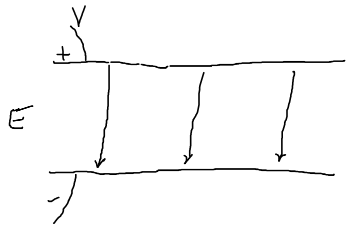
2-27-2019: Capacitors in Circuits

Wednesday, February 27, 2019 9:07 AM

$$V = \frac{q}{C}, \quad q = CV, \quad C = \frac{q}{V}$$

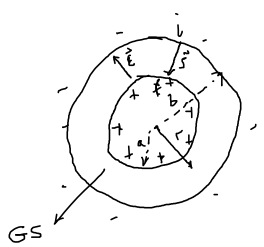
Now let's talk a bit about parallel-plate capacitors…



$$E = \frac{V}{d}$$

$$C = \frac{\varepsilon_0 A}{d} \leftarrow \text{PARALLEL-PLATE CAPACITOR}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}$$



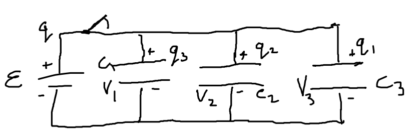
$$\varepsilon_0 E(2\pi r)L = q$$

$$E = \frac{q}{2\pi\varepsilon_0 r L}$$

$$V = V_f - V_i = -\int \vec{E}\cdot d\vec{s} = \int_a^b \frac{q}{2\pi\varepsilon_0 r L}\,dr = \frac{q}{2\pi\varepsilon_0 L}\int_a^b \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left|\frac{b}{a}\right|$$

$$C = \frac{q}{V} = \frac{\cancel{q}}{\frac{\cancel{q}}{2\pi\varepsilon_0 L}\ln\left|\frac{b}{a}\right|} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{a}{b}\right)}$$

For a sphere… $E = \frac{q}{4\pi\varepsilon_0 r^2}$, $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$, $C = \frac{q}{V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$

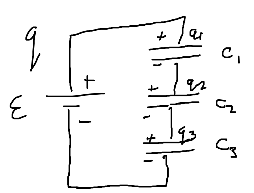


$$q = q_1 + q_2 + q_3$$

$$\varepsilon = V_1 = V_2 = V_3$$

$$C_{eq} = \frac{q}{\varepsilon} = \frac{q_1 + q_2 + q_3}{\varepsilon} = \frac{q_1}{V_1} + \frac{q_2}{V_2} + \frac{q_3}{V_3}$$

$$C_{eq} = C_1 + C_2 + C_3 \Rightarrow \boxed{C_{eq} = \sum_{i=1}^{n} C_i}$$ n capacitors in parallel.



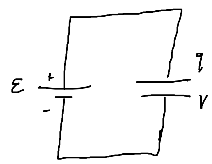
$$q_1 = q_2 = q_3 = q$$

$$\varepsilon = V = V_1 + V_2 + V_3,$$

DIVIDE BOTH NUM. & DENOM. BY $q = q_1 = q_2 = q_3$.

$$C_{eq} = \frac{Q}{V} = \frac{q}{V_1 + V_2 + V_3} = \frac{1}{\frac{V_1}{q_1} + \frac{V_2}{q_2} + \frac{V_3}{q_3}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$$\boxed{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$ n capacitors in series



$$\mathcal{E} = \frac{dU}{dq} \qquad dU = \mathcal{E}\,dq$$

$$U = \int \mathcal{E}\,dq = \int \frac{q}{C}\,dq$$

$$= \frac{1}{C}\int q\,dq = \frac{1}{C}\,\frac{1}{2}q^2 = \frac{q^2}{2C}$$

$$\boxed{U = \frac{q^2}{2C} = \frac{1}{2}qV = \frac{1}{2}CV^2}$$