Q1: (a) $L > d$, the spring is stretched.

(b) $L-d$ represents the amount of stretching in the spring. The corresponding force by the spring is $k(L-d)$. If this force matches F_2 (equilibrium), then

$$F_2 = m_2 g = k(L-d) \Rightarrow k = \frac{m_2 g}{L-d}$$

(c) $(L-d)$ is now the amplitude of the motions.

$$x = A\cos(\omega t + \phi) \ ; \qquad \dot{x} = -\omega A \omega \sin(\omega t + \phi)$$

$\dot{x}(t=0) = 0 \Rightarrow \phi = 0$ or π, but has to be 0 for $A > 0$

$\Rightarrow x(t=0) = A = L-d$

Frequency is $\omega_0^2 = \frac{k}{m_1} = \frac{m_2}{m_1}\frac{g}{L-d} \Rightarrow$ period $T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{(L-d)}{g}\frac{m_1}{m_2}}$

Q2 (PS2 6, King 3.10)

(a) $K = \frac{1}{2}m\dot{x}^2 = \frac{1}{2}m\omega^2 A^2(\omega)\sin^2(\omega t - \delta)$

$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2 A^2(\omega)\cos^2(\omega t - \delta) \qquad \left(\omega_0 = \frac{k}{m}\right)$

$E = U + K = \frac{1}{2}A^2(\omega)m\left[\omega^2\sin^2(\omega t - \delta) + \omega_0^2\cos^2(\omega t - \delta)\right]$

(b) If $\omega = \omega_0$, then $E(t) = \frac{1}{2}m\omega_0^2 A^2(\omega)\underbrace{\left[\sin^2(\omega t - \delta) + \cos^2(\omega t - \delta)\right]}_{=1}$

(c) Time-averaged KE: (i.e., avg over one cycle):

$\overline{K} = \frac{1}{2}m\omega^2 A^2(\omega)\frac{1}{T}\int_0^T \sin^2(\omega t - \delta)\,dt \qquad$ with $T = \frac{2\pi}{\omega}$

$\Rightarrow \overline{K} = \frac{1}{4}m\omega^2 A^2(\omega)$

$$\bar{E} = \frac{1}{2} m A^2(\omega)\left[\omega^2 \underbrace{\frac{1}{T}\int_0^T \sin^2(\omega t-\delta)\,dt}_{=1/2} + \omega_0^2 \underbrace{\frac{1}{T}\int_0^T \cos^2(\omega t-\delta)\,dt}_{=1/2}\right]$$

$$\Rightarrow \quad \bar{E} = \frac{1}{4} m A^2(\omega)\left(\omega^2+\omega_0^2\right)$$

$$\Rightarrow \frac{\bar{K}}{\bar{E}} = \frac{\omega^2}{\omega_0^2+\omega^2} = \frac{1}{1+\omega_0^2/\omega^2} = f(\omega)$$

d) $\bar{U} = \bar{K} \Rightarrow \bar{E} = 2\bar{K} \Rightarrow \dfrac{1}{1+\frac{\omega_0^2}{\omega^2}} = \dfrac{1}{2} \Rightarrow \omega = \omega_0$

e) $\bar{E} = \dfrac{1}{2} m A^2(\omega)\left(\omega^2+\omega_0^2\right) = \dfrac{1}{4}\cancel{m}\dfrac{F_0^2}{m^{\cancel{2}}}\dfrac{(\omega^2+\omega_0^2)}{(\omega_0^2-\omega^2)+\frac{b^2}{m^2}\omega^2}$

Q3:

a) Only force felt by m_1 is spring.

$m_1\ddot{x}_1 = +k(x_2-x_1-l)$ (if $x_2-x_1=l$, no force; if x_2 is increased, everything else equal, force is >0)

$m_2\ddot{x}_2 = -k(x_2-x_1-l)$ (opposite of above).

$\ddot{x} = \ddot{x}_2 - \ddot{x}_1 \Rightarrow m_1 m_2 \ddot{x}_2 - m_2 m_1 \ddot{x}_1 = -(m_2+m_1)kx$

$m_1 m_2 \ddot{x} = -(m_2+m_1)kx \Rightarrow \ddot{x} + \omega_0^2 x = 0$ with $\omega_0^2 = \dfrac{k}{\mu}$

$\mu = \dfrac{m_1 m_2}{m_1+m_2}$

divide by $m_1 m_2$

b) $k = \mu\omega_0^2 = \dfrac{1.67\times10^{-27}\times 23\times 35}{23+35}\times\left(1.14\times10^{13}\times 2\pi\right)^2 \approx 119\ \mathrm{N\,m^{-1}}$