2-4-2019: Electric Field and Flux

Monday, February 4, 2019 3:04 PM

$E \propto \frac{1}{r^2}$ Point charge

$E \propto \frac{1}{r}$ Infinite straight line

$E \propto \frac{1}{r^3}$ dipole

How can we find dA? for a circle…

$dq = \sigma dA$; $A = \pi r^2$ ✓

$dq = \sigma (2\pi r) dr$ $\quad \frac{dA}{dr} = 2\pi r \Rightarrow dA = (2\pi r) dr$

$$dE_{disk} = \frac{z\sigma(2\pi r)dr}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} \Rightarrow E_{disk} = \int_0^R \frac{z\sigma(2\pi r)dr}{4\pi\varepsilon_0(z^2+r^2)^{3/2}} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}$$

We can learn about the electric field of an infinite sheet from the ring… let's take the limit where R goes to infinity…

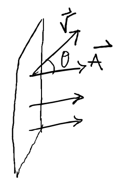
$$\lim_{R\to\infty} \vec{E}_{disk} = \lim_{R\to\infty}\left(\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}\right) = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

This is our electric field for an infinite sheet! Notice that it doesn't depend on how far away we are (constant).

Gaussian surface is an imaginary surface that could be any shape. We often use a Gaussian surface to look at flux due to a field (in this case an electric field). If we want the calculation to be simple, we need to make the surface as simple as possible to use symmetry.

Let's introduce the idea of flux by talking about a quantity, flow (m^3/s).

Normal Vector of Area A



$\vec{v}$ $\frac{m}{s}$

$\vec{A}$ m^2 , So

$\Phi = vA$ (WHEN $\vec{v} \perp$ PLANE)

$\Phi = v\cos\theta A$ ($\vec{v} \angle \theta$)

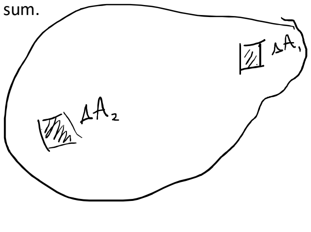
$\Phi = \emptyset$ (WHEN $\vec{v} \perp \vec{A}$)

FLUX

$\Phi = \vec{v}\cdot\vec{A}$

$\Phi_E = \vec{E}\cdot\vec{A}$

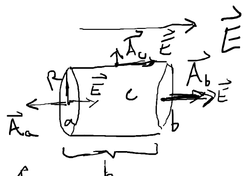
Consider this surface. Here, we can break up the surface into many different little areas. So, our flux will be the discrete sum.



$$\Phi = \sum \vec{E}\cdot d\vec{A}$$

Notice that flux is a scalar! Each of these dot products will be a scalar, and their sum will be a scalar.

Units: $\frac{N}{C}m^2$



$\vec{E}$ <- Uniform electric field

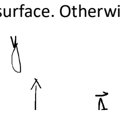
ϕ_{NET} going through A . . .

$A = A_a + A_b + A_c$

$$\phi_{NET} = \phi_a + \phi_b + \phi_c = EA_a\cos 180° + EA_b\cos\emptyset° + \cancel{EA_c\cos 90°}^{\,\emptyset}$$

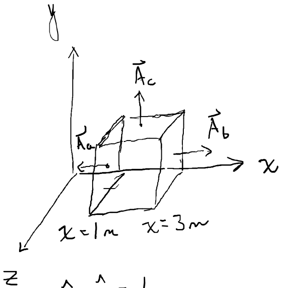
$$= -EA_a + EA_b = -E(\pi R^2) + E(\pi R^2) = \emptyset$$

Note: always choose the area vector to point in a direction that is pointing away/outward from the surface. Otherwise, you might get the wrong sign (and thus the wrong answer)!



$$\vec{E} = 3.\emptyset x\,\hat{i} + 4.\emptyset\,\hat{j} \;\; N/C$$

surface. Otherwise, you might get the wrong sign (and thus the wrong answer).



$$\vec{E} = 3.0x\,\hat{i} + 4.0\,\hat{j} \ \text{N/C}$$

Given this surface and field, find the three fluxes below…

$$\phi_{LEFT},\quad \phi_{RIGHT},\quad \phi_{TOP}$$

$$\Phi = \vec{E}\cdot\vec{A} = \oint \vec{E}\cdot d\vec{A}$$

Closed surface

$\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$

$$\Phi_{LEFT} = \int \vec{E}\cdot d\vec{A}$$

$$= \int (3.0x\,\hat{i} + 4.0\,\hat{j})\cdot(-\hat{i})\,dA$$

$$= \int(-3.0x + 0)\,dA = -\int 3.0x\,dA = -\int(3.0)(1.0)\,dA = -3.0\int dA = (-3.0)(4.0) = \boxed{-12\ \frac{N\cdot m^2}{C}}$$

$$\Phi_{RIGHT} = \boxed{36\ \frac{N\cdot m^2}{C}}$$

$$\Phi_{TOP} = \int \vec{E}\cdot d\vec{A} = \int\left((3x\,\hat{i} + 4\,\hat{j})\cdot\hat{j}\right)dA = \int 4\,dA = 4(4) = \boxed{16\ \frac{N\cdot m^2}{C}}$$

$$\Phi_{FRONT} = \Phi_{BACK} = 0$$

$\Phi_{NET} \neq 0 \Rightarrow$ THERE IS A CHARGE