How do we write and interpret them?

Grammar Rules

1. (Yosiphon and Mjolsness, 2009); 2. (Mjolsness 2019)

- Simplified DGG graph notation[2], where λ is a label vector:

$$G\langle\langle\lambda\rangle\rangle \longrightarrow G'\langle\langle\lambda'\rangle\rangle$$ with ρ_r or solving $\dot{x} = v$

$$(\bigcirc_1 \text{—} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2) \rangle\!\rangle$$

$$\longrightarrow (\bigcirc_1 \text{—} \bigcirc_3 \text{—} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3) \rangle\!\rangle$$

$$\textbf{with} \quad \hat{\rho}_{\text{grow}} \times H(\|\boldsymbol{x}_2 - \boldsymbol{x}_1\|; L_{\text{div}})$$

$$\textbf{where} \begin{cases} \boldsymbol{x}_3 = \boldsymbol{x}_2 - (\boldsymbol{x}_2 - \boldsymbol{x}_1)\gamma \\ \\ \boldsymbol{u}_3 = \frac{\boldsymbol{x}_3 - \boldsymbol{x}_2}{\|\boldsymbol{x}_3 - \boldsymbol{x}_2\|} \end{cases}$$

Stochastic Growth Rule:

- ρ_r is a propensity function i.e. probability per unit time and can be rewritten in the form of a rate of input labels and sampling function of output labels given input labels.

1. Left-hand Side (LHS)
2. Right-hand Side (RHS)
3. Rate part of ρ_r which determines how often the rewrite occurs.
4. Sampling part of ρ_r which determines the new labels after a rule rewrite occurs.

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Graph Rewrites

Example of how Graph Rewrites in DGGML Work and Operator Semantics

Left-hand Side (LHS) **Right-hand Side (RHS)**

Rewrite operations:

1. Delete edge (1, 2)
2. Create node 3
3. Create edge (3, 1)
4. Create edge (3, 2)

Category theoretic approaches can be found in (Lowe, 1993) and (Ehrig et al., 1973)