2-22-2019: Resistance, KCL, and KRL

Friday, February 22, 2019 9:02 AM

Let's review what we learned last time…

$$V = iR \;, \quad \vec{E} = \rho \vec{J}$$

$$P = \frac{dU}{dt} = iV = i^2 R = \frac{V^2}{R}$$

$$R_{eq} = \sum_{i=1}^{n} R_i$$ (for "n" resistors in-series)

Kirchhoff's Loop Rule:

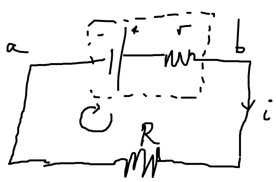
resistors: following i '$-iR$'

against i '$+iR$'

emf: '+' to '−' $-\mathcal{E}$

'−' to '+' $+\mathcal{E}$

Last time, we looked at a non-ideal emf generator. What would be the current through the following system?



KLR: $+\mathcal{E} - ir - iR = 0$

$$i_a = \frac{\mathcal{E}}{r+R}$$

OHM:

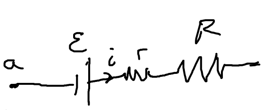
$$i_a = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{r+R}$$

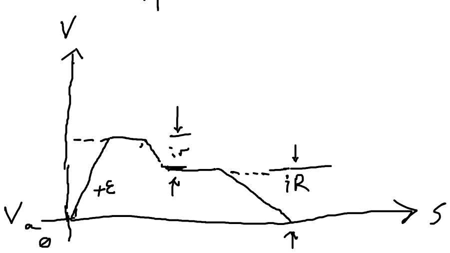
What's the potential drop across the non-ideal battery?

$$V_a + \mathcal{E} - ir = V_b$$

$$V_b - V_a = \mathcal{E} - ir$$

$$\boxed{V_{ba} = \mathcal{E} - ir}$$





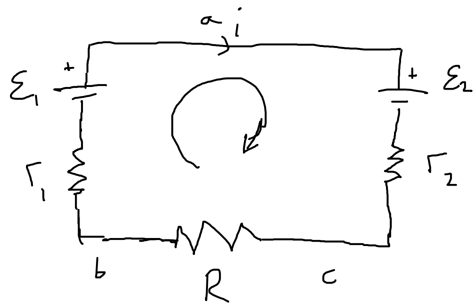
What is the power dissipated by each element?

$$P_{\mathcal{E}} = i\mathcal{E}$$

$$P_r = i^2 r \neq \frac{\mathcal{E}^2}{r}$$

$$P_R = i^2 R \neq \frac{\mathcal{E}^2}{R}$$

What is the current in the circuit below?

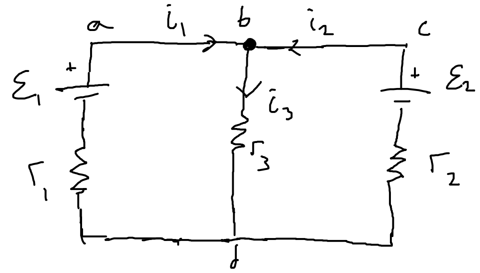


KLR

$$-\mathcal{E}_2 - ir_2 - iR - ir_1 + \mathcal{E}_1 = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2}$$

Suppose the above circuit had a junction in the center like below…

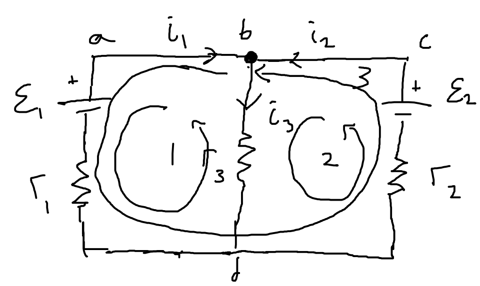


At point b, we can make use of another of Kirchhoff's rules: Kirchhoff's Current Rule…

KCL

$$i_1 + i_2 = i_3$$

Let's now apply KRL to this system by using three loops…



KRL:

1 $\quad \mathcal{E}_1 + i_3 R_3 + i_1 R_1 = \varnothing$

2 $\quad \mathcal{E}_2 + i_2 R_2 + i_3 R_3 = \varnothing$

3 $\quad -\mathcal{E}_1 + \mathcal{E}_2 + i_2 R_2 - i_1 R_1 = 0$

Notice that the third result is the sum of the smaller two loops (the KRL solution is a superposition of loops 1 and 2).

Let's take a look at a circuit that with resistors in parallel…

i i'

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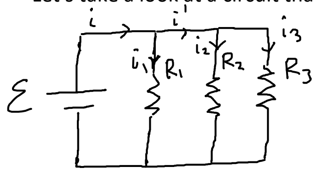

$$V_1 = V_2 = V_3 = \mathcal{E}$$

$$i_1 R_1 = i_2 R_2 = i_3 R_3 = \mathcal{E}$$

$$i' = i_2 + i_3$$

$$i = i' + i_1 = i_1 + i_2 + i_3 = \frac{\cancel{\mathcal{E}}}{R_1} + \frac{\cancel{\mathcal{E}}}{R_2} + \frac{\cancel{\mathcal{E}}}{R_3} = \frac{\cancel{\mathcal{E}}}{R_{eq}}$$

$$\Rightarrow \frac{1}{R_{eq}} = \sum_i^n \frac{1}{R_i}$$