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Number Systems

and their types, uses and significance

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Introduction

What is a number?

A number is a **mathematical object** used to *count*, *measure*, and *label*. Numbers serve as the fundamental elements of mathematics and have a wide variety of forms and uses depending on the context.[1]

Number Systems

Number systems are the various ways of representing numbers. Each system has its own structure, set of rules, and operations. It is a mathematical notation for representing numbers of a given set, consistently using digits or other symbols. It allows us to perform arithmetic operations like addition, subtraction, multiplication, and division.[2]

1 (Number, n.d.)
2 (GeeksforGeeks, 2024)

History

The Babylonian Civilization

The first true written positional numeral system is generally attributed to the ancient Babylonian civilization. It emerged around 3000 BCE influenced by earlier Sumerian systems. It was primarily used in ancient Mesopotamia for trade, astronomy, and record-keeping[3] and was based on a **sexagesimal (base-60)** system.[4]

fig. 1: The sexagesimal system and their symbols

The **sexagesimal system** was an ancient system of counting, calculation, and numerical notation that used powers of 60 much as the decimal system uses powers of 10. Rudiments of the ancient system survive in vestigial form in our division of the hour into 60 minutes and the minute into 60 seconds.[5]

Although it is a positional system, it **lacks a true zero**, which can complicate calculations and representations. The absence of zero means that the system relies on spacing to indicate the absence of a value, which can be **less intuitive**.[3]

3 (Tatomir & Chesnutt, 2023)
4 (Friberg, 2019)
5 (Powell, 2008)

History

Hindu-Arabic System

While the Babylonian's Sexagesimal system was the first true positional numeral system, the Hindu-Arabic system was also one of the number systems that influenced modern mathematics and also introduced the number 0.[6]

The **Hindu-Arabic numeral system** is a decimal (base-10) system, meaning it uses ten distinct symbols to represent numbers. Each position in a number corresponds to a power of ten, allowing for the representation of large numbers with relatively few symbols.

Developed between the **1st** and **4th centuries CE** in India and later spread to the Middle East and Europe by the 9th century. It became widely adopted in Europe during the 13th century due to the works of mathematicians like Fibonacci.[6.1]

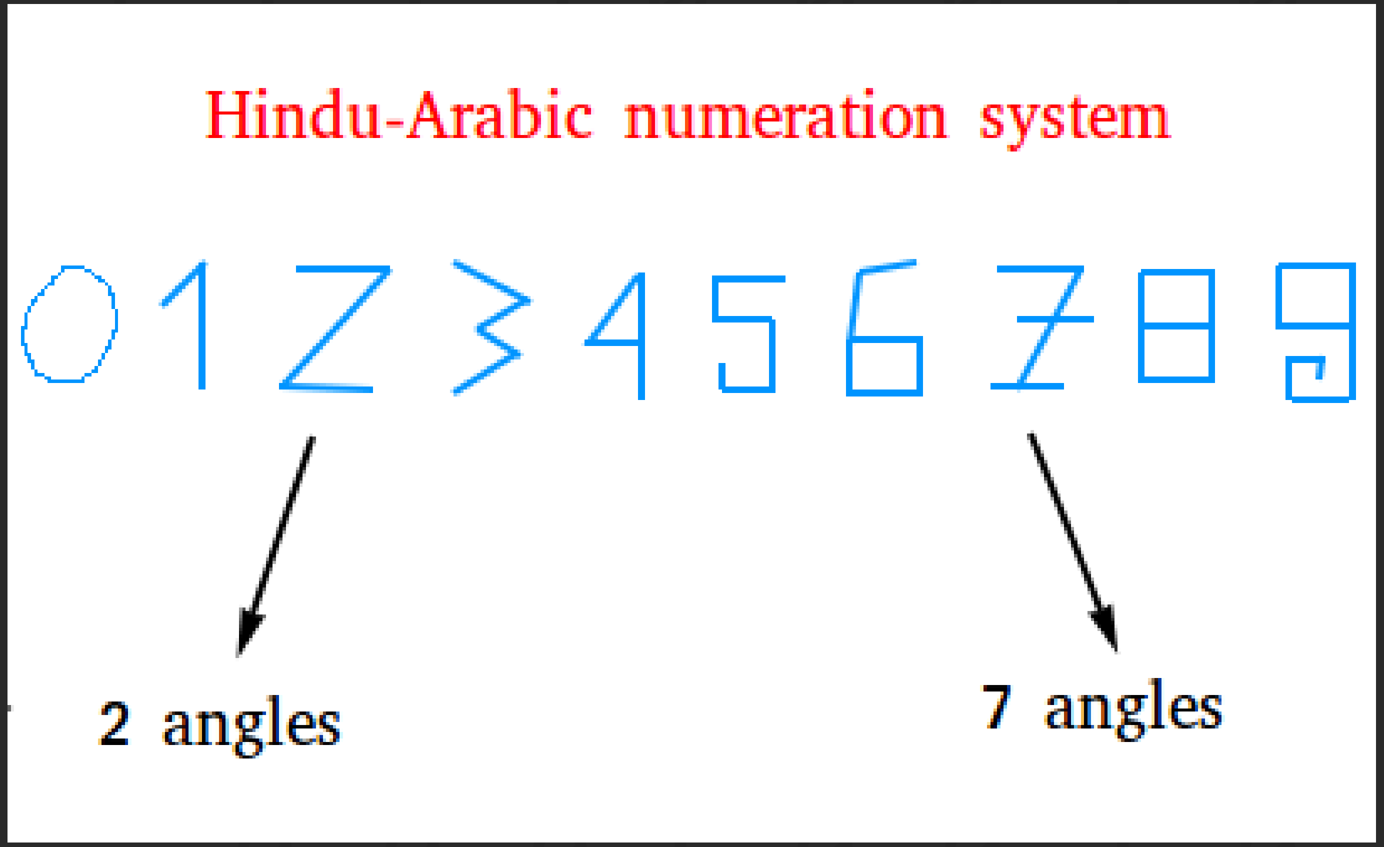

fig. 2: the Hindu-Arabic numeration system

6 (A Brief History of Numbers: How 0-9 Were Invented, 2023)
6.1 (The Editors of Encyclopaedia Britannica, 2024)

Elements of the Number System

(Lippman, n.d.)

The Base

Every number system is based on a specific **base** or *radix*, which determines how many unique digits or symbols are used in that system. The base also defines how place values increase or decrease as you move left or right in a number.[7]

There are **four** commonly used type of bases, and they are:

- **Binary (Base 2)** - 0s and 1s
- **Decimal (Base 10)** - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- **Octal (Base 8)** - 0, 1, 2, 3, 4, 5, 6, 7
- **Hexadecimal (Base 16)** - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Digits

These are the basic symbols used in the system. The number of unique digits equals the base of the number system.

7 (Virnuls, n.d.)

Elements of the Number System

(Lippman, n.d.)

Positional Notation

Positional notation (or place-value notation) is a method for representing numbers in which the position of each digit within a number determines its value. The value of a digit depends not only on the digit itself but also on its position or place in the number, and the base (or radix) of the number system being used. It uses a set of digits to represent the value of a number. The product of each digit and its place value indicates the value of the digit. The sum of those products is the value of the number.

Place Value

Each digit's value depends on its position within a number. Moving left in a number increases the value by powers of the base, and moving right decreases the value (for decimal fractions).

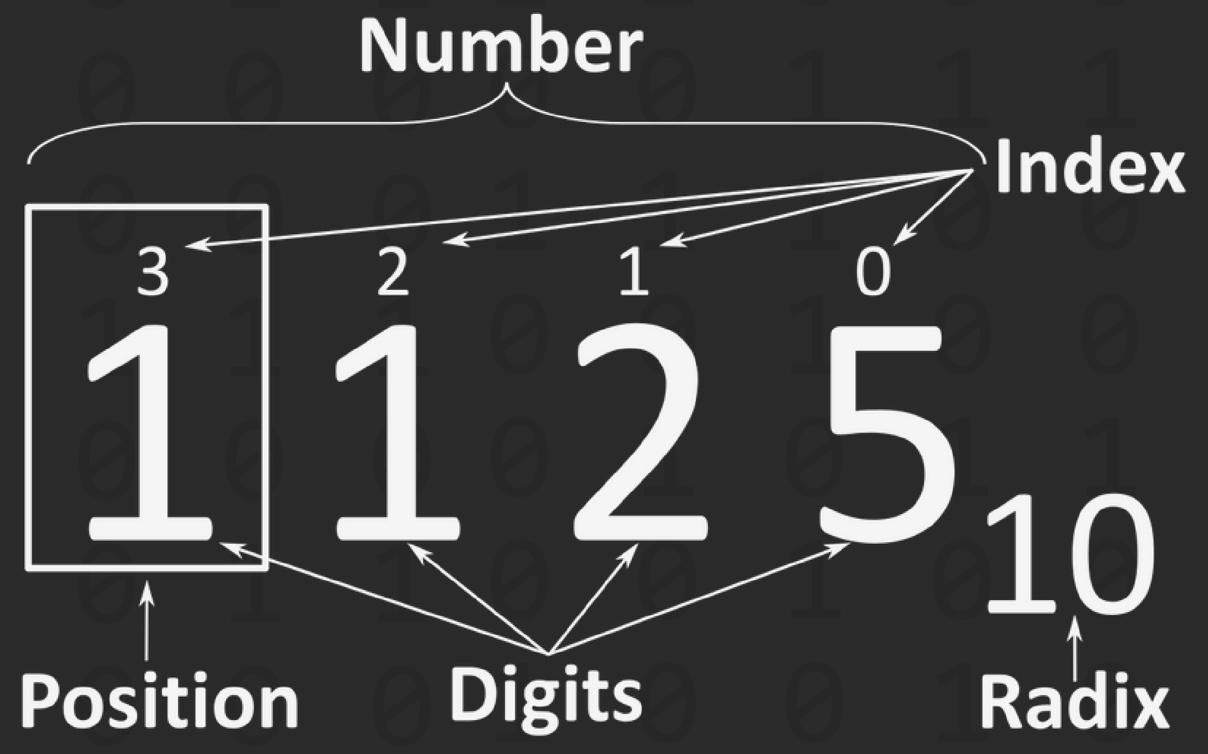

fig. 3: The number 1125 as Base 10 with its corresponding positional notation

Types of Number Systems

(Jha, 2021)

There are 4 commonly used number systems in computing:

Decimal `{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }`
Base **10**

Binary `{ 0, 1 }`
Base **2**

Octal `{ 0, 1, 2, 3, 4, 5, 6, 7 }`
Base **8**

Hexadecimal `{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }`
Base **16**

- *Remember, the amount of digits in a number system corresponds to it's* ***radix*** *or* ***base*** *number.*

Decimal Base 10

The decimal number system, also called the **base-10** number system, is the standard system for denoting integer and non-integer numbers. It is the most widely used number system[8] in the world. The decimal number system has 10 symbols, mainly 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The position of every digit has a weight which is a power of 10.

As shown in Figure 3, we can see the number **5319** as an example of a base 10 number. Every digit is multiplied by 10 raised to the corresponding position number (which is also called its *weight*), and this process repeats for all the digits in the decimal number. The figure proves the derivation of a decimal number by adding the product of each operation.

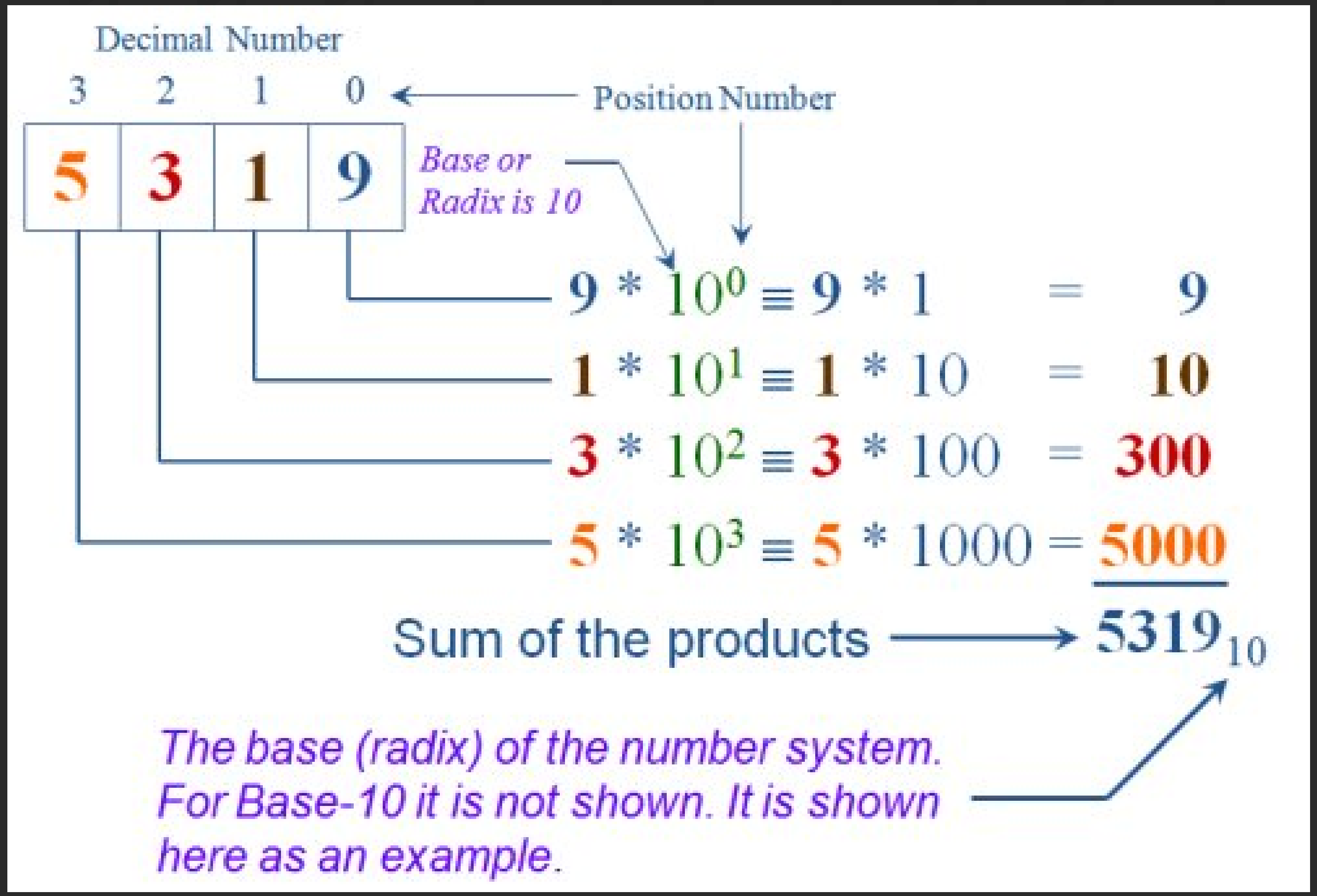

fig. 4: decimal number with the position and weight of each digit shown and how the value of the number is derived

8 (Brioli & Newton, 2023)

Decimal Base 10

Uses and Significance

Applications in Computing

While computers primarily operate in binary, decimal is still fundamental in user interfaces and data representation. For example, many programming languages convert binary to decimal to present results that are easy for humans to interpret.[9]

Scientific Calculations

The decimal system is vital in scientific notation, where large or small numbers are expressed in terms of powers of ten. This system facilitates accurate and manageable representations of complex measurements, like atomic distances or astronomical data. [10]

Daily Transactions

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9 (Princeton University, 2019) 10 (Adam & Adam, 2022)
11 (Gill, 2024) 12 (Marchant, 2024)

Binary Base 2

The binary number system is a way of representing numbers using only two digits: 0 and 1. It is a **base-2 system**, meaning each position in a binary number represents a power of 2. Unlike the decimal system, which uses ten digits (0-9), binary only uses two, making it essential in computing because digital systems like computers process data in binary form.[13]

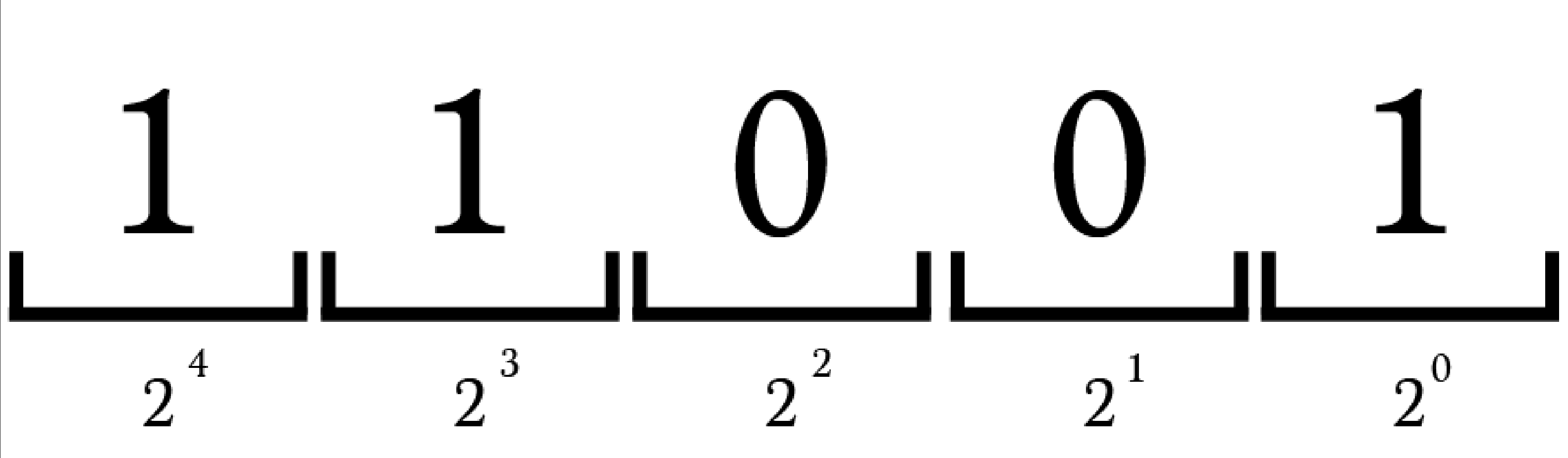

fig. 5: a binary number with the position and weight of each digit

In figure 5, we can see the binary number **11001**. Since it's the **Base-2** system, that means that we multiply the digits by 2 raised to the corresponding position number (similar to that of a decimal number). Hence, our first digit (starting from the right) or also known as the **Least Significant Bit**, is multiplied by 2^0, and the second digit by 2^1, so on and so forth.

We disregard zero since 0 multiplied with anything still equates to zero. Therefore, we are left with 1x2^0, 1x2^3 and 1x2^4. Adding all of these products together gives us the **base 10 conversion** of this binary which is **25.**

13 (Brioli et al., 2023)

Binary Base 2

Uses and Significance

Data Representation in Modern Computing

All forms of data in computers, including text, images, and audio, are encoded in binary. Each piece of information is represented as a series of bits (0s and 1s), allowing for efficient storage and processing. For example, a 16-bit color depth can represent 65,536 different colors on a screen.[13]

Logic Gates and Circuits

Binary is the foundation of digital circuits, where electronic components represent binary states (on/off). Logic gates operate on binary inputs to produce binary outputs, allowing for complex computations and operations in devices like CPUs.[14]

Universal Language for Machines

Binary serves as a universal language for computers, allowing different systems and devices to communicate effectively. This universality is crucial for interoperability in an increasingly digital world.[15]

13 (Brioli et al., 2023)
14 (Awati, 2022)
15 (Ranjan, 2023)

Octal Base 8

The octal number system is a base-8 numbering system that uses eight digits: 0, 1, 2, 3, 4, 5, 6, and 7. Each position in an octal number represents a power of 8. [16]

In the decimal system, each place is a power of ten. For example:

$$\mathbf{74}_{10} = (\mathbf{7} \times 10^1) + (\mathbf{4} \times 10^0)$$

In the octal system, each place is a power of ten. For example:

$$\mathbf{112}_8 = (\mathbf{1} \times 8^2) + (\mathbf{1} \times 8^1) + (\mathbf{2} \times 8^0)$$

In converting octal to binary, octal numbers can be represented by 3 binary digits. Since three binary digits can represent a maximum of 7, hence it can represent each and every octal number since it only goes up to 7.

Octal Number	Binary Number
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

16 (Vedantu, n.d.)

Octal Base 8

Uses and Significance

Historical Use in Computing

Octal was widely used in early computing systems, particularly in minicomputers and mainframes, because it provides a more compact representation of binary numbers. Each octal digit corresponds to three binary digits, making it easier to read and write. An example of these is the **UNIVAC 1050** and **PDP-8**.

File Permissions in Unix/Linux

In Unix and Linux operating systems, file permissions are often represented in octal format. Each digit in an octal number corresponds to a set of permissions for the owner, group, and others. [17]

17 (Nersc, n.d.)

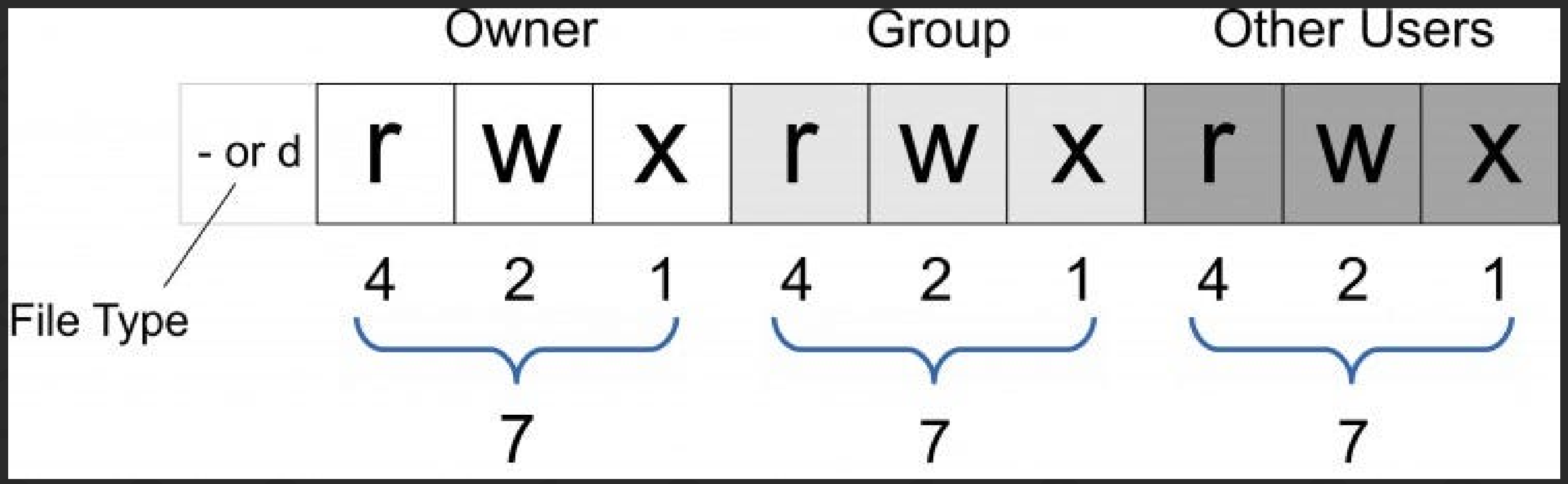

fig. 6: Unix/Linux file permission system with a max digit of 7

Hexadecimal Base 16

Base-16 (Hexadecimal) is a positional number system that uses 16 as its base. It is widely used in computer science and digital electronics because it offers a compact way to represent large binary numbers. In the hexadecimal system, each digit can represent one of 16 values, ranging from 0 to 15.[18]

Digits in Hexadecimal

- The first 10 digits (0–9) are the same as in the decimal system.
- However, the next 6 digits (10-15) are represented as follows:

Digit	Hex Representation
10	A
11	B
12	C
13	D
14	E
15	F

18 (Christensson, 2023)

Hexadecimal Base 16

Uses and Significance

Memory Addressing

In programming and computer architecture, hexadecimal is often used to represent memory addresses. This is because it provides a more manageable way to express large binary numbers, which can become cumbersome in binary form.

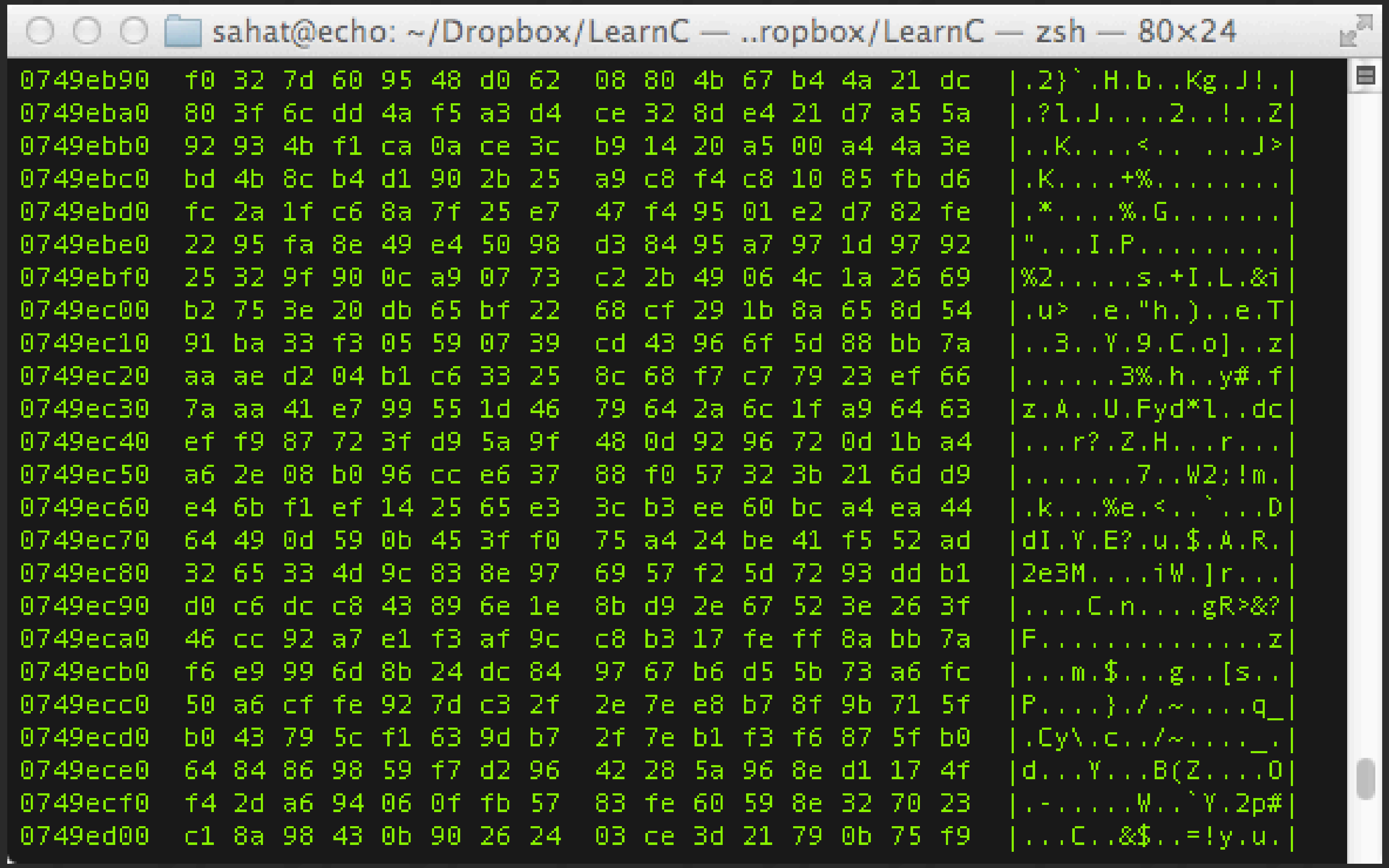

fig. 7: an example of a memory hexdump, the raw data of a program's functions, variables etc. stored in memory with hex addresses.

Hexadecimal Base 16

Uses and Significance

Color Representation

Hexadecimal is widely used in web design to represent colors. Colors in HTML and CSS are defined using a # followed by six hexadecimal digits, where the first two digits represent red, the next two represent green, and the last two represent blue (RGB). For example, #FF5733 represents a specific shade of orange.

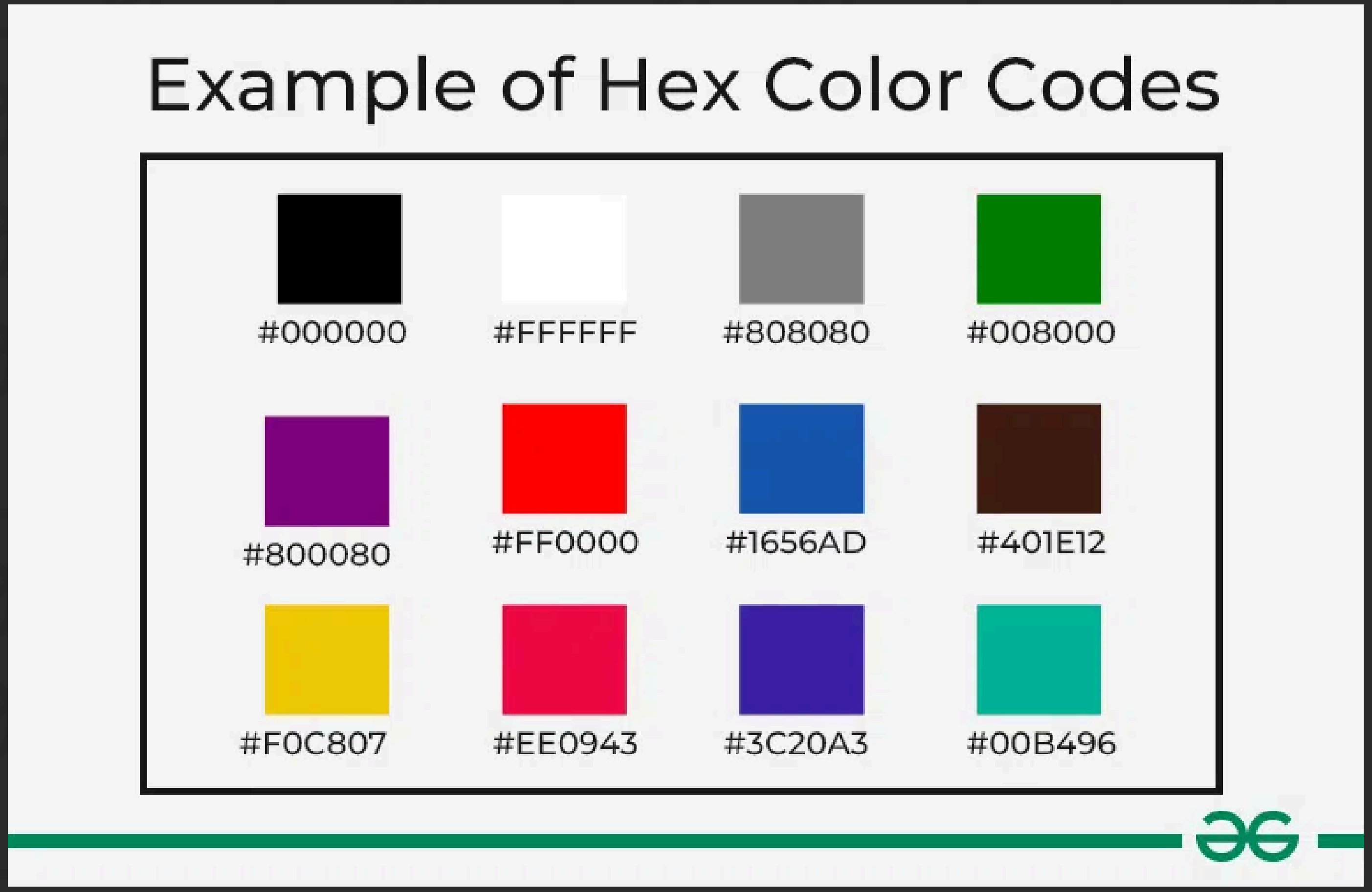

fig. 8: an example of hex color codes used in web design

Analysis

Number Systems have already existed thousands of years ago. From the earliest mesopotamian civilizations to Hindu-arabic civilizations, number systems have paved the way to represent the mathematical world around us. The sexagesimal system was one of the first invented positional numeral system by the Babylonians, and then came the Hindu-arabic system with the introduction of 0. The Hindu-arabic system was later spread to Middle-east and Europe and became widely adopted due to the works of mathematicians like Fibonacci. In present time, we have many number systems that exist. However, there are 4 that stand out especially in the discipline of Computing. Those 4 are the decimal number system, binary number system, octal number system and hexadecimal number system. Each system has a specific use case and significance in computing. The decimal number system represents 10 numbers ranging from 0-9. They are used in daily transactions, scientific calculations and applications. The decimal system is easy to interpret by a human person. Next is the binary number system, which only comprises of 0s and 1s. They are the universal language of machines, specifically computing machines. The mobile devices, computer devices and any computer accessory we know today communicate through binary. Files, images, audios and text that are stored in these devices are all encoded in binary. Since the number system only contains 0s and 1s, it allows for efficient storage and processing. The core of a computer, CPU, is made up of logic gates, which stores states of on/off, which can be represented by 1s and 0s. However, to represent large binary values, earlier computers such as the UNIVAC and PDP-8 used the Octal numbering system. The octal numbering system consists of 8 digits, ranging from 0-7. Each digit in the octal number system can be represented by 3 binary digits, or also called bits. File permissions in UNIX/LINUX systems later years also utilized the Octal number system for their file permissions. The hexadecimal system contains the most amount of digits, ranging from 0 to 15. Digits starting from 10 to 15 are represented with letters, A-F, respectively. Similar to the octal number system, each hexadecimal digit can be represented by 4 binary digits, allowing for greater number of bits to be stored. Memory addresses in computers use hexadecimal format, as well as color representation in web development. These 4 number systems make up the world of computing we know of today.

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Figures

Fig 1. https://mathematicalmysteries.org/wp-content/uploads/2022/08/babylonian-number-system.jpg

Fig 2. https://www.basic-mathematics.com/hindu-arabic-numeration-system.html

Fig 3. https://upload.wikimedia.org/wikipedia/commons/7/78/Positional_notation_glossary-en.svg

Fig 4. https://theplctutor.com/images/base10weightsmall.jpg

Fig 5. https://medium.com/i-math/decoding-binary-numbers-50b8fd914908

Fig 6. https://medium.com/@omerorhan87/unix-file-permissions-a9372f3b1568

Fig 7. https://stackoverflow.com/questions/12434579/how-to-find-the-address-of-variable-in-a-memory-hexdump

Fig 8. https://www.geeksforgeeks.org/hex-color-codes/