

0101
1001
0101

Number Systems

and their types, uses and significance

Matt Erron G. Cabarrubias
BSCS-1

Introduction

What is a number?

1

A number is a **mathematical object** used to *count, measure, and label*. Numbers serve as the fundamental elements of mathematics and have a wide variety of forms and uses depending on the context.¹

Number Systems

Number systems are the various ways of representing numbers. Each system has its own structure, set of rules, and operations. It is a mathematical notation for representing numbers of a given set, consistently using digits or other symbols. It allows us to perform arithmetic operations like addition, subtraction, multiplication, and division.²

.00

8

64

¹ (Number, n.d.)

² (GeeksforGeeks, 2024)

History

The Babylonian Civilization

The first true written positional numeral system is generally attributed to the ancient Babylonian civilization. It emerged around 3000 BCE influenced by earlier Sumerian systems. It was primarily used in ancient Mesopotamia for trade, astronomy, and record-keeping³ and was based on a **sexagesimal (base-60)** system.⁴

1	፩	፪	፫	፬	፭	፮	፯	፱	፲	፳	፴	፵	፶	፷	፸	፹	፻	፼	፽	፿	፿
2	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪	፪
3	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫	፫
4	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬	፬
5	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭	፭
6	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮	፮
7	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯	፯
8	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺	፺
9	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻	፻
10	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼	፼

fig. 1: The sexagesimal system and their symbols

The **sexagesimal system** was an ancient system of counting, calculation, and numerical notation that used powers of 60 much as the decimal system uses powers of 10. Rudiments of the ancient system survive in vestigial form in our division of the hour into 60 minutes and the minute into 60 seconds.⁵

Although it is a positional system, it **lacks a true zero**, which can complicate calculations and representations. The absence of zero means that the system relies on spacing to indicate the absence of a value, which can be **less intuitive**.³

3 (Tatomir & Chesnutt, 2023)

4 (Friberg, 2019)

5 (Powell, 2008)

History

Hindu-Arabic System

While the Babylonian's Sexagesimal system was the first true positional numeral system, the Hindu-Arabic system was also one of the number systems that influenced modern mathematics and also introduced the number 0.⁶

The **Hindu-Arabic numeral system** is a decimal (base-10) system, meaning it uses ten distinct symbols to represent numbers. Each position in a number corresponds to a power of ten, allowing for the representation of large numbers with relatively few symbols.

Developed between the **1st** and **4th centuries CE** in India and later spread to the Middle East and Europe by the 9th century. It became widely adopted in Europe during the 13th century due to the works of mathematicians like Fibonacci.^{6.1}

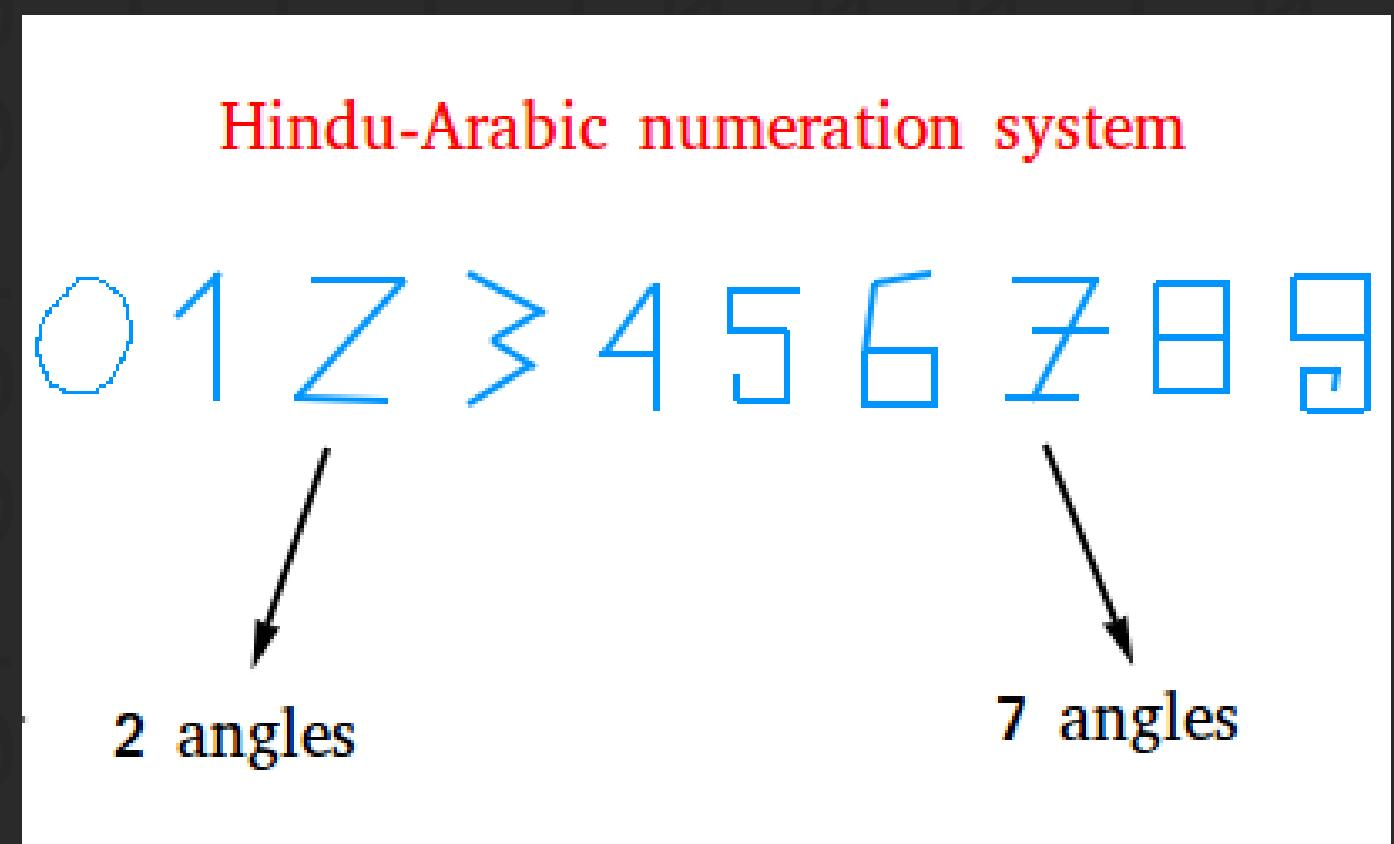


fig. 2: the Hindu-Arabic numeration system

6 (A Brief History of Numbers: How 0-9 Were Invented, 2023)

6.1 (The Editors of Encyclopaedia Britannica, 2024)

Elements of the Number System

(Lippman, n.d.)

The Base

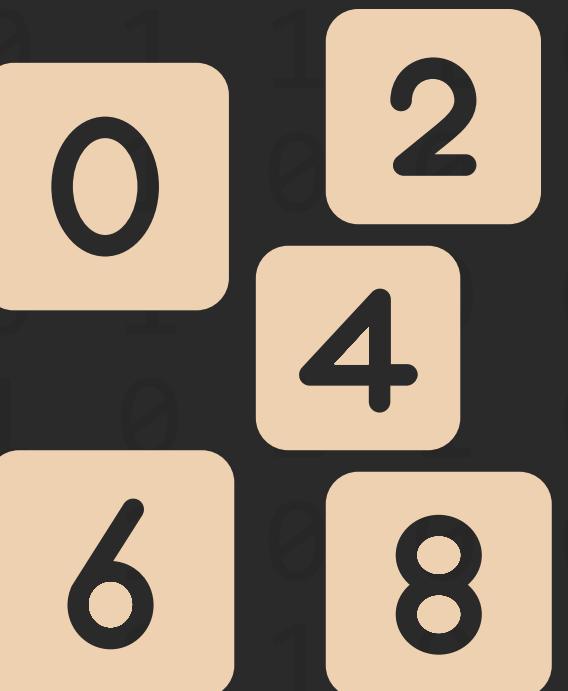
Every number system is based on a specific **base** or *radix*, which determines how many unique digits or symbols are used in that system. The base also defines how place values increase or decrease as you move left or right in a number.⁷

There are **four** commonly used type of bases, and they are:

- **Binary (Base 2)** - 0s and 1s
- **Decimal (Base 10)** - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- **Octal (Base 8)** - 0, 1, 2, 3, 4, 5, 6, 7
- **Hexadecimal (Base 16)** - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Digits

These are the basic symbols used in the system. The number of unique digits equals the base of the number system.



7 (Virnuls, n.d.)

Elements of the Number System

(Lippman, n.d.)

Positional Notation

Positional notation (or place-value notation) is a method for representing numbers in which the position of each digit within a number determines its value. The value of a digit depends not only on the digit itself but also on its position or place in the number, and the base (or radix) of the number system being used. It uses a set of digits to represent the value of a number. The product of each digit and its place value indicates the value of the digit. The sum of those products is the value of the number.

Place Value

Each digit's value depends on its position within a number. Moving left in a number increases the value by powers of the base, and moving right decreases the value (for decimal fractions).

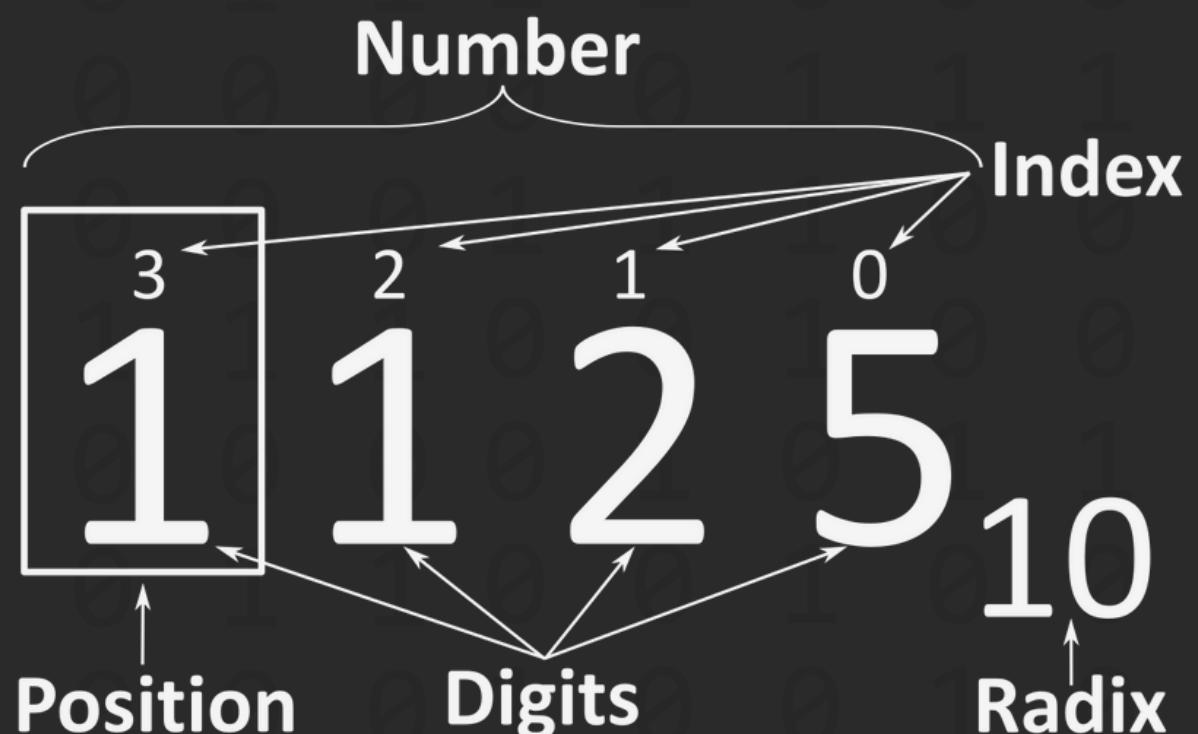


fig. 3: The number 1125 as Base 10 with its corresponding positional notation

Types of Number Systems

(Jha, 2021)

There are 4 commonly used number systems in computing:

Decimal { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
Base 10

.00 01
10 8 0x

- Remember, the amount of digits in a number system corresponds to its **radix** or **base** number.

Binary { 0, 1 }
Base 2

Octal { 0, 1, 2, 3, 4, 5, 6, 7 }
Base 8

Hexadecimal { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }
Base 16

Decimal Base 10

The decimal number system, also called the **base-10** number system, is the standard system for denoting integer and non-integer numbers. It is the most widely used number system⁸ in the world. The decimal number system has 10 symbols, mainly 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The position of every digit has a weight which is a power of 10.

As shown in Figure 3, we can see the number **5319** as an example of a base 10 number. Every digit is multiplied by 10 raised to the corresponding position number (which is also called its *weight*), and this process repeats for all the digits in the decimal number. The figure proves the derivation of a decimal number by adding the product of each operation.

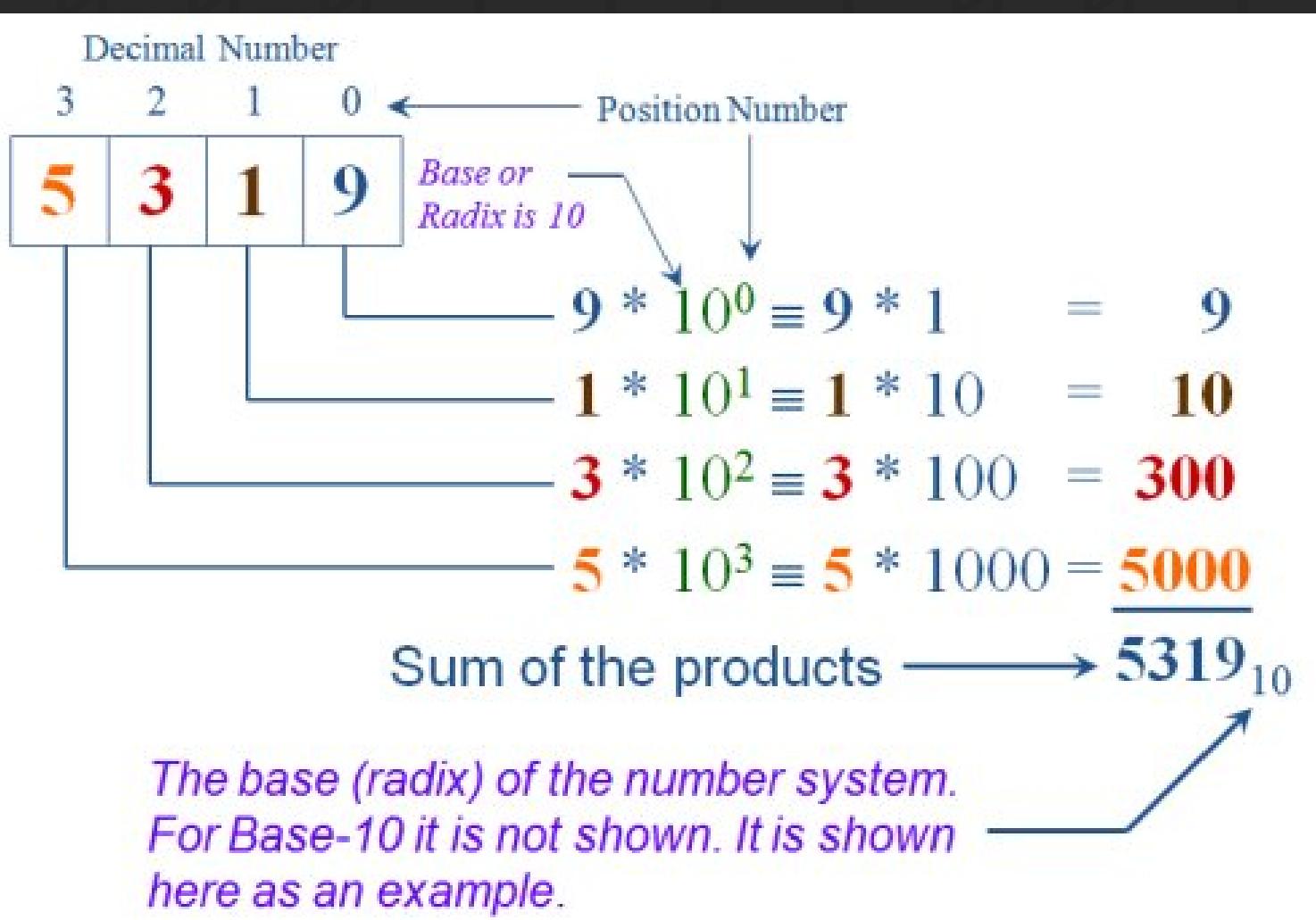


fig. 4: decimal number with the position and weight of each digit shown and how the value of the number is derived

Decimal Base 10

Uses and Significance

Applications in Computing

While computers primarily operate in binary, decimal is still fundamental in user interfaces and data representation. For example, many programming languages convert binary to decimal to present results that are easy for humans to interpret.⁹

Scientific Calculations

The decimal system is vital in scientific notation, where large or small numbers are expressed in terms of powers of ten. This system facilitates accurate and manageable representations of complex measurements, like atomic distances or astronomical data.¹⁰

Daily Transactions

The decimal system is vital in scientific notation, where large or small numbers are expressed in terms of powers of ten. This system facilitates accurate and manageable representations of complex measurements, like atomic distances or astronomical data.^{11 12}

⁹ (Princeton University, 2019) ¹⁰ (Adam & Adam, 2022)

¹¹ (Gill, 2024) ¹² (Marchant, 2024)

Binary Base 2

The binary number system is a way of representing numbers using only two digits: 0 and 1. It is a **base-2 system**, meaning each position in a binary number represents a power of 2. Unlike the decimal system, which uses ten digits (0-9), binary only uses two, making it essential in computing because digital systems like computers process data in binary form.¹³

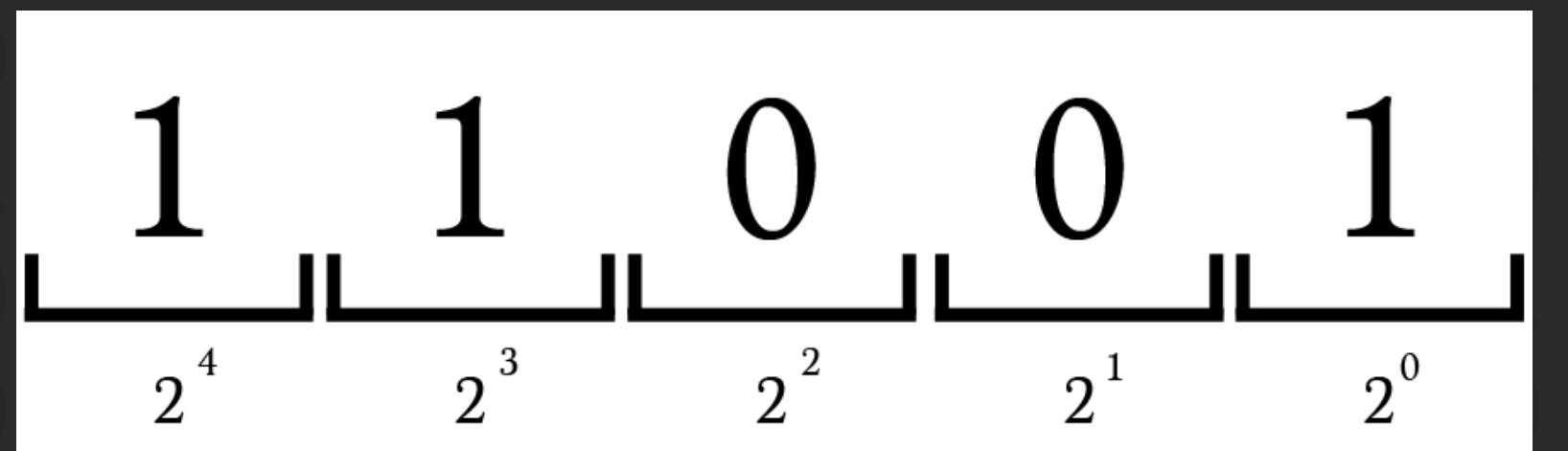


fig. 5: a binary number with the position and weight of each digit

In figure 5, we can see the binary number **11001**. Since it's the **Base-2** system, that means that we multiply the digits by 2 raised to the corresponding position number (similar to that of a decimal number). Hence, our first digit (starting from the right) or also known as the **Least Significant Bit**, is multiplied by 2^0 , and the second digit by 2^1 , so on and so forth.

We disregard zero since 0 multiplied with anything still equates to zero. Therefore, we are left with 1×2^0 , 1×2^3 and 1×2^4 . Adding all of these products together gives us the **base 10 conversion** of this binary which is **25**.

13 (Brioli et al., 2023)

Binary Base 2

Uses and Significance

Data Representation in Modern Computing

All forms of data in computers, including text, images, and audio, are encoded in binary. Each piece of information is represented as a series of bits (0s and 1s), allowing for efficient storage and processing. For example, a 16-bit color depth can represent 65,536 different colors on a screen.¹³

Logic Gates and Circuits

Binary is the foundation of digital circuits, where electronic components represent binary states (on/off). Logic gates operate on binary inputs to produce binary outputs, allowing for complex computations and operations in devices like CPUs.¹⁴

Universal Language for Machines

Binary serves as a universal language for computers, allowing different systems and devices to communicate effectively. This universality is crucial for interoperability in an increasingly digital world.¹⁵

13 (Brioli et al., 2023)

14 (Awati, 2022)

15 (Ranjan, 2023)

Octal Base 8

The octal number system is a base-8 numbering system that uses eight digits: 0, 1, 2, 3, 4, 5, 6, and 7. Each position in an octal number represents a power of 8.¹⁶

In the decimal system, each place is a power of ten. For example:

$$74_{10} = (7 \times 10^1) + (4 \times 10^0)$$

In the octal system, each place is a power of ten. For example:

$$112_8 = (1 \times 8^2) + (1 \times 8^1) + (2 \times 8^0)$$

In converting octal to binary, octal numbers can be represented by 3 binary digits. Since three binary digits can represent a maximum of 7, hence it can represent each and every octal number since it only goes up to 7.

¹⁶ (Vedantu, n.d.)

Octal Number	Binary Number
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Octal Base 8

Uses and Significance

Historical Use in Computing

Octal was widely used in early computing systems, particularly in minicomputers and mainframes, because it provides a more compact representation of binary numbers. Each octal digit corresponds to three binary digits, making it easier to read and write. An example of these is the **UNIVAC 1050** and **PDP-8**.

File Permissions in Unix/Linux

In Unix and Linux operating systems, file permissions are often represented in octal format. Each digit in an octal number corresponds to a set of permissions for the owner, group, and others.¹⁷

	Owner			Group			Other Users		
File Type	r	w	x	r	w	x	r	w	x
- or d	4	2	1	4	2	1	4	2	1
				7			7		7

¹⁷ (Nersc, n.d.)

fig. 6: Unix/Linux file permission system with a max digit of 7

Hexadecimal Base 16

Base-16 (Hexadecimal) is a positional number system that uses 16 as its base. It is widely used in computer science and digital electronics because it offers a compact way to represent large binary numbers. In the hexadecimal system, each digit can represent one of 16 values, ranging from 0 to 15.¹⁸

Digits in Hexadecimal

- The first 10 digits (0–9) are the same as in the decimal system.
- However, the next 6 digits (10–15) are represented as follows:

Digit	Hex Representation
10	A
11	B
12	C
13	D
14	E
15	F

¹⁸ (Christensson, 2023)

Hexadecimal Base 16

Uses and Significance

Memory Addressing

In programming and computer architecture, hexadecimal is often used to represent memory addresses. This is because it provides a more manageable way to express large binary numbers, which can become cumbersome in binary form.

```
○ ○ ○ └ sahat@echo: ~/Dropbox/LearnC — ..ropbox/LearnC — zsh — 80x24
0749eb90 f0 32 7d 60 95 48 d0 62 08 80 4b 67 b4 4a 21 dc | .2)`^H.b..Kg.J!.|_
0749eba0 80 3f 6c dd 4a f5 a3 d4 ce 32 8d e4 21 d7 a5 5a | .?l.J....2...!..Z|
0749ebb0 92 93 4b f1 ca 0a ce 3c b9 14 20 a5 00 a4 4a 3e | ..K....<... .J>|
0749ebc0 bd 4b 8c b4 d1 90 2b 25 a9 c8 f4 c8 10 85 fb d6 | .K....+%.....|
0749ebd0 fc 2a 1f c6 8a 7f 25 e7 47 f4 95 01 e2 d7 82 fe | .*....%.G.....|
0749ebf0 22 95 fa 8e 49 e4 50 98 d3 84 95 a7 97 1d 97 92 | "...I.P.....|
0749ebf0 25 32 9f 90 0c a9 07 73 c2 2b 49 06 4c 1a 26 69 | %2.....s.+I.L.&i|
0749ec00 b2 75 3e 20 db 65 bf 22 68 cf 29 1b 8a 65 8d 54 | .u> ..e."h.)..e.T|
0749ec10 91 ba 33 f3 05 59 07 39 cd 43 96 6f 5d 88 bb 7a | ..3..Y.9.C.o]..z|
0749ec20 aa ae d2 04 b1 c6 33 25 8c 68 f7 c7 79 23 ef 66 | .....3%.h..y#.f|
0749ec30 7a aa 41 e7 99 55 1d 46 79 64 2a 6c 1f a9 64 63 | z.A..U.Fyd*l..dc|
0749ec40 ef f9 87 72 3f d9 5a 9f 48 0d 92 96 72 0d 1b a4 | ....r?.Z.H...r...|
0749ec50 a6 2e 08 b0 96 cc e6 37 88 f0 57 32 3b 21 6d d9 | .....7..W2;!m..|
0749ec60 e4 6b f1 ef 14 25 65 e3 3c b3 ee 60 bc a4 ea 44 | .k...%e.<...`...D|
0749ec70 64 49 0d 59 0b 45 3f f0 75 a4 24 be 41 f5 52 ad | dI.Y.E?.u.$.A.R..|
0749ec80 32 65 33 4d 9c 83 8e 97 69 57 f2 5d 72 93 dd b1 | 2e3M....iW.]r...|
0749ec90 d0 c6 dc c8 43 89 6e 1e 8b d9 2e 67 52 3e 26 3f | ....C.n....gR>&?|
0749eca0 46 cc 92 a7 e1 f3 af 9c c8 b3 17 fe ff 8a bb 7a | F.....|.....z|
0749ecb0 f6 e9 99 6d 8b 24 dc 84 97 67 b6 d5 5b 73 a6 fc | ...m.$...g..[s..|
0749ecc0 50 a6 cf fe 92 7d c3 2f 2e 7e e8 b7 8f 9b 71 5f | P.....)../~....q_|
0749ecd0 b0 43 79 5c f1 63 9d b7 2f 7e b1 f3 f6 87 5f b0 | .Cy\..c.../~/...._|
0749ece0 64 84 86 98 59 f7 d2 96 42 28 5a 96 8e d1 17 4f | d...Y...B(Z....0|
0749ecf0 f4 2d a6 94 06 0f fb 57 83 fe 60 59 8e 32 70 23 | ..-....W..`Y.2p#|
0749ed00 c1 8a 98 43 0b 90 26 24 03 ce 3d 21 79 0b 75 f9 | ...C..&$..=!y.u.|
```

fig. 7: an example of a memory hexdump, the raw data of a program's functions, variables etc. stored in memory with hex addresses.

Hexadecimal Base 16

Uses and Significance

Color Representation

Hexadecimal is widely used in web design to represent colors. Colors in HTML and CSS are defined using a # followed by six hexadecimal digits, where the first two digits represent red, the next two represent green, and the last two represent blue (RGB). For example, #FF5733 represents a specific shade of orange.



fig. 8: an example of hex color codes used in web design

Analysis

Number Systems have already existed thousands of years ago. From the earliest mesopotamian civilizations to Hindu-arabic civilizations, number systems have paved the way to represent the mathematical world around us. The sexagesimal system was one of the first invented positional numeral system by the Babylonians, and then came the Hindu-arabic system with the introduction of 0. The Hindu-arabic system was later spread to Middle-east and Europe and became widely adopted due to the works of mathematicians like Fibonacci. In present time, we have many number systems that exist. However, there are 4 that stand out especially in the discipline of Computing. Those 4 are the decimal number system, binary number system, octal number system and hexadecimal number system. Each system has a specific use case and significance in computing. The decimal number system represents 10 numbers ranging from 0-9. They are used in daily transactions, scientific calculations and applications. The decimal system is easy to interpret by a human person. Next is the binary number system, which only comprises of 0s and 1s. They are the universal language of machines, specifically computing machines. The mobile devices, computer devices and any computer accessory we know today communicate through binary. Files, images, audios and text that are stored in these devices are all encoded in binary. Since the number system only contains 0s and 1s, it allows for efficient storage and processing. The core of a computer, CPU, is made up of logic gates, which stores states of on/off, which can be represented by 1s and 0s. However, to represent large binary values, earlier computers such as the UNIVAC and PDP-8 used the Octal numbering system. The octal numbering system consists of 8 digits, ranging from 0-7. Each digit in the octal number system can be represented by 3 binary digits, or also called bits. File permissions in UNIX/LINUX systems later years also utilized the Octal number system for their file permissions. The hexadecimal system contains the most amount of digits, ranging from 0 to 15. Digits starting from 10 to 15 are represented with letters, A-F, respectively. Similar to the octal number system, each hexadecimal digit can be represented by 4 binary digits, allowing for greater number of bits to be stored. Memory addresses in computers use hexadecimal format, as well as color representation in web development. These 4 number systems make up the world of computing we know of today.

References

Articles

- A Brief History of Numbers: How 0-9 Were Invented. (2023, July 19). Casio Calculators. <https://education.casio.co.uk/blog/a-brief-history-of-numbers/>
- Awati, R. (2022, May 25). binary. WhatIs. <https://www.techtarget.com/whatis/definition/binary>
- Brioli, D., Karsner, D., & Boddie, K. (2023, November). Binary Number System. Study.com. Retrieved September 15, 2024, from <https://study.com/academy/lesson/binary-number-system-application-advantages.html>
- Christensson, P. (2023, May 16). Hexadecimal. <https://techterms.com/definition/hexadecimal>
- Jha, A. C. (2021). Positional Number System. NUTA Journal, 7(1–2), 2616–017x. <https://www.nepjol.info/index.php/nutaj/issue/view/2501>
- Lippman, D. (n.d.). The Positional System and Base 10 | Mathematics for the Liberal Arts. <https://courses.lumenlearning.com/waymakermath4libarts/chapter/the-positional-system-and-base-10/>
- Nersc. (n.d.). Unix File Permissions - NERSC Documentation. <https://docs.nersc.gov/filesystems/unix-file-permissions/>
- Ranjan, R. (2023, July 24). History, advantages, and applications of binary language and how binary translators translate it? TechDotMatrix. <https://www.techdotmatrix.com/2020/03/advantages-and-applications-of-binary-language/>
- The Editors of Encyclopaedia Britannica. (2024, August 2). Hindu-Arabic numerals | History & Facts. Encyclopedia Britannica. <https://www.britannica.com/topic/Hindu-Arabic-numerals>
- Vedantu. (n.d.). Octal Number System. VEDANTU. <https://www.vedantu.com/math/octal-number-system>

References

Articles

- Adam, H., & Adam, M. (2022). Basics: number systems, dual numbers and codes. In Springer eBooks (pp. 3–15). https://doi.org/10.1007/978-3-662-65254-1_1
- Friberg, J. (2019). Three thousand years of sexagesimal numbers in Mesopotamian mathematical texts. *Archive for History of Exact Sciences*, 73(2), 183–216. <https://doi.org/10.1007/s00407-019-00221-3>
- GeeksforGeeks. (2024, August 5). Number System in Maths. GeeksforGeeks. <https://www.geeksforgeeks.org/number-system-in-maths/>
- Gill, S. (2024, March 10). How Did Decimals Evolve And Why Do We Need Them? ScienceABC. <https://www.scienceabc.com/pure-sciences/how-did-decimals-evolve-and-why-did-we-come-up-with-it.html>
- Marchant, J. (2024). The decimal point is 150 years older than historians thought. *Nature*. <https://doi.org/10.1038/d41586-024-00473-2>
- number, n. meanings, etymology and more | Oxford English Dictionary. (n.d.). https://www.oed.com/dictionary/number_n
- Powell, M. A. (2008). Sexagesimal system. In Springer eBooks (pp. 1998–1999). https://doi.org/10.1007/978-1-4020-4425-0_9055
- Princeton University. (2019). Number Systems and Number Representations. Princeton University. https://www.cs.princeton.edu/courses/archive/spr19/cos217/lectures/05_NumberSystems.pdf
- Tatomir, J., & Chesnutt, B. (2023, November). Ancient number systems. Study.com. <https://study.com/academy/lesson/basics-of-ancient-number-systems.html>
- Virnuls, A. (n.d.). Number Bases. Computing and ICT in a Nutshell. <https://www.advanced-ict.info/mathematics/bases.html>

References

Figures

Fig 1. <https://mathematicalmysteries.org/wp-content/uploads/2022/08/babylonian-number-system.jpg>

Fig 2. <https://www.basic-mathematics.com/hindu-arabic-numeration-system.html>

Fig 3. https://upload.wikimedia.org/wikipedia/commons/7/78/Positional_notation_glossary-en.svg

Fig 4. <https://theplctutor.com/images/base10weightsmall.jpg>

Fig 5. <https://medium.com/i-math/decoding-binary-numbers-50b8fd914908>

Fig 6. <https://medium.com/@omerorhan87/unix-file-permissions-a9372f3b1568>

Fig 7. <https://stackoverflow.com/questions/12434579/how-to-find-the-address-of-variable-in-a-memory-hxdump>

Fig 8. <https://www.geeksforgeeks.org/hex-color-codes/>