



UNIVERSITY OF TORONTO  
FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION: April 20, 2023 9:30 AM - 12:00 PM

DURATION: 2½ hours

First Year - Engineering Science  
ECE159H1S - Fundamentals of Electric Circuits

First name (please write as legibly as possible within the boxes)

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Last name

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Student Number

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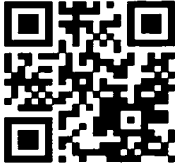
Calculator Type: 2 (All non-programmable calculators permitted)

Exam Type: C (One 8.5" x 11" double-sided entirely hand-written aid sheet is allowed)

Examiner: M. Stickel

1. DO NOT disconnect any pages. Leave this booklet stapled.
2. Where appropriate, include your final answers for the questions in the boxes provided. Make sure to include units if your answer is numeric.
3. The last page is left blank and can be used for rough work or extra space. It will NOT be reviewed unless you clearly direct the marker to your work on this page.
4. Ensure you respect all academic integrity policies and guidelines. Your work must be entirely your own.
5. Take a deep breath and relax!

Question	Marks
Q1	20
Q2	20
Q3	20
Q4	20
Q5	20
BONUS	5
Total	100

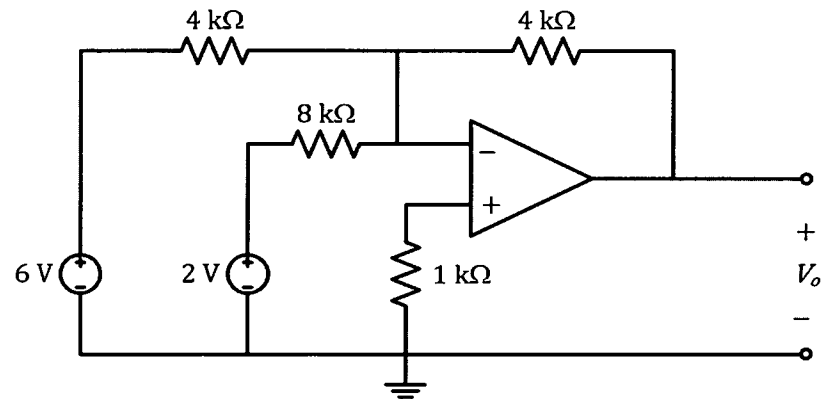


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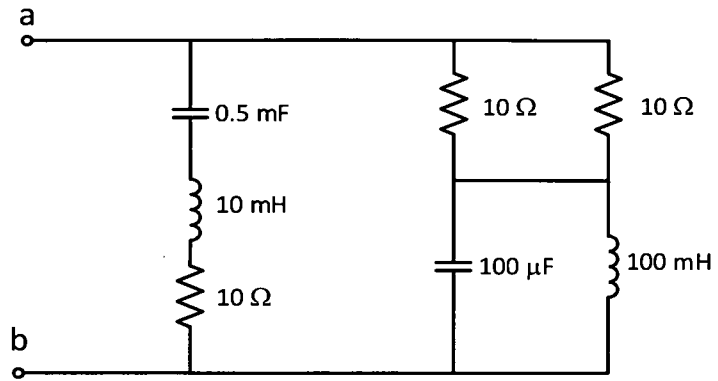
- (6 marks) 1. (a) For the op-amp circuit below, determine the value of the output voltage  $V_o$ . Make sure to show all of your work.



$V_o =$  \_\_\_\_\_

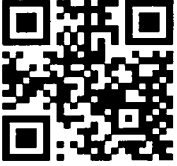


- (6 marks) 1. (b) Find the equivalent impedances at the terminals  $a - b$  at the frequencies  $\omega_1 = 0$  rad/s and  $\omega_2 = 500$  rad/s for the circuit shown below. Express your final answers in polar form.

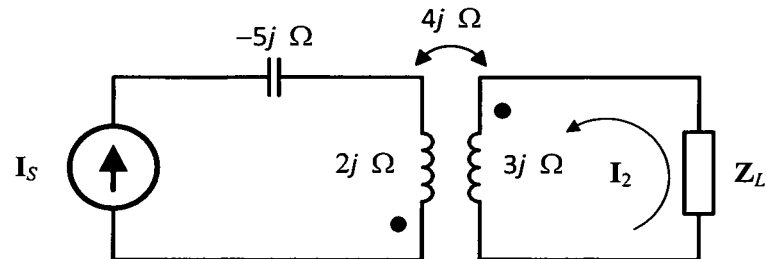


$$Z_{eq}(\omega_1) = \underline{\hspace{2cm}}$$

$$Z_{eq}(\omega_2) = \underline{\hspace{2cm}}$$



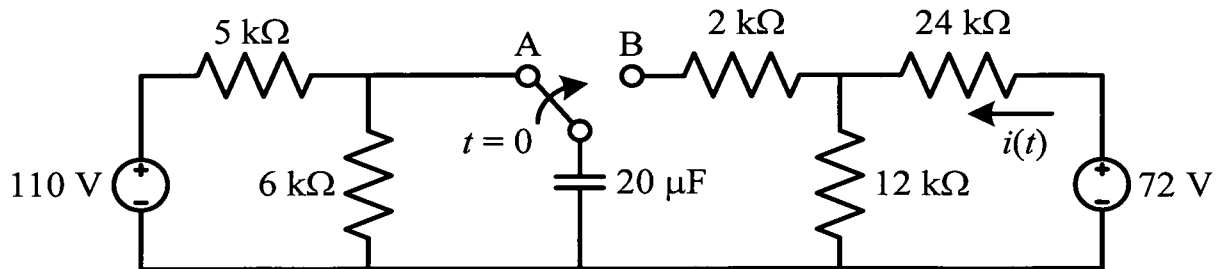
- (8 marks) 1. (c) Determine the impedance  $Z_L$  in the circuit below, such that the phasor current  $I_2$  has twice the amplitude and the same phase as  $I_S$ . Is the impedance capacitive or inductive in nature?



$Z_L =$  \_\_\_\_\_  
Capacitive or Inductive



- [20 marks] 2. For the circuit shown below, the switch has been in position “A” for a long time. At  $t = 0$  s the switch is moved instantaneously to position “B”.



- (3 marks) (a) Find the energy stored in the capacitor just before the switch moves.

$w_C(t = 0-) = \underline{\hspace{2cm}}$

- (4 marks) (b) Find the values of  $v_C(0+)$  and  $i(0+)$ .

$v_C(0+) = \underline{\hspace{2cm}}$

$i(0+) = \underline{\hspace{2cm}}$



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2. continued

(2 marks)

(c) Find the values of  $v_C(\infty)$  and  $i(\infty)$ .

$v_C(\infty) =$  \_\_\_\_\_

$i(\infty) =$  \_\_\_\_\_

(2 marks)

(d) Find the time constant of the circuit for  $t > 0$ .

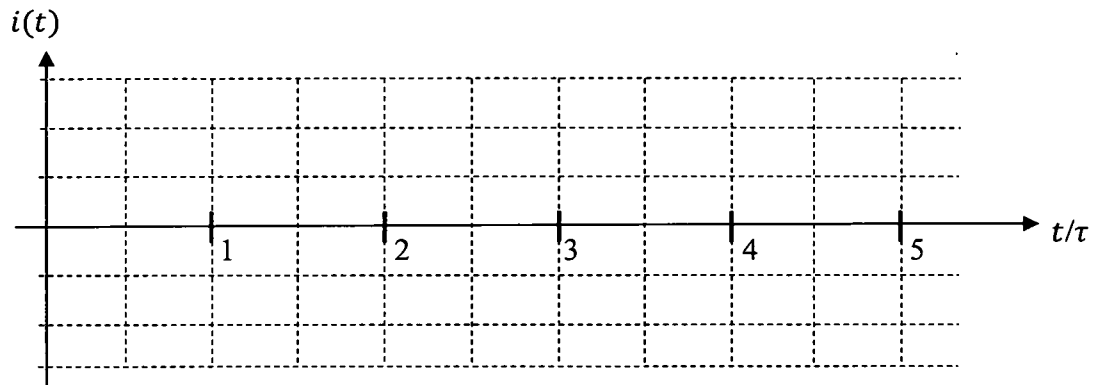
$\tau =$  \_\_\_\_\_

(5 marks)

(e) Find the expression for  $i(t)$  for  $t > 0$  s and sketch this current for the first  $5\tau$  on the figure below.



2. (e) continued



$i(t) =$  \_\_\_\_\_

(2 marks)

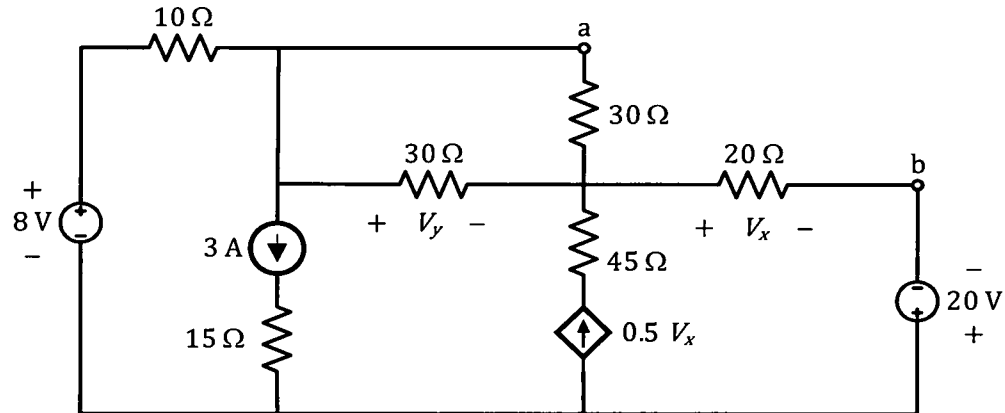
- (f) Does the capacitor absorb or supply energy after the switch is moved to position B? Justify your answer.

(2 marks)

- (g) If this circuit had an inductor instead of a capacitor where the capacitor is connected, how would that have changed the final value of the current  $i(t)$ ?



[20 marks] 3. For the circuit shown below:



- (10 marks) (a) Use mesh analysis to show that  $V_x = \frac{8}{41} \text{ V} = 0.195 \text{ V}$  and  $V_y = -\frac{54}{41} \text{ V} = -1.317 \text{ V}$ . If you would like, you may apply any initial simplifications to the circuit that you think appropriate before applying mesh analysis.



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3. (a) continued



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- (10 marks) 3. (b) Find the Thévenin equivalent of the circuit as seen across the terminals  $a - b$ . What is the maximum power the circuit can transfer to a matched load placed across the terminals  $a - b$ ? What is that matched load?

*Hint:* The results from part (a) might be useful for part of this problem.



3. (b) continued

$V_{th} =$  \_\_\_\_\_

$R_{th} =$  \_\_\_\_\_

$R_{LMatched} =$  \_\_\_\_\_

$P_{max} =$  \_\_\_\_\_

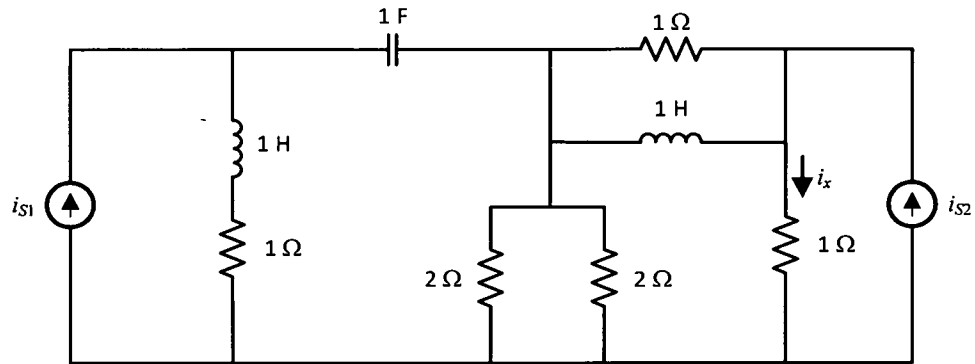


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[20 marks] 4. In the circuit below,  $i_{S1}(t) = 2 \cos(t + 60^\circ)$  A and  $i_{S2}(t) = 3$  A.



(14 marks) (a) Determine  $i_x$  in the steady-state.

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4. (a) continued



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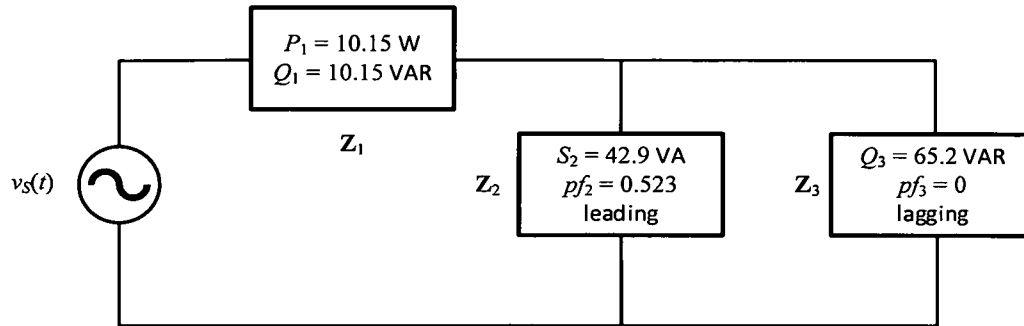
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- (6 marks) 4. (b) Determine the expression for the power delivered to the circuit by the current source  $i_{s1}(t)$  (i.e.,  $p_{s1}(t)$ ), and find the average value of  $p_{s1}(t)$  and the power factor for the circuit at  $\omega = 1$  rad/s. *Hint:* Remember that  $\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$



- [20 marks] 5. Below is a circuit with three impedances that are described in terms of their complex power characteristics. The circuit is excited by a source  $v_s(t) = 225 \cos(120\pi t)$  V.



- (14 marks) (a) Determine the total complex power provided by the source,  $S_{tot}$ , the power factor for the entire circuit, the value of the impedance  $Z_2$ , and the current leaving the source,  $i_s(t)$ .



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5. (a) continued

$$S_{tot} = \underline{\hspace{2cm}}$$

$$pf_{tot} = \underline{\hspace{2cm}}$$

$$Z_2 = \underline{\hspace{2cm}}$$

$$i_s(t) = \underline{\hspace{2cm}}$$

(6 marks)

- (b) In order to correct the power factor for the entire circuit to be 0.95 *leading*:
- How much reactive power would need to be added to the circuit, such that the real power for the entire circuit remains the same? Draw the original and new power triangles.
  - What is the value of the capacitor that would be added to the circuit to correct this power factor? Indicate on the circuit where you would place this capacitor to properly correct the power factor.

$$Q_{added} = \underline{\hspace{2cm}}$$

$$C_{added} = \underline{\hspace{2cm}}$$

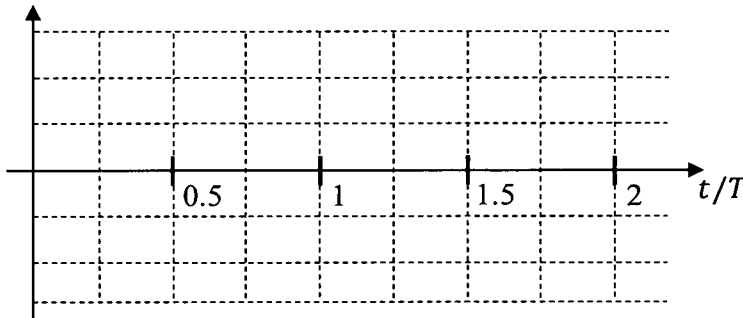
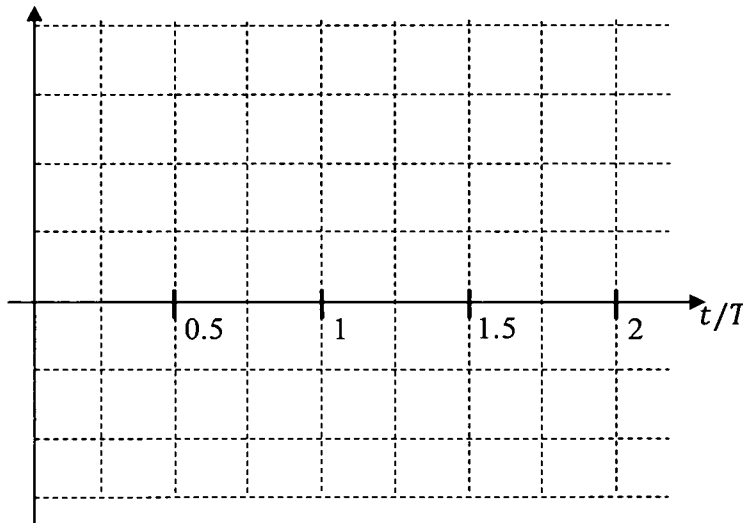
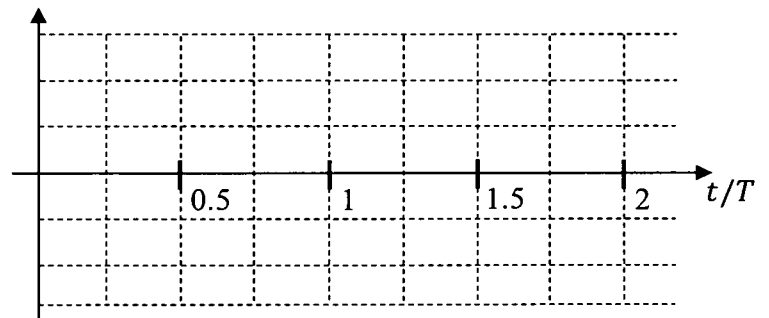
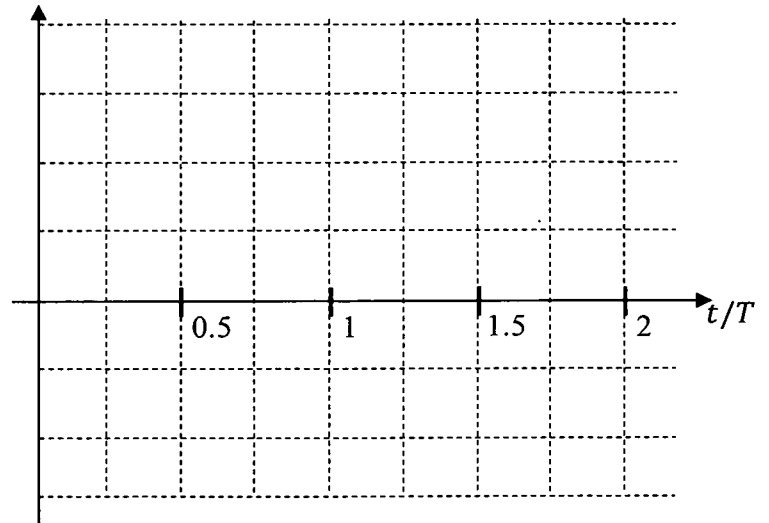


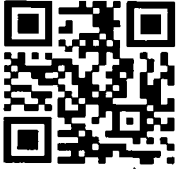


(5 marks)

**Bonus Question**

Consider in more detail the situation in Question 5 above. In the figures below, sketch the source voltage, current and power for the two power factor cases. These do not need to be exact drawings, reasonable sketches are all that is needed. Clearly indicate or describe the primary differences between the two cases in terms of the voltage, current and power waveforms.

**Case A: Original Power Factor** $v_{SA}(t), i_{SA}(t)$  $p_{SA}(t)$ **Case B: Corrected Power Factor** $v_{SB}(t), i_{SB}(t)$  $p_{SB}(t)$ 



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