MAT 194 MOCK MIDTERM I

Prepared by David Simons and Marc Goudge (NΨ 1T6) for Peer Assisted Study Sessions.
We imagine this will be somewhat representative of your midterm, but make no guarantees!

1) Find f'(x) for the following equations. Note, a, b ϵ **R**

a) $f(x) = a^6 + b^6 + x$ b) $f(x) = bx^{a^{b^a}}$ c) $f(x) = \sin(\cos(\sin(\cos(x))))$

d) $f(x) = [x^2 + (1 - 3x)^5]^3$ e) $f'(2x) = x^2$

2)

a) Evaluate the following limits. If the limit does not exist, explain.

i. $\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x}$ ii. $\lim_{x \to -2} \frac{2 - |x|}{2+x}$

iii. $\lim_{h \to 0} \frac{\sqrt[4]{16+h} - 2}{h}$ iv. $\lim_{x \to \infty} \frac{\cos(2x^2)}{2x^2}$

b) Using the definition of the derivative, find $\frac{d}{dx}\cos(x)$

3)

a) Provide an ϵ-δ type argument to show that the $\lim_{x \to 2} \frac{3}{x+1} = 1$

b) Without using the previous result, provide an ϵ-δ type argument to show that the $\lim_{x \to 2} \frac{3}{x+1} \neq 2$

4) A runner sprints around a circular track of radius 100 at a constant speed of 7 m/s. The runner's friend is standing at a distance 200 from the center of the track. How fast is the distance between the friends changing when the distance between them is 200 m? Do not simplify. (Hint: the cosine law states that $c^2 = a^2 + b^2 - 2ab\cos(\alpha)$, where α is the angle between sides a and b.)

5) Sketch the curve $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$ indicating all important features.

6) At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

7) Two runners start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed.

8) If *f* is differentiable at *a*, where $a > 0$, evaluate the following limit in terms of $f'(a)$

$$\lim_{x \to a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}$$