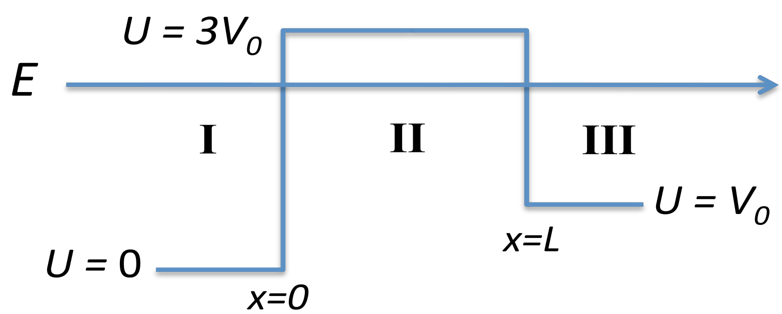
PHY294 Term Test #1 (February 9th 2015)

- **75 minutes (closed book, no calculator, one single-sided page of hand-written notes is allowed)**
- **All the questions are equally weighted (except Extra Credit question #5)**
- **Note the helpful identities and integrals on the back page**

1. Consider a 1D simple harmonic oscillator of mass m and spring constant k:
(a) Use the uncertainty principle to show that the particle's minimum energy is ~ $\hbar\omega_0/2$, where $\omega_0=(k/m)^{1/2}$.
(b) Let this oscillator be in the ground state. Show that although it is not an eigenstate of either p^2 or x^2, it is an eigenstate of the combined operator $p^2/2m + kx^2/2$, consistent with energy being well-defined. Note: the ground-state wave function $\psi(x) = A \exp(-ax^2/2)$, where $A=(a/\pi)^{1/4}$ and $a=(mk/\hbar^2)^{1/2}$.

2. A particle of mass m is in a 1D infinite-potential square well between $x = -L$ and $x = L$:
(a) Derive $\psi(x)$ and sketch $|\psi(x)|^2$ for the three lowest-energy eigenstates, and determine these energies.
(b) Initially the particle is in the lowest-energy state. Suddenly the well is tripled in width symmetrically about x=0. Calculate the probability of finding the particle in the 3rd-lowest state of the widened well.

3. A flux of electrons with energy E strikes a potential barrier as shown below (Note: $3V_0 > E > V_0$):
(a) Write down the proper form of the wave function for each region (I, II, III), in terms of: $E, V_0, \hbar, L, m_e$.
(b) State all the boundary conditions, and express (need not solve) the transmission probability T in terms of: $E, V_0, \hbar, L, m_e$ and the coefficients of the wave functions.



4. Consider two examples of electrons in motion:
(a) A fast-moving electron whose DeBroglie wavelength is equal to its Compton wavelength $\lambda_c = h/m_e c$. What are the velocity and kinetic energy of this electron? (Give your answers in terms of: h, m_e, c.)
(b) A slow-moving wave packet at t=0 is represented by $\psi(x) = A \exp[(-x^2/2\sigma^2)+i(p_0 x/\hbar)]$; A, σ, p_0 are constants. Show that the probability current $j(x) = |\psi(x)|^2 p_0/m_e$, and the expectation value of momentum $\langle p\rangle = p_0$.
Note: $j(x) = -\frac{i\hbar}{2m}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$

5. Extra Credit (half-weight):
In Question **3(b)**, if $E = 3V_0$, solve for the reflection probability R in terms of: $V_0, \hbar, L, m_e$.

Identities and Integrals:

$$\sin 2\theta = 2 \cdot \sin\theta \cdot \cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\int_0^\pi \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad m, n \text{ integers}$$

$$\int_0^\pi \cos mx \cos nx dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad m, n \text{ integers}$$

$$\int_0^\pi \sin mx \cos nx \, dx = \begin{cases} 0 & \text{if } m + n \text{ even} \\ \dfrac{2m}{m^2 - n^2} & \text{if } m + n \text{ odd} \end{cases}$$

$$\int_{-\infty}^{\infty} e^{-ax^2} \, \mathrm{d}x = \sqrt{\frac{\pi}{a}} \quad (a > 0)$$

$$\int_{-\infty}^{\infty} x e^{-a(x-b)^2} \, \mathrm{d}x = b\sqrt{\frac{\pi}{a}}$$