2-20-2019: More Ohm's Law and Kirchhoff's Loop Rule

Wednesday, February 20, 2019 9:07 AM

current: $i = \frac{dq}{dt}$ $\qquad \vec{j} = \frac{i}{A}$

$\vec{V}_d = \frac{\vec{j}}{ne}$ $\qquad R = \frac{V}{i}\ (\Omega)$ $\qquad \rho = \frac{E}{\vec{j}} \Rightarrow R = \frac{\rho L}{A}$

Remember that we can only use these equations in an isotropic material (meaning that the resistivity is the same in both directions). If we don't have that, we will get a conductivity tensor.

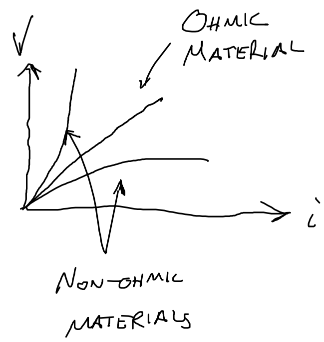
$\rho = \frac{E}{\vec{j}}$, $\qquad \sigma = \frac{1}{\rho}$

RESISTIVITY $\qquad$ CONDUCTIVITY

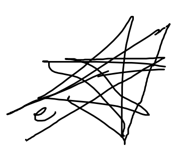
Some materials obey Ohm's law very well, but others do not.

$\boxed{R = \frac{V}{i}}$ — OR — $\boxed{\rho = \frac{E}{\vec{j}}}$

$V \propto i$

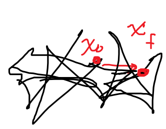


If there isn't a potential across the wire, the electrons will engage in a "random walk" behavior, with a net velocity that is zero.



$V_{eff} = 10^{6}\ m/s$

However, when there's a potential difference, the electron will move in the preferred direction over time **on average**.



$V_d = \frac{x_f - x_0}{\Delta t} \approx 10^{-5}\ m/s$

V_{eff} STILL $\approx 10^{6}\ m/s$

$$F = ma = qE \Rightarrow a = \frac{qE}{m} = \frac{eE}{m}$$

accel. for an e^-.

$$V_1 = a\tau = \left(\frac{eE}{m}\right)\tau = \frac{\vec{j}}{ne} \Rightarrow E = \boxed{\frac{m}{e^2 n \tau}}\vec{j}$$

$$V_d = a\tau = \left(\frac{eE}{m}\right)\tau = \frac{\vec{J}}{ne} \Rightarrow E = \boxed{\frac{m}{e^2 n \tau}}\vec{J}$$

This is the mean-free time.

$$\rho = \frac{m}{e^2 n \tau}$$

Now let's take a look at how to calculate power in these systems…

$$P = \frac{du}{dt} \qquad du = dq\,V$$

$$= \frac{dq\,V}{dt} = iV \quad (\text{Watt}) \; (W) \quad ; \quad 1\,W = 1\left(\frac{J}{s}\right)$$

$$i = \frac{V}{R} \Rightarrow P = \frac{V^2}{R} \quad ; \quad V = iR \Rightarrow P = i^2 R$$

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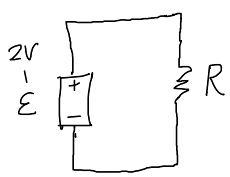
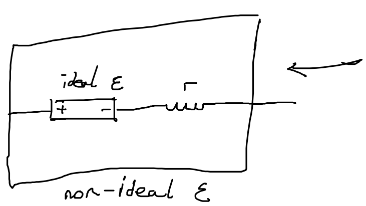
emf: electromotive force

An electromotive force can be generated by anything that is "pumping out charges" such as a generator, a battery, solar panels, etc. Most emf sources are not ideal, which is to say that they do not provide a fixed voltage under all conditions.

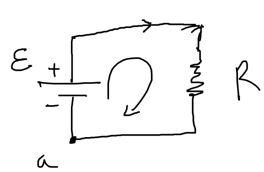
$$\mathcal{E} = \frac{dW}{dq} \quad \left(\frac{J}{C} = V\right)$$





We can think of a non-ideal emf generator as a system that contains an ideal emf and some internal resistance, "r".

A simple circuit example…



$$R = \frac{\mathcal{E}}{i} \qquad i = \frac{\mathcal{E}}{R}$$

Kirchhoff's Loop Rule:

- In a closed loop, the change in potential should be zero.

$$V_a + \mathcal{E} - iR = V_a \Rightarrow \mathcal{E} - iR = 0 \Rightarrow \mathcal{E} = iR$$

Conventions:

emf '+' to '−'

$$\Delta V = -\mathcal{E}$$

'−' to '+'

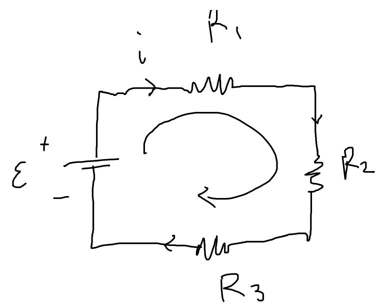
$$\Delta V = +\mathcal{E}$$

Resistor following i

$$\Delta V = -iR$$

against i,

$$\Delta V = +iR$$



$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$\mathcal{E} = i(R_1 + R_2 + R_3) \Rightarrow \boxed{i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}}$$

$$V = V_1 + V_2 + V_3$$