The Dynamical Graph Grammar Formalism (Extended)

What are DGGs anyway?

- The DGG formalism is a declarative modeling language L:
    1. A compositional map $\Psi : L \longrightarrow S$ that maps all syntactically valid models $M \in L$ into some space S of dynamical systems.
    2. Conditionally valid or conditionally approximate valid families of Abstract Syntax Tree Transformations.
- Rules map to operators where $\Psi(M) = W(M)$
- The master equation, $\frac{d}{dt}P(t) = W \cdot P(t)$, represents the time evolution of a continuous-time Markov process with formal solution is $P(t) = e^{tW} \cdot P(0)$.
    - Hard to solve analytically! So, we need help!

- Let $\hat{W}_r \equiv \hat{W}_{LHS_r \to RHS_r}$ be an operator that specifies the nonnegative flow of probability between states under each rule r then:
    1. $W = \sum_r W_r$ (rule operators sum up)
    2. $W_r \equiv \hat{W}_r - D_r$ (rules conserve probability)
    3. $D_r = \text{diag}(1 \cdot \hat{W}_r)$ (total probability outflow per state)

 More information can found in (Mjolsness 2022, 2019, 2013, 2010)

Grammar Rules (Extended)

How do we write and interpret them?

- History, dynamic grammars[1]:

$$\{\tau_{\alpha(p)}[x_p] \,|\, p \in L_r\}_* \longrightarrow \{\tau_{\beta(q)}[x_q] \,|\, q \in R_r\}_*$$

with $\rho_r([x_p], [y_q])$

$$\{\tau_{\alpha(p)}[x_p] \,|\, p \in L_r = R_r\}_* \longrightarrow \{\tau_{\beta(q)}[x_q] \,|\, q \in R_r = L_r\}_*$$

solving $\{\dfrac{dx_{p,j}}{dt} = v_{p,j}([x_k]) \,|\, p, j\}$.

- Rate function factorization[1]:

$$\rho_r([x_p]) \equiv \int \rho_r([x_p], [y_q])\, \Delta[y_q]$$

$$P([y_q] \mid [x_p]) \equiv \frac{\rho_r([x_p], [y_q])}{\rho_r([x_p])}$$

$$\rho_r([x_p], [y_q]) \equiv \rho_r([x_p]) \times P([y_q] \mid [x_q])$$

- Simplified DGG graph notation[2], where λ is a label vector:

$$G\langle\langle\lambda\rangle\rangle \longrightarrow G'\langle\langle\lambda'\rangle\rangle \quad \text{with } \rho_r \text{ or solving } \dot{x} = v$$

Stochastic Growth Rule:

Left-hand Side (LHS)	$(\bigcirc_1 - \bullet_2)\langle\langle(\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2)\rangle\rangle$
Right-hand Side (RHS)	$\longrightarrow (\bigcirc_1 - \bigcirc_3 - \bullet_2)\langle\langle(\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3)\rangle\rangle$
ρ_r in factored form, where new labels are sampled in the where clause.	**with** $\hat{\rho}_{\text{grow}} \times H(\|\boldsymbol{x}_2 - \boldsymbol{x}_1\|; L_{\text{div}})$ **where** $\begin{cases} \boldsymbol{x}_3 = \boldsymbol{x}_2 - (\boldsymbol{x}_2 - \boldsymbol{x}_1)\gamma \\ \boldsymbol{u}_3 = \frac{\boldsymbol{x}_3 - \boldsymbol{x}_2}{\|\boldsymbol{x}_3 - \boldsymbol{x}_2\|} \end{cases}$

1. (Yosiphon and Mjolsness, 2009); 2. (Mjolsness 2019)