Convex Hulls and Jensen's Inequality

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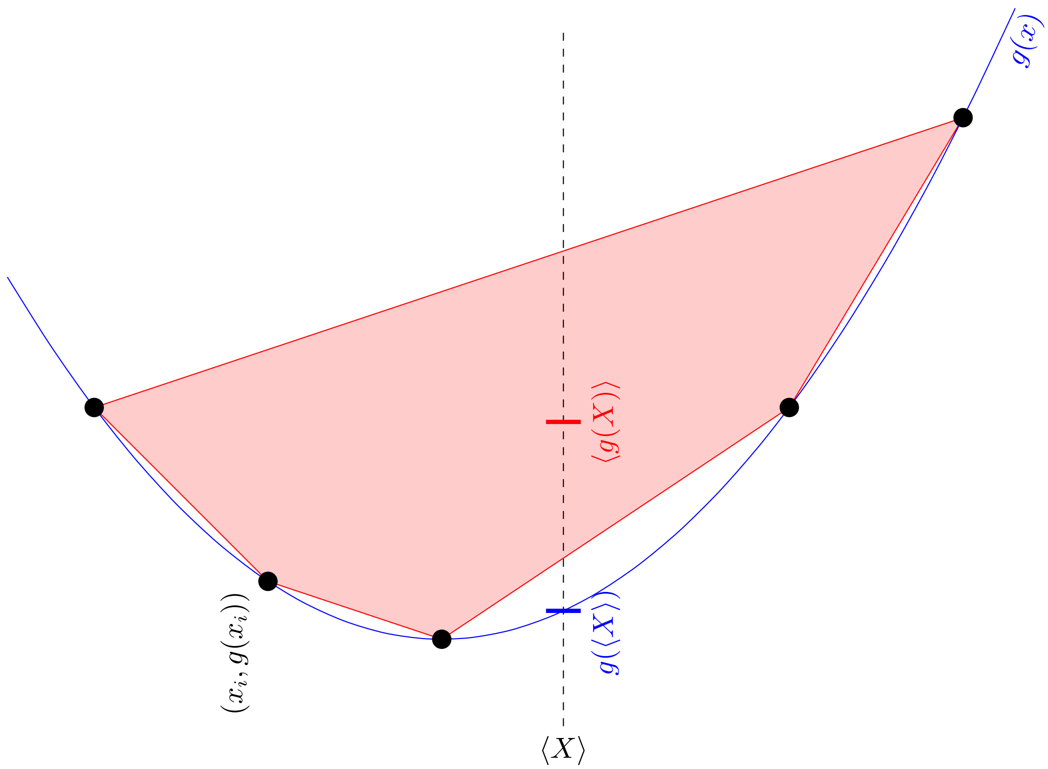
Let X be a random variable and $\langle X \rangle$ its expected value. In the case where X is equally likely to be any of the values $x_1, \ldots, x_n$, $\langle X \rangle$ is simply the arithmetic mean $\frac{1}{n}\sum_{i=1}^{n} x_i$.

Jensen's inequality says that $g(\langle X \rangle) \le \langle g(X) \rangle$ for any convex (i.e., concave up) function g. Taking $g(x) = x^2$, one obtains the inequality between the arithmetic mean and the root mean square [**1**].



Jensen's inequality says that $g(\langle X \rangle) \ge \langle g(X) \rangle$ for any concave (i.e., concave down) function g. Taking $g(x) = \log x$, one obtains the AM-GM inequality [**1**].

References

1. S. Kung, Harmonic, geometric, arithmetic, root mean inequality, *College Mathematics Journal* **21** (1990) 227.