Chapter 1 //

Verifikasi & solusi (mendifferensialkan)

$y = \frac{c}{x} \rightarrow$ ODE $x\, y' = -y$

$y' = \frac{dy}{dx} = (-1)\, c x^{-2} = \frac{-c}{x^2}$

$y' = \frac{-c}{x^2}$ // $\rightarrow y' = \frac{-c}{x^2}$ kalikan $x \rightarrow y' = \frac{-c}{x} \rightarrow x y' = \frac{-c}{x}$

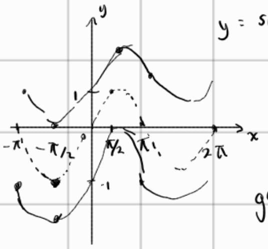
$\boxed{x y' = -y}$

benar //

Solusi kurva ODE

ODE $y' = \cos x \rightarrow y = \int \cos x \, dx$ → konstanta

$y = \sin x + c$



jika $c = -1, 0, 1$

general solution

kondisi awal / batas

ODE $y' = f(x, y)$, $y(x_0) = y_0$

jika x_0 , maka y_0

Contoh :

$y' = 3y \rightarrow \frac{dy}{dx} = 3y$

$\frac{dy}{y} = 3\,dx$

$\int \frac{dy}{y} = \int 3 \, dx$

$\ln y + c = 3x + c$

$\ln y = 3x + c_1$

$e^{\ln y} = e^{3x + c_1}$

$y = e^{3x} \cdot e^{c_1}$ — konstanta

$\boxed{y = k e^{3x}}$ //

Solusi umum

Menemukan Solusi khusus

↳ misal diketahui $y(0) = 5$

$y_0 = k e^{3 x_0}$

$5 = k e^{3 \cdot 0}$

$5 = k e^0$

$5 = k \cdot 1 \rightarrow k = 5$ //

$\rightarrow y = 5 e^{3x}$ dengan initial state

metode pemisahan variabel

→ Contoh soal

$g(y)\, y' = f(x)$

$g(y) \frac{dy}{dx} = f(x)$

$\int g(y)\, dy = \int f(x)\, dx$

$g(y) = f(x) + c$

1.) $y' = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x}$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln|y| = \ln|x| + c$ → konstanta di sebelah kanan aja asalnya kedua ruas ada

↳ $e^{\ln|y|} = e^{\ln|x| + c}$

$y = x \cdot e^c = x \cdot c_1$

$y = c_1 \cdot x$ //

$y' = \frac{2x^2}{3y^3}$

$\frac{dy}{dx} = \frac{2x^2}{3y^3}$

$\int 3y^3 \, dy = \int 2x^2 \, dx$

$\frac{3}{4} y^4 = \frac{2}{3} x^3 + c$

$9 y^4 = 8 x^3 + c$ //

$y' = -2y$

$\frac{dy}{dx} = -2y$

$\int \frac{dy}{y} = \int -2\, dx$

$\ln|y| = -2x + c$

↳ $e^{\ln|y|} = e^{-2x + c}$

$y = e^{-2x} e^c$

$y = c_1 \cdot e$ //

Contoh metode pemisahan untuk penyelesaian persamaan diferensial

Sebuah bahan radioaktif memiliki waktu paruh 3,6 hari untuk meluruh menjadi setengahnya. Jika massa awal adalah 1 gram, tentukan massa bahan tersebut setelah 1 hari.

Persamaan waktu peluruhan

$\therefore$ 3,6 hari waktu paruh
1 gram massa awal

1.) Modelling $y' = -ky$

$$\frac{dy}{dx} = -ky$$
$$\frac{dy}{y} = -k\,dx$$
$$\int \frac{dy}{y} = \int -k\,dx$$
$$\ln|y| = -kx + c$$
$$e^{\ln|y|} = e^{-kx+c}$$
$$y = C_1 e^{-kx}$$

$x = 0 \rightarrow$ awal
$y = 1$
$1 = y(0) = C_1 e^0 = C_1$
$C_1 = 1$
$\hookrightarrow \boxed{y = e^{-kx}}$

$\hookrightarrow \ln|y| = \ln(e^{-kx})$
$\ln y = -kx$
$k = -\dfrac{\ln y}{x}$

ketika tinggal setengah $\rightarrow$ butuh waktu 3,6 hari
$x = 3{,}6$ hari
$y = \frac{1}{2} y_0 = \frac{1}{2} \cdot 1 = \frac{1}{2}$

$$k = -\frac{\ln 1/2}{3{,}6} = 0{,}19$$

$\hookrightarrow y = e^{-0{,}19x} \rightarrow$ setelah 1 hari berapa gram?

$x = 1 \rightarrow y = e^{-0{,}19} = 0{,}83$ gram

Metode Reduksi dan Pemisahan untuk penyelesaian persamaan differensial

$2xy\,y' = y^2 - x^2 \longrightarrow y' = \dfrac{y^2 - x^2}{2xy}$

$2xy\,y' - y^2 = -x^2$

$\hookrightarrow \boxed{2yy' - \dfrac{y^2}{x} = -x}$

$$y' = \frac{y^2}{2xy} - \frac{x^2}{2xy}$$
$$y' = \frac{y}{2x} - \frac{x}{2y}$$

$\hookrightarrow \boxed{u = \dfrac{y}{x}}$

$$y' = \frac{u}{2} - \frac{1}{2u}$$
$$u'x + u = \frac{u}{2} - \frac{1}{2u}$$
$$u'x = \frac{u}{2} - \frac{1}{2u} - u$$
$$u'x = -\frac{u}{2} - \frac{1}{2u}$$
$$u'x = -\frac{(u^2+1)}{2u}$$

$\hookrightarrow \dfrac{du}{dx}x = -\dfrac{(u^2+1)}{2u}$

$\hookrightarrow \dfrac{2u\,du}{u^2+1} = -\dfrac{dx}{x}$ $\quad \boxed{y' = u'x + u}$

$\hookrightarrow \displaystyle\int \frac{2u\,du}{u^2+1} = \int -\frac{dx}{x}$

$\boxed{\begin{aligned} &y' = u'x + ux' \\ &y' = u'x + u\frac{dx}{dx} \\ &y' = u'x + u \end{aligned}}$

Reduksi pemisahan variabel

$y' = f\left(\frac{y}{x}\right)$ $\quad \boxed{\frac{y}{x} = u}$ $\quad y' = f(u)$

$y = ux \rightarrow y' = u'x + u$
$f(u) = u'x + u$

$f(u) - u = \dfrac{du}{dx}x$

$$\boxed{\frac{f(u) - u}{du} = \frac{x}{dx}}$$

$\hookrightarrow \int \frac{\cancel{2u}}{u^2+1} \frac{d(u^2+1)}{\cancel{2u}} = \ln|u^2+1| \longleftarrow \int \frac{2u\, du}{u^2+1} = \int -\frac{du}{x}$

$\ln|u^2+1| = -\ln|x| + c$

$e^{\ln|u^2+1|} = e^{\ln|\frac{1}{x}| + c}$

$u^2+1 = \frac{1}{x} \cdot c_1 \qquad u = \frac{y}{x}$

$\left(\frac{y}{x}\right)^2 + 1 = \frac{c_1}{x}$

$\frac{y^2}{x^2} + 1 = \frac{c_1}{x} \rightarrow y^2 + x^2 = c_1 x$ //

Review pemisahan Variabel

Contoh

$xy' = y - x \qquad \boxed{u = \frac{y}{x}}$

$y' = \frac{y-x}{x} \qquad \boxed{\frac{du}{F(u)-u} = \frac{dx}{x}}$

$y' = \frac{y}{x} - 1$

$y' = u - 1$

$\boxed{F(u) = u - 1}$

$\hookrightarrow$ masukkan persamaan

$\frac{du}{u-1-u} = \frac{dx}{x}$

$\int \frac{du}{-1} = \int \frac{dx}{x}$

$-u = \ln|x| + c$

$-\frac{y}{x} = \ln|x| + c$

$y = -x\ln|x| - cx$ //

lanjut part 2 (persamaan eusan diferensial)