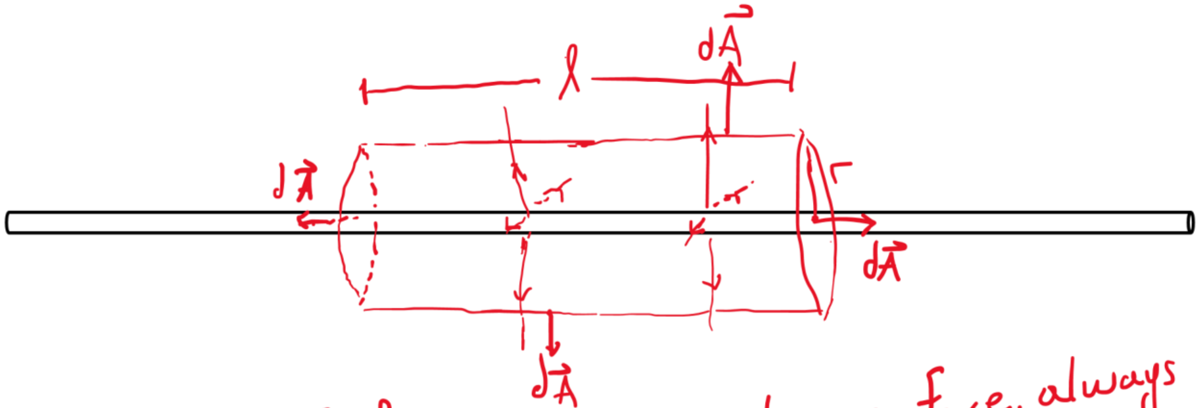
Electric Flux and Gauss's Law

Calculate the electric field for the following distribution of charge…

An infinite wire with a linear charge density, λ (lambda).



$Q_{enclosed} = \lambda \ell$

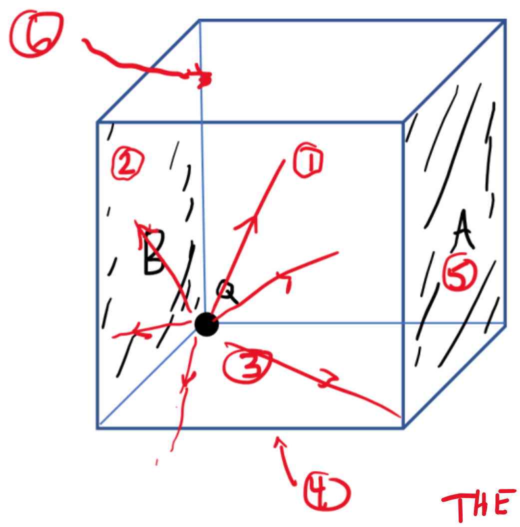
symmetric, uniform along surface, always $\vec{E} \parallel d\vec{A}$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = EA_{cyl} = \frac{Q_{enc}}{\varepsilon_0}$$

$$E(2\pi r \cancel{\ell}) = \frac{\lambda \cancel{\ell}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}}$$

WE NEGLECT END CAPS B/C $\Phi = \emptyset$ WHEN $d\vec{A} \perp \vec{E}$

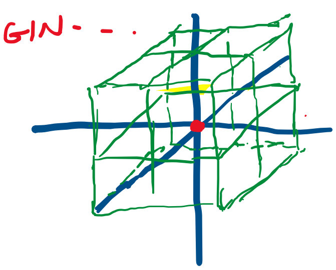
Now, let's find the flux on each of the two labeled sides of this box (where there is a point charge that is *just* inside of the box in the bottom left) …



ESSENTIALLY, WE ARE IMAGINING THAT THE POINT CHARGE IS AT THE POSITION:

$$(x, y, z) \rightarrow (0, 0, 0).$$

IMAGINE A CUBE CENTERED AROUND THE ORIGIN...



(NOTICE THAT ONE-EIGHTH OF THIS CUBE IS THE SAME AS OUR ORIGINAL CUBE.)

THE REASON WE ARE CONSIDERING THIS SECOND CASE IS BECAUSE NOW WE HAVE SYMMETRY. WE HAVE $6 \times 4 = \underline{24}$ QUARTER-FACES, EACH W/ EQUAL FLUX BY SYMMETRY. TOTAL FLUX IS $\frac{Q}{\varepsilon_0}$.

SO EACH FACE RECEIVES A FLUX $\frac{Q}{24\varepsilon_0}$.

FACES 4, $\underline{5}$, AND 6 WILL THUS RECEIVE $\boxed{\phi = \frac{Q}{24\varepsilon_0}}$.

BUT WHAT ABOUT FACES 1, 2, AND 3?

WE KNOW THAT $\phi = \frac{Q}{\varepsilon_0}$ HERE BY GAUSS.

WE ALSO HAVE THAT $\phi_4 = \phi_5 = \phi_6 = \frac{Q}{24\varepsilon_0}$.

SO, $\phi_{total} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6$

BY SYMMETRY, $\phi_1 = \phi_2 = \phi_3 \Rightarrow \phi_1 + \phi_2 + \phi_3 = 3\phi_2$.

$$\phi_{tot} = 3\phi_2 + 3\left(\frac{Q}{24\varepsilon_0}\right) = \frac{Q}{\varepsilon_0}$$

$$3\phi_2 = \frac{Q}{\varepsilon_0}\left(1 - \frac{1}{8}\right) \Rightarrow \phi_2 = \frac{Q}{3\varepsilon_0}\left(\frac{7}{8}\right) = \boxed{\frac{7Q}{24\varepsilon_0}}$$