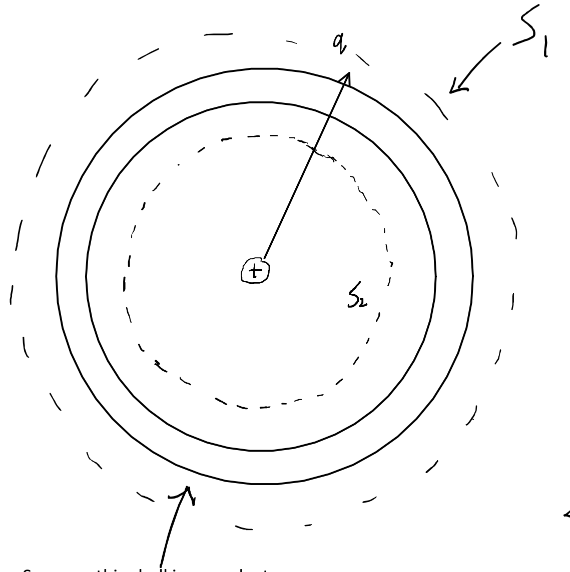
2-8-2019: Gauss's Law and Spheres

Friday, February 8, 2019 9:05 AM



Suppose this shell is a conductor.

Shell Theorem for Gauss's Law:

$$F_{Net} = 0$$

$\underline{S_1}$ (OUTSIDE SHELL)

$$\varepsilon_0 E\,(4\pi R^2) = q \Rightarrow E = \frac{q}{4\pi\varepsilon_0 R^2}$$

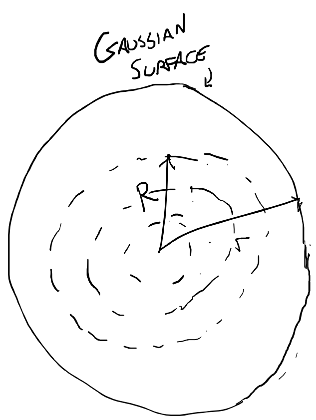
$\Leftrightarrow$ point charge q

$\underline{S_2}$ (INSIDE SHELL)

$$\varepsilon_0 E\left(4\pi R^2\right) = q_{enc} = 0$$

$$\Rightarrow E = 0$$

Consider this uniform "cloud" of uniform electron density...



$$\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi R^3} \quad \left(\frac{C}{m^3}\right)$$

$$\varepsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{enc}$$

WELL, FOR $r > R$...

$$\varepsilon_0 E\left(4\pi r^2\right) = q \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}}$$

AND FOR $r < R$...

$$\varepsilon_0 E\left(4\pi r^2\right) = q_{enc} = \rho V_{GS} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{q}{\cancel{\frac{4}{3}\pi} R^3}\cdot \cancel{\frac{4}{3}\pi} r^3 = q\left(\frac{r}{R}\right)^3$$

$$= \varepsilon_0 E 4\pi r^2 = q\frac{r^{\cancel{3}}}{R^3}$$

$\varepsilon_0 E \, 4\pi r^2 \; r R^3$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0}\cdot\frac{r}{R^3}}$$

Example of a conducting shell,