Last name, first name (print legibly):____________________

Student # (print legibly):____________________

Q1:____Q2:____Q3:____Q4:____Q5:____Q6:____

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

October 26, 2017

Instructor – Professor W.R. Cluett

Closed book.

Allowable calculators:

- Sharp EL-520X
- Sharp EL-520W
- Casio FX-991
- Casio FX-991EX
- Casio FX-991ES Plus
- Casio FX-991MS

All questions of equal value.

All work to be marked <u>must</u> appear on front of page. Use back of page for rough work only.

Given information:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

$$proj_{\vec{d}}\vec{u} = \frac{\vec{u}\cdot\vec{d}}{\|\vec{d}\|^2}\vec{d}$$

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Q1: Consider the following two lines in R^3:

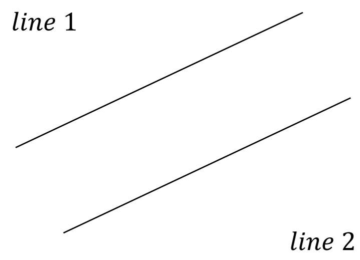
$$\overrightarrow{x_1} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad (line\ 1)$$

$$\overrightarrow{x_2} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} + s\begin{bmatrix} 5 \\ 2.5 \\ 2.5 \end{bmatrix} \quad (line\ 2)$$

a) Prove that these two lines are parallel.

b) Prove that these two lines are not the same line.

c) Using projections, find the minimum distance between these two lines. To receive full marks, you must include a sketch as part of your solution.



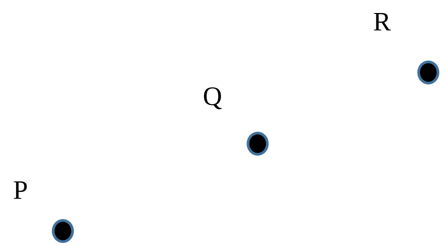
Extra page for Q1(c)

Q2: Assume that P, Q and R are collinear points in R^3 and let $\overrightarrow{OP} = \vec{p}, \overrightarrow{OQ} = \vec{q}$ and $\overrightarrow{OR} = \vec{r}$ where O denotes the origin.

Show that:

$$(\vec{p} \times \vec{q}) + (\vec{q} \times \vec{r}) + (\vec{r} \times \vec{p}) = \vec{0}$$

To receive full marks, you must include a sketch as part of your solution.



Extra page for Q2

Q3: Let $\vec{w}$ be a fixed vector in R^3. Define a transformation $\vec{v} = cross_{\vec{w}}\vec{u}$ where:

$$cross_{\vec{w}}\vec{u} = \vec{w} \times \vec{u}$$

a) Use the properties of the cross product, rather than the definition of cross product, to verify that $cross_{\vec{w}}\vec{u}$ is a linear transformation.

b) Determine the matrix M associated with this linear transformation, i.e. $\vec{v} = M\vec{u}$.

c) Without trying to find the determinant of matrix M, do you believe the matrix derived in part (b) has an inverse? Give a geometric explanation of your answer.

Q4: This question has two separate parts, (a) and (b). Both parts are related to eigenvalues and eigenvectors.

a) Let $A = \begin{bmatrix} -3 & 1 & -1 \\ 8 & -3 & 8 \\ 8 & -1 & 6 \end{bmatrix}$. Determine whether these two vectors, $\vec{v}$ and $\vec{w}$, are eigenvectors and, if they are, determine the corresponding eigenvalues:

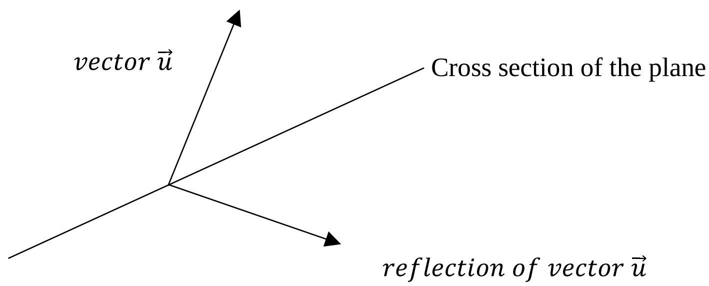
$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

b) Suppose that $\vec{u}$ is an eigenvector of both matrix B and C with corresponding eigenvalue λ for B and corresponding eigenvalue α for C. Show that $\vec{u}$ is an eigenvector of (B+C) and of BC and determine the corresponding eigenvalues.

Q5: Consider the reflection of the vector $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ through a plane that has scalar equation:

$$x + 2y + 3z = 0$$

Find the vector obtained by this reflection.



Extra page for Q5

Q6: Consider the following 3x4 matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

a) Determine the reduced normal form (RNF) and rank of this matrix A.

b) Use Gaussian elimination to solve the following homogeneous system:

$$A\vec{x} = \vec{0}$$

assuming that the unknown variables in $\vec{x}$ are given by x_1, x_2, x_3, x_4.

c) Geometrically, what does the solution in part (b) represent?