Against Gold: An Analysis

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1 Introduction and Motivation

Since my childhood, human behavior has intrigued me. Throughout my upbringing, I have explored various disciplines, each expanding my horizons in incomparable ways. However, Daniel Kahneman's book "Thinking, Fast and Slow" was a game-changer. The studies in the book showed that people behave irrationally, and Kahneman presented these findings in a concrete manner. Concepts like System 1, System 2, and the Peak-End Rule resonated with my biological background. After discovering this book, my interest rapidly shifted toward economics. I loved it. This field was both a social science and allowed for numerical analysis. My passion grew so much that during my undergraduate studies, pursuing a master's degree in economics became one of my dreams.

After starting my master's degree in Data Science, I continued to develop myself in finance. I read numerous finance books. If you ask me what I learned from all these books, I would say that the phrase "We know nothing" has far deeper meanings than it appears, and that the best investment strategy is to buy an index fund and hold it for as long as possible. The first lesson is not the focus of this paper, so I will leave that for another discussion. The second lesson, regarding stock market investments, forms the basis of this study because it contradicts our traditional investment method of buying gold and storing it under the mattress. People around me generally avoid stock market investments, thinking of them as gambling. Gold, on the other hand, is a safe haven—it never loses its value.

But how true is this belief? Does our cultural wisdom hold, or are the books on investment correct? In this study, I aimed to analyze this question. To make my study more comprehensive, I also examined whether the buy-sell strategy is more advantageous than the buy-hold strategy. The data used primarily consists of the S&P 500 index [1][2] and gold prices [3] from 1950 to 2023. Additionally, I utilized data such as the Consumer Price Index (CPI) [4], SPY ETF prices [5], and S&P 500 dividend yield [6] for further analysis. For buy-sell simulations, I considered the "Exponential Moving Average (EMA) Crossover Strategy." Lastly, those interested in reviewing the code used during my research can access it via THIS link.

My research questions (RQ) are as follows:

- **RQ1:** Are the annual returns of gold and stock investments statistically different?
- **RQ2:** Does the purchase price matter in 20-year investments?
- **RQ3:** Are the average returns of investment instruments over a 20-year period different?
- **RQ4:** Does the buy-sell strategy yield more profit than the buy-hold strategy?

1.1 Datasets

In this study, price records of gold and the S&P 500 index from 1950 to 2023, price records of the SPY ETF since its inception, annual S&P 500 dividend yield, and annual CPI values were used. Gold, SPY, and S&P 500 prices are available in four different forms: opening, closing, highest, and lowest (Table 1). Although there are small differences between them, only the closing price was used in calculations to eliminate the effect of intraday price fluctuations.

Table 1: S&P 500 Prices

Date	Closing	Opening	Highest	Lowest
04/25/2024	5048.42	5019.88	5057.75	4990.58
04/24/2024	5071.63	5084.86	5089.48	5047.02

CPI data is announced on a monthly basis and is used to measure inflation [7]. The CPI dataset contains the CPI value for each year, which represents the average of the CPI values announced within that year (Table 2).

Companies may distribute a portion of their profits to shareholders at certain times of the year to attract investors. This is called a dividend. Since each company's stock price is different, comparing distributed dividends on a nominal basis is not accurate. This has led to the creation of a measure called dividend yield. It is also possible to receive dividends when investing in the S&P 500 index. The dividend yield dataset shows the historical dividend yield of the S&P 500 index (Table 3).

Table 2: CPI Data

Year	CPI
2022	292.7
2023	304.7

Table 3: Dividend Yield Data

Year	Dividend Yield
Dec 31, 2018	2.09%
Dec 31, 2017	1.84%

2 Preliminary Preparation

This section will discuss the statistical concepts used during the study to provide the necessary theoretical background for the reader.

Normal Distribution

The normal distribution forms the foundation of statistical science. This is because it is quite effective in explaining uncomplicated phenomena such as height and possesses several properties that make it useful for inference. These can be summarized as follows:

- The normal distribution is symmetric.
- The mean and standard deviation are sufficient to define a normal distribution.
- The area under the normal distribution curve contains 68% within one standard deviation from the mean and 95% within two standard deviations.
- The mode, median, and mean of a normal distribution are identical.

Paired T-Test

The paired t-test is a parametric test used to compare whether the means of two dependent groups differ from each other [8]. To calculate the test statistic, the mean and standard deviation of the differences must first be determined. If we denote the differences of all paired samples as D, the sample mean for a sample size of n is calculated as in Equation (1):

$$\bar{d} = \frac{D}{n} \tag{1}$$

To calculate the unbiased sample standard deviation, each paired sample's difference must be compared to this mean:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}} \tag{2}$$

After computing the sample mean and standard deviation of the differences, the test statistic is calculated in the final step:

$$t = \frac{\bar{d} - 0}{\hat{\sigma}/\sqrt{n}} \tag{3}$$

Using the test statistic and degrees of freedom, the p-value is automatically obtained from the t-distribution. When applying a two-tailed paired t-test, the relevant hypotheses are as follows:

- H_0: The mean difference between the compared groups is 0.
- H_1: The mean difference between the compared groups is not 0.

If the obtained p-value is below the critical p-value, H_0 is rejected. In this study, the critical p-value for all tests was set at 0.05. In cases where multiple hypotheses were tested, the Bonferroni Correction was applied.

Shapiro-Wilk Test

The Shapiro-Wilk test evaluates whether a distribution follows a normal distribution [9]. As a distribution approaches normality, the test statistic approaches 1. The hypotheses for this test are as follows:

- H_0 : The given distribution is normal.
- H_1 : The given distribution is not normal.

Wilcoxon Signed-Rank Test

The Wilcoxon Signed-Rank test is used to compare paired samples when the normality assumption is not met [10]. It assumes that the distributions being compared are similar and makes a comparison between their medians. The corresponding hypotheses are as follows:

- H_0 : $\text{Median}_A - \text{Median}_B = 0$
- H_1 : $\text{Median}_A - \text{Median}_B \neq 0$

3 Methods

This section details the mathematical methods behind three different calculations: annual return calculations, long-term return analyses, and the EMA Crossover strategy.

3.1 Annual Return Calculations

The choice of the starting month for annual returns affects the results (Table A1). For example, the real return of the S&P 500 Index between January 1950 and January 1951 is 23.9%, while between June 1950 and June 1951, it is only 14%. To standardize return variability, percentage returns for each month can be calculated and averaged. Alternatively, using the average of annual closing prices yields similar results to computing individual annual returns for each month and averaging them (Pearson correlation coefficient = 0.999 for both investment vehicles). Consequently, annual average closing prices (AAC) were used for return calculations.

The nominal annual proportional returns (NPR) of different investment vehicles can be calculated using Equation (4):

$$\text{NPR}_{i,i-1} = \frac{\text{AAC}_i - \text{AAC}_{i-1}}{\text{AAC}_{i-1}} \times 100 \tag{4}$$

Since inflation erodes the value of money over time, annual proportional real returns (PRR) must be adjusted for inflation. The inflation factor (IF) is used for this purpose and is obtained by taking the ratio of the CPI values of the two examined years.

$$\text{IF}_{i,j} = \frac{\text{CPI}_i}{\text{CPI}_j} \tag{5}$$

In this context, Equation (4) is adjusted using IF to calculate real returns.

$$\text{PRR}_{i,i-1} = \frac{\text{AAC}_i - \text{AAC}_{i-1} \times \text{IF}_{i,i-1}}{\text{AAC}_{i-1} \times \text{IF}_{i,i-1}} \times 100 \tag{6}$$

As mentioned in the Data Sets section, dividend income contributes to the total earnings of a stock market investor. The dividend yield (DY) is obtained by dividing the gross dividend by the stock price.

$$\text{DY}_i = \left(\frac{\text{Gross Dividend}}{\text{Stock Price}} \right)_i \times 100 \tag{7}$$

When investing in the S&P 500 index, it is also possible to receive dividends. If an investor does not reinvest dividends into their portfolio, annual proportional real returns are calculated using Equation (6). If dividends are reinvested, Equation (8) is used.

$$\mathrm{PRR}_{i,i-1} = \left(\frac{\mathrm{AAC}_i + \mathrm{AAC}_{i-1} \times \mathrm{IF}_{i,i-1} \times \mathrm{DY}_{i-1} - \mathrm{AAC}_{i-1} \times \mathrm{IF}_{i,i-1}}{\mathrm{AAC}_{i-1} \times \mathrm{IF}_{i,i-1}} \right) \times 100 \tag{8}$$

Since gold investors do not receive dividends, Equation (6) was used to calculate their annual returns.

3.2 Calculation of Long-Term Investment Returns

In this section, the proportional real returns obtained by investors who invested in gold and the S&P 500 index via the SPY ETF over twenty-year periods are examined. Before discussing the methodology followed in long-term investment simulations, it is necessary to mention the fundamental assumptions of the study:

- Since it is not possible for investors to invest directly in the S&P 500 index, they have used the SPY ETF for this purpose.
- Although the SPY ETF was actually established in 1993, it can be used in simulations for all years, and its unit price is always 0.1 times the index price ($\mu_{ratio} = 0.1$ & $\sigma_{ratio} = 0.00027$).
- The deduction coefficient for the SPY ETF's management expenses (ME) is 0.00095 and remains constant for all years.
- No deductions are applied to the dividends received by index investors.
- There are no buying or selling commissions for investment instruments.
- Investors attempting to profit through trading adopt the EMA Crossover strategy and never deviate from this strategy.
- Investors following a buy-sell strategy do not invest their money elsewhere when they are not in a position.
- Investors adopting the buy-sell strategy do not enter short positions when the strategy is in a sell position.

3.2.1 Twenty-Year ETF Buy-and-Hold Simulation

The twenty-year buy-and-hold simulation is based on the scenario where investors randomly purchase 2 ETFs at 10 different times in the initial year. Accordingly, the total nominal costs (NC) of investors in all simulations can be calculated using Equation (9). The variable F in the equation corresponds to the unit price of the ETF.

$$\mathrm{NC}_i = 2 \times \sum_{k=1}^{10} F_{k,random} \tag{9}$$

The annual growth rate (AGR) of a portfolio can be found using Equation (10) when dividends are not reinvested.

$$\mathrm{AGR}_i = \frac{\mathrm{AAC}_i}{\mathrm{AAC}_{i-1}} - \mathrm{ME} \tag{10}$$

During the study, the nominal portfolio value without dividends (PV) obtained from simulations was calculated using Equation (11).

$$\mathrm{PV}_{i+20,\text{without dividends}} = \mathrm{PV}_i \times \prod_{i+1}^{i+20} (\mathrm{AGR}_i - \mathrm{ME}) \qquad i = 1950, 1951..., 2003 \tag{11}$$

On the other hand, a minor adjustment was made to Equation (11) during simulations to calculate the nominal portfolio value with dividends. This approach assumes that all dividends can be fully reinvested independently of ETF prices.

$$\mathrm{PV}_{i+20,\text{with dividends}} = \mathrm{PV}_i \times \prod_{i+1}^{i+20} (\mathrm{AGR}_i + \mathrm{DY}_{i-1} - \mathrm{ME}) \qquad i = 1950, 1951..., 2003 \tag{12}$$

3.2.2 Twenty-Year Gold Buy-and-Hold Simulation

The steps followed in the twenty-year gold buy-and-hold simulation are similar to those in the twenty-year ETF buy-and-hold simulation. In this simulation, it is also assumed that the gold investor purchases 2 ounces of gold at 10 different random times, so their nominal cost is calculated using Equation (9). The only difference is that the variable F now corresponds to the price per ounce of gold.

The nominal value of gold after twenty years is found by multiplying the amount of gold held by the average gold price twenty years later. Since the simulation assumes that the investor purchases a total of 20 ounces at the start, the nominal portfolio value for gold investors is calculated using Equation (13).

$$\mathrm{PV}_{i+20} = 20 \times \mathrm{AAC}_{i+20} \qquad i = 1950, 1951..., 2003 \tag{13}$$

Using the relevant CPI values, the nominal values obtained from Equations (9), (11), (12), and (13) can be adjusted for inflation. In this study, to standardize all simulations, total portfolio values and costs were adjusted to the 2023 CPI value.

3.3 EMA Crossover Buy-Sell Simulation

3.3.1 EMA Crossover Strategy

The EMA Crossover strategy is based on the comparison of two exponential moving averages (EMA) of different lengths. The fundamental logic is to take positions by measuring the buying or selling appetite of recent traders compared to those with a longer-term perspective. In this study, the EMA Crossover strategy was applied using (3-5) and (5-8) settings (Figure 1). The reason for preferring short-term moving averages is to ensure that investors applying the buy-sell strategy act as differently as possible from long-term investors. Since transactions were simulated based on daily closing prices, EMAs were also constructed based on daily closing prices. A buy signal is generated when the short-term EMA crosses above the long-term EMA, whereas a sell signal occurs when the opposite happens.

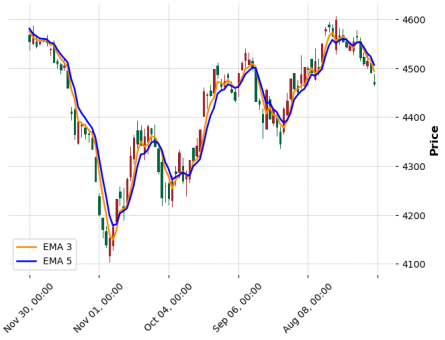


Figure 1: EMA Crossover Strategy

3.3.2 EMA Crossover Simulations

The EMA Crossover simulation starts with the investor purchasing 20 ETFs upon receiving the first buy signal. Additionally, upon receiving a sell signal, the investor liquidates all their ETFs. When selling their ETFs, management expenses

are deducted proportionally to the number of days the position was held, and the remaining cash serves as capital for future buy-sell transactions (Example of Buy-Sell Breakdown). At the end of simulations, it is assumed that an investor with an open position closes it at the last day's price of the year.

The simulations run on a yearly basis and have been repeated from 1950 to 2023. To compare the returns of the buy-sell strategy, a control group was formed in which an investor buys 20 ETFs at the average price of the first month of the year and sells them at the average price of the last month. The returns of the control group are calculated by subtracting the purchase cost and management expenses from the revenue obtained from selling. Since the primary aim of the buy-sell strategy is to profit from price fluctuations, dividend income is excluded from the total return calculations for both groups.

4 Test Results and Findings

This section addresses each of the research questions mentioned in the Introduction and Motivation section and presents the findings to the reader.

4.1 Investment Instruments and Annual Returns

To calculate the annual returns of gold and the S&P 500 index, the formulas discussed in the Annual Return Calculations section were applied to the data, and the resulting histograms are shown in Figure 2. As can be seen from the histograms, although the average returns of investment instruments are close to each other, their return distributions are quite different. In other words, the annual return of the S&P 500 index is much more stable compared to gold. Thus, for investors who are uncomfortable with excessive price fluctuations, investing in the S&P 500 index may be a more suitable option.

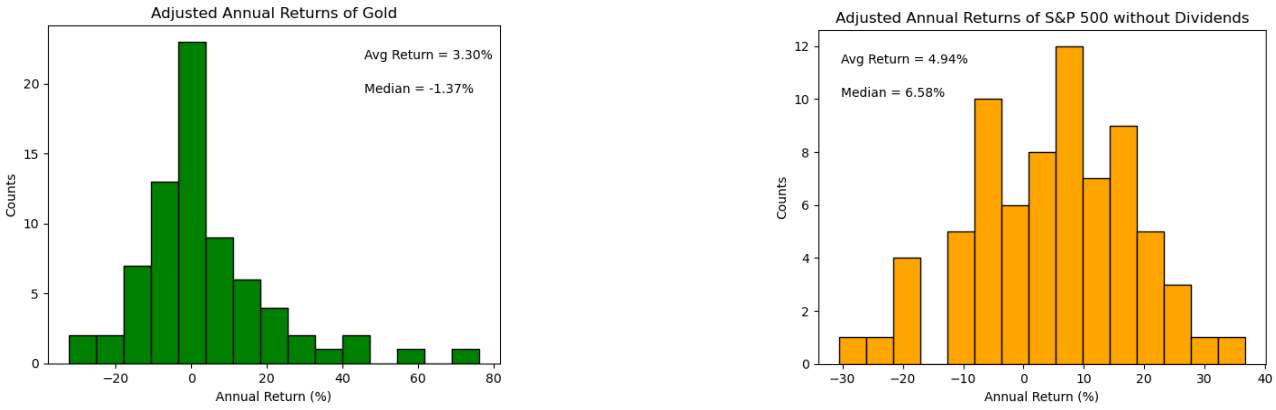


Figure 2: Real Returns of Investment Instruments

RQ1: Are the annual returns of gold and stock market investments statistically different from each other?

To test whether the annual returns of the investment instruments used in the study differ from each other, the Wilcoxon Signed-Rank Test was applied. According to the test results, it was found that the returns derived from the price movements of gold and the S&P 500 index are not significantly different from each other ($p = 0.15$).

As previously mentioned, investors in the S&P 500 index earn returns not only from price movements but also from dividends periodically distributed by companies. When dividends are taken into account, the annual real return of the S&P 500 index rises to 8.07% (Table 4). This return is significantly higher than the annual real return of gold.

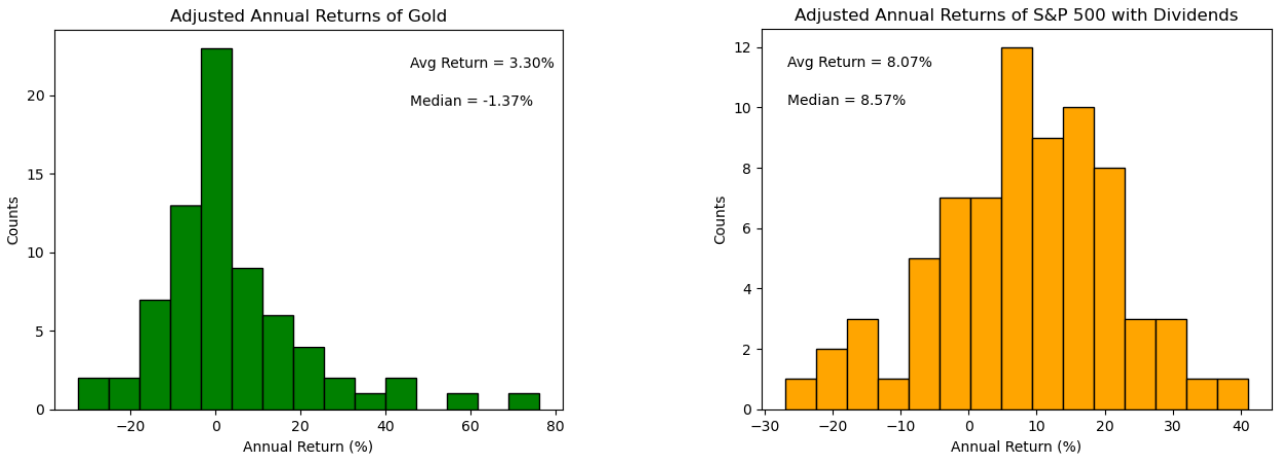


Figure 3: Real Returns of Investment Instruments (2)

Table 4: Annual Returns

Investment Instrument	Mean	Median
Gold	3.30	-1.37
S&P 500 (Excluding Dividends)	4.94	6.58
S&P 500 (Including Dividends)	8.07	8.57

4.2 Long-Term Returns of Investment Instruments

In the previous section, the annual returns of gold and the S&P 500 were compared, revealing that when only price movements are considered, the annual returns of these two investment instruments are not significantly different from each other. However, when dividends are included, the opposite conclusion emerges. In this section, the returns of a 20-year investment in the S&P 500 via the SPY ETF are compared with the returns of a gold investment.

RQ2: Does the purchase price matter in 20-year investments?

Before comparing the returns, it is necessary to determine how much the purchase price affects the total earnings of an investor. For this purpose, 20-year investments in gold and the SPY ETF from 1950 to 2023 were simulated 30 times each, and the standard deviations of the obtained proportional returns were examined. Table 5 presents the five periods with the highest standard deviations from the simulation results for 20-year investments in gold and the SPY ETF.

Table 5: Standard Deviations of Gold and S&P 500 Proportional Returns

Period	Standard Deviation
(1972, 1992)	28.45
(2003, 2023)	19.87
(1968, 1988)	17.84
(1973, 1993)	17.30
(1969, 1989)	16.77

Period	Standard Deviation
(1980, 2000)	2.72E-13
(1979, 1999)	2.32E-13
(1981, 2001)	2.16E-13
(1982, 2002)	1.96E-13
(1978, 1998)	1.92E-13

The results indicate that in 20-year investments, the purchase price of gold has a significantly greater impact on the profits obtained compared to the purchase price of the S&P 500. Therefore, when comparing the long-term returns of investment instruments, only one simulation was conducted per period for stock market investments. On the other hand, when calculating gold returns, 30 simulations were conducted for each period, and the averages of these returns were considered during the comparison.

RQ3: Are the average returns provided by investment instruments over a 20-year period different from each other?

To determine the returns of 20-year investments in gold and the S&P 500 index, the formulas discussed in the Calculation of Long-Term Investment Returns section were applied, and the results are shown in Figure 4. Consistent with the findings in the Investment Instruments and Annual Returns section, while the averages are similar, the range of returns for gold is observed to be broader compared to the S&P 500 (respectively [-78%, 523%] and [-42%, 465%]).

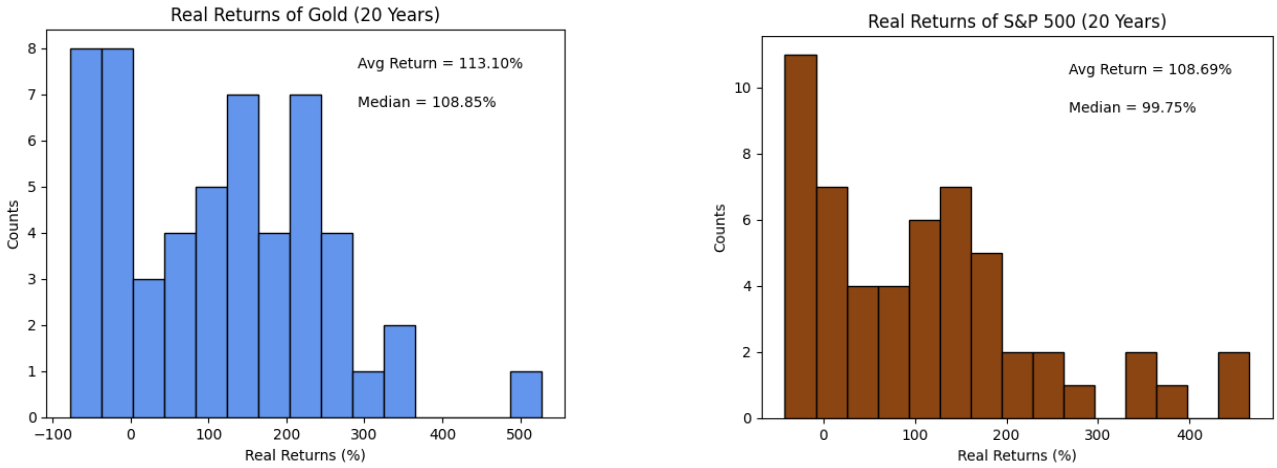


Figure 4: Real Long-Term Returns of Investment Instruments

The long-term return of the S&P 500 excluding dividends was found to be statistically indistinguishable from that of gold ($p = 0.68$). However, when dividends are considered, the average real return of the S&P 500 rises to 261% (Table 6), which is significantly higher than the long-term real return of gold.

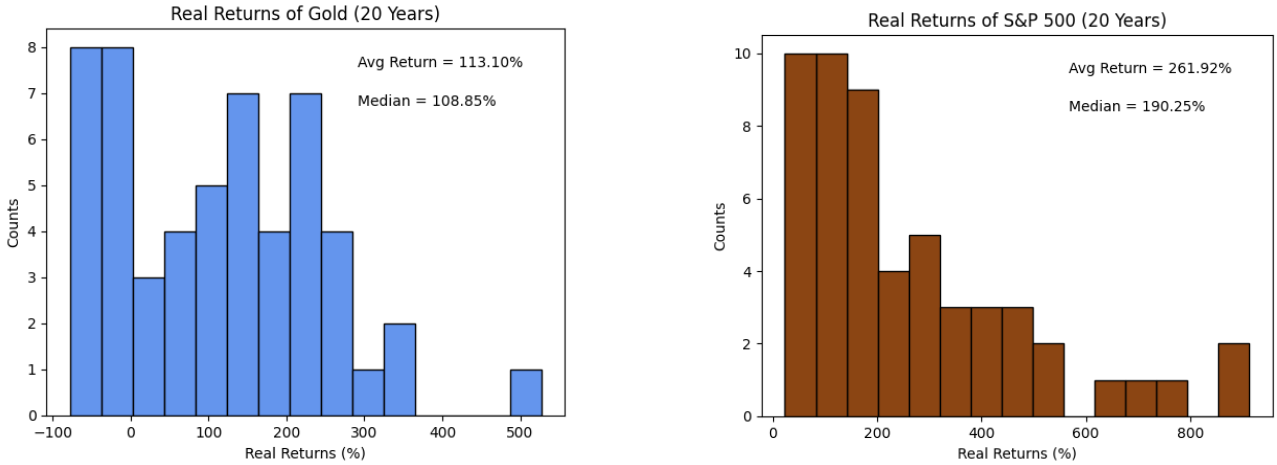


Figure 5: Real Long-Term Returns of Investment Instruments (2)

Table 6: Long-Term Returns

Investment Instrument	Mean	Median
Gold	113.10	108.85
S&P 500 (Excluding Dividends)	108.69	99.75
S&P 500 (Including Dividends)	261.92	190.25

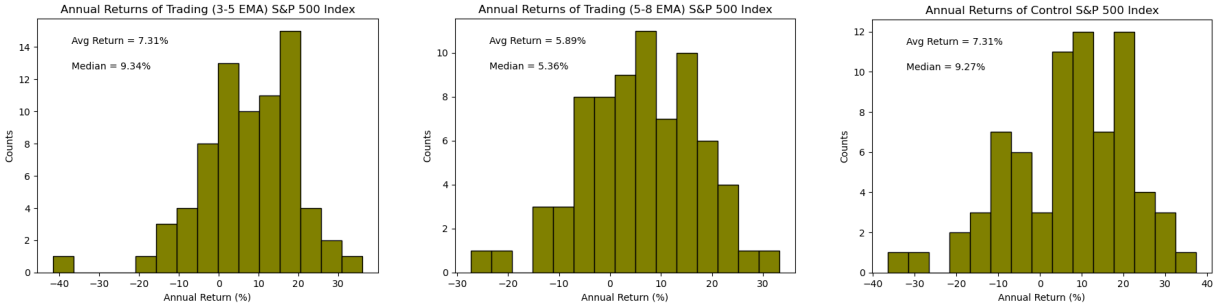
Finally, it was observed that 20-year investments in the S&P 500 always provided a positive real return across all time periods (Figure 5).

4.3 Profiting from Investment Instruments Using Buy-Sell Strategies

In this section, the statistical significance of profits obtained from daily transactions using the EMA Crossover buy-sell strategy is compared with the profits generated from a buy-and-hold strategy. The methodology explained in the EMA Crossover Buy-Sell Simulation section was followed for the buy-sell simulations.

4.3.1 Buy-Sell Transactions in the S&P 500 Index

RQ4: Does the buy-sell method yield higher returns than the buy-and-hold method?



(a) (3-5) EMA (b) (5-8) EMA (c) Control Group

Figure 6: Annual Returns of Different Buy-Sell Strategies in the S&P 500

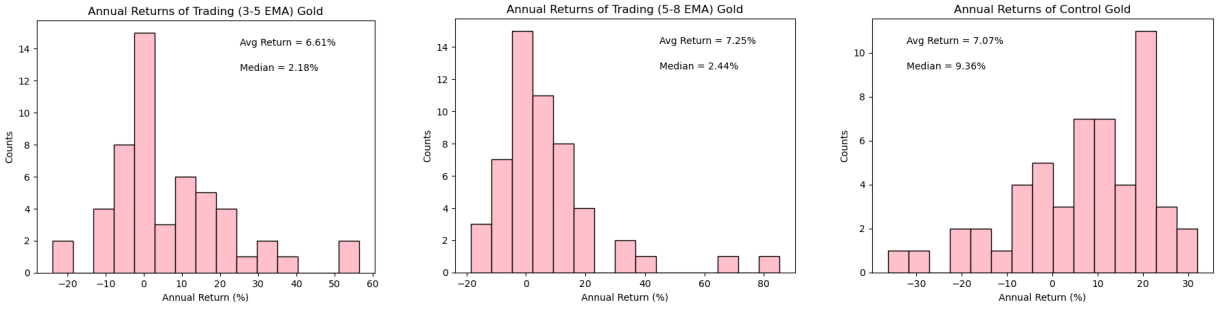
The return histograms of the (3-5) EMA, (5-8) EMA, and control group are shown in Figure 6. The results for (3-5) EMA are closely aligned, while the (5-8) EMA results are slightly lower compared to the other two.

Table 7: Comparison of Different Buy-Sell Strategies in the S&P 500

Comparison	Applied Test	Null Hypothesis (H_0) Rejected/Not Rejected
(3-5) EMA vs (5-8) EMA	Wilcoxon	Rejected
Control vs (3-5) EMA	Wilcoxon	Not Rejected
Control vs (5-8) EMA	Paired t-test	Not Rejected

Findings on whether the distributions differ from each other are summarized in Table 7. A statistically significant difference was only found between the (3-5) and (5-8) EMA strategies. However, neither strategy demonstrated a significant advantage over the control group.

4.3.2 Buy-Sell Transactions in Gold



(a) (3-5) EMA (b) (5-8) EMA (c) Control Group

Figure 7: Annual Returns of Different Buy-Sell Strategies in Gold

The application of the simulations described in the previous section to gold is shown in Figure 7. The annual returns of all three strategies are very close to each other and statistically insignificant (Table 8).

Table 8: Comparison of Different Buy-Sell Strategies in Gold

Comparison	Applied Test	Null Hypothesis (H_0) Rejected/Not Rejected
(3-5) EMA vs (5-8) EMA	Wilcoxon	Not Rejected
Control vs (3-5) EMA	Wilcoxon	Not Rejected
Control vs (5-8) EMA	Wilcoxon	Not Rejected

5 Conclusion and Potential Improvements

This study examined the returns of investments in gold and the S&P 500 index through various approaches. It was found that in both long-term and annual returns, the S&P 500 index provided more stable and higher returns when dividends were included (RQ3, RQ1). The purchase price was observed to be more significant for gold investments compared to stock market investments (RQ2). The income obtained through buy-sell strategies was found to be insignificant for both investment instruments (RQ4).

In this study, it was assumed that long-term investors fully benefited from dividend income. However, in reality, a certain amount of tax is deducted from dividends, and the obtained dividend income may not be fully reflected in ETF prices. This situation may have resulted in an overestimation of long-term investor earnings.

There are various options available for traders, and these may be more suitable for their objectives. Additionally, it is well known that traders frequently use leverage and trade within shorter time frames. Moreover, traders generally adopt more complex buy-sell strategies and test their historical performance. These factors could be considered in future studies to create more realistic simulations.

Finally, the finding that the S&P 500 provides higher returns does not necessarily make gold a poor investment. In a future study, I will also discuss the role of gold in an investment portfolio.

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A Appendices

Table A1: Annual Returns by Month

Period	Month	(%)Real Return
(1950, 1951)	1	23.909
(1950, 1951)	2	25.936
(1950, 1951)	3	22.955
(1950, 1951)	4	21.114
(1950, 1951)	5	17.749
(1950, 1951)	6	14.03
(1950, 1951)	7	24.382
(1950, 1951)	8	22.555
(1950, 1951)	9	21.501
(1950, 1951)	10	16.506
(1950, 1951)	11	13.781
(1950, 1951)	12	17.389

Example of Buy-Sell Breakdown

Date: 1950-01-04
Bought 20 ETFs at a price of: 1.685
Capital invested in this trade: 33.7
Cash in hand: 0

Date: 1950-01-13
Sold 20 ETFs at a price of: 1.66
Capital gained from trade: 33.34
Expense payment: 0.00078
Net cash: 33.33

Date: 1950-01-19
Bought 19.0 ETFs at a price of: 1.68
Capital invested in this trade: 32.05
Cash in hand: 1.28