ErinAhernHW21

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A particle moves in a periodic box of length $L = 10$. Consider a position-space wavefunction $\psi(x)$ which is unity in the interval $x_1 = 2 \leq x \leq x_2 = 3$ and zero for all other values of x. Approximate this wave function as a Fourier series with $2\ell + 1$ terms with $-\ell \leq m \leq \ell$ and plot the result for ℓ =10, 20, 40, 80, 100 and 200.

[15]:
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import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import quad
# assign constants
L=20 # size of region in which particle moves.
x2=4 # upper limit of psi(x)
x1=2 # lower limit of psi(x)
l_max=200 #(-l \le m \le +l) largest value of l used below
Dx = 0.01 # Accuracy for plotting position
Nx = int(L/Dx)

psi_tilde = np.zeros((2*l_max+1),dtype=complex)
# Calculate psi_tilde for many values of m, not all of which must be used.

#partA

x = 1
psi_og = 1 - np.cos(2*np.pi*(x-x1)/(x2-x1))
psi_squ = lambda x: (1 - np.cos(2*np.pi*(x-x1)/(x2-x1)))**2
prob_tot = quad(psi_squ, x1, x2)[0]

N = (1 /prob_tot)**.5
print("a)")
print("N = " + str(round(N,4)))


#partB
for x in range (0, 2):
    for m in range(-l_max,l_max+1):
        fr = lambda x: N*(1 - np.cos(2*np.pi*(x-x1)/(x2-x1)))*np.cos(2*np.
↪pi*m*x/L) # Real part of given wave function multiplied by inversse Fourier␣
↪factor.
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        fi = lambda x: N*(1 - np.cos(2*np.pi*(x-x1)/(x2-x1)))*np.sin(2*np.
↪pi*m*x/L)  # Imaginary part of given wave function multiplied by inversse␣
↪Fourier factor.
        Ir = quad(fr, x1, x2)[0] # Perform the inverse Fourier transform␣
↪integral for the real part
        Ii = -quad(fi, x1, x2)[0]  # Perform the inverse Fourier transform␣
↪integral for the imaginary part
        psi_tilde[m] = (Ir+1j*Ii)/L**0.5

# Reconstruct psi
psi = np.zeros((Nx+1),dtype=complex) # array to hold the complex value of psi(x)
psi_mag = np.zeros((Nx+1)) # array to hold the magnitude of psi(x)
x = np.zeros((Nx+1)) # array to hold the values of the position used for␣
↪plotting
def wf(l): # define a function that reconstructs the wave function from 2l+1␣
↪terms in the Fourier series.
    for n in range(0,Nx+1):
        x[n]=n*Dx
        psi[n] = 0+0j
        for m in range(-l,l+1):
            psi[n] = psi[n]+psi_tilde[m]*np.e**(+2j*np.pi*(m-l)*x[n]/L)/L**0.5
        psi_mag[n] = (psi.real[n]**2+psi.imag[n]**2)**0.5
    return psi_mag
MPSI = np.zeros((Nx+1))
fig = plt.figure(figsize=(10,4))
ax = fig.add_subplot(1,1,1)
case=0 # Use this integer below to shift the position of each example to make␣
↪them easier to compare.
MPSI = wf(100)
print("b)")
ax.plot(x,MPSI,'g-') #Add a curve described by the arrays x and psi.real to␣
↪Plot, choose a red solid curve.
plt.show()

#partC
psi_complex = np.zeros((Nx+1),dtype=complex)
psi_magnitude = np.zeros((Nx+1))
position = np.zeros((Nx+1))
def wf(l):# function using 2l+1 terms in the Fourier series
    fig = plt.figure(figsize=(10,4))
    ax = fig.add_subplot(1,1,1)
    for t in range(0,7) :
        for n in range(0,Nx+1):
            x[n]=n*Dx
            psi[n] = 0+0j
            for m in range(-l,l+1):
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                psi[n] = psi[n]+psi_tilde[m]*np.e**(+2j*np.pi*(m)*x[n]/L)/L**0.
↪5*np.e**((-1j*t*.2)*((4*((np.pi)**2)*((m)**2))/(L**2))/2)
            psi_mag[n] = (psi.real[n]**2+psi.imag[n]**2)**0.5
        ax.plot(x,psi_mag)
print("c)")

wf(100)
plt.show()

#partD

L=80 # region size
x2=4 # upper limit
x1=2 # lower limit
l_max=100
Dx = 0.01
Nx = int(L/Dx)
psi_tilde = np.zeros((2*l_max+1),dtype=complex)
for m in range(-l_max,l_max+1):
    fr = lambda x: N*(1 - np.cos(2*np.pi*(x-x1)/(x2-x1)))*np.cos(2*np.pi*m*x/L)␣
↪# Real part of given wave function multiplied by inversse Fourier factor.
    fi = lambda x: N*(1 - np.cos(2*np.pi*(x-x1)/(x2-x1)))*np.sin(2*np.pi*m*x/L)␣
↪# Imaginary part of given wave function multiplied by inversse Fourier␣
↪factor.
    Ir = quad(fr, x1, x2)[0] # Perform the inverse Fourier transform integral␣
↪for the real part
    Ii = -quad(fi, x1, x2)[0]  # Perform the inverse Fourier transform integral␣
↪for the imaginary part
    psi_tilde[m] = (Ir+1j*Ii)/L**0.5

psi = np.zeros((Nx+1),dtype=complex)
psi_magnitude = np.zeros((Nx+1))
x = np.zeros((Nx+1))
def wf(l):
    fig = plt.figure(figsize=(10,4))
    ax = fig.add_subplot(1,1,1)
    for t in range(0,7) :
        for n in range(0,Nx+1):
            x[n]=n*Dx
            psi[n] = 0+0j
            for m in range(-l,l+1):
                psi[n] = psi[n]+psi_tilde[m]*np.e**(+2j*np.pi*(m)*x[n]/L)/L**0.
↪5*np.e**((-1j*t*.2)*(((4*((np.pi)**2)*((m)**2))/(L**2))/2+120*((2*np.pi*m)/
↪(L))/2+3600))
            psi_magnitude[n] = (psi.real[n]**2+psi.imag[n]**2)**0.5
        ax.plot(x,psi_magnitude)
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```
print("d)")

wf(100)
plt.show()
```

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a)
N = 0.5774
b)
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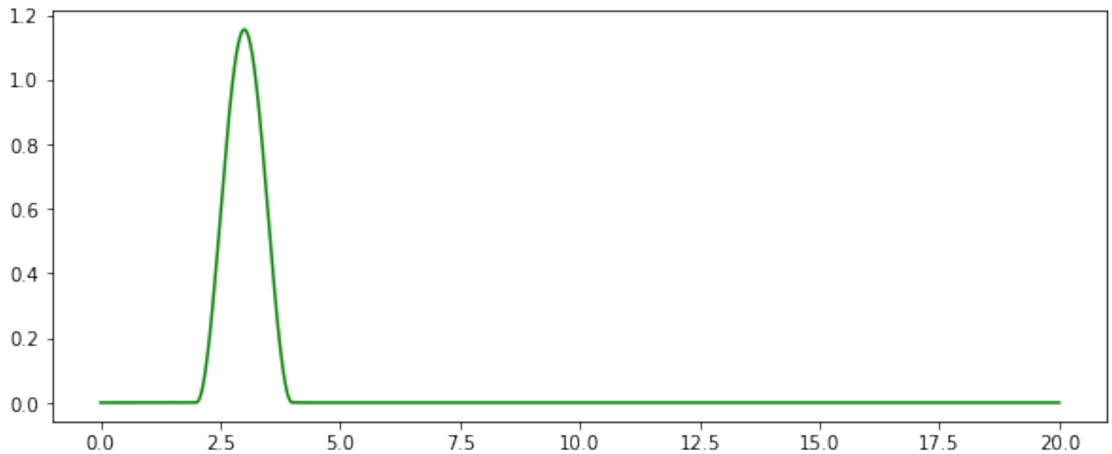


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c)
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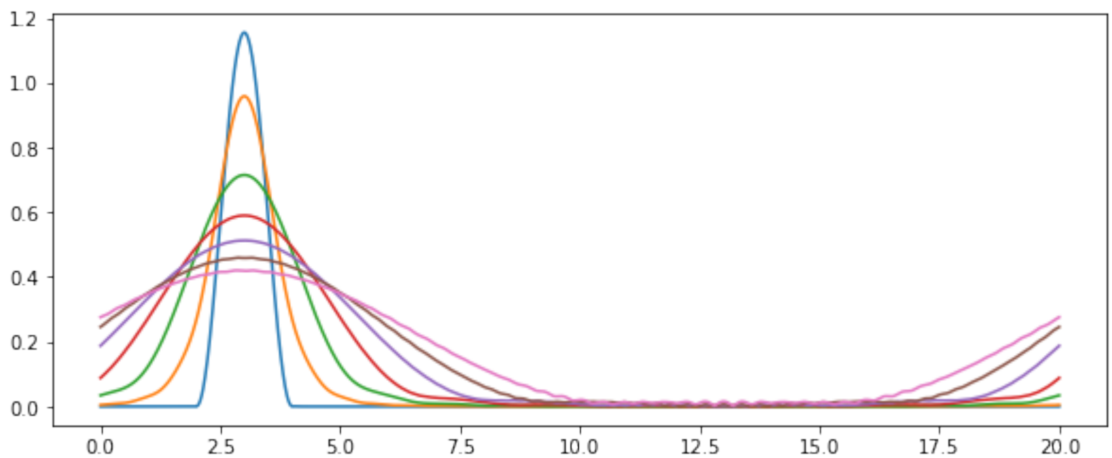


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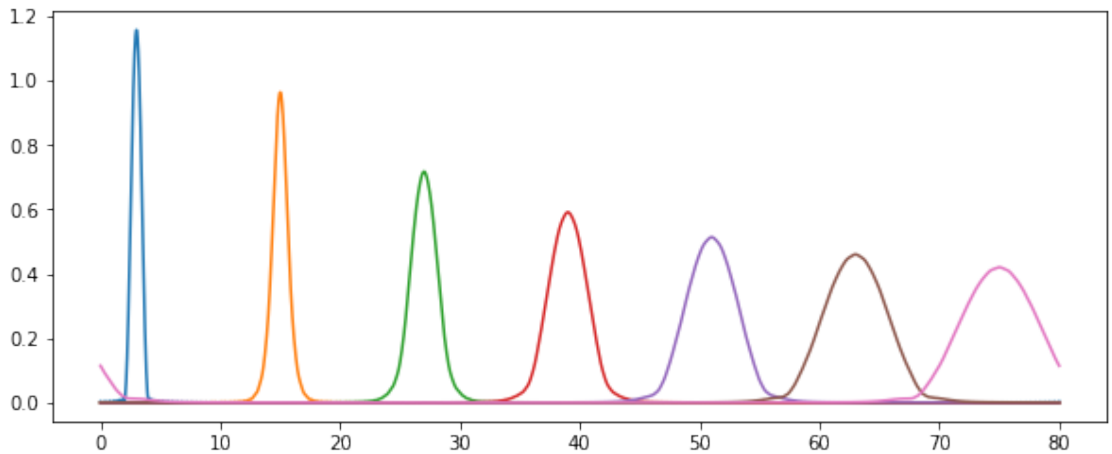
d)
```


1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
60
70
80

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[ ]:
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