

1 hr : 3 qrs, done in Sun, Jan 19 4:00

Separation of Variables

2.15) $\frac{dy}{dx} = \sec y \tan x$

$$\frac{dy}{\sec y} = \tan x dx$$

$$\cos y dy = \frac{\sin x}{\cos x} dx$$

$$\int \cos y dy = - \int \frac{d(\cos x)}{\cos x}$$

$$\sin y = - \ln \cos x + c$$

$$\ln \cos x + \sin y = c //$$

Integrating factors

solve a.) $(y+x^4)dx - xdy = 0$

$$ydx - xdy + x^4 dx = 0$$

→ Integrating factor $\frac{ydx - xdy}{x^4} = -d\left(\frac{y}{x}\right)$

$$\hookrightarrow \frac{ydx - xdy}{x^4} + x^4 dx = 0$$

$$\hookrightarrow -d\left(\frac{y}{x}\right) + x^4 dx = 0$$

$$-d\left(\frac{y}{x}\right) + d\left(\frac{x^5}{5}\right) = 0$$

$$d\left(-\frac{y}{x} + \frac{x^5}{5}\right) = 0$$

$$-\frac{y}{x} + \frac{x^5}{5} = c //$$

Linear equations

2.19) Solve $x \frac{dy}{dx} - 2y = x^3 \cos 4x$

$$\hookrightarrow x \frac{dy}{dx} = x^3 \cos 4x + 2y$$

$$\frac{dy}{dx} = x^2 \cos 4x + \frac{2y}{x}$$

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \cos 4x$$

→ Integrating factor form

$$\frac{dy}{dx} + Py = Q$$

$$\hookrightarrow x^{-2} \frac{dy}{dx} - 2x^{-3}y = \cos 4x$$

$$\hookrightarrow \frac{d}{dx}(x^{-2}y) = \cos 4x$$

$$\hookrightarrow \text{after integrating } x^{-2}y = \frac{1}{4} \sin 4x + c$$

$$y = \frac{1}{4} x^2 \sin 4x + c x^2 //$$