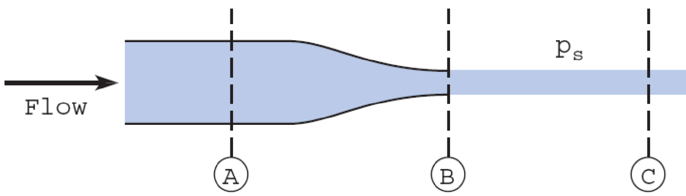
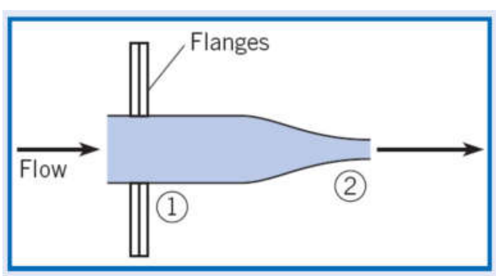
Note:

Nozzles: When a fluid flows a nozzle, we typically assume velocity is uniform at each cross-sections. If the nozzle exhausts into the atmosphere, the pressure of the free jet (flow exiting the nozzle) is atmospheric. Hence, for instance, pressure at section B below is atmospheric. Applying Bernoulli equation between sections A and B will provide an equation for the pressure at section A. This pressure will exert a force of magnitude pA, where p is the pressure at the centroid of section A.



Problem 1: The sketch shows air flowing through a nozzle. The inlet pressure is P_1 = 105 kPa, abs., and the air exhausts into the atmosphere, where the pressure is 101.3 kPa, abs. The nozzle has an inlet diameter of 60 mm and an exit diameter of 10 mm, and the nozzle is connected to the supply pipe by flanges. Find the air speed at the exit of the nozzle and the force required to hold the nozzle stationary. Assume the air has a constant density of 1.22 kg/m^3. Neglect the weight of the nozzle.



Solution: The control volume is shown in the sketch below. A stationary control volume and a stationary reference frame are selected. Also, the Bernoulli equation can be applied between sections 1 and 2. Since the fluid is a gas, elevation changes are negligible. Also, P_2 = 0 kPa gage. Hence the Berboullu equation simplifies to

$$P_1 + \rho V_1^2/2 = \rho V_2^2/2 \qquad (1)$$

The continuity equation gives

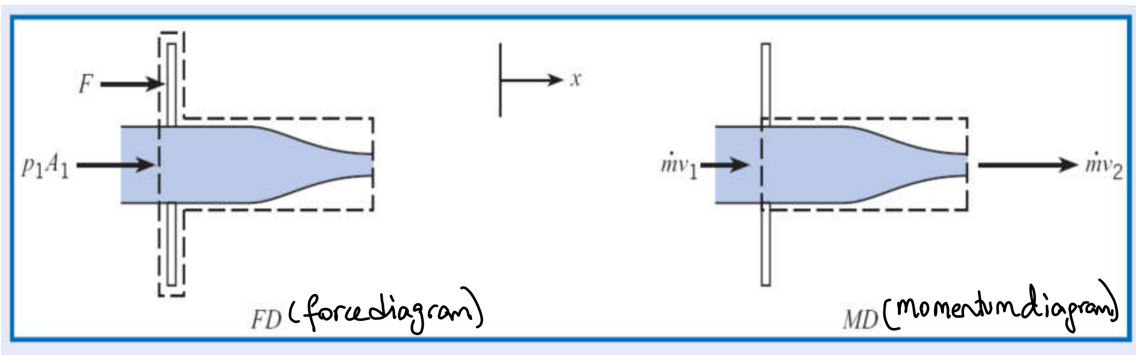
$$V_1 A_1 = V_2 A_2 \Rightarrow V_1 = V_2 \frac{A_2}{A_1} \qquad (2)$$

Combining equations (1) & (2):

$$P_1 + \rho \frac{V_2^2}{2}\left(\frac{d_2}{d_1}\right)^4 = \rho \frac{V_2^2}{2}$$

Rearranging;

$$V_2 = \sqrt{\frac{2P_1}{\rho\left(1-\left(\frac{d_2}{d_1}\right)^4\right)}} \quad (3)$$



The force diagram (FD) in previous page shows the forces acting on the control volume. There exists a force due to pressure p_1A_1 and a force F, which is net force acting on the flange of the nozzle. Since the direction of F is unknown, it was assumed to act in the positive direction. At the end if the result is positive it really acts in positive directions, if we find it negative then it is the opposite of what we assume now… From the force diagram:

$$\sum F_x = F + p_1 A_1$$

There is no momentum accumulation because the flow is steady and the nozzle is stationary. Therefore, the momentum equation for steady, uniform flow distribution (1D), incompressible flow can be used:

$$\sum \vec{F} = \dot{m}_2 \vec{V}_2 - \dot{m}_1 \vec{V}_1$$

From continuity equation, mass flow rate is constant: $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Then, the above equation becomes:

$$\sum \vec{F} = \dot{m}\,(\vec{V}_2 - \vec{V}_1)$$

The problem involves only one direction, the x direction. So, we can write;

$$\underbrace{\Sigma F_x}_{F + p_1 A_1} = \dot{m}(V_2 - V_1)$$

Solving the force from the momentum equation results in:

$$\boxed{F = \dot{m}(V_2 - V_1) - p_1 A_1}$$

To begin the calculations, V_2 is found from the previously derived equation (3):

$$V_2 = \sqrt{\frac{2 \times (105 - 101.3) \times 1000 \text{ Pa}}{(1.22 \text{ kg/m}^3)(1 - (10/60)^4)}} = 77.9 \text{ m/s}$$

NOTE: We are using gage pressure as then the pressure force at A_2 becomes zero since jet is exposed to atmosphere...

Then V_1 is:

$$V_1 = V_2 \frac{A_2}{A_1} = (77.9 \text{ m/s})(10/60)^2 = 2.16 \text{ m/s}$$

Mass flow rate is:

$$\dot{m} = \rho A_2 V_2 = (1.22 \text{ kg/m}^3)(\pi \times 0.01^2/4 \text{ m}^2)(77.9 \text{ m/s})$$
$$= 7.46 \times 10^{-3} \text{ kg/s}$$

The net rate of momentum flow is:

$$\dot{m}(V_2 - V_1) = (7.46 \times 10^{-3} \text{ kg/s})(77.9 - 2.16) \text{ m/s} = 0.57 \text{ N}$$

The force due to pressure is calculated using gage pressure:

$$p_1 A_1 = [(105 - 101.3) \times 1000 \text{ Pa}][\pi \times 0.06^2/4 \text{ m}^2] = 10.46 \text{ N}$$

The net force is:

$$F = -p_1 A_1 + \dot{m}(V_2 - V_1)$$
$$= -10.46 \text{ N} + 0.57 \text{ N}$$

$F = -9.89\,N$

Because F is found negative, the direction is opposite to the direction assumed on the force diagram. Hence, the force acts in the negative x direction.

Note:

Vanes: A vane is a structural component, typically thin, that is used to turn a fluid jet or be turned by a fluid jet. Examples include a blade in a turbine and a sail on a ship. Figure below shows a flat vane impacted by a jet of fluid. A typical control volume is also shown. In analyzing flow over a vane, it is common to apply Bernoulli equation and to neglect changes in elevation. Since the pressure is constant (atmosphetic pressure), $V_1=V_2=V_3$. Another common assumption unless otherwise mentioned is that viscous forces are often negligible.

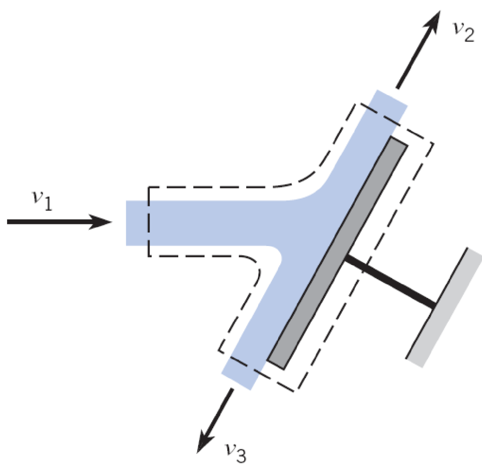
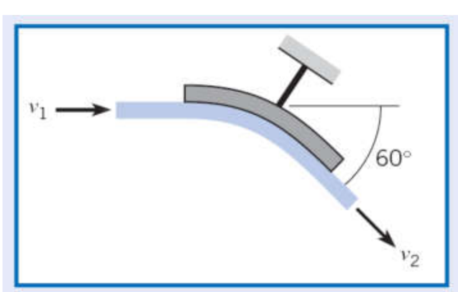
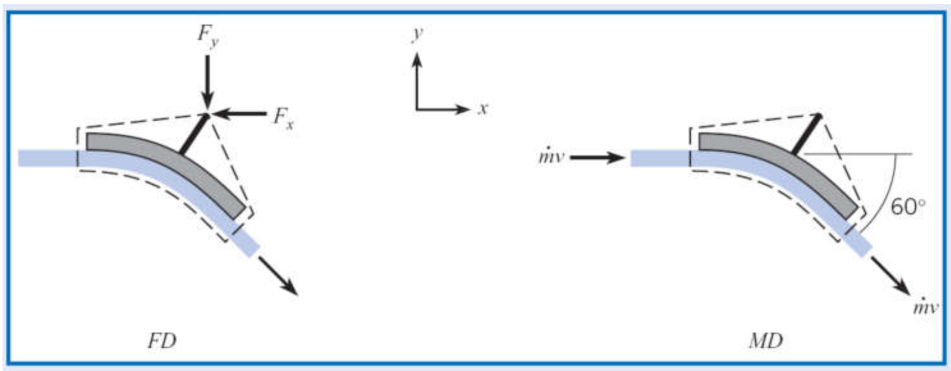


Figure shows a fluid jet striking a flat vane.

Problem 2: A water jet s deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 1 m/s and a diameter of 2 cm. Find the force exerted by the jet on the vane. Neglect the influence of gravity.



Solution to Problem 2: The control volume is shown by dashed lines in the sketch below. A stationary control volume and stationary reference frame is selected. Application of the Bernoulli equation shows that the inlet and exit speeds are equal since pressure is constant (atmospheric pressure): $V_1=V_2$. Also, the inlet and exit mass flow rates are equal. $\dot{m}_1 = \dot{m}_2 = \dot{m}$



This problem involves two directions, the x and y directions. We can select the momentum equation as either the two component equations in x and y directions or the vector equation. We will use here the vector form. In the force diagram (FD) above, the force term is the force needed to hold the vane stationary.

$$\underbrace{\sum \vec{F}}_{\text{force acting on the control volume}} = -F_x \hat{i} - F_y \hat{j} \quad \text{(from the force diagram)}$$

Since the vane is stationary and Control Volume is stationary, the flow is steady. The momentum diagram shows one inward momentum flow and one outward momentum flow.

$$\dot{m}\vec{V}_{in} = \dot{m} V \hat{i}$$

$$\dot{m}\vec{V}_{out} = [\dot{m} V \cos 60^\circ \hat{i} - \dot{m} V \sin 60 \hat{j}]$$

So the net momentum flow is:

$$\dot{m}\vec{V}_{out} - \dot{m}\vec{V}_{in} = (\dot{m} V \cos 60^\circ - \dot{m} V)\hat{i} - (\dot{m} V \sin 60^\circ)\hat{j}$$

$$\underbrace{\sum \vec{F}}_{-F_x \hat{i} - F_y \hat{j}} = \underbrace{\dot{m}\vec{V}_{out} - \dot{m}\vec{V}_{in}}$$

Equating force and net momentum flow terms we get:

$$-F_x\hat{i} - F_y\hat{j} = (\dot{m}V\cos 60^\circ - \dot{m}V)\hat{i} - (\dot{m}V\sin 60^\circ)\hat{j}$$

$\hat{i}$: $\quad -F_x = \dot{m}V\cos 60 - \dot{m}V$

$\hat{j}$: $\quad -F_y = -\dot{m}V\sin 60^\circ$

$\dot{m}$ is the rate at which mass is crossing the control surface:

$$\dot{m} = \rho A V$$

$$F_x = \rho \underset{\uparrow\ \frac{\pi d^2}{4}}{A} V^2 (1 - \cos 60^\circ) = 1000 \times \pi \times \frac{(0.02)^2}{4}(1 - \cos 60^\circ) = \ldots\ldots$$ (Please find the answer)

$$F_y = \rho A V^2 \sin 60^\circ = 1000 \times \pi \times \frac{(0.02)^2}{4} \sin 60^\circ = \ldots$$ (Please find the answer!

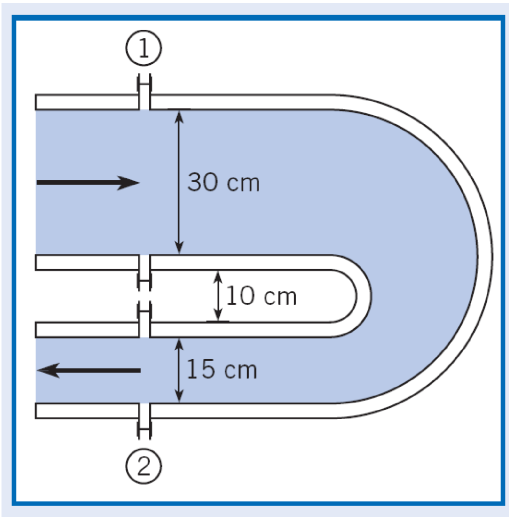
The force of the jet on the vane $\vec{F}_{jet}$ is opposite in direction to the force $\vec{F}$ acting on the control volume. Therefore; $\vec{F}_{jet} = F_x\hat{i} + F_y\hat{j}$

Please calculate numeric values from above!

Note:

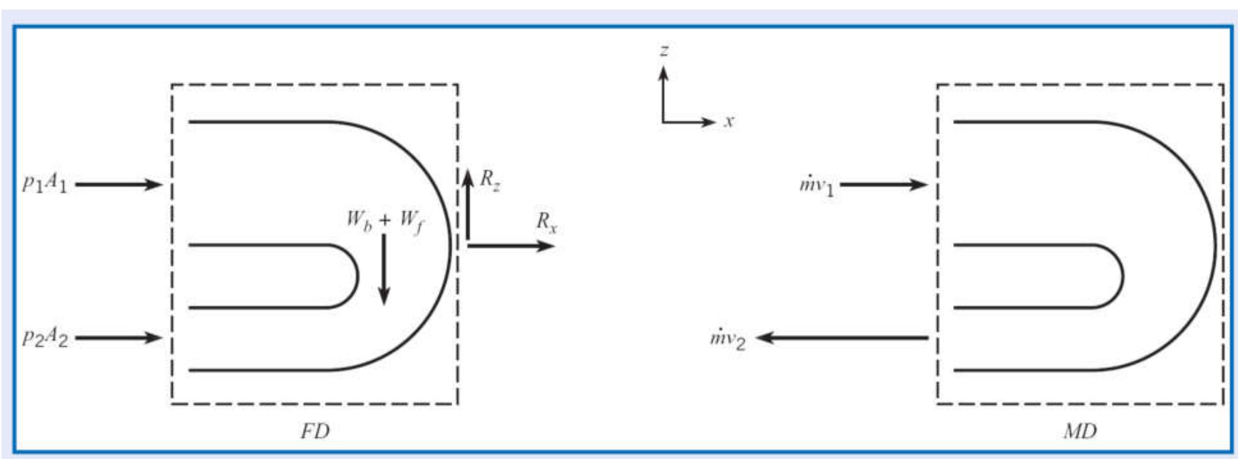
Pipes: Unless otherwise stated, it is often common to assume the velocity in a pipe nearly constant across each cross section of the pipe (or use the average velocity value at each cross-section). Also, the force acting on a pipe cross section is given by PA, where P is the pressure at the centroid of area and A is area.

Problem 3: Water flows through a 180° reducing bend, as shown. The discharge is 0.25 m^3/s, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is 0.10 m^3, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500N.



Solution: The control volume is shown in the sketch below. A stationary reference frame was selected. The flow is steady, and continuity gives:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$



Velocities may be found using continuity:

$$\dot{\forall} = A_1 V_1 = A_2 V_2$$

The Bernoulli equation can be used to related pressure at sections 1 and 2:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

The force diagram shows weight, pressure forces and the net force due to the connections at sections 1 and 2. From the force diagram:

$$\sum F_x = P_1 A_1 + P_2 A_2 + R_x$$
$$\sum F_z = R_z - W_b - W_f$$

Where W_b is the weight of the bend and W_f is the weight of the fluid within the bend.

Momentum accumulation is zero because the fluid is steady and the bend is stationary.

$$\dot{m}_{out} \vec{V}_{out} = \dot{m}(-V_2)\hat{i} = -\dot{m} V_2 \hat{i}$$
$$\dot{m}_{in} \vec{V}_{in} = \dot{m}(V_1)\hat{i} = \dot{m} V_1 \hat{i}$$

From the momentum equation we get:

$$\sum F_x \hat{i} + \sum F_z \hat{k} = \dot{m}_{out} \vec{V}_{out} - \dot{m}_{in} \vec{V}_{in}$$
$$(P_1 A_1 + P_2 A_2 + R_x)\hat{i} + (R_z - W_b - W_f)\hat{k} = -\dot{m}(V_2 + V_1)\hat{i}$$

$\hat{i}$: $\quad P_1 A_1 + P_2 A_2 + R_x = -\dot{m}(V_2 + V_1)$

$\hat{k}$: $\quad R_z - W_b - W_f = 0$

Speeds are given by:

$$V_1 = \frac{\dot{\forall}}{A_1} = \frac{0.25 \text{ m}^3/\text{s}}{\frac{\pi}{4} \times 0.3^2 \text{ m}^2} = 3.54 \text{ m/s}$$
$$V_2 = \frac{\dot{\forall}}{A_2} = \frac{0.25 \text{ m}^3/\text{s}}{\frac{\pi}{4} \times 0.15^2 \text{ m}^2} = 14.15 \text{ m/s}$$

Mass flow rate is:

$$\dot{m} = \rho \dot{\forall} = \left(1000 \frac{\text{kg}}{\text{m}^3}\right)\left(0.25 \frac{\text{m}^3}{\text{s}}\right) = 250 \text{ kg/s}$$

The net outward momentum flow rate is:

$$\dot{m}(V_2 + V_1) = (250 \text{ kg/s})(14.5 + 3.54)(\text{m/s}) = 4420 \text{N}$$

Pressure at section 2 is given by Bernoulli equation:

$$P_2 = P_1 + \frac{\rho(V_1^2 - V_2^2)}{2} + \rho g(z_1 - z_2)$$

$$p_2 = 150\ \text{kPa} + \frac{(1000)(3.54^2 - 14.15^2)\ \text{Pa}}{2} + (9810)(0.325)\ \text{Pa}$$

$$p_2 = 59.3\ \text{kPa}$$

R_x is given by $R_x = -(p_1 A_1 + p_2 A_2) - \dot{m}(V_2 + V_1)$

The net pressure force is.

$$p_1 A_1 + p_2 A_2 = (150\ \text{kPa})(\pi \times 0.3^2/4\ \text{m}^2) + (59.3\ \text{kPa})(\pi \times 0.15^2/4\ \text{m}^2)$$
$$= 11.6\ \text{kN}$$

The x component of the support force is:

$$R_x = -(p_1 A_1 + p_2 A_2) - \dot{m}(V_2 + V_1)$$
$$= -(11.6\ \text{kN}) - (4.42\ \text{kN})$$
$$= -16.0\ \text{kN}$$

and the z component is:

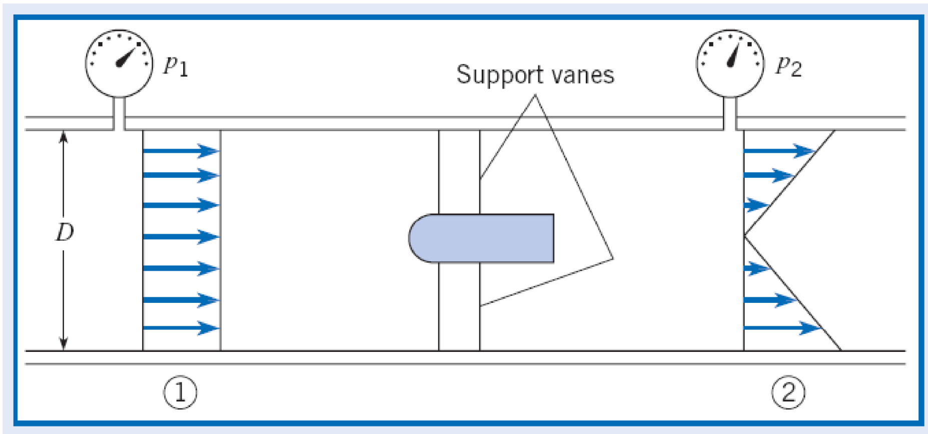
$$R_z = W_b + W_f$$
$$= 500\ \text{N} + (9810\ \text{N/m}^3)(0.1\ \text{m}^3)$$
$$= 1.48\ \text{kN}$$

Note:

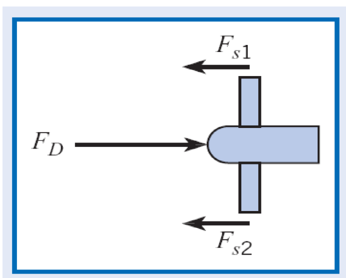
Non-uniform Velocity Distribution:

The previous examples were cases wherein it was assumed that the velocity across each flow section was constant. The next example involves a nouniform velocity distribution across a flow section.

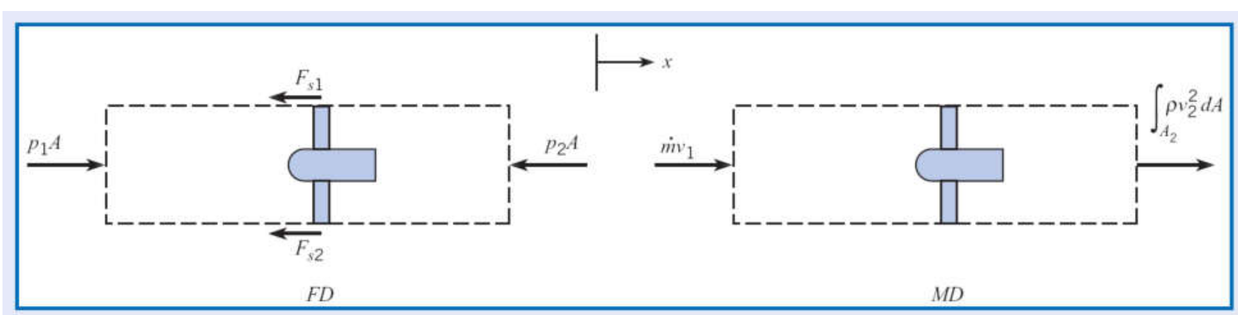
Problem 4: The drag force of a bullet-shaped device may be measured using a wind tunnel. The tunnel is round with a diameter of 1m, the pressure at section 1 is 1.5 kPa gage, the pressure at section 2 is 1.0 kPa gage, and air density is 1.0 kg/m^3. At the inlet, the velocity is uniform with a magnitude of 30 m/s. At the exit, the velocity varies linearly as shown in the sketch. Determine the drag force on the device and support vanes. Neglect viscous resistance at the wall, and assume pressure is uniform across sections 1 and 2.



Solution: A free-body diagram of the device is shown below. The drag force F_D is balanced by forces on the support vanes, so $F_D = F_{s1}+F_{s2}$.



The sketch below shows the control volume. A stationary reference frame was selected. The velocity distribution across the exit section is nonuniform.



On the force diagram, shear stress and weight are omitted, and the pressure forces are shown. At the two locations where the control surface passes through the support vanes, the surface forces are F_{s1} and F_{s2}. From the force diagram:

$$\sum F_x = p_1 A_1 - p_2 A_2 - (F_{s1} + F_{s2})$$

Letting $F_D = F_{s1} + F_{s2}$ results in

$$\sum F_x = p_1 A_1 + p_2 A_2 - F_D$$

The momentum diagram shows an inward momentum flow and an outward momentum flow. Each momentum flow term was analyzed using $\iint_A V\rho \vec{V}\cdot d\vec{A}$. Across the inlet (section 1), variables on the integral are: V = V_1,

$$\vec{V}_1 = V_1\hat{i}$$
$$\vec{A}_1 = -A\hat{i}$$

Because these variables are constants, the integral simplifies to

$$\iint_{A_1} V\rho \vec{V}\cdot d\vec{A} = V_1\rho(-V_1A) = -\dot{m}V_1$$

Across the exit (section 2), variables in the integral are:

$$V = V_2$$
$$\vec{V}_2 = V_2\hat{i}$$
$$d\vec{A} = dA\,\hat{i}$$

And the outward momentum flow is

$$\iint_{A_2} V\rho \vec{V}\cdot d\vec{A} = \iint_{A_2} V_2\rho(\vec{V}_2\cdot d\vec{A}) = \iint_{A_2} \rho V_2^2\, dA$$

Where V_2 cannot be factored out of the integral because it varies across the area: V_2=V_2(r). From the momentum diagram, then, the net momentum flow is given by:

$$\underset{\underset{\text{control surface}}{\uparrow}}{\oiint_{CS}} V\rho \vec{V}\cdot d\vec{A} = \iint_{A_2} \rho V_2^2\, dA - \dot{m}V_1$$

The momentum equation is:

$$\Sigma F_x = \oiint_{CS} V\rho \vec{V}\cdot d\vec{A}$$

Substituting forces and momentum terms into the momentum equation results in

$$P_1A_1 - P_2A_2 - F_D = \int_{A_2} \rho V_2^2\, dA - \dot{m}V_1$$

From here:

$$F_D = A_1(p_1 - p_2) + \dot{m}V_1 - \int_{A_2} \rho V_2^2 \, dA$$

The net pressure force is:

$$\dot{m}V_1 = \rho A_1 V_1^2 = (1.0 \text{ kg/m}^3)(\pi \times 0.5^2 \text{ m}^2)(30^2 \text{ m}^2/\text{s}^2) = 706.8 \text{ N}$$

To evaluate the outward momentum flow, we need an equation for $V_2(r)$. Since the outlet velocity distribution is linear, $V_2(r) = V_{max}(r/r_0)$, where V_{max} is the velocity near the wall, and r_0 is the outer radius of the tunnel. To evaluate V_{max}, we use continuity.

$$\dot{V}_1 = \dot{V}_2$$

$$A_1 V_1 = \int_{A_2} V_2(r) \, dA$$

$$A_1 V_1 = \int_0^{r_0} V_{max}\left(\frac{r}{r_0}\right) 2\pi r \, dr$$

$$(\pi \times 0.5^2 \text{ m}^2)(30 \text{ m/s}) = \int_0^{0.5} V_{max}\left(\frac{r}{0.5}\right) 2\pi r \, dr$$

which can be solved to show that $V_{max} = 45$ m/s. Evaluating the exit momentum flow gives:

$$\int_{A_2} \rho V_2^2 \, dA = \rho \int_0^{0.5} V_{max}^2 \left(\frac{r}{0.5}\right)^2 2\pi r \, dr$$

$$= (1.0 \text{ kg/m}^3)(45^2 \text{ m}^2/\text{s}^2)(0.5^2 \text{ m}^2)(\pi/2)$$

$$= 795.2 \text{ N}$$

The drag force is:

$$F_D = A_1(p_1 - p_2) + \dot{m}V_1 - \int_{A_2} \rho V_2^2 \, dA$$

$$= 392.7\text{N} + 706.8\text{ N} - 795.2\text{ N} = 304 \text{ N}$$

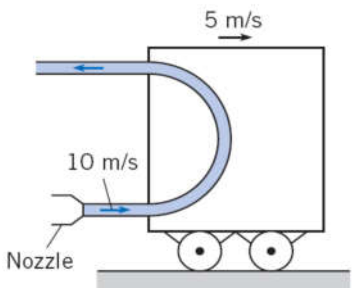
Note:

Moving Control Volumes:

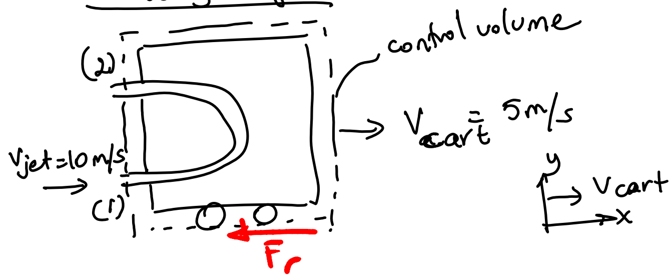
When an object us moving, it is often convenient to define a control volume that moves with the object. As discussed in the course notes, the velocity $\vec{V}$ in the momentum equation must be relative to an inertial reference frame. The reference frame selected may move with the control volume, or it can be independent of the control volume. In the momentum equation, each mass flow rate is calculated using the velocity with respect to the control surface. In other words, the term $\dot{m}$ gives the rate at which mass passes in or out of the control volume. Mass flow rate is independent of the reference frame that is selected to evaluate$\vec{V}$.

A cart is moving along a track at a constant velocity of 5 m/s as shown. Water (with density ρ = 1000 kg/m^3) issues from a nozzle at 10 m/s and is deflected through 180° by a vane on the cart. The cross-sectional area of the nozzle is 0.0012 m^2. Calculate the resistive force on the cart.

(Note: Assume the resistive force is caused primarily by rolling resistance (bearing friction, etc); therefore, the resistive force acts on the wheels at the ground surface)



Free body diagram



Let's select $\begin{cases} \text{moving control volume} \\ \& \\ \text{moving reference frame with the control volume:} \end{cases}$

x momentum equation: $\hat{i}$: $\Sigma F_x = \dot{m}\,(V_2 - V_1)$

$\dot{m}$: with respect to CV.

V_2, V_1 : with respect to reference frame

$\vec{V}_1 = \vec{V}_{jet} - \vec{V}_{cart} = 5\,\hat{i}$ (m/s) ; $\vec{V}_2 = -5\,\hat{i}$ m/s (from continuity)

$\dot{m} = \rho A V = 1000 \times 0.0012 \times 5 = 6$ kg/s

Let's assume $\vec{F}_r = -F_r\,\hat{i}$, if the direction of $\vec{F}_r$ is to the left we will get positive value for F_r, if the direction of $\vec{F}_r$ is to the right, we will get negative value of F_r.

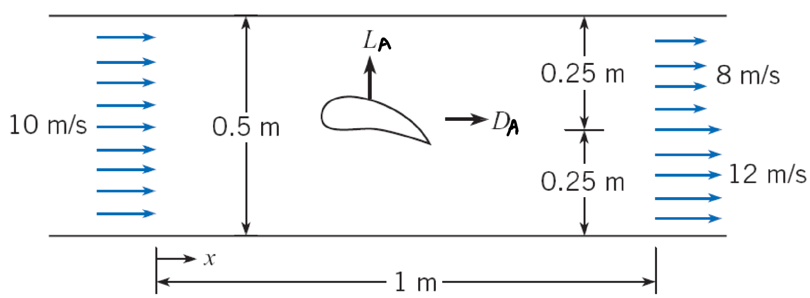
$-F_r = 6\,(-5 - 5) \Rightarrow F_r = 60$ N (Since the value of F_r is positive, the direction of $\vec{F}_r$ is to the left)

$$\boxed{\vec{F}_r = -60\,\hat{i}\ \text{(N)}}$$

Problem: An engineer is measuring the lift and drag on an airfoil section mounted in a two-dimensional wind tunnel. The wind tunnel is 0.5 m high and 0.5 m deep (into the paper). The upstream wind velocity is uniform at 10 m/s, and the downstream velocity is 12 m/s and 8 m/s as shown. The vertical component of velocity is zero at both stations. The test section is 1m long. The engineer measures the pressure distribution in the tunnel along the upper and lower walls and finds:

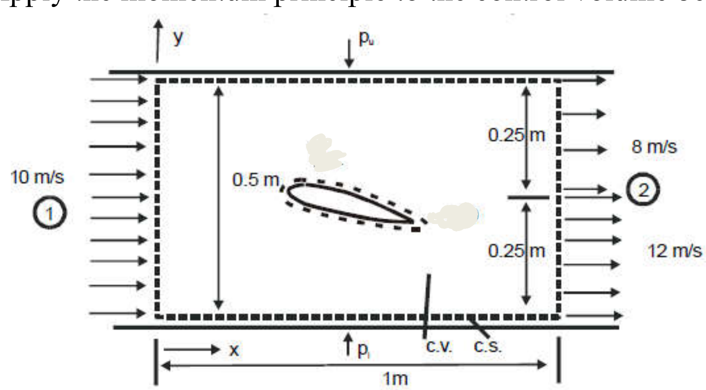
$$P_u = 100 - 10x - 20x(1-x) \qquad \text{(Pa, gage)}$$
$$P_l = 100 - 10x + 20x(1-x) \qquad \text{(Pa, gage)}$$

where x is the distance in meters measured from the beginning of the test section. The gas density is homogeneous throughout and equal to 1.2 kg/m^3. The lift and drag are the vectors indicated on the figure, and are shown with symbols L and D respectively. The forces acting on the fluid are in the opposite direction to these vectors. Find the lift and drag forces acting on the airfoil section.

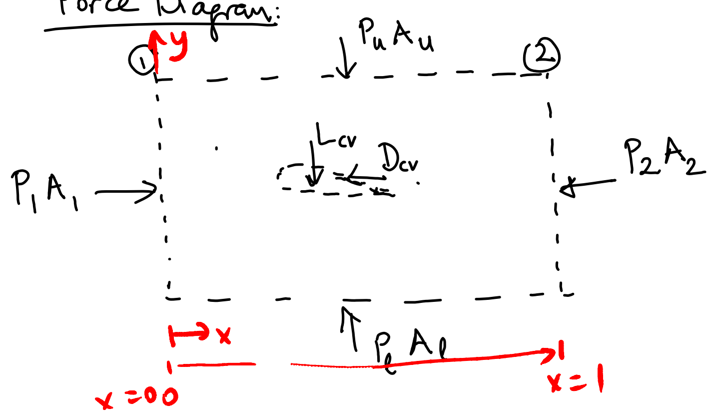


Solution:

Apply the momentum principle to the control volume below:



Force Diagram:



$$\boxed{\sum \vec{F} = \sum \dot{m}_{out} \vec{V}_{out} - \sum \dot{m}_{in} \vec{V}_{in}}$$

$P_1 = P_u(x=0) = P_\ell(x=0) = 100$ (Pa, gage)

$P_2 = P_u(x=1) = P_\ell(x=1) = 100 - 10 = 90$ (Pa, gage)

$$\boxed{\sum \vec{F} = (P_1 A_1 - P_2 A_2 - D_{cv})\hat{i} + \left(-L_{cv} \, 0.5\int_{x=0}^{x=1} P_u \, dx + 0.5 \int_{x=0}^{x=1} P_\ell \, dx\right)\hat{j}}$$

Force term.

$$\int_{x=0}^{x=1} P_u \, dx = \int_{x=0}^{x=1} (100 - 10x - 20x + 20x^2) \, dx = 100x - \frac{30x^2}{2} + \frac{20x^3}{3} \Big|_0^{x=1} = 85 + \frac{20}{3}$$

$$\int_{x=0}^{x=1} P_\ell \, dx = \int_{x=0}^{x=1} (100 \underbrace{-10x + 20x}_{10x} - 20x^2) \, dx = 100x + \frac{10x^2}{2} - \frac{20x^3}{3} \Big|_0^{x=1} = 105 - \frac{20}{3}$$

$$\sum \dot{m}_{out} \vec{V}_{out} = \underbrace{\overbrace{(1.2)}^{\rho} \overbrace{(0.25 \times 0.5)}^{\text{Area}} \overbrace{(8)}^{\text{velocity}}}_{\dot{m}} \underbrace{(8\hat{i})}_{\vec{V}} + \underbrace{\overbrace{(1.2)}^{\text{density}} \overbrace{(0.25 \times 0.5)}^{\text{Area}} (12)}_{\dot{m}} \underbrace{(12\hat{i})}_{\vec{V}}$$

$$= (1.2)(0.25 \times 0.5)(208)\hat{i} = 31.2\hat{i}$$

$$\sum \dot{m}_{in} \vec{V}_{in} = \underbrace{\overbrace{(1.2)}^{\text{density}} \overbrace{(0.5 \times 0.5)}^{\text{area}} (10)}_{\dot{m}_{in}} \, 10\hat{i} = 30\hat{i}$$

$$\sum \vec{F} = \sum \dot{m}_{out} \vec{V}_{out} - \sum \dot{m}_{in} \vec{V}_{in}$$

$$(P_1 A_1 - P_2 A_2 - D_{cv})\hat{i} + \left(-L_{cv} - 0.5\int_{x=0}^{1} P_u \, dx + 0.5 \int_{x=0}^{x=1} P_\ell \, dx\right)\hat{j} = 31.2\hat{i} - 30\hat{i}$$

$\hat{i}$: From x component of momentum equation.

$$(P_1 A_1 - P_2 A_2 - D) = 1.2$$

$$D_{cv} = P_1 A_1 - P_2 A_2 - 1.2$$

$$= 100 \times 0.5 \times 0.5 - 90 \times 0.5 \times 0.5 - 1.2$$

$\boxed{D_{cv} = 1.3 \text{ N}}$ ⇒ found positive. So to the C.V. the force $D_{cv} = -1.3\,\hat{i}$ acts.

Force acting on the airfoil should have the same magnitude but opposite direction so: $\boxed{D_A = 1.3\,\hat{i}}$ is the drag force acting on the airfoil.

$\hat{j}$: From y component of momentum equation.

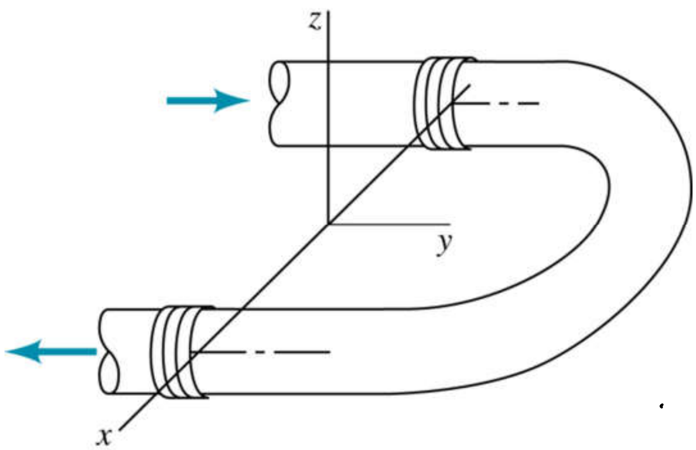
$$-L_{cv} - 0.5 \underbrace{\int_{x=0}^{x=1} P_u \, dx}_{\left(85 + \frac{20}{3}\right)} + 0.5 \underbrace{\int_{x=0}^{x=1} P_e \, dx}_{\left(105 - \frac{20}{3}\right)} = 0$$

$$L_{cv} = 0.5 \left[-85 - \frac{20}{3} + 105 - \frac{20}{3} \right]$$

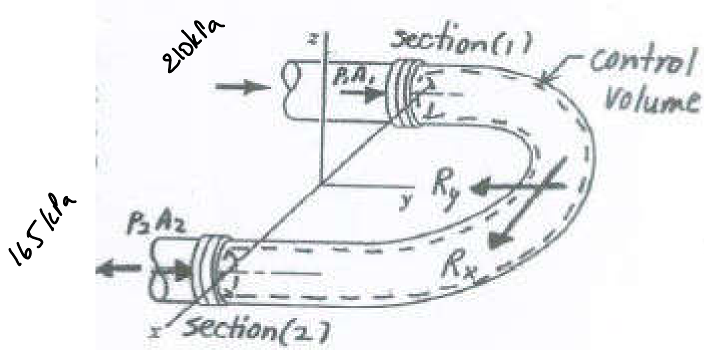
$L_{cv} = 3.33 \text{ N}$ ⇒ The answer is positive. So our assumption of the force direction acting on the C.V is correct direction $(-3.33\,\hat{j})$ acts on the control volume. Same magnitude but opposite direction force acts as a lift force on the ~~control volume~~ AIRFOIL

$$\boxed{L_A = 3.33\,\hat{j}}$$

Problem: Water flows through a horizontal 180° pipe bend as illustrated in Figure below. The flow cross section area is constant at a value of 9000mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the x and y components of the anchoring force needed to hold the bend in place. (The density of water is 1000kg/m^3)



Solution:



$$\sum \vec{F} = \dot{m}(\vec{V}_2 - \vec{V}_1)$$

$$\sum \vec{F} = R_x\hat{i} - R_y\hat{j} + (P_2A_2 + P_1A_1)\hat{j}$$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2.$$

$$\vec{V}_2 = -V_2\hat{j}$$

$$\vec{V}_1 = V_1\hat{j}$$

$$R_x\hat{i} - R_y\hat{j} + P_1A_1\hat{j} + P_2A_2\hat{j} = \rho_1 A_1 V_1(-V_2\hat{j} - V_1\hat{j})$$

$V_1 = V_2 = V = 15\ m/s \qquad A_1 = A_2 = A = 9000\ mm^2$

$$R_x\hat{i} + (-R_y + P_1A + P_2A)\hat{j} = \rho AV^2(-2\hat{j})$$

$\hat{i}$: $\boxed{R_x = 0}$

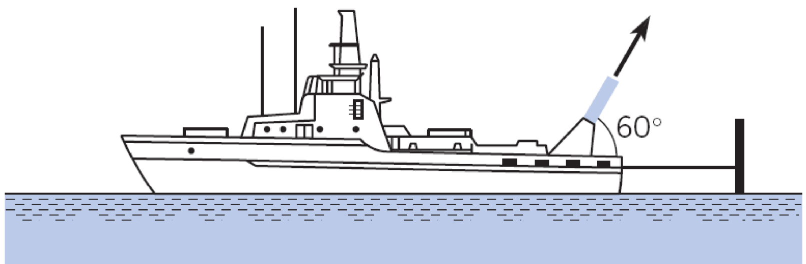
$\hat{j}$: $-R_y + P_1A + P_2A = \rho AV^2(-2) \Rightarrow R_y = 2\rho AV^2 - (P_2 + P_1)A$

$$R_y = 2\times 1000 \times \frac{9000}{1000^2}\times 15^2 - (165 + 210)\times 1000 \times \frac{9000}{1000^2}$$

$$\boxed{R_y = 7425\ N}$$

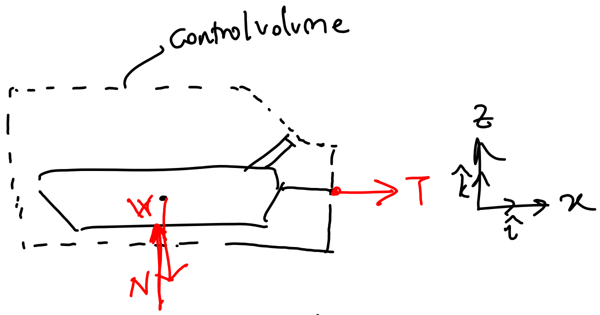
Question:

A firehose on a boat is producing a 0.1-m-diameter water jet with a steady speed of V=30 m/s. The boat is held stationary by a horizontal cable attached to a pier as shown in the figure. Calculate the tension in the cable. (Take the density of water as ρ=1000kg/m^3.)



Solution:

Free-body diagram:



$$\sum \vec{F} = \dot{m}(\vec{V}_{out} - \underbrace{\cancel{\vec{V}_{in}}}_{0})$$

$$\sum \vec{F} = \dot{m}\, \vec{V}_{out}$$

$$\vec{V}_{out} = V\cos 60\, \hat{i} + V \sin 60\, \hat{k}$$

Momentum balance in x direction:

x component of velocity

$$\sum F_x = \dot{m}\, V_x$$

$$T = \dot{m}\, V \cos 60$$

$\dot{m} = \rho A V$

$T = \dot{m} V \cos 60 \Rightarrow T = \rho A V^2 \cos 60$

$= (1000)\pi\left(\frac{0.1}{4}\right)^2 (30)^2 \cos 60^\circ$

$\boxed{T \cong 3{,}534 \text{ N}}$