Value at Risk (VaR)

What is VaR?

Value at Risk (VaR) is a statistical measure that quantifies the potential loss in value of a portfolio or an investment over a specified time period for a given confidence level.

It answers the question:
"What is the maximum expected loss over a given time horizon at a certain confidence level?"

For example, a 1-day 95% VaR of $1 million means:
There is a 95% confidence that the portfolio will not lose more than $1 million in one day.

Key Components of VaR

1. **Time horizon:**
   The period over which risk is measured (e.g., 1 day, 10 days, 1 month).
2. **Confidence level:**
   The probability with which the loss will not exceed VaR (e.g., 95%, 99%).
3. **Loss amount:**
   The estimated maximum loss that won't be exceeded with the specified confidence.

Techniques to Calculate VaR

There are several approaches to compute VaR. Each has its advantages and limitations.

1. Historical Simulation VaR

Concept

- Uses actual historical returns to simulate potential future losses.
- Sorts historical portfolio returns and picks the percentile loss corresponding to the confidence level.

Steps

1. Collect historical returns for the portfolio or asset.
2. Order these returns from worst to best.
3. For 95% VaR, find the 5th percentile loss in historical returns.
4. Multiply this loss by the portfolio value to get VaR.

Pros

- Simple and intuitive.
- Does not assume any particular distribution.
- Reflects actual observed market behavior including fat tails.

Cons

- Assumes the future resembles the past (stationarity).
- Limited by the size and quality of historical data.
- Cannot capture extreme events not in the dataset.

2. Variance-Covariance (Parametric) VaR

Concept

- Assumes portfolio returns are normally distributed.
- Uses portfolio mean and variance to estimate VaR analytically.

Pros

- Fast and easy to compute.
- Requires only mean, variance, and covariance.

Cons

- Assumes normal distribution, which may underestimate risk (ignores fat tails).
- Less accurate for portfolios with options or non-linear assets.

3. Monte Carlo Simulation VaR

Concept

- Simulates a large number of possible future portfolio values by modeling asset returns with a stochastic process (e.g., Geometric Brownian Motion).
- Uses simulations to empirically generate a distribution of portfolio losses.

Steps

1. Define a model for asset price dynamics (e.g., GBM).
2. Simulate many possible future price paths.
3. Calculate portfolio values for each path.
4. Determine loss distribution and pick percentile loss for VaR.

Pros

- Highly flexible, can model complex assets and dynamics.
- Can incorporate fat tails, skewness, and other realistic features.
- Works well for portfolios with non-linear payoffs (options).

Cons

- Computationally intensive.
- Requires modeling assumptions and parameter estimation.
- Model risk if assumptions are wrong.

4. Other Approaches

- **Extreme Value Theory (EVT) VaR:** Focuses on modeling the tail of the loss distribution to better capture rare extreme losses.
- **Conditional VaR (CVaR) / Expected Shortfall:** Measures the expected loss given that the loss exceeds the VaR threshold, addressing some VaR limitations.
- **Filtered Historical Simulation:** Enhances historical simulation by filtering returns through volatility models like GARCH.