Assignment-2 (Ordinary Differential Equations and its applications)

1. Form the differential equation by eliminating the arbitrary constants from each of the following equations:

   a. $x^3 - 3x^2y = c$

   b. $y = A\sin(Bx+C)$

   c. $y = ae^{3x} + be^{-x}$

   d. family of circles having their centres on the y-axis.

2. A bacterial population B is known to have a rate of growth $\propto$ to B itself. If between noon and 2PM the population triples, at what time, no controls being exerted, should B become 100 times what it was at noon.

3. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What would be the value of N after $1\frac{1}{2}$ hours.

4. Radium decomposes at a rate $\propto$ the quantity of radium present. Suppose that it is found that in 25 years approximately 1.1% of a certain quantity of radium has decomposed. Determine approximately how long will it take for one-half of the original amount of radium to decompose.

5. If half life of Uranium is 1500 years. (a) Find percentage of original amount that will remain after 4500 years, (b) Find in how many years will only $\frac{1}{10}$ of the original amount remain?

6. If a substance cools from 370K to 330K in 10 mts, when the temprature of the surrounding air is 290K, find the temprature of the substance after 40 mts.

7. Water at temprature 10°C takes 5 min to warm up to 20°C in a room at temprature 40°C. (a) Find the temprature after 20 min; after $\frac{1}{2}$ hr, (b) when will the temprature be 25°C?

8. A generator having emf 100 volts is connected in series with a 10 ohm resistor and an inductor of 2 henries. If the switch is closed at a time $t=0$, determine the current at time $t>0$.

9. Determine the charge and current at time $t>0$ in a RC- circuit with $R=10$, $C=2\times10^{-4}$, $E=100$ V given that $Q(t=0)=0$.

10. Solve the (RL- circuit) equation $L\frac{dI}{dt}+RI=E(t)$ when

(a) $E(t)=E_0$ and the initial current is I_0.

(b) Solve the problem when $L=3$ henries, $R=15$ ohm, emf is the 60 cycle sine wave of amplitude 110 volts and $I(t=0)=0$.

11. A decaying emf $E=200e^{-5t}$ is connected in series with a 20 ohm resistor and 0.01 farad capacitor. Find the charge and current at any time t assuming $Q=0$ at $t=0$. Show that the charge reaches a maximum, calculate it and find the time when it is reached.