MAT292 – Fall 2021

Quiz – October 4, 2021

Time alotted: 50 minutes

Full Name _____

Student Number _____

Email _____@mail.utoronto.ca

Signature _____

DO NOT OPEN
until instructed to do so

NO CALCULATORS ALLOWED
and no cellphones or other electronic devices

DO NOT DETACH ANY PAGES

This quiz contains 6 pages (including this title page). Once the quiz starts, make sure you have all of them.

In the first section, only answers and sometimes brief justifications are required.

In the second section, justify your answers fully.

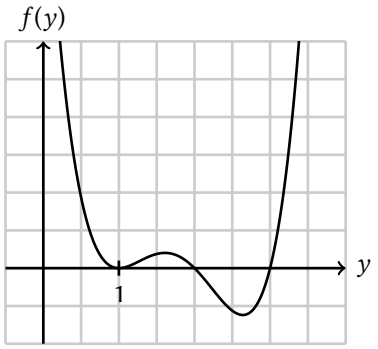
You can use pages 5–6 to complete questions. In such a case, **MARK CLEARLY** that your answer "continues on page X" **AND** indicate on the additional page which questions you are answering.

	Short answer				**Long answer**		
Question	Q1	Q2	Q3	Q4	Q6	Q7	**Total**
Marks	3	1	2	4	12	12	**34**

GOOD LUCK! YOU GOT THIS!

SECTION I Provide the final answer. Justify briefly only when asked.

1. **(3 marks)** Find and classify the equilibrium points of the differential equation $\frac{dy}{dt} = f(y) = (y-1)^2(y-2)(y-3)$ whose phase plot is given on the right. You do **NOT** need to justify your answers.



Equilibrium	Classification

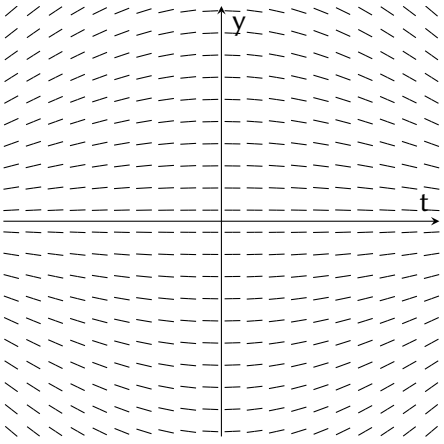
2. **(1 mark)** Consider again the ODE $\frac{dy}{dt} = f(y) = (y-1)^2(y-2)(y-3)$ from the previous question. If $y(t)$ is a solution and $y(0.5) = 2.8$, write the limit in the box on the right. You do **NOT** need to justify.

$$\lim_{t\to\infty} y(t) =$$ ☐

3. **(2 marks)** Assume $\phi(t)$ and $\psi(t)$ both solve the same first-order non-homogeneous linear ODE and that $a \in \mathbb{R}$. Then $\phi(t) + a\psi(t)$ is also a solution to this ODE. This statement is... ◯ TRUE ◯ FALSE

Choose true or false above, then justify:

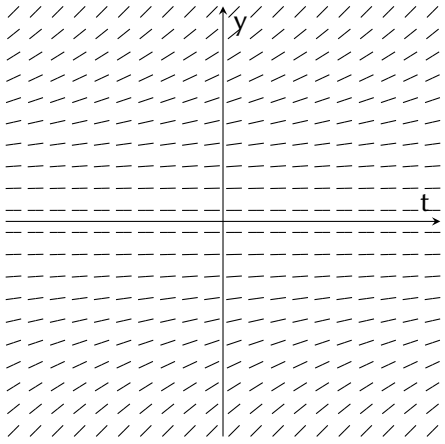
4. **(4 marks)** Below are two direction fields of two ODEs. For each plot and each row, make a choice. You do **NOT** need to justify. "Can't tell" means that by visual inspection it is impossible to tell.



The 1st-order ODE generating this direction field is...

◯ autonomous ◯ non-autonomous ◯ can't tell

◯ linear ◯ non-linear ◯ can't tell



The 1st-order ODE generating this direction field is...

◯ autonomous ◯ non-autonomous ◯ can't tell

◯ linear ◯ non-linear ◯ can't tell

SECTION II **Justify** all your answers.

5. For this question, consider the ODE $(t-2)y' + g(y) = 1$.
Assume that $g(y)$ has a continuous derivative for all values of y.

(a) (3 marks) For this part only, you can assume that $g(y) = 2y$. Find the solution such that $y(0) = 1$.

Solution to the IVP: $y(t) =$

For parts b/c/d, you do NOT know $g(y)$. You only know that it has a continuous derivative for all y.

True means that you can guarantee it is true. **False** means that you can guarantee it is false. **Undecidable** means that, given the information available to you, it is impossible to tell if the statement is true or false.

The correct choice alone is not worth points. All points are given for the explanation.

(b) (4 marks) "Given an initial value $y(0) = c$, we can find a unique solution to the IVP."
This statement is... ◯ True ◯ False ◯ Undecidable
Choose and explain:

(c) (3 marks) "Given an initial value $y(0) = c$, we can find $y(1)$."
This statement is... ◯ True ◯ False ◯ Undecidable
Choose and explain:

(d) (2 marks) "Given an initial value $y(2) = c$, we can find a solution to the IVP."
This statement is... ◯ True ◯ False ◯ Undecidable
Choose and explain:

6. There is about to be a zombie outbreak in a remote village in Northern Ontario.
There are 1000 individuals in the village. Every individual is either a zombie or a human.
The number of zombies in the village is denoted by $Z(t)$, where t is measured in days.

(a) (4 marks) At $t = 0$, half the town are zombies. The number of zombies follows logistic growth with rate $r = 1.5$. But there is a second effect, in addition to the effects of the logistic growth: The zombie spell is starting to wear off. One zombie is turning back to human on the first day, two zombies are turning back on the second day, three zombies are turning back on the third day and so on.

Model this scenario as an IVP. Explain the details. You do NOT need to solve the IVP.

IVP:

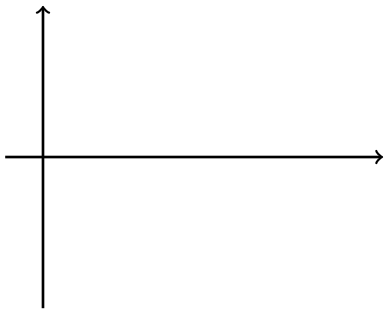
We will now consider a different model. **Ignore** anything from part (a) for the rest of the question.

(b) (4 marks) At $t = 0$, there is only one human left in town. As it turns out, she is a magic healer. Her name is Zara and she casts healing spells, turning zombies back into humans. The healing rate is equal to the share of humans (for example if half of all individuals are humans, Zara heals half of the zombies each day; if a quarter of all individuals are humans, Zara heals a quarter of the zombies each day; and so on).

Model this scenario as an IVP. Explain the details. You do NOT need to solve the IVP.

IVP:

(c) (4 marks) Draw a phase plot for the model in part **(b)**. Make sure to label the axes. Using the plot (and without solving the ODE!), answer this question: In the first days, is the number of zombies healed each day increasing or decreasing?



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