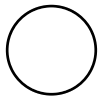
CHE 260 FINAL - 2021

(1)



$D = 0.01\ m$

$T_i = 80°C.$ $\qquad T_f = 60°C.$

$T_\infty = 20°C.$

$k = 401 \frac{W}{m°C}, \quad C_p = 385\ J/kg°C, \quad \rho = 8933\ kg/m^3$

$h_i = 4.44(60)^{5/4} = 741 \frac{W}{m^2 °C} \Rightarrow Bi = \frac{h_i D}{k} = \frac{741 \times 0.01}{401} = 0.02$

$\Rightarrow$ use lumped capacitance approximation.

$$m C_p \frac{dT}{dt} = -h A_s (T - T_\infty)$$

$$= -4.44 A_s (T - T_\infty)^{5/4}$$

$$\Rightarrow \int_{T_i}^{T_f} \frac{d(T-T_\infty)}{(T-T_\infty)^{5/4}} = -\frac{4.44(\pi D^2)}{\rho \left(\frac{\pi D^3}{6}\right) C_p} \int_0^{t_f} dt$$

$$\Rightarrow -4(T-T_\infty)^{-1/4} \Big|_{T_i}^{T_f} = -\frac{4.44 \times 6}{\rho C_p D} t_f$$

$$\Rightarrow \left[(T_f - T_\infty)^{-1/4} - (T_i - T_\infty)^{-1/4}\right] = \frac{26.64}{\rho C_p D} t_f$$

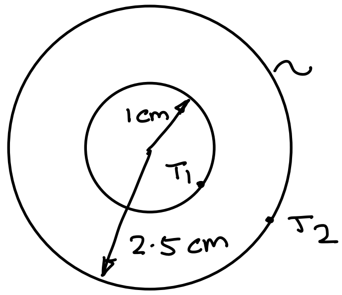
$$\Rightarrow t_f = \frac{\rho C_p D}{6.66} \left[\frac{1}{(T_f - T_\infty)^{1/4}} - \frac{1}{(T_i - T_\infty)^{1/4}}\right]$$

$$\Rightarrow t_f = \frac{8933 \frac{kg}{m^3} \times 385\ J/kg°C \times 0.01\ m.}{6.66} \left[\frac{1}{(60-20)^{1/4}} - \frac{1}{(80-20)^{1/4}}\right]$$

$$t_f = 198\ s$$

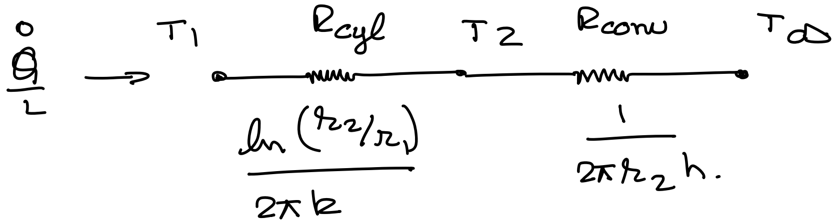
(2)

$T_\infty = 27^\circ C$



$h = 120\ W/m^2\,^\circ C.$

$$\frac{\dot{Q}}{L} = 2\pi r_1 \dot{q} = 2\pi (0.01\,m)\, 10^5 \frac{W}{m^2}.$$
$$= 6283\ W/m.$$



$$\frac{T_2 - T_\infty}{R_{conv}} = \frac{\dot{Q}}{L}$$

$$\Rightarrow \frac{\dot{Q}}{L} = 2\pi r_2 h\,(T_2 - T_\infty)$$

$$\Rightarrow 6283\ W/m = 2\pi(0.025\,m)\left(120 \frac{W}{m^2\,^\circ C}\right)(T_2 - 27^\circ C)$$

$$\Rightarrow T_2 = 360.3^\circ C.$$

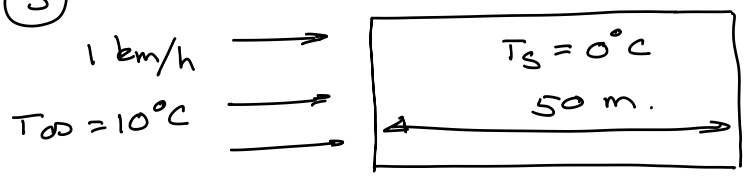
$$\frac{T_1 - T_2}{R_{cyl}} = \frac{\dot{Q}}{L}.$$

$$\Rightarrow \frac{\dot{Q}}{L} = \frac{2\pi k\,(T_1 - T_2)}{\ln(r_2/r_1)}$$

$$\Rightarrow 6283 = \frac{2\pi\,(2.2\ W/m\,^\circ C)\,(T_1 - 360.3)}{\ln(2.5\,cm/1\,cm)}$$

$$\Rightarrow T_1 = 776.8^\circ C.$$

③



$$\dot{m} = \rho_{ice} A \frac{dL}{dt}$$

L - thickness of iceberg
$\dot{m}$ - melting rate

$$\dot{Q} = \dot{q} A = \dot{m}\, h_{sl}.$$

$$\Rightarrow \quad h A (T - T_\infty) = \rho_{ice} A \frac{dL}{dt} \cdot h_{sl}.$$

$$\Rightarrow \quad \frac{dL}{dt} = \frac{h (T - T_\infty)}{\rho_{ice}\, h_{sl}}$$

Properties of water at 5°C

$\rho = 999.9°c$, $\mu = 1.519 \times 10^{-3} \frac{kg}{ms}$, $k = 0.571 \frac{W}{m°C}$, $Pr = 11.2$

$$V_\infty = 1 \frac{km}{h} \times \frac{1000 \text{ m/km}}{3600 \text{ s/h}} = 0.278 \text{ m/s}$$

$$Re = \frac{\rho V_\infty L}{\mu} = \frac{999.9 \text{ kg/m}^3 \times 0.278 \frac{m}{s} \times 50\text{ m}}{1.519 \times 10^{-3} \text{ kg/ms}} = 9.15 \times 10^6$$

$$Nu = (0.037\, Re^{0.8} - 871)\, Pr^{1/3}$$

$$Nu = (0.037\,(9.15\times10^6)^{0.8} - 871)(11.2)^{1/3} = 28747$$

$$h = Nu \frac{k}{L} = \frac{28747 \times 0.571 \text{ W/m°C}}{50 \text{ m}} = 328.3 \frac{W}{m^2 °C}$$

$$\Rightarrow \frac{dL}{dt} = \frac{328.3 \frac{W}{m^2 °C} (10-0)}{917 \text{ kg/m}^3 \times 333.4 \times 10^3 \text{ J/kg}} = 1.07 \times 10^{-5} \text{ m/s}$$

$$\frac{dL}{dt} = 38.5 \text{ mm/h.}$$

(4) Properties of air at 60°C.

$\nu = 1.896\times10^{-5}\ m^2/s$; $k = 0.02808\ \frac{W}{mK}$; $Pr = 0.7202$.

$$Re = \frac{V_\infty L}{\nu} = \frac{5\ m/s \times 0.06\ m}{1.896\times10^{-5}\ m^2/s} = 1.58\times10^4$$

$$Nu = 0.664\ Re^{0.5}\ Pr^{1/3} = 0.664\,(1.58\times10^4)^{0.5}\,(0.7202)^{1/3}$$

$$Nu = 74.8$$

$$\Rightarrow\ h = Nu\ k/L = 74.8 \times \frac{0.02808\ W/m^\circ C}{0.06\ m} = 35.0\ W/m^2\,^\circ C$$

For fin $\quad P = 2(0.06\,m) + 2(0.001\,m) = 0.122\ m.$

$$A_c = 0.06\ m \times 0.001\ m = 6\times10^{-5}\ m^2.$$

For Al, $k = 237\ W/m^\circ C$.

$$a = \sqrt{\frac{hP}{kA_c}} = \left(\frac{35\ W/m^2\,^\circ C \times 0.122\,m}{237\ \frac{W}{m^\circ C}\times 6\times10^{-5}\,m^2}\right)^{1/2} = 17.3\ m^{-1}$$

$$L_c = L + \frac{t}{2} = 0.006\ m + \frac{0.001\ m}{2} = 0.0065\ m.$$

$$\dot{Q}_{fin} = \sqrt{hPkA}\,(T_b - T_\infty)\tanh aL_c$$

$$\dot{Q}_{fin} = \left(35\ \frac{W}{m^2\,^\circ C}\times 0.122\,m \times 237\ \frac{W}{m^\circ C}\times 6\times10^{-5}\,m^2\right)^{1/2}(100-20)\tanh(17.3\times0.0065)$$

$$\dot{Q}_{fin} = 2.21\ W$$

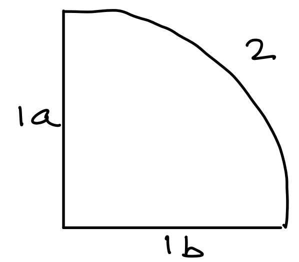
$$\dot{Q}_{fin,\ total} = 6\times 2.21\ W = 13.3\ W.$$

$$\dot{Q}_{unfin,\ total} = h\,A_{unfin}\,(T_b - T_\infty)$$

$$= 35\ \frac{W}{m^2\,^\circ C}\times 5\,(0.06\times0.01)(100-20) = 8.4\,W$$

$$\dot{Q}_{total} = 13.3 + 8.4 = 21.7\ W.$$

5)



$$F_{1a\to 1b} = 1 - \sin\left(\frac{90^\circ}{2}\right)$$

$$= 0.2929.$$

$$F_{1a\to 1b} + F_{1a\to 2} = 1.$$

$$\Rightarrow F_{1a\to 2} = 1 - 0.2929 = 0.7071$$

$$\Rightarrow F_{2\to 1a} = \frac{A_{1a}}{A_2} F_{1a\to 2}$$

$$= \frac{R}{\pi R/2} \times 0.7071 = 0.4502$$

By symmetry $F_{2\to 1b} = 0.4502.$

$$\Rightarrow F_{2\to 1} = 0.9003$$

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{\pi R/2}{2R} \times 0.9003 = 0.7071$$

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_2 F_{21}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{5.67\times10^{-8}(1200^4 - 500^4)}{\frac{1-0.5}{2(0.1)0.5} + \frac{1}{\frac{\pi(0.1)}{2}0.9003} + \frac{1-0.9}{\frac{\pi(0.1)}{2}\times0.9}}$$

$$= 8923 \text{ W/m}.$$