CHE 260: THERMODYNAMICS AND HEAT TRANSFER

FINAL EXAMINATION FOR HEAT TRANSFER

20th DECEMBER 2019

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3	Q4	Q5	Total
20	20	20	15	25	100

INSTRUCTIONS

1. This examination is closed textbook, closed internet, closed all communication devices.
2. One aid sheet of size 8.5" x 11" (both sides) is permitted.
3. Any non-communicating, non-programming, non-graphing calculator is permissible.
4. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
5. Answers written in pencil will NOT be re-marked. This is University policy.
6. For all problems, you must present the solution process in a step by step fashion for partial marks.
7. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY.**

Q.1. [20 points] WHO KILLED MR. McDUCK?

Mr. McDuck is discovered murdered at 9:00 pm in the big refrigerator of his office by the floor janitor, Duckworth. A police unit, headed by Inspector McQuack, a former Eng. Sci. graduate who aced CHE260, arrives at 9:30 pm with the forensic team. At that time, Inspector McQuack asks the medical examiner accompanying the police unit to measure the surface temperature on Mr. McDuck's waist and the temperature near the center of his body (Model the body as a 28 cm diameter, 1.8 m long cylinder). These temperatures are found to be 28°C and 15°C, respectively.

Center Surface

The office is secured by a lock that needs to be swapped with an ID card upon entry and exit. Inspector McQuack demands to see the electronic log sheet for the lock, and is given the following table by security personnel:

PERSON	LOG IN	LOG OUT
Mr. McDuck	6:00 am	-
Mr. Glomgold	6:30 am	11:00 am
Ms. De Spell	10:00 am	5:00 pm
Mrs. Beagle	2:00 pm	7:00 pm
Mr. Duckworth	8:45 pm	9:00 pm
Mr. Duckworth	9:30 pm	9:45 pm

The temperature inside the refrigerator is maintained at 4°C, and the initial temperature of the body is 37°C. Assuming the thermal conductivity and the thermal diffusivity of the body to be 0.62 W/m°C and 1.5×10^{-7} m^2/s, respectively, answer the following questions:

(a) **[13 points]** What is Mr. McDuck's time of death? [Note: the heat transfer coefficient, h, is to be determined as part of the solution to this problem [part (c)]].

(b) **[2 points]** Who are Inspector McQuack's likely suspects? Explain.

(c) **[2 points]** What is the heat transfer coefficient for convective heat transfer between the body and the air in the refrigerator?

(d) **[3 points]** How much total heat (kJ) is lost from Mr. McDuck's body from the time of death to 9:30 pm?

Q.2. [20 points] FIN PROBLEM

A 0.25 cm thick, 20 cm high, and 10 cm long circuit board houses 80 closely-spaced logic chips on one side, each dissipating 0.1 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 30 W/m·°C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 25°C, with a heat transfer coefficient of 15 W/(m^2 °C).

(a) **[5 points]** Determine the steady state temperatures on the two sides of the circuit board. Use the resistance network approach and sketch the circuit.

(b) **[2 points]** Is it worthwhile to add a fin block (heat sink) to the board? Explain.

(c) **[13 points]** Now, a 0.2 cm thick, 20 cm high and 10 cm long aluminium plate (k = 237 W/m-K) with 400, 3-cm-long aluminium <u>pin fins of rectangular profile</u> each of base diameter 0.5 cm is attached to the back side of the circuit board with a 0.02 cm thick epoxy adhesive (k = 1.5 W/m-K). At steady state, determine

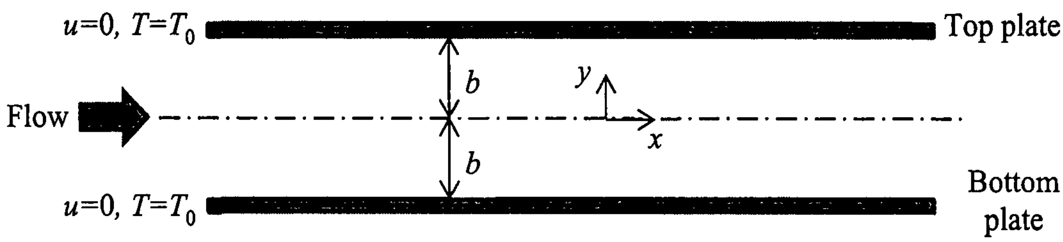
(a) the new temperatures on the two sides of the circuit board, and

(b) the effectiveness of the fin block.

Again, use the resistance network approach and sketch the circuit. Assume that the same heat transfer coefficient as part (a) is applicable in this configuration.

Q.3. [20 points] HEATING OF A LIQUID BY VISCOUS DISSIPATION

In this problem, you will examine a simplified model for the temperature distribution developed in a flowing liquid due to an effect called viscous heating, which manifests when highly viscous oils or polymers are sheared at sufficiently large rates.



Consider the pressure-driven flow of a liquid between two parallel plates separated by a distance $2b$. The walls of the plate are fixed (zero velocity), and are maintained at a temperature of T_0. The flow is unidirectional in the x-direction. The velocity in the x-direction is u, which is a function of the co-ordinate y perpendicular to the plane of plates and measured from the centerline. Due to symmetry, we consider only the interval $y \in [0,b]$. At steady state and under fully developed conditions , the governing equation for u is

$$\mu \frac{d^2u}{dy^2} = -G, \tag{1}$$

with the boundary conditions,

$$\left.\frac{du}{dy}\right|_{y=0} = 0, \text{ and } \left.u\right|_{y=b} = 0. \tag{2}$$

In Eq. 1, G is the pressure gradient (Pa/m) driving the flow between the channels, and is a constant for this problem. μ is the viscosity (Pa-s) of the liquid.

The steady state, fully developed temperature distribution, $T(y)$, obeys the following familiar equation:

$$k\frac{d^2T}{dy^2} = -\mu\left(\frac{du}{dy}\right)^2. \tag{3}$$

The term on the LHS is the conductive term, and the term on the RHS is due to viscous heating of the sheared liquid, which is a volumetric heat source in this problem.

The boundary conditions for the temperature distribution are

$$\left.\frac{dT}{dy}\right|_{y=0} = 0, \text{ and } \left.T\right|_{y=b} = T_0. \qquad (4)$$

Answer the following questions:

(a) **[1 point]** Make a complete list of all the parameters in this problem (i.e. all variables other than u, T and x).

(b) **[4 points]** Render the governing equations and boundary conditions for u (Eqs. 1 and 2) dimensionless. Do not solve the equations yet! Present a summary of the dimensionless equations for the velocity distribution.

(c) **[5 points]** Render the governing equations and boundary conditions for temperature T (Eqs. 3 and 4) dimensionless. Again, do not solve the equations. Present a summary of the dimensionless equations for the temperature distribution.

(d) **[3 points]** Report the temperature scale and velocity scale you got from parts (b) and (c). What are the dependences of these scales on the parameters listed in part (a)? List the dimensionless parameter(s) that emerge in the scaled equations. Compare the number of dimensionless parameters to the number of parameters in the original, dimensional problem, noted in part (a).

(e) **[3 points]** Solve the dimensionless ODE + boundary conditions from part (b) to obtain the dimensionless velocity profile.

(f) **[4 points]** Solve the dimensionless ODE + boundary conditions from part (c) to obtain the dimensionless temperature profile.

Q. 4. [15 points] SURFACE TEMPERATURE OF A LIGHT BULB

Consider a 10-cm-diameter spherical, 100-W light bulb cooled by a fan that blows air at 30°C to the bulb at a velocity of 2 m/s. The surrounding surfaces are also at 30°C and the emissivity of the glass surface is 0.9. Assuming that 10 percent of the energy passes through the glass bulb as light with negligible absorption, and the rest of the energy is absorbed and dissipated by the bulb itself (i.e. 90% of the electrical energy is converted to heat), determine the equilibrium surface temperature of the bulb. You will need to determine the convective heat transfer coefficient using a suitable correlation.

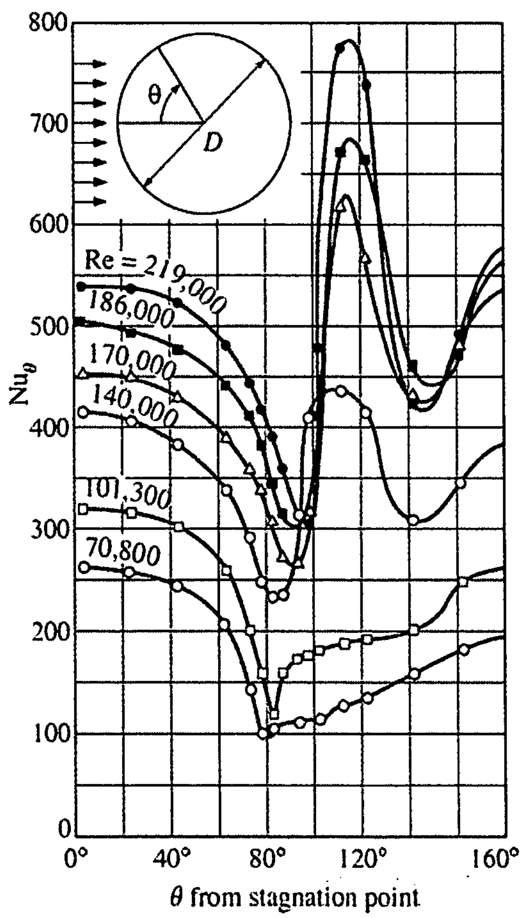
Use the following properties of air: $k = 0.02588$ W/m-K, $\nu = 1.608 \times 10^{-5}$ m²/s, $\mu_S = 2.181 \times 10^{-5}$ kg/m-s, $\mu_\infty = 1.872 \times 10^{-5}$ kg/m-s, Pr = 0.7282.

The Stefan Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴

Note: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$.

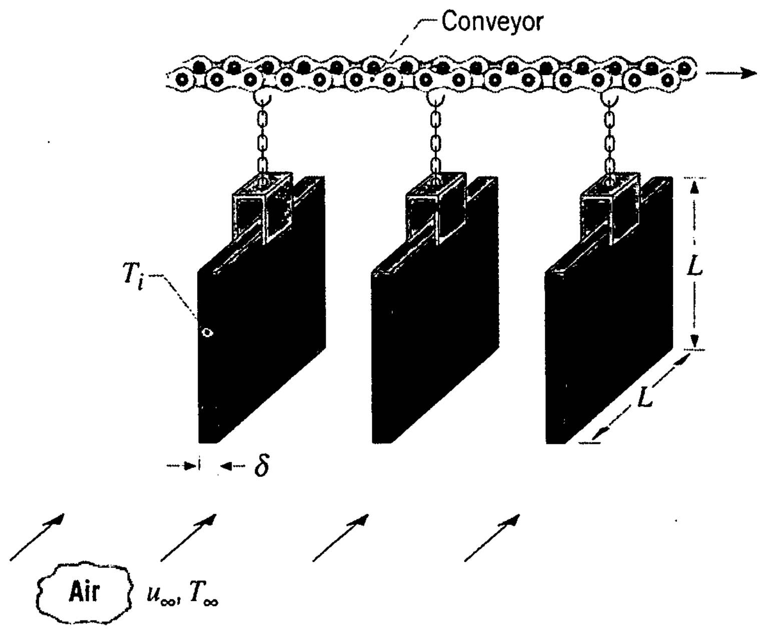
Q.5A. [4 points] LOCAL HEAT TRANSFER COEFFICIENT FOR FLOW PAST A CYLINDER

Consider the uniform flow of a cold fluid over a hot cylinder of diameter D. Graphs of the local Nusselt number as a function of θ are shown in the adjacent picture for different Reynolds numbers. Explain the mechanisms behind the variations of h with θ for the smallest and the largest Reynolds numbers shown in the picture.



Q.5B. [13 points] COOLING OF PLATES ON A CONVEYOR BELT

Hot square steel plates of side L = 1 m and thickness δ = 3 mm are initially at a temperature of T_i = 300°C. The plates are mounted on a conveyor belt, which carries the plates into a cooling chamber. There, air at T_∞ = 25°C blows at a velocity of u_∞ = 20 m/s parallel to the lateral surfaces of the plates as shown in the figure. The properties of the steel plates are k_{plate} = 49.2 $W/m \cdot K$, C_{plate} = 549 $J/kg \cdot K$, and ρ_{plate} = 7832 kg/m^3.



Your task: Find the temperature profile in the plate after 5 min and 20 min.

Note: You will need to calculate the heat transfer coefficient using an appropriate correlation. Use the following properties of air: ν = 30.4×10^{-6} m^2/s, k = 0.0361 W/m·K, and Pr = 0.688.

Q.5C. [3 points] DIMENSIONLESS NUMBERS IN HEAT TRANSFER

The definitions of the Nusselt number and the Biot numbers appear to be the same. Explain then, why two separate dimensionless numbers are required. Don't just give the definitions of the numbers. To get credit, explain what they mean physically.

Q.5D. [5 points] CRUSHED ICE OR ICE CUBES?

Choose one of (a) through (c) for the first question, and justify it with one of (1) through (5). Full credit if and only if answers to both questions are correct.

You have a glass of tea in a well-insulated cup that you would like to cool off before drinking. You also have 2 ice cubes to use in the cooling process and an equivalent mass of crushed ice. Assuming no energy is lost from the tea into the room, which form of ice (cubes or crushed) added to your tea will give a lower final drink temperature?

(a) The crushed ice (b) The ice cubes

(c) Either will lower the drink temperature the same amount

Because...

(1) Energy transfer is proportional to the mass of ice used

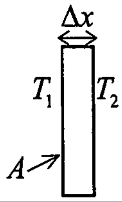
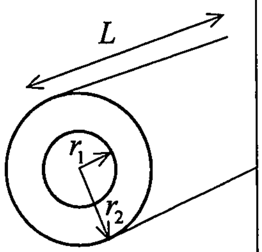
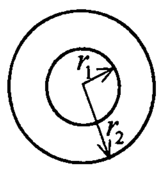
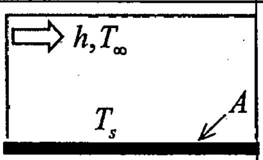
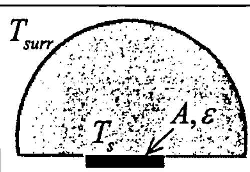
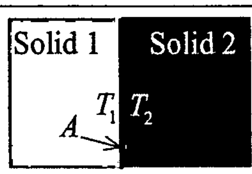
(2) Crushed ice will melt faster and will transfer energy from the tea faster

(3) Ice cubes contain less energy per mass than crushed ice so tea will cool more

(4) Ice cubes have a higher heat capacity than crushed ice

(5) Crushed ice has more surface area so energy transfer rate will be higher

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\Delta x}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi kL}$
Spherical shell (Hollow sphere)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \dfrac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \dfrac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \dfrac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \dfrac{1}{\varepsilon\sigma A\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)}$
Thermal contact resistance		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{R_c}{A}$ (R_c has units of °C-m²/W)

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot \left(k \nabla T \right) + \dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_x = -k \frac{\partial T}{\partial x}, \quad \dot{q}_y = -k \frac{\partial T}{\partial y}, \quad \dot{q}_z = -k \frac{\partial T}{\partial z}$

<u>Constant k:</u> $$\rho C \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{S}$$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta}\left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_r = -k \frac{\partial T}{\partial r}, \quad \dot{q}_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}, \quad \dot{q}_z = -k \frac{\partial T}{\partial z}$

<u>Constant k:</u> $$\rho C \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \dot{S}$$

SPHERICAL CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_r = -k \frac{\partial T}{\partial r}, \quad \dot{q}_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}, \quad \dot{q}_\phi = -k \frac{1}{r \sin\theta} \frac{\partial T}{\partial \phi}$

<u>Constant k:</u> $\rho C \frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \varphi^2} \right] + \dot{S}$

TABLE 11–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

TABLE 11–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 11–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Total heat transferred:

$$\frac{Q}{Q_{\max}} = 1-\theta_0 \frac{\sin\lambda_1}{\lambda_1} \qquad \text{Plane wall}$$

$$= 1-2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1} \qquad \text{Cylinder}$$

$$= 1-3\theta_0 \frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} \qquad \text{Sphere}$$

Here, $Q_{\max} = mC(T_\infty - T_i)$, where m and C and the mass and the specific heat capacity, respectively, of the solid. θ_0 is a dimensionless center temperature.

NUSSELT NUMBER CORRELATIONS

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\text{Re}_{x,cr} = \frac{\rho V x_{cr}}{\mu} = 5\times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \dfrac{0.664}{\text{Re}_x^{1/2}}, \qquad \text{Re}_x < 5\times 10^5$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}, \qquad \text{Pr} > 0.6$$

Turbulent: $C_{f,x} = \dfrac{0.059}{\text{Re}_x^{1/5}}, \qquad 5\times 10^5 \le \text{Re}_x \le 10^7$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}, \qquad 0.6 \le \text{Pr} \le 60, \quad 5\times 10^5 \le \text{Re}_x \le 10^7$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \dfrac{1.33}{\text{Re}_L^{1/2}}, \qquad \text{Re}_L < 5\times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}}, \qquad 5\times 10^5 \le \text{Re}_L \le 10^7$

Combined: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}} - \dfrac{1742}{\text{Re}_L}, \qquad 5\times 10^5 \le \text{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62\log\dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\text{Nu} = \dfrac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3}, \qquad \text{Re}_L < 5\times 10^5$

Turbulent:

$$\text{Nu} = \frac{hL}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3}, \qquad 0.6 \le \text{Pr} \le 60, \quad 5\times 10^5 \le \text{Re}_L \le 10^7$$

Combined:

$$\text{Nu} = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3}, \qquad 0.6 \le \text{Pr} \le 60, \quad 5\times 10^5 \le \text{Re}_L \le 10^7$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for}\ \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for}\ \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \dfrac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$

Turbulent: $h = \dfrac{5[1-(\xi/x)^{9/10}]}{(1-\xi/L)}\,h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\text{Nu}_x = 0.453\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}$

Turbulent: $\text{Nu}_x = 0.0308\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\text{Nu}_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1+(0.4/\text{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\text{Nu}_{sph} = \frac{hD}{k} = 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \text{Re} \le 80{,}000$ and $0.7 \le \text{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

Efficiency and surface areas of common fin configurations

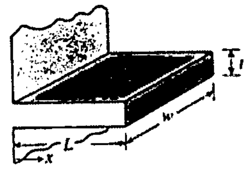
Straight rectangular fins

$m = \sqrt{2h/kt}$

$L_c = L + t/2$

$A_{fin} = 2wL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

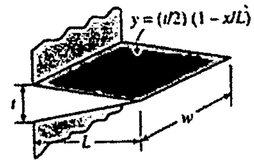


Straight triangular fins

$m = \sqrt{2h/kt}$

$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$

$$\eta_{fin} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



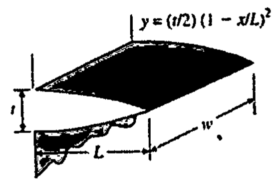
Straight parabolic fins

$m = \sqrt{2h/kt}$

$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$

$C_1 = \sqrt{1 + (t/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



Circular fins of rectangular profile

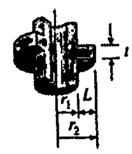
$m = \sqrt{2h/kt}$

$r_{2c} = r_2 + t/2$

$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$

$$\eta_{fin} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



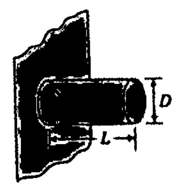
Pin fins of rectangular profile

$m = \sqrt{4h/kD}$

$L_c = L + D/4$

$A_{fin} = \pi DL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



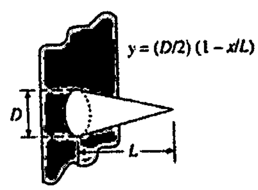
Pin fins of triangular profile

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$

$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

$I_2(x) = I_0(x) - (2/x)I_1(x)$ where $x = 2mL$



Pin fins of parabolic profile

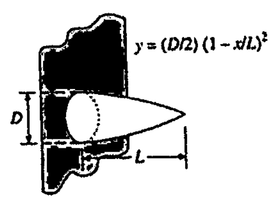
$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi L^3}{8D}\left[C_3C_4 - \frac{L}{2D}\ln(2DC_4/L + C_3)\right]$

$C_3 = 1 + 2(D/L)^2$

$C_4 = \sqrt{1 + (D/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$



Pin fins of parabolic profile (blunt tip)

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$

$$\eta_{fin} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$