Towards A Synthetic Formulation of Multiparty Session Types

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Background and Motivation

A Crash Course on Classic Multiparty Session Types

What is wrong with this code?

```go
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    workerChs := make([]chan int, ubound)
    errCh := make(chan error)
    for i := 0; i < ubound; i++ {
      workerChs[i] = make(chan int)
      go Worker(i+1, workerChs[i], errCh)
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubour
    work
    errCh
    for
      wo
      go
    }
    var
    for
      sel
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

DEADLOCK!

ORPHAN MESSAGES!

NO RESOURCE CLEANUP!

...

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    worke
    errCh
    for i
      wor
      go
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

Master needs to guarantee that all Workers are notified when there is an error.

Key Idea

Multiparty Session Types prevent you from writing the code in the previous slide by enforcing syntactically that process implementations follow a given specification.

In a nutshell:

1. Global types: protocol specifications among a fixed number of different *roles*.
2. Role: sets of interactions that processes can do in a protocol.
3. Local types: protocol specifications *from the point of view of a single role*.
4. Projection: a *partial function* that extracts a *local type* given a *global type* and a *role*.

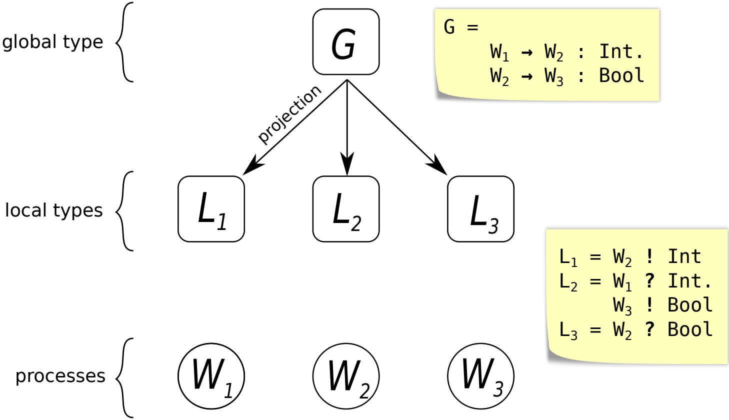
5. Well-formedness: guarantees **deadlock-freedom**, usually defined in terms of *projectability*.

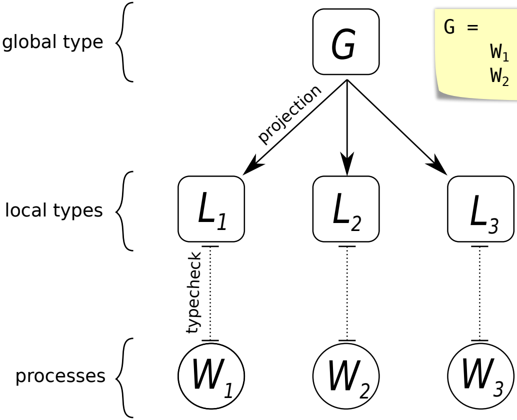

processes
W_1
W_2
W_3

global type G

```
G =
    W₁ → W₂ : Int.
    W₂ → W₃ : Bool
```

processes W_1 W_2 W_3

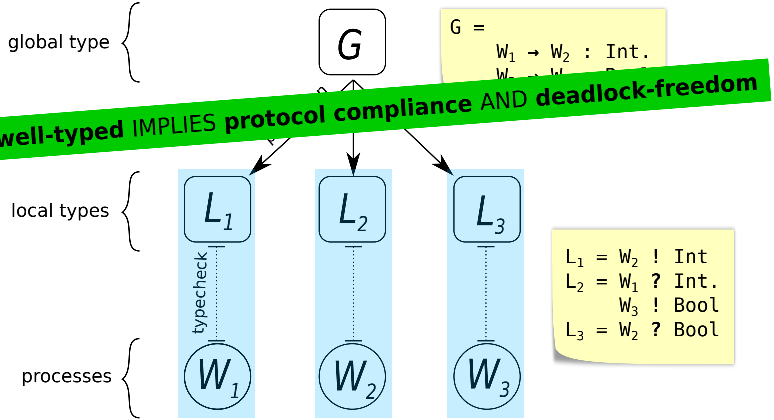

global type
local types
processes
G
projection
L_1
L_2
L_3
W_1
W_2
W_3
G =
$W_1 \to W_2$: Int.
$W_2 \to W_3$: Bool
$L_1 = W_2$! Int
$L_2 = W_1$? Int.
W_3 ! Bool
$L_3 = W_2$? Bool




```
G =
    W₁ → W₂ : Int.
    W₂ → W₃ : Bool
```

```
L₁ = W₂ ! Int
L₂ = W₁ ? Int.
     W₃ ! Bool
L₃ = W₂ ? Bool
```

well-typed IMPLIES **protocol compliance** AND **deadlock-freedom**



Global and Local Types

Roles			$\mathsf{p}, \mathsf{q}, \ldots$	
Sorts	S	$:=$	$\texttt{bool} \mid \texttt{nat} \mid \cdots$	Basic data types.
Global Types	G	$:=$	$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i \in I}$	Message communication.
		$\mid$	$\mu X.G$	Recursion.
		$\mid$	X	Recursion variable.
		$\mid$	$\varnothing$	End of protocol.
Local Types	L	$:=$	$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i \in I}$	Send message.
		$\mid$	$\mathsf{q}?\{\ell_i(S_i).L_i\}_{i \in I}$	Receive message.
		$\mid$	$\mu X.G$	Recursion.
		$\mid$	X	Recursion variable.
		$\mid$	$\varnothing$	End of protocol.

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \land \qquad\qquad \land\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\qquad\qquad \land\, \mathsf{r} = \mathsf{q} \land \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \upharpoonright \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \land \mathsf{r} \neq \mathsf{q} \land \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \upharpoonright \mathsf{r} = \begin{cases} \mu X.G \upharpoonright \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \upharpoonright \mathsf{r} = X \qquad \varnothing \upharpoonright \mathsf{r} = \varnothing$$

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad\qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\qquad\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \restriction \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \restriction \mathsf{r} = \begin{cases} \mu X.G \restriction \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \restriction \mathsf{r} = X \qquad \varnothing \restriction \mathsf{r} = \varnothing$$

$$\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J}$$
$$= \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \quad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\quad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \upharpoonright \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

It gets complicated very quickly!

$$\mu X.G \upharpoonright \mathsf{r} = \begin{cases} \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \upharpoonright \mathsf{r} = X \qquad \varnothing \upharpoonright \mathsf{r} = \varnothing$$

$$\begin{aligned} &\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J} \\ &\quad = \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J} \end{aligned}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

What is the point of $\sqcap$?

Consider the following protocol
– this is similar to the behaviour of the previous Go code snippet:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

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Projecting r

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\mathtt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$=$$

What is the point of $\sqcap$?

Consider the following protocol
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Projecting r

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\texttt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$= \mu X.\mathsf{q}?\left\{ \begin{array}{ll} \mathsf{REQ}(\texttt{bool}).X \\ \mathsf{END}() \quad .\mathsf{done} \end{array} \right\}$$

Processes and Typing

$$
\begin{array}{llcll}
\text{Process} & P & := & \mathtt{p}\,!\,\ell\langle e\rangle.P & \text{Send a message.} \\
 & & \mid & \sum_{i\in I} \mathtt{p}?\ell_i(x_i).P_i & \text{Receive a message.} \\
 & & \mid & \mathtt{if}\, e\, \mathtt{then}\, P\, \mathtt{else}\, P' & \text{Conditional process.} \\
 & & \mid & \mathtt{rec}\, X\, .\, P & \text{Recursive process.} \\
 & & \mid & X & \text{Recursion variable.} \\
 & & \mid & \mathtt{done} & \text{Inactive process.}
\end{array}
$$

Process Typing (simplified)

Once we have local types, process typing is simple:

T-SEND

$$\frac{\Gamma \vdash P : L_i \qquad \Gamma \vdash e : S_i \qquad i \in I}{\Gamma \vdash \mathsf{q} \,!\, \ell_i\langle e\rangle.P : (\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I})}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : L_i \qquad \forall i \in I}{\Gamma \vdash \sum_{i\in I} \mathsf{p}?\ell_i(x_i).P_i : (\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I})}$$

Process Typing (simplified)

Once

Local types and processes are so similar that some developments omit them, and projection produces directly processes.

T-SEND

$$\frac{\Gamma \vdash P : L_i \qquad \Gamma \vdash e : S_i \qquad i \in I}{\Gamma \vdash \mathsf{q} \,!\, \ell_i\langle e\rangle.P : (\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I})}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : L_i \qquad \forall i \in I}{\Gamma \vdash \sum_{i\in I} \mathsf{p}?\ell_i(x_i).P_i : (\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I})}$$

Process Typing (simplified)

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Once

Deconfined Global Types for Asynchronous Sessions

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T-SEND

$\Gamma \vdash P :$ … $\in I$

$\Gamma \vdash \mathsf{q} \,!\, \ell$ … $.L_i\}_{i \in I})$

Problems with Classic Formulation

1. **Too syntactic:**
   - Processes and local types must align
   - Too restrictive, rules out correct processes
   - ...
2. **Unnecessarily complex:**
   - Hard to implement/mechanise, e.g.:
     - Use of runtime coinductive global types: Our PLDI 2021 paper
     - Complex graph-based representation of MPST: Jacobs et al. (2022)
     - Graph-based reasoning and decision procedure for the equality of recursive types: Tirore et al. (2023)
   - Hard to extend
3. **Imprecise** about the uses of coinduction

Example of Imprecision in Classic MPST

“We identify $\mu X.G$ with $[\mu X.G/X]G$”

This is a common statement in proofs about MPST, which clearly specifies an equirecursive formulation, but...

1. The rules still refer to open global types with variables X
2. The rules specify when and how to unfold $\mu X.G$ – if we are using equirecursion, $\mu.$ should not be in the syntax ouf our language!

Moreover, this “identification” of a global type and its unfolding is not powerful enough. E.g.

$$\mathsf{p} \to \mathsf{q} : \mathsf{p}' \to \mathsf{q}' : G \neq \mathsf{p}' \to \mathsf{q}' : \mathsf{p} \to \mathsf{q} : G$$

This forces the use of tedious syntactic proofs about how the swapping of unrelated actions does not affect the protocol.

A Few Attempts at Simplifying the Theory

Deconfined Global Types for Asynchronous Sessions

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A Few Attempts at Simplifying the Theory

Less Is More: Multiparty Session Types Revisited

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A Few Attempts at Simplifying the Theory

Less Is More: Multiparty Session Types Revisited

Less is More Revisited
Association with Global Multiparty Session Types

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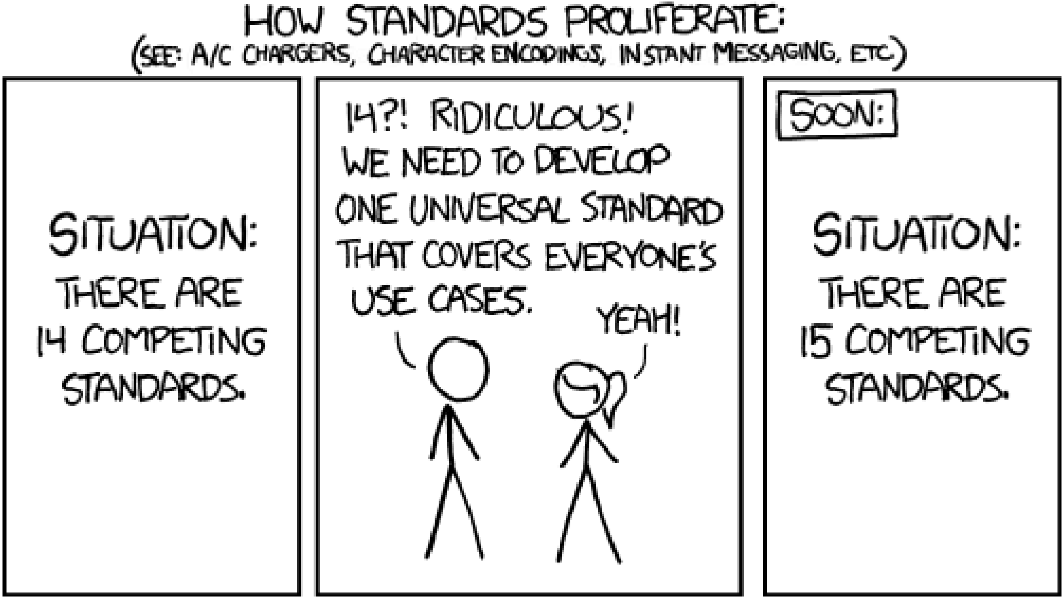

https://xkcd.com/927/

Our Approach: Synthetic Typing

Synthetic Behavioural Typing: Sound, Regular Multiparty Sessions via Implicit Local Types

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Our Approach: Synthetic Typing

Goals:

- "Free" typing from being tied up to the syntax of local types.
- Avoid projection/merging/etc.
- A formal description of equality between global types to replace informally equating global types to their unfolding.
- Well-formedness/deadlock-freedom is decided by typeability.
- Mechanisation in Agda.

Towards Synthetic MPST (WIP)

New (Synthetic) Core Typing Rules

New judgement : $\Gamma \vdash P : G \upharpoonright \mathsf{p}$

T-SEND

$$\frac{\Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q}\, !\, \ell\langle e \rangle . P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p} \qquad \forall\, G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}{\Gamma \vdash \sum_{i \in I} \mathsf{q} ? \ell_i(x_i) . P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\Gamma \vdash P : G' \upharpoonright \mathsf{r} \qquad \forall\, G \setminus \alpha = G' \text{ s.t. } \mathsf{r} \notin \mathsf{parts}(\alpha)}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

Synthetic, in that G' occurs only in the premise, not in the conclusion. G' needs to be *synthesised* by using the rules of the operational semantics of global types (Jongmans and Ferreira, 2023).

New

What is wrong with these rules?

T-SEND

$$\frac{\Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle.P : G \upharpoonright \mathsf{p}}$$

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New

Hint: the problem is in these rules

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : G' \restriction \mathsf{p} \qquad \forall\ G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \restriction \mathsf{p}}$$

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New

Hint 2: the problem is the same in both rules, let's focus on this one

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p} \qquad \forall\, G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

New (Synthetic) Core Typing Rules

What happens if G does not allow p to receive from q?

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p} \qquad \boxed{\forall\, G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

New (Synthetic) Core Typing Rules

This was a "rookie" mistake ... We cannot allow rules to be vacuously true!

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p} \qquad \boxed{\forall\, G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\alpha, G) = \exists G',\ G \setminus \alpha = G'$
– this means that an interaction α is "ready" (i.e. can happen) in G.
Let $\mathcal{W}(\mathsf{r}, G) = \exists \alpha,\ \mathcal{R}(\alpha, G) \wedge \mathsf{r} \notin \mathsf{parts}(\alpha)$
– this means that r can "wait" for another (possibly unrelated) interaction in G.

$$\frac{\Gamma \vdash P : G' \restriction \mathsf{p} \qquad G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle.P : G \restriction \mathsf{p}}\ \text{T-SEND}$$

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T-RECV

$$\frac{\exists (j \in I), \mathcal{R}(\mathsf{q} \xrightarrow{\ell_j(S_j)} \mathsf{p}, G) \qquad \Gamma, x_i : S_i \vdash P_i : G' \restriction \mathsf{p} \qquad \forall\ G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \restriction \mathsf{p}}$$

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$$\frac{\boxed{\exists (j \in I), \mathcal{R}(\mathsf{q} \xrightarrow{\ell_j(S_j)} \mathsf{p}, G)} \qquad \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p} \qquad \forall\ G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP
$$\frac{\boxed{\mathcal{W}(\mathsf{r}, G)} \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{r} \qquad \forall\ G \setminus \alpha = G' \text{ s.t.} \mathsf{r} \notin \mathsf{parts}(\alpha)}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\alpha, G) = \exists G',\ G \setminus \alpha = G'$
– this means that an interaction α is "ready" (i.e. can happen) in G.
Let $\mathcal{W}(\mathsf{r}, G) = \exists \alpha,\ \mathcal{R}(\alpha, G) \wedge \mathsf{r} \notin \mathsf{parts}(\alpha)$
– this mea

- The rules look more complex than with a syntactic approach, but computing $G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ is entirely mechanical by using the semantics of global types.
- The proof of subject reduction is greatly simplified with this formulation.
- There is no need of projection/merging.

$$\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \restriction \mathsf{p}$$

T-SKIP

$$\frac{\boxed{\mathcal{W}(\mathsf{r}, G)} \qquad \Gamma \vdash P : G' \restriction \mathsf{r} \qquad \forall\, G \setminus \alpha = G' \text{ s.t.} \mathsf{r} \notin \mathsf{parts}(\alpha)}{\Gamma \vdash P : G \restriction \mathsf{r}}$$

Semantics

The semantics of global types is defined in a standard way.

Although the current semantics is synchronous, this does not prevent us from defining an asynchronous semantics for processes.

It deals with recursion: in our typing rules we do not need to deal with recursion variables or global type unfolding – a true equirecursive formulation in our type system.

$$\frac{j \in I}{\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i \in I} \setminus \mathsf{p} \xrightarrow{\ell_j(S_j)} \mathsf{q} = G_j} \qquad \frac{[\mu X.G/X]G \setminus \alpha = G'}{\mu X.G \setminus \alpha = G'}$$

$$\frac{\forall (i \in I), G_i \setminus \alpha = G'_i \qquad \mathsf{parts}(\alpha) \cap \{\mathsf{p}, \mathsf{q}\} = \varnothing}{\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i \in I} \setminus \alpha = \mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G'_i\}_{i \in I}}$$

Global Type Bisimilarity

We use a coinductive definition of **strong bisimilarity**:

$G_1 \sim G_2$ iff:

- $\forall \alpha,\ G_1 \setminus \alpha = G_1' \Rightarrow \exists G_2',\ G_2 \setminus \alpha = G_2' \wedge G_1' \sim G_2'$
- $\forall \alpha,\ G_2 \setminus \alpha = G_2' \Rightarrow \exists G_1',\ G_1 \setminus \alpha = G_1' \wedge G_1' \sim G_2'$

It is straightforward that $[\mu X.G/X]G \sim \mu X.G$

Global Type Bisimilarity

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G_1 ∼

- \

- \

We **never** use syntactic equality,
in our type system, only $G \sim G'$

It is straightforward that $[\mu X.G/X]G \sim \mu X.G$

Example

Consider again:

$$G = \mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() \quad\quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

We are going to typecheck a process implementing role r...

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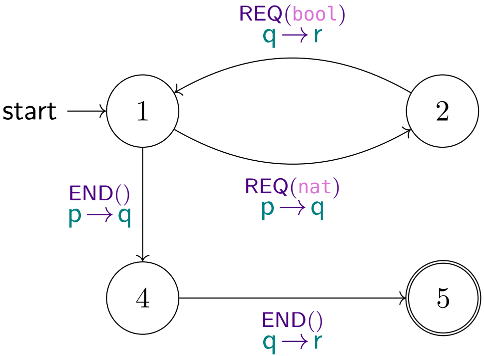
We are going to typecheck a process implementing role r...
but first, let's get rid of the syntax for G!

Example: Semantic View of Global Types

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\texttt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\texttt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$



Example: Semantic View of Global Types

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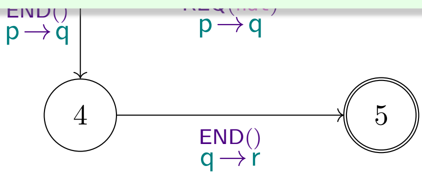
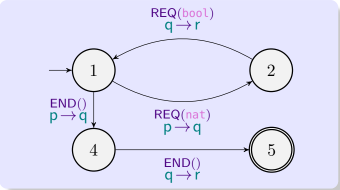
(Small parenthesis, and shameless advertising: I am working with Jonah – and hopefully joining efforts with Francisco, Marco Carbone, Alceste Scalas, any of you that is interested ... – on automating the mechanisation of this semantic view of LTS in Coq/OCaml.)



Example: Process & Typing



$$P = \sum \left\{ \begin{array}{l} \text{q?REQ}(x).\text{print}(x).\ \text{rec}\, X\, .\sum \left\{ \begin{array}{l} \text{q?REQ}(x).\text{process}(x).\ X \\ \text{q?END}(\_).\text{done} \end{array} \right\} \\ \text{q?END}(\_).\text{done} \end{array} \right\}$$

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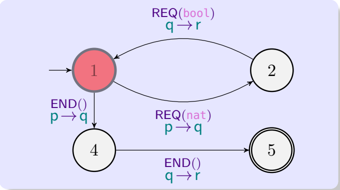

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Goal: $\boxed{\cdot \vdash P : 1 \upharpoonright \mathsf{r}}$ – for simplicity, this example uses LTS state numbers as global types.

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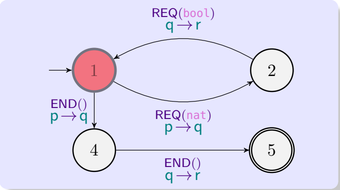

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Goal: $\boxed{\cdot \vdash P : 1 \upharpoonright \mathsf{r}}$ – for simplicity, this example uses LTS state numbers as global types.

–We have a $\sum$, so we can only apply either T-RECV or T-SKIP. At 1, r cannot receive from q, so we must use T-SKIP.

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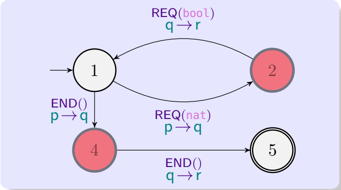

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Two cases: $1 \to 2$, and $1 \to 4$

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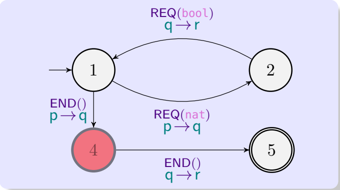

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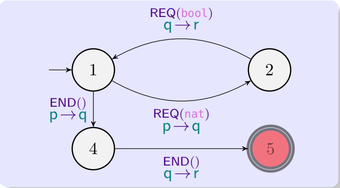
Two cases: $1 \to 2$, and $\boxed{1 \to 4}$

– We have that $4 \setminus \overset{\mathsf{END}()}{\mathsf{q} \to \mathsf{r}} = 5$

– At 5, r can no longer take any action in G, so done is well typed.

Example: Process & Typing



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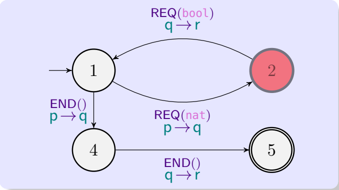

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Two cases: $\boxed{1 \to 2}$, and $1 \to 4$

– We have that $2 \setminus \underset{\mathsf{q}\to\mathsf{r}}{\mathsf{REQ}(\mathtt{bool})} = 1$

– We transition back to 1.

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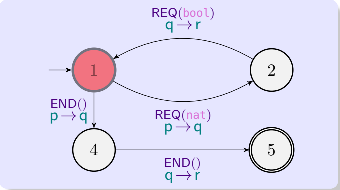

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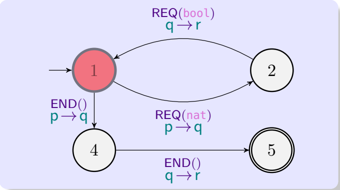

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With a $\mathtt{rec}\ X\ .$, we need to remember the state of the protocol, $\boxed{1}$. Whenever we jump back to X, we will check that we are again in a bisimilar state.

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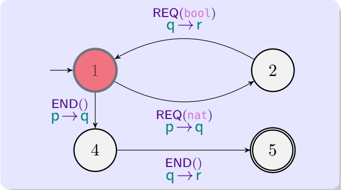

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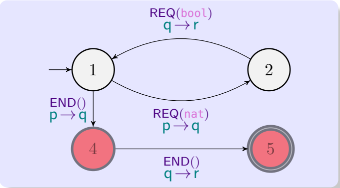

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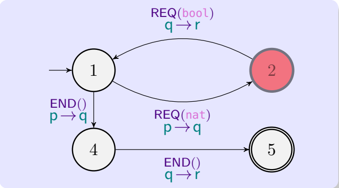

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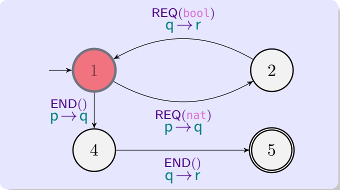

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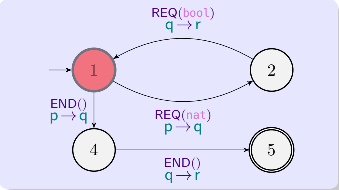

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– We landed in the same state where we used recursion.

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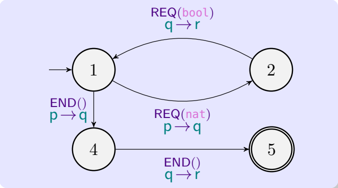

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We finished building our type derivation: the process is well typed

Properties of Synthetic MPST

Some key lemmas:

- If $G \sim G'$ and $\Gamma \vdash P : G \upharpoonright \mathsf{r}$ then $\Gamma \vdash P : G' \upharpoonright \mathsf{r}$
- If $G \setminus \alpha = G'$, with $\mathsf{r} \notin \alpha$, and $\Gamma \vdash P : G \upharpoonright \mathsf{r}$, then $\Gamma \vdash P : G' \upharpoonright \mathsf{r}$

These are needed for proving progress and preservation.
If $\mathcal{M}$ is a collection of processes that implement all of the roles in G:

- If $\vdash \mathcal{M} : G$ and $\mathcal{M} \longrightarrow \mathcal{M}'$, then there exists G' and α such that $G \setminus \alpha = G'$ and $\vdash \mathcal{M}' : G'$
- If $\vdash \mathcal{M} : G$ and G is not ended, then there exists $\mathcal{M}'$ such that $\mathcal{M} \longrightarrow \mathcal{M}'$.

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If $\mathcal{M}$ is a

- If $\vdash$
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I am going to be annoying again ...

One of the above lemmas is wrong! It should be obvious which one ... But why?

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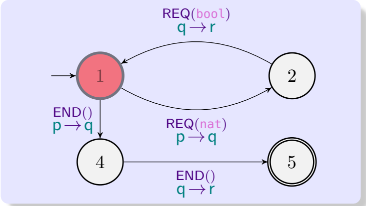

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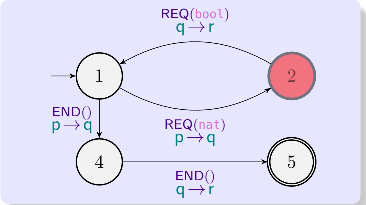

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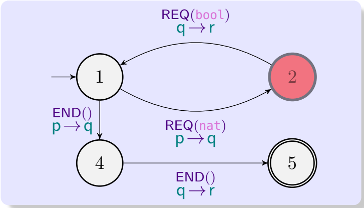

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Recursion state X: 2

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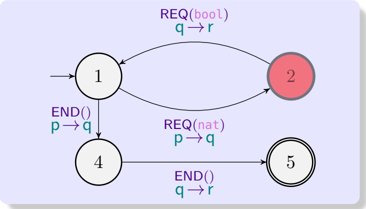

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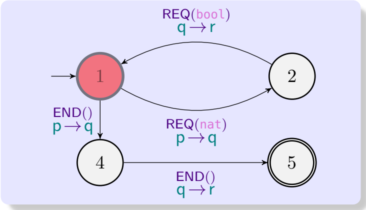

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Recursion state X: 2

We should be at 2, but we are at 1!

Wrap Up

Benefits of Synthetic Typing

1. Decoupling behavioural typing from the syntactic objects that describe the protocols.
2. No need for complex projections, merging, ...
3. As long as the protocol specifications satisfy certain required properties, they can be extended without affecting the typing, or the progress and preservation of the type system.
4. (Hopefully) easier integration in a mainstream programming language: we would need to walk throught the AST, and step through the semantics of the protocol as needed.

TODO

We reached (somewhat) stable definitions in our Agda mechanisation, but we need to fix (or reformulate) the following (incorrect) property:
If $G \setminus \alpha = G'$, with $\mathsf{r} \notin \alpha$, and $\Gamma \vdash P : G \restriction \mathsf{r}$, then $\Gamma \vdash P : G' \restriction \mathsf{r}$

We still need to show that type inhabitation subsumes common well-formedness criteria for global types.