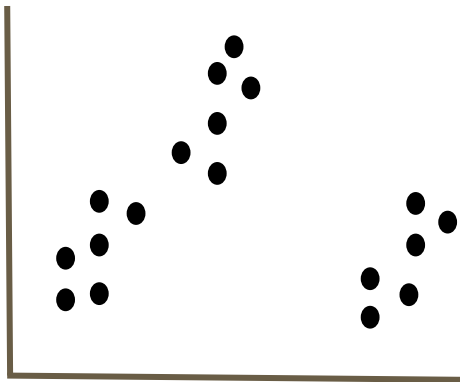
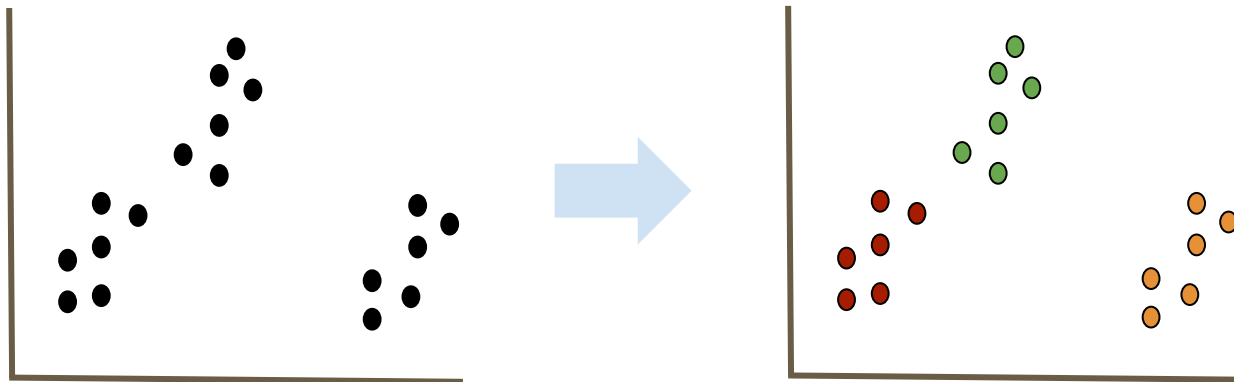

Clustering - Kmeans

— Boston University CS 506 - Lance Galletti —

What is a Clustering



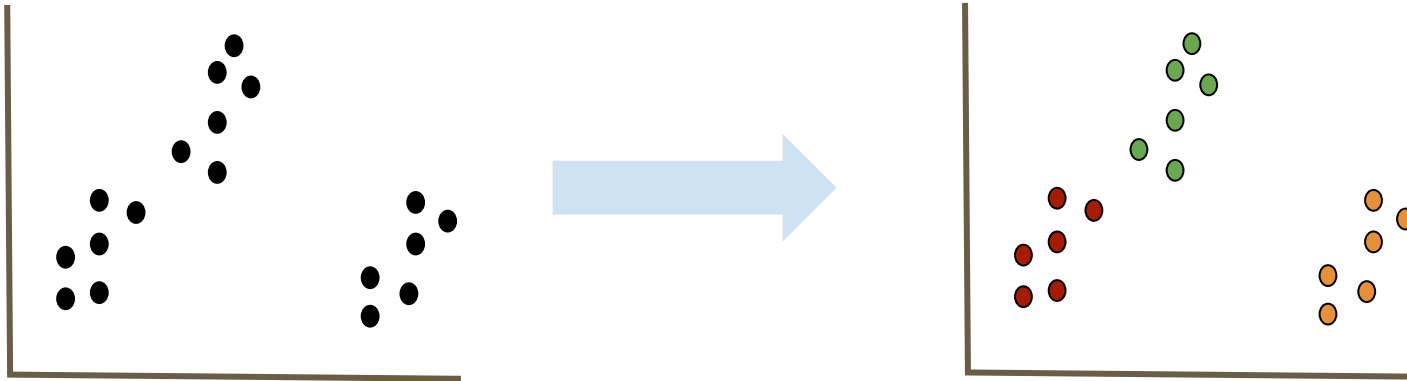
What is a Clustering



What is a Clustering

A clustering is a grouping / assignment of objects (data points) such that objects in the same group / cluster are:

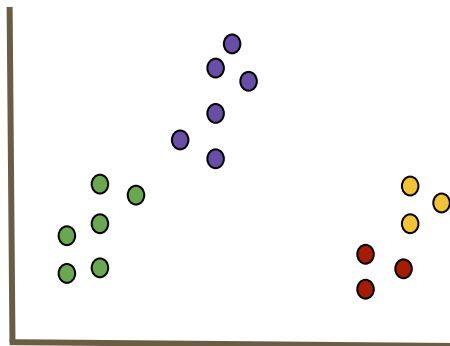
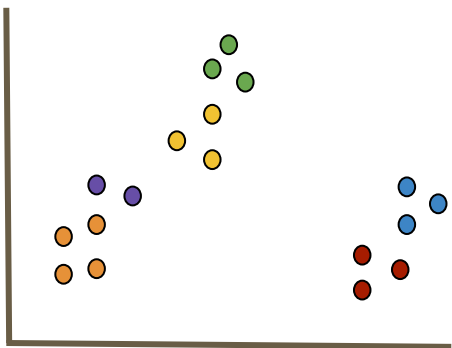
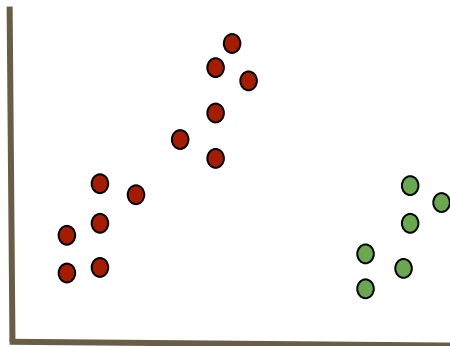
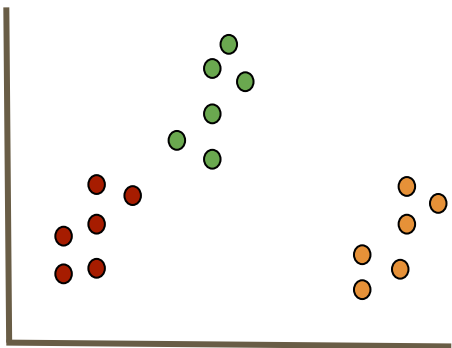
- similar to one another
- dissimilar to objects in other groups



Applications

- Outlier detection / anomaly detection
 - Data Cleaning / Processing
 - Credit card fraud, spam filter etc.
- Feature Extraction
- Filling Gaps in your data
 - Using the same marketing strategy for similar people
 - Infer probable values for gaps in the data (similar users could have similar hobbies, likes / dislikes etc.)

Clusters can be Ambiguous



Types of Clusterings

Partitional

Each object belongs to exactly one cluster

Hierarchical

A set of nested clusters organized in a tree

Density-Based

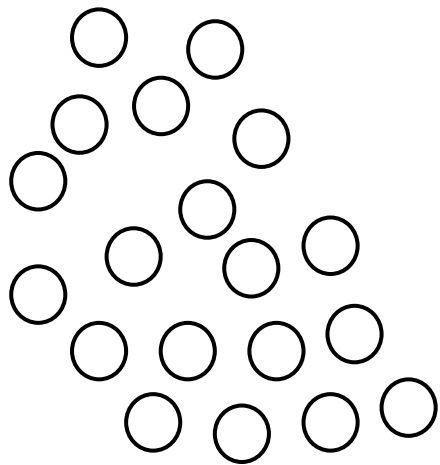
Defined based on the local density of points

Soft Clustering

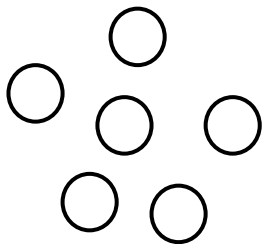
Each point is assigned to every cluster with a certain probability

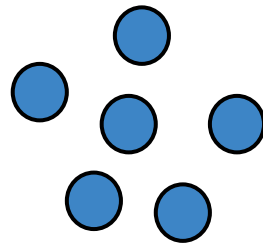
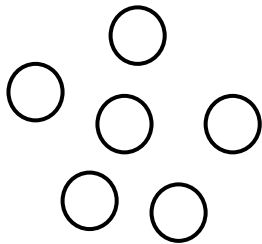
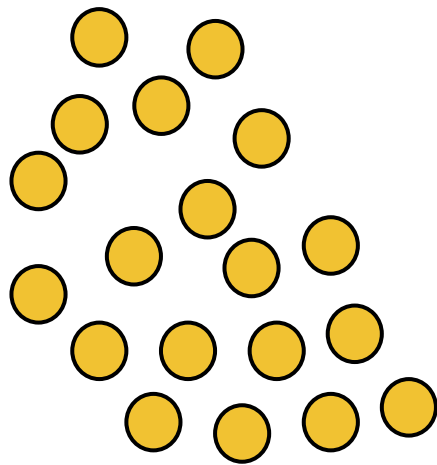
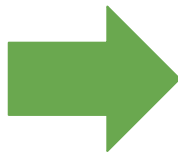
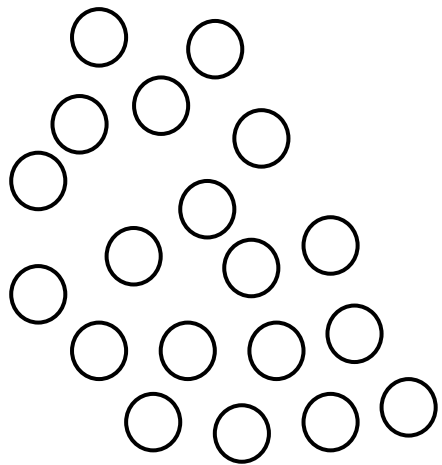
Partitional Clustering

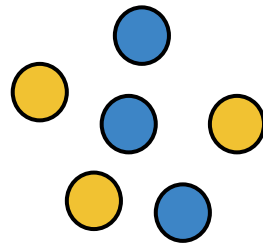
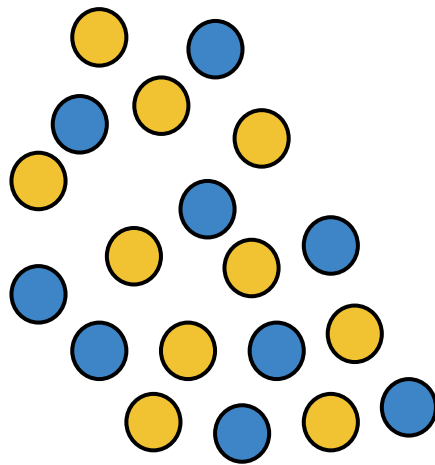
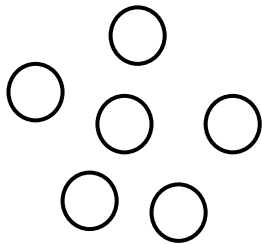
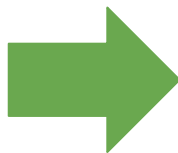
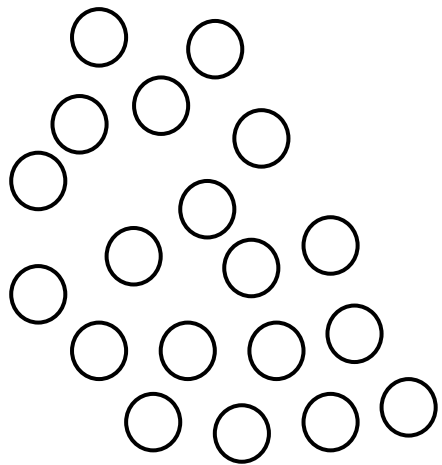
Partitional Clustering

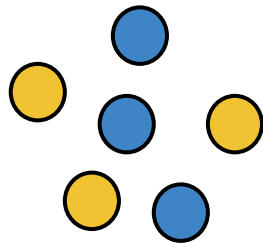
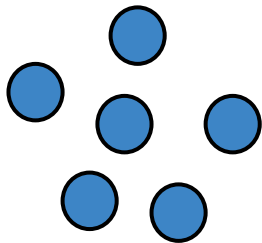
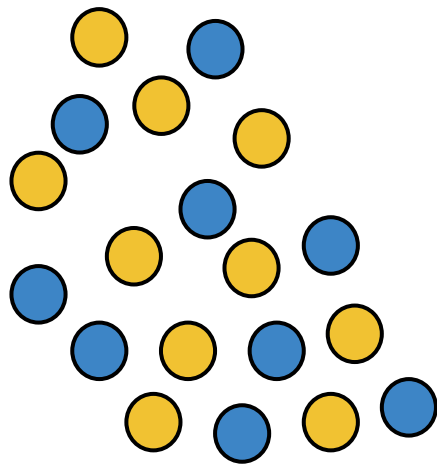
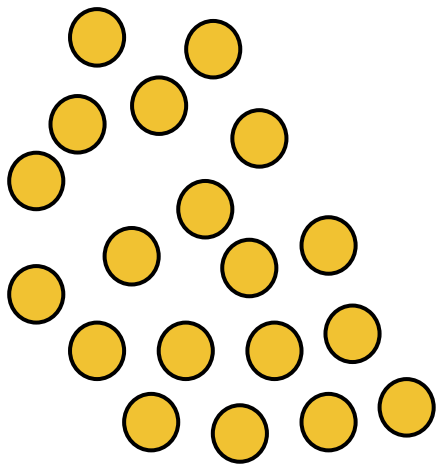


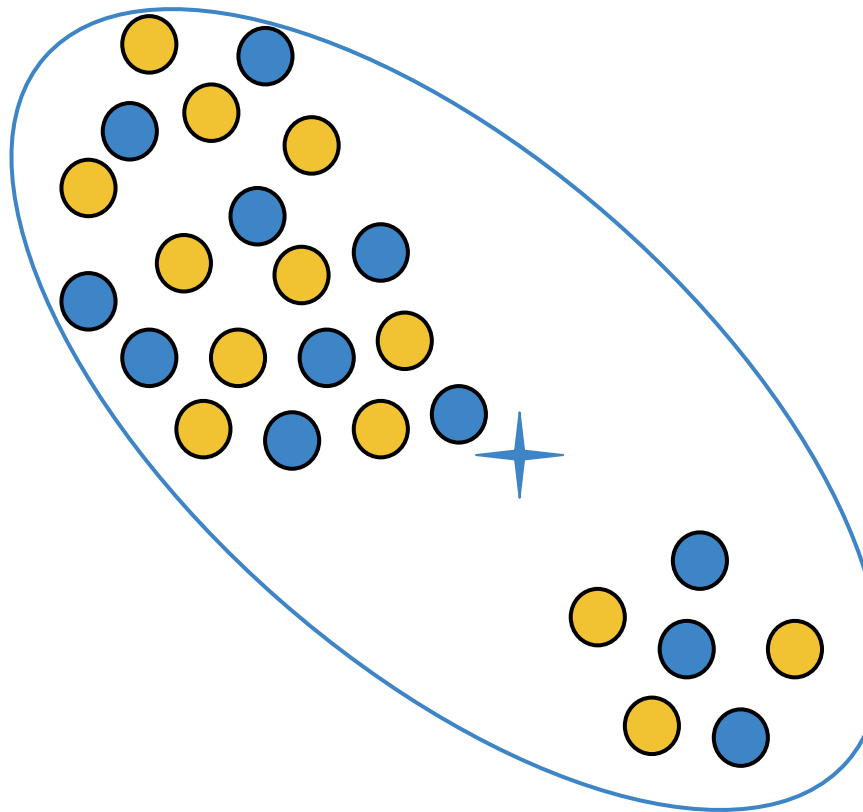
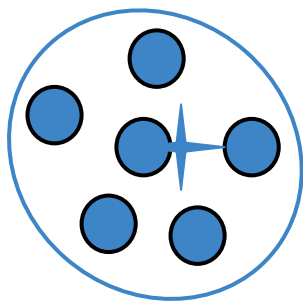
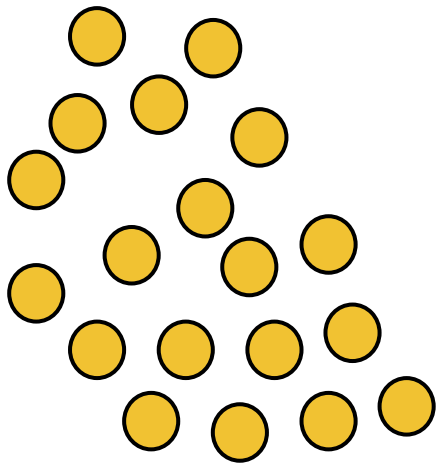
Goal: partition dataset into k partitions

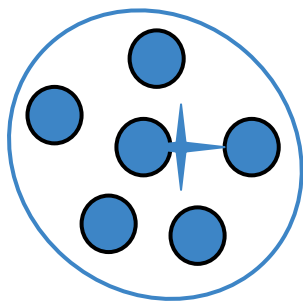
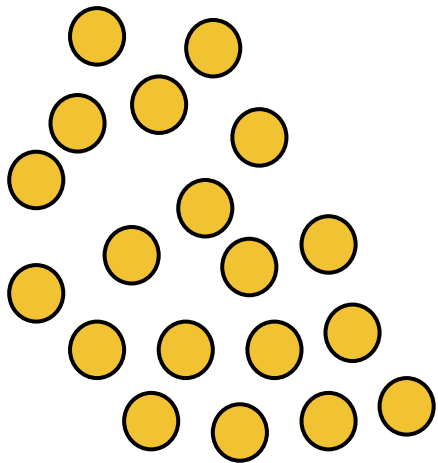




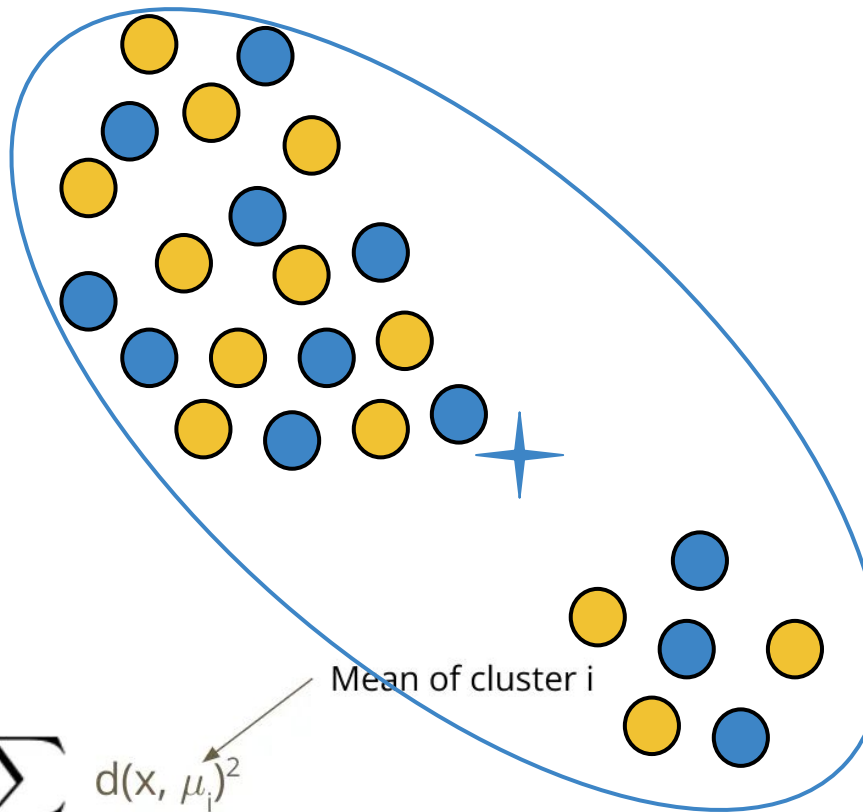








$$\frac{1}{|C_i|} \sum_{x \in C_i}$$



Mean of cluster i

Cluster i

Cost Function

$$\sum_i^k \sum_{x \in C_i} d(x, \mu_i)^2$$

- Way to evaluate and compare solutions
- Hope: can find some algorithm that find solutions that make the cost small

K-means

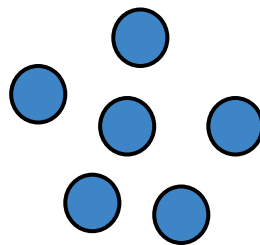
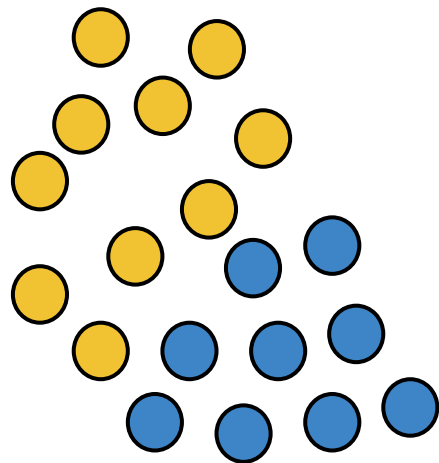
Given $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$ our dataset, \mathbf{d} the euclidean distance, and \mathbf{k}

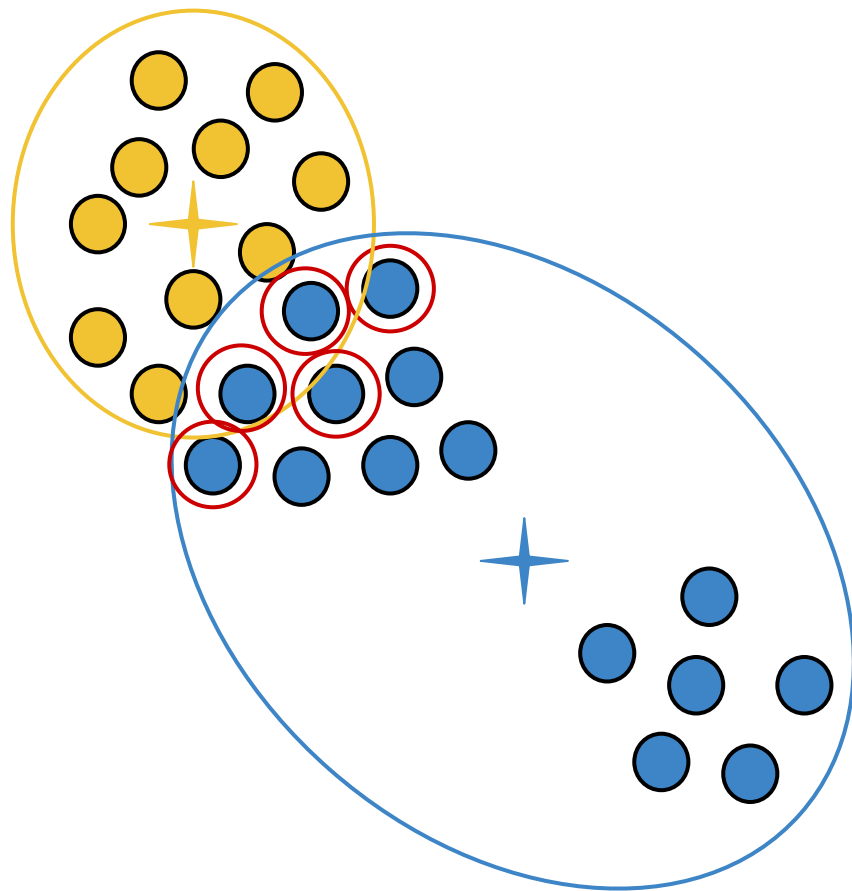
Find \mathbf{k} centers $\{\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_k\}$ that minimize the **cost function**:

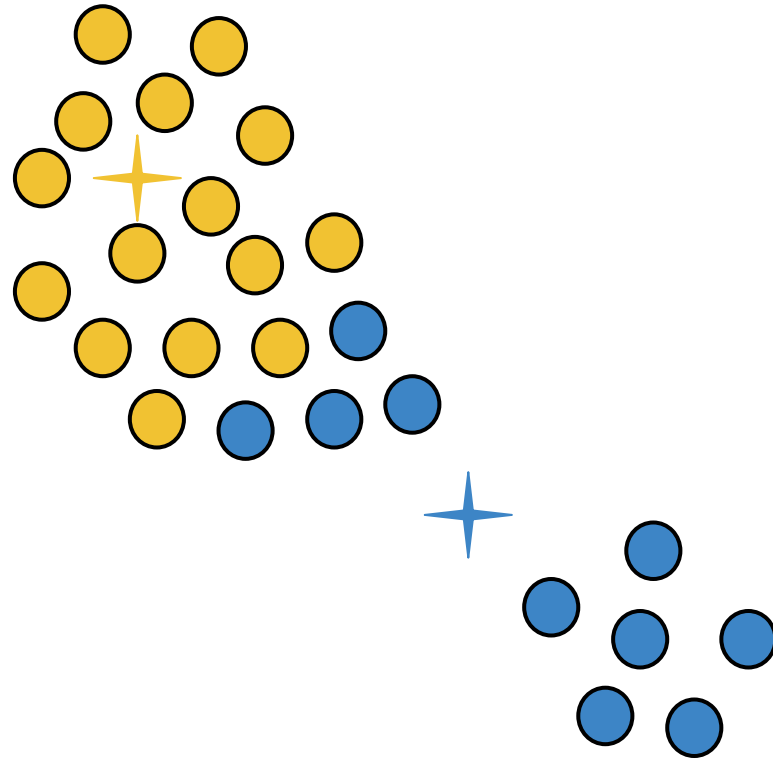
$$\sum_i^k \sum_{x \in C_i} d(x, \mu_i)^2$$

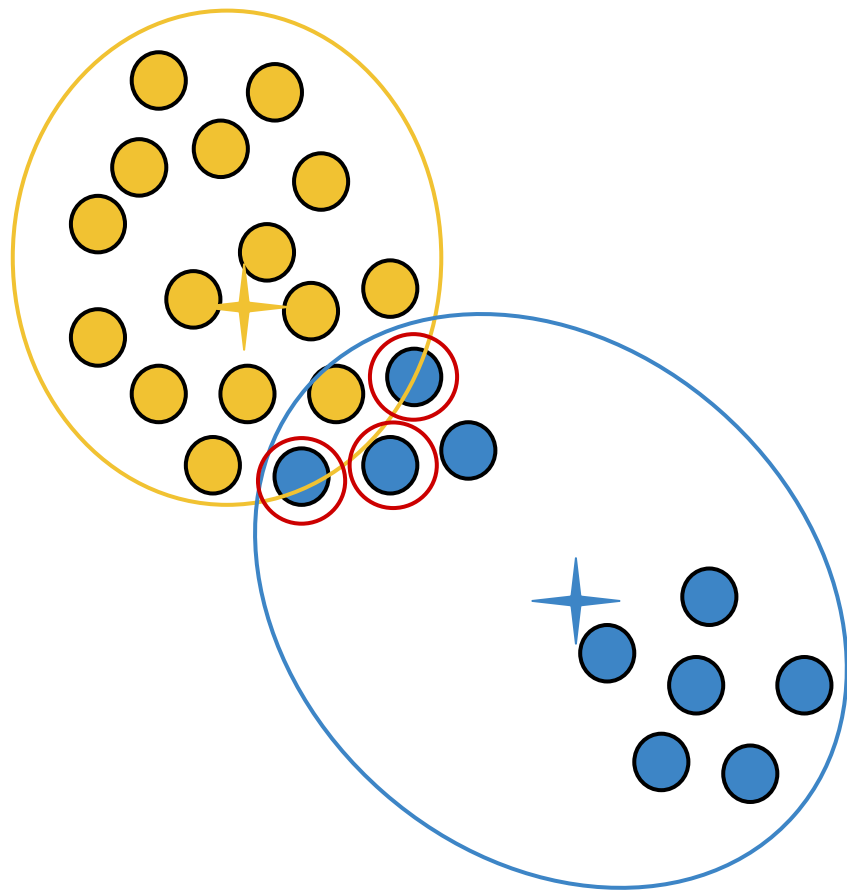
When $\mathbf{k}=1$ and $\mathbf{k}=n$ this is easy. Why?

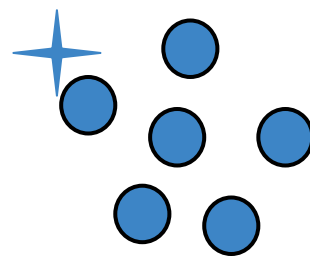
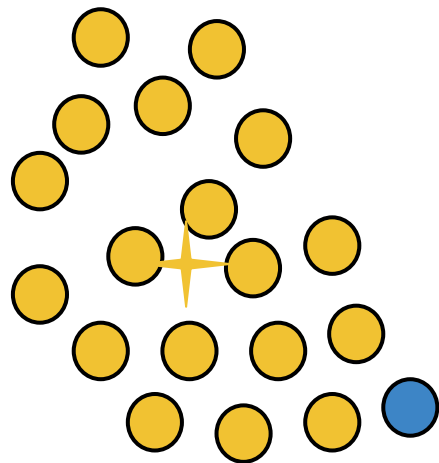
When \mathbf{x}_i lives in more than 2 dimensions, this is a very difficult (**NP-hard**) problem

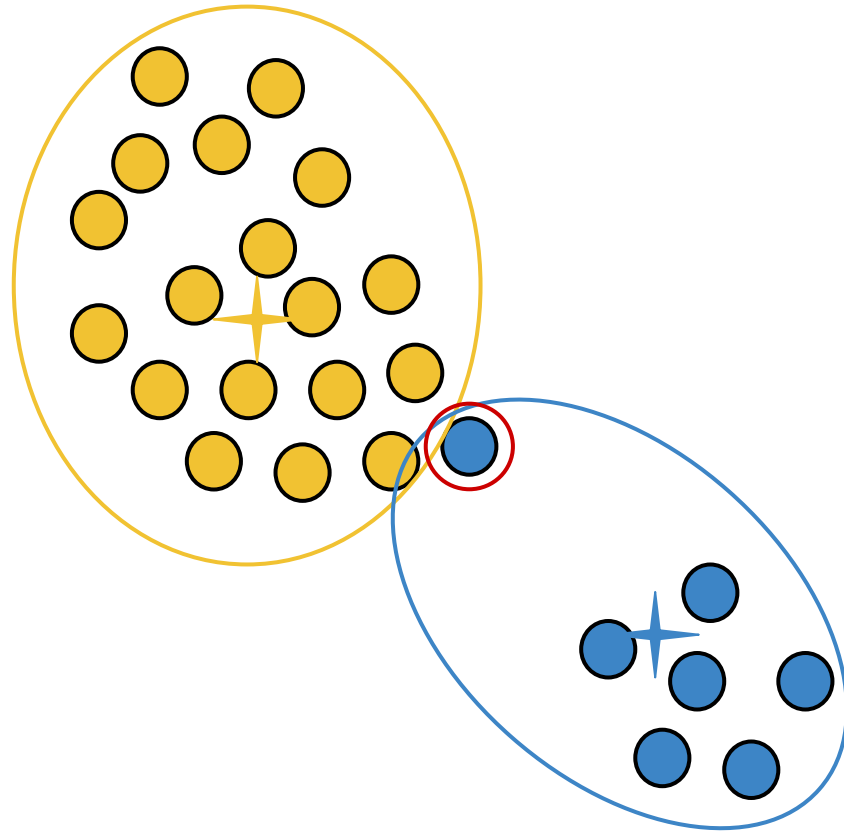


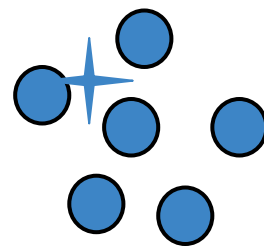
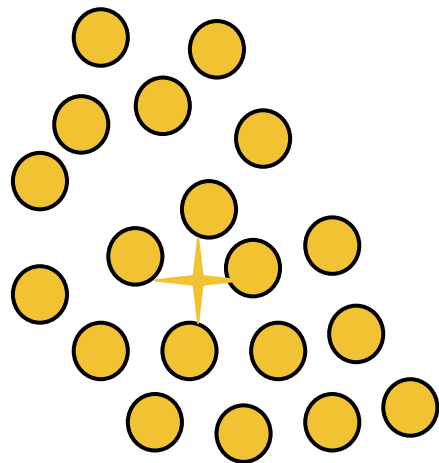






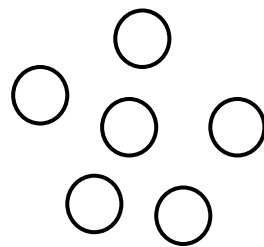
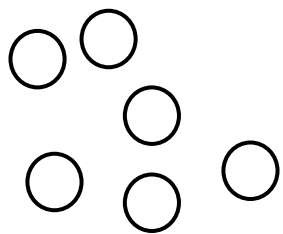
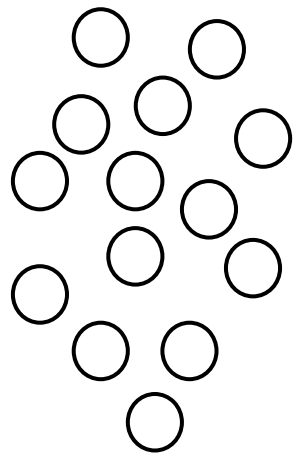


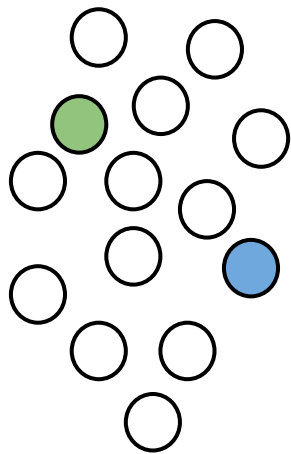




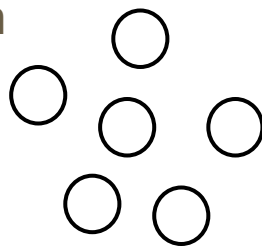
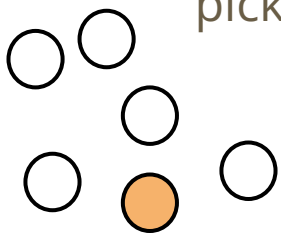
K-means - Lloyd's Algorithm

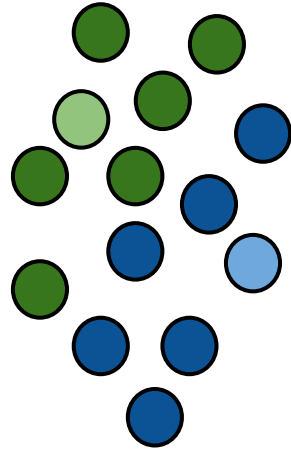
1. Randomly pick k centers $\{\mu_1, \dots, \mu_k\}$
2. Assign each point in the dataset to its closest center
3. Compute the new centers as the means of each cluster
4. Repeat 2 & 3 until convergence



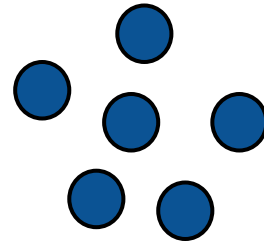
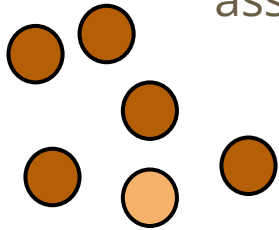


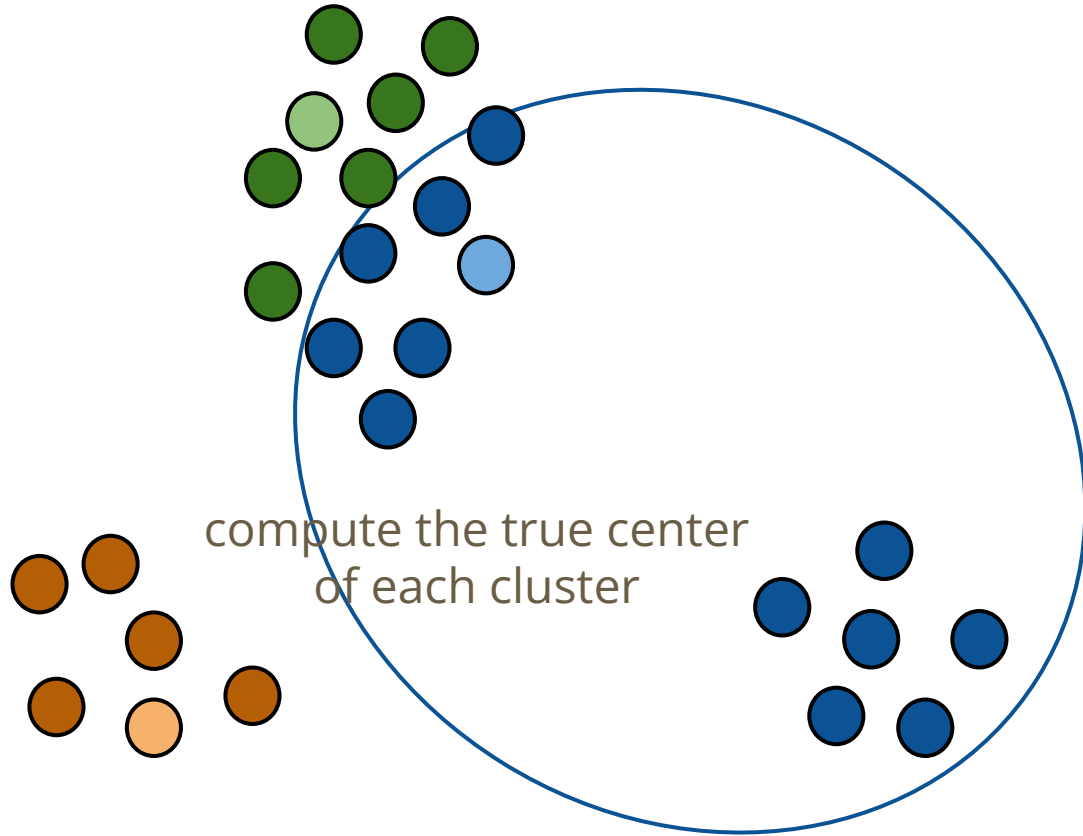
pick k centers at random

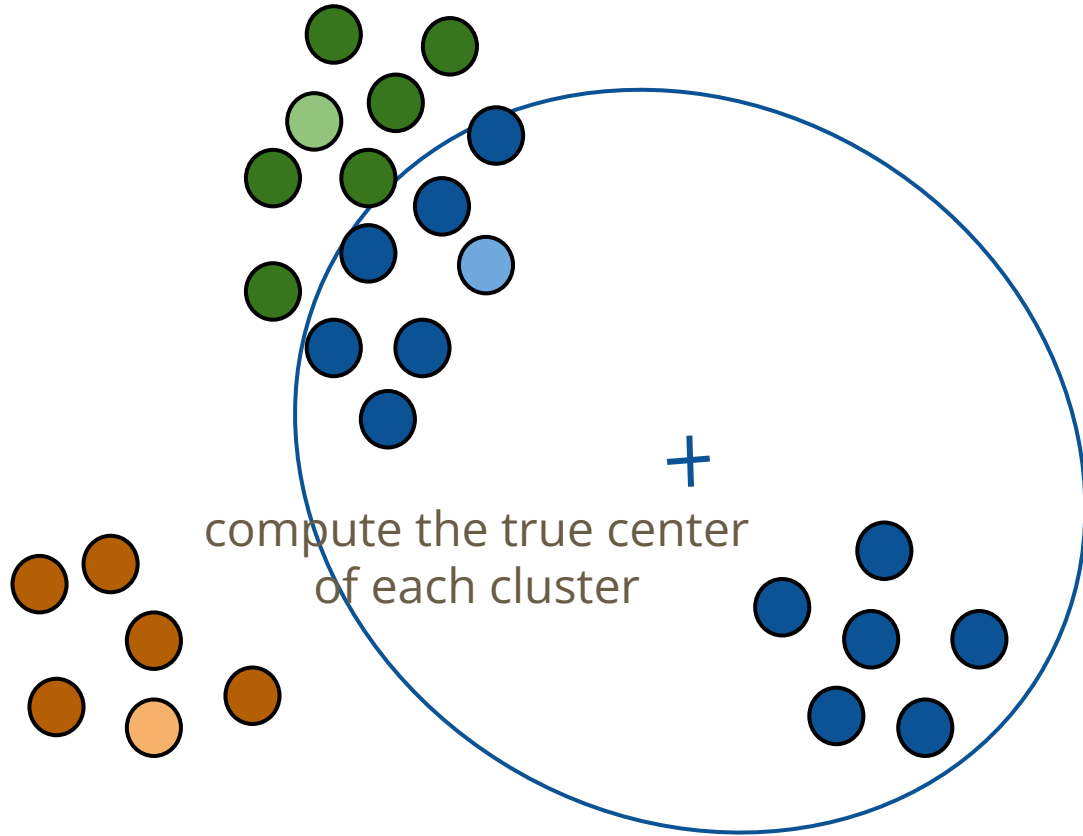


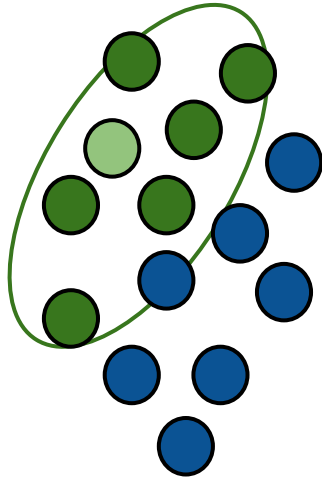


assign points to closest
center



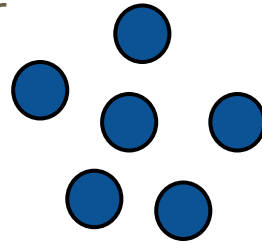
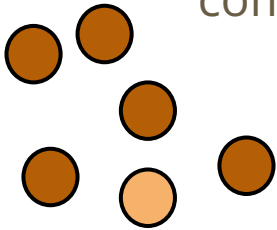


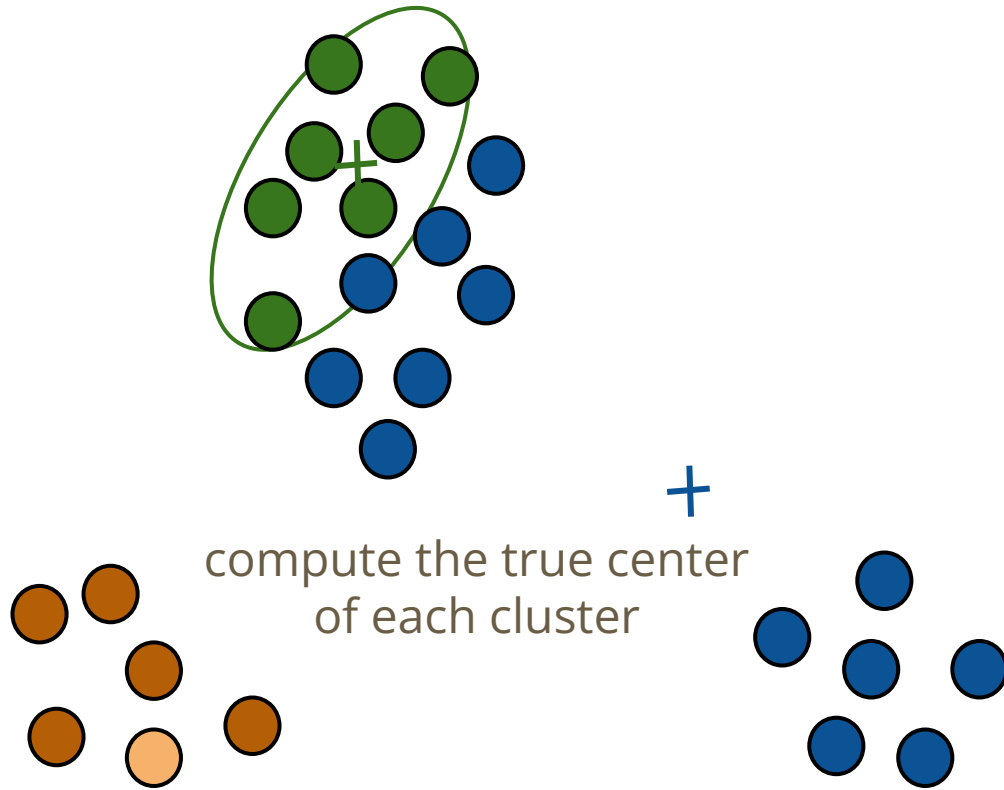


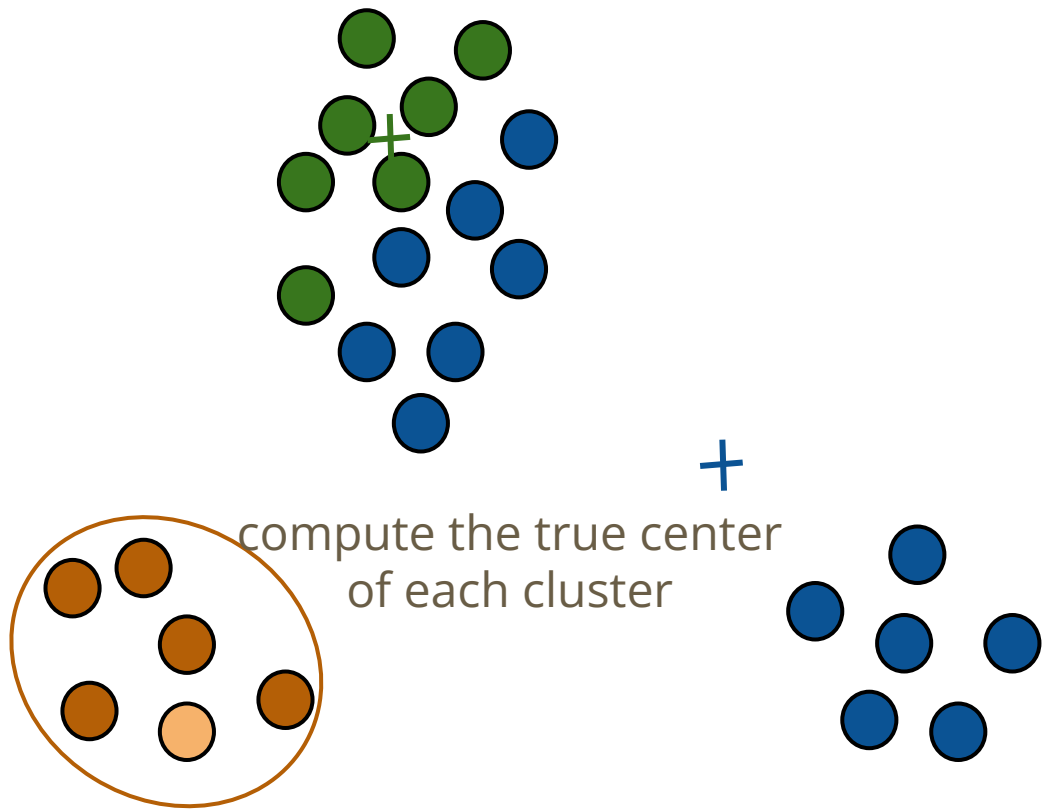


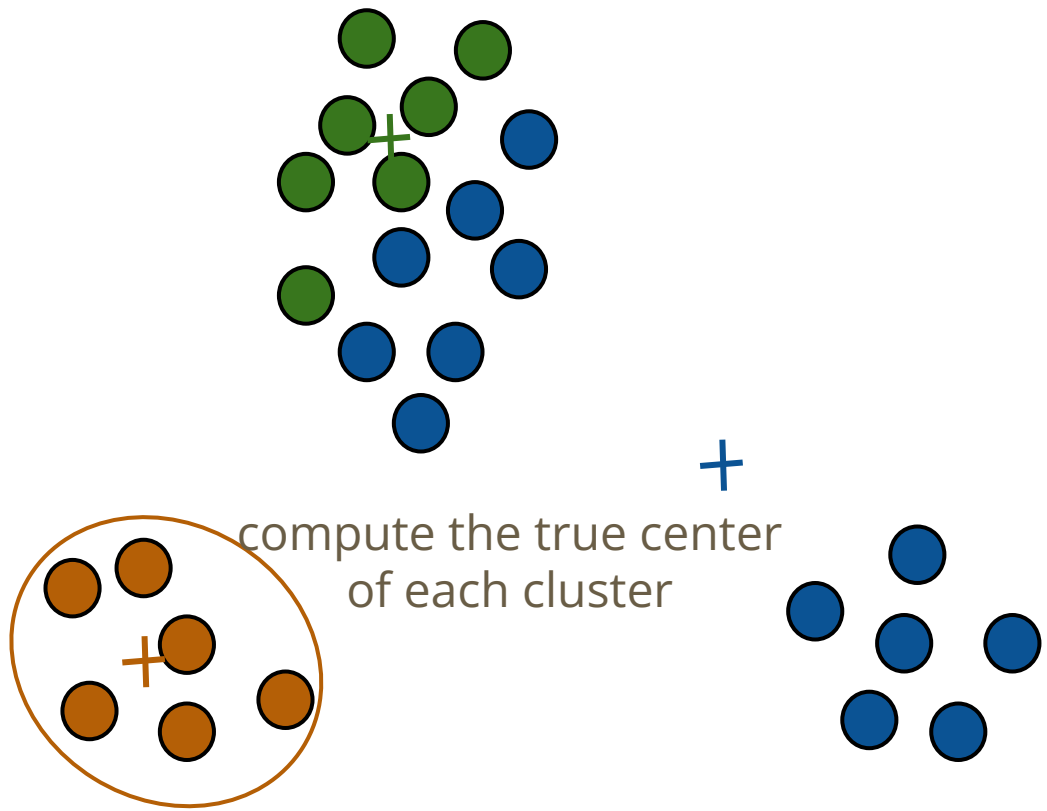
+

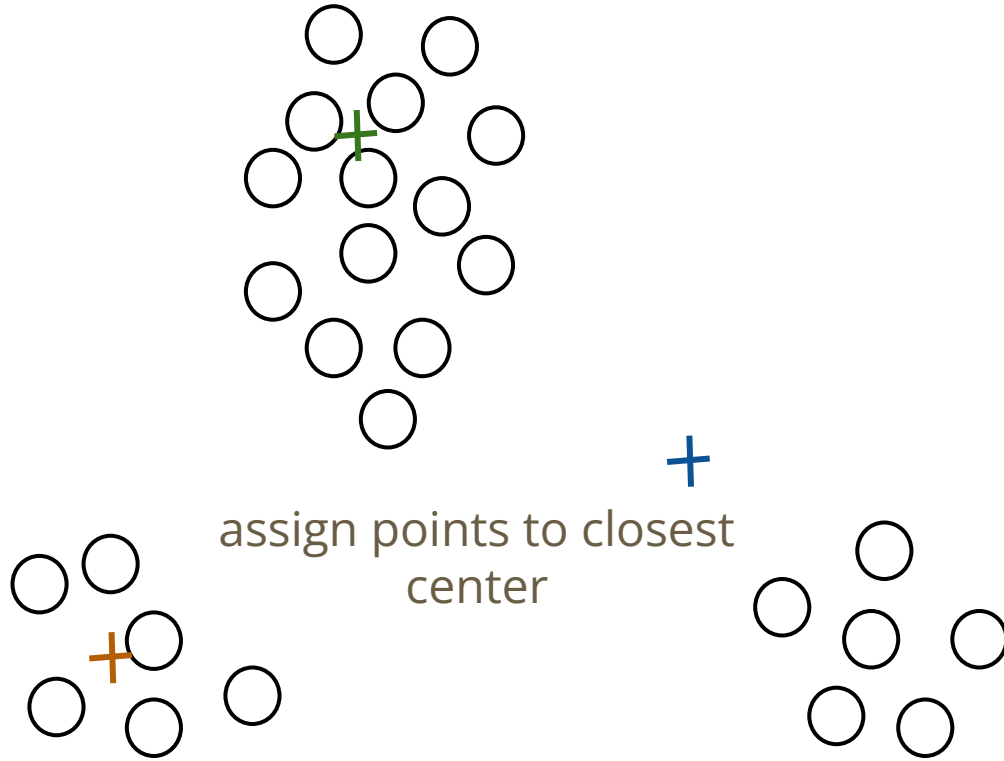
compute the true center
of each cluster

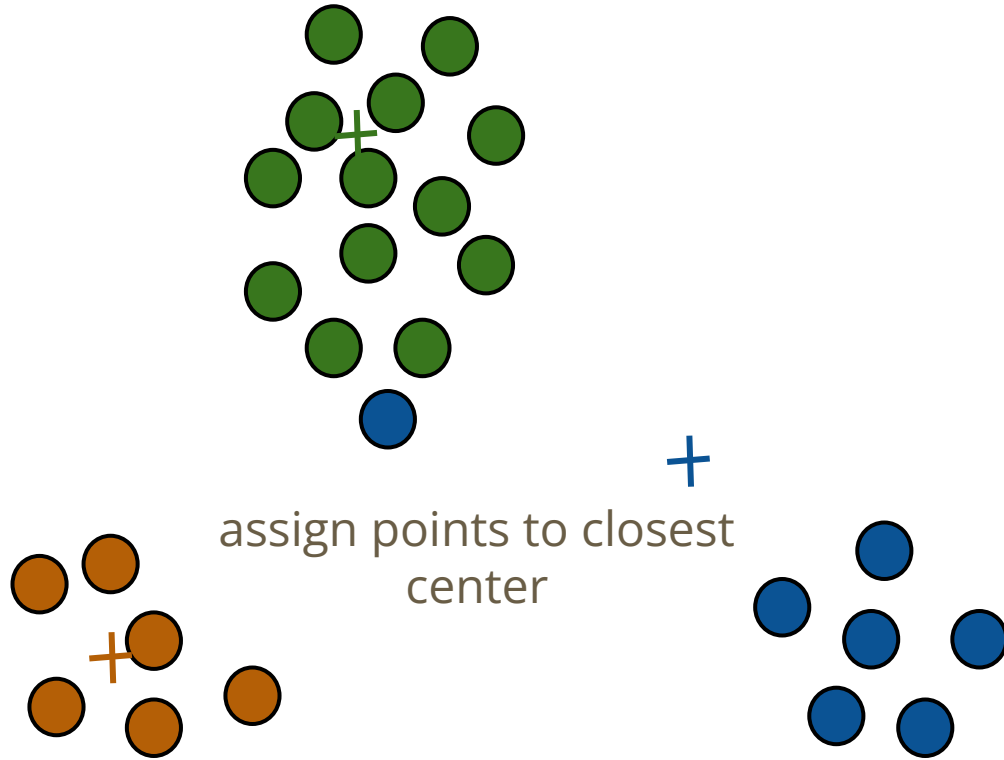


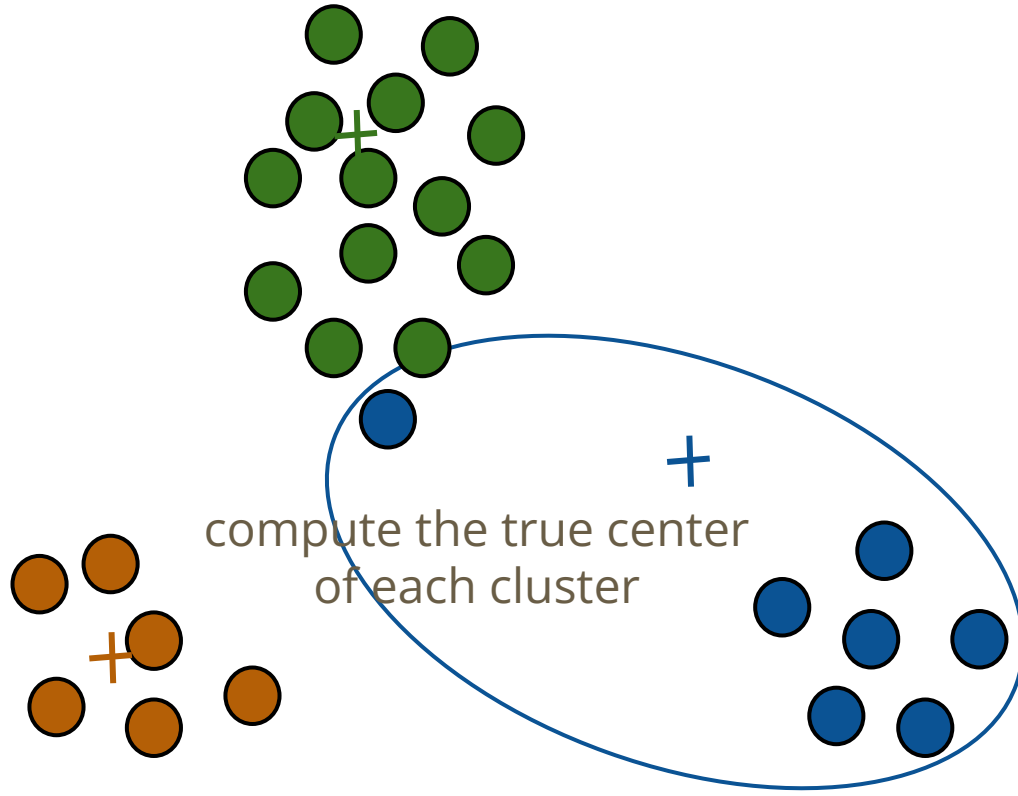


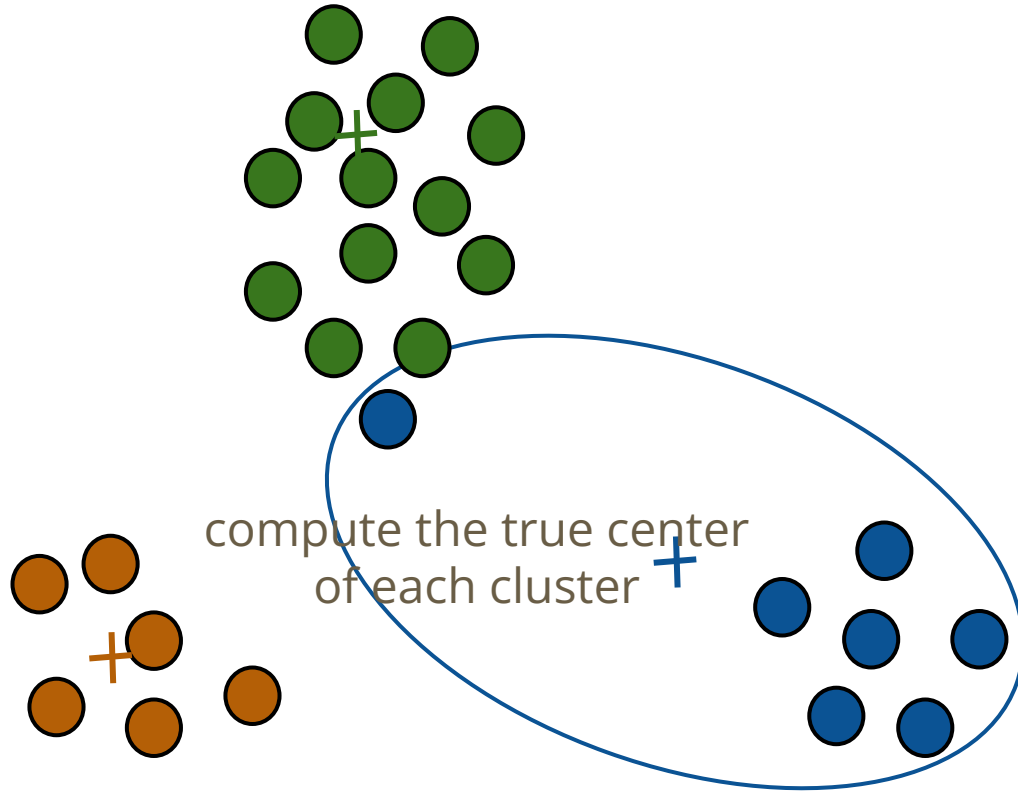


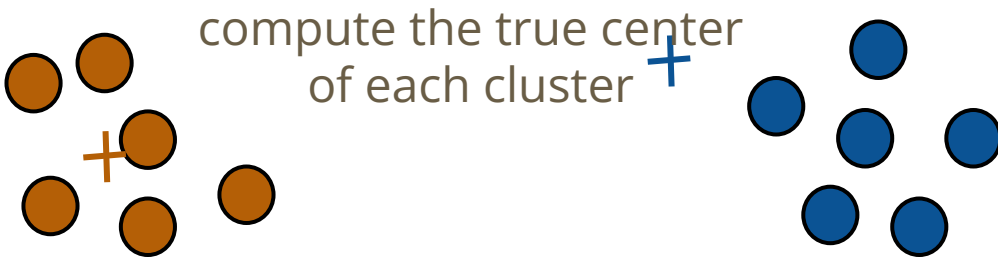
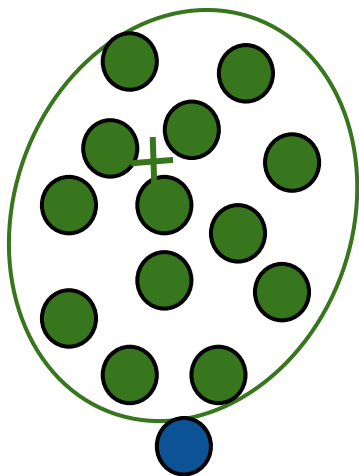


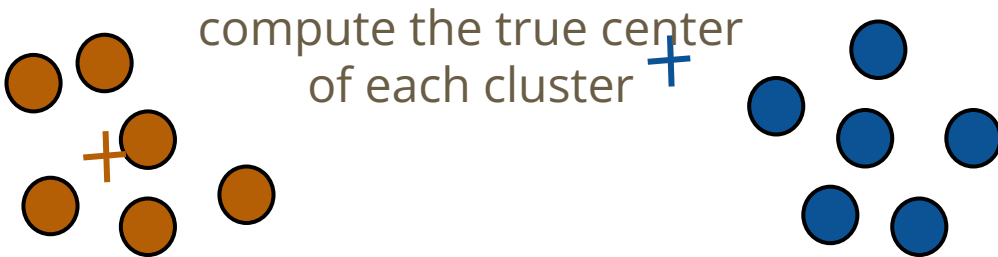
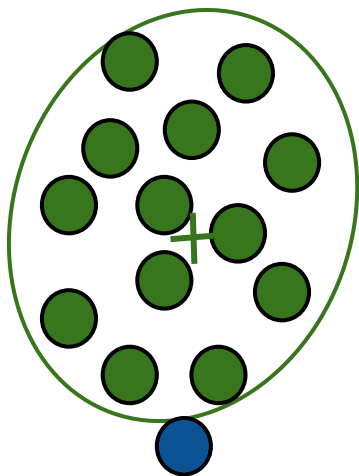


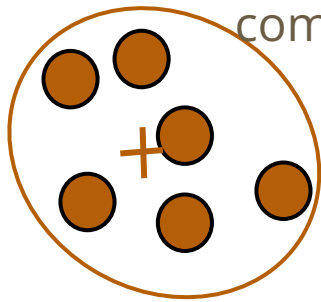
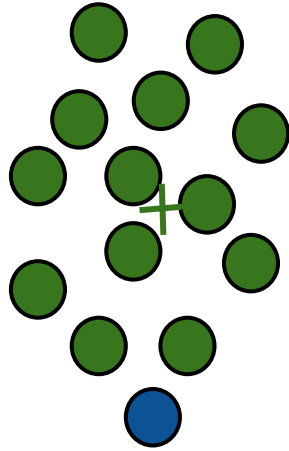




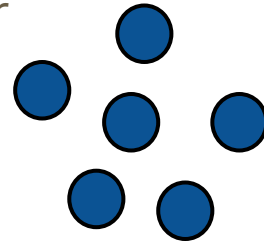


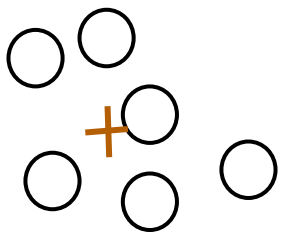
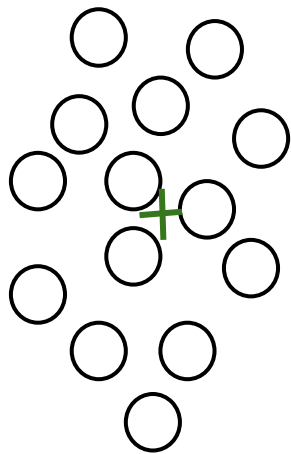




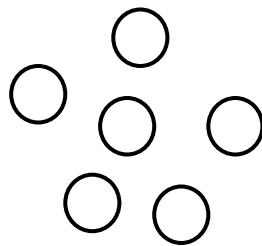


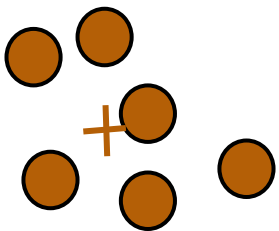
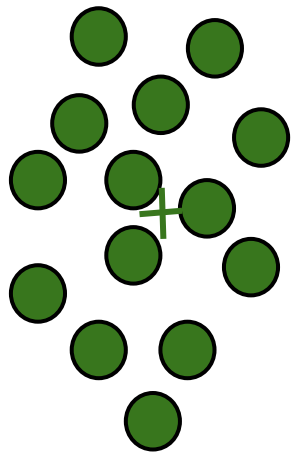
compute the true center
of each cluster +



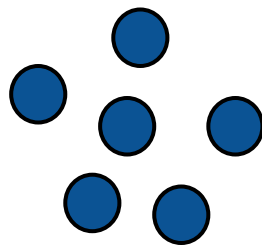


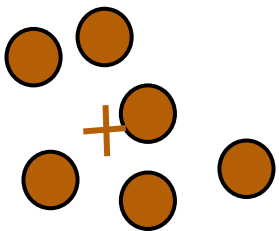
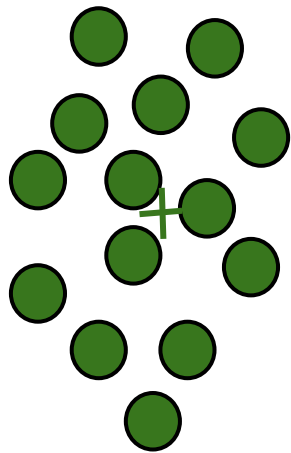
+



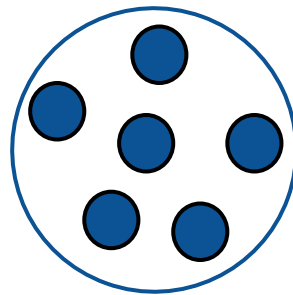


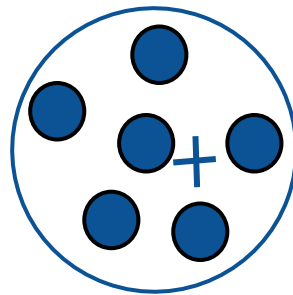
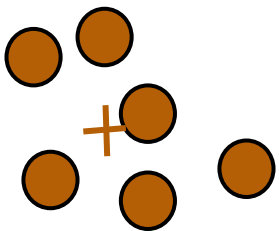
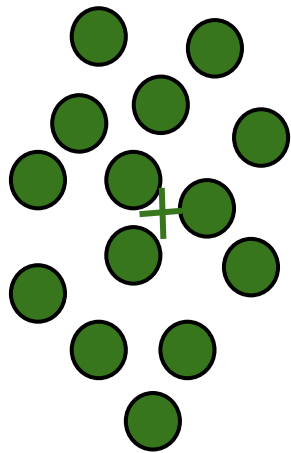
+

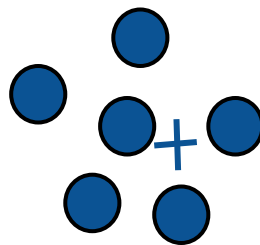
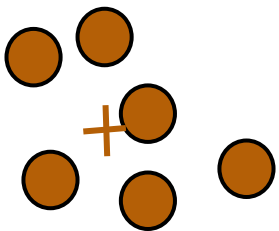
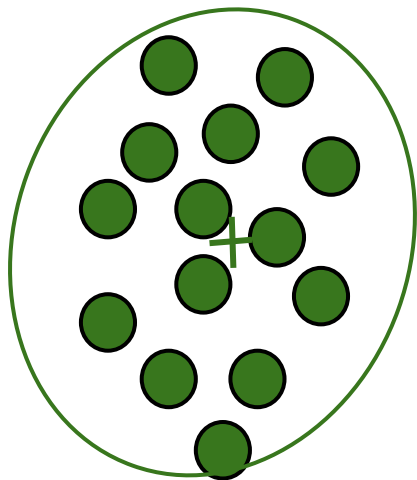


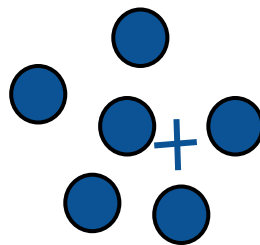
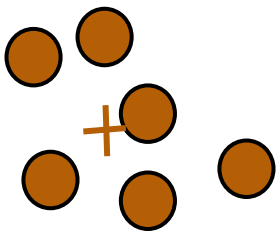
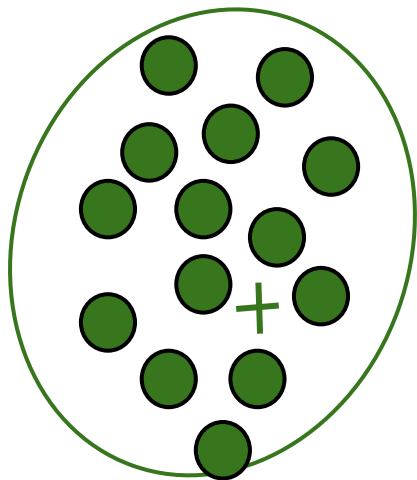


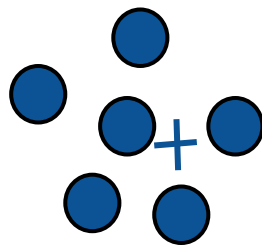
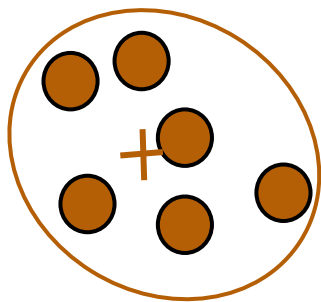
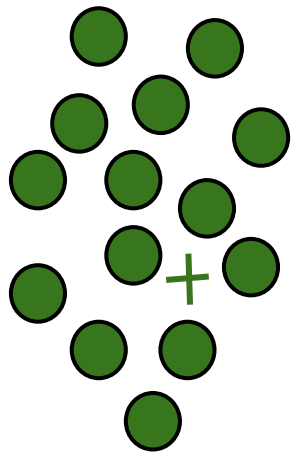
+

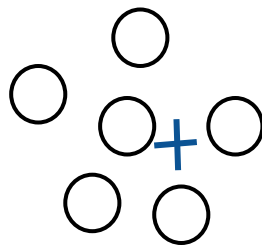
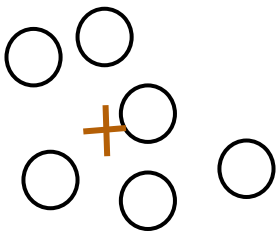
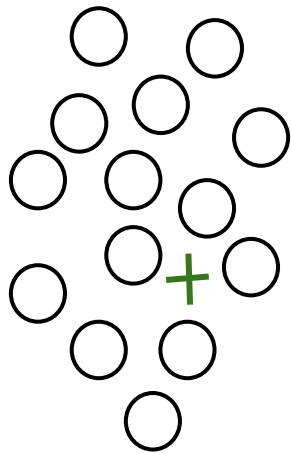


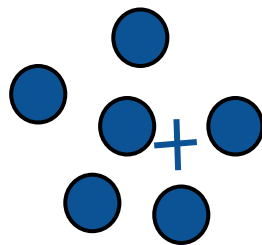
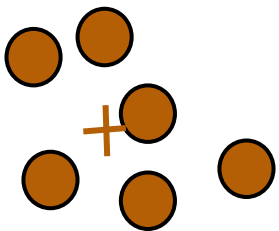
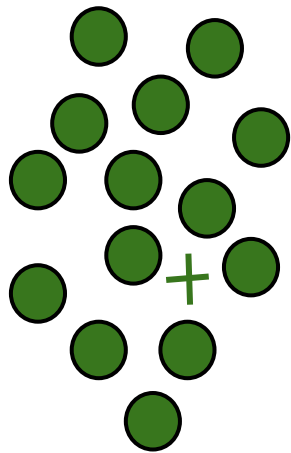


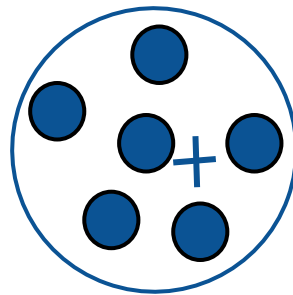
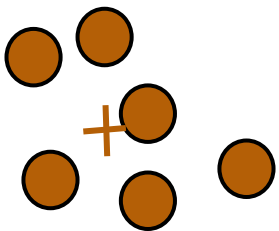
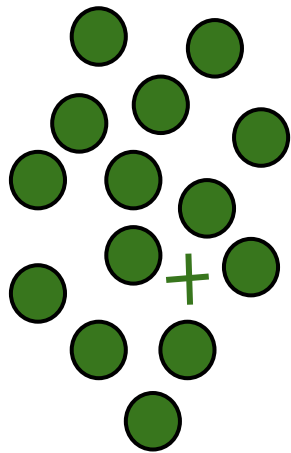


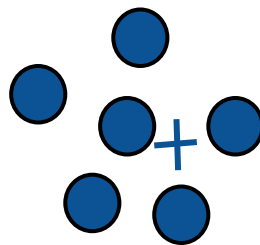
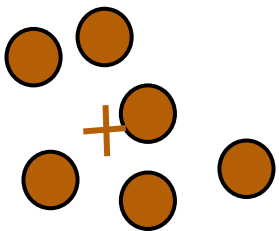
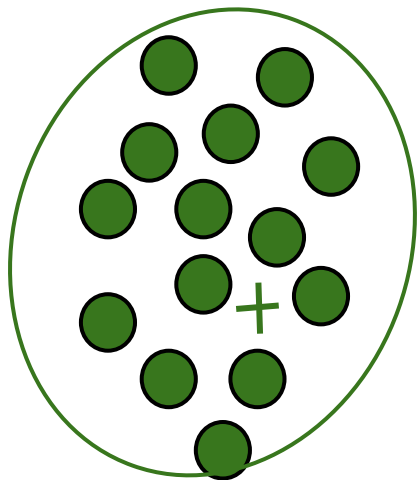


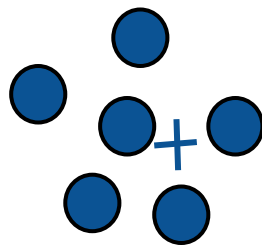
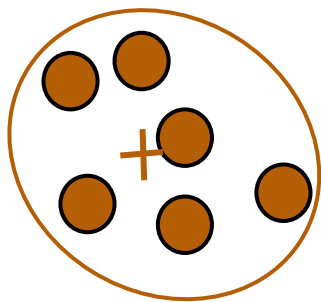
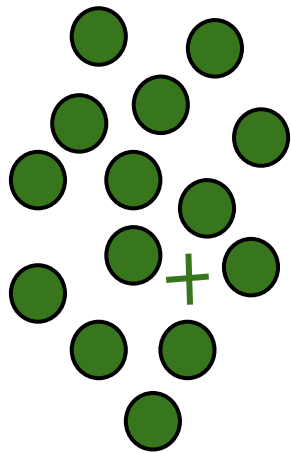


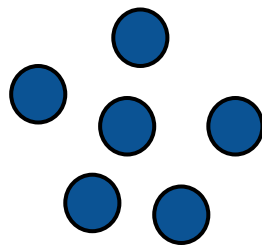
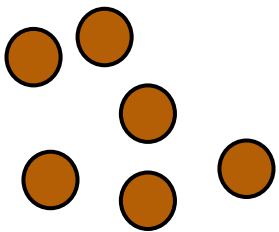
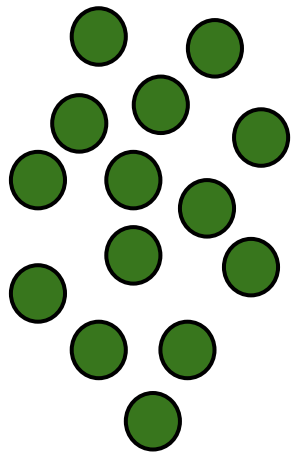






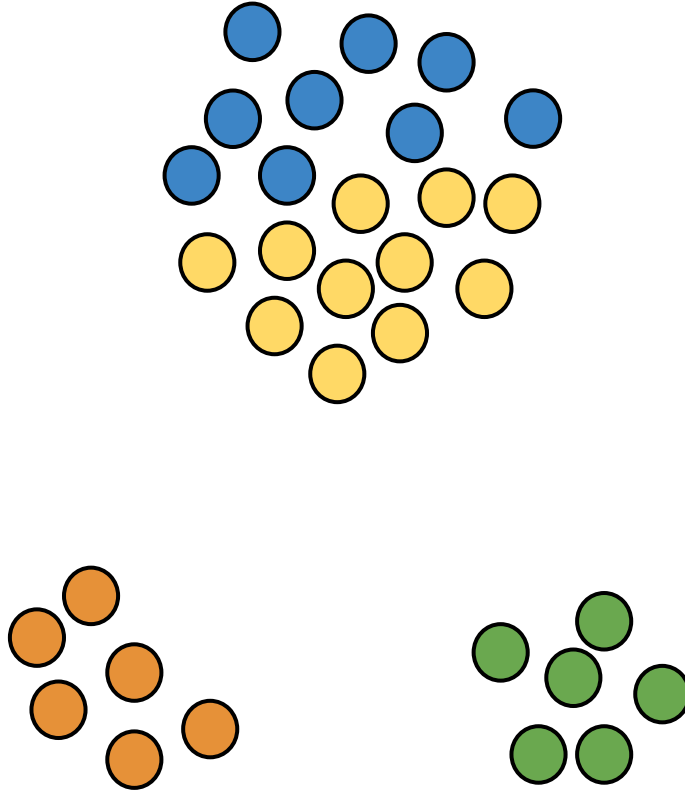




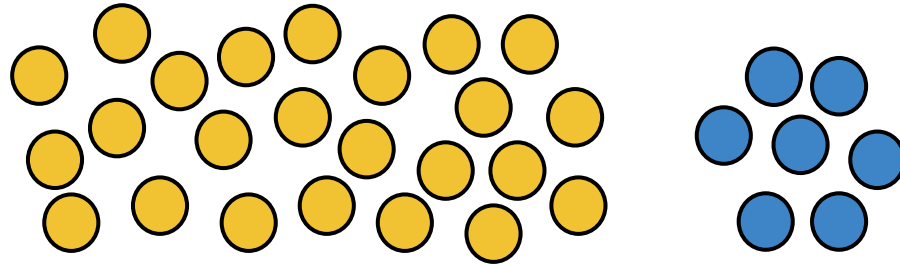


Questions

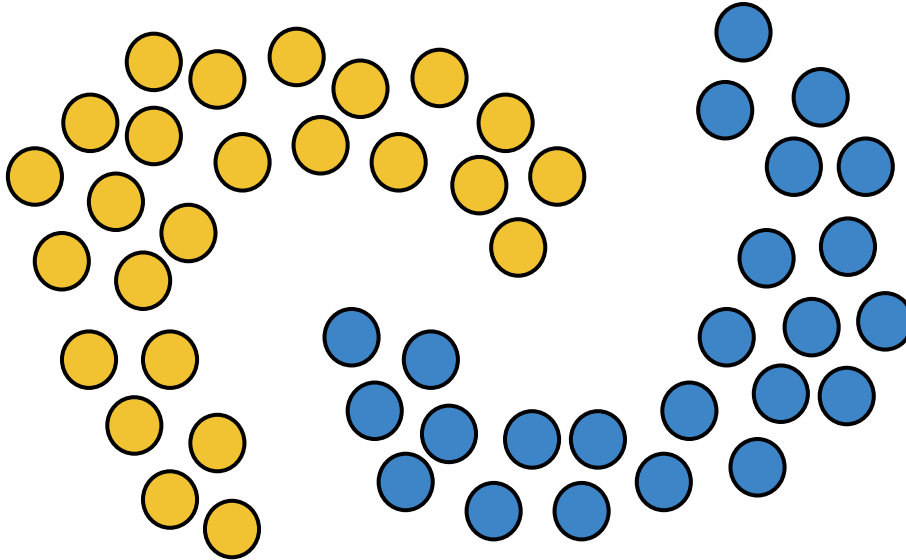
**Q1: Is this a possible
output of Kmeans?**



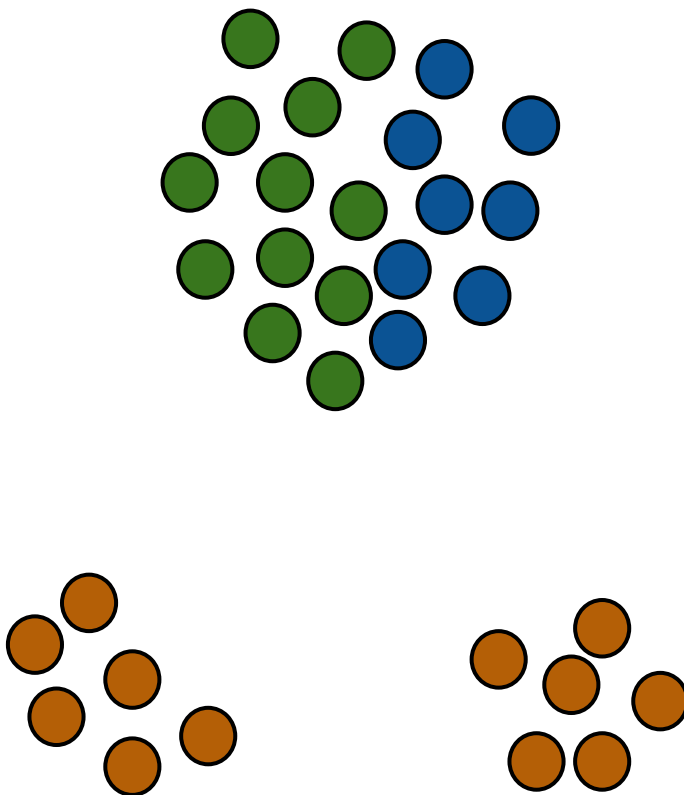
Q2: Is this a possible output of Kmeans?

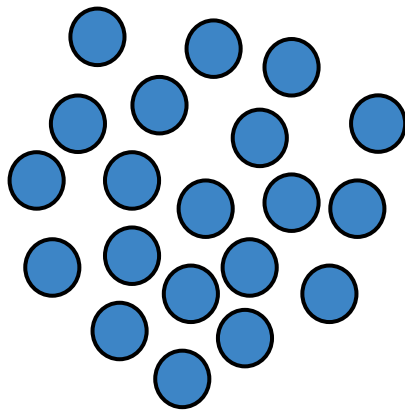


Q3: Is this a possible output of Kmeans?



**Q4: Is this a possible
output of Kmeans?**





Q5: Is this a possible output of Kmeans?

