Assignment 3 – Applications to Partial differentiation

Q1. Use Taylor's theorem to expand $f(x,y) = x^2 + xy + y^2$ in powers of $(x-1)$ and $(y-2)$.

2. Expand $f(x,y) = e^y \ln(1+x)$ in powers of x and y.

3. Expand $f(x,y) = \cos x \cos y$ at $(0,0)$ in powers of x and y.

4. Expand $\tan^{-1}(y/x)$ by taylor's series about $(1,1)$ and hence find the value of $\tan^{-1}(0.9/1.1)$ approximately.

5. Using differential calculus, calculate the approximate value of $f(1.997)$ where $y(x) = x^4 - 2x^3 + 9x + 7$

6. The time T of a complete Oscillation of a simple Pendulum of length L is governed by the equation $T = 2\pi\sqrt{\frac{L}{g}}$, where g is a constant. find the error in T, when error in L is 2%.

7. The diameter and height of a right circular cylinder are measured to be 5 and 8 inches respectively. If each of these dimensions may be in error of ± 0.1 each, find the relative percentage error in volume of the cylinder.

8. what are the advantages of Lagrange's method over the ordinary method of calculating maxima and minima?

Q 9. Find the minimum and maximum values of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

10. Find the shortest distance from the Origin to the surface $xyz^2 = 2$

11. Find the shortest distance from the origin to the plane $x - 2y - 2z = 3$

12. Find the Volume of the largest rectangular parallelopiped with edges parallel to the axes, that can be inscribed in the (i) sphere $x^2+y^2+z^2=a^2$
(ii) ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

13. Find the dimensions of a rectangular box, with open top, so that the total surface area of the box is a minimum, given that the volume of the box is constant say V.

14. Find the minimum and the maximum distances from the origin to the curve $3x^2+4xy+6y^2=140$

15. Find the minimum value of $x^2+y^2+z^2$ subject to the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

16. Find the extreme value of $\sqrt{x^2+y^2}$ when $13x^2 - 10xy + 13y^2 = 72$

17. Find the maximum and minimum distance of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$ using Lagrange's Method