Midterm Test: PHY 294
[3 problems, 15 points, 60 mins] (11 Feb 2019, 9:30am-10:30am)

Useful formulae

Units, Constants, Identities

$$\begin{aligned} 1\text{eV} &= 1.6 \times 10^{-19}\text{J} \\ \hbar &= 1.05 \times 10^{-34} kg\ m^2/s \\ \mathrm{e}^{ix} &= \cos x + i \sin x \end{aligned} \tag{1}$$

Harmonic oscillator frequency

$$\Omega = \sqrt{\frac{K}{m}} \quad (K = \text{spring constant})$$

Time-dependent Schrodinger equation in 1D

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x)\psi(x,t) \tag{2}$$

Time-independent Schrodinger equation in 1D

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) = E\psi(x) \tag{3}$$

Harmonic oscillator

$$\begin{aligned} E_n &= (n + \frac{1}{2})\hbar\Omega \qquad (n = 0, 1, 2 \ldots) \\ \psi_0(x) &= (\frac{m\Omega}{\pi\hbar})^{1/4} \exp\left[-\frac{m\Omega x^2}{2\hbar}\right] \\ \psi_1(x) &= (\frac{m\Omega}{\pi\hbar})^{1/4} \sqrt{\frac{m\Omega}{2\hbar}}\ 2x \exp\left[-\frac{m\Omega x^2}{2\hbar}\right] \end{aligned}$$

Particle in a 1D box: $0 < x < L$

$$\begin{aligned} E_n &= \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad (n = 1, 2 \ldots) \\ \psi_n(x) &= \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L} \end{aligned}$$

Fourier transforms

$$\begin{aligned} \tilde{\psi}(k) &= \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{2\pi}}\ \mathrm{e}^{-ikx}\ \psi(x) \\ \psi(x) &= \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\pi}}\ \mathrm{e}^{+ikx}\ \tilde{\psi}(k) \end{aligned}$$

Integrals

$$\begin{aligned} \int_{-\infty}^{\infty} dx\ \mathrm{e}^{-\alpha x^2} &= \sqrt{\frac{\pi}{\alpha}} \\ \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}}\ \mathrm{e}^{-\alpha x^2}\ \mathrm{e}^{-ikx} &= \sqrt{\frac{1}{2\alpha}}\ \mathrm{e}^{-\frac{k^2}{4\alpha}} \\ \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}}\ x\mathrm{e}^{-\alpha x^2}\ \mathrm{e}^{-ikx} &= -\sqrt{\frac{1}{2\alpha}}\ \frac{ik}{2\alpha}\ \mathrm{e}^{-\frac{k^2}{4\alpha}} \end{aligned}$$

Name: **Number:** **Signature:**

1. Consider a particle in a box, confined to a region $-L < x < L$, with infinitely high walls. (5 points)

   (i) Find the wavefunction for the second excited state (i.e., $n = 3$).
   (ii) What is the corresponding energy?
   (iii) What would be the frequency of a photon emitted if the particle jumps to the state $n = 1$?

2. Consider a particle in the first excited state of a harmonic oscillator, with the normalized wavefunction

$$\psi(x) = (\frac{m\Omega}{\pi\hbar})^{1/4}\sqrt{\frac{m\Omega}{2\hbar}}\ 2x\ \exp\left[-\frac{m\Omega x^2}{2\hbar}\right]$$

Determine the momentum wavefunction $\tilde{\psi}(k)$ for this particle, and the most probable outcomes when you measure its momentum. (5 points)

3. Consider a potential step at $x = 0$, where the potential energy drops as (5 points)

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ \\ -V_0 & \text{for } x > 0 \end{cases}$$

where $V_0 > 0$. Sketch the potential energy as a function of x. For a particle incident from the left with energy $E > 0$, the most general wavefunction for $x < 0$ is of the form $A_1 \mathrm{e}^{ik_1x} + B_1\mathrm{e}^{-ik_1x}$, and the most general wavefunction for $x > 0$ is $A_2\mathrm{e}^{ik_2x}$. What are k_1 and k_2 in terms of $E, V_0, m, \hbar$? Using boundary conditions, and solving, determine the ratio $\frac{B_1}{A_1}$. For $V_0 = 4E$, determine the reflection coefficient $\frac{|B_1|^2}{|A_1|^2}$.