Ohm's Law and Kirchhoff's Voltage Rule

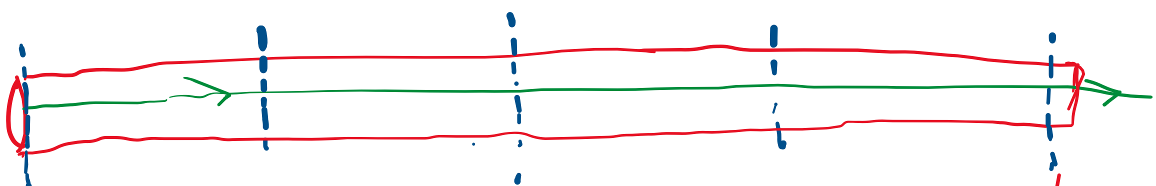
Using the observation that electric field in a conductor is proportional to current density (a.k.a. Ohm's Law), can you derive the formula V=IR?

OHM'S LAW SAYS THAT $\vec{J} \propto \vec{E}$.

$$\Rightarrow \vec{J} = \sigma \vec{E}$$

CONDUCTIVITY.

IN A CLASSICAL WIRE, FIELD/POTENTIAL LINES ARE LIKE SO - - -



$$V = E l$$

$$E = \frac{V}{l}$$

WE ALSO KNOW THAT CURRENT DENSITY IS RELATED TO CURRENT AS FOLLOWS. - .

$\vec{J} = \frac{i}{\vec{A}}$. So, $\vec{J} = \sigma \vec{E} \Rightarrow \frac{i}{\vec{A}} = \sigma\left(\frac{V}{\vec{l}}\right)$. $\vec{A} \parallel \vec{l}$.

So, $i l = \sigma A V \Rightarrow V = i\left(\frac{l}{\sigma A}\right)$. $\sigma \equiv \frac{1}{\rho}$.

$$\Rightarrow V = i\left(\frac{\rho l}{A}\right). \quad R \equiv \frac{\rho l}{A} \Rightarrow \boxed{V = iR}$$

Suppose I have two wires:

- **one has a radius of $r_1 = 0.100\ m$, a length of $l_1 = 0.100\ m$, and a resistivity of $\rho_1 = 5.00\ \Omega * m$.**
- **a second wire has a radius of $r_2 = 0.200\ m$, a length of $l_2 = 0.190\ m$, and a resistivity of $\rho_2 = 5.30\ \Omega * m$.**

Which wire has a greater resistivity? Which has a greater resistance?

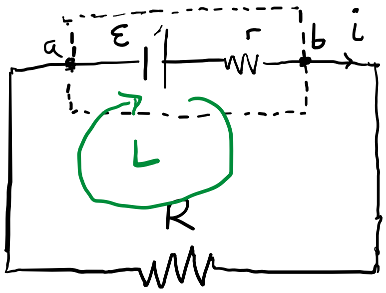
- THE SECOND WIRE HAS A GREATER RESISTIVITY $(\rho_2 > \rho_1)$.

$$R_1 = \frac{\rho_1 l_1}{A_1} = \frac{\rho_1 l_1}{\pi r_1^2} = \frac{(5.00\ \Omega m)(.100 m)}{\pi (.100 m)^2} = 15.9\ \Omega$$

$$R_2 = \frac{\rho_2 l_2}{A_2} = \frac{\rho_2 l_2}{\pi r_2^2} = \frac{(5.30\ \Omega m)(.190 m)}{\pi (.200 m)^2} = 8.01\ \Omega$$

- THE FIRST WIRE HAS A GREATER RESISTANCE $(R_1 > R_2)$

Consider the circuit below (which contains a non-ideal battery).



What is the potential difference between points a and b?

LET'S USE KIRCHHOFF'S LOOP RULE.

$\underline{L}$: $\mathcal{E} - ir - iR = 0$

$$\mathcal{E} = i(r+R) \Rightarrow i = \frac{\mathcal{E}}{r+R}.$$

NOW, $V_{ab} = iR$ (ALSO $\mathcal{E} - iR$ ... SAME RESULT)

$$\boxed{\Rightarrow V_{ab} = \frac{\mathcal{E}R}{r+R}}$$