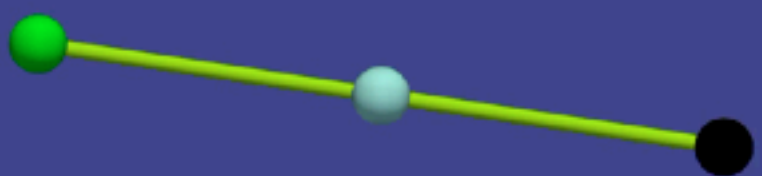
Grammar Rules

Example Deterministic and Stochastic Rules from CMA DGG

11



Node Type

Negative Intermediate Positive

Figure 3: Example of the two rules combined for our approximation of a growing microtubule.

$$(\bigcirc_1 \text{---} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1)(\boldsymbol{x}_2, \boldsymbol{u}_2) \rangle\!\rangle$$

$$\longrightarrow (\bigcirc_1 \text{---} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2 + d\boldsymbol{x}_2, \boldsymbol{u}_2) \rangle\!\rangle$$

$$\textbf{solving} \quad d\boldsymbol{x}_2/dt = v_{plus} \times \boldsymbol{u}_2$$

Deterministic Growth Rule[1]:

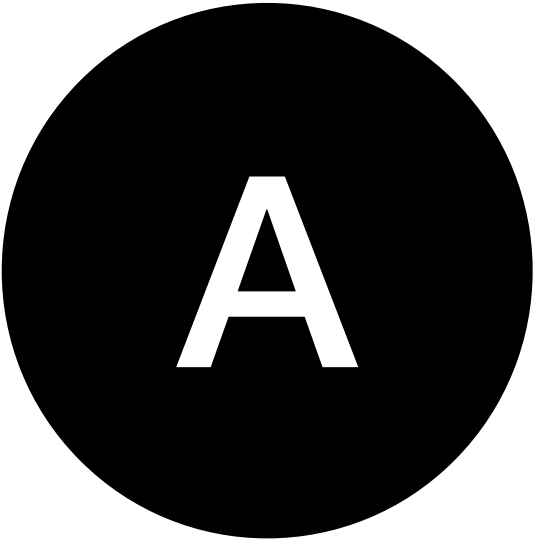
$$(\bigcirc_1 \text{---} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2) \rangle\!\rangle$$

$$\longrightarrow (\bigcirc_1 \text{---} \bigcirc_3 \text{---} \bullet_2) \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3) \rangle\!\rangle$$

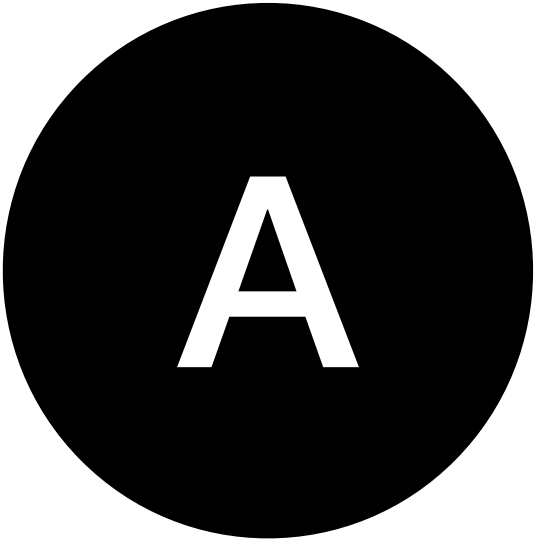
$$\textbf{with} \quad \hat{\rho}_{\text{grow}} \times H(\|\boldsymbol{x}_2 - \boldsymbol{x}_1\|; L_{\text{div}})$$

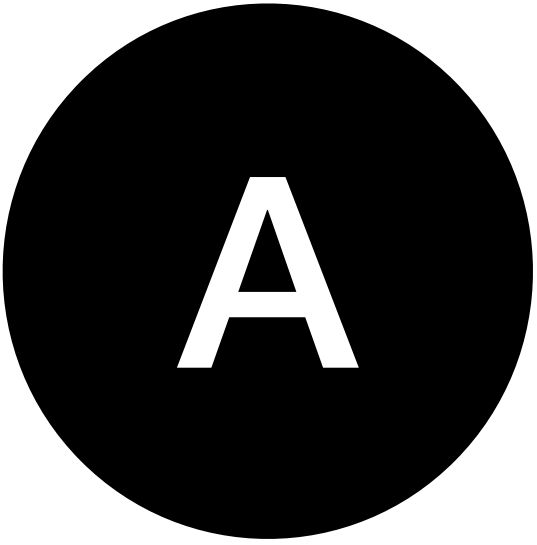
$$\textbf{where} \begin{cases} \boldsymbol{x}_3 = \boldsymbol{x}_2 - (\boldsymbol{x}_2 - \boldsymbol{x}_1)\gamma \\ \boldsymbol{u}_3 = \frac{\boldsymbol{x}_3 - \boldsymbol{x}_2}{\|\boldsymbol{x}_3 - \boldsymbol{x}_2\|} \end{cases}$$

Stochastic Growth Rule[1]:


A

1. More examples in thesis and (Medwedeff and Mjolsness, 2023)


A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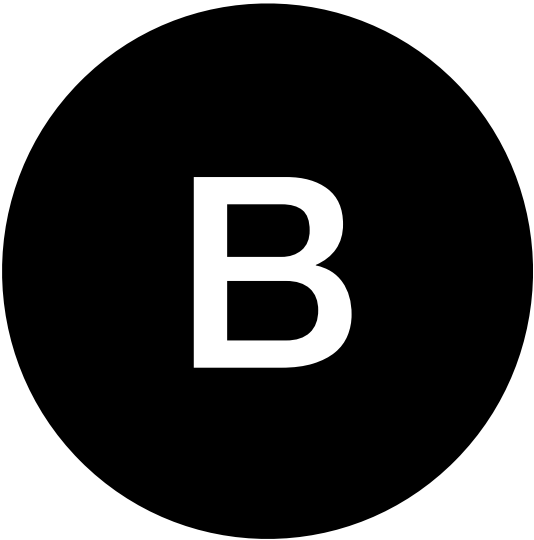
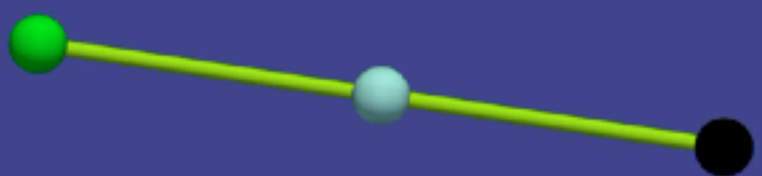

B

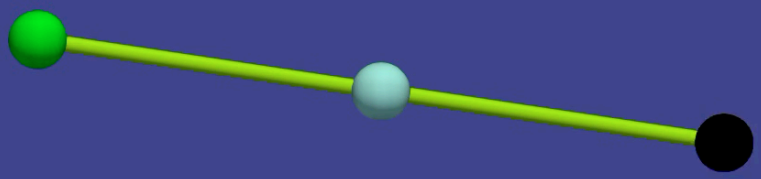
i.e. A transforms to A with a rewrite as a label update via ODE!

i.e. A transforms to B with a rewrite as a label updates in the form of graph structure changes!



Node Type

Negative Intermediate Positive



Node Type

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Grammar Rules

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Stochastic Growth Rule[1]:

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$$\longrightarrow (\bigcirc_1 \text{—} \bigcirc_3 \text{—} \bullet_2)\,\langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3) \rangle\!\rangle$$

$$\textbf{with} \quad \hat{\rho}_{\text{grow}} \times H(\|\boldsymbol{x}_2 - \boldsymbol{x}_1\|; L_{\text{div}})$$

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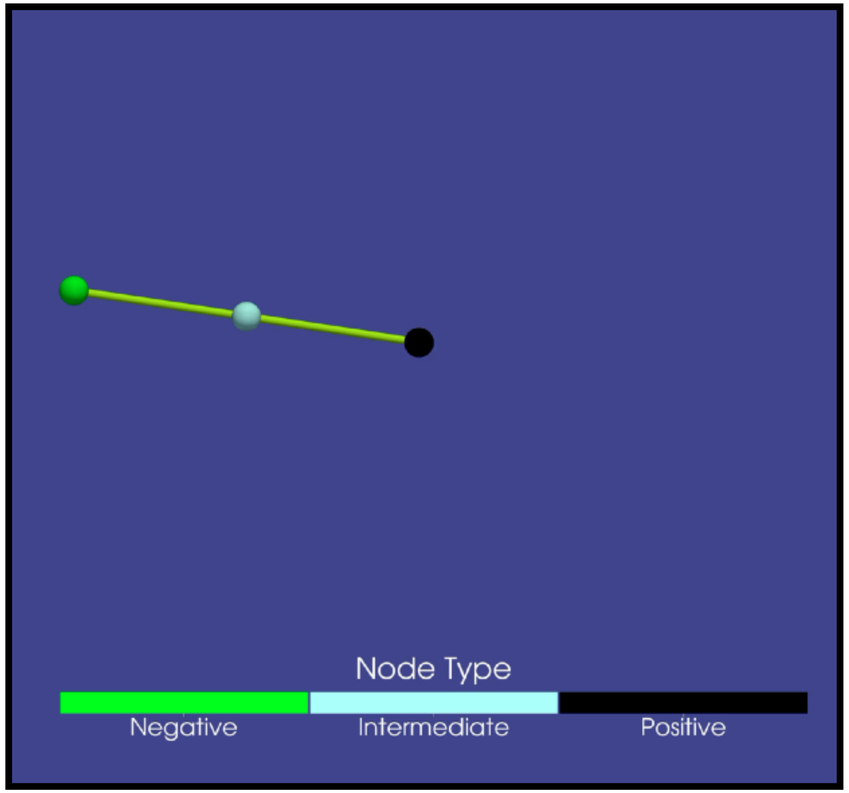


Figure 3: Example of the two rules combined for our approximation of a growing microtubule.

1. More examples in thesis and (Medwedeff and Mjolsness, 2023)

Grammar Rules

Muitcomponent Rules

- The left and right side graphs may have more than one connected component.
- A set of nodes forms a ***connected component*** in an undirected graph if any node in the set can reach any other node in the set by traversing edges.

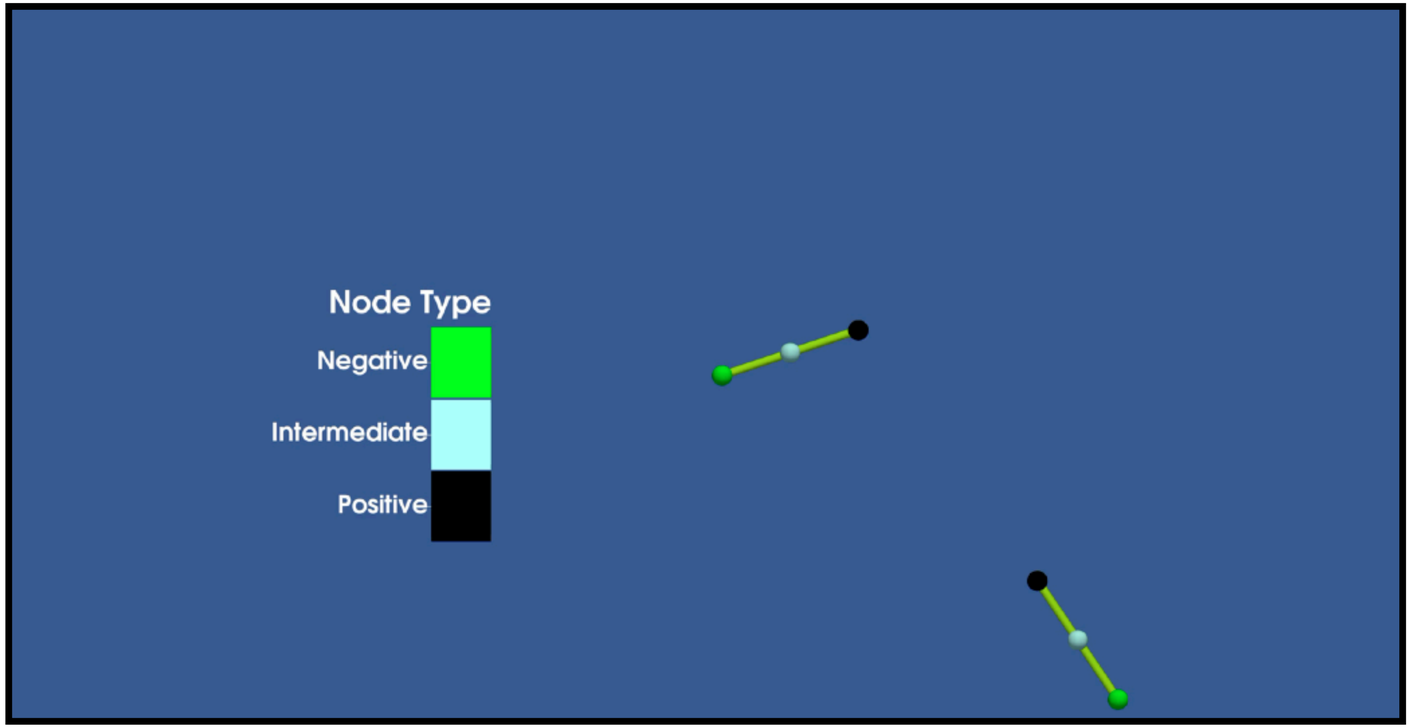
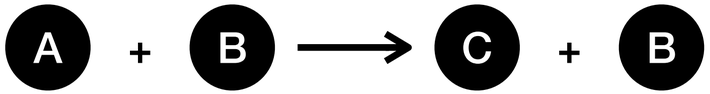


Figure 4: Example of two Microtubules colliding. The graphs are spatially embedded, and the collision is spatially local!



Think of A, B, C as short hand for connected components of the graphs on the left and right.

Stochastic Collision Induced Catastrophe Rule[1]:

$$(\ \bigcirc_1 \text{—} \bullet_2,\ \bigcirc_3 \text{—} \bigcirc_4)\ \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3), (\boldsymbol{x}_4, \boldsymbol{u}_4) \rangle\!\rangle$$

$$\longrightarrow (\ \bigcirc_1 \text{—} \blacksquare_2,\ \bigcirc_3 \text{—} \bigcirc_4)\ \langle\!\langle (\boldsymbol{x}_1, \boldsymbol{u}_1), (\boldsymbol{x}_2, \boldsymbol{u}_2), (\boldsymbol{x}_3, \boldsymbol{u}_3), (\boldsymbol{x}_4, \boldsymbol{u}_4) \rangle\!\rangle$$

(Propensity ρ_r excluded for clarity)

1. More examples in thesis.