Q1:

a) One result that we did not discuss in class is that for square matrices, an inverse on one side is automatically an inverse on the other side. In other words, if $CD = I$ then automatically $DC = I$ and D is C^{-1}. Using this result, if matrix B is the inverse of matrix A^2, show that $AB = BA$.

b) If the product $M = ABC$ of three square matrices is invertible, then A, B and C are invertible. Find a formula for B^{-1} that involves M^{-1}, A and C.

c) There are sixteen different 2 by 2 matrices whose entries are 1's and 0's. How many of them are invertible?

Q1

a) GIVEN B IS THE INVERSE OF A^2, THEN

$$B(A^2)=(A^2)B=I$$

$$\therefore\ (BA)A=A(AB)=I$$

$$\therefore\ BA=AB=A^{-1}$$

b) GIVEN $M=ABC$ IS INVERTIBLE, THEN

$$M^{-1}=C^{-1}B^{-1}A^{-1}$$

$$\therefore\ B^{-1}A^{-1}=CM^{-1}$$

$$\therefore\ B^{-1}=CM^{-1}A$$

c) $\overset{\times}{\begin{bmatrix}0&0\\0&0\end{bmatrix}}$

$\overset{\times}{\begin{bmatrix}1&0\\0&0\end{bmatrix}}, \overset{\times}{\begin{bmatrix}0&1\\0&0\end{bmatrix}}, \overset{\times}{\begin{bmatrix}0&0\\1&0\end{bmatrix}}, \overset{\times}{\begin{bmatrix}0&0\\0&1\end{bmatrix}}$

$\overset{\times}{\begin{bmatrix}1&1\\0&0\end{bmatrix}}, \overset{\checkmark}{\begin{bmatrix}1&0\\0&1\end{bmatrix}}, \overset{\times}{\begin{bmatrix}1&0\\1&0\end{bmatrix}}, \overset{\checkmark}{\begin{bmatrix}0&1\\1&0\end{bmatrix}} \overset{\times}{\begin{bmatrix}0&0\\1&1\end{bmatrix}}, \overset{\times}{\begin{bmatrix}0&1\\0&1\end{bmatrix}}$

$\overset{\checkmark}{\begin{bmatrix}1&1\\1&0\end{bmatrix}}, \overset{\checkmark}{\begin{bmatrix}1&1\\0&1\end{bmatrix}}, \overset{\checkmark}{\begin{bmatrix}1&0\\1&1\end{bmatrix}}, \overset{\checkmark}{\begin{bmatrix}0&1\\1&1\end{bmatrix}}$

$\overset{\times}{\begin{bmatrix}1&1\\1&1\end{bmatrix}}$

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad-bc$$

✓ INDICATES INVERTIBLE ($\det \neq 0$)
X INDICATES NOT INVERTIBLE ($\det = 0$)

A TOTAL OF 6 ARE INVERTIBLE.