Naive Bayes

Boston University CS 506 - Lance Galletti

Conditional Probability

Recall

$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$

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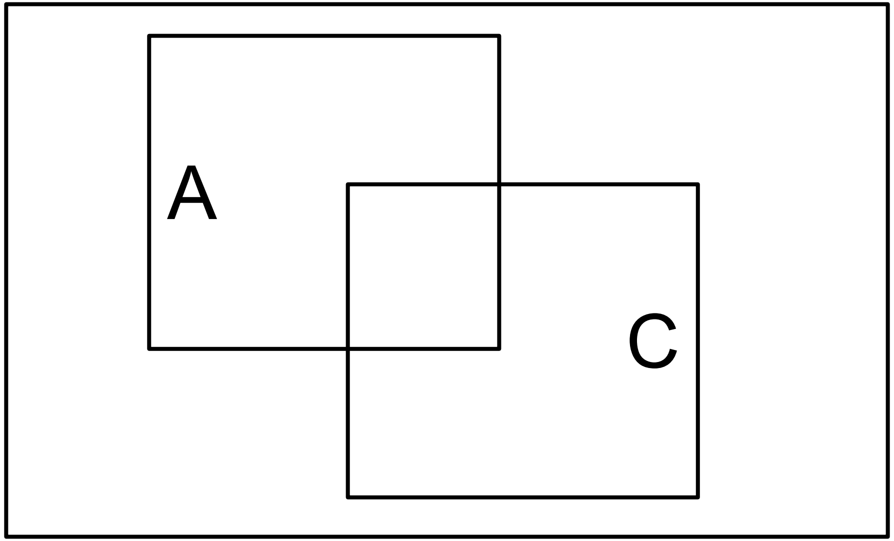

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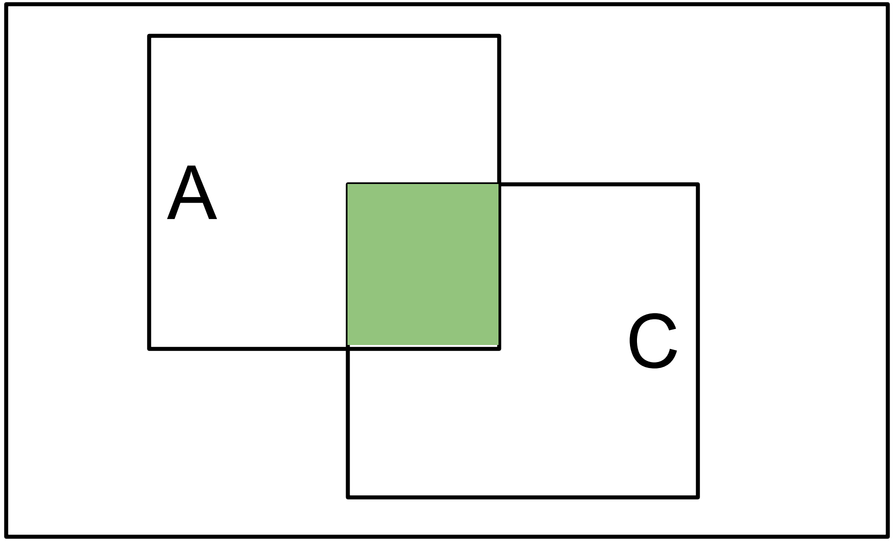

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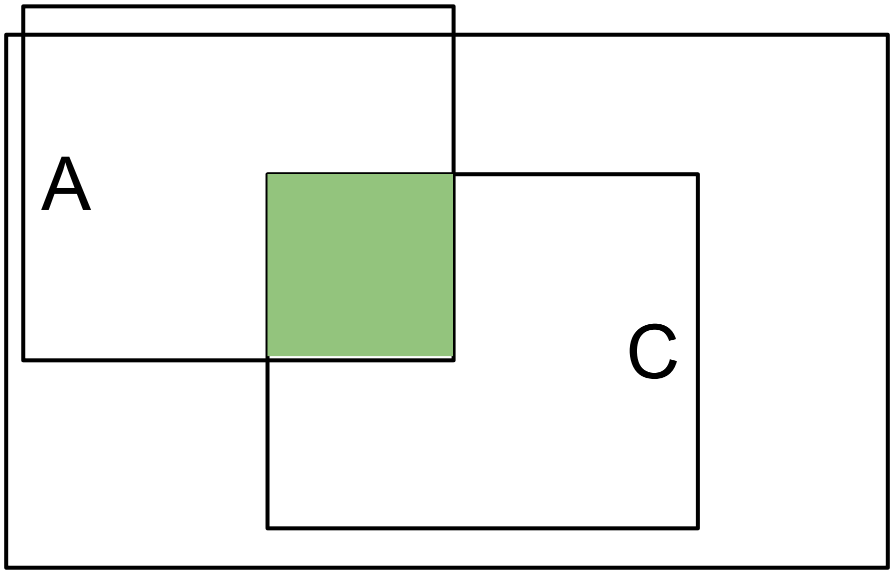

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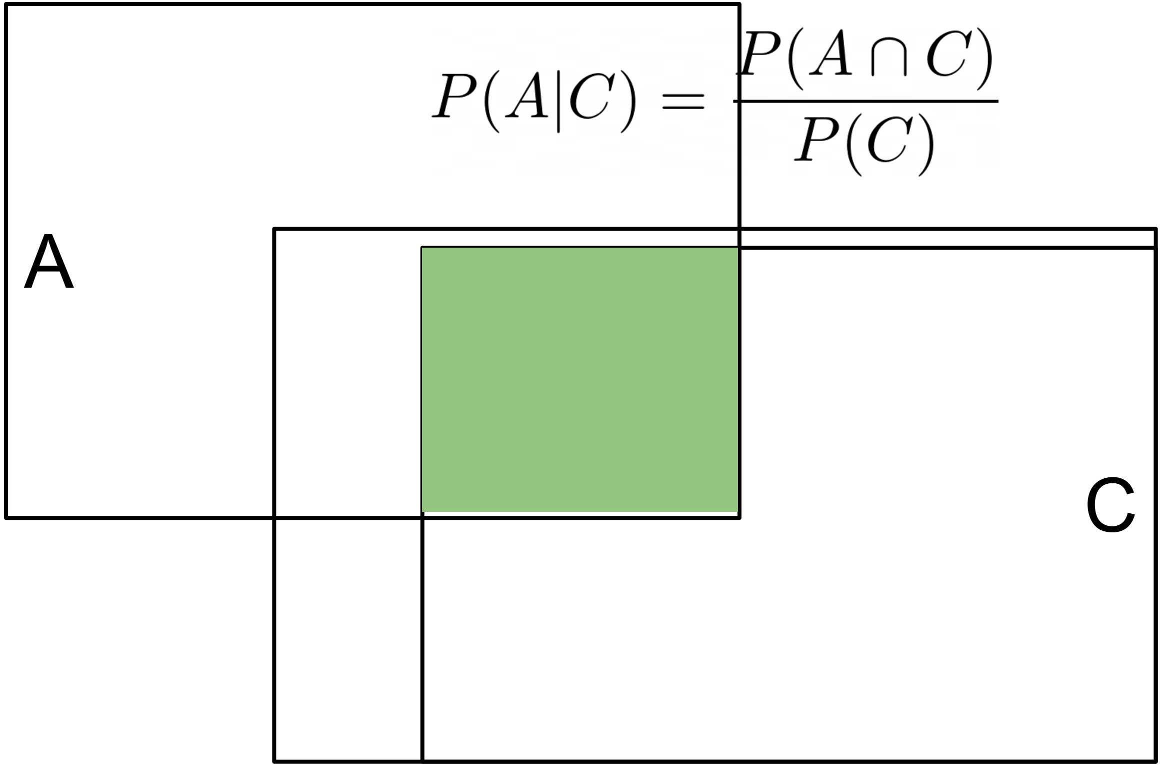

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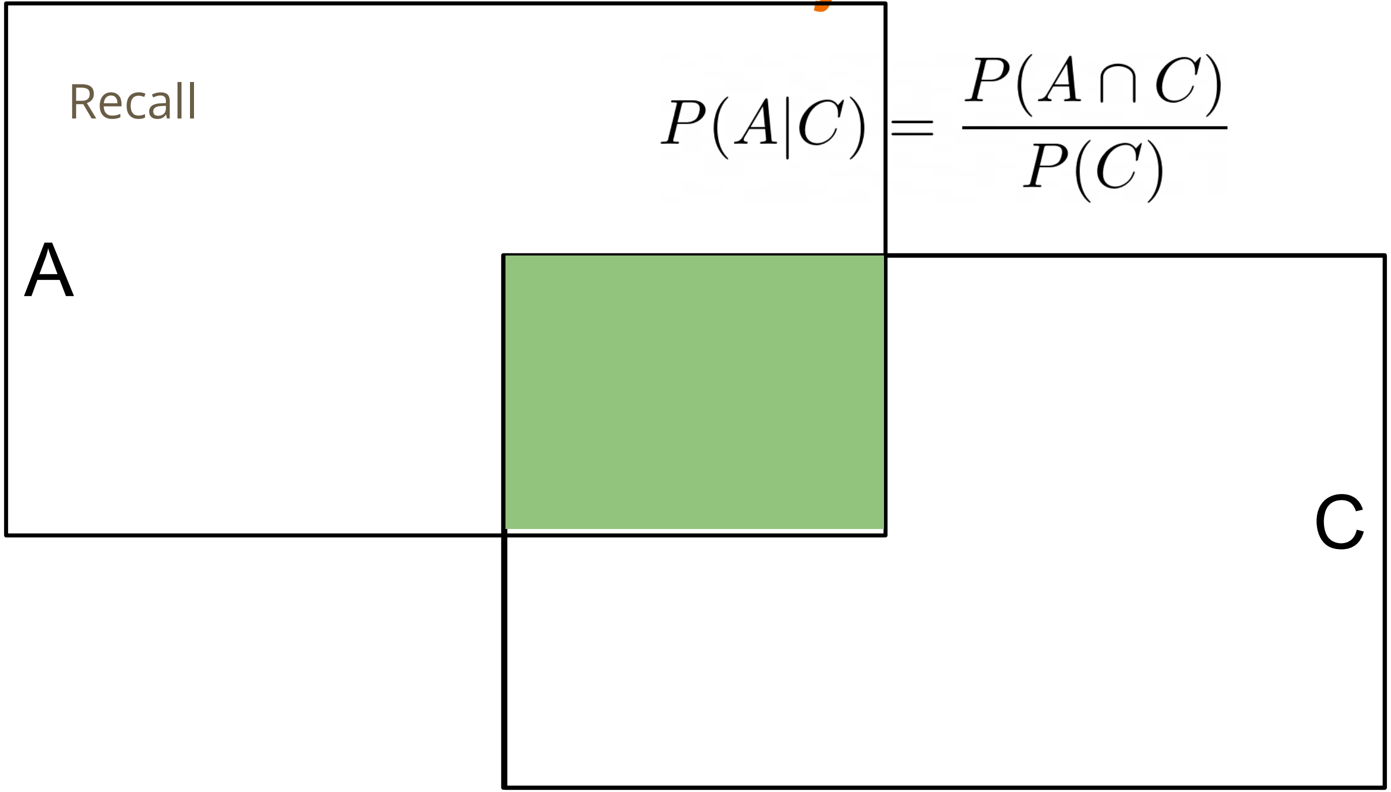

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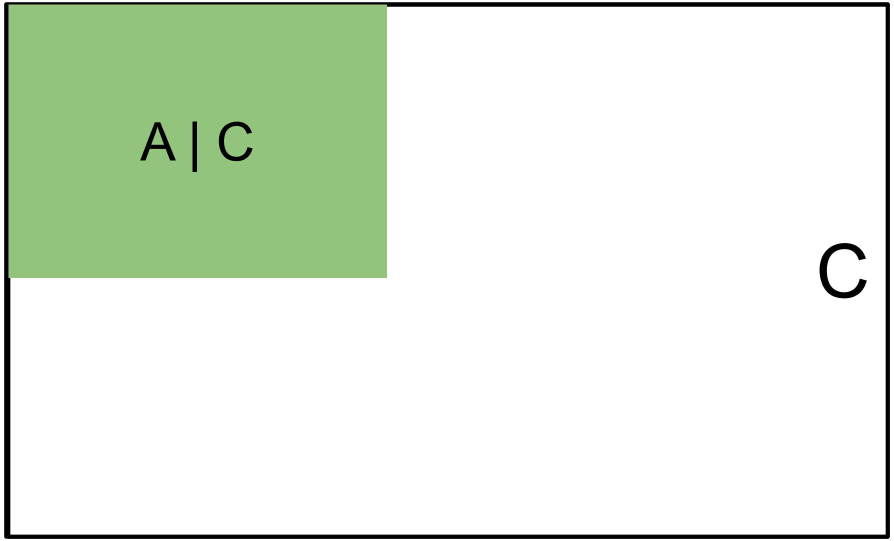

Bayes Theorem

$$P(A|C) = \frac{P(C|A)P(A)}{P(C)}$$

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Example

Given:

- Meningitis causes a stiff neck 50% of the time

If a patient has a stiff neck, what is the probability that they have meningitis?

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$$P(M|S) = \frac{P(S|M)P(M)}{P(S)}$$

Example

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- Meningitis causes a stiff neck 50% of the time

If a patient has a stiff neck, what is the probability that they have meningitis?

$$P(M|S) = \frac{P(S|M)P(M)}{P(S)}$$

Example

Given:

- Meningitis causes a stiff neck 50% of the time
- Prior probability of any patient having meningitis is 1/50,000
- Prior probability of any patient having a stiff neck is 1/20

If a patient has a stiff neck, what is the probability that they have meningitis?

$$P(M|S) = \frac{P(S|M)P(M)}{P(S)}$$

Bayesian Classifiers

Given an unknown example:

$(A_1 = a_1, A_2 = a_2, \ldots, A_m = a_m)$

Predict the class C that maximizes $P(C \mid A_1 = a_1, A_2 = a_2, \ldots, A_m = a_m)$

Bayesian Classifiers

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Example: **binary class {yes, no}**

To classify unseen record **(marital status = “married”, income = 100k)**

Bayesian Classifiers

Predict the class C that maximizes $P(C \mid A_1 = a_1, A_2 = a_2, \ldots, A_m = a_m)$

Example: binary class {yes, no}

To classify unseen record **(marital status = "married", income = 100k)**

1. Compute **P(yes | marital status = "married" and income = 100k)**
2. Compute **P(no | marital status = "married" and income = 100k)**
3. Compare and predict the class that has the highest prob given the attribute values

Bayesian Classifiers

How do we estimate $P(C \mid A_1 = a_1, A_2 = a_2, \ldots, A_m = a_m)$ from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n)$$

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$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n)$$

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

Bayesian Classifiers

How do we estimate $P(C \mid A_1 = a_1, A_2 = a_2, \dots, A_m = a_m)$ from the data?

$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

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Does not depend on C

Bayesian Classifiers

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Maximizing $P(C|A_1A_2 \dots A_n)$ is equivalent to maximizing

Bayesian Classifiers

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$$P(C|A_1 \cap A_2 \cap \cdots \cap A_n) = \frac{P(A_1 \cap A_2 \cap \cdots \cap A_n|C)P(C)}{P(A_1 \cap A_2 \cap \cdots \cap A_n)}$$

Maximizing $P(C|A_1A_2 \dots A_n)$ is equivalent to maximizing the numerator on the right

Bayesian Classifiers

So how to we estimate $P(A_1A_2\ldots A_n|C)P(C)$ from the data?

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P(C) is easy - why?

Bayesian Classifiers

So how to we estimate $P(A_1A_2 \ldots A_n|C)P(C)$ from the data?

P(C) is easy we can just count how many instances of each class we have

But $P(A_1A_2 \ldots A_n|C)$ is tricky because it requires knowing the **joint distribution** of the attributes...

Bayesian Classifiers

Can we make some assumptions about the attributes in order to simplify the problem?

Bayesian Classifiers

Assume that $A_1A_2 \ldots A_n$ are independent!

Then

$P(A_1A_2 \ldots A_n | C) = P(A_1 | C)\ P(A_2 | C) \ldots P(A_n | C)$

Can we estimate $P(A_j | C)$ from the data?

Bayesian Classifiers

Assume that $A_1A_2 \ldots A_n$ are independent!

Then

$P(A_1A_2 \ldots A_n | C) = P(A_1 | C)\ P(A_2 | C) \ldots P(A_n | C)$

Can we estimate $P(A_j | C)$ from the data?

Yes! Just count the occurrence of A_j for that class.

Example

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
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$P(C = Yes) = 3/10$

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No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)

Refund	Marital Status	Income	Class
No	Divorced	90k	Yes
No	Single	85k	Yes
No	Single	90k	Yes

P(Marital Status = “Single” | C = Yes)
= 2/3

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Married” | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Divorced	90k	Yes
No	Married	60k	No
Yes	Divorced	220k	No
No	Single	85k	Yes
No	Married	75k	No
No	Single	90k	Yes

P(Marital Status = “Married” | C = No)

Refund	Marital Status	Income	Class
Yes	Single	125k	No
No	Married	100k	No
No	Single	70k	No
Yes	Married	120k	No
No	Married	60k	No
Yes	Divorced	220k	No
No	Married	75k	No

P(Marital Status = “Married” | C = No)
= 4/7

Form

Refund	Marital Status	Income	Class
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No	Single	85k	Yes
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No	Single	90k	Yes

$P(\text{Income} = 120k \mid C = \text{No})$

Continuous Attributes

- Binning / 2-way or multi-way split
  - Create new attribute for each bin
  - Issue is that these attributes are no longer independent
- Pdf estimation
  - Assume attribute follows a particular distribution (example: normal)
  - Use data to estimate the parameters of the distribution

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Assume normal distribution

$P(\text{Income} = 120\text{k} \mid C = \text{No})$

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Assume normal distribution

$P(\text{Income} = 120k \mid C = \text{No})$

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Assume normal distribution

P(Income = 120k | C = No)

Sample mean = 110

Sample variance = 2975

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Assume normal distribution

P(Income = 120k | C = No)

Sample mean = 110

Sample variance = 2975

$$P(Income = 120|No) = \frac{1}{\sqrt{2\pi}(54.54)} e^{-\frac{(120-110)^2}{2(2975)}} = .0072$$

Putting it all together

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Test Record:
X = (Refund = No, Married, Income = 120k)

- P(X | No) =

- P(X | Yes) =

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Test Record:
X = (Refund = No, Married, Income = 120k)

- P(X | No) = P(Refund = No | No) P(Married | No) P(Income=120k | No) = 4/7 * 4/7 * .0072 = .0024
- P(X | Yes) =

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- P(X | Yes) = P(Refund = No | Yes) P(Married | Yes) P(Income=120k |Yes) = 1 * 0 * 1.2 * 10^{-9} = 0

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- P(X | Yes) = P(Refund = No | Yes) P(Married | Yes) P(Income=120k |Yes) = 1 * 0 * 1.2 * 10^{-9} = 0

Since P(X|No)P(No) > P(X|Yes)P(Yes)
=> predict No

Limitation

If one of the conditional probabilities is zero, the entire expression becomes zero...

Original estimate of $P(A_i \mid C) = N_{ic} / N_c$

Laplace estimate : $P(A_i \mid C) = (N_{ic} + 1) / (N_c + \text{constant})$

Question

Can you use Naive Bayes to predict class C based on the following two features:

1. Weight
2. Height