Introduction to Magnetism

Find the cross-product and the dot-product for all combinations of the following vectors…

$$\vec{a} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$$

$$\vec{b} = -\hat{\imath} + 7\hat{k}$$

$$\vec{c} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

Dot-Product: $\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x} = \sum_i x_i y_i$

So, $\vec{a}\cdot\vec{b} = (3)(-1)+(5)(0)+(-2)(7) = \boxed{-17} = \vec{b}\cdot\vec{a}$

$\vec{a}\cdot\vec{c} = (3)(2)+(5)(-1)+(-2)(2) = 6-10 = \boxed{-4} = \vec{c}\cdot\vec{a}$

$\vec{b}\cdot\vec{c} = (-1)(2)+(0)(-1)+(7)(2) = \boxed{12} = \vec{c}\cdot\vec{b}$

Cross-Product: $\vec{x}\times\vec{y} = -(\vec{y}\times\vec{x}) = \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & -2 \\ -1 & 0 & 7 \end{vmatrix} = (5\cdot 7 - 0\cdot(-2))\hat{\imath} - (3\cdot 7 - (-1)\cdot(-2))\hat{\jmath} + (3\cdot 0 - (-1)\cdot 5)\hat{k}$$
$$= 35\hat{\imath} - 19\hat{\jmath} + 5\hat{k}$$

$\Rightarrow \vec{b}\times\vec{a} = -35\hat{\imath} + 19\hat{\jmath} - 5\hat{k}$

$$\vec{a}\times\vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & -2 \\ 2 & -1 & 2 \end{vmatrix} = (5\cdot 2 - (-1)\cdot(-2))\hat{\imath} - (3\cdot 2 - 2\cdot(-2))\hat{\jmath} + (3\cdot(-1) - 2\cdot 5)\hat{k}$$
$$= 8\hat{\imath} - 10\hat{\jmath} - 13\hat{k}$$

$\Rightarrow \vec{c}\times\vec{a} = -8\hat{\imath} + 10\hat{\jmath} + 13\hat{k}$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 7 \\ 2 & -1 & 2 \end{vmatrix} = (0\cdot 2-(-1)\cdot 7)\hat{i} - (2\cdot(-1)-2\cdot 7)\hat{j}$$

$$+((-1)\cdot(-1)-2\cdot 0)\hat{k}$$

$$= 7\hat{i} + 16\hat{j} + \hat{k}$$

$$\Rightarrow \vec{c}\times\vec{b} = -7\hat{i} - 16\hat{j} - \hat{k}$$

A particle has an initial velocity of $v_o = \left(+5.00\,\frac{m}{s}\right)\hat{x}$ in a uniform field of $B_o = \left(-2.00\,\frac{m}{s}\right)\hat{y}$. Describe the velocity of the particle as a function of time.

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = ma$$

$$\Rightarrow \frac{dv}{dt} = \frac{q}{m}(5\hat{x} \times -2\hat{y}) = \frac{-10q}{m}\hat{z}$$

$$\int_0^t \frac{dv}{dt}\,dt = \int_0^t \frac{-10q}{m}\hat{z}\,dt \Rightarrow \int_{v_0}^{v(t)} dv = -10\frac{q}{m}t\hat{z}$$

$$v(t) = v_0 - \frac{10q}{m}t\,\hat{z} = \boxed{5.00\,\hat{x} - \frac{10qt}{m}\hat{z}}$$