Towards A Synthetic Formulation of Multiparty Session Types

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Background and Motivation

A Crash Course on Classic Multiparty Session Types

What is wrong with this code?

```go
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    workerChs := make([]chan int, ubound)
    errCh := make(chan error)
    for i := 0; i < ubound; i++ {
      workerChs[i] = make(chan int)
      go Worker(i+1, workerChs[i], errCh)
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubour
    work
    errCh
    for
      wo
      go
    }
    var
    for
      sel
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

DEADLOCK!

ORPHAN MESSAGES!

NO RESOURCE CLEANUP!

...

What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    worke
    errCh
    for i
      wor
      go
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

Master needs to guarantee that all Workers are notified when there is an error.

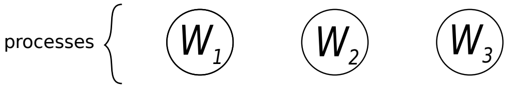
Key Idea

Multiparty Session Types prevent you from writing the code in the previous slide by enforcing syntactically that process implementations follow a given specification.

In a nutshell:

1. Global types: protocol specifications among a fixed number of different *roles*.
2. Role: sets of interactions that processes can do in a protocol.
3. Local types: protocol specifications *from the point of view of a single role*.
4. Projection: a *partial function* that extracts *local type* given a *global types* and a *role*.

5. Well-formedness: guarantees **deadlock-freedom**, usually defined in terms of *projectibility*.


processes
W_1
W_2
W_3

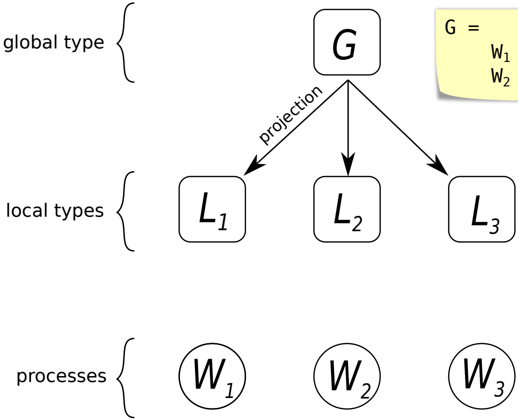
global type

G

```
G =
    W₁ → W₂ : Int.
    W₂ → W₃ : Bool
```

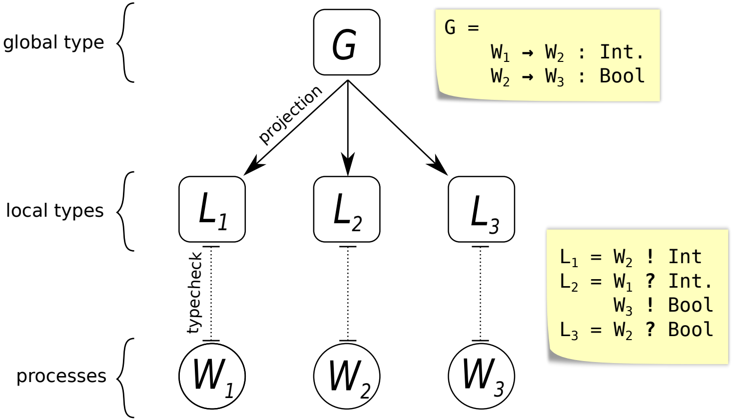
processes

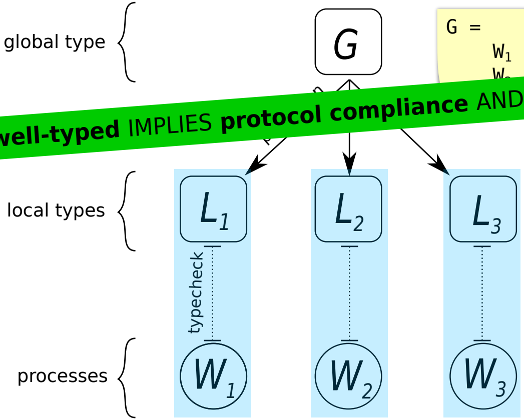
W_1 W_2 W_3




```
G =
    W₁ → W₂ : Int.
    W₂ → W₃ : Bool
```

```
L₁ = W₂ ! Int
L₂ = W₁ ? Int.
     W₃ ! Bool
L₃ = W₂ ? Bool
```


global type
local types
processes
G
projection
typecheck
L1
L2
L3
W1
W2
W3
G =
W1 → W2 : Int.
W2 → W3 : Bool
L1 = W2 ! Int
L2 = W1 ? Int.
W3 ! Bool
L3 = W2 ? Bool



well-typed IMPLIES **protocol compliance** AND **deadlock-freedom**

$$G = W_1 \to W_2 : \mathtt{Int}.$$

$$
\begin{aligned}
L_1 &= W_2\ !\ \mathtt{Int}\\
L_2 &= W_1\ ?\ \mathtt{Int}.\\
    &\ W_3\ !\ \mathtt{Bool}\\
L_3 &= W_2\ ?\ \mathtt{Bool}
\end{aligned}
$$

Global and Local Types

Roles			$\mathsf{p}, \mathsf{q}, \ldots$	
Sorts	S	$:=$	$\mathtt{bool} \mid \mathtt{nat} \mid \cdots$	Basic data types.
Global Types	G	$:=$	$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i \in I}$	Message communication.
		$\mid$	$\mu X.G$	Recursion.
		$\mid$	X	Recursion variable.
		$\mid$	$\varnothing$	End of protocol.
Local Types	L	$:=$	$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i \in I}$	Send message.
		$\mid$	$\mathsf{q}?\{\ell_i(S_i).L_i\}_{i \in I}$	Receive message.
		$\mid$	$\mu X.G$	Recursion.
		$\mid$	X	Recursion variable.
		$\mid$	$\varnothing$	End of protocol.

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \restriction \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \restriction \mathsf{r} = \begin{cases} \mu X.G \restriction \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \restriction \mathsf{r} = X \qquad \varnothing \restriction \mathsf{r} = \varnothing$$

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \upharpoonright \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \upharpoonright \mathsf{r} = \begin{cases} \mu X.G \upharpoonright \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \upharpoonright \mathsf{r} = X \qquad \varnothing \upharpoonright \mathsf{r} = \varnothing$$

$$\begin{aligned} &\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J} \\ &\quad = \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J} \end{aligned}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \upharpoonright \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

It gets complicated very quickly!

$$\mu X.G \upharpoonright \mathsf{r} = \begin{cases} \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \upharpoonright \mathsf{r} = X \qquad \varnothing \upharpoonright \mathsf{r} = \varnothing$$

$$\begin{aligned} &\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J} \\ &\quad = \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J} \end{aligned}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

What is the point of $\sqcap$?

Example:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\texttt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\texttt{bool}).X \\ \mathsf{END}() \quad\quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

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Projecting r

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\mathtt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$=$$

What is the point of $\sqcap$?

Example:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

Projecting r

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\mathtt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$= \mu X.\mathsf{q}?\left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() & .\mathsf{done} \end{array} \right\}$$

Processes and Typing

$$
\begin{array}{llcll}
\text{Process} & P & := & \mathtt{p} \,!\, \ell\langle e\rangle.P & \text{Send a message.} \\
& & \mid & \sum_{i\in I} \mathtt{p}?\ell_i(x_i).P_i & \text{Receive a message.} \\
& & \mid & \mathtt{if}\; e \;\mathtt{then}\; P \;\mathtt{else}\; P' & \text{Conditional process.} \\
& & \mid & \mathtt{rec}\; X \,.\, P & \text{Recursive process.} \\
& & \mid & X & \text{Recursion variable.} \\
& & \mid & \mathtt{done} & \text{Inactive process.}
\end{array}
$$

Process Typing (simplified)

Once we have local types, process typing is simple:

T-SEND

$$\frac{\Gamma \vdash P : L_i \qquad \Gamma \vdash e : S_i \qquad i \in I}{\Gamma \vdash \mathsf{q} \,!\, \ell_i\langle e\rangle.P : (\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I})}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : L_i \qquad \forall i \in I}{\Gamma \vdash \sum_{i\in I} \mathsf{p}?\ell_i(x_i).P_i : (\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I})}$$

Problems with Classic Formulation

1. **Too syntactic:**
   - Processes and local types must align
   - Too restrictive, rules out correct processes
   - ...
2. **Unnecessarily complex:**
   - Hard to implement/mechanise, e.g.:
     - Use of runtime coinductive global types: Our PLDI 2021 paper
     - Complex graph-based representation of MPST: Jacobs et al. (2022)
     - Graph-based reasoning and decision procedure for the equality of recursive types: Tirore et al. (2023)
   - Hard to extend

A Few Attempts at Simplifying the Theory

Less Is More: Multiparty Session Types Revisited

ALCESTE SCALAS, Imperial College London, UK
NOBUKO YOSHIDA, Imperial College London, UK

A Few Attempts at Simplifying the Theory

Le

ALC
NO

Less is More Revisited

Association with Global Multiparty Session Types

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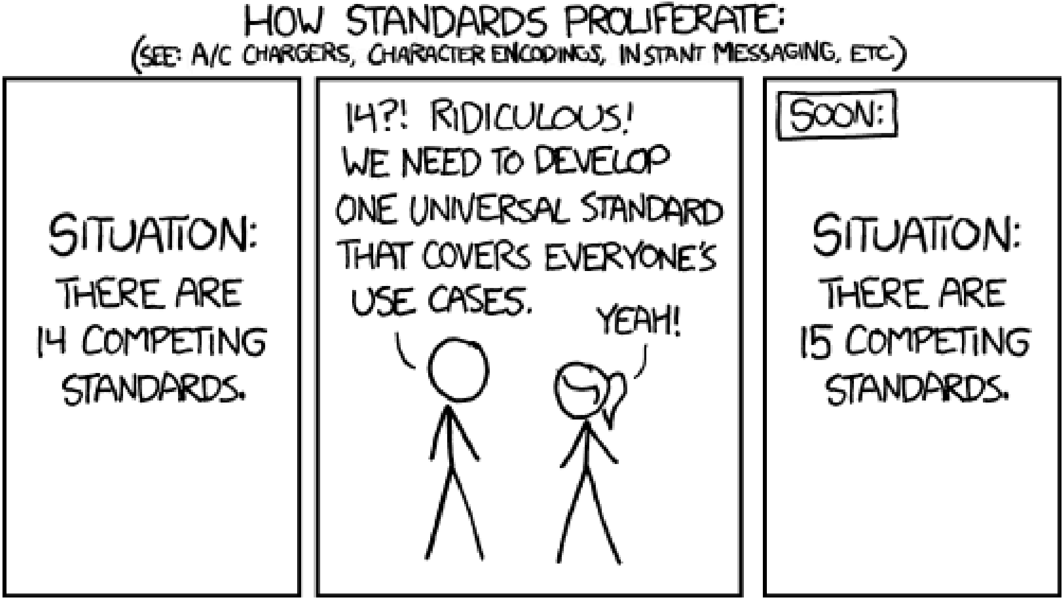
HOW STANDARDS PROLIFERATE:
(SEE: A/C CHARGERS, CHARACTER ENCODINGS, INSTANT MESSAGING, ETC)
SITUATION:
THERE ARE 14 COMPETING STANDARDS.
14?! RIDICULOUS! WE NEED TO DEVELOP ONE UNIVERSAL STANDARD THAT COVERS EVERYONE'S USE CASES.
YEAH!
SOON:
SITUATION:
THERE ARE 15 COMPETING STANDARDS.

Our Approach: Synthetic Typing

Synthetic Behavioural Typing: Sound, Regular Multiparty Sessions via Implicit Local Types

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Our Approach: Synthetic Typing

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Mu
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Fran
Depar

Goals:

- "Free" typing from being tied up to the syntax of local types.
- Avoid projection/merging/etc.
- A formal description of equality between global types to replace informally equating global types to their unfolding.
- Well-formedness/deadlock-freedom is decided by typeability.
- Mechanisation in Agda.

Towards Synthetic MPST (WIP)

New (Synthetic) Core Typing Rules

New judgement : $\Gamma \vdash P : G \upharpoonright \mathsf{p}$

T-SEND

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q}\,!\,\ell\langle e \rangle.P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

Synthetic, in that G' occurs only in the premise, not in the conclusion. G' needs to be *synthesised* by using the rules of the operational semantics of global types.

New (Synthetic) Core Typing Rules

What is wrong with these rules?

T-SEND

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle.P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \;\text{ we have }\; \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \;\text{ we have }\; \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

New (Synthetic) Core Typing Rules

Hint: the problem is in these rules

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \restriction \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \restriction \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \text{ we have } \Gamma \vdash P : G' \restriction \mathsf{r}}{\Gamma \vdash P : G \restriction \mathsf{r}}$$

New (Synthetic) Core Typing Rules

Hint 2: the problem is the same in both rules, let's focus on this one

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \restriction \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \restriction \mathsf{p}}$$

New (Synthetic) Core Typing Rules

What happens if G does not allow q to receive from p?

T-RECV

$$\frac{\boxed{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'} \quad \text{we have} \quad \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathsf{p}, \mathsf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists(i \in I) G', G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ – this means that an interaction between p and q is "ready" (i.e. can happen) in G.

$$\text{T-SEND}\ \frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle.P : G \upharpoonright \mathsf{p}}$$

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T-RECV

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$$\text{T-SKIP}\quad \frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i\in I}, G) \wedge (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \qquad \forall(i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathsf{p}, \mathsf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists (i \in I) G', G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ – this means that an interaction between p and q is "ready" (i.e. can happen) in G.

T-SEND

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle . P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \qquad \forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\begin{array}{c}\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \wedge (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \\ \forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}\end{array}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

(Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathsf{p}, \mathsf{q}, \{\ell_i(S_i)\}_{i\in I}, G) = \exists(i \in I)G', G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ – this means that an interaction between p and q is "ready" (i.e. can happen) in G.

- The rules look more complex than with a syntactic approach, but computing $G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ is entirely mechanical by using the semantics of global types.
- The proof of subject reduction is greatly simplified (more in a few slides) with this formulation.
- No need of projection/merging.

T-RECV

$\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell$ ... $\upharpoonright \mathsf{p}$

T-SKIP

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i\in I}, G) \wedge (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

Semantics

Lemmas

Wrap Up

A Crash Course on Classic Multiparty Session Types

Benefits of Synthetic Typing

TODO