

Towards A Synthetic Formulation of Multiparty Session Types

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Background and Motivation

A Crash Course on Classic Multiparty Session Types

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
    for {
        ubound := <-reqCh
        workerChs := make([]chan int, ubound)
        errCh := make(chan error)
        for i := 0; i < ubound; i++ {
            workerChs[i] = make(chan int)
            go Worker(i+1, workerChs[i], errCh)
        }
        var res []int
        for i := 0; i < ubound; i++ {
            select {
            case sql := <-workerChs[i]:
                res = append(res, sql)
            case err := <-errCh:
                cErrCh <- err
            }
            return
        }
    }
    respCh <- res}}
```

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
    for {
        about
        work
        errCh
        for
            wo
            go
        }
        var
        for
            sel
            case sql := <-workerChs[i]:
                res = append(res, sql)
            case err := <-errCh:
                cErrCh <- err
            return
        }}
    respCh <- res}}
```

DEADLOCK!

ORPHAN MESSAGES!

NO RESOURCE CLEANUP!

...

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
    for {
        ubound := <-reqCh
        worke
        errCh
        for i
            wor
            go
    }
    var res []int
    for i := 0; i < ubound; i++ {
        select {
            case sql := <-workerChs[i]:
                res = append(res, sql)
            case err := <-errCh:
                cErrCh <- err
                return
        }
    }
    respCh <- res}}
```

Master needs to guarantee that all Workers are notified when there is an error.

Key Idea

Multiparty Session Types prevent you from writing the code in the previous slide by enforcing syntactically that process implementations follow a given specification.

In a nutshell:

1. Global types: protocol specifications among a fixed number of different *roles*.
2. Role: sets of interactions that processes can do in a protocol.
3. Local types: protocol specifications *from the point of view of a single role*.
4. Projection: a *partial function* that extracts *local type* given a *global types* and a *role*.
5. Well-formedness: guarantees **deadlock-freedom**, usually defined in terms of *projectibility*.

processes { W_1 W_2 W_3 }

global type {

G

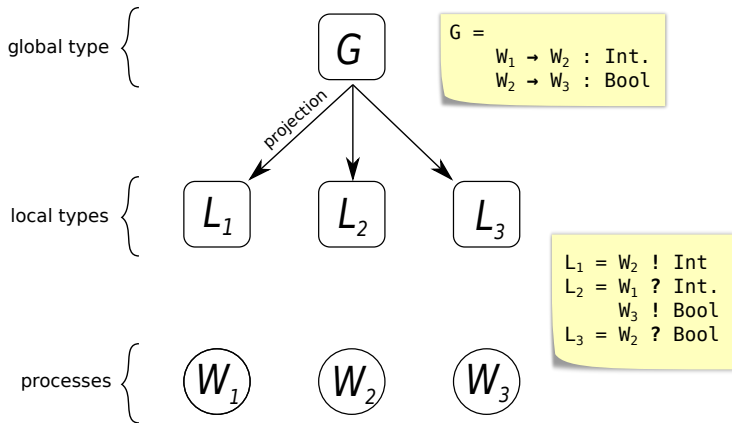
$G =$
 $W_1 \rightarrow W_2 : \text{Int.}$
 $W_2 \rightarrow W_3 : \text{Bool}$

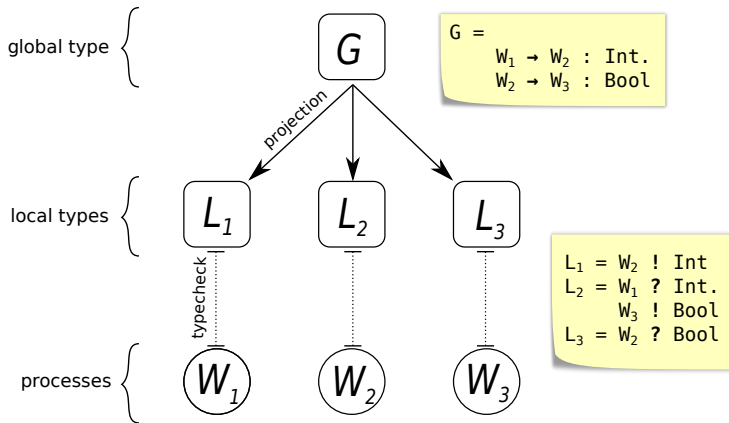
processes {

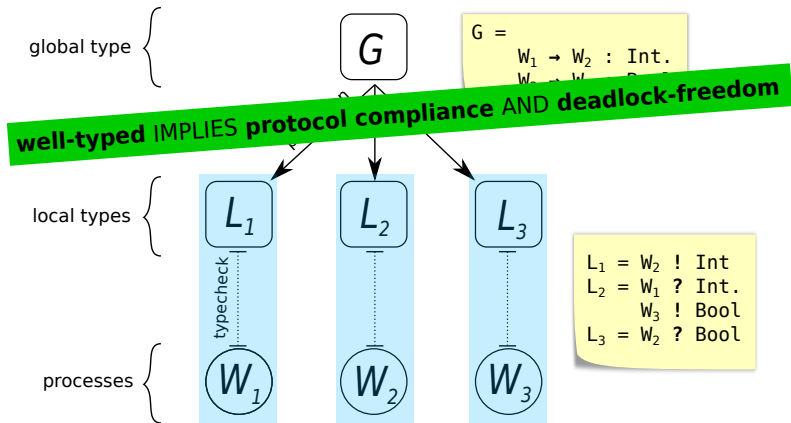
W_1

W_2

W_3







Global and Local Types

Roles	p, q, \dots	
Sorts	$S := \text{bool} \mid \text{nat} \mid \dots$	Basic data types.
Global Types	$G :=$ $\quad p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I}$ $\quad \mid$ $\quad \mu X.G$ $\quad \mid$ $\quad X$ $\quad \mid$ $\quad \emptyset$	Message communication. Recursion. Recursion variable. End of protocol.
Local Types	$L :=$ $\quad p!\{\ell_i(S_i).L_i\}_{i \in I}$ $\quad \mid$ $\quad q?\{\ell_i(S_i).L_i\}_{i \in I}$ $\quad \mid$ $\quad \mu X.G$ $\quad \mid$ $\quad X$ $\quad \mid$ $\quad \emptyset$	Send message. Receive message. Recursion. Recursion variable. End of protocol.

Projection

$$p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = \begin{cases} q! \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (r = p \wedge \quad \wedge p \neq q) \\ p? \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (\quad \wedge r = q \wedge p \neq q) \\ \sqcap_{i \in I} (G_i \upharpoonright r) & (r \neq p \wedge r \neq q \wedge p \neq q) \end{cases}$$

$$\mu X.G \upharpoonright r = \begin{cases} \mu X.G \upharpoonright r & (r \in G) \\ \emptyset & (r \notin G) \end{cases} \quad X \upharpoonright r = X \quad \emptyset \upharpoonright r = \emptyset$$

Projection

$$p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = \begin{cases} q! \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (r = p \wedge \wedge p \neq q) \\ p? \{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (\wedge r = q \wedge p \neq q) \\ \sqcap_{i \in I} (G_i \upharpoonright r) & (r \neq p \wedge r \neq q \wedge p \neq q) \end{cases}$$

$$\mu X.G \upharpoonright r = \begin{cases} \mu X.G \upharpoonright r & (r \in G) \\ \emptyset & (r \notin G) \end{cases} \quad X \upharpoonright r = X \quad \emptyset \upharpoonright r = \emptyset$$

$$\begin{aligned} & p? \{\ell_i(S_i).L_i\}_{i \in I} \sqcap p? \{\ell_j(S_j).L'_j\}_{j \in J} \\ &= p? \{\ell_i(S_i).L_i\}_{i \in I \setminus J} \cup \{\ell_j(S_j).L'_j\}_{j \in J \setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i \in I \cap J} \end{aligned}$$

$$p! \{\ell_i(S_i).L_i\}_{i \in I} \sqcap p! \{\ell_i(S_i).L'_i\}_{i \in I} = p! \{\ell_i(S_i).L_i \sqcap L'_i\}_{i \in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \quad L \sqcap L = L$$

Projection

$$p \rightarrow q : \{\ell_i(S_i).G_i\}_{i \in I} \upharpoonright r = \begin{cases} q!\{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (r = p \wedge \quad \wedge p \neq q) \\ p?\{\ell_i(S_i).G_i \upharpoonright r\}_{i \in I} & (\quad \wedge r = q \wedge p \neq q) \\ \sqcap_{i \in I} (G_i \upharpoonright r) & (r \neq p \wedge r \neq q \wedge p \neq q) \end{cases}$$

It gets complicated very quickly!

$$\mu X.G \upharpoonright r = \begin{cases} \emptyset & (r \notin G) \end{cases} \quad \Delta \upharpoonright r = \Delta \quad \emptyset \upharpoonright r = \emptyset$$

$$\begin{aligned} & p?\{\ell_i(S_i).L_i\}_{i \in I} \sqcap p?\{\ell_j(S_j).L'_j\}_{j \in J} \\ &= p?\{\ell_i(S_i).L_i\}_{i \in I \setminus J} \cup \{\ell_j(S_j).L'_j\}_{j \in J \setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i \in I \cap J} \end{aligned}$$

$$p!\{\ell_i(S_i).L_i\}_{i \in I} \sqcap p!\{\ell_i(S_i).L'_i\}_{i \in I} = p!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i \in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \quad L \sqcap L = L$$

What is the point of \sqcap ?

Example:

$$\mu X. p \rightarrow q : \left\{ \begin{array}{l} \text{REQ}(\text{nat}).q \rightarrow r : \text{REQ}(\text{bool}).X \\ \text{END}() \quad .q \rightarrow r : \text{END}().\text{done} \end{array} \right\}$$

What is the point of \sqcap ?

Example:

$$\mu X. p \rightarrow q : \left\{ \begin{array}{l} \text{REQ}(\text{nat}).q \rightarrow r : \text{REQ}(\text{bool}).X \\ \text{END}() \quad .q \rightarrow r : \text{END}().\text{done} \end{array} \right\}$$

Projecting r

$$\mu X. (q? \text{REQ}(\text{bool}).X) \sqcap (q? \text{END}().\emptyset)$$

=

What is the point of \sqcap ?

Example:

$$\mu X. p \rightarrow q : \left\{ \begin{array}{l} \text{REQ}(\text{nat}).q \rightarrow r : \text{REQ}(\text{bool}).X \\ \text{END}() \quad .q \rightarrow r : \text{END}().\text{done} \end{array} \right\}$$

Projecting r

$$\begin{aligned} & \mu X. (q? \text{REQ}(\text{bool}).X) \sqcap (q? \text{END}().\emptyset) \\ &= \mu X. q? \left\{ \begin{array}{l} \text{REQ}(\text{bool}).X \\ \text{END}() \quad .\text{done} \end{array} \right\} \end{aligned}$$

Processes and Typing

Process	P	$:=$	$p!l\langle e \rangle.P$	Send a message.
			$\sum_{i \in I} p?l_i(x_i).P_i$	Receive a message.
			$\text{if } e \text{ then } P \text{ else } P'$	Conditional process.
			$\text{rec } X.P$	Recursive process.
			X	Recursion variable.
			done	Inactive process.

Process Typing (simplified)

Once we have local types, process typing is simple:

T-SEND

$$\frac{\Gamma \vdash P : L_i \quad \Gamma \vdash e : S_i \quad i \in I}{\Gamma \vdash \mathbf{q} ! \ell_i \langle e \rangle . P : (\mathbf{p} ! \{\ell_i(S_i) . L_i\}_{i \in I})}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : L_i \quad \forall i \in I}{\Gamma \vdash \sum_{i \in I} \mathbf{p} ? \ell_i(x_i) . P_i : (\mathbf{p} ? \{\ell_i(S_i) . L_i\}_{i \in I})}$$

Problems with Classic Formulation

1. Too syntactic:

- Processes and local types must align
- Too restrictive, rules out correct processes
- ...

2. Unnecessarily complex:

- Hard to implement/mechanise, e.g.:
 - Use of runtime coinductive global types: Our PLDI 2021 paper
 - Complex graph-based representation of MPST: Jacobs et al. (2022)
 - Graph-based reasoning and decision procedure for the equality of recursive types: Tirore et al. (2023)
- Hard to extend

A Few Attempts at Simplifying the Theory

Less Is More: Multiparty Session Types Revisited

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A Few Attempts at Simplifying the Theory

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ALC
NO

Less is More Revisited

Association with Global Multiparty Session Types

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HOW STANDARDS PROLIFERATE:

(SEE: A/C CHARGERS, CHARACTER ENCODINGS, INSTANT MESSAGING, ETC.)

SITUATION:
THERE ARE
14 COMPETING
STANDARDS.

14?! RIDICULOUS!
WE NEED TO DEVELOP
ONE UNIVERSAL STANDARD
THAT COVERS EVERYONE'S
USE CASES.



YEAH!

SOON:

SITUATION:
THERE ARE
15 COMPETING
STANDARDS.

Our Approach: Synthetic Typing

Synthetic Behavioural Typing: Sound, Regular Multipart Sessions via Implicit Local Types

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Our Approach: Synthetic Typing

Synthetic Typing: A New Paradigm in Type Theory

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Sun

Depar

Centr

Frar

Depar

Goals:

- “Free” typing from being tied up to the syntax of local types.
- Avoid projection/merging/etc.
- A formal description of equality between global types to replace informally equating global types to their unfolding.
- Well-formedness/deadlock-freedom is decided by typeability.
- Mechanisation in Agda.

Towards Synthetic MPST (WIP)

New (Synthetic) Core Typing Rules

New judgement : $\Gamma \vdash P : G \upharpoonright p$

T-SEND

$$\frac{G \setminus p \xrightarrow{\ell(S)} q = G' \quad \Gamma \vdash P : G' \upharpoonright p \quad \Gamma \vdash e : S}{\Gamma \vdash q ! \ell(e).P : G \upharpoonright p}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus q \xrightarrow{\ell_i(S_i)} p = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i).P_i : G \upharpoonright p}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus q \xrightarrow{\ell_i(S_i)} p = G' \text{ and } p \neq r \wedge q \neq r \text{ we have } \Gamma \vdash P : G' \upharpoonright r}{\Gamma \vdash P : G \upharpoonright r}$$

Synthetic, in that G' occurs only in the premise, not in the conclusion. G' needs to be *synthesised* by using the rules of the operational semantics of global types (Jongmans and Ferreira, 2023).

New (Synthetic) Core Typing Rules

What is wrong with these rules?

T-SEND

$$\frac{G \setminus \overset{\ell(S)}{p \rightarrow q} = G' \quad \Gamma \vdash P : G' \upharpoonright p \quad \Gamma \vdash e : S}{\Gamma \vdash q ! \ell\langle e \rangle . P : G \upharpoonright p}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{q \rightarrow p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i) . P_i : G \upharpoonright p}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{q \rightarrow p} = G' \text{ and } p \neq r \wedge q \neq r \text{ we have } \Gamma \vdash P : G' \upharpoonright r}{\Gamma \vdash P : G \upharpoonright r}$$

New (Synthetic) Core Typing Rules

Hint: the problem is in these rules

T-RECV

$$\frac{\forall(i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{q \rightarrow p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i). P_i : G \upharpoonright p}$$

T-SKIP

$$\frac{\forall(i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{q \rightarrow p} = G' \text{ and } p \neq r \wedge q \neq r \text{ we have } \Gamma \vdash P : G' \upharpoonright r}{\Gamma \vdash P : G \upharpoonright r}$$

New (Synthetic) Core Typing Rules

Hint 2: the problem is the same in both rules, let's focus on this one

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \frac{\ell_i(S_i)}{q \rightarrow p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i). P_i : G \upharpoonright p}$$

New (Synthetic) Core Typing Rules

What happens if G does not allow p to receive from q ?

T-RECV

$$\frac{\boxed{\forall (i \in I) \text{ s.t. } G \setminus \frac{\ell_i(S_i)}{q \rightarrow p} = G'} \quad \text{we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i). P_i : G \upharpoonright p}$$

New (Synthetic) Core Typing Rules

This was a “rookie” mistake ... We cannot allow rules to be vacuously true!

T-RECV

$$\frac{\boxed{\forall (i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{q \rightarrow p} = G'} \quad \text{we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright p}{\Gamma \vdash \sum_{i \in I} q ? \ell_i(x_i). P_i : G \upharpoonright p}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathbf{p}, \mathbf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \overset{\ell_i(S_i)}{\mathbf{p} \rightarrow \mathbf{q}} = G'$ – this means that an interaction between \mathbf{p} and \mathbf{q} is “ready” (i.e. can happen) in G .

T-SEND

$$\frac{G \setminus \overset{\ell(S)}{\mathbf{p} \rightarrow \mathbf{q}} = G' \quad \Gamma \vdash P : G' \upharpoonright \mathbf{p} \quad \Gamma \vdash e : S}{\Gamma \vdash \mathbf{q} ! \ell \langle e \rangle . P : G \upharpoonright \mathbf{p}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathbf{p}, \mathbf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \mathbf{p} \xrightarrow{\ell_i(S_i)} \mathbf{q} = G'$ – this means that an interaction between \mathbf{p} and \mathbf{q} is “ready” (i.e. can happen) in G .

T-RECV

$$\frac{\mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathbf{q} \xrightarrow{\ell_i(S_i)} \mathbf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathbf{p}}{\Gamma \vdash \sum_{i \in I} \mathbf{q} ? \ell_i(x_i). P_i : G \upharpoonright \mathbf{p}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathbf{p}, \mathbf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \overset{\ell_i(S_i)}{\mathbf{p} \rightarrow \mathbf{q}} = G'$ – this means that an interaction between \mathbf{p} and \mathbf{q} is “ready” (i.e. can happen) in G .

T-SKIP

$$\frac{\begin{array}{l} \mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \wedge (\mathbf{r} \notin \{\mathbf{p}, \mathbf{q}\}) \\ \forall(i \in I) \text{ s.t. } G \setminus \overset{\ell_i(S_i)}{\mathbf{q} \rightarrow \mathbf{p}} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathbf{r} \end{array}}{\Gamma \vdash P : G \upharpoonright \mathbf{r}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathbf{p}, \mathbf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \mathbf{p} \xrightarrow{\ell_i(S_i)} \mathbf{q} = G'$ – this means that an interaction between \mathbf{p} and \mathbf{q} is “ready” (i.e. can happen) in G .

T-SEND

$$\frac{G \setminus \mathbf{p} \xrightarrow{\ell(S)} \mathbf{q} = G' \quad \Gamma \vdash P : G' \upharpoonright \mathbf{p} \quad \Gamma \vdash e : S}{\Gamma \vdash \mathbf{q} ! \ell\langle e \rangle . P : G \upharpoonright \mathbf{p}}$$

T-RECV

$$\frac{\mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathbf{q} \xrightarrow{\ell_i(S_i)} \mathbf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathbf{p}}{\Gamma \vdash \sum_{i \in I} \mathbf{q} ? \ell_i(x_i) . P_i : G \upharpoonright \mathbf{p}}$$

T-SKIP

$$\frac{\mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \wedge (r \notin \{\mathbf{p}, \mathbf{q}\}) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathbf{q} \xrightarrow{\ell_i(S_i)} \mathbf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright r}{\Gamma \vdash P : G \upharpoonright r}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathbf{p}, \mathbf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \mathbf{p} \xrightarrow{\ell_i(S_i)} \mathbf{q} = G'$ – this means that an interaction between \mathbf{p} and \mathbf{q} is “ready” (i.e. can happen) in G .

- The rules look more complex than with a syntactic approach, but computing $G \setminus \mathbf{q} \xrightarrow{\ell_i(S_i)} \mathbf{p} = G'$ is entirely mechanical by using the semantics of global types.
- The proof of subject reduction is greatly simplified (more in a few slides) with this formulation.
- No need of projection/merging.

T-RECV

$\mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G)$

\mathbf{p}

T-SKIP

$$\frac{\mathcal{R}(\mathbf{q}, \mathbf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \wedge (r \notin \{\mathbf{p}, \mathbf{q}\}) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathbf{q} \xrightarrow{\ell_i(S_i)} \mathbf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright r}{\Gamma \vdash P : G \upharpoonright r}$$

Semantics

Examples

Global Type Bisimilarity

Properties of Synthetic MPST

Wrap Up

A Crash Course on Classic Multiparty Session Types

Benefits of Synthetic Typing

TODO