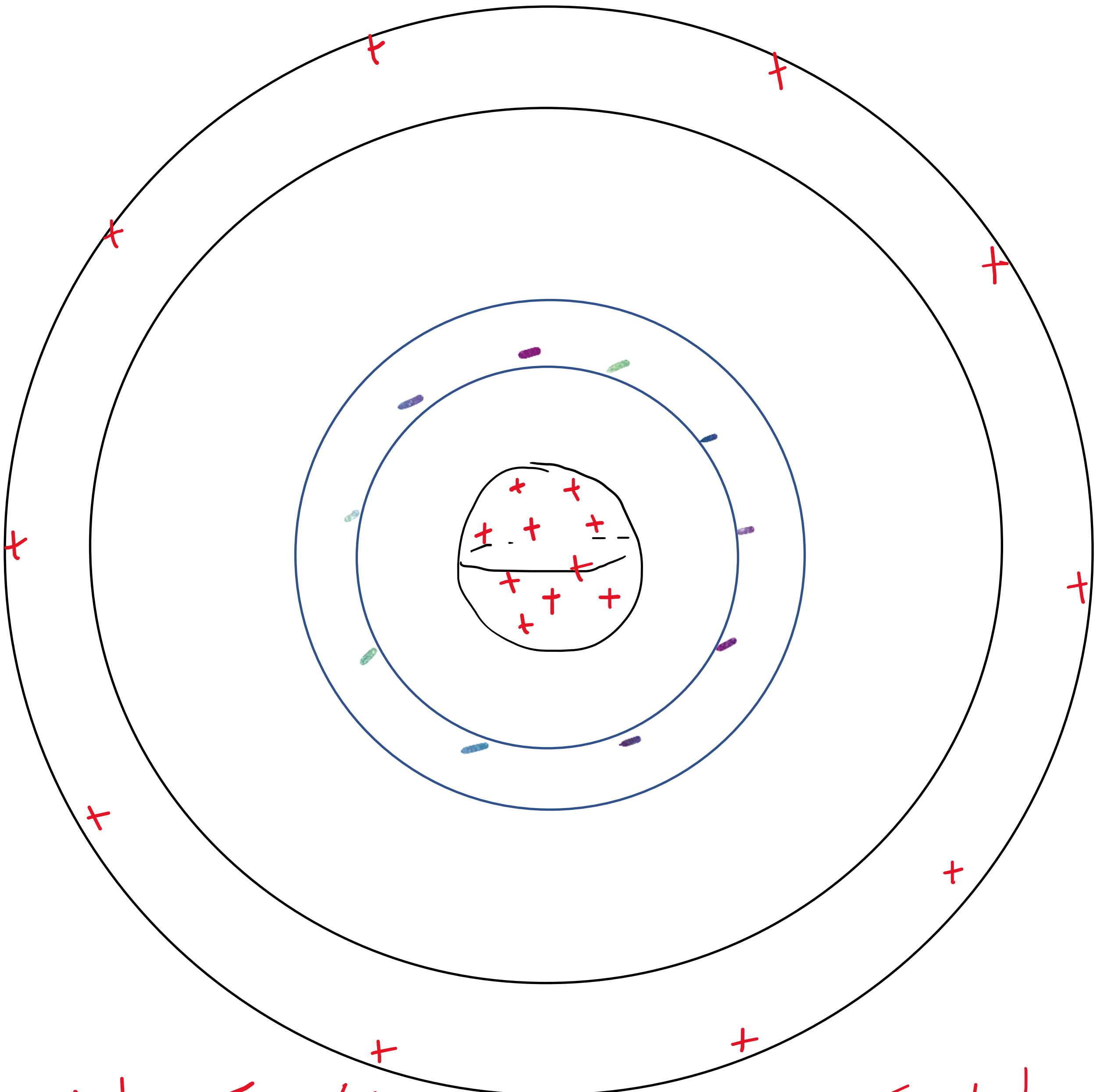
Gauss's Law and Spherical Charge Densities

1. Suppose that there exists the following system of spheres and shells, where the inner sphere is non-conducting and has a uniformly distributed net charge of +Q, the blue shell is conducting and has a net charge of -Q, and the black outer-most shell has a net charge of +Q. Draw the charge distribution on the system below.



non-conducting: $E_{inside} \neq 0$

conducting: $E_{inside} = 0$ ALWAYS

HERE, $E = \dfrac{Q_{enclosed}}{A_{Gaussian}\,\varepsilon_0}$.

2. The sphere below has a radius of R and a charge of Q, is non-conducting, and has a volumetric charge distribution according to the following function:

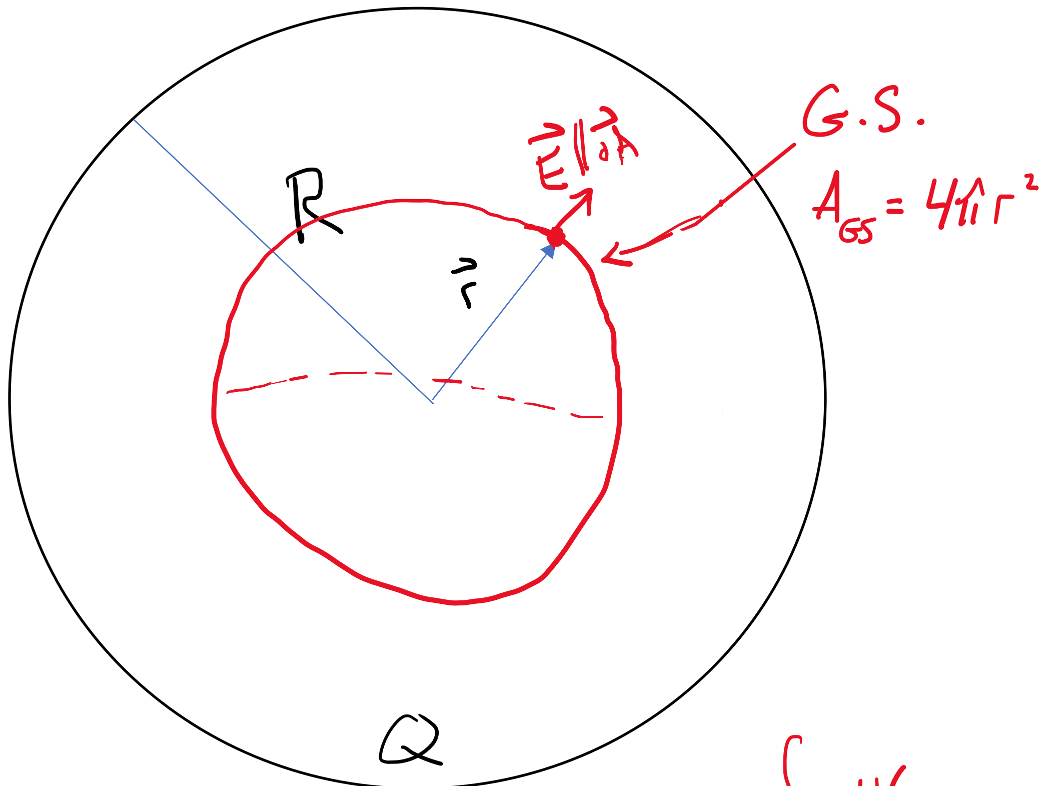
$$\rho = c\,r; \{r < R, c > 0\}$$

where c is a constant, and r is the radial distance from the center of the sphere.

Using Gauss's Law, find the electric field for points inside of the sphere as a function of r.

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2 dr$



So, $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow EA = \frac{Q_{in}}{\varepsilon_0} = \frac{\int \rho\, dV}{\varepsilon_0}$

$$E(r) = \frac{\int \rho\, dV}{A\,\varepsilon_0} = \frac{\int_0^r (c\,r)(4\pi r^2)\,dr}{4\pi r^2 \varepsilon_0} = \frac{\cancel{4\pi} c \left(\frac{r^3}{3}\Big|_0^r\right)}{\cancel{4\pi} r^2 \varepsilon_0}$$

$$\vec{E}(r) = \frac{c\,r^{\cancel{3}}}{3\cancel{r^2}\varepsilon_0}\hat{r} = \boxed{\frac{c\,r}{3\varepsilon_0}\hat{r}}$$