Q4:

Compute L and U for the matrix A given by

$$A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

To receive full marks for this part you must show the steps of your decomposition.

Near the end of your decomposition, you will have one entry in L that is undefined because the corresponding entry of A is equal to zero. Derive the value for this undefined entry of L and then show that $A = LU$.

Q4:

$$A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix} \qquad L = \begin{bmatrix} \cdot & 0 & 0 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix} R1 * \frac{1}{a} \qquad = \begin{bmatrix} a & 0 & 0 \\ \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & b & b \\ 0 & b & b+c \end{bmatrix} R2 - R1 * a \qquad = \begin{bmatrix} a & 0 & 0 \\ a & \cdot & 0 \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & b & b+c \end{bmatrix} R2 * \frac{1}{b} \qquad = \begin{bmatrix} a & 0 & 0 \\ a & b & 0 \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & c \end{bmatrix} R3 - R2 * b \qquad = \begin{bmatrix} a & 0 & 0 \\ a & b & 0 \\ \cdot & b & \cdot \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} R3 * \frac{1}{c} \qquad = \begin{bmatrix} a & 0 & 0 \\ a & b & 0 \\ \cdot & b & c \end{bmatrix}$$

WHAT ABOUT THE ENTRY (3,1) IN L?

SINCE ENTRY (3,1) IN MATRIX A IS EQUAL TO THE DOT PRODUCT OF ROW 3 OF L WITH COLUMN 1 OF U, THEN

$$0 = (\cdot)(1) + (b)(0) + (c)(0)$$

$\therefore$ ENTRY (3,1) IN MATRIX A IS ZERO.

$$\therefore \quad L = \begin{bmatrix} a & 0 & 0 \\ a & b & 0 \\ 0 & b & c \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LU = \begin{bmatrix} a & 0 & 0 \\ a & b & 0 \\ 0 & b & c \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

$$= A$$