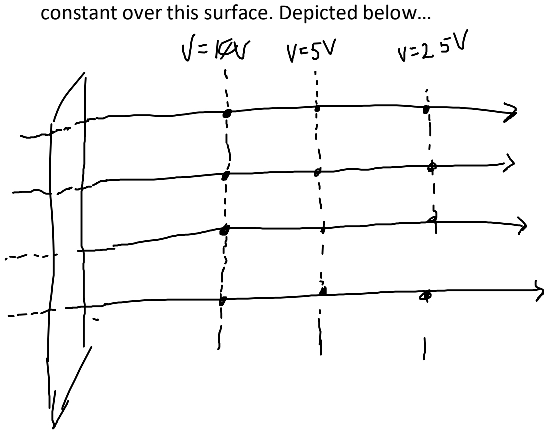
2-13-2019: Electric Potential, Continued

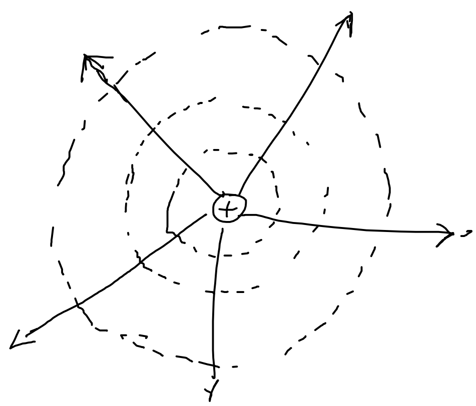
Wednesday, February 13, 2019 9:03 AM

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = \frac{-W_E}{q}$$

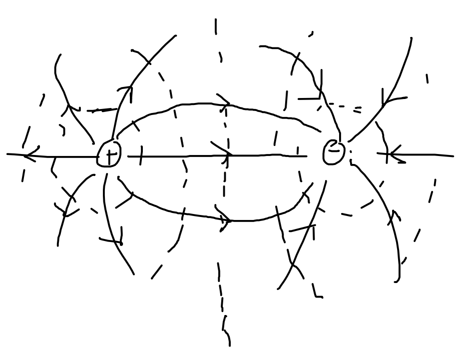
Consider the infinite sheet. For adjacent electric field lines, we can draw a point on each that is the same distance from the sheet. This line of charges will form an "equipotential surface," where the potential is constant over this surface. Depicted below…



We can create an analogous situation with some point charge…



Or with a dipole!

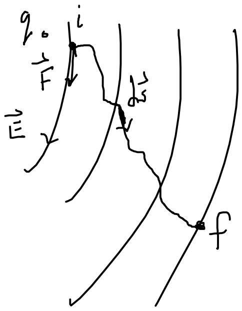


$$\Delta W_{net} = 0$$

This is true because any component of the electric field that isn't perpendicular to the motion will factor into the work! So,

equipotential lines will always be perpendicular to the field lines.

Let's find an expression for calculating the potential difference using some positive test charge…



$$dw = \vec{F}\cdot d\vec{s} = q_0 \vec{E}\cdot d\vec{s}$$

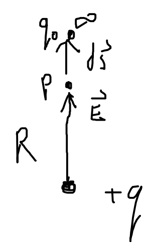
$$W = \int_i^f q_0 \vec{E}\cdot d\vec{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E}\cdot d\vec{s}$$

$(V_\infty = 0)$

$$\boxed{V_p = -\int_i^f \vec{E}\cdot d\vec{s}}$$

Can we use this integral to find the potential at some point P from a point charge?



$$V_f - \cancelto{0}{V_i} = -\int_i^f \vec{E}\cdot d\vec{s}, \quad \vec{E} \parallel d\vec{s} \Rightarrow -\int_i^f E\,ds\cos\theta = -\int_R^\infty E\,dr \qquad \theta = 0$$

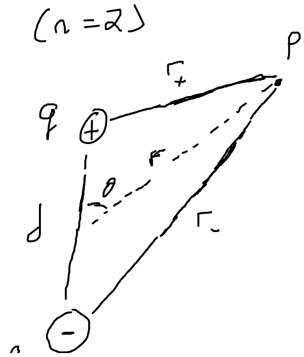
FOR POINT CHARGE… $E = \dfrac{q_0}{4\pi r^2}$

$$= -\int_R^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$

$$\boxed{V_p = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

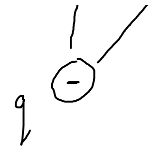
A system of "n" charged particles…

$(n = 2)$



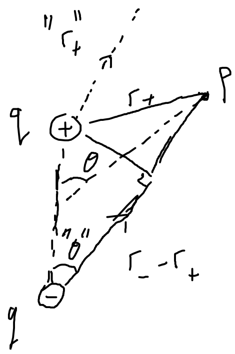
$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}} \leftarrow \text{SUPERPOSITION.}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r - r_+}\right)$$



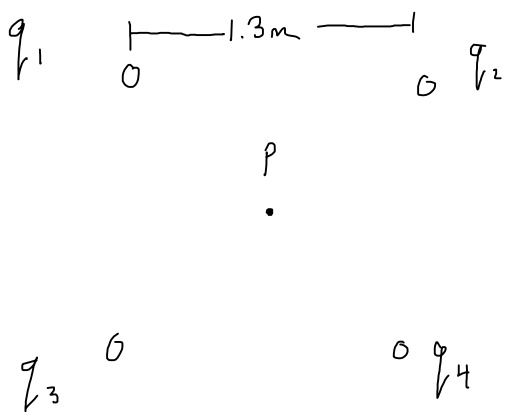
Now, let's find the potential in the limit case where r >> d.

$r_- \parallel r_+$



$$r_- - r_+ = d\cos\theta$$

$$\left(\frac{q}{4\pi\varepsilon_0}\right)\left(\frac{d\cos\theta}{r^2}\right) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$



$q_1 = +12nC$ $\quad$ $q_2 = -24nC$

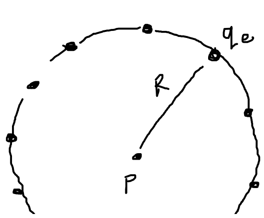
$q_3 = +31nC$ $\quad$ $q_4 = +17nC$

$$V_p = V_1 + V_2 + V_3 + V_4$$

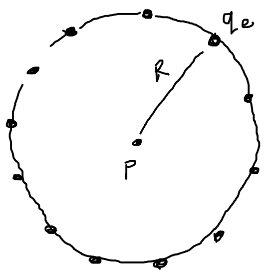
$$= \frac{1}{4\pi\varepsilon_0}\left(\qquad \right)\times 10^{-9}$$

$$= 350\,V$$

What is the potential at the center of this sphere?



$$V_p = -12 \times \frac{e}{4\pi\varepsilon_0 R}$$

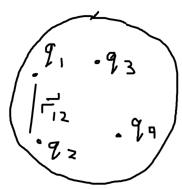


$$V_p = -12 \times \frac{e}{4\pi\varepsilon_0 R}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}\hat{r} \quad , \quad \vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{U}{q_0} \Rightarrow W = U = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}$$

For a system of charges, we should calculate the work by considering each pair of charges (like a combination).



$$W = U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \cdots + \frac{q_3 q_4}{r_{34}}\right)$$