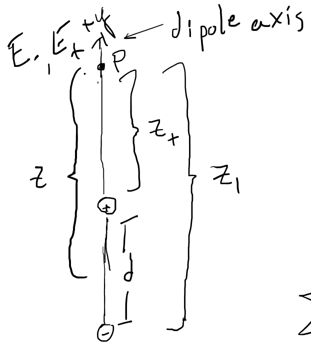
2/1/2019: Calculating Electric Fields

Friday, February 1, 2019 3:01 PM



$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{z_+^2}\hat{y} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(z-\frac{d}{2})^2}\hat{y}$$

$$\vec{E}_- = \frac{1}{4\pi\varepsilon_0}\frac{q}{z_-^2}(-\hat{y}) = \frac{-1}{4\pi\varepsilon_0}\frac{q}{(z+\frac{d}{2})^2}\hat{y}$$

$$\sum\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(z-\frac{d}{2})^2} - \frac{1}{(z+\frac{d}{2})^2}\right)\hat{y} = \frac{q}{4\pi\varepsilon_0}\left(\frac{(z+\frac{d}{2})^2-(z-\frac{d}{2})^2}{(z^2-\frac{d^2}{4})^2}\right)\hat{y}$$

$$\sum\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{zd}{(z^2+(\frac{d}{2})^2)^2}\hat{y}$$

Now let's look at limiting cases…

Multiply here by 1/z^4 on both sides of fraction.

$(z >> d)$.

$$\lim_{\frac{d}{z}\to 0}\sum\vec{E} = \lim_{\frac{d}{z}\to 0}\left(\frac{q}{4\pi\varepsilon_0}\frac{zd}{(z^2-\frac{d^2}{2^2})^2}\right)\cdot\frac{\frac{1}{z^4}}{\frac{1}{z^4}} = \lim_{\frac{d}{z}\to 0}\left(\frac{q}{4\pi\varepsilon_0}\cdot\frac{\frac{d}{z^3}}{(1-\frac{d^2}{z^4})}\right) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{d}{z^3}$$

This quantity will evaluate to zero in the limit.

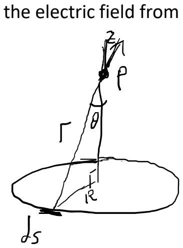
How do electric fields from different source charges differ?

$E \propto \frac{1}{r^2}$ (POINT CHARGE)

$E \propto \frac{1}{r^3}$ (DIPOLE CHARGE)

Name	Traditional Variable	Units
Point Charge	q	C (coulomb)
Line Charge (Linear Density)	λ	C/m (coulomb per meter)
Surface Charge (Surface Density)	σ	C/m^2 (coulomb per meter squared)
Volume Density	ρ	C/m^3 (coulomb per cubic meter)

Let's look at the electric field from a ring of charge…



$$dq = \lambda ds$$

$$dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda ds}{z^2+R^2}\cdot\frac{z}{\sqrt{z^2+R^2}}$$

$$E_x = \int_0^{2\pi R}\frac{1}{4\pi\varepsilon_0}\cdot\frac{z}{(z^2+R^2)^{3/2}}\lambda ds = \frac{1}{\cancel{4\pi}\varepsilon_0}\cdot\frac{(\cancel{2\pi}R)z\lambda}{(z^2+R^2)^{3/2}} = \frac{\lambda R z}{\varepsilon_0(z^2+R^2)^{3/2}}\hat{z}$$

$(z >> R)$.

– OR –

$$\lambda = \frac{q}{2\pi R} \Rightarrow E_x = \frac{q\cancel{R}z}{2\pi\cancel{R}\varepsilon_0(z^2+R^2)^{3/2}}\hat{z}$$

$$\boxed{\frac{qz}{\ldots}\hat{z}}$$

$\lambda = 2\pi R$

$$\boxed{= \frac{q z}{2\pi\epsilon_0 (z^2+R^2)^{3/2}} \hat{z}}$$

$(z >> R)$.

$$E = \frac{q}{4\pi\epsilon_0 z^2} \quad \propto \frac{1}{z^2}$$

Steps for these kinds of problems:

1. Look at ds, and write dq in terms of ds.
2. Then, write a formula for electric field.
3. See what symmetries exist… maybe we only have to worry about x-direction (like in the ring case).