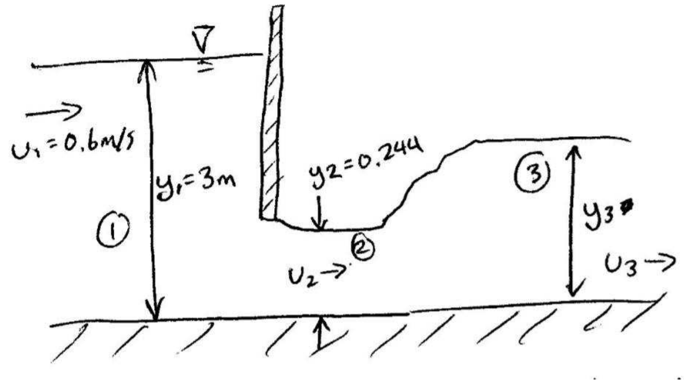
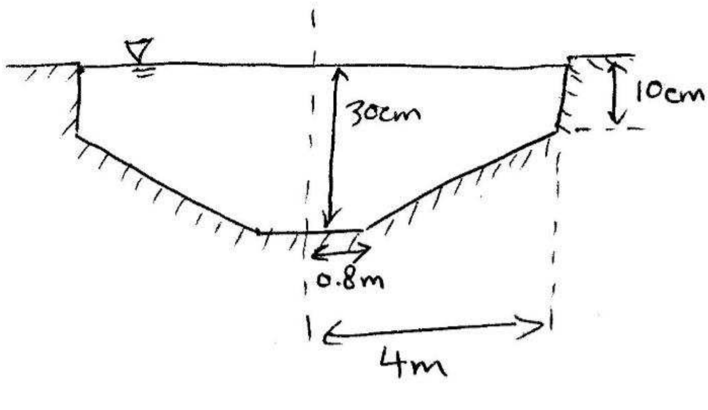
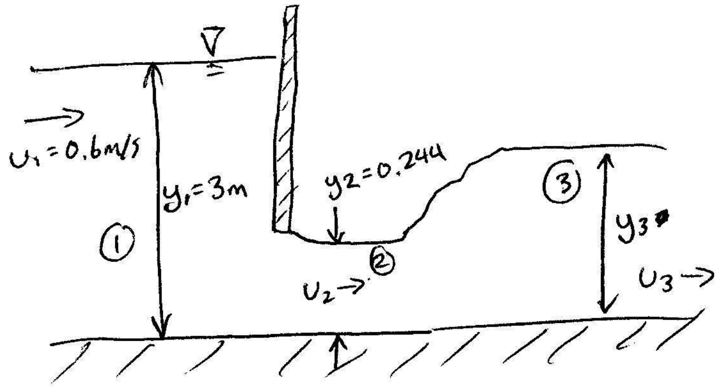
1) A hydraulic jump downstream of a sluice gate is shown in the image.

- Find the Froude number at the three points (1), (2), (3). Comment on the state of the flow at each location.
- Find the water height, y_3, and the water speed U_3, after the hydraulic jump.



2) A child throws a pebble into the centre of a circular pool. The bottom of the pool is shaped as shown in the image. Calculate the time for the first ripple to travel to the edge of the pool.





A hydraulic jump downstream of a sluice gate is shown in the image. Find the Froude numbers at locations 1, 2 and 3. Find the height y_3 and ~~dept~~ water speed U_3 after the hydraulic jump. State if the flow is subcritical, critical, or supercritical at each location.

Froude number @ 1 $\quad Fr_1 = \frac{U_1}{\sqrt{g y_1}} = \frac{0.6}{\sqrt{9.81 \times 3}} = 0.11 < 1 \therefore$ SUBCRITICAL

$$U_1 y_1 = U_2 y_2$$

$$U_2 = \frac{U_1 y_1}{y_2} = \frac{0.6 \times 3}{0.244} = 7.38 m/s$$

Froude number @ 2

$$Fr_2 = \frac{U_2}{\sqrt{g y_2}} = \frac{7.38}{\sqrt{9.81 \times 0.244}} = 4.77 \quad \longrightarrow \quad Fr_2 = 4.77 > 1 \therefore \text{SUPERCRITICAL}$$

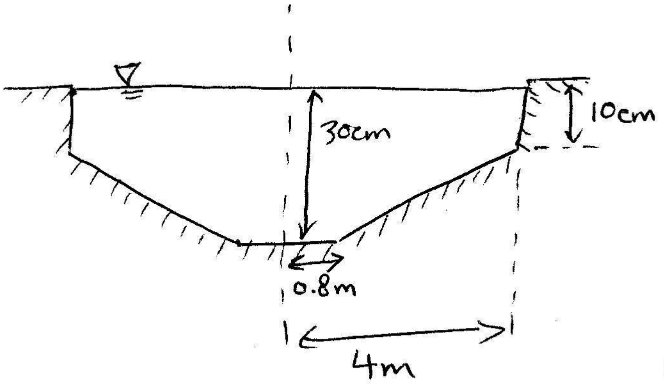
Across the hydraulic jump

$$\frac{y_3}{y_2} = \frac{1}{2}\left[\sqrt{1 + 8 Fr_2^2} - 1\right] = \frac{1}{2}\left[\sqrt{1 + 8 \times 4.77^2} - 1\right] = 6.26$$

$$y_3 = 6.26 \, y_2 = 6.26 \times 0.244 = 1.528 \, m$$

$$U_3 = \frac{U_2 y_2}{y_3} = \frac{7.38 \times 0.244}{1.528} = 1.18 m/s$$

$$Fr_3 = \frac{U_3}{\sqrt{g y_3}} = \frac{1.18}{\sqrt{9.81 \times 1.528}} = 0.3 < 1 \therefore \text{SUBCRITICAL}$$



FIND:

Time for first ripple to travel to the edge of a circular pool.

Assume: wave is small compared to depth so it can be approximated by $U = \sqrt{gy}$

The depth, y, changes with radial distance, r.

The time dt for the wave to travel the short distance dr is:

$$dt = \frac{dr}{U}$$

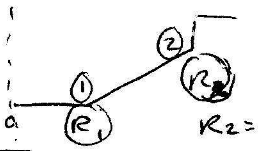
So the total time is the integral:

$$t = \int_0^t dt = \int_0^R \frac{dr}{U}$$

As the pool floor is flat in the centre then linearly changes with radius, we will split the right-hand side into two integrals.

$$t = \int_0^{R_1} \frac{dr}{U_1} + \int_{R_1}^{R_2} \frac{dr}{U_2}$$

$$U_1 = \sqrt{g y_1} \qquad U_2 = \sqrt{g\left[y_1 - (y_1 - y_2)\left(\frac{r - R_1}{R_2 - R_1}\right)\right]}$$



$R_2 = 4m$

$R_1 = 0.8m$

$y_1 = 0.3m$

$y_2 = 0.1m$

$$\Rightarrow t = \frac{R_1}{\sqrt{g y_1}} + \frac{2}{\sqrt{g}}\left(\frac{R_2 - R_1}{\sqrt{y_1} + \sqrt{y_2}}\right) = \frac{0.8}{\sqrt{9.81 \times 0.3}} + \frac{2}{\sqrt{9.81}}\left(\frac{4 - 0.8}{\sqrt{0.3} + \sqrt{0.1}}\right) = 2.83\,s$$