1

$$\frac{dp}{dz} = -\rho g \Rightarrow \frac{dp}{dz} = \frac{-p}{RT} g$$

we assume the temperature changes linearly with height as follows $T \approx T_0 - Bz$

where T_0 = sea level temperatu[re]
B = lapse rate $\left[\frac{dT}{dz}\right]$
z = elevation above sea level

Integrate the equation

$$\int_{P_a}^{P} \frac{1}{p}\, dp = \frac{-g}{R} \int_0^z \frac{1}{T_0 - Bz}\, dz$$

atmospheric pressure @ sea level

$$\Rightarrow \ln(p)\Big|_{P_a}^{P} = \frac{-g}{R}\left[-\frac{\ln(T_0 - Bz)}{B}\Big|_0^z \right]$$

$$\Rightarrow \ln\frac{P}{P_a} = \frac{g}{RB}\left[\ln\left(\frac{T_0 - Bz}{T_0}\right)\right]$$

$$\Rightarrow \frac{P}{P_a} = \left(1 - \frac{Bz}{T_0}\right)^{g/RB}$$

To find the elevation where $\frac{P}{P_a} = 0.5$, rearrange the equation:

$$z = \frac{T_0}{B}\left(1 - 0.5^{\frac{RB}{g}}\right) = \frac{288.15}{0.00645}\left(1 - 0.5^{\frac{287 \times 0.00645}{9.81}}\right) = 5477\,m$$

② $P = P_a\left(1 - \frac{Bz}{T_0}\right)^{g/RB}$

Temperature increases from 20°C → 45° over 3000 m descent

∴ Lapse rate (B) is 0.008333 K/m

At the surface, $P_a = 100\,000$ Pa

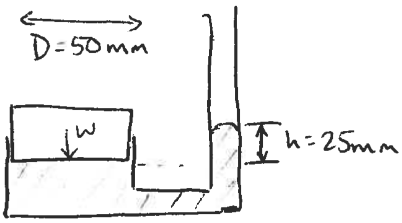
$T_0 = 293.15$ K

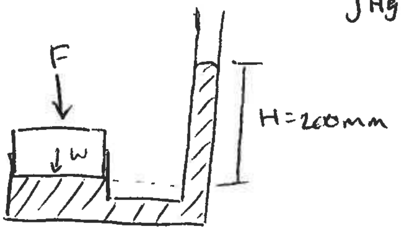
At $z = -3000$ m

$$P = 100\,000\left(1 - \frac{(0.008333 \times -3000)}{293.15}\right)^{\frac{9.81}{287 \times 0.008333}} = 139889 \text{ Pa}$$

$\rho_{Hg} = SG\, \rho_{H_2O} = 13540$

③





For the first case, the weight of the piston is equal to the hydrostatic force of the mercury in the tube. Pa acts on both bodies, so can be ignored (gauge pressures)

$W = F_1$

$W = \rho_{Hg}\, g\, h\, A$

For the second case, the weight of the piston, plus the force on the piston are equal to the hydrostatic force due to the column of mercury.

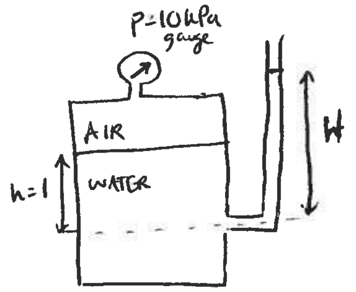
$W + F = F_2$

$\Rightarrow F = F_2 - W = F_2 - F_1 = \rho_{Hg} g H A - \rho_{Hg} g h A$

$\Rightarrow F = \rho_{Hg}\, g\, \frac{\pi D^2}{4}(H - h)$

$$F = \rho_{Hg}\, g\, \frac{\pi D^2}{4}(H-h) = 13540 \times 9.81 \times \left(\frac{200-25}{1000}\right) \times \frac{\pi\, 0.05^2}{4} = 45.6 \text{ N}$$

(4)

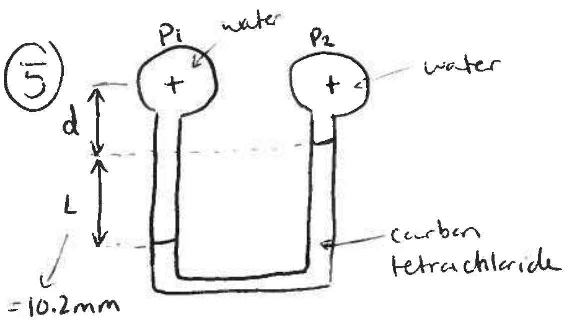


The pressure on the left side (main tank) is equal to the hydrostatic pressure plus the air pressure. The pressure on the right side (tube) is equal to the hydrostatic pressure of the water column. Atmospheric pressure is the same on the water column in the tube as the reference pressure for the gauge, hence it can be ignored.

$$P + \rho g h_1 = \rho g H$$

$$H = \frac{P + \rho g h_1}{\rho g} = \frac{10\,000 + 1000 \times 9.81 \times 1}{1000 \times 9.81} = 2.02\text{m}$$

(5)



Equating pressures on each arm of manometer

$$P_1 + \rho_w g (d+L) = P_2 + \rho_w g d + \rho_{CT} g L$$

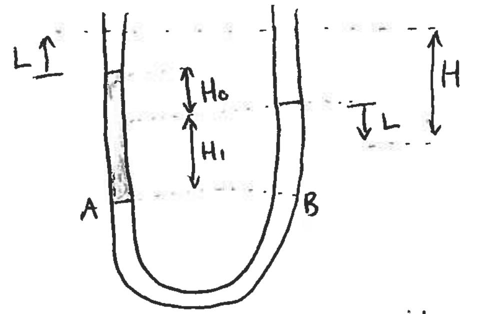
$$P_2 - P_1 = \rho_w g (d+L) - \rho_w g d - \rho_{CT} g L$$

$$P_2 - P_1 = \rho_w g L - \rho_{CT} g L = g L (\rho_w - \rho_{CT})$$

$$P_2 - P_1 = (1000 - 1595) \times 9.81 \times 0.0102 = -59.54 \text{ Pa}$$

This states that the pressure P_2 is less than P_1, as expected from the manometer.

(6)



when pressure is applied to the right tube, the water in that tube will drop by distance L, whilst the kerosene in the left tube will increase in height by distance L.

Hence, under the applied gauge pressure (ΔP) the elevation difference is

$$H = H_0 + 2L$$

Since points A and B are at the same height in the same fluid, they must have equal pressure.

$$P_A = \rho_k g (H_0 + H_1) \qquad P_B = \rho_w g H_1$$

equating: $\rho_k g (H_0 + H_1) = \rho_w g H_1$

divide by ρ_w and g, then solving for H_1 gives

$$H_1 = \frac{SG_k \times H_0}{1 - SG_k} = \frac{0.82 \times 20\,mm}{1 - 0.82} = 91.11\,mm$$

Now when the pressure (ΔP) is applied to the right hand tube

$$P_A = \rho_k g (H_0 + H_1) + \rho_w g L \qquad P_B = \Delta P + \rho_w g (H_1 - L)$$

setting equal and dividing by ρ_w and g gives

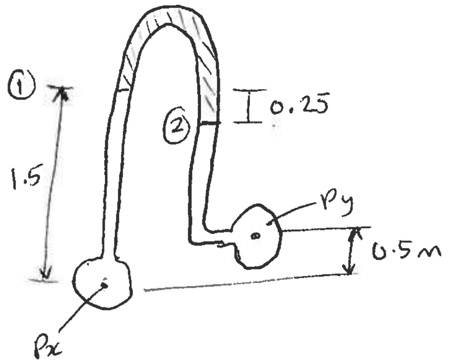
$$SG_k (H_0 + H_1) + L = \frac{\Delta P}{\rho_w g} + H_1 - L$$

$$\Rightarrow L = \frac{1}{2}\left(\frac{\Delta P}{\rho_w g} + H_1 - SG_k (H_0 + H_1)\right) = \frac{1}{2}\left(\frac{98}{1000 \times 9.81} + \frac{91.11}{1000} - 0.82\left(\frac{20 + 91.11}{1000}\right)\right)$$

$$L = 5\,mm$$

Now $H = H_0 + 2L = 20\,mm + 2 \times 5\,mm = 30\,mm$

⑦



$SG_0 = 0.9$

$h_1 = 1.5m$
$h_2 = 0.5m$
$h_3 = 0.25m$

Px to ① $\rightarrow P_x = P_1 + \rho_w g h_1$

① to ② $\rightarrow P_1 = P_2 - SG_0 \rho_w g h_3$

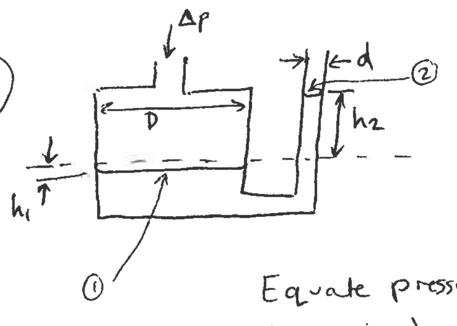
② to Py $\rightarrow P_2 = P_y - \rho_w g (h_1 - h_2 - h_3)$

Adding these three equations together gives

$$P_x - P_y = \rho_w g (h_2 + h_3) - SG_0 \rho_w g h_3$$

$$P_x - P_y = 1000 \times 9.81 (0.5 + 0.25) - 0.9 \times 1000 \times 9.81 \times 0.25 = 5150 \text{ Pa}$$

⑧



$D = 50mm$
$d = 5mm$
$\Delta p = 100 Pa$
$\rho = 1000 kg/m^3$

pressure @ ① = P_1
pressure @ equal height as ① in the small tube is:
$P_2 + \rho g h_2 + \rho g h_1$

Equate pressures at reservoir level:

$$P_1 = P_2 + \rho g (h_2 + h_1)$$

we know that the volume of water is constant in whole system, therefore:

$$\frac{\pi D^2}{4} h_1 = \frac{\pi d^2}{4} h_2 \Rightarrow h_1 = h_2 \left(\frac{d}{D}\right)^2 = h_2 \left(\frac{5}{50}\right)^2 = 0.01 h_2$$

$$P_1 - P_2 = \Delta p = \rho g (h_2 + 0.01 h_2) \Rightarrow h_2 = \frac{\Delta p}{1.01 \rho g} = \frac{100}{1.01 \times 1000 \times 9.81} = 0.01009m = (10.09mm)$$

with $h_2 = 10.09mm$
$h_1 = 0.01 \times 10.09 = 0.1009mm$

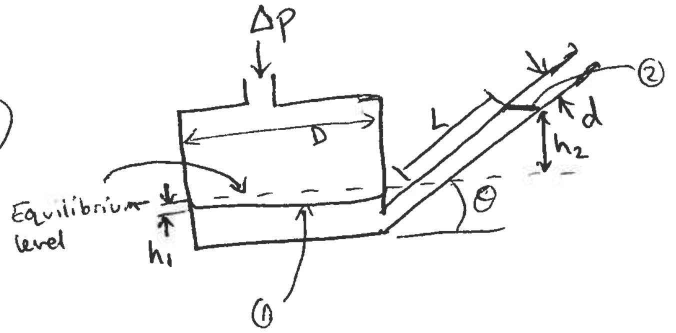
Total height change $= h_1 + h_2 = 10.19mm$

for standard U-tube $\Delta p = \rho g \Delta h \Rightarrow \Delta h = \frac{\Delta p}{\rho g}$

$\Delta h = \frac{100}{9.81 \times 1000} = 10.19mm$

The overall deflection/height change is equal!!

9



Pressure at ① = P_1

pressure at same height in the inclined tube is: $P_2 + \rho g h_2 + \rho g h_1 \equiv P_2 + \rho g (h_2 + h$

As these two are the same vertical location in the same fluid, they are equal in pressure.

$$\therefore \quad P_1 = P_2 + \rho g (h_1 + h_2)$$

$$\Rightarrow P_1 - P_2 = \rho g (h_1 + h_2)$$

To eliminate h_1 from this equation, we recognize that the volume of the liquid in the manometer is constant.

$$\frac{\pi D^2}{4} h_1 = \frac{\pi d^2}{4} L \quad \Rightarrow \quad h_1 = L\left(\frac{d}{D}\right)^2$$

Also we know that $h_2 = L \sin\theta$, therefore

$$P_1 - P_2 = \rho g \left[L\left(\frac{d}{D}\right)^2 + L\sin\theta \right] = \rho g L \left[\left(\frac{d}{D}\right)^2 + \sin\theta \right]$$

$$\Rightarrow L = \frac{P_1 - P_2}{\rho g \left[\left(\frac{d}{D}\right)^2 + \sin\theta \right]} = \frac{\Delta p}{\rho g \left[\left(\frac{d}{D}\right)^2 + \sin\theta \right]}$$

Eqn for liquid deflection due to Δp

For a standard U-tube manometer, the height due to Δp is given by:

$$\Delta p = \rho g h \Rightarrow h = \frac{\Delta p}{\rho g}$$

As we want a 5:1 increase in L, that means $\frac{L}{h} = 5$

$$\frac{L}{h} = 5 = \frac{\frac{\Delta p}{\rho g\left[\left(\frac{d}{D}\right)^2 + \sin\theta\right]}}{\frac{\Delta p}{\rho g}} = \frac{1}{\left(\frac{d}{D}\right)^2 + \sin\theta}$$

To find the angle θ of the tube to get the 5:1 increase in L compared to the standard U-tube, rearrange the equation to find θ:

$$5 = \frac{1}{\left(\frac{d}{D}\right)^2 + \sin\theta} \Rightarrow \sin\theta = \frac{1}{5} - \left(\frac{d}{D}\right)^2 \Rightarrow \theta = \sin^{-1}\left(\frac{1}{5} - \left(\frac{8}{96}\right)^2\right)$$

$$\theta = 11.13^\circ$$

(10) From question (9) the deflection along the inclined tube can be calculated from:

$$L = \frac{\Delta P}{\rho g\left(\left(\frac{d}{D}\right)^2 + \sin\theta\right)} = \frac{350}{1000 \times 9.81\left(\left(\frac{8}{96}\right)^2 + \sin(11.13)\right)} = 178.4\,mm$$

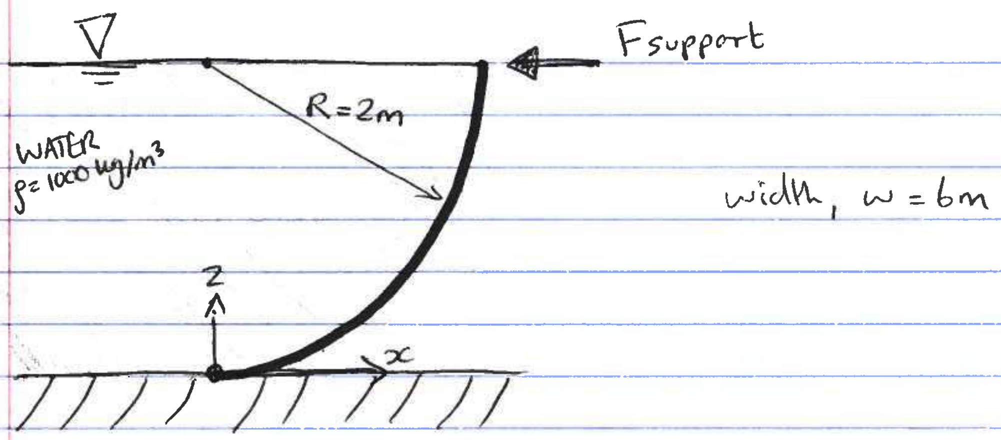
we know that the change in L should be 5 times greater than the vertical height difference. $\Delta h = \frac{178.4}{5} = 35.68\,mm$ Match!

Let us check with the definition of $\Delta h = h_1 + h_2 = L\left(\frac{d}{D}\right)^2 + L\sin\theta = 35.68\,mm$

Now for a standard U-tube manometer, the Δh can be found from $\Delta P = \rho g \Delta h \Rightarrow \Delta h = \frac{\Delta P}{\rho g} = \frac{350}{1000 \times 9.81} = 35.68\,mm$

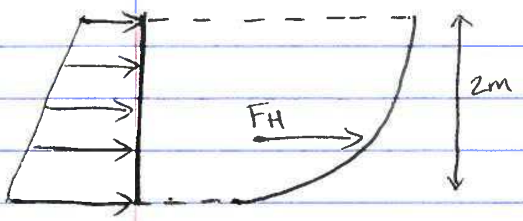
This shows that the vertical change in height is the same for both the U-tube and Inclined-tube manometers, however by inclining the 2nd tube in the latter equipment and measuring the variation in water level along the tube, a significantly better resolution can be obtained.

(11)

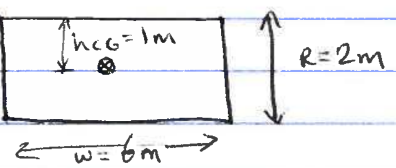


To calculate the force Fsupport, we need to determine the horizontal force on the gate due to the pressure of the water.

The horizontal component of force on a curved surface equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.



The projected plane is rectangular in shape



The force on this plate is equal to the vertical height from the free surface to the projected plane centroid, multiplied by the specific gravity and projected area.

$$F_H = \gamma h_{CG} A_{proj} = \rho g h_{CG} A_{proj} = 1000 \times 9.81 \times 1 \times (2 \times 6) = 117\,720\ N$$

To ensure static equilibrium, the sum of the horizontal forces must equal zero.

$$\therefore F_H - F_{support} = 0 \implies F_H = F_{support}$$

$$F_{support} = 117\ 720 \text{ N}$$ ← This is the same solution as we previously found.

THIS SOLUTION IS NOT ALWAYS THE CASE SO:

----- EXTENSION -----

To verify this further, let us look at all the forces and calculate the moment at the hinge.

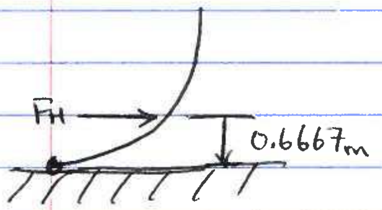
So, F_H acts at the centre of pressure of the projected plane. This is calculated from

$$\boxed{I_{xx_c} = \frac{bd^3}{12}}$$

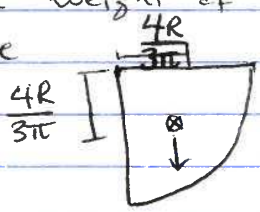
$$y_{cp} = \frac{I_{xx\_c}}{y_{cG}A} + y_{cG} = \frac{\left(\frac{wR^3}{12}\right)}{y_{cG}A} + y_{cG}$$

$$y_{cp} = \frac{4}{1\times 2\times 6} + 1 = 1.333\text{m}$$ from the centre of the circle.

From the origin at the hinge $y_{cp} = 2 - 1.333 = 0.6667\text{m}$



Now the vertical force is equal to the weight of the water above the gate



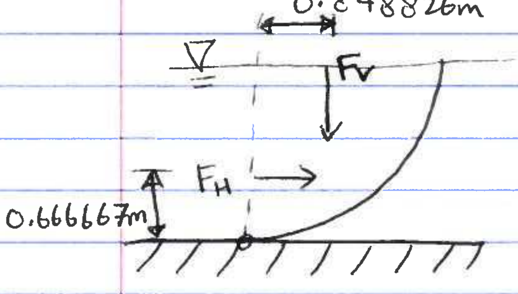
For a quarter circle, the Area $A = \frac{\pi R^2}{4}$

And the centroid is located at $\frac{4R}{3\pi}$ from the circle centre.

The force due to the weight of the water acts through the centroid and is given by:

$$F_V = \gamma A b = \rho g \frac{\pi R^2}{4} b = 1000 \times 9.81 \times \pi \times \frac{2^2}{4} \times 6 = 184\ 914 \text{ N}$$

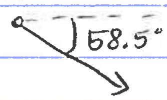
The location of this force is at $\frac{4R}{3\pi} = \frac{4 \times 2}{3\pi} = 0.848826\,m$ from the circle centre.



So the total force acting on the gate is:

$$F = \sqrt{F_H^2 + F_V^2} = 219206\ N$$

acting at 58.5° from the free surface/horizontal



The total moment acting around the hinge should be zero for static equilibrium. Therefore we can calculate the $F_{support}$ value:

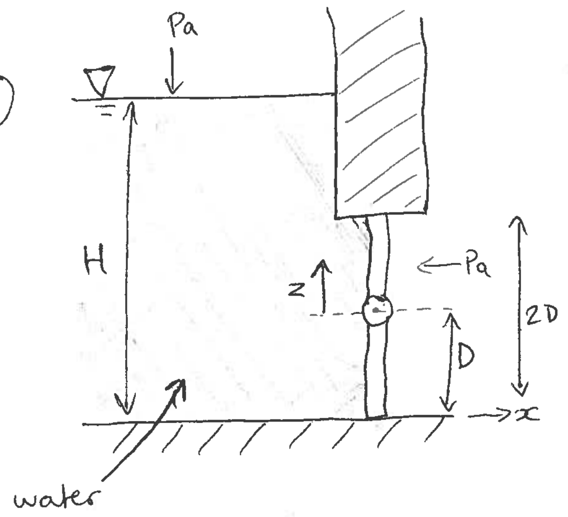
$$\circlearrowleft^{+} M = 0 \qquad -F_V \times x - F_H \times z + F_{support} \times R = 0$$

$$F_{support} = \frac{(184914 \times 0.848826) + (117720 \times 0.666667)}{2}$$

$$F_{support} = 117720\ N$$

SAME AS THE HORIZONTAL FORCE FOUND EARLIER!

(12)



Define the hinge of the gate as $z=0$

The pressure at any point on the gate is equal to the weight of the water above that point, applied over a given area.

This pressure is:

$$P = \rho g (H - D - z)$$

Atmospheric pressure has been ignored as it applies to both the water free surface and the gate

If we look at the hydrostatic force on a strip of area dz, we get:

$$d\vec{F}_p = p w dz \, \hat{\imath} \quad \Rightarrow \quad dF_p = \rho g w (H-D-z)\, dz$$

We can now calculate the moments around the hinge dM, knowing that counterclockwise moments are positive M^{+}

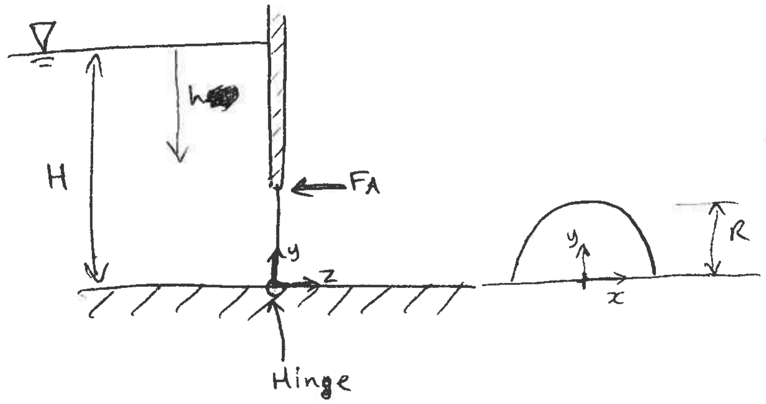
$$dM = -z\, dF_p = -z\rho g w (H-D-z)\, dz = -\rho g w[(H-D)z - z^2]\, dz$$

Integrate over the gate to get the total moment

$$M = -\rho g w \int_{-D}^{D} [(H-D)z - z^2]\, dz = -\rho g w \left[\frac{H-D}{2} z^2 - \frac{1}{3} z^3 \right]\Bigg|_{-D}^{D} = \frac{2}{3} \rho g w D^3$$

Independant of H

(13)

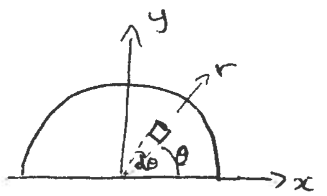


We know that the moments around the hinge must equal zero

$M_z = 0$ which means that the moment produced by the force $F_A \cdot R$ must equal the sum of all elemental moments caused by the gate pressure.

$$F_A \cdot R = \int y \cdot p \, dA \qquad \text{where } p = \rho g h$$

where $h = H - y$



$$dA = r\,dr\,d\theta$$
$$y = r\sin\theta$$

$$F_A = \frac{1}{R}\int_0^{\pi}\int_0^{R} r\sin\theta \, \rho g (H - r\sin\theta)\, r\,dr\,d\theta = \frac{\rho g}{R}\int_0^{\pi}\int_0^{R} (Hr^2\sin\theta - r^3\sin^2\theta)\,dr\,d\theta$$

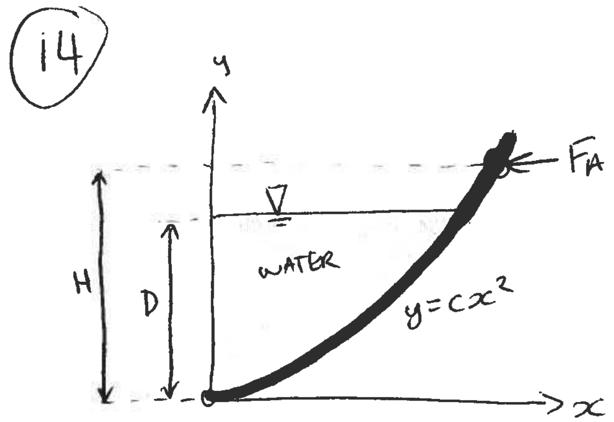
$$= \frac{\rho g H R^2}{3}\int_0^{\pi} \sin\theta \, d\theta - \frac{\rho g R^3}{4}\int_0^{\pi} \sin^2\theta \, d\theta$$

$$= \frac{2\rho g H R^2}{3} - \frac{\pi \rho g R^3}{8} = \rho g R^2 \left(\frac{2}{3}H - \frac{\pi R}{8}\right)$$

when $H = 25\,m$
$R = 10\,m$
$\rho = 1000\,kg/m^3$
$g = 9.81\,m/s^2$

$$F_A = \frac{2 \times 1000 \times 9.81 \times 25 \times (10)^2}{3} - \frac{\pi \times 1000 \times 9.81 \times (10)^3}{8} = 12\ 497\ 622\,N$$

(14)



width $b = 2$

$c = 0.25\ m^{-1}$

$D = 2\ m$

$H = 3\ m$

when $y = D \rightarrow x = \sqrt{\frac{D}{c}}$

$p = \rho g h$

a) Magnitude and line of action of the vertical component of hydrostatic force

$$F_v = \int p\, dA_y \qquad dA_y = b\, dx$$

$p = \rho g h$ where $h = D - y$
and $y = cx^2$
$\therefore h = D - cx^2$

$$\Rightarrow F_v = \int_0^{\sqrt{\frac{D}{c}}} \rho g (D - cx^2) b\, dx = \rho g b \int_0^{\sqrt{\frac{D}{c}}} (D - cx^2)\, dx$$

$$F_v = \frac{2}{3} \rho g b \frac{D^{3/2}}{c^{1/2}} = \frac{2}{3} \times 1000 \times 9.81 \times 2 \times \frac{2^{3/2}}{\sqrt{0.25}} = 73992\ N$$

To find the line of action of this force

$$F_v\, x' = \int x\, dF_v \quad \Rightarrow x' = \frac{1}{F_v} \int x\, dF_v = \frac{1}{F_v} \int x\, p\, dA_y$$

$$x' = \frac{1}{F_v} \int_0^{\sqrt{\frac{D}{c}}} x \rho g (D - cx^2) b\, dx = \frac{\rho g b}{F_v} \int_0^{\sqrt{\frac{D}{c}}} (Dx - cx^3)\, dx = \frac{\rho g b}{F_v} \frac{D^2}{4c}$$

$$x' = \frac{1000 \times 9.81 \times 2 \times 2^2}{73992 \times 4 \times 0.25} = 1.061\ m$$

So $F_v = 73992\ N$ acting at $x' = 1.061\ m$

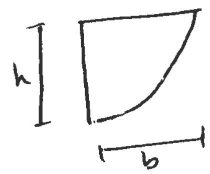
This could also have been solved by calculating the weight of the water above the gate.

This is calculated as:

$$F_{V_2} = \rho g \underbrace{Ab}_{\text{volume of water}}$$

For a parabola, the area is equal to

$$A = \frac{2bh}{3}$$



$$A = \frac{2}{3}\sqrt{\frac{D}{c}}\,D$$

$$F_{V_2} = \rho g \frac{2}{3}\sqrt{\frac{D}{c}}\,Db = 1000 \times 9.81 \times \frac{2}{3} \times \sqrt{\frac{2}{0.25}} \times 2 \times 2$$

$$F_{V_2} = 73992\ \text{N}$$

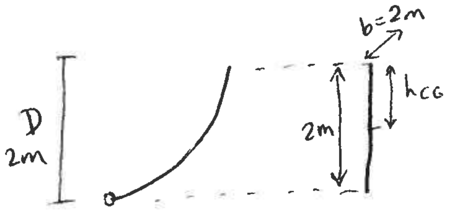
This acts through the centroid of the water, located at $x_2' = \frac{3}{8}\sqrt{\frac{D}{c}}$

which equals $x_2' = \frac{3}{8}\sqrt{\frac{2}{0.25}} = 1.061\ \text{m}$

These values are equal to the values obtained from the pressure integration!

b)

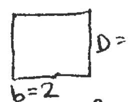
Now to find the horizontal force of the water on the gate we will project the gate onto the y plane and calculate the equivalent pressure at the centroid of the projected plane



$$F_H = \underbrace{P_{CG}}_{\text{pressure at centroid}} \overbrace{A}^{\text{Area of projected plane}} = \rho g h_{CG} bD$$

centroid is at $h_{CG} = \frac{D}{2}$

$$F_H = \rho g \frac{D}{2} bD = \rho g \frac{D^2}{2} b = 1000 \times 9.81 \times \frac{2^2}{2} \times 2 = 39240\ \text{N}$$



The line of action is the centre of pressure, $h_{CP} = \frac{I_{xx}}{A\,h_{CG}} + h_{CG}$

$$I_{xx} = \frac{bD^3}{12} = \frac{2 \times 2^3}{12}$$

$$I_{xx} = \frac{4}{3}$$

$$h_{CP} = \frac{(4/3)}{2 \times 2 \times 1} + 1 = \frac{4}{3}\ \text{m}$$ from the free surface

The position from the hinge is $D - h_{CP} = 2 - \frac{4}{3} = \frac{2}{3}\ \text{m}$

So knowing where the vertical and horizontal forces due to the water act and their magnitude, we can calculate F_{A_H} & F_{A_V} required to keep the gate in equilibrium

→ For horizontal force at A:

Take moments about the hinge

$$F_{A_H} \cdot H - F_V \cdot x' - F_H \cdot (D - h_{cp}) = 0$$

$$F_{A_H} = \frac{F_V \cdot x' + F_H \cdot (D - h_{cp})}{H} = \frac{73992 \times 1.061 + 39240 \times (2 - \frac{4}{3})}{3} = 34889 N$$

→ For Vertical force at A:

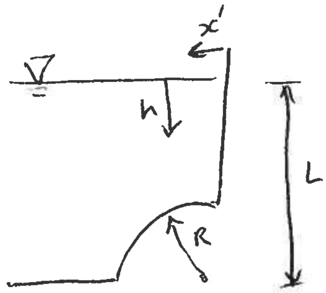
$$F_{A_V} \cdot L - F_V \cdot x' - F_H \cdot (D - h_{cp}) = 0$$

where $L = \sqrt{\frac{H}{c}}$ from eqn $y = cx^2$

$L = 3.464 m$

$$F_{A_V} = \frac{F_V \cdot x' + F_H \cdot (D - h_{cp})}{L} = \frac{73992 \times 1.061 + 39240 \times (2 - \frac{4}{3})}{3.464} = 30215 N$$

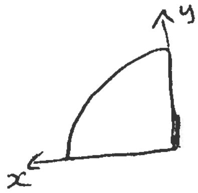
(15)



$R = 4m$
$L = 10\,m$
$b = 10m$

Vertical component of hydrostatic force

$$P = \rho g h$$



Eqn of circle
$x^2 + y^2 = R^2$
so $y = \sqrt{R^2 - x^2}$

To find h calculate $L - y$ where $y = \sqrt{R^2 - x^2}$

$\therefore h = L - \sqrt{R^2 - x^2}$

$$F_v = \int p \, \underbrace{dA_y}_{b\,dx} = \int_0^R \rho g (L - \sqrt{R^2 - x^2})\, b\, dx = \rho g b R (L - R\tfrac{\pi}{4})$$

$$F_v = 1000 \times 9.81 \times 10 \times 4 (10 - \tfrac{4\pi}{4}) = 2691239\ N$$

↳ This can also be found from calculating the weight of the water above the cut out. Practically, this is achieved by calculating the weight of the whole cube, then subtracting the semicircular volume weight from it.

To find the line of action, we equate the moment due to the vertical force with the sum of all the infinitesimal moments due to the ~~point~~ elemental vertical forces.

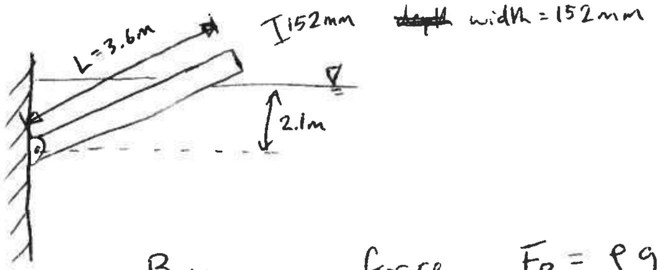
$$x' \cdot F_v = \int x \, dF_v \qquad \text{where} \quad dF_v = \rho g b (L - \sqrt{R^2 - x^2}) dx$$

$$x' = \frac{1}{F_v} \int_0^R x \rho g b (L - \sqrt{R^2 - x^2}) dx = \frac{1}{R(L - R\frac{\pi}{4})} \int_0^R (Lx - x\sqrt{R^2 - x^2}) dx$$

$$x' = \frac{1}{R(L - R\frac{\pi}{4})} \left[\frac{1}{2} L R^2 - \frac{1}{3} R^3 \right] = \frac{R}{L - \frac{\pi R}{4}} \left(\frac{L}{2} - \frac{R}{3} \right) = \frac{4}{10 - \frac{4\pi}{4}} \left(\frac{10}{2} - \frac{4}{3} \right)$$

$$x' = 2.14m$$

(16)



$P = \rho g h$

Buoyancy force $F_B = \rho g V$

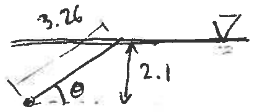
$F_B = \rho g V = 1000 \times 9.81 \times (0.152 \times 0.152 \times x) = 226.7x$

The moment caused by the weight will equal the moment caused by the buoyancy force

$M \circlearrowleft +$ $\quad M = -\underbrace{mg}_{W} \cos\theta \frac{L}{2} + F_B \cos\theta \frac{x}{2} = 0$

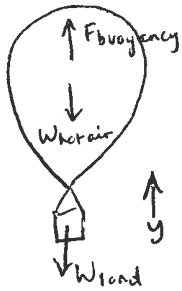
$-670 \times \frac{3.6}{2} + 226.7x \times \frac{x}{2} = 0 \quad \Rightarrow x = 3.26\,m$

when the water surface is $y = 2.1\,m$ above the hinge



$\theta = \sin^{-1}\left(\frac{2.1}{3.26}\right) = 40.1°$

(17)



$V = 9061\,m^3$

$T_{atm} = 8.9°C$

$T_{hotair} = 71.1°C$

$P_{atm} = 101352\,Pa$

Buoyancy force: $F_B = \rho_{atm}\, g\, V$

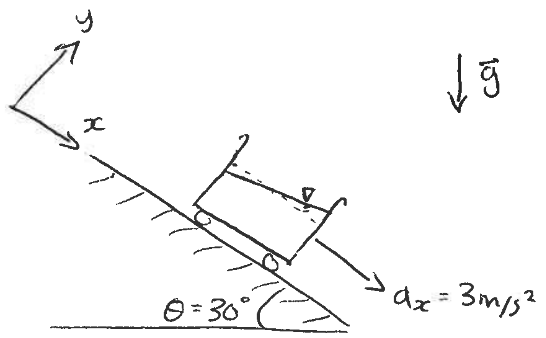
All forces in y (vertical)

$$\sum F_y = 0 = F_B - W_{hotair} - W_{Load} = \rho_{atm}\, gV - \rho_{hotair}\, gV - mg$$

$$m = V(\rho_{atm} - \rho_{hotair}) = V\frac{P_{atm}}{R}\left(\frac{1}{T_{atm}} - \frac{1}{T_{hotair}}\right)$$

$$m = \frac{9061 \times 101352}{287}\left(\frac{1}{8.9+273} - \frac{1}{71.1+273}\right) = 2052\ kg$$

(18)



The basic equation governing this situation is : $-\vec{\nabla}p + \rho\vec{g} = \rho\vec{a}$

In components

$$-\frac{\partial p}{\partial x} + \rho g_x = \rho a_x \qquad -\frac{\partial p}{\partial y} + \rho g_y = \rho a_y \qquad -\frac{\partial p}{\partial z} + \rho g_z = \rho a_z$$

$$a_x = 3 \qquad a_y = 0 \qquad a_z = 0$$

$$g_x = g\sin\theta \qquad g_y = -g\cos\theta \qquad g_z = 0$$

$$-\frac{\partial p}{\partial x} + \rho g \sin\theta = \rho a_x \qquad -\frac{\partial p}{\partial y} - \rho g\cos\theta = 0 \qquad -\frac{\partial p}{\partial z} = 0$$

So the problem is only a function of x and y : $p = p(x, y)$

The total differential of $p = p(x,y)$ is

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$

At the free surface, pressure is equal to atmosphere, which is a constant and as we are looking at gauge pressure we can make it 0. So to find the slope, rearrange the eqn.

$$0 = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy \Rightarrow \frac{dy}{dx} = \frac{-\left(\frac{\partial p}{\partial x}\right)}{\left(\frac{\partial p}{\partial y}\right)} = \frac{\rho g\sin\theta - \rho a_x}{\rho g\cos\theta} = \frac{g\sin\theta - a_x}{g\cos\theta}$$

$$\frac{dy}{dx} = \frac{9.81 \times \sin 30 - 3}{9.81 \times \cos 30} = 0.224$$

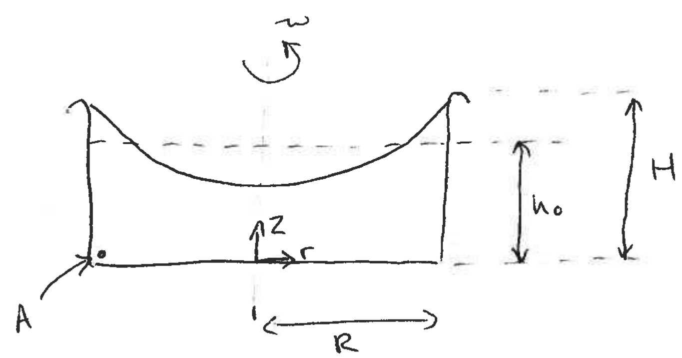
This gives a water surface angle ($\tan^{-1}(0.224) = 12.6°$) relative to the ~~[illegible]~~ x-axis

To have a water free surface parallel to the x-axis, the slope $\frac{dy}{dx}$ must equal zero

$$\therefore \frac{dy}{dx} = 0 = \frac{g\sin\theta - a_x}{g\cos\theta} \Rightarrow a_x = g\sin\theta = 9.81\sin 30 = 4.905\frac{m}{s^2}$$

So an acceleration of $a_x = 4.905\frac{m}{s^2}$ would cause the free surface to have a ~~constant~~ constant y height

19



$h_o = 0.1\,m$

$H = 0.15\,m$

$R = 0.5\,m$

$$p = p_0 + \frac{\rho \omega^2 r^2}{2} - \rho g z$$

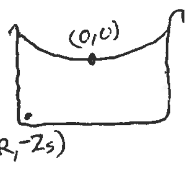
$$Z_s = h_o + \frac{\omega^2}{4g}(2r^2 - R^2)$$

At the outer edge of the container, r=R and $Z_s = H$

$$\therefore \quad Z_s = h_o + \frac{\omega^2}{4g}(2R^2 - R^2) = h_o + \frac{\omega^2 R^2}{4g}$$

$$\Rightarrow \omega = \sqrt{\frac{(Z_s - h_o)\,4g}{R^2}} = \sqrt{\frac{(0.15-0.1)\times 4\times 9.81}{(0.5)^2}} = 2.8014\ \text{rad/s}$$

To calculate the pressure at the bottom corner A, it is more convenient to make the axis origin the free surface at bottom of the depression



A(R, −Zs)

The height of the depression is equal to the height of the free (unrotated) surface minus rise of fluid at the edges. ($H - h_o = 0.15 - 0.1 = 0.05\,m$). This comes from: at $r=0$ $\quad Z_s = h_o - \frac{\omega^2 R^2}{4g} = 0.1 - \frac{2.8014^2 \times 0.5^2}{4\times 9.81} = 0.05\,m$

so pressure is: $p = 0 + \frac{13400 \times 2.8014^2 \times 0.5^2}{2} - 13400 \times 9.81 \times (-0.05) = \underline{19\,718\ Pa}$

This compares to $p = \rho g z = 13400 \times 9.81 \times 0.1 = 13\,145$ Pa for stationary container