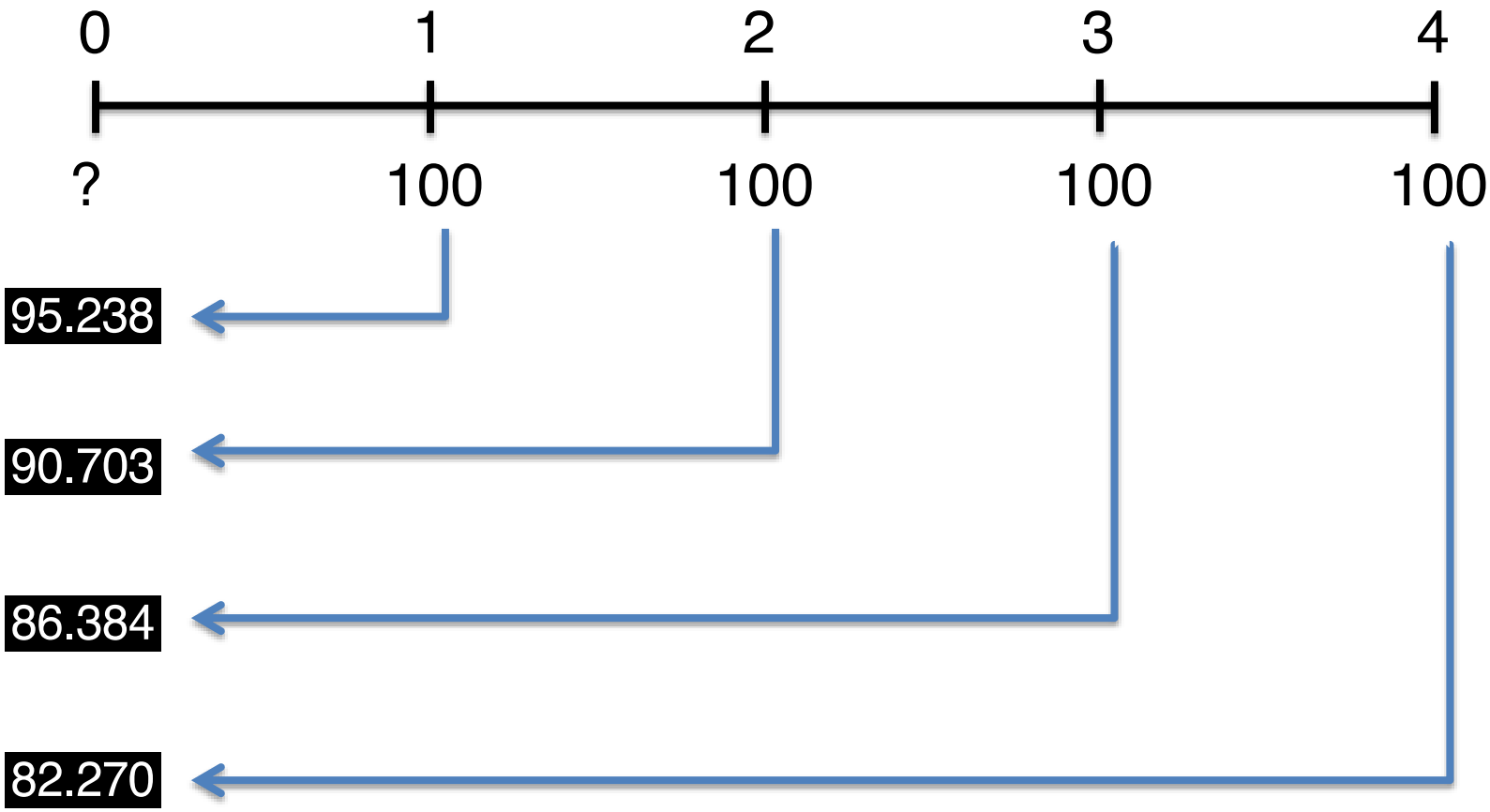
# Example – Savings

## Step 2: Move CFs back in time to today



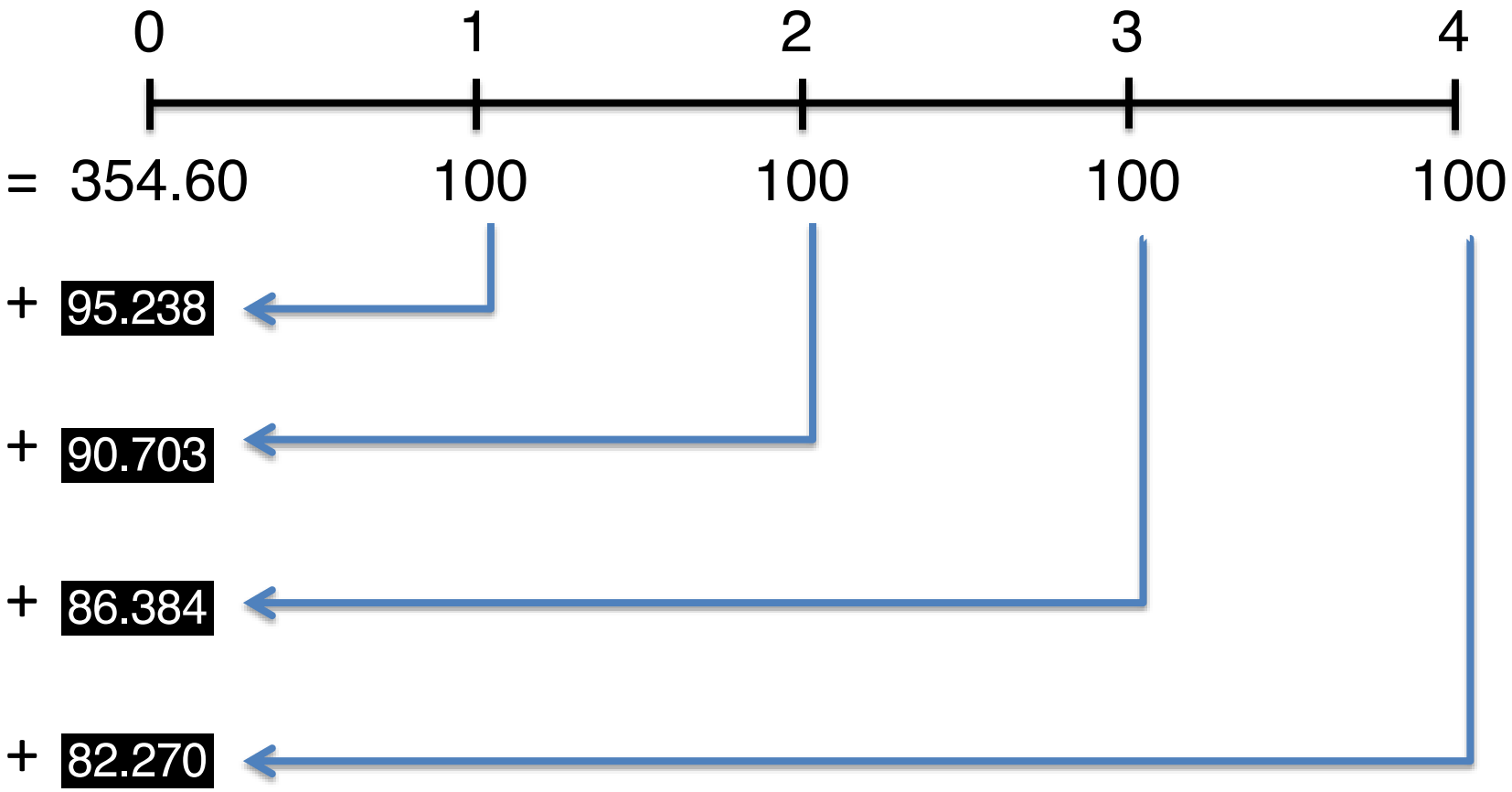
# Example – Savings

## Step 2: Move CFs back in time to today

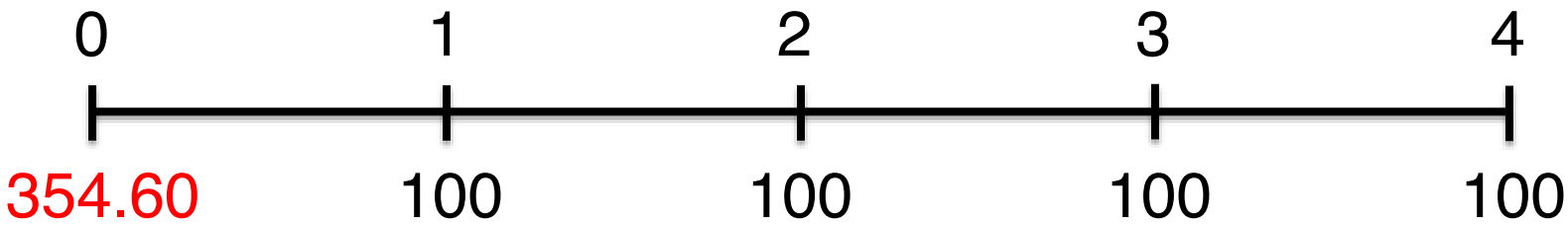


# Example – Savings

## Step 3: Add up CFs (all in time 0 units)

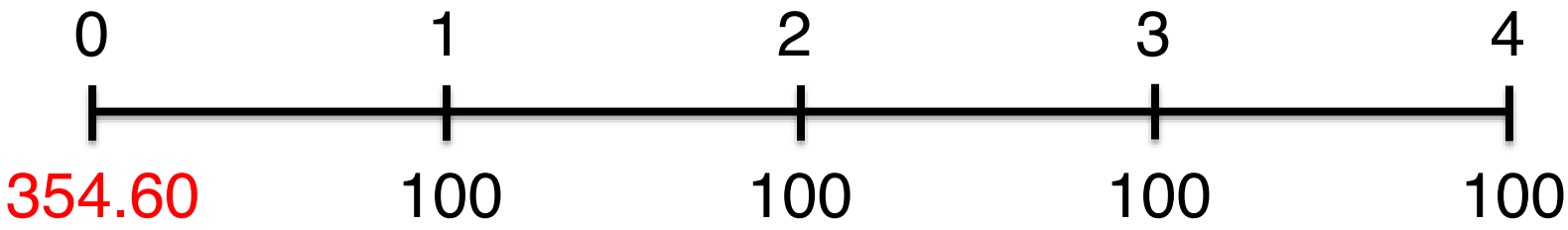


# Example – Savings



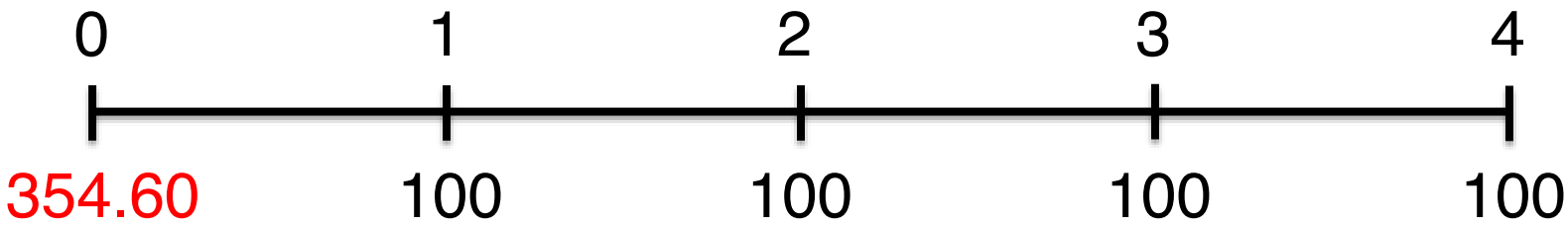

Interpretation 1: We need $354.60 today in an account earning 5% each year so that we can withdraw $100 at the end of each of the next four years

# Example – Savings




Interpretation 2: The present value of $100 received at the end of each of the next four years is $354.60 when the discount rate is 5%.

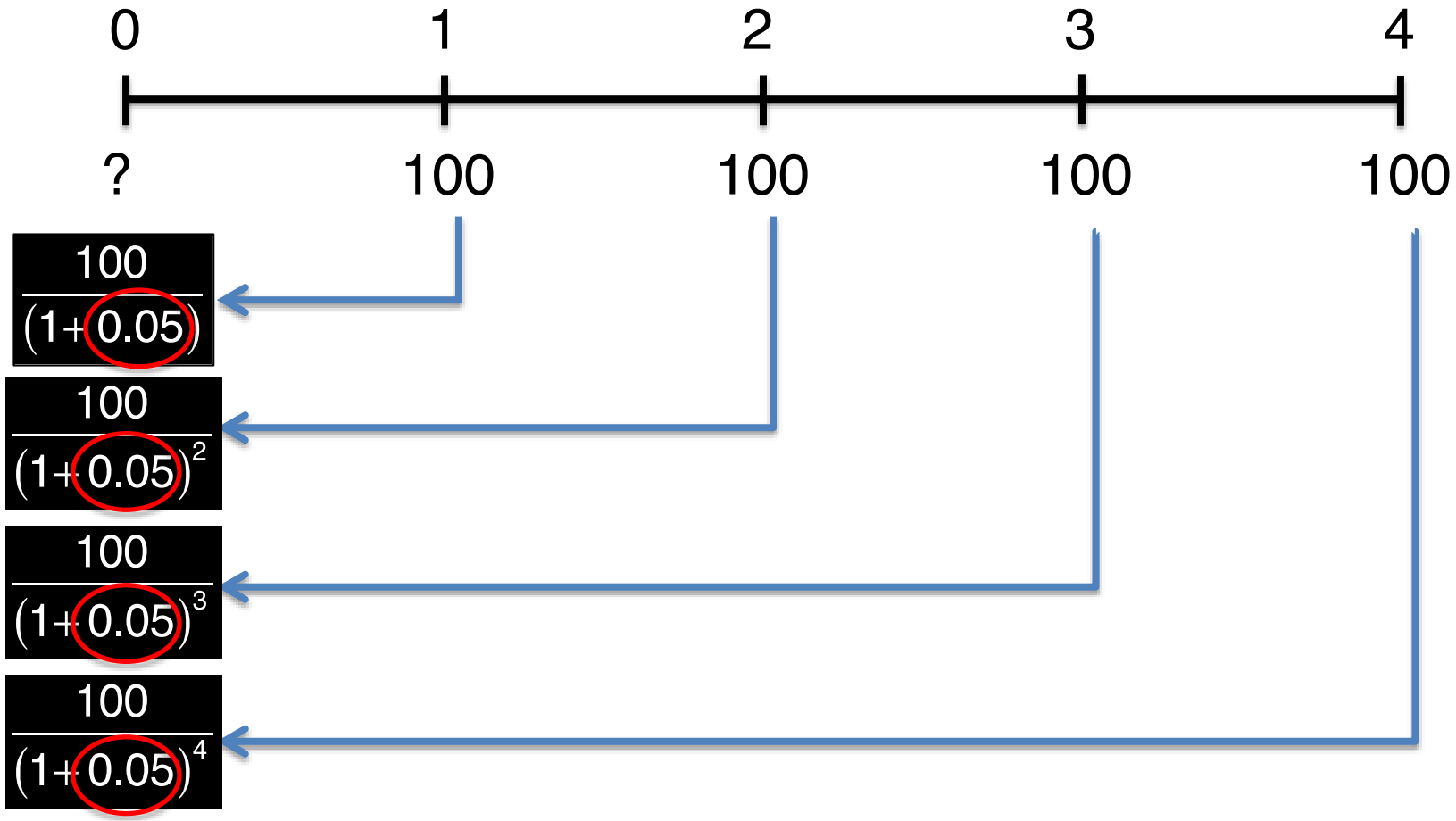
# Example – Savings



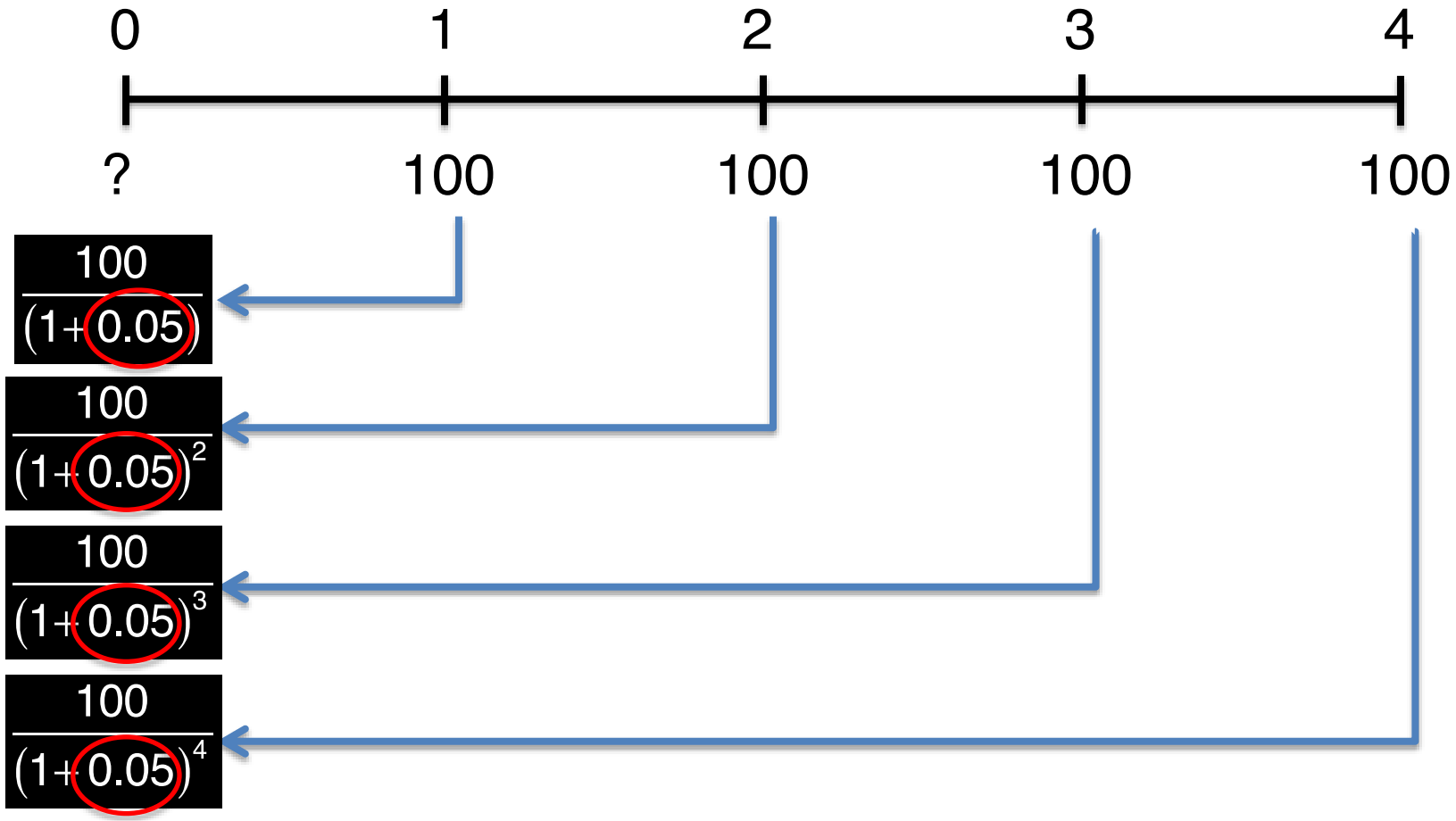

Interpretation 3: Today's price for a contract that pays $100 at the end of each of the next four years is $354.60 when the discount rate is 5%.

Comment: We are assuming that the discount rate, $R$, is constant over time.

# Comment: We are assuming that the discount rate, *R*, is constant over time.



# Comment: We are assuming that the discount rate, *R*, is constant over time.




Common assumption but still an *assumption*

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawl Balance | Withdrawal | Post-Withdrawl Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawal Balance | Withdrawal | Post-Withdrawal Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | | $354.60 \times 0.05$ | |

*Activity happens at end of the period

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawal Balance | Withdrawal | Post-Withdrawal Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | $372.32 | | |

=

354.60 + 17.73

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawal Balance | Withdrawal | Post-Withdrawal Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | $372.32 | | |

=

$$PV_0(\$372.32) = \$372.32 \times (1+0.05)^{-1} = \$354.60$$

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawl Balance | Withdrawal | Post-Withdrawal Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | $372.32 | $100.00 | |

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawl Balance | Withdrawal | Post-Withdrawl Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | $372.32 | $100.00 | $272.32 |

=

372.32 – 100

# Example 2 – Savings (Account)

| Year | Interest | Pre-Withdrawal Balance | Withdrawal | Post-Withdrawal Balance |
|---|---|---|---|---|
| 0 | | | | $354.60 |
| 1 | $17.73 | $372.32 | $100.00 | $272.32 |
| 2 | $13.62 | $285.94 | $100.00 | $185.94 |
| 3 | $9.30 | $195.24 | $100.00 | $95.24 |
| 4 | $4.76 | $100.00 | $100.00 | $0.00 |

# Summary

# Lessons

- Never add/subtract cash flows from different time periods

- Use (i.e., multiply by) discount factor to change cash flows' time units

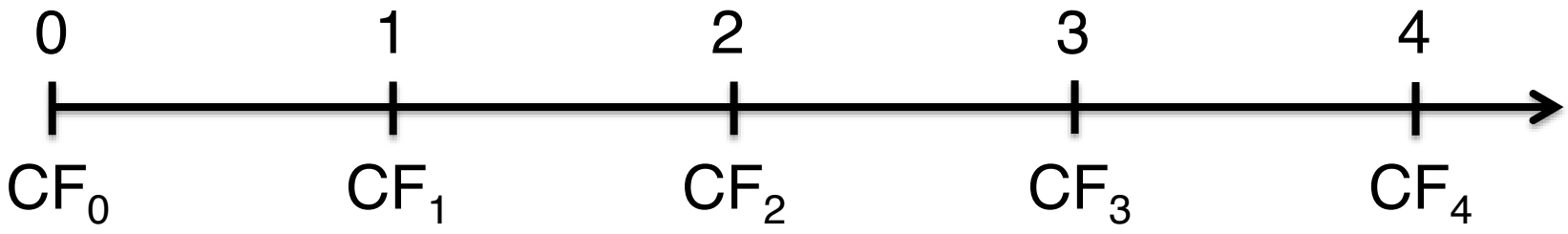
$$(1+R)^t$$

$t < 0$ moves CF back in time (discounting)

$t > 0$ moves CF forward in time (compounding)

# Lessons

- Use a time line to help formulate problems



# Lessons

- Present value as of time $s$ of a cash flow at time $t > s$ is denoted, $PV_s$ ($CF_t$)
  - Tells us the value future cash flows
  - Tells us the price of a claim to those cash flows

# Coming up next

- Compounding