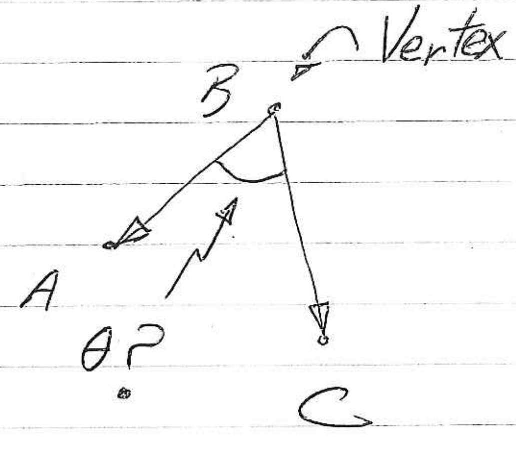
Q1:



FIND $\vec{BA}$ AND $\vec{BC}$

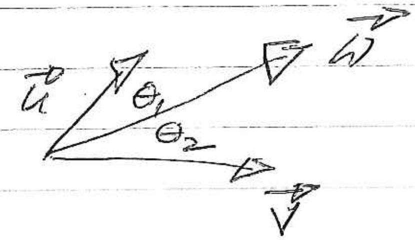
$$\vec{BA} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \quad \|\vec{BA}\| = \sqrt{6}$$

$$\vec{BC} = \begin{bmatrix} 6 \\ 1 \\ 10 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix} \quad \|\vec{BC}\| = \sqrt{66}$$

$$\cos\theta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\|\|\vec{BC}\|} = \frac{\begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}}{\sqrt{6}\sqrt{66}} = \frac{-17}{\sqrt{396}}$$

$$\theta = 2.59 \text{ rad} \ (149 \text{ degrees})$$

Q2:



$$\vec{w} = l\vec{u} + k\vec{v}$$
$$l = \|\vec{v}\|$$
$$k = \|\vec{u}\|$$

CAN WE SHOW THAT $\theta_1 = \theta_2$?

$$\cos\theta_1 = \frac{\vec{u}\cdot\vec{w}}{\|\vec{u}\|\|\vec{w}\|} = \frac{\vec{u}\cdot(\|\vec{v}\|\vec{u} + \|\vec{u}\|\vec{v})}{\|\vec{u}\|\|\vec{w}\|}$$

$$= \frac{\|\vec{v}\|\|\vec{u}\|^2 + \|\vec{u}\|\,\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{w}\|}$$

$$= \frac{\|\vec{v}\|\|\vec{u}\| + \vec{u}\cdot\vec{v}}{\|\vec{w}\|}$$

SIMILARLY,

$$\cos\theta_2 = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|} \ldots = \frac{\|\vec{v}\|\|\vec{u}\| + \vec{u}\cdot\vec{v}}{\|\vec{w}\|}$$

$\therefore \cos\theta_1 = \cos\theta_2$

$\therefore \theta_1 = \theta_2$

$\therefore$ $\vec{w}$ BISECTS $\vec{u}$ AND $\vec{v}$.

Q3:

TAKE THE VECTOR ASSOCIATED WITH THE DIAGONAL TO BE:

$$\vec{V}_1 = \begin{bmatrix} 1 \\ 1 \\ \sqrt{2} \end{bmatrix}$$

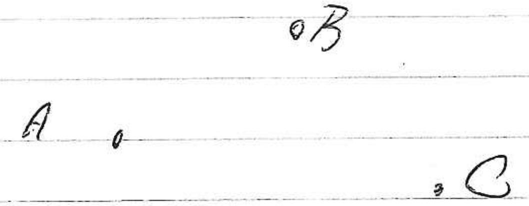
TAKE THE VECTOR ASSOCIATED WITH THE LONGEST SIDE TO BE:

$$\vec{V}_2 = \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

$$\cos\theta = \frac{\vec{V}_1 \cdot \vec{V}_2}{\|\vec{V}_1\| \|\vec{V}_2\|} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \theta = \frac{\pi}{4} \text{ rad } (45°)$$

P4:



AREA OF TRIANGLE ABC $= \frac{1}{2}$ (AREA OF PARALLELOGRAM FORMED BY $\vec{AB}$ AND $\vec{AC}$)

$$= \frac{1}{2} \left\| \vec{AB} \times \vec{AC} \right\|$$

$$\vec{AB} = \begin{bmatrix} 6 \\ -4 \end{bmatrix} \qquad \vec{AC} = \begin{bmatrix} 6 \\ 14/3 \end{bmatrix}$$

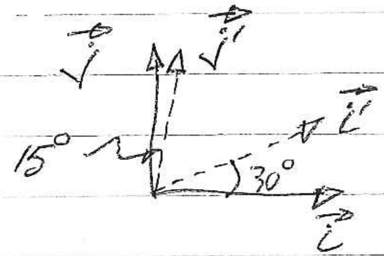
IN 3-D, $\vec{AB} = \begin{bmatrix} 6 \\ -4 \\ 0 \end{bmatrix}$; $\vec{AC} = \begin{bmatrix} 6 \\ 14/3 \\ 0 \end{bmatrix}$

$$\vec{AB} \times \vec{AC} = \begin{bmatrix} 0 \\ 0 \\ 52 \end{bmatrix}$$

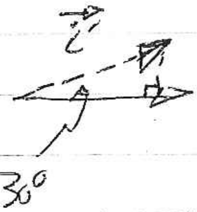
$\therefore$ AREA OF TRIANGLE ABC $= \frac{1}{2}(52) = 26$

Q5:

i)



ii) COORDINATES OF $\vec{i'}$?



$\|\vec{i'}\| = 1$ (UNIT VECTOR)

$$\therefore \quad \vec{i'} = \begin{bmatrix} \cos 30° \\ \sin 30° \end{bmatrix}$$

iii) COORDINATES OF $\vec{j'}$?

SIMILARLY,

$$\vec{j'} = \begin{bmatrix} \sin 15° \\ \cos 15° \end{bmatrix}$$

iv) WE WANT TO SOLVE FOR a AND b SUCH THAT:

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = a\vec{i'} + b\vec{j'}$$

$$= a \begin{bmatrix} \cos 30° \\ \sin 30° \end{bmatrix} + b \begin{bmatrix} \sin 15° \\ \cos 15° \end{bmatrix}$$

$$\therefore \quad 2 = a\cos 30^\circ + b\sin 15^\circ$$
$$1 = a\sin 30^\circ + b\cos 15^\circ$$

2 EQUATIONS IN 2 UNKNOWNS. SOLVING FOR a AND b,

$$a \approx 2.37$$

$$b \approx -.190$$

$\therefore \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ IS EQUIVALENT TO $\begin{bmatrix} 2.37 \\ -.190 \end{bmatrix}$

IN THE NEW COORDINATE SYSTEM.

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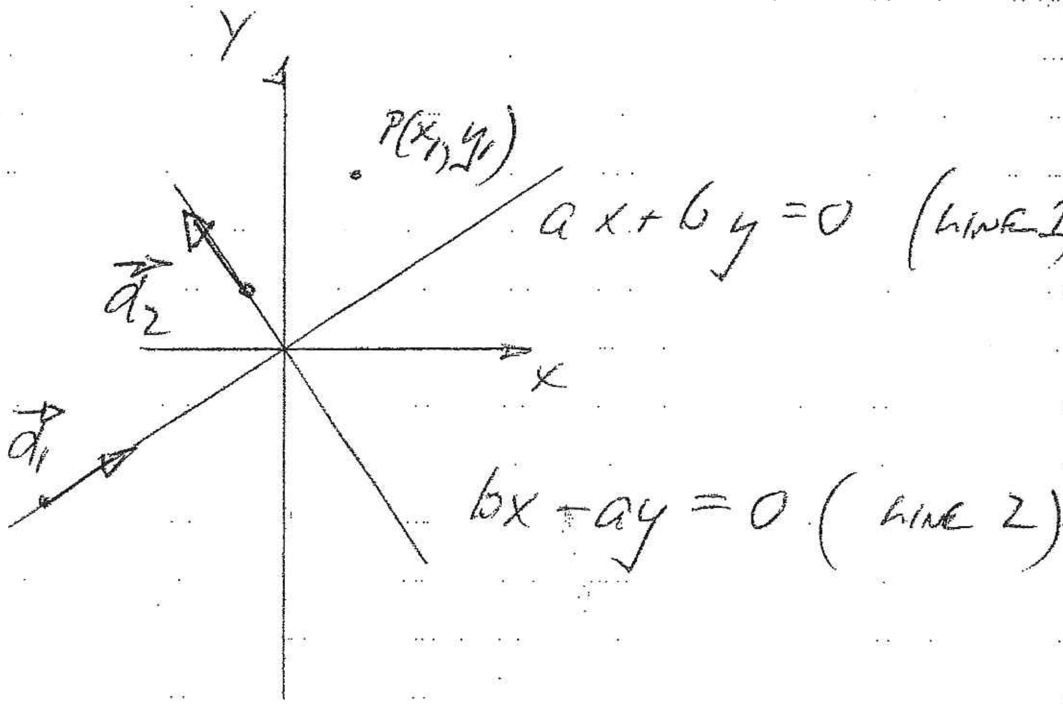

TO OBTAIN THE DISTANCES OF P TO THE TWO LINES, WE WILL PROJECT $\vec{OP}$ ONTO THE TWO DIRECTION VECTORS.

LINE 1: $ax + by = 0$

$$\text{SLOPE} = -\frac{a}{b}$$

$$\text{TAKE } \vec{d}_1 = \begin{bmatrix} b \\ -a \end{bmatrix}$$

LINE 2: $bx - ay = 0$

$$\text{SLOPE} = \frac{b}{a}$$

$$\text{TAKE } \vec{d}_2 = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\text{proj}_{\vec{d}_1}\vec{OP} = \frac{\vec{OP}\cdot\vec{d}_1}{\|\vec{d}_1\|}\frac{\vec{d}_1}{\|\vec{d}_1\|}$$

$$\text{MAGNITUDE} = \left|\frac{\vec{OP}\cdot\vec{d}_1}{\|\vec{d}_1\|}\right| = \frac{\left|\begin{bmatrix}x_1\\y_1\end{bmatrix}\cdot\begin{bmatrix}b\\-a\end{bmatrix}\right|}{\sqrt{a^2+b^2}} = \frac{|bx_1-ay_1|}{\sqrt{a^2+b^2}}$$

$$\text{proj}_{\vec{d}_2}\vec{OP} = \frac{\vec{OP}\cdot\vec{d}_2}{\|\vec{d}_2\|}\frac{\vec{d}_2}{\|\vec{d}_2\|}$$

$$\text{MAGNITUDE} = \left|\frac{\vec{OP}\cdot\vec{d}_2}{\|\vec{d}_2\|}\right| = \frac{\left|\begin{bmatrix}x_1\\y_1\end{bmatrix}\cdot\begin{bmatrix}a\\b\end{bmatrix}\right|}{\sqrt{a^2+b^2}} = \frac{|ax_1+by_1|}{\sqrt{a^2+b^2}}$$

∴ SUM OF SQUARES OF THE DISTANCES

$$= \frac{(bx_1-ay_1)^2}{a^2+b^2} + \frac{(ax_1+by_1)^2}{a^2+b^2}$$

$$= \dots$$

$$= \frac{(a^2+b^2)x_1^2+(a^2+b^2)y_1^2}{a^2+b^2} = x_1^2+y_1^2$$

$$= \|\vec{OP}\|^2$$

Q7:

TAKE: $\vec{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

THEN: $\vec{u} \times \vec{v} = \begin{bmatrix} y_1 z_2 - y_2 z_1 \\ -(x_1 z_2 - x_2 z_1) \\ x_1 y_2 - x_2 y_1 \end{bmatrix}$

AND: $$\vec{u} \cdot (\vec{u} \times \vec{v}) = x_1(y_1 z_2 - y_2 z_1) - y_1(x_1 z_2 - x_2 z_1) + z_1(x_1 y_2 - x_2 y_1)$$

$$= \cancel{x_1 y_1} z_2 - \cancel{x_1 y_2} z_1 - \cancel{x_1 y_1} z_2 + \cancel{x_2 y_1} z_1 + \cancel{x_1 y_2} z_1 - \cancel{x_2 y_1} z_1$$

$$= 0$$

Q8:

$$\|\vec{u} \times \vec{v}\|^2 = (y_1 z_2 - y_2 z_1)^2 + (x_1 z_2 - x_2 z_1)^2 + (x_1 y_2 - x_2 y_1)^2$$

$$= y_1^2 z_2^2 - 2 y_1 y_2 z_1 z_2 + y_2^2 z_1^2$$
$$+ x_1^2 z_2^2 - 2 x_1 x_2 z_1 z_2 + x_2^2 z_1^2$$
$$+ x_1^2 y_2^2 - 2 x_1 x_2 y_1 y_2 + x_2^2 y_1^2$$

NEW TERMS ADDED AND THEN SUBTRACTED

$$= \underbrace{x_1^2x_2^2 + y_1^2y_2^2 + z_1^2z_2^2} + \left(x_1^2y_2^2 + x_1^2z_2^2 + x_2^2y_1^2 + y_1^2z_2^2 + x_2^2z_1^2 + y_2^2z_1^2\right)$$

$$- \left(\overbrace{x_1^2x_2^2 + y_1^2y_2^2 + z_1^2z_2^2} + \left(2x_1x_2y_1y_2 + 2x_1x_2z_1z_2 + 2y_1y_2z_1z_2\right)\right)$$

$$= \left(x_1^2 + y_1^2 + z_1^2\right)\left(x_2^2 + y_2^2 + z_2^2\right) - \left(x_1x_2 + y_1y_2 + z_1z_2\right)^2$$

$$= \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u}\cdot\vec{v})^2$$

Q9:

FALSE

LET $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$\vec{v} + \vec{w} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{u} \cdot (\vec{v} + \vec{w}) = 0$ $\vec{u}$ IS ORTHOGONAL TO $\vec{v} + \vec{w}$

$\vec{u} \cdot \vec{v} = 1 \neq 0$ $\therefore$ $\vec{u}$ IS NOT ORTHOGONAL TO $\vec{v}$

$\vec{u} \cdot \vec{w} = -1 \neq 0$ $\therefore$ $\vec{u}$ IS NOT ORTHOGONAL TO $\vec{w}$

Q10:

TRUE

GIVEN $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2$

$\|\vec{u}+\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$

$= \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u}\cdot\vec{v}$

$\therefore \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u}\cdot\vec{v} = \|\vec{u}\|^2 + \|\vec{v}\|^2$

$\therefore \vec{u}\cdot\vec{v} = 0$

$\therefore$ $\vec{u}$ AND $\vec{v}$ ARE ORTHOGONAL.

Q11:

FALSE

LET $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$\therefore$ $\vec{u}$ AND $\vec{v}$ ARE PARALLEL

$\|\vec{u}+\vec{v}\| = 0$

$\|\vec{u}\| + \|\vec{v}\| = 1 + 1 = 2$

$\therefore \|\vec{u}+\vec{v}\| \neq \|\vec{u}\| + \|\vec{v}\|$