Kirchhoff's Laws and Capacitors

Three capacitors of equal capacitance are arranged in a circuit in parallel with the only other element in the circuit: a battery with emf of 5V. What is the charge on each of the capacitor plates? Suppose one of these capacitors is removed. What happens to the charge on each plate?

$$C_1 = \frac{Q_1}{V_1} = \frac{Q_1}{\mathcal{E}} = C \Rightarrow Q_1 = C\mathcal{E} = 5C$$

$$C_2 = \frac{Q_2}{V_2} = \frac{Q_2}{\mathcal{E}} = C \Rightarrow Q_2 = C\mathcal{E} = 5C$$

$$C_3 = \frac{Q_3}{V_3} = \frac{Q_3}{\mathcal{E}} = C \Rightarrow Q_3 = C\mathcal{E} = 5C$$

$V = V_1 = V_2 = V_3 = \mathcal{E}$

$Q = Q_1 = Q_2 = Q_3 = 5C$.

$C = C_1 = C_2 = C_3$

IF ONE IS REMOVED,

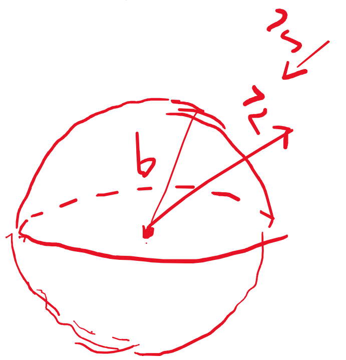
$V = V_1 = V_2 = \mathcal{E}$. $C_1 = C_2 = C$.

$$C_1 = \frac{Q_1}{V_1} = \frac{Q_1}{\mathcal{E}} = C \Rightarrow \boxed{Q_1 = 5C}.$$

$$C_2 = \frac{Q_2}{V_2} = \frac{Q_2}{\mathcal{E}} = C \Rightarrow \boxed{Q_2 = 5C}.$$

SO, THE CHARGE REMAINS THE SAME!

Derive the capacitance of a conducting spherical shell.



$(r < b)$.

$$E = \emptyset \Rightarrow V_{ab} = \emptyset$$

$dr = -ds$.

$(r > b)$.

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}.$$

$$\Rightarrow V_{b\infty} = -\int_b^\infty \vec{E}\cdot d\vec{s} = \int_b^\infty E\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\int_b^\infty \frac{dr}{r^2} = \frac{-q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right]_b^\infty$$

$$= \frac{-q}{4\pi\varepsilon_0}\left(\frac{1}{\infty} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0 b}$$

$$C = \frac{Q}{V} = \frac{\cancel{q}}{\frac{\cancel{q}}{4\pi\varepsilon_0 b}} = \boxed{4\pi\varepsilon_0 b}$$