UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAM – DECEMBER 2023

First Year Engineering Science

CIV102F –Structures & Materials – An Introduction to Engineering Design

Permissible Aids: Non programmable calculator, printed or handwritten notes and marked quiz/assigns

Examiner – E.C. Bentz

NAME: Solution [redacted]

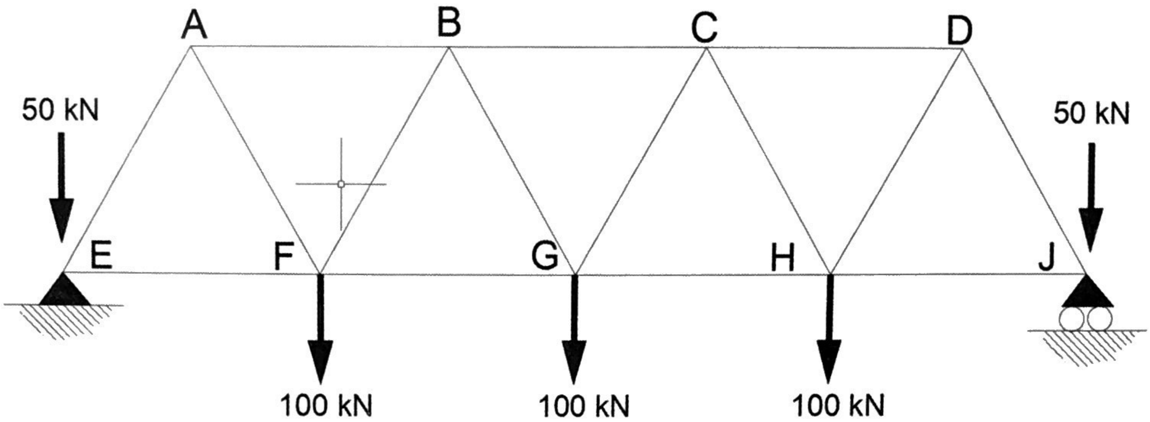
ID Number: [redacted]

Important Instructions

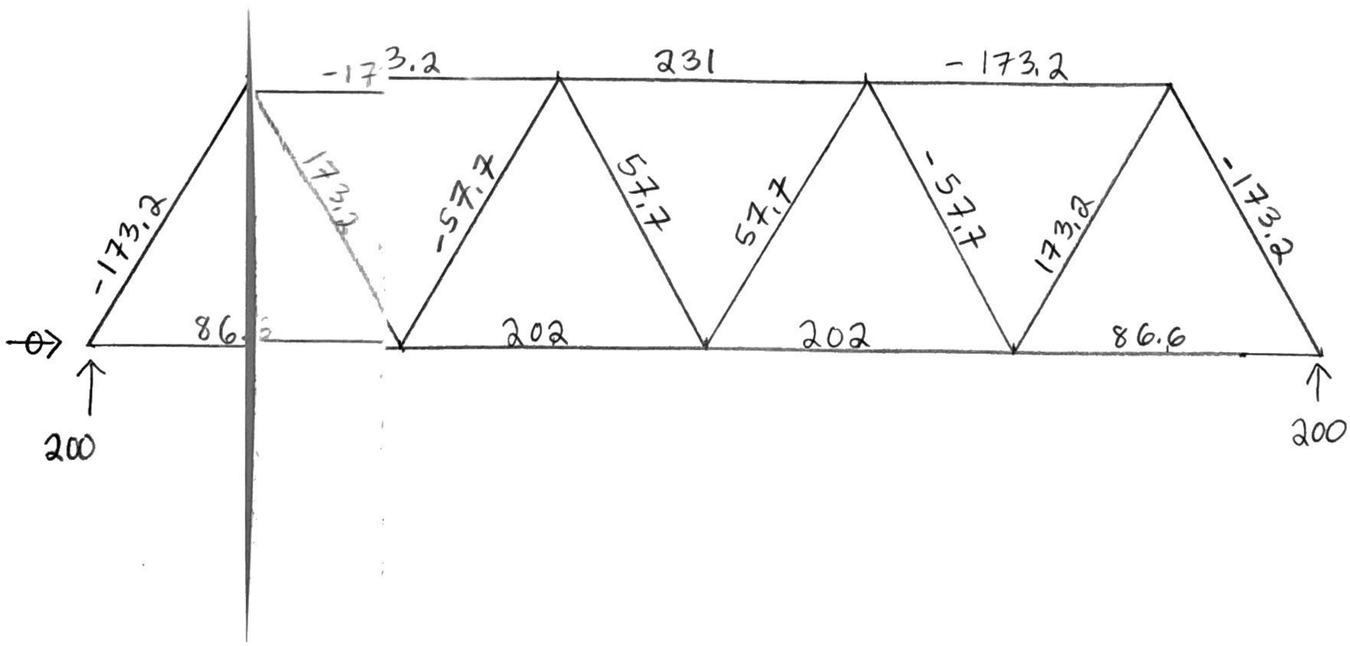
1. You have 2.5 hours to complete this exam. Use your time wisely.
2. There are four questions on the exam. Attempt all questions. Any questions left blank will receive a mark of zero. Part marks will be awarded for incomplete answers.
3. Report all final answers to standard CIV102 accuracy (4 digits if first digit is 1, 3 otherwise)
4. If you need more space, use the back of the pages or extra pages at the end but indicate which page leads to which and which page each one came from so I don't get lost.
5. Write neatly and draw a box around your final answer.

Question Number	Earned Grade	Maximum Grade
1		28
2		29
3		30
4		21
Total		108

1. The following bridge is made of steel HSS members with a yield stress of 350 MPa. **Each member is 5000 mm long** so the total bridge length is 20 metres from E to J. Answer the following questions about it. (28 marks total)



1(a) Solve for the reactions and internal forces in this truss. Indicate the forces on the drawing by writing numbers alongside the members: positive for tension, negative for compression, all in kN (10 marks)



1(b) Given that all members in the truss are HSS 127x127x9.5 ($A=4240\ mm^2$, $I = 9.47\times10^6\ mm^4$), is the truss safe against the loading using the safety factors we use in this course? Determine a factor by which all the shown loads could be increased or decreased to make the structure right on the boundary of being safe and not being safe. (6 marks)

Axial Tension

$F_{Demand} = 202\ kN$

$F_{capacity} = \sigma_y A$

$= (350\ MPa)(4240\ mm^2)$

$= 1484\ kN$

$FOS = \dfrac{F_{capacity}}{F_{Demand}} = \dfrac{1484\ kN}{202\ kN} = 7.35 > 2$

$\Rightarrow$ Safe

Axial Compression

$F_{Demand} = 231\ kN$

$F_{capacity} = 1484\ kN$

$FOS = \dfrac{1484\ kN}{231\ kN} = 6.42 > 2$

$\Rightarrow$ Safe

Buckling

$F_{Demand} = 231\ kN$

$F_{capacity} = \dfrac{\pi^2 EI}{L^2}$

$= \dfrac{\pi^2 (200\,000\ MPa)(9.47\times10^6\ mm^4)}{(5000\ mm)^2}$

$= 748\ kN$

$FOS = \dfrac{748\ kN}{231\ kN} = 3.24 > 3 \Rightarrow$ Safe

$\boxed{\text{Truss is safe}}$

Scale Factor

$FOS = \dfrac{F_{capacity}}{(Factor)(F_{Demand})}$

$\Rightarrow Factor = \dfrac{F_{capacity}}{(FOS)(F_{Demand})}$

For tension:

$Factor = \dfrac{1484\ kN}{(2)(202\ kN)}$

$= 3.67$

For Compression:

$Factor = \dfrac{1484\ kN}{(2)(231\ kN)}$

$= 3.21$

For Buckling:

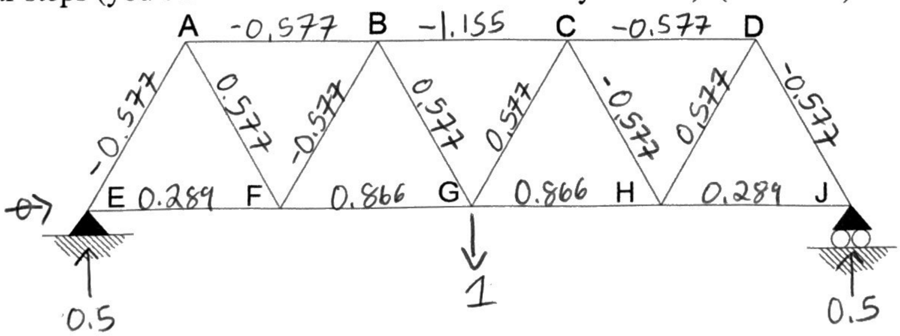
$Factor = \dfrac{748\ kN}{(3)(231\ kN)}$

$= 1.079$

$\boxed{\text{Loads can be increased by a factor of } 1.079.}$

1(c) Solve for the vertical displacement of this truss node G. Use the table on the next page to summarize your steps (you can also use the extra columns if you want). (10 marks)

All forces are in kN.



Member	P Member Force (kN)	Area (mm^2)	Length (mm)			ΔL	* (kN)	(2
AB	-173.2	4240	5000			1.021	-0.577	0.58
BC	-231	4240	5000			-1.362	-1.155	1.573
CD	-173.2	4240	5000			-1.021	-0.577	0.589
EF	86.6	4240	5000			0.511	0.289	0.148
FG	202	4240	5000			1.191	0.866	1.031
GH	202	4240	5000			1.191	0.866	1.031
HJ	86.6	4240	5000			0.511	0.289	0.148
EA	-173.2	4240	5000			-1.021	-0.577	0.589
AF	173.2	4240	5000			1.021	0.577	0.589
FB	-57.7	4240	5000			-0.340	-0.577	0.196
BG	57.7	4240	5000			0.340	0.577	0.196
GC	57.7	4240	5000			0.340	0.577	0.196
CH	-57.7	4240	5000			-0.340	-0.577	0.196
HD	173.2	4240	5000			1.021	0.577	0.589
DJ	-173.2	4240	5000			-1.021	-0.577	0.589

$$\Delta L = \frac{PL}{AE} = \frac{(P \times 1000\ N)(5000\ mm)}{(4240\ mm^2)(200\,000\ MPa)} = 5.90\times10^{-3} P\ mm$$

$\Sigma = 8.25$

$W = (\Delta L)(P^*)$

$(1)(\Delta) = 8.25$

$\Rightarrow \boxed{\Delta = 8.25\ mm}$

1(d) What is your best guess of the natural period of vibration and natural frequency of vibration of this bridge? (2 marks)

Loads on F, G, & H are symmetric

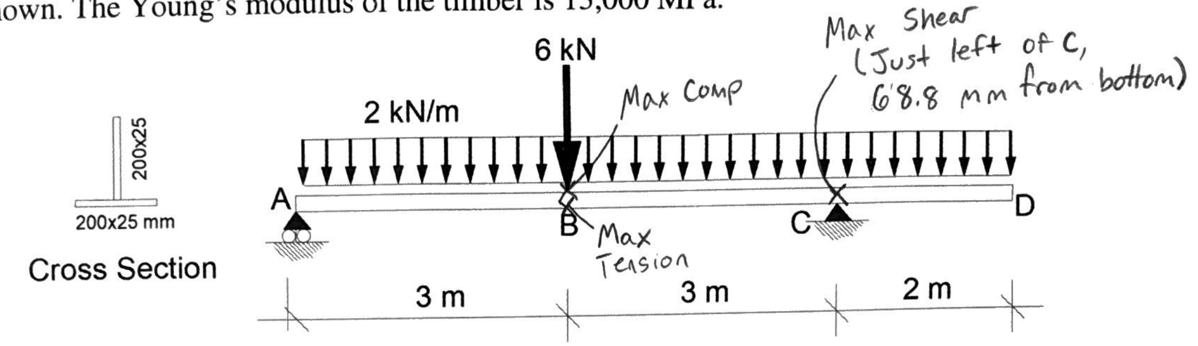
$$\Rightarrow f_n = \frac{17.76}{\sqrt{\Delta}} = \frac{17.76}{\sqrt{8.25}}$$

$$\boxed{f_n = 6.18\ Hz}$$

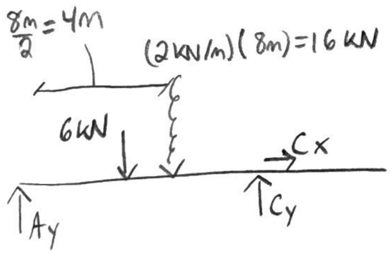
$$T_n = \frac{1}{f_n} = \frac{1}{6.18}$$

$$\Rightarrow \boxed{T_n = 0.1618\ sec}$$

2: The following 8 metre long beam was made by gluing two wooden boards together as shown in the cross section. It is subjected to a uniform load over the entire length of the beam in addition to a point load as shown. The Young's modulus of the timber is 13,000 MPa.



2(a) Determine the reactions for this beam (3 marks)



$\overset{+}{\rightarrow} \Sigma F_x = \boxed{C_x = 0}$

$\circlearrowleft + \ \Sigma M_A = (6 kN)(3m) + (16 kN)(4m) - ((C_y)(6m)) = 0$

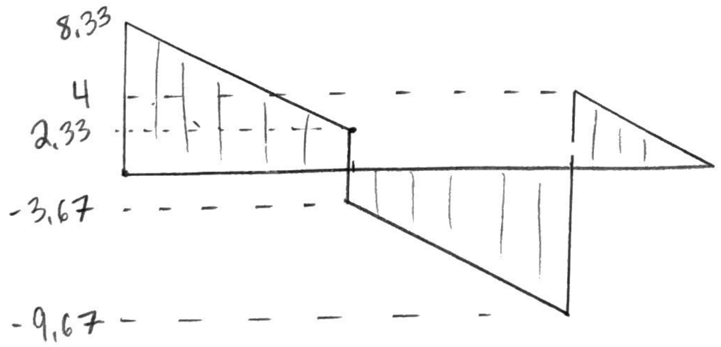
$\Rightarrow C_y = +13.67 kN = \boxed{13.67 kN \uparrow}$

$+\uparrow \Sigma F_y = A_y - 6 kN - 16 kN + 13.67 kN = 0$

$\Rightarrow A_y = +8.33 kN = \boxed{8.33 kN \uparrow}$

2(b) Determine and draw the SFD and BMD for this beam. Show your sign convention and all important values on the drawings. (8 marks)

Diagrams are not to scale.

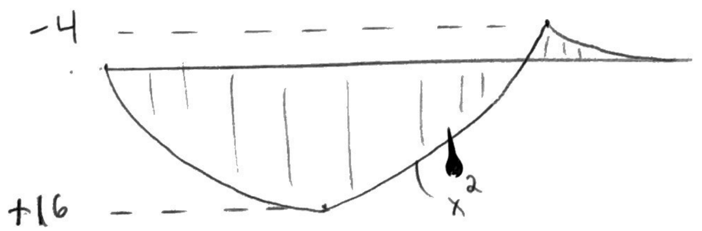


+

V (kN) ↑ + ↓

-

-



M (kNm) ↻ + ↺
(Tension side)

+

2(c) Determine the depth of the centroidal axis and the second moment of area of the cross section. (5 marks) All values are in mm

$$\bar{y} = \frac{(200)(25)(12.5) + (200)(25)(125)}{(200)(25)(2)} = \boxed{68.8 \text{ mm}}$$

$$I = \frac{(200)(25)^3}{12} + (200)(25)(68.8-12.5)^2 + \frac{(25)(200)^3}{12} + (200)(25)(125-68.8)^2$$

$$\boxed{= 48.5 \times 10^6 \text{ mm}^4}$$

2(d) Determine the maximum values of tensile stress and compressive stress in the beam as well as the maximum shear stress. Indicate on the drawing at the start of this question where these maxima and minima are. Finally, calculate the shear stress on the glue joint. (10 marks)

At B:

$$\sigma_T = \frac{M y_{bot}}{I} = \frac{(16\times10^6 \text{ N mm})(68.8\text{ mm})}{48.5\times10^6 \text{ mm}^4} = \boxed{22.6 \text{ MPa}}$$

$$\sigma_c = \frac{-M y_{top}}{I} = \frac{-(16\times10^6 \text{ N mm})(225-68.8\text{ mm})}{48.5\times10^6} = \boxed{-51.5 \text{ MPa}}$$

At C:

$$\sigma_T = \frac{M y_{top}}{I} = \frac{(4\times10^6)(225-68.8)}{48.5\times10^6}$$

$$= 12.9 \text{ MPa}$$

$$\sigma_c = -\frac{M y_{bot}}{I} = \frac{-(4\times10^6)(68.8)}{48.5\times10^6} = -5.66 \text{ MPa}$$

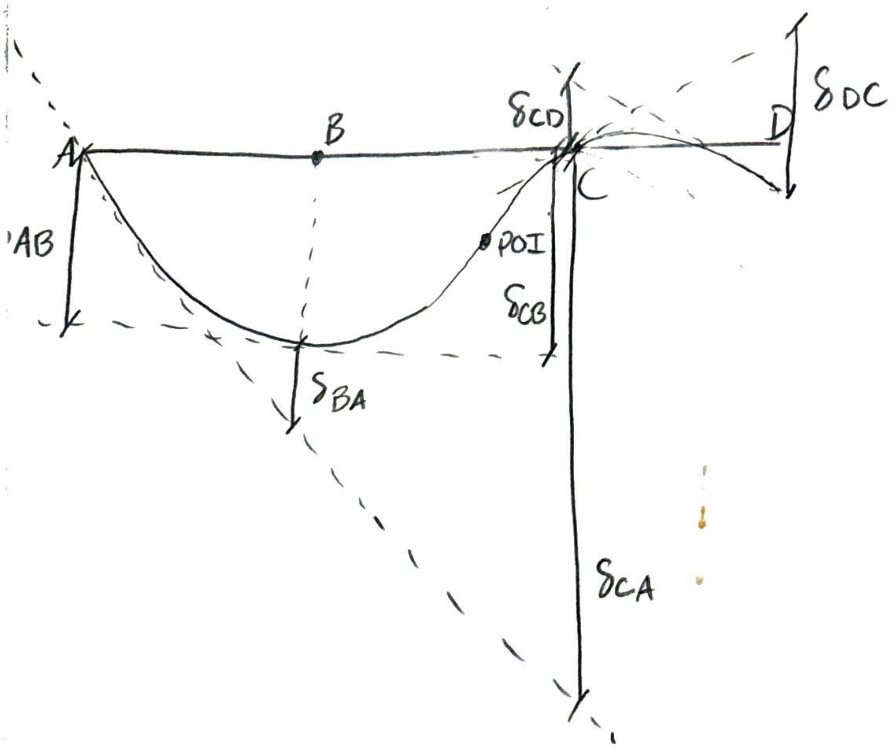
$$Q_{max} = (25\text{mm})(225-68.8\text{ mm})^2(0.5) = 305\times10^3 \text{ mm}^3$$

$$\tau_{max} = \frac{V Q_{max}}{I b_{web}} = \frac{(9.67\times10^3 \text{ N})(305\times10^3 \text{ mm}^3)}{(48.5\times10^6 \text{ mm}^4)(25\text{mm})} = \boxed{2.43 \text{ MPa}}$$

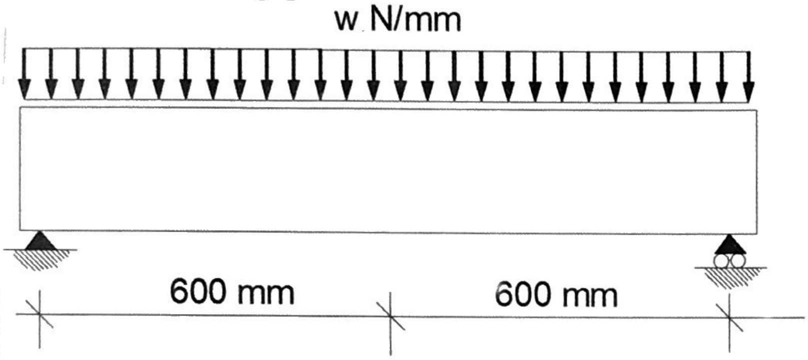
$$Q_{glue} = (200\text{ mm})(25\text{mm})(68.8-12.5\text{ mm}) = 281\times10^3 \text{ mm}^3$$

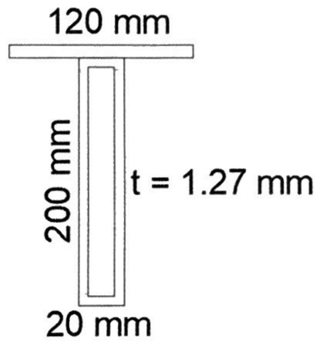
$$\tau_{glue} = \frac{V Q_{glue}}{I b_{glue}} = \frac{(9.67\times10^3 \text{ N})(281\times10^3 \text{ mm}^3)}{(48.5\times10^6 \text{ mm}^4)(25\text{ mm})} = \boxed{2.24 \text{ MPa}}$$

Do not draw the curvature diagram or do any calculations for this, but draw a sketch of the [illegible]laced shape of the beam and carefully draw and label the following values on this plot: δ_{AB}, δ_{BA}, δ[illegible]$_{A}$, δ_{CB}, δ_{CD}, δ_{DC}. (3 marks)



) Based on one of the winning beams for this year's design-build project on matboard, the following esign has been suggested to you to analyze. The train loading to be considered should be approximated s a uniform load along the entire span length specified as w in units of Newtons per millimetre of ength. The cross section is a narrow box with a single layer of matboard fully glued on top as shown. : also contains diaphragms every 100 mm along the length. The material properties are shown below. nswer the following questions:





Cross Section

Compressive strength = 6 MPa
Tensile strength = 30 MPa
Young's Modulus = 4000 MPa

Thickness = 1.27 mm
Poisson's ratio = 0.2
Shear strength = 4 MPa

Glue shear strength = 2 MPa

3(a) Calculate the centroidal location and second moment of area of this cross section (4 marks)

All values are in mm

$$\bar{y} = \frac{(1?)(1.27)(200.635)+(200)(20)(100)-(200-2.54)(20-2.54)(100)}{(120)(1.27)+(200)(20)-(200-2.54)(20-2.54)}$$

$$= \frac{5812}{05}$$

$$\boxed{= 121.8 \text{ mm}}$$

$$I = \frac{(20)(1.27)^3}{12} + (120)(1.27)(200.635-121.8)^2 + \frac{(20)(200)^3}{12} + (20)(200)(121.8-100)^2$$

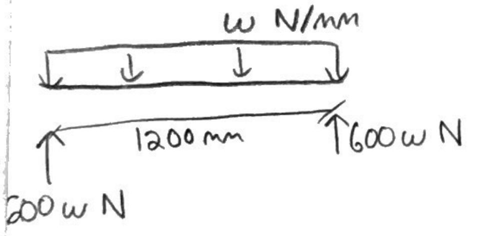
$$- \frac{(20-2.54)(200-2.54)^3}{12} - (20-2.54)(200-2.54)(121.8-100)^2$$

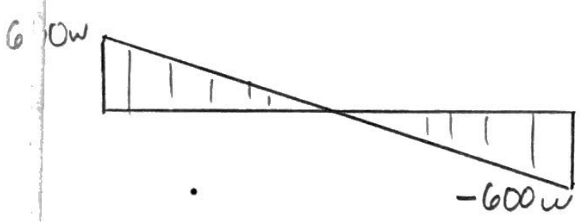
$$= 20 + 948\times10^3 + 13.33\times10^6 + 1.894\times10^6 - 11.20\times10^6 - 1.633\times10^6$$

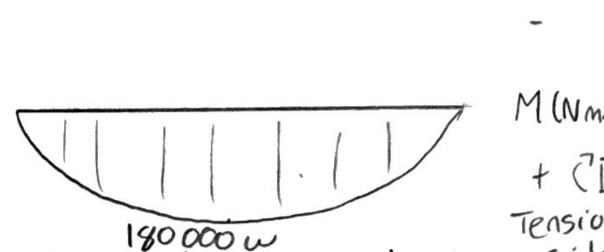
$$\boxed{= 3.34\times10^6 \text{ mm}^4}$$

3(b) Draw and appropriately label the SFD and BMD for this member as a function of w. (4 marks)

Diagrams are not to scale







3(c) Find and explain the value of w needed to cause failure by maximum tension or compression stress of the material due to bending. (4 marks)

$$\sigma_T = \frac{M\, y_{bot}}{I}$$

$$30\ MPa = \frac{(180\,000w\ Nmm)(121.8\ mm)}{3.34 \times 10^6\ mm^4}$$

$$\Rightarrow \boxed{w_{Tension} = 4.57\ N/mm}$$

$$\sigma_C = \frac{M\, y_{top}}{I}$$

$$6\ MPa = \frac{(180\,000w\ Nmm)(201.27 - 121.8\ mm)}{3.34 \times 10^6\ mm^4}$$

$$\Rightarrow \boxed{w_{compression} = 1.401\ N/mm}$$

3(d) Find the maximum value of w to cause the member to fail by shear stresses reaching the shear strength and also the value required to cause the glue to fail in shear. (4 marks)

$$Q_{max} = (20mm)(121.8\ mm)^2(0.5) - (20 - 2.54mm)(121.8 - 1.27\ mm)^2(0.5)$$
$$= 21.5 \times 10^3\ mm^3$$

$$\tau_{max} = \frac{V\, Q_{max}}{I\, b_{web}} \Rightarrow \frac{(600w\ N)(21.5 \times 10^3\ mm^3)}{(3.34 \times 10^6\ mm^4)(2.54mm)} = 4\ MPa$$

$$\Rightarrow \boxed{w_{shear} = 2.63\ N/mm}$$

$$Q_{glue} = (120mm)(1.27\ mm)(200.635 - 121.8\ mm)$$
$$= 12.02 \times 10^3\ mm^3$$

$$\tau_{glue} = \frac{V\, Q_{glue}}{I\, b_{glue}} \Rightarrow 2\ MPa = \frac{(600w\ N)(12.02 \times 10^3\ mm^3)}{(3.34 \times 10^6\ mm^4)(120\ mm)}$$

$$\Rightarrow \boxed{w_{glue} = 18.53\ N/mm}$$

3(e) Find the values of w to cause failure of the member by buckling in each cases that applies to this design. Explain your steps (6 marks)

Case 1.

$$\frac{(180000w\ Nmm)(201.27-121.8\ mm)}{3.34\times10^6\ mm^4} = \frac{4\pi^2(4000\ MPa)}{12(1-0.2^2)}\left(\frac{1.27mm}{20-1.27mm}\right)^2 \Rightarrow \boxed{w_{case\ 1} = 14.71\ N/mm}$$

Case 2:

$$\frac{(180000w)(201.27-121.8)}{3.34\times10^6} = \frac{(0.425)\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{50.635}\right)^2 \Rightarrow \boxed{w_{case\ 2} = 0.214\ N/mm}$$

Case 3:

$$\frac{(180000w)(200-0.635-121.8)}{3.34\times10^6} = \frac{6\pi^2(4000)}{12(1-0.2^2)}\left(\frac{1.27}{200-0.635-121.8}\right)^2 \Rightarrow \boxed{w_{case\ 3} = 1.317\ N/mm}$$

Case 4:

$$\frac{(600w)(21.5\times10^3)}{(3.34\times10^6)(2.54)} = \frac{5\pi^2(4000)}{12(1-0.2^2)}\left[\left(\frac{1.27}{200-1.27}\right)^2+\left(\frac{1.27}{100}\right)^2\right] \Rightarrow \boxed{w_{case\ 4} = 2.28\ N/mm}$$

3(f) what is your best estimate of the value of w required to make the member fail and why? (3 marks)

The expected w causing failure is 0.214 N/mm due to buckling of the sides of the top flange. This makes sense since the edges are unsupported on one side, allowing easier buckling. These portions are also rather long.

3(g) In point form, provide a critique of the design of this bridge based on what you know about bridges from all you have seen this term. Provide me with two good things and three bad things. (5 marks)

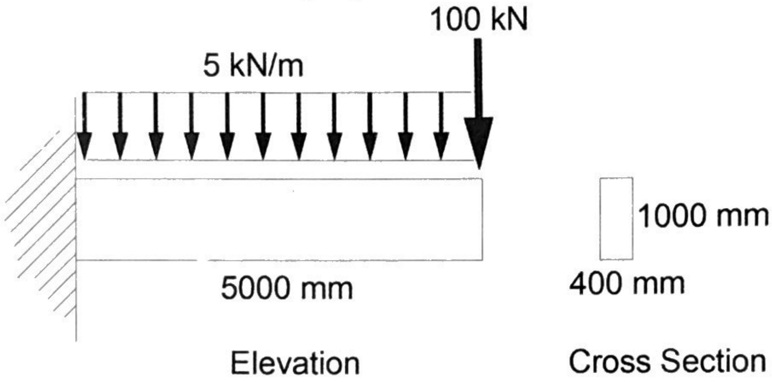
Good

- Very tall, leading to high I
- Very thin, leading to increased strength against case 1 buckling

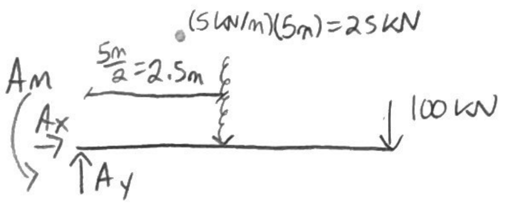
Bad

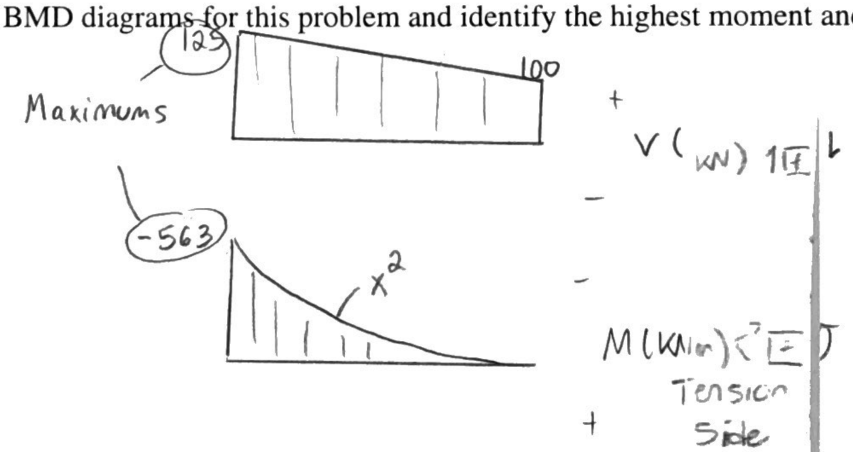
- Wheels will not be over the webs, which might cause bending failure in the edges of the top flange
- Very tall, may fall over if the load isn't centred
- Thin middle region may make it difficult to install diaphragms properly

4: You have been asked to complete a design for a 5 metre long reinforced concrete cantilever that will hold a heavy advertising sign at its end for a newly planned downtown store. A preliminary design suggested a 400 mm wide by 1000 mm tall beam should work for the shown loading. The initial flexural design (which could be wrong) suggested that the flexural steel can be 4-35M reinforcing bars ($A_s = 4000\ mm^2$) with an effective depth d of 900 mm. The store owner is concerned about embodied carbon and sustainability and so has requested a steel strength $f_y = 500$ MPa and the concrete strength f_c' of only 20 MPa as these should have lower carbon footprints. The shown loading includes the weight of the concrete beam as well as the additional signage that the beam will also support.



4(a) Draw and label the SFD and BMD diagrams for this problem and identify the highest moment and shear force. (4 marks)





$\xrightarrow{+} \Sigma F_x = A_x = 0$

$+\uparrow \Sigma F_y = A_y - 25\ kN - 100\ kN = 0$

$\Rightarrow A_y = 125\ kN \uparrow$

$\Sigma M_A = -A_M + (25\ kN)(2.5m) + (100\ kN)(5m) = 0 \Rightarrow A_M = 563\ kNm$; Diagrams are not to scale

4(b) Determine the applied stress in the steel reinforcement due to the applied loads as well as the maximum stress in the concrete. Are these values safe for these low-carbon materials using the safety factors we used in the course? (4 marks)

$E_c = 4500\sqrt{f_c'} = 20.1\times10^3\ MPa$

$n = \frac{E_s}{E_c} = \frac{200000\ MPa}{20.1\times10^3\ MPa} = 9.94$

$\rho = \frac{A_s}{bd} = \frac{(4)(1000\ mm^2)}{(400mm)(900mm)} = 0.01111$

$k = \sqrt{[(9.94)(0.01111)]^2 + (2)(9.94)(0.01111)} - (9.94)(0.01111)$

$= 0.372$

$jd = d(1 - \frac{1}{3}k) = 900\ mm(1 - \frac{0.372}{3})$

$788\ mm$

$f_s = \frac{M}{A_s jd} = \frac{563\times10^6\ Nmm}{(4)(1000\ mm^2)(788mm)} = 170\ MPa$

$0.6 f_y = (0.6)(500\ MPa) = \boxed{300\ MPa} > f_s \Rightarrow$ Safe

$f_c = \frac{k}{1-k}\frac{M}{nA_s jd} = \frac{0.372}{(1-0.372)}\frac{563\times10^6\ Nmm}{(9.94)(4)(1000\ mm^2)(788mm)}$

$= 10.6\ MPa$

$0.5 f_c' = (0.5)(20\ MPa) = \boxed{10\ MPa} < f_c$

$\Rightarrow \boxed{UNSAFE}$

4(c) Perform the shear design for this beam as was described in class. Explain your steps. If you need stirrups, use a 10M bar size. (a 10M bar has area of 100 mm^2) (10 marks)

$V_{max} = 0.125 f'_c b_w jd = (0.125)(20\ MPa)(400\ mm)(788\ mm) = 788\ kN > 125\ kN$ ✓

Check V_c without stirrups

$$V_c = \frac{(0.5)(230)\sqrt{f'_c}\, b_w jd}{1000 + 0.9d} = \frac{(0.5)(230)\sqrt{20\ MPa}\,(400\ mm)(788\ mm)}{1000 + (0.9)(900\ mm)} = 89.5\ kN < 125\ kN \ \text{X}$$

$\Rightarrow$ Stirrups needed

Minimum reinforcement spacing with 2 leg stirrups ($A_v = 200\ mm^2$)

$$s \leq \frac{A_v f_y}{0.06\sqrt{f'_c}\, b_w} = \frac{(200\ mm^2)(500\ MPa)}{(0.06)\sqrt{20\ MPa}\,(400\ mm)} = 932\ mm$$

Maximum $s = 0.7jd$ or $600\ mm$

$S_{max} = (0.7)(788\ mm) = 551\ mm \Rightarrow$ Use $\boxed{s = 550 mm}$

$V_c = (0.5)(0.18)\sqrt{f'_c}\, b_w jd = (0.5)(0.18)\sqrt{20\ MPa}\,(400\ mm)(788\ mm) = 126.9\ kN$

$$V_s = \frac{0.6 A_v f_y jd \cot 35°}{s} = \frac{(0.6)(200\ mm^2)(500\ MPa)(788\ mm)}{(550\ mm)\tan 35°} = 122.8\ kN$$

$V_{total} = V_c + V_s = 126.9\ kN + 122.8\ kN = 250\ kN > 125\ kN$ ✓

4(d) Summarize your design with a sketch of the side view of the beam showing all reinforcement (3 mark) Sketch is not to scale