# Towards A Synthetic Formulation of Multiparty Session Types

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# Background and Motivation

A Crash Course on Classic Multiparty Session Types

# What is wrong with this code?

```go
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    workerChs := make([]chan int, ubound)
    errCh := make(chan error)
    for i := 0; i < ubound; i++ {
      workerChs[i] = make(chan int)
      go Worker(i+1, workerChs[i], errCh)
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

# What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    uboun
    work
    errCh
    for
      wo
      go
    }
    var
    for
      sel
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

DEADLOCK!

ORPHAN MESSAGES!

NO RESOURCE CLEANUP!

...

# What is wrong with this code?

```
func Worker(n int, resp chan int, err chan error) { ... }
func Master(reqCh chan int, respCh chan []int, cErrCh chan error) {
  for {
    ubound := <-reqCh
    worke
    errCh
    for i
      wor
      go
    }
    var res []int
    for i := 0; i < ubound; i++ {
      select {
      case sql := <-workerChs[i]:
        res = append(res, sql)
      case err := <-errCh:
        cErrCh <- err
        return
      }}
    respCh <- res}}
```

Master needs to guarantee that all Workers are notified when there is an error.

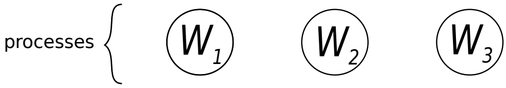
# Key Idea

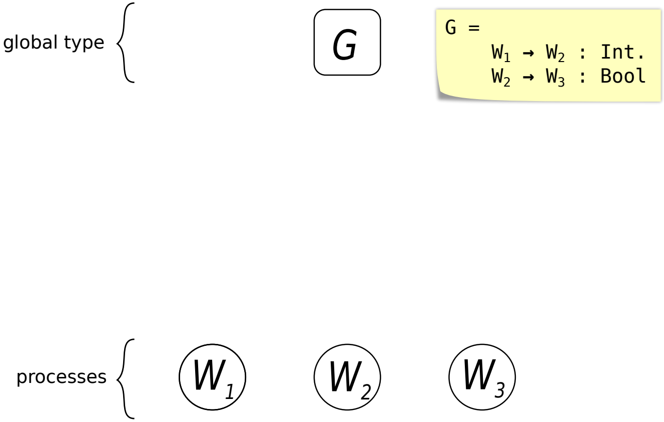
Multiparty Session Types prevent you from writing the code in the previous slide by enforcing syntactically that process implementations follow a given specification.

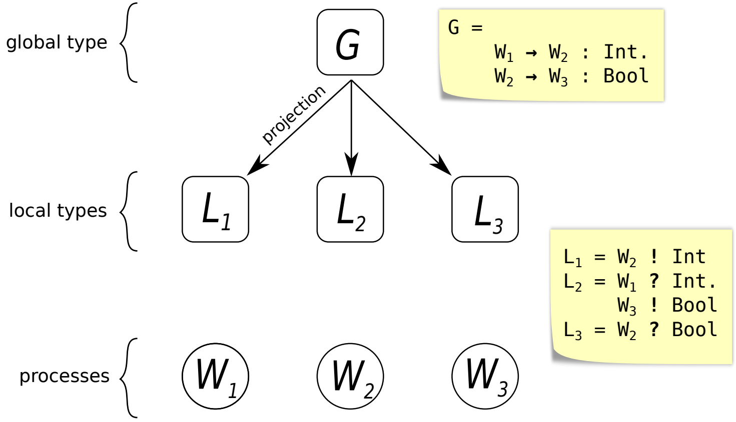
**In a nutshell:**

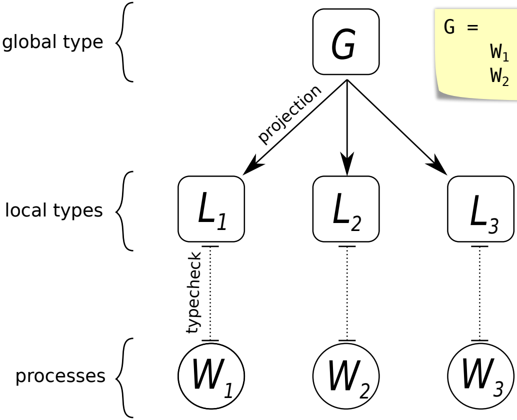
1. Global types: protocol specifications among a fixed number of different *roles*.
2. Role: sets of interactions that processes can do in a protocol.
3. Local types: protocol specifications *from the point of view of a single role*.
4. Projection: a *partial function* that extracts *local type* given a *global types* and a *role*.

5. Well-formedness: guarantees **deadlock-freedom**, usually defined in terms of *projectibility*.


processes
$W_1$
$W_2$
$W_3$

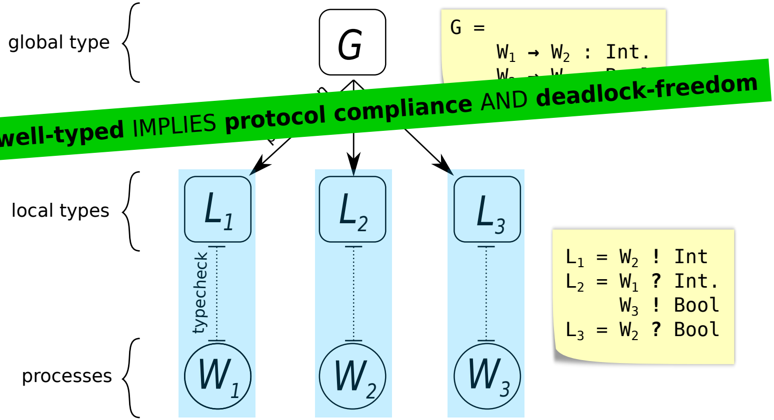

global type
G
G =
W1 → W2 : Int.
W2 → W3 : Bool
processes
W1
W2
W3


global type
local types
processes
$G$
projection
$L_1$
$L_2$
$L_3$
$W_1$
$W_2$
$W_3$
G =
$W_1 \to W_2$ : Int.
$W_2 \to W_3$ : Bool
$L_1 = W_2$ ! Int
$L_2 = W_1$ ? Int.
$W_3$ ! Bool
$L_3 = W_2$ ? Bool




```
G =
    W₁ → W₂ : Int.
    W₂ → W₃ : Bool
```

```
L₁ = W₂ ! Int
L₂ = W₁ ? Int.
     W₃ ! Bool
L₃ = W₂ ? Bool
```


global type
local types
processes
G
$L_1$
$L_2$
$L_3$
$W_1$
$W_2$
$W_3$
typecheck
G =
$W_1 \rightarrow W_2$ : Int.
$L_1 = W_2$ ! Int
$L_2 = W_1$ ? Int.
$W_3$ ! Bool
$L_3 = W_2$ ? Bool
**well-typed** IMPLIES **protocol compliance** AND **deadlock-freedom**

# Global and Local Types

| | | | | |
|---|---|---|---|---|
| Roles | | | $\mathsf{p}, \mathsf{q}, \ldots$ | |
| Sorts | $S$ | $:=$ | $\mathtt{bool} \mid \mathtt{nat} \mid \cdots$ | Basic data types. |
| Global Types | $G$ | $:=$ | $\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i \in I}$ | Message communication. |
| | | $\mid$ | $\mu X.G$ | Recursion. |
| | | $\mid$ | $X$ | Recursion variable. |
| | | $\mid$ | $\varnothing$ | End of protocol. |
| Local Types | $L$ | $:=$ | $\mathsf{p}!\{\ell_i(S_i).L_i\}_{i \in I}$ | Send message. |
| | | $\mid$ | $\mathsf{q}?\{\ell_i(S_i).L_i\}_{i \in I}$ | Receive message. |
| | | $\mid$ | $\mu X.G$ | Recursion. |
| | | $\mid$ | $X$ | Recursion variable. |
| | | $\mid$ | $\varnothing$ | End of protocol. |

# Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad\qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\qquad\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \restriction \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \restriction \mathsf{r} = \begin{cases} \mu X.G \restriction \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \restriction \mathsf{r} = X \qquad \varnothing \restriction \mathsf{r} = \varnothing$$

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# Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \upharpoonright \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \upharpoonright \mathsf{r}\}_{i\in I} & (\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \upharpoonright \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

$$\mu X.G \upharpoonright \mathsf{r} = \begin{cases} \mu X.G \upharpoonright \mathsf{r} & (\mathsf{r} \in G) \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \upharpoonright \mathsf{r} = X \qquad \varnothing \upharpoonright \mathsf{r} = \varnothing$$

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$$\begin{aligned} &\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J} \\ &\quad = \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J} \end{aligned}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

# Projection

$$\mathsf{p} \to \mathsf{q} : \{\ell_i(S_i).G_i\}_{i\in I} \restriction \mathsf{r} = \begin{cases} \mathsf{q}!\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\mathsf{r} = \mathsf{p} \wedge \qquad \wedge\, \mathsf{p} \neq \mathsf{q}) \\ \mathsf{p}?\{\ell_i(S_i).G_i \restriction \mathsf{r}\}_{i\in I} & (\qquad \wedge\, \mathsf{r} = \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \\ \sqcap_{i\in I}(G_i \restriction \mathsf{r}) & (\mathsf{r} \neq \mathsf{p} \wedge \mathsf{r} \neq \mathsf{q} \wedge \mathsf{p} \neq \mathsf{q}) \end{cases}$$

**It gets complicated very quickly!**

$$\mu X.G \restriction \mathsf{r} = \begin{cases} \ldots \\ \varnothing & (\mathsf{r} \notin G) \end{cases} \qquad X \restriction \mathsf{r} = X \qquad \varnothing \restriction \mathsf{r} = \varnothing$$

---

$$\begin{aligned} &\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}?\{\ell_j(S_j).L'_j\}_{j\in J} \\ &\quad = \mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I\setminus J} \cup \{\ell_j(S_j).L'_j\}_{j\in J\setminus I} \cup \{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I\cap J} \end{aligned}$$

$$\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I} \sqcap \mathsf{p}!\{\ell_i(S_i).L'_i\}_{i\in I} = \mathsf{p}!\{\ell_i(S_i).L_i \sqcap L'_i\}_{i\in I}$$

$$\mu X.L \sqcap \mu X.L' = \mu X.(L \sqcap L') \qquad L \sqcap L = L$$

# What is the point of $\sqcap$?

Example:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

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Example:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\mathtt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\mathtt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

Projecting r

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\mathtt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$=$$

# What is the point of $\sqcap$?

Example:

$$\mu X.\mathsf{p} \to \mathsf{q} : \left\{ \begin{array}{ll} \mathsf{REQ}(\texttt{nat}).\mathsf{q} \to \mathsf{r} : \mathsf{REQ}(\texttt{bool}).X \\ \mathsf{END}() \quad .\mathsf{q} \to \mathsf{r} : \mathsf{END}().\mathsf{done} \end{array} \right\}$$

Projecting $\mathsf{r}$

$$\mu X.(\mathsf{q}?\mathsf{REQ}(\texttt{bool}).X) \sqcap (\mathsf{q}?\mathsf{END}().\varnothing)$$

$$= \mu X.\mathsf{q}?\left\{ \begin{array}{ll} \mathsf{REQ}(\texttt{bool}).X \\ \mathsf{END}() \quad .\mathsf{done} \end{array} \right\}$$

# Processes and Typing

$$
\begin{array}{llcll}
\text{Process} & P & := & \mathsf{p}\,!\,\ell\langle e\rangle.P & \text{Send a message.} \\
 & & \mid & \sum_{i\in I} \mathsf{p}?\ell_i(x_i).P_i & \text{Receive a message.} \\
 & & \mid & \mathtt{if}\, e\, \mathtt{then}\, P\, \mathtt{else}\, P' & \text{Conditional process.} \\
 & & \mid & \mathtt{rec}\, X\, .\, P & \text{Recursive process.} \\
 & & \mid & X & \text{Recursion variable.} \\
 & & \mid & \mathtt{done} & \text{Inactive process.}
\end{array}
$$

# Process Typing (simplified)

Once we have local types, process typing is simple:

T-SEND

$$\frac{\Gamma \vdash P : L_i \qquad \Gamma \vdash e : S_i \qquad i \in I}{\Gamma \vdash \mathsf{q} \,!\, \ell_i\langle e\rangle.P : (\mathsf{p}!\{\ell_i(S_i).L_i\}_{i\in I})}$$

T-RECV

$$\frac{\Gamma, x_i : S_i \vdash P_i : L_i \qquad \forall i \in I}{\Gamma \vdash \sum_{i\in I} \mathsf{p}?\ell_i(x_i).P_i : (\mathsf{p}?\{\ell_i(S_i).L_i\}_{i\in I})}$$

# Problems with Classic Formulation

1. **Too syntactic:**
   - Processes and local types must align
   - Too restrictive, rules out correct processes
   - ...
2. **Unnecessarily complex:**
   - Hard to implement/mechanise, e.g.:
     - Use of runtime coinductive global types: Our PLDI 2021 paper
     - Complex graph-based representation of MPST: Jacobs et al. (2022)
     - Graph-based reasoning and decision procedure for the equality of recursive types: Tirore et al. (2023)
   - Hard to extend

# A Few Attempts at Simplifying the Theory

**Less Is More: Multiparty Session Types Revisited**

ALCESTE SCALAS, Imperial College London, UK
NOBUKO YOSHIDA, Imperial College London, UK

# A Few Attempts at Simplifying the Theory

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ALC
NO

## Less is More Revisited
### Association with Global Multiparty Session Types

Nobuko Yoshida( ) and Ping Hou

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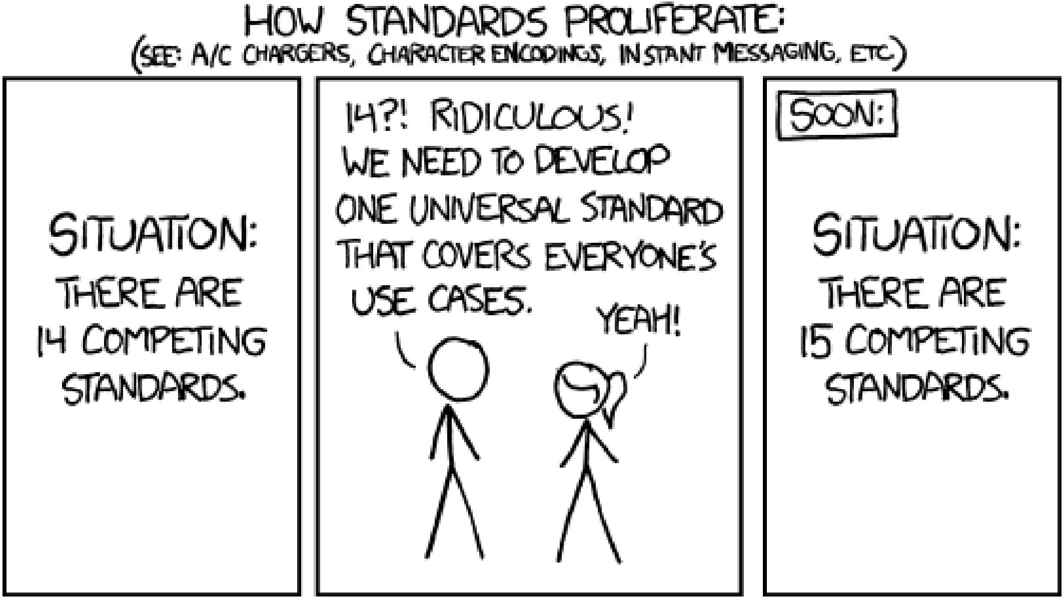


https://xkcd.com/927/

# Our Approach: Synthetic Typing

**Synthetic Behavioural Typing: Sound, Regular Multiparty Sessions via Implicit Local Types**

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Centrum Wiskunde & Informatica (CWI), NWO-I, Amsterdam, The Netherlands

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# Our Approach: Synthetic Typing

Goals:

- “Free” typing from being tied up to the syntax of local types.
- Avoid projection/merging/etc.
- A formal description of equality between global types to replace informally equating global types to their unfolding.
- Well-formedness/deadlock-freedom is decided by typeability.
- Mechanisation in Agda.

# Towards Synthetic MPST (WIP)

# New (Synthetic) Core Typing Rules

**New judgement** : $\Gamma \vdash P : G \upharpoonright \mathsf{p}$

T-SEND

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q}\,!\,\ell\langle e\rangle.P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

Synthetic, in that $G'$ occurs only in the premise, not in the conclusion. $G'$ needs to be *synthesised* by using the rules of the operational semantics of global types.

# New (Synthetic) Core Typing Rules

**What is wrong with these rules?**

T-SEND

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \upharpoonright \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e\rangle.P : G \upharpoonright \mathsf{p}}$$

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \ \text{ we have } \ \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \ \text{ we have } \ \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

# New (Synthetic) Core Typing Rules

**Hint: the problem is in these rules**

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \;\text{ we have }\; \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ and } \mathsf{p} \neq \mathsf{r} \wedge \mathsf{q} \neq \mathsf{r} \;\text{ we have }\; \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

# New (Synthetic) Core Typing Rules

**Hint 2: the problem is the same in both rules, let's focus on this one**

T-RECV

$$\frac{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \quad \text{we have} \quad \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

# New (Synthetic) Core Typing Rules

What happens if $G$ does not allow p to receive from q?

T-RECV

$$\dfrac{\boxed{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'} \quad \text{we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

# New (Synthetic) Core Typing Rules

This was a "rookie" mistake ... We cannot allow rules to be vacuously true!

$$\textsf{T-RECV}\quad \frac{\boxed{\forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'} \quad \text{we have} \quad \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

# (Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathsf{p}, \mathsf{q}, \{\ell_i(S_i)\}_{i \in I}, G) = \exists (i \in I) G', G \setminus \mathsf{p} \xrightarrow{\ell_i(S_i)} \mathsf{q} = G'$ – this means that an interaction between $\mathsf{p}$ and $\mathsf{q}$ is "ready" (i.e. can happen) in $G$.

$$\frac{G \setminus \mathsf{p} \xrightarrow{\ell(S)} \mathsf{q} = G' \qquad \Gamma \vdash P : G' \restriction \mathsf{p} \qquad \Gamma \vdash e : S}{\Gamma \vdash \mathsf{q} \,!\, \ell\langle e \rangle . P : G \restriction \mathsf{p}} \;\text{T-SEND}$$

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T-RECV

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i\in I}, G) \qquad \forall(i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i\in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

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T-SKIP

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i\in I}, G) \land (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \qquad \forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

# (Hopefully) Fixed Typing Rules

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T-SEND

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T-RECV

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \qquad \forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma, x_i : S_i \vdash P_i : G' \upharpoonright \mathsf{p}}{\Gamma \vdash \sum_{i \in I} \mathsf{q}?\ell_i(x_i).P_i : G \upharpoonright \mathsf{p}}$$

T-SKIP

$$\frac{\begin{array}{c}\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i \in I}, G) \wedge (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \\ \forall (i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}\end{array}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

# (Hopefully) Fixed Typing Rules

Let $\mathcal{R}(\mathsf{p}, \mathsf{q}, \{\ell_i(S_i)\}_{i\in I}, G) = \exists(i \in I)G', G \setminus \mathsf{p} \xrightarrow{\ell_i(S_i)} \mathsf{q} = G'$ – this means that an interaction between $\mathsf{p}$ and $\mathsf{q}$ is "ready" (i.e. can happen) in $G$.

- The rules look more complex than with a syntactic approach, but computing $G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G'$ is entirely mechanical by using the semantics of global types.
- The proof of subject reduction is greatly simplified (more in a few slides) with this formulation.
- No need of projection/merging.

T-RECV

$\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell$ ... $\upharpoonright \mathsf{p}$

T-SKIP

$$\frac{\mathcal{R}(\mathsf{q}, \mathsf{p}, \{\ell_i(S_i)\}_{i\in I}, G) \wedge (\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}) \quad \forall(i \in I) \text{ s.t. } G \setminus \mathsf{q} \xrightarrow{\ell_i(S_i)} \mathsf{p} = G' \text{ we have } \Gamma \vdash P : G' \upharpoonright \mathsf{r}}{\Gamma \vdash P : G \upharpoonright \mathsf{r}}$$

# Semantics

# Examples

# Global Type Bisimilarity

# Properties of Synthetic MPST

# Wrap Up

A Crash Course on Classic Multiparty Session Types

# Benefits of Synthetic Typing

# TODO