

CHE 260: THERMODYNAMICS AND HEAT TRANSFER

FINAL EXAMINATION FOR HEAT TRANSFER

14th DECEMBER 2017

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3	Q4	Q5	Total
15	15	15	15	15	75

INSTRUCTIONS

1. This examination is closed textbook, closed internet, closed all communication devices.
2. One aid sheet of size 8.5" x 11" aid sheet (both sides) is permitted.
3. Any non-communicating, non-programming, non-graphing calculator is permissible.
4. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
5. Answers written in pencil will NOT be re-marked. This is University policy.
6. For all problems, you must present the solution process in a step by step fashion for partial marks.
7. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY.**

Q.1. [15 points] HOT DOG OR LAUNDRY?

On a weekend afternoon, Mr. X, lazing around in the lounge of his home, suddenly feels hungry. He goes to his refrigerator looking for food, and finds a cylindrical hot dog of diameter 2 cm. The hot dog is at a uniform temperature of 4°C . Mr. X happily takes the hot dog out and places it into his convection oven preheated to a temperature of 150°C . The heat transfer coefficient for heat exchange between the oven and the hot dog surface is $300 \text{ W/m}^2\text{ }^{\circ}\text{C}$. 10 minutes later, Mr. X's wife yells at him, reminding him that he has been postponing doing the laundry for a week. Afraid of overcooking the hot dog, Mr. X immediately pulls the hot dog out of the oven, places it on a plate in the lounge, and then proceeds to take the clothes to the common laundry room of his building. The convective heat transfer coefficient for heat exchange between the hot dog and the air in the lounge is $5 \text{ W/m}^2\text{ }^{\circ}\text{C}$. The air in the lounge is maintained at 25°C .

Taking the thermal conductivity, specific heat capacity and density of the hot dog to be $0.5 \text{ W/m }^{\circ}\text{C}$, $4180 \text{ J/kg }^{\circ}\text{C}$ and 990 kg/m^3 , respectively, and assuming the heating and cooling of the hot dog to be axisymmetric, invariant along the height of the hot dog, and purely due to heat exchange with surrounding air, answer the following questions:

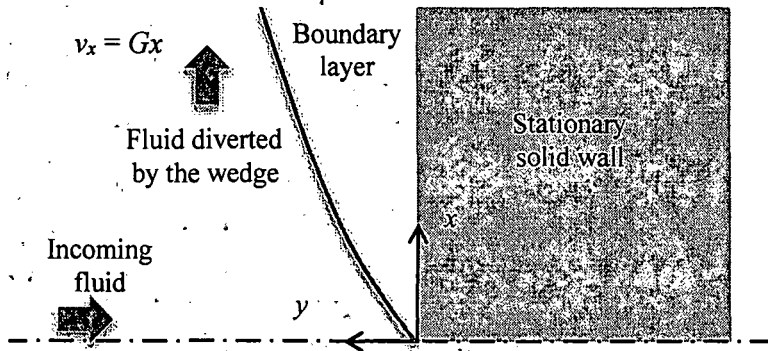
- (a) **[9 points]** What are the center and surface temperatures of the hot dog after Mr. X pulls it out of the oven?
- (b) **[6 points]** If Mr. X returns home from the laundry room in 60 min, what will be the center and surface temperatures of the hot dog at that time?

Q.2A. [8 points] HEAT TRANSFER RATE FROM DRAG

A streamlined body is moving at a velocity of 1 m/s against air at an ambient temperature of 25°C . The body is maintained at a temperature of 70°C . The drag force acting on the object is 2.1 N . What is the rate of heat transfer between the body and fluid? For air, use a density of 1.1 kg/m^3 , a specific heat capacity (at constant pressure) of $1007 \text{ J/kg }^{\circ}\text{C}$, a momentum diffusivity of $1.7 \times 10^{-5} \text{ m}^2/\text{s}$, and a Prandtl number of 0.72 , and assume these values to be constant over the temperature range of concern.

Q.2B. [7 points] FLOW PAST A WALL

Consider the flow of air directly against a plane wall as shown in the adjacent figure (note: only the top half of the geometry is shown). As air hits the wall and turns around to flow past it, a steady two-



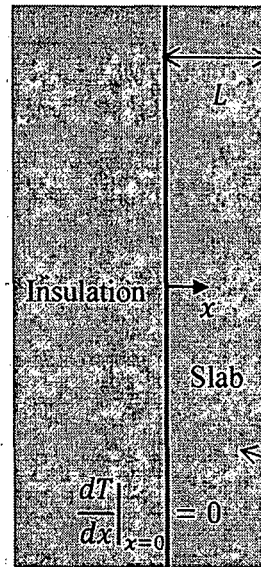
dimensional laminar 'boundary layer' develops on the wall. If the velocity outside the boundary layer is given by $v_x = Gx$, where G is a constant called the strain rate (s^{-1}), how will the momentum boundary layer thickness vary along the x -direction? If there is a temperature difference between the wall and air, how will the thermal boundary layer thickness vary along the x -direction? [Hint: Think about the Prandtl number].

Q.3. [15 points] HEATING UP A STEEL SLAB

Consider a thin, plane stainless steel slab of thickness $L=5$ cm and surface area 4 m², density $\rho = 8010$ kg/m³ and specific heat capacity $C = 490$ J/kg°C. The face of the slab at $x = 0$ is insulated i.e. $\left. \frac{dT}{dx} \right|_{x=0} = 0$,

while the other face is exposed to a hot fluid at 200°C . The slab is initially at a uniform temperature of 10°C . After 4 min,

the temperatures at $x = 0$ and $x = L$ are 45.9°C and 90.4°C , respectively. Answer the following questions:



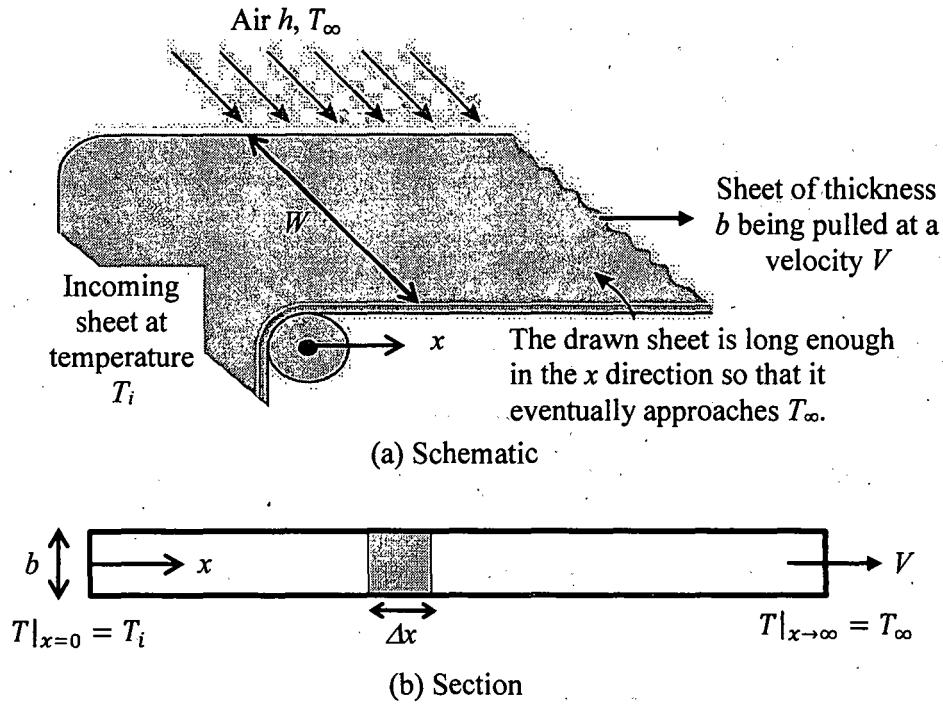
Hot fluid at
 $T_\infty = 200^\circ\text{C}$

$$-k \left. \frac{dT}{dx} \right|_{x=L} = h(T|_{x=L} - T_\infty)$$

$$\left. \frac{dT}{dx} \right|_{x=0} = 0 \quad T|_{t=0} = T_i = 10^\circ\text{C}$$

- [12 points] Calculate the thermal diffusivity and thermal conductivity of the slab, and the convective heat transfer coefficient.
- [3 points] What is the total heat exchanged between the wall and the fluid?

Q.4. [15 points] COOLING DOWN A SHEET METAL OR PLASTIC



In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment. Typically the process is a continuous one, with a sheet of thickness b and width W pulled out at a velocity V by rollers [see Fig. (a)]. The cooling is carried out by a moving airstream at a temperature T_∞ . Assume that heat transfer coefficient corresponding to the heat exchange between the sheet and the air is constant at all locations on the sheet and equal to h . Also assume that the sheet being pulled is long enough that the temperature of the sheet approaches the ambient temperature T_∞ for asymptotically large x .

- (a) **[4 points]** Perform a steady state energy balance over an element of the sheet thickness Δx [see Fig. (b)]. Ignore temperature variations over the thickness and width of the sheet. Take the limit as $\Delta x \rightarrow 0$ to obtain the following ordinary differential equation (ODE):

$$k \frac{d^2 T}{dx^2} - \rho C V \frac{dT}{dx} - \frac{2h}{b} (T - T_\infty) = 0. \quad (1)$$

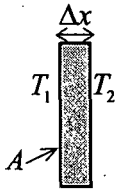
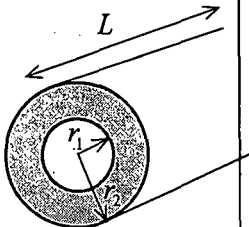

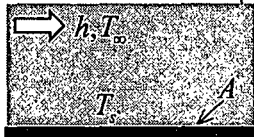

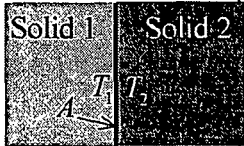
- (b) **[5 points]** Given the boundary conditions $T|_{x=0} = T_i$ and $T|_{x \rightarrow \infty} = T_\infty$, scale the ODE and the boundary conditions to obtain their dimensionless versions. You will get two possible choices for the length scale in the x direction. Which two terms of the governing equation are you equating when you make either choice? How many dimensionless parameters do you get, and in what limits of these dimensionless parameters are the two possible choices for the length scale valid? **NOTE:** In normalizing the ODE, use the prefactor or scale of the term corresponding to Newton's law of cooling, as this term is always significant in determining the temperature distribution for this problem. **[Bonus: 2 points – Why is this true?]**
- (c) **[2 points]** Consider the case when the sheet is made of aluminium ($k = 237 \text{ W/m}^\circ\text{C}$, $\rho = 2700 \text{ kg/m}^3$, $C = 900 \text{ J/kg}^\circ\text{C}$). If $b = 2 \text{ mm}$, $h = 41 \text{ W/m}^2^\circ\text{C}$, $V = 10 \text{ } \mu\text{m/s}$. what is the approximate length scale in the x direction over which the temperature will fall to the ambient air temperature? Which two terms in the governing equation balance each other to produce this length scale? What would your responses change to, if $V = 0.1 \text{ m/s}$, everything else remaining the same?
- (d) **[4 points]** Integrate the ODE in Eq. (1) and apply the boundary conditions to get the temperature distribution. From the solution, determine the length scale over which the temperature decreases to the ambient value (i.e. get the inverse of the coefficient of x in the exponent). Is this scale consistent with the results in part (b)?

Q.5. [15 points] STEADY STATE TEMPERATURE OF COPPER SPHERE

A pure copper sphere of radius 5 mm and an emissivity of 0.45 is suspended in a large furnace, with walls at a uniform temperature of 500°C surrounding the sphere. Air flows over the sphere at a temperature of 900°C and a velocity of 8 m/s . What is the steady state temperature of the sphere? In your calculations, determine the convective heat transfer coefficient using the following properties: $\mu_\infty = 4.67 \times 10^{-5} \text{ N}\cdot\text{s/m}^2$, $\nu = 0.000157 \text{ m}^2/\text{s}$, $k = 0.075 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 0.73$; Air: $\mu_s = 4.27 \times 10^{-5} \text{ N}\cdot\text{s/m}^2$. The Stefan Boltzmann constant is $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$. **Note:** The Newton-Raphson iterative formula for finding the root

x^* of a function $f(x)$, such that $f(x^*) = 0$, is
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\Delta x}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$
Spherical shell (Hollow sphere)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \frac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}}$	$R_{\text{rad}} = \frac{1}{\epsilon \sigma A (T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})}$
Thermal contact resistance		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{R_c}{A}$ (R_c has units of °C·m²/W)

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + \dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_x = -k \frac{\partial T}{\partial x}$, $\dot{q}_y = -k \frac{\partial T}{\partial y}$, $\dot{q}_z = -k \frac{\partial T}{\partial z}$

Constant k:
$$\rho C \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \dot{S}$$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(k r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(k \frac{\partial T}{\partial \theta} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_r = -k \frac{\partial T}{\partial r}$, $\dot{q}_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$, $\dot{q}_z = -k \frac{\partial T}{\partial z}$

Constant k:
$$\rho C \frac{\partial T}{\partial t} = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] + \dot{S}$$

SPHERICAL CO-ORDINATE SYSTEM

$$\rho C \frac{\partial T}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(k r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(k \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left(k \frac{\partial T}{\partial \phi} \right) + \dot{S}$$

Conductive flux components: $\dot{q}_r = -k \frac{\partial T}{\partial r}$, $\dot{q}_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$, $\dot{q}_\phi = -k \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}$

Constant k:
$$\rho C \frac{\partial T}{\partial t} = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + \dot{S}$$

TABLE 11-1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_0 and a sphere of radius r_0 subjected to convection from all surfaces.*

Geometry	Solution	λ_n 's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x / L)$	$\lambda_n \tan \lambda_n = \text{Bi}$ ✓
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r / r_0)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x / L)}{\lambda_n x / L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

TABLE 11-2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres ($\text{Bi} = hL/k$ for a plane wall of thickness $2L$, and $\text{Bi} = hr_0/k$ for a cylinder or sphere of radius r_0)

Bi	Plane Wall		Cylinder		Sphere	
	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 11-3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	-0.0968	-0.4708
2.8	-0.1850	-0.4097
3.0	-0.2601	-0.3391
3.2	-0.3202	-0.2613

Total heat transferred:

$$\begin{aligned}\frac{Q}{Q_{\max}} &= 1 - \theta_0 \frac{\sin \lambda_1}{\lambda_1} && \text{Plane wall} \\ &= 1 - 2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1} && \text{Cylinder} \\ &= 1 - 3\theta_0 \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3} && \text{Sphere}\end{aligned}$$

Here, $Q_{\max} = mC(T_{\infty} - T_i)$, where m and C are the mass and the specific heat capacity, respectively, of the solid. θ_0 is a dimensionless center temperature.

NUSSELT NUMBER CORRELATIONS

Transition from laminar to turbulent occurs at the critical Reynolds number of

$$Re_{x,c} = \frac{\rho V x_c}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

$$\begin{aligned}\text{Laminar: } C_{f,x} &= \frac{0.664}{Re_x^{1/2}}, \quad Re_x < 5 \times 10^5 \\ Nu_x &= \frac{h_x x}{k} = 0.332 Re_x^{0.5} Pr^{1/3}, \quad Pr > 0.6\end{aligned}$$

$$\begin{aligned}\text{Turbulent: } C_{f,x} &= \frac{0.059}{Re_x^{1/5}}, \quad 5 \times 10^5 \leq Re_x \leq 10^7 \\ Nu_x &= \frac{h_x x}{k} = 0.0296 Re_x^{0.8} Pr^{1/3}, \quad 0.6 \leq Pr \leq 60 \\ &\quad 5 \times 10^5 \leq Re_x \leq 10^7\end{aligned}$$

The average friction coefficient relations for flow over a flat plate are:

$$\begin{aligned}\text{Laminar: } C_f &= \frac{1.33}{Re_L^{1/2}}, \quad Re_L < 5 \times 10^5 \\ \text{Turbulent: } C_f &= \frac{0.074}{Re_L^{1/5}}, \quad 5 \times 10^5 \leq Re_L \leq 10^7 \\ \text{Combined: } C_f &= \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L}, \quad 5 \times 10^5 \leq Re_L \leq 10^7 \\ \text{Rough surface, turbulent: } C_f &= \left(1.89 - 1.62 \log \frac{x}{L}\right)^{-2.5}\end{aligned}$$

The average Nusselt number relations for flow over a flat plate are:

$$\begin{aligned}\text{Laminar: } Nu &= \frac{hL}{k} = 0.664 Re_L^{0.5} Pr^{1/3}, \quad Re_L < 5 \times 10^5 \\ \text{Turbulent: } Nu &= \frac{hL}{k} = 0.037 Re_L^{0.8} Pr^{1/3}, \quad 0.6 \leq Pr \leq 60 \\ &\quad 5 \times 10^5 \leq Re_L \leq 10^7\end{aligned}$$

Combined:

$$Nu = \frac{hL}{k} = (0.037 Re_L^{0.8} - 871) Pr^{1/3}, \quad 0.6 \leq Pr \leq 60 \\ 5 \times 10^5 \leq Re_L \leq 10^7$$

For isothermal surfaces with an unheated starting section of length ξ , the local Nusselt number and the average convection coefficient relations are

$$\begin{aligned}\text{Laminar: } Nu_x &= \frac{Nu_x(\text{for } \xi=0)}{[1 - (\xi/x)^{3/4}]^{1/3}} = \frac{0.332 Re_x^{0.5} Pr^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}} \\ \text{Turbulent: } Nu_x &= \frac{Nu_x(\text{for } \xi=0)}{[1 - (\xi/x)^{9/10}]^{1/9}} = \frac{0.0296 Re_x^{0.8} Pr^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}} \\ \text{Laminar: } h &= \frac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L} h_{x=L} \\ \text{Turbulent: } h &= \frac{5[1 - (\xi/x)^{9/10}]}{(1 - \xi/L)} h_{x=L}\end{aligned}$$

These relations are for the case of isothermal surfaces. When a flat plate is subjected to uniform heat flux, the local Nusselt number is given by

$$\begin{aligned}\text{Laminar: } Nu_x &= 0.453 Re_x^{0.5} Pr^{1/3} \\ \text{Turbulent: } Nu_x &= 0.0308 Re_x^{0.8} Pr^{1/3}\end{aligned}$$

The average Nusselt numbers for cross flow over a cylinder and sphere are

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re}{282,000}\right)^{4/5}\right]^{1/4}$$

which is valid for $Re Pr > 0.2$, and

$$Nu_{sph} = \frac{hD}{k} = 2 + [0.4 Re^{1/2} + 0.06 Re^{2/3}] Pr^{0.4} \left(\frac{\mu_{\infty}}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \leq Re \leq 80,000$ and $0.7 \leq Pr \leq 380$. The fluid properties are evaluated at the film temperature $T_f = (T_{\infty} + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_{∞} (except for μ_s , which is evaluated at the surface temperature T_s) in the case of a sphere.