AER210F VECTOR CALCULUS AND FLUID MECHANICS

Quiz 2

26 October 2015 8:50 am - 9:50 am

Closed Book, No aid sheets, No calculators

Instructor: J. W. Davis

Last Name: ______________________

Given Name: ______________________

Student #: ______________________

Tutorial/TA: ______________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	10	
2	10	
3	10	
4	10	
5	14	
TOTAL	54	/ 50

Note: The following integrals may be useful.

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C; \qquad \int \sin^2\theta\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dR; \qquad \oiint_S \vec{F}\cdot\hat{n}\,dS = \iiint_V \nabla\cdot\vec{F}\,dV; \qquad \oint_C \vec{F}\cdot d\vec{r} = \iint_S \nabla\times\vec{F}\cdot d\vec{S}$$

1) Let R be the region in the first quadrant bounded by the parabolas: $x - y^2 = 0$, $x - y^2 = -4$, $x + y^2 = 9$, and $x + y^2 = 16$. Use a coordinate transformation to evaluate $\iint_R xy\, dR$. Provide a sketch of the original region in the x-y plane, and the new region in the u-v plane.

(10 marks)

2) Verify Green's theorem for the line integral $\oint_C x^2 y\,dx + e^y\,dy$, where C is the triangle with vertices (0, 1), (0, 0) and (1, 1).

3) The fundamental theorem of calculus as applied to volume integrals gives the following results: for a function $f(x, y, z)$ which is continuous over a volume V enclosed by a surface S, if $\hat{n} = n_x\hat{i} + n_y\hat{j} + n_z\hat{k}$ is the unit normal on S pointing to the exterior of V, then

$$\int_V \frac{\partial f}{\partial x} dV = \int_S f\hat{n}\cdot\hat{i}\, dS \; ; \quad \int_V \frac{\partial f}{\partial y} dV = \int_S f\hat{n}\cdot\hat{j}\, dS \; ; \quad \int_V \frac{\partial f}{\partial z} dV = \int_S f\hat{n}\cdot\hat{k}\, dS$$

Use this result to derive the Gradient and Divergence Theorems.

(10 marks)

4) Evaluate the surface integral $\int_S \vec{F} \cdot d\vec{S}$ for the vector field $\vec{F} = x\hat{i} - y\hat{j} + z\hat{k}$, where S consists of the paraboloid $x = y^2 + z^2$, $0 \le x \le 1$, and the disk $y^2 + z^2 \le 1$, $x = 1$.

(10 marks)

5) Verify the Stokes' theorem for the vector field $\bar{F}(x,y,z) = z\hat{i} + 2xz\,\hat{j} + xy\,\hat{k}$ over the part of the plane $2x + 4y + z = 8$ in the first octant. Provide a sketch of the surface and the boundary curve.

(14 marks)