Graph Rewrites

Example of how Graph Rewrites in DGGML Work and Operator Semantics

- When reading from right to left:
  - all vertices and vertex labels of the LHS graph are annihilated (destroyed) in an arbitrary order.
  - Next all edges are annihilated in any order, then all vertices and vertex labels of the RHS graph are created in any order.
  - After, all edges of the RHS are created in any order.
  - The final operators on the first line are the dangling edge erasure operators that enforce that edges must connect active vertices.

10

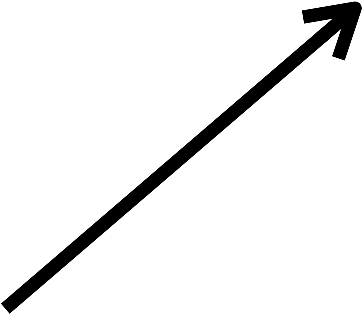
Category theoretic approaches can be found in (Lowe, 1993) and (Ehrig et al., 1973)

Left-hand Side (LHS) **Right-hand Side (RHS)**

$$(\circ_1 - \bullet_2) \longrightarrow (\circ_1 - \circ_3 - \bullet_2)$$

Rewrite operations:

1. Delete edge (1, 2)
2. Create node 3
3. Create edge (3, 1)
4. Create edge (3, 2)



- Formally, graph rewrites can be expressed using operators (Mjolsness 2019):

$$\hat{W}_r \propto \rho_r(\lambda, \lambda') \sum_{\langle i_1 \dots i_k \rangle_{\neq}} \left[\left(\prod_{p \in B_r} \prod_{i \neq i_q | \forall q \in \overline{B}_r p} E_{i_p\, i} \right) \left(\prod_{p \in C_r} \prod_{i \neq i_q | \forall q \in \overline{C}_r p} E_{i\, i_p} \right) \right]$$
$$\times \left[\left(\prod_{p', q' \in \mathrm{rhs}(r)} \left(\hat{a}_{i_{p'} i_{q'}} \right)^{g'_{p'q'}} \right] \left[\left(\prod_{p' \in \mathrm{rhs}(r)} \left(\hat{a}_{i_{p'} \lambda'_{p'}} \right)^{h_{p'}} \right]$$
$$\times \left[\left(\prod_{p, q \in \mathrm{lhs}(r)} \left(a_{i_p i_q} \right)^{g_{pq}} \right] \left[\left(\prod_{p \in \mathrm{lhs}(r)} \left(a_{i_p \lambda_p} \right)^{h_p} \right]$$

Graph Rewrites

Example of how Graph Rewrites in DGGML Work and Operator Semantics

- Formally, graph rewrites can be expressed using operators (Mjolsness 2019):

$$\hat{W}_r \propto \rho_r(\lambda, \lambda') \sum_{\langle i_1 \ldots i_k \rangle_{\neq}} \left[\left(\prod_{p \in B_r} \prod_{i \neq i_q | \forall q \in \overline{B}_r p} E_{i_p\, i} \right) \left(\prod_{p \in C_r} \prod_{i \neq i_q | \forall q \in \overline{C}_r p} E_{i\, i_p} \right) \right]$$

$$\times \left[\left(\prod_{p', q' \in \mathrm{rhs}(r)} \left(\hat{a}_{i_{p'} i_{q'}} \right)^{g'_{p'q'}} \right) \right] \left[\left(\prod_{p' \in \mathrm{rhs}(r)} \left(\hat{a}_{i_{p'} \lambda'_{p'}} \right)^{h_{p'}} \right) \right]$$

$$\times \left[\left(\prod_{p, q \in \mathrm{lhs}(r)} \left(a_{i_p i_q} \right)^{g_{pq}} \right) \right] \left[\left(\prod_{p \in \mathrm{lhs}(r)} \left(a_{i_p \lambda_p} \right)^{h_p} \right) \right]$$

Left-hand Side (LHS)		**Right-hand Side (RHS)**
$(\circ_1 - \bullet_2)$	$\longrightarrow$	$(\circ_1 - \circ_3 - \bullet_2)$

Rewrite operations:

1. Delete edge (1, 2)
2. Create node 3
3. Create edge (3, 1)
4. Create edge (3, 2)

- When reading from right to left:
  - all vertices and vertex labels of the LHS graph are annihilated (destroyed) in an arbitrary order.
  - Next all edges are annihilated in any order, then all vertices and vertex labels of the RHS graph are created in any order.
  - After, all edges of the RHS are created in any order.
  - The final operators on the first line are the dangling edge erasure operators that enforce that edges must connect active vertices.

Category theoretic approaches can be found in (Lowe, 1993) and (Ehrig et al., 1973)

Grammar Rules

Example Deterministic and Stochastic Rules from CMA DGG

$$A \longrightarrow A \quad \longleftrightarrow$$

i.e. A transforms to A with a rewrite as a label update via ODE!

Deterministic Growth Rule[1]:

$$(\bigcirc_1 - \bullet_2)\langle\!\langle(\boldsymbol{x}_1,\boldsymbol{u}_1)(\boldsymbol{x}_2,\boldsymbol{u}_2)\rangle\!\rangle$$

$$\longrightarrow (\bigcirc_1 - \bullet_2)\langle\!\langle(\boldsymbol{x}_1,\boldsymbol{u}_1),(\boldsymbol{x}_2 + d\boldsymbol{x}_2,\boldsymbol{u}_2)\rangle\!\rangle$$

$$\textbf{solving} \quad d\boldsymbol{x}_2/dt = v_{plus} \times \boldsymbol{u}_2$$

$$A \longrightarrow B \quad \longleftrightarrow$$

i.e. A transforms to B with a rewrite as a label updates in the form of graph structure changes!

Stochastic Growth Rule[1]:

$$(\bigcirc_1 - \bullet_2)\langle\!\langle(\boldsymbol{x}_1,\boldsymbol{u}_1),(\boldsymbol{x}_2,\boldsymbol{u}_2)\rangle\!\rangle$$

$$\longrightarrow (\bigcirc_1 - \bigcirc_3 - \bullet_2)\langle\!\langle(\boldsymbol{x}_1,\boldsymbol{u}_1),(\boldsymbol{x}_2,\boldsymbol{u}_2),(\boldsymbol{x}_3,\boldsymbol{u}_3)\rangle\!\rangle$$

$$\textbf{with} \quad \hat{\rho}_{\text{grow}} \times H(\|\boldsymbol{x}_2 - \boldsymbol{x}_1\|; L_{\text{div}})$$

$$\textbf{where} \quad \begin{cases} \boldsymbol{x}_3 = \boldsymbol{x}_2 - (\boldsymbol{x}_2 - \boldsymbol{x}_1)\gamma \\ \boldsymbol{u}_3 = \frac{\boldsymbol{x}_3 - \boldsymbol{x}_2}{\|\boldsymbol{x}_3 - \boldsymbol{x}_2\|} \end{cases}$$

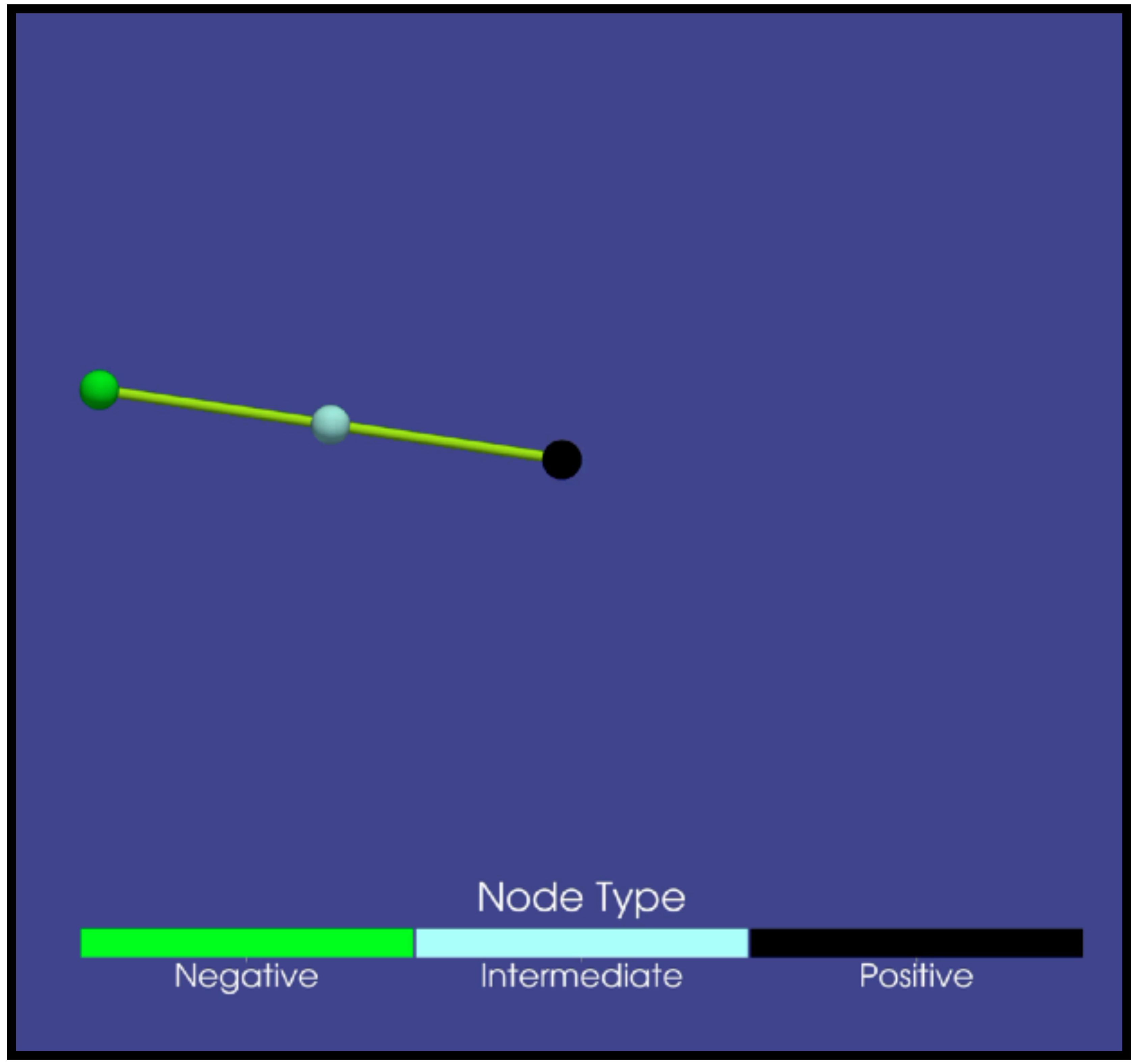


Figure 3: Example of the two rules combined for our approximation of a growing microtubule.

1. More examples in thesis and (Medwedeff and Mjolsness, 2023)