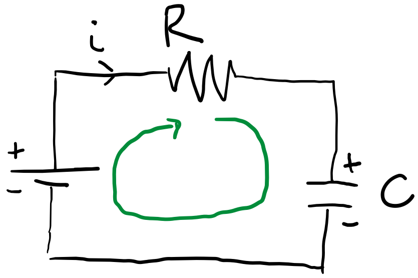
RC Circuits

Derive the solutions for the charge, voltage, and current of a capacitor in a simple RC circuit.



$$i \equiv \frac{dq}{dt}$$

From KLR

$$\varepsilon - iR - \frac{q}{C} = 0$$

$$\Rightarrow \varepsilon - \frac{dq}{dt}R - \frac{q}{C} = 0 \Rightarrow \frac{dq}{dt}R = \varepsilon - \frac{q}{C} \Rightarrow \frac{dq}{dt}\left(\frac{R}{\varepsilon - \frac{q}{C}}\right) = 1$$

$$\int_0^{q(t)} \frac{1}{\frac{\varepsilon}{R} - \frac{q}{RC}} \frac{dq}{dt}\, dt = \int_0^t 1\, dt \quad \cdot \quad u = \frac{\varepsilon}{R} - \frac{q}{RC}, \quad du = -\frac{dq}{RC} \Rightarrow \int_{\frac{\varepsilon}{R}}^{\frac{\varepsilon}{R} - \frac{q(t)}{RC}} \frac{-RC\, du}{u} = \int_0^t dt$$

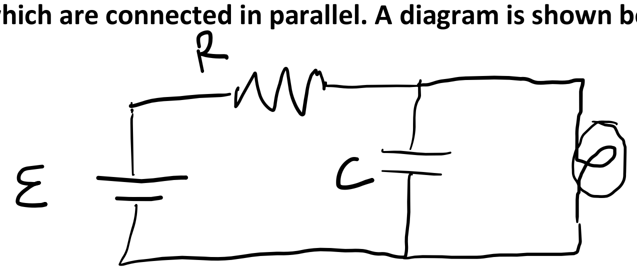
$$\Rightarrow -RC \ln|u| \Big|_{\frac{\varepsilon}{R}}^{\frac{\varepsilon}{R} - \frac{q(t)}{RC}} = t \Rightarrow \ln\left|\frac{\frac{\varepsilon}{R} - \frac{q(t)}{RC}}{\frac{\varepsilon}{R}}\right| = -\frac{t}{RC} \Rightarrow 1 - \frac{q(t)}{C\varepsilon} = e^{-\frac{t}{RC}}$$

$$\boxed{q(t) = C\varepsilon\left(1 - e^{-t/RC}\right)}$$

$$i(t) = \frac{dq(t)}{dt} = -\cancel{C}\varepsilon\left(-\frac{1}{R\cancel{C}}\right)e^{-t/RC} = \boxed{\frac{\varepsilon}{R}e^{-t/RC}}$$

$$V_C(t) = \frac{q}{C} = \frac{C\varepsilon\left(1 - e^{-t/RC}\right)}{C} = \boxed{\varepsilon\left(1 - e^{-t/RC}\right)}$$

A circuit is designed with a battery and a variable resistor in series with a capacitor and lightbulb, which are connected in parallel. A diagram is shown below…



A multi-meter measures the voltage across the capacitor over time. Sketch a graph of this behavior, and explain each trend.