Пример 3 Т11

$H_0: \xi \sim p_0(x) = 1\{(0;1)\}$

$H_1: \xi \sim p_1(x) = \frac{e}{e-1} e^{-x} \{(0;1)\}$

а) $n=1$ α

$L = \frac{L_1}{L_0} = \frac{\frac{e}{e-1} e^{-x}}{1} \geq C$

$e^{-x} \geq B \rightarrow x \leq A$

$P(x \leq A \mid H_0) = \alpha$

$\int_0^A 1 dx = A = \alpha$

$G: x \leq \alpha$

$\alpha_1 = \alpha$

$W = P(x \leq A \mid H_1) = \int_0^\alpha \frac{e}{e-1} e^{-x} dx =$

$= \frac{e}{e-1}(1 - e^{-\alpha})$

$\alpha_2 = 1 - W$

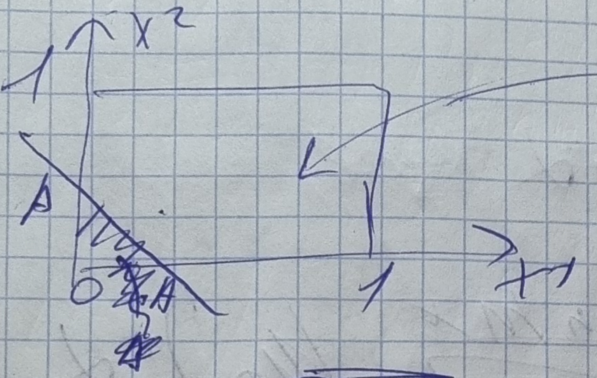
в) $n=2$

$$L = \frac{L_1}{L_0} = \frac{\left(\frac{e}{e-1}\right)^2 \cdot e^{-x_1} e^{-x_2}}{1 \cdot 1} \geq c$$

$$e^{-(x_1+x_2)} \geq h$$

$$x_1 + x_2 \leq A$$

$$P(x_1+x_2 \leq A \mid H_0) = \alpha$$



$$L_0 = 1$$

$$\iint\limits_{x_1+x_2 \leq A} 1\, dx = \frac{A^2}{2} = \alpha$$

$$A = \sqrt{2\alpha}$$

$$G_{кр}: \ x_1 + x_2 \leq \sqrt{2\alpha}$$

$$\alpha_1 = \alpha$$

$$W = P(x_1 + x_2 \leq A \mid H_1) = \iint\limits_{x_1+x_2 \leq A} \left(\frac{e}{e-1}\right)^2 e^{-x_1-x_2}\, dx_1 dx_2$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A dx_1 \int_0^{A-x_1} e^{-x_1} \cdot e^{-x_2}\, dx_2 = \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1}\left(1 - e^{-(A-x_1)}\right) dx_1$$

$$= \left(\frac{e}{e-1}\right)^2 \int_0^A e^{-x_1} - e^{-A} = \left(\frac{e}{e-1}\right)^2 \left(1 - e^{-A} - Ae^{-A}\right)$$

$$\alpha_2 = 1 - W$$

г) $$L = \frac{L_1}{L_0} = \prod \frac{p_1(x_i)}{p_0(x_i)} \geq C$$

$$\ln L = \sum \ln \frac{p_1(x_i)}{p_0(x_i)} \geq \ln C$$

$$\frac{\sum \eta_i - n M\eta}{\sqrt{n D_\eta}} \rightsquigarrow N(0,1)$$

$$P(\ln L \geq \ln c \mid H_0) = \alpha$$

$$\eta = \ln\left(\frac{e}{e-1} e^{-x}\right) = \ln \frac{e}{e-1} - x$$

$$\ln L = \sum \ln \frac{e}{e-1} - \sum x_i \geq \ln c$$

$$G: \sum x_i \leq A$$

$$P\left(\sum x_i \leq A \mid H_0\right) = \alpha$$

$$P\left(\frac{\sum x_i - n Mx}{\sqrt{n Dx}} \leq \frac{A - n Mx}{\sqrt{n Dx}} \mid H_0\right) = \alpha$$

$$Mx = \frac{1}{2}$$

$$Dx = \frac{1}{12}(b-a)^2$$

$$G: \sum x_i \leq n \frac{1}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$\frac{A - n\frac{1}{2}}{\sqrt{n \cdot 1/12}} = u_\alpha$$

$$A = n\frac{1}{2} + u_\alpha \sqrt{\frac{n}{12}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\sum x_i \leq A \mid H_1\right) = P\left(\frac{\sum x_i - n Mx}{\sqrt{n Dx}} \leq \frac{A - n M_0}{\sqrt{n D_0}} \mid H_1\right)$$

$$Mx = \int_0^1 x \frac{e}{e-1} e^{-x} dx = \frac{e-2}{e-1}$$

$$Mx^2 = \frac{2e-5}{e-1}$$

$$D_x = \frac{e^2-3e+1}{(e-1)^2}$$

$$W = \int_{-\infty}^{b} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

$$b = \frac{\frac{n}{2} + u_\alpha \sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \frac{e^2-3e+1}{(e-1)^2}}} = \frac{n\overbrace{\left(\frac{1}{2} - \frac{e-2}{e-1}\right)}^{0,082} + u_\alpha\sqrt{\frac{n}{12}}}{}$$

$W \to 1$ $\quad$ состоит.

$n \to \infty$

$\alpha = 0,05$ $\quad$ $n=1$ $\quad$ $x \le 0,05$ $\quad$ $W = 0,077$ $\quad$ α

$n=2$ $\quad$ $x_1+x_2 < 0,56$ $\quad$ $W = 0,102$ $\quad$ $\alpha =$

$\alpha = 0,78$

$n=10$ $\quad$ $\sum x_i \le 3,5$ $\quad$ $W = 0,22$

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d) $G: x_{min} < C$

$P(x_{min} < C | H_0) = \alpha$

$H_0: \xi \sim R(0;1)$

$\xi \sim F_0(x) = x$

$\xi_{min} \sim 1 - (1 - F_0(x))^n$

$P(x_{min} < C) = 1 - (1 - \underbrace{F_0(C)}_{=C})^n = \alpha$

$C = 1 - \sqrt[n]{1-\alpha}$

$G: x_{min} < 1 - \sqrt[n]{1-\alpha}$

$\alpha 1 = \alpha$

$W = P(\vec{x}_n \in G | H_1) = P(x_{min} < C | H_1)$

$F_1(x) = \int_0^x \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}(1 - e^{-x})$

$W = 1 - (1 - F_1(C))^n = 1 - (1 - \frac{e}{e-1}(1 - e^{\sqrt[n]{1-\alpha} - 1}))^n$

$\alpha 2 = 1 - W$

Проверка на сост.

$n \to \infty$

$W = 1 - (1 - \frac{e}{e-1}(1 - e^{\sqrt[n]{1-\alpha} - 1}))^n = 1 - (1 - \frac{e}{e-1}(1 -$

$$-e^{-1}\cdot e^{e\frac{1}{n}\cdot\ln(1-\alpha)}\Big)\Big)^n = 1-\Big(1-\frac{e}{e-1}\Big(1-1-\frac{1}{n}\ln(1-\alpha)$$

$$-O\Big(\frac{1}{n}\Big)\Big)\Big)\Big)^n = 1-\Big(1+\frac{e}{e-1}\frac{1}{n}\ln(1-\alpha)+O\Big(\frac{1}{n}\Big)\Big)^n$$

$$\xrightarrow[n\to\infty]{} 1-e^{\frac{e\ln(1-\alpha)}{e-1}} = 1-(1-\alpha)^{\frac{e}{e-1}} \not\to 1$$

$\Rightarrow$ не сост.