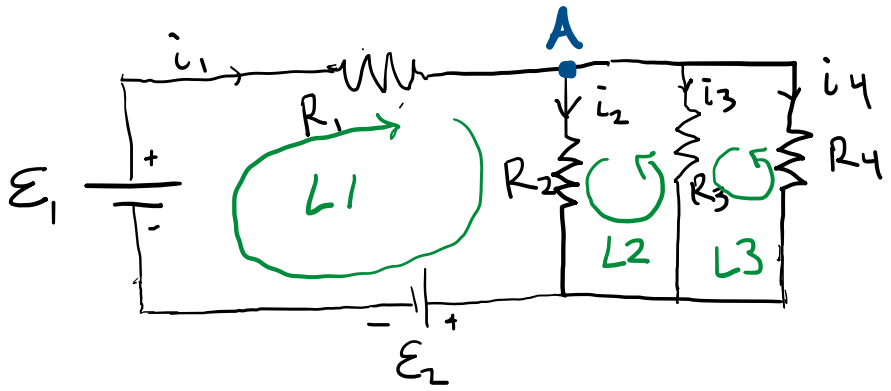
Complex Circuits

Find the value of each of the currents in terms of the given variables for the circuit below…



LET'S START BY MAKING KIRCHHOFF LOOPS...

L1: $-i_1R_1 - i_2R_2 - \mathcal{E}_2 + \mathcal{E}_1 = 0$

L2: $i_3R_3 - i_2R_2 = 0 \Rightarrow i_3R_3 = i_2R_2$

L3: $i_4R_4 - i_3R_3 = 0 \Rightarrow \underline{i_2R_2 = i_3R_3 = i_4R_4}$

THE FACT THAT $V_2 = V_3 = V_4$ MAKES SENSE FROM OUR KNOWLEDGE OF PARALLEL CIRCUITS.

Now lets do a Kirchhoff Junction (A)

$$i_1 R_1 = i_2 R_2 + i_3 R_3 + i_4 R_4$$

From our KVR, $i_2 R_2 = i_3 R_3 = i_4 R_4$.

So, $i_1 R_1 = 3 i_2 R_2$

$$i_1 R_1 + \frac{i_1 R_1}{3} - \mathcal{E}_2 + \mathcal{E}_1 = 0$$

$$\Rightarrow \left(\frac{4}{3} R_1\right) i_1 = \mathcal{E}_2 - \mathcal{E}_1 \Rightarrow i_1 = \frac{3}{4}\left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{R_1}\right)$$

$$\Rightarrow i_2 R_2 = \frac{1}{4}(\mathcal{E}_2 - \mathcal{E}_1) = i_3 R_3 = i_4 R_4.$$

So,

$$i_2 = \frac{1}{4}\left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{R_2}\right), \quad i_3 = \frac{1}{4}\left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{R_3}\right), \quad i_4 = \frac{1}{4}\left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{R_4}\right)$$