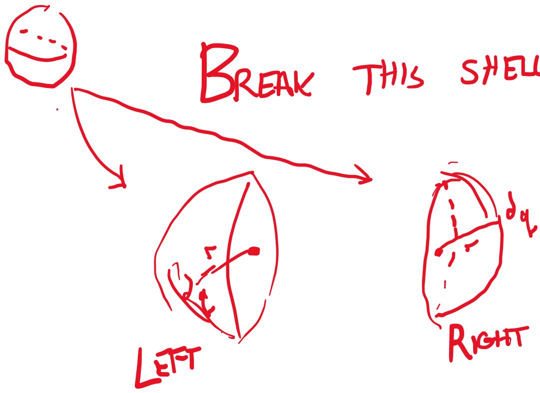
Electric Field: Continuous Charge Distribution

A spherical shell of radius r has a uniformly distributed charge, Q. Show that the net electric field on the center of the shell is zero.

BREAK THIS SHELL IN HALF...



$$E = \int \frac{k\,dq}{r^2}\,\hat{r}$$

REMEMBER, r IS FIXED HERE!

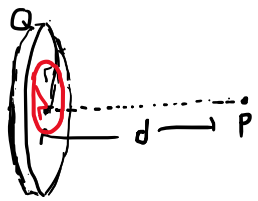
$\hat{r}$ IS OPP.

$$\vec{E} = \vec{E}_{LEFT} + \vec{E}_{RIGHT} = \frac{k}{r^2}\int dq\,(+1) + \frac{k}{r^2}\int dq\,(-1)$$

$$= \frac{k}{r^2}\left(\frac{Q}{2}\right) - \frac{k}{r^2}\left(\frac{Q}{2}\right) = \boxed{\emptyset} \checkmark$$

HALF CHARGE IN EACH HEMISPHERE

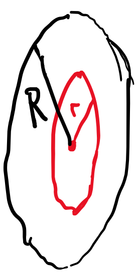
A metal (conducting) disk of radius r has a uniformly distributed charge, Q. Find the electric field for a point at a distance d along the axis normal to the center of the disk (shown below…)



LET'S USE THE RESULT FROM LECTURE, THAT

$$E_{ring} = \frac{q\,d}{4\pi\varepsilon_0 (d^2+r^2)^{3/2}}$$

THINK ABOUT THE DISC AS MANY RINGS...



INTEGRATE RINGS FROM $r=0 \rightarrow r=R$.

$$dE_{ring}(r) = \frac{dq\,d}{4\pi\varepsilon_0 (d^2+r^2)^{3/2}}$$

$$dq = (2\pi r)\lambda\,dr \quad \text{SUBSTITUTE...}$$

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$$E_{disk} = \int_0^R dE_{ring}(r)\,dr = \int_0^R \frac{(\cancel{2\pi} r)\lambda d\,dr}{\underset{2}{\cancel{4\pi}}\varepsilon_0 (d^2+r^2)^{3/2}} = \frac{\lambda d}{4\varepsilon_0}\left[-\frac{1}{2(d^2+r^2)^{1/2}}\right]_0^R$$

$$\boxed{E_{disk} = \frac{\lambda}{2\varepsilon_0}\left(1 - \frac{d}{(d^2+R^2)^{1/2}}\right)}$$