Automated analysis of algorithms implemented on top of TBD

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Abstract—The principle of incremental programming has been well known for many years. While the automatic transformation of sequential programs into efficient incremental programs is not solved in general, a number of platforms for incremental computation have been created. In this document, we inspect the incremental computation platform TBD. First, we summarize known principles like directed dependency graphs, execution traces, intrinsic trace distance and trace stability, then we create an program model which enables us to apply those principles to TBD programs. We also present an algorithm to calculate the intrinsic trace distance in practice. Finally, we show how we can make automatic optimizations to programs by analyzing the dependency graph.

I. Introduction

This section is going to describe the purpose of incremental computation and also explain why the concept of incremental computation is useful for speeding up computations. Furthermore, used and referenced terminology should be outlined.

A. Approaches to incremental computation

This subsection briefly describes various approaches to incremental computation, and outline their strengths and weaknesses.

These approaches include:

- Providing a platform or framework for incremental programming, utilizing
 - o function caching. [1] [2]
 - o formal manipulation of the program. [3]
 - o differential data flow. [4]
 - a combination of multiple approaches, like memorization and execution traces. [5] [6] [7]
- Providing a high-level abstraction, like an incremental database. [9]
- Deriving an incremental program from a non-incremental, non-functional program. [10]
- Deriving an incremental program from a nonincremental, functional program. [11]

II. FUNDAMENTAL WORK

The work of U. Acar et al[12] describes the theoretical and practical concept of incremental computing using memorization and DDGs in detail. Since these concepts are important for understanding this writing, they are summarized in this section. First, general terms are explained briefly, then, a short theoretical outline for the term of stable algorithms is provided.

A. Change Propagation, Dynamic Dependece Graphs and Memorization

Change Propagation refers to the task of re-evaluating certain parts of a program as soon as the input data changes. The target of this re-evaluation is to update the output accordingly, as if the whole program would have been re-evaluated. A change propagation algorithm is responsible for selecting the function calls in the program, which have to be re-executed. When the program is executed for the first time, no change propagation happens. This execution is called the *Initial Run* [12].

A Dynamic Dependence Graph (DDG) can be described as a data structure, which holds a directed graph for tracking control and data dependencies during execution [12], whereas the nodes of the graph are usually function calls in the program. In contrast to Static Dependence Graphs [?], DDGs are mutated during change propagation and therefore adjusted to the new program structure.

Memorization is the concept of storing intermediate results and re-using them during change propagation. Memorization can be combined with DDGs, by inserting memorization nodes into the graph. During change propagation, this can lead to a significant performance increase, because entire sub-trees of call graph and their corresponding results can be re-used [12].

B. Execution Traces

A trace is a theoretical construct which can be described as an ordered tree, whereas nodes represent function calls during the program execution [12]. While similar to the DDG, a trace tracks no data dependencies. A trace usually resembles the call tree of a program.

Each node v is uniquely described by a tag, consisting of

- the function being called, fun(v)
- the arguments of the function call, arqs(v)
- the values read in the body of the function, reads(v)
- the values returned to the function from its callees, returns(v)
- the weight of the function, w(v), which is equal to its execution time.

Two nodes v and v' are equal, denoted $v \equiv v'$, if fun(v) = fun(v'), args(v) = args(v'), reads(v) = reads(v') and returns(v) = returns(v').

C. Trace distance

Trace distance can basically be described as an edit distance between two execution traces [12] [15].

To find the minimum trace distance of two traces T and T^\prime , the so called cognates relation can be used.

A set of cognates C is a relation of two traces T and T' with the set of nodes V and V', so that

- $C \subset V \times V'$
- for each $(v, v') \in V : v \equiv v'$
- no node is paired with more than one node.

All nodes of both traces T and T' can be colored either blue, yellow or red. Nodes which have a cognate are colored blue. Nodes of T without a cognate are colored yellow. Nodes of T' without a cognate are colored red.

The trace distance $\delta(T,T')$ between two traces T and T' can now be calculated by summing up the weights of all yellow and red nodes. This can be formally written as:

$$\delta(T, T') = \sum_{y \in Y} w(y) + \sum_{r \in R} w(r)$$

In this equation, Y denotes the set of all yellow vertices and R is the set of all red vertices.

If the cognate relation C is maximal, the intrinsic distance is minimal, denoted as δ^{min} . Also, a maximal cognate relation can be found using a naive greedy algorithm [12] [15].

D. Stable algorithms

This subsection describes the concepts of stable algorithms, intrinsic trace distance and their relationship.

Also, this section should emphasis that the intrinsic trace distance forms a lower bound for the time needed by change propagation during an update. [12]

III. TBD

The TBD platform is a framework for incremental computation currently being developed at Carnegie Mellon University (CMU). TBD follows the approach of memorization combined with directed dependency graphs (DDGs), as throughly described in [12]. Also, parallel computing is supported. The framework allows a programmer to write software using TBDs programming interface, while TBD automatically takes care about invoking the correct functions for change propagation, in case of an update of the input data. [?] The framework is being developed in the Scala language, which enables us to exploit the reflection capabilities of Scala for analysis [13] [14]. The source code of TBD is available at https://github.com/twmarshall/tbd.

A. Programming interface

TBD needs to keep track of reads and writes of variables in the program. To accomplish this, TBD wraps all values relevant for change propagation into so called *modifiables* or short *Mods*. TBD automatically wraps all input data into Mods.

```
def mod[T](
  initializer: Dest[T] => Changeable[T]
): Mod[T]
```

Fig. 1. Signature of the mod method

```
def write[T](
    dest: Dest[T],
    value: T
): Changeable[T]
```

Fig. 2. Signature of the write method

1) mod: To create Mods, for example as result of the program execution, TBD provides a method mod. The declaration of mod can be seen in listing 1. The mod method calls a function parameter initializer with a destination or Dest as argument. The value written to the Dest by the function parameter is then stored in the Mod, which is returned by the mod method. Requireing the return type of Changeable simply enforces that a write is the last operation inside initializer.

2) write: To write to a Dest, TBD provides a write method. The signature of write can be found in listing 2. The write method simply takes a Dest and a value, and writes the value to the given Dest. The write method returns a Changeable.

3) read: The values from within modifiables have to be read explicitely. For this purpose, TBD provides a read method, which accepts a Mod as parameter and then calls a function parameter reader with the value of the Mod as first argument. The signature can be seen in in listing 3. For read, the function parameter reader also has to return a Changeable. Reads without an enclosed write are not useful, since the read method my not modify values outside of it's scope.

Listing 4 shows a very simple example, which adds two Mods of type integer. First, mod is called to create a Dest for the result, then the values of mod1 and mod2 are read. The values of mod1 and mod2 are then added and written to dest. The nesting of functions as seen in the example is typical for TBD.

Since all programs consist of *read*, *write* and *mod* functions, and all Modifiables have to be explicitly written, TBD is able to construct a DDG from monitoring the calls to the corresponding functions.

4) memo: As we already mentioned, TBD not only utilizes DDGs, but also memorization. To accomplish memorization, tbd provides a method to create so-called *Lifts*, which in turn provide a method for memorization, memo. The memo method accepts a list of parameters, which are used to match

```
def read[T, U <: Changeable[_]](
   mod: Mod[T],
   reader: T => U
): U
```

Fig. 3. Signature of the read method

Fig. 4. A basic example, utilizing read, write, and mod

```
def memo(
    args: List[_],
    func: () => T
): T
```

Fig. 5. Signature of the memo method

this memo call and a function parameter func. A Lift can be described as memorization context. Calling memo with the same parameters as any previous call on the same Lift will yield the same result, without evaluation func. If there is no match, func will be called and the result will be stored for future memorization. In general, it is important to not share Lift objects between unrelated function calls, but to preserve the same Lift for all calls to the same function. The signature of memo can be seen in listing 5.

A typical use case for memorization is list processing. A typical example is shown in 6. First, we define a class for list nodes and the properties value of type integer and next. Note that the class is immuteable. Next, we define a function, incrementalList, which initializes a lift and calls a recursive function, incrementRecursive with the head of the list and the created lift. The latter function maps each list node to a list node with value increased by one. This is done by first creating a Dest dest for the new ListNode. Then, the current node is read from it's modifiable. If the current node is null, the end of the list is reached and null can be written to dest. If the current node is not null, the value is read, increased, and written again to create the Mod newValue, similar to the example in listing 4.

Then, *incrementRecursive* is called recursiveley with the next node as parameter. The call to *incrementRecursive*, however, is enclosed in a memo operation, with the next node as parameter. If a change propagation happens now, TBD is not going to recursiveley call all reads again, but will stop as soon as a memo match occours. This is typically the case as soon as the recursion reaches an unchanged list element.

In the end, a new list node is constructed from the results and returned.

5) par: The last crucial method offered by TBD is a method to execute code in parallel, par. The par method takes two function parameters one and two, where each

```
def par[T, U](
   one: TBD => T,
   two: TBD => U
): Tuple2[T, U]
```

Fig. 7. Signature of the par method

function parameter is executed on a septerate worker thread with septerate TBD objects. The par method blocks util both workers are finished. The signature of par is shown in listing 7

B. Constraints and responsibilities

During change propagation, TBD re-evaluates all read calls that read modifiables which have changed, in the same order they were called during the initial run. Obviously, the functions invoked read, mod, memo and par may not write variables outside of their scope, or they will easily break change propagation. They have to be side effect free. If, for example, a static variable is written from within a function called by read, and then used somewhere else in the program, the system has no way to propagate a change of this variable.

Furthermore, all functions called have to be deterministic. Calling the same function with the same parameters has to lead to the same return value or the same value written to a dest. Otherwise, memorization will not be useable in the program.

For each function parameter passed to the functions read or mod, the last operation executed in that function parameter has to be a write. This is enforced by requireing the return type of Changeable for function parameters.

[TODO: describe the order of function calls.]

IV. A THEORETICAL MODEL FOR TBD

While we can retain the definition of a trace, we have to adjust the definition of nodes and node equality for our purpose, so we can use it in the next section to construct an algorithm for trace distance caluclation.

A. Trace node equality and similarity

As described in section III, TBD provides read, mod, write, memo and par methods to the developer. Instead of creating an execution trace out of all functions in the program, we restrict ourselfs to a trace consisting of only these functions. It should be noted, that, since we require each function to be side-effect free and determinisite, we could theoretically omit write nodes in the DDG, since they directly depend on their corresponding parent nodes. However, including these nodes can provide useful insights during debugging.

Definition 1 (Trace nodes) Let each node in our execution trace represent a read, mod, write memo or par function. We annotate each node with a tuple of the following values:

- the node type t, which can have the values read, mod, write, memo or par
- a node tag, a sequence of labels which has a different structure depending on the node type

```
class ListNode(_value: Mod[Int], _next: Mod[ListNode]) {
   val value = _value
   val next = _next
}
def incrementList(tbd: TBD, head: Mod[ListNode]): Mod[ListNode] = {
    val lift = tbd.makeLift()
    incrementRecursive (tbd, head, lift)
}
def incrementRecursive(tbd: TBD, current: Mod[ListNode], lift: Lift[ListNode])
    : Mod[ListNode] = {
    tbd.mod((dest: Dest[ListNode]) => {
        tbd.read(current) (current => {
            if(current == null) {
                tbd.write(dest, null)
            } else {
                val newValue = tbd.mod((destValue: Dest[Int]) => {
                    tbd.read(current.value) (value => {
                      tbd.write(destValue, value + 1)
                    })
                })
                val newNext = lift.memo(List(current.next), () => {
                    incrementRecursive(tbd, current.next, lift)
                })
                tbd.write(dest, new ListNode(newValue, newNext))
            }
        })
    })
```

Fig. 6. A basic example, utilizing memo

Depending on the node type, we define the following node tags:

Definition 2 Let the tag for read nodes consist of (a, fun), whereas

- a is the value of the modifiable being read
- fun is the reader function being called

Definition 3 Let the tag for mod nodes consist of mod(fun), whereas

• fun is the initialzer function being called

Definition 4 Let the tag for write nodes consist of write(a, d), whereas

- a is the value being written
- d is the destination where a is being written to

Definition 5 Let the tag for memo nodes consist of $memo((a_1, ..., a_n), fun)$, whereas

- $(a_1,...,a_n)$ is the list of values to memo match against
- fun is the function being called

Definition 6 Let the tag for par nodes consist of $par(fun_1, fun_2)$, whereas

- fun_1 is the first function being called
- fun_2 is the second function being called

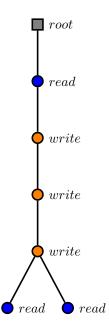
Given these definitions, we now re-define equality of nodes.

Definition 7 (Node equality) Let a node A and B be equal, iff the node type of A, t_a , equals the node type of B, t_b , and the tag of A equals the tag of B.

We only compare the the tag if the node type already matches. Therefore, we can simply compare each element in the tag of A with it's counterpart in the tag of B.

The tag can consist of objects, value types, modifiables or functions. For functions, showing equality is not solveable in general [16]. With the constraints of TBD programs, however, we are able to create a sufficient equality defintion for our purpose.

Definition 8 (Function execution equality for TBD traces) A function execution fun_a and a function execution fun_b are equal, iff all of the following conditions apply:



- 1) fun_a and fun_b refer to the same symbol in the source code.
- 2) all arguments are equal.
- 3) all free variables bound from an outer scope are equal.

The requirement for side-effect free and deterministic functions leads to the conclusion, that all subcalls to other functions, including any writes, are going to be equal if the function is invoked with the same parameters. We have to take care of free variables in the function, however, since they might influence the behavior of the program. A prime example would be a read nested within another read, whereas the inner read accesses the value provided by the outer read, which can be seen in listing $\ref{eq:condition}$?? If the value of mod1 changes in the example the inner function performing the addition of v1 and v2 is not going to be equal anymore, therefore the read node of the inner read has changed, even the value of mod2 stays the same.

For comparing values or objects inside the tag, function parameters or closed free variables we use *deep equality*. Modifiables, however should be compared by reference equality. The reason for doing so is to ensure correctness even with complex types, for example like arrays, nested lists or objects. For modifiables, the change propagation algorithm takes care of changed values, and automatically calls all subcalls which are affected. The case where the modifiable itself was recreated forms an exception, where we would have to re-execute all reads which would access this modifiable. This leads to the following formal definition:

Definition 9 (Object equality for TBD traces) A pritive value p is equal to a primitive value k iff p and k have the same type and the same value.

A modifiable x is equal to a modifiable y iff x and y refer to the same object in memory.

An object A with ordered properties $(a_1,...,a_n)$ is equal to an object B with ordered properties $(b_1,...,b_n)$ iff A and

B have the same type, and a_i equals $b_i \ \forall i \in [1, n]$. Properties can be other objects, modifiables or primitives.

With these definition of trace node equality, we can keep the definition of *Cognates* and *TraceDistance* from [12].

V. AN INTRINSIC TRACE DISTANCE ALGORITHM FOR TBD

While [12] already outlines a greedy algorithm for calculating the intrinsic trace distance, there are details we have to take care of for an implementation.

A. Implementing node equality

While equality of nodes is defined in section IV-A, it still remains open how a equality is implemented. For value types or objects we can use the *equals* method provided by the Scala platform or define our own overload of *equals*, if needed.

For comparing anonymous functions passed to *read*, *memo*, *mod*, and *par*, we have to compare the abstract syntax trees (ASTs) of functions, all parameters, and all arguments, and all free variables bound from an outer scope, as described in definition 8. To accomplish this task, we can utilize the Scala macro API [13]. Basically, the Scala macro API enables us to define macros written in Scala, which are executed during compile time. From within these macros, we can access and modify the AST of our program.

To gather the necassary information for comparing anonymous functions during runtime, we replace the implementations of read, memo, mod and par with macros, which extract interesting information and create a tag from it. Then, the macros generate code which calls the original function and passes the original parameters and the tag.

[Include sample of implementation]

The symbol of the anonymous function is well known to the Scala macro, hence easy to extract. The arguments of the function are also well known for all methods provided by TBD, so they can be easily added to the tag.

Finding free variables which are bound from an outer scope is not straight-forward, because at the macro expansion step, the scala compiler has no knowledge about whether a symbol is a function or a variable, or from where it is bound. To extract only the correct symbols, we first create a list of all symbols which occour in the anonymous function F and store them in a set $V = (v_1, ..., v_n)$. Then, for each v_i , $i \in [1..n]$, we iterate over all ancestors of v_i in the AST of F. If we find a variable definition or parameter which defines a symbol with the same name as v_i , we know that v_i is not bound from an outer scope, so we remove it from our list V.

Then, we iterate over all ancestors in the AST of the outmost enclosing scope of F. This scope is the class in which F is defined in most cases. If we find an ancestor which defines a variable or parameter, we add the symbol of that variable to a set $D=(d_1,...,d_m)$. Finally, we compute the set $U=(u_1,...,u_k)=V\cap D$, whereas equality of elements in V and D is defined by equality of the symbol name. The set U now contains only symbols, which are used in F, defined somewhere outside F and are variables.

Now, we generate code to add the name and value of each symbol u_i to a Scala list, whereas the list is then added to the tag. The tag is passed to the original function, which adds it to the corresponding node in the DDG.

By applying the described technique, we are now able to create a tag, which can be used to compare nodes which depend on anonymous functions for equality.

B. Implementing the intrinsic distance algorithm

Given all nodes in two traces T_1 and T_2 , including their tag, the trace distance can be computed like described in [12].

For a naive greedy algorithme, we create a tree- or hashset S_1 , which holds all nodes from T_1 . Then, we test for each node in T_2 , if a node with an equal tag existed in S_1 . If so, we remove the node from S_1 .

When all nodes have been tested, the intrinsic trace distance is given by the size of the set $|S_1|$ plus the count of nodes from T_2 which were not contained in S_1 .

C. Proof of correctness

[We shall also proof the correctness of our algorithm. It is of importance that we take the changes of the program model outlined in the previous section into account.]

VI. AUTOMATIC OPTIMIZATION OF PROGRAMS

This section is going to describe how analyzing the Directed Dependency Graph (DDG) can be used for automatic optimization. For accomplishing this task we can utilize the following features of the DDG:

- Caller/callee dependencies.
- Dependencies of modifiables¹.
- Dependencies of bound variables which are not modifiables.

Furthermore, using the intrinsic distance algorithm, we can recognize which nodes are deleted, inserted or retained [12], which can be used to optimize the program to accomplish faster change propagation.

The exact contents described in this chapter are still to be determined, based on our findings. Possible approaches include, but may not be limited to:

- Function call reordering.
- Insertion of explicit memorization calls.
- Detection of cascading updates, which could be omitted.

VII. EVALUATION

This section is going to demonstrate the usefulness of the described techniques using real-world algorithms, like map, reduce and quicksort.

Basically, it is shown how it is possible to optimize a classic implementation (without memorization) of each algorithm, so that change propagation time lies within the same complexity class as the theoretical lower bound for updates for this algorithm.

VIII. CONCLUSION

This section will conclude and summarize with the findings of this work.

A. Future work

The final section briefly outlines problems encountered but not solved during the writing of this thesis, as well as encourages future research on interesting issues of incremental computation.

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¹Pointer-like variables which have to be explicitly read and written, and therefore support automatic change propagation

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