Math 504: Advanced Linear Algebra

Hugo Woerdeman, with edits by Darij Grinberg*

October 4, 2021 (unfinished!)

Contents

Math 504 Lecture 7

1. Schur triangularization (cont'd)

1.1. Sylvester's equation (cont'd)

Last time, we proved the $\mathcal{V} \Longrightarrow \mathcal{U}$ direction of the following fact:

Theorem 1.1.1. Let A be an $n \times n$-matrix, and let B be an $m \times m$-matrix (both with complex entries). Let C be an $n \times m$-matrix. Then, the following statements are equivalent:

- $\mathcal{U}$: There is a **unique** matrix $X \in \mathbb{C}^{n \times m}$ such that $AX - XB = C$.
- $\mathcal{V}$: We have $\sigma(A) \cap \sigma(B) = \varnothing$.

Here, $\sigma(A)$ denotes the **spectrum** of a matrix A (that is, the set of all eigenvalues of A).

*Drexel University, Korman Center, 15 S 33rd Street, Philadelphia PA, 19104, USA