University of Toronto
Faculty of Applied Science and Engineering
Final Examination, April, 2014

STA286S: Probability and Statistics

Examiners: B. Donmez and K. Knight

Name:____

Student Number:__________________________________

Circle your **lecture** and **tutorial** sections:

Lecture (circle one)	Tutorial (circle one)	Time	Location	TA
LEC 01 (Prof. Knight)	TUT01	Mon 1-2 pm	BA2159	Maryam Merrikhpour
	TUT02	Mon 1-2 pm	BA3008	Karen Wong
	TUT03	Th 5-6 pm	BA2155	Karen Wong
	TUT04	Wed 1-2 pm	WB144	Wayne Giang
LEC 02 (Prof. Donmez)	TUT05	Mon 2-3 pm	BA3116	Maryam Merrikhpour
	TUT06	Fri 10-11 am	BA3008	Zheng Li
	TUT07	Tues 1-2 pm	BA3008	Zheng Li
	TUT08	Tues 1-2 pm	BA3012	Wayne Giang

Instructions:

- **Time allowed:** 2 ½ hours.
- **Aids:** a non-programmable calculator and a double-sided A4 size aid sheet.
- There are **8 questions** and **20 pages** including this page. The last 4 pages contain the probability tables and an empty page as scrap paper. You can rip them off if you want.
- If you run out of space, there is extra space provided at the end of each question.
- If you do not understand a question, or are having some other difficulty, do not hesitate to ask your instructor or TA for clarification.
- Points for each question are indicated in parentheses. Total points: 100.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**
Max	**14**	**10**	**14**	**10**	**15**	**10**	**15**	**12**	**100**
Score									

GOOD LUCK !

Question 1: A discrete random variable X has a moment generating function

$$M(t) = E[\exp(tX)] = \frac{1}{2} + \frac{1}{6}\exp(t) + \frac{1}{3}\exp(4t)$$

(a) (4 pts) Find the mean and variance of X.

$$\boldsymbol{E(X) = \left.\frac{dM(t)}{dt}\right|_{t=0} = \left(\frac{1}{6}\exp(t) + \frac{4}{3}\exp(4t)\right)_{t=0} = 3/2}$$

$$\boldsymbol{E(X^2) = \left.\frac{d^2M(t)}{dt^2}\right|_{t=0} = \left(\frac{1}{6}\exp(t) + \frac{16}{3}\exp(4t)\right)_{t=0} = 33/6}$$

$$\boldsymbol{Var(X) = E(X^2) - [E(X)]^2 = \frac{33}{6} - \frac{9}{4} = 13/4}$$

(b) (5 pts) Suppose that X and Y are independent random variables with a common distribution whose moment generating function is $M(t)$ and define $Z = X + Y$. What is the moment generating function of Z?

$$\boldsymbol{M_Z(t) = M_X(t)M_Y(t)}$$

$$\boldsymbol{M_Z(t) = \left(\frac{1}{2} + \frac{1}{6}\exp(t) + \frac{1}{3}\exp(4t)\right)^2 = \frac{1}{4} + \frac{1}{6}\exp(t) + \frac{1}{36}\exp(2t) + \frac{1}{3}\exp(4t) + \frac{1}{9}\exp(5t) + \frac{1}{9}\exp(8t)}$$

(c) (5 pts) Find the probability distribution of Z, that is, compute $P(Z = z)$ for all possible values of z.

$$\boldsymbol{M_Z(t) = \sum e^{tz}P(Z = z)}$$

z	**0**	**1**	**2**	**4**	**5**	**8**
P(Z=z)	**1/4**	**1/6**	**1/36**	**1/3**	**1/9**	**1/9**

Question 2

Suppose that X is a Binomial random variable $n = 100$ and p unknown and we want to test $H_o: p = 0.4$ against $H_1: p > 0.4$. We reject H_o if $X \geq 48$. Use normal approximations for this question (both in part (a) and part (b)).

(a) (5 pts) Find the significance level (α) for this test.

$$\boldsymbol{\alpha = P(reject|H_o\ true) = P(X \geq 48|p = 0.4) = 1 - P(X \leq 47|p = 0.4)}$$

$$\boldsymbol{= 1 - P\left(Z \leq \frac{47 + 0.5 - 100 \times 0.4}{\sqrt{100 \times 0.4 \times 0.6}}\right) = 1 - P\left(Z \leq \frac{7.5}{\sqrt{24}}\right) = 1 - P(Z \leq 1.5309) = P(Z > 1.5309)}$$

$$\boldsymbol{\approx 0.063}$$

(b) (5 pts) Suppose that the true value of p is 0.55. Find the power of the test for this value of p.

$$\boldsymbol{Power = P(reject|H_1\ true) = P(X \geq 48|p = 0.55) = 1 - P(X \leq 47|p = 0.55)}$$

$$\boldsymbol{= 1 - P\left(Z \leq \frac{47 + 0.5 - 100 \times 0.55}{\sqrt{100 \times 0.55 \times 0.45}}\right) = 1 - P\left(Z \leq \frac{-7.5}{\sqrt{24.75}}\right) = 1 - P(Z \leq -1.50756)}$$

$$\boldsymbol{= P(Z > -1.50756) \approx 0.934}$$

Question 3

Suppose that X and Y are continuous random variables with joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} x+y & if\ 0 \le x \le 1\ and\ 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

(a) (4 pts) Find the marginal density of X.

$$\boldsymbol{f(x) = \int_0^1 (x+y)\, dy = xy + \frac{y^2}{2}\bigg|_0^1 = x + \frac{1}{2}\ \ , 0 \le x \le 1}$$

(b) (5 pts) Find the conditional expected value $E(Y|X=x)$ for $0 \le x \le 1$.

$$\boldsymbol{f_{Y|X=x}(y) = \frac{f_{X,Y}(x,y)}{f(x)} = \frac{x+y}{x+\frac{1}{2}}\ \ , 0 \le y \le 1\ and\ 0 \le x \le 1}$$

$$\boldsymbol{E(Y|X=x) = \int_0^1 y\frac{x+y}{x+\frac{1}{2}} dy = \frac{1}{\left(x+\frac{1}{2}\right)}\int_0^1 xy + y^2\, dy = \frac{\frac{x}{2}+\frac{1}{3}}{x+\frac{1}{2}}}$$

(c) (5 pts) Suppose that we define $U = \min(X,Y)$ and $V = \max(X,Y)$. Find the joint probability density function of (U,V). (Hint: Note that for $0 \le u < v \le 1$, $P(U > u, V \le v) = P(u < X \le v, u < Y \le v)$.).

$$\boldsymbol{P(U \le u, V \le v) = P(V \le v) - P(U > u, V \le v) = P(X \le v, Y \le v) - P(u < X \le v, u < Y \le v)}$$

$$\boldsymbol{= \int_0^v \int_0^v x + y\, dxdy - \int_u^v \int_u^v x + y\, dxdy = v^3 - (v^3 - u^2v - uv^2 + u^3) = u^2v + uv^2 - u^3}$$

$$\boldsymbol{for\ \ 0 \le u < v \le 1}$$

$$\boldsymbol{f_{U,V}(u,v) = \frac{\partial^2 F(u,v)}{\partial u \partial v} = 2u + 2v, for\ \ 0 \le u < v \le 1}$$

2nd method
When X<Y, we have the following one-to-one transformation: U=X, V=Y

$$\boldsymbol{J = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1}$$

When Y<X, we have the following one-to-one transformation: U=Y, V=X

J=$\begin{vmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{vmatrix} = -\mathbf{1}$

$$\boldsymbol{f_{U,V}(u,v) = (u+v)|1| + (v+u)|-1| = 2u + 2v, for\ \ 0 \le u < v \le 1}$$

Question 4

A wine importer held a wine-tasting party whose guests could be classified as either experts or novices at wine identification; 20% of the guests were experts and the remaining 80% were novices. Assume that a novice has a 0.4 probability of correctly identifying a wine while an expert has a 0.9 probability of correct identification, and that each identification is made independently of the others.

(a) (4 pts) What is the probability that a novice will identify at least 3 of the 5 wines correctly?

This is binomial with n=5 and p=0.4

$$\boldsymbol{P(X \geq 3) = \binom{5}{3} 0.4^3 \times 0.6^2 + \binom{5}{4} 0.4^4 \times 0.6^1 + \binom{5}{5} 0.4^5 \times 0.6^0 = 0.31744}$$

(b) (6 pts) Suppose that a guest identifies 4 of the 5 wines correctly. What is the probability that this guest is an expert?

$$\boldsymbol{P(X = 4 |\, Y = Expert) = \binom{5}{4} 0.9^4 \times 0.1^1 = 0.32805}$$

$$\boldsymbol{P(X = 4 |\, Y = Novice) = \binom{5}{4} 0.4^4 \times 0.6^1 = 0.0768}$$

$$\boldsymbol{P(Y = Expert | X = 4) = \frac{P(X = 4 | Y = Expert) P(Y = Expert)}{P(X = 4 | Y = Expert) P(Y = Expert) + P(X = 4 | Y = Novice) P(Y = Novice)}}$$

$$\boldsymbol{= \frac{0.32805 \times 0.2}{0.32805 \times 0.2 + 0.0768 \times 0.8} = \frac{0.06561}{0.06561 + 0.06144} = 0.516411}$$

Question 5

Suppose that $X_1, \dots, X_{200}$ are independent continuous random variables with common density function

$$f(x) = \begin{cases} 2(1-x) & for\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

(a) (4 pts) Compute the mean and variance of X_i.

$$\boldsymbol{E(X) = \int_0^1 2x(1-x)dx = \frac{1}{3}}$$
$$\boldsymbol{E(X^2) = \int_0^1 2x^2(1-x)dx = \frac{1}{6}}$$
$$\boldsymbol{Var(X) = E(X^2) - [E(X)]^2 = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}}$$

(b) (5 pts) Define $S = X_1 + \cdots + X_{200}$. Using an appropriate method, find an approximation for $P(S > 70)$.

CLT. *S* is approximately normal with mean = $\boldsymbol{200 \times \frac{1}{3}}$ and variance = $\boldsymbol{200 \times \frac{1}{18}}$

$$\boldsymbol{P(S > 70) = P\left(Z > \frac{70 - 66.6}{\sqrt{11.11}}\right) = P\left(Z > \frac{70 - 66.6}{\sqrt{11.11}}\right) = P(Z > 1.02) = 0.1539}$$

(c) (6 pts) Now define

$$Y_i = \begin{cases} 1 & if\ X_i > 0.9 \\ 0 & if\ X_i \le 0.9 \end{cases}$$

and set $T = Y_1 + \cdots + Y_{200}$. Compute $P(T \ge 3)$ either exactly or using an appropriate approximation method.

$$\boldsymbol{P(X_i \le 0.9) = \int_0^{0.9} 2(1-x)dx = 0.99}$$

***T* is binomial with p = 0.01, n=200**

<u>Exact</u>

$\boldsymbol{P(T \ge 3) = 1 - P(T < 3) = 1 - \binom{200}{0} 0.01^0 \times 0.99^{200} - \binom{200}{1} 0.01^1 \times 0.99^{199} -}$
$\boldsymbol{\binom{200}{2} 0.01^2 \times 0.99^{198} = 0.3233}$

Approximate

We can use the Poisson approximation to calculate this probability with Poisson mean of np=200x0.01=2.

$$\boldsymbol{P(T \geq 3) = 1 - P(T < 3) = 1 - \sum_{k=0}^{k=2} \frac{e^{-2}2^k}{k!} = 1 - 0.696384 = 0.3233}$$

Question 6

Suppose that $X_1, \dots, X_n$ are independent continuous random variables with common probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & for\ 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Where $\theta > 0$ is an unknown parameter.

(a) (5 pts) Find the maximum likelihood estimator of θ based on $X_1, \dots, X_n$.

The likelihood function is given by

$$\boldsymbol{L(\theta|x_1, \dots, x_n) = \prod_1^n \theta (x_i)^{\theta-1} = \theta^n \prod_1^n (x_i)^{\theta-1}}$$

The log-likelihood function is given by

$$\boldsymbol{l(\theta) = ln\theta^n + (\theta-1)lnx_1 + (\theta-1)lnx_2 + \cdots = nln\theta + (\theta-1)\sum_1^n ln\, x_i}$$

Further, taking derivative of *l*(θ) with respect to θ and setting it to 0 we get:

$$\boldsymbol{l'(\theta) = \frac{n}{\theta} + \sum_1^n ln\, x_i = 0}$$

$$\boldsymbol{\hat{\theta} = -\frac{n}{\sum_1^n ln\, x_i}}$$

Checking second derivative:

$$\boldsymbol{l''(\theta) = -\frac{n}{\theta^2}}$$

$$\boldsymbol{l''(\hat{\theta})} < 0$$

Therefore, $\boldsymbol{\hat{\theta} = -\frac{n}{\sum_1^n ln\, x_i}}$ **is the MLE of θ.**

(b) (5 pts) Suppose we want to test the null hypothesis $H_o: \theta = 1$ versus the alternative $H_1: \theta > 1$ using the test statistic $\bar{X} = \frac{1}{n}(X_1 + \cdots + X_n)$.

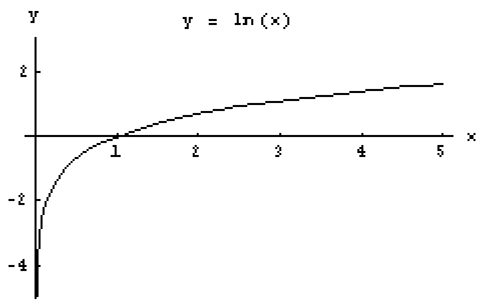
For a test with level α, would we reject H_o when

(I) $\bar{X} > some\ k_\alpha$

or when

(II) $\bar{X} < some\ k_\alpha$?

Justify your answer.



The values of $\bar{X}$ can be between 0 and 1. Larger values of $\bar{X}$ would generate larger positive values of $\hat{\theta} = -\frac{n}{\sum_1^n \ln x_i}$, and would support the alternative more than smaller values of $\bar{X}$. Thus, we would reject when $\bar{X} > some\ k_\alpha$.

Question 7

Two different methods (A and B) have been devised to reduce the time spent in transferring materials from one location to another. Each approach is tried several times, and the times to completion (in hours) are recorded below:

```
Method A: 8.3  7.1  7.8  8.9  8.7  7.2  6.9  6.7
Method B: 7.8  8.1  8.3  8.5  7.6  8.6  7.4
```

The mean and standard deviation for each method are also calculated:
$\bar{x}_A = 7.70$, $\bar{x}_B = 8.04$, $s_A = 0.850$, $s_B = 0.458$

(a) (5 pts) Assuming that the data for each method are normally distributed with the same variance, construct a 95% confidence interval for the difference in mean times to completion between methods A and B.

Use the pooled two sample *t* CI.

The CI is $(\bar{x} - \bar{y}) \pm t_{(n_1+n_2-2);\alpha/2} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

The confidence level 1- α = 0.95 and so $\alpha/2 = 0.025$.

$$t_{(n_1+n_2-2);\alpha/2} = t_{13;0.025} = 2.16$$

The pooled estimate of the common variance is:

$$s_p^2 = \frac{(n_1-1)s_x^2 + (n_2-1)s_y^2}{n_1+n_2-2} = \frac{(7)(0.850)^2 + (6)(0.458)^2}{8+7-2} = 0.4858526$$

Substituting into the formula we get:

$$(7.70-8.04) \pm (2.16) \cdot \sqrt{0.4858526\left(\frac{1}{8}+\frac{1}{7}\right)} = (-1.119\,, 0.439)$$

(b) (5 pts) Is there evidence that the mean time to completion differs for the two methods? Test the null hypothesis of no difference versus the two-sided alternative using a significance level $\alpha = 0.05$. (Assume as in part (a) that the data are normally distributed with equal variances).

No, because the CI includes 0

(c) (5 pts) In parts (a) and (b), we assumed that the data are normally distributed with equal variances. Using the data and summary statistics given above, assess the equal variance assumption for $\alpha = 0.1$.

$H_0 : \sigma_x^2 = \sigma_y^2$ **vs** $H_a : \sigma_x^2 \neq \sigma_y^2$

The test statistic is: $F_{stat} = \frac{s_x^2}{s_y^2} = \frac{(0.850)^2}{(0.458)^2} = 3.444$**, it has a** $F_{(7,6)}$ **distribution.**

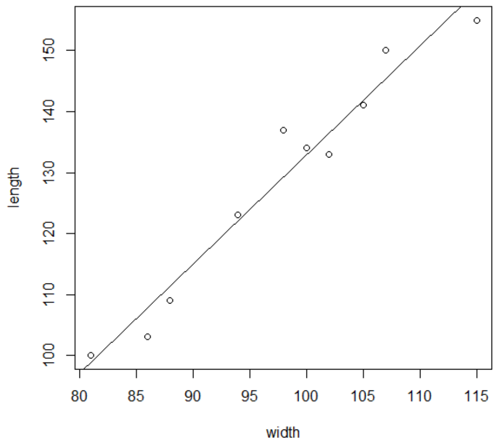
Using α = 0.1, we reject H_0 if:

$$F_{stat} > F_{(7,6);0.05} = 4.21 \text{ or } F_{stat} < F_{(7,6);0.95} = \frac{1}{F_{(6,7);0.05}} = \frac{1}{3.87} = 0.2584$$

Conclusion: since F_{stat} **is in the acceptance region**

we fail to reject H_0 and conclude that there is no difference in variances.

Question 8: The following plot and summary statistics are based on measurements on length and width (both in mm) of 10 painted female turtles (*Chrysemys picta marginta*):



$\overline{x} = 97.6$

$\overline{y} = 128.5$

$\sum_1^{10}(y_i - \overline{y})^2 = 3316.5$

$\sum_1^{10}(x_i - \overline{x})^2 = 986.4$

$\sum_1^{10}(x_i - \overline{x})(y_i - \overline{y}) = 1768$

The following model is fit to the data: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$

where x_i and y_i is the width and the length, respectively, of the i$^{\text{th}}$ turtle and $\epsilon_i\ iid\ N(0, \sigma^2)$.

(a) (3 pts) Find the least squares estimate β_1.

$$\boldsymbol{b_1 = \frac{\sum_{i=1}^{10}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{10}(x_i - \overline{x})^2} = \frac{1768}{986.4} = 1.7924}$$

(b) (5 pts) Find a 95% confidence interval for β_1. Do the data support the claim that the length of a painted female turtle does depend on its width? Briefly explain why?

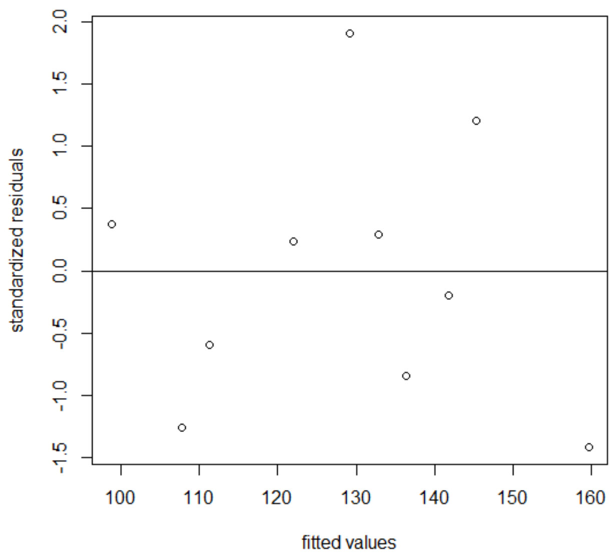
$$\boldsymbol{s^2 = \frac{S_{yy} - b_1 S_{xy}}{n-2} = \frac{3316.5 - 1.7924 \times 1768}{8} = 18.4421}$$

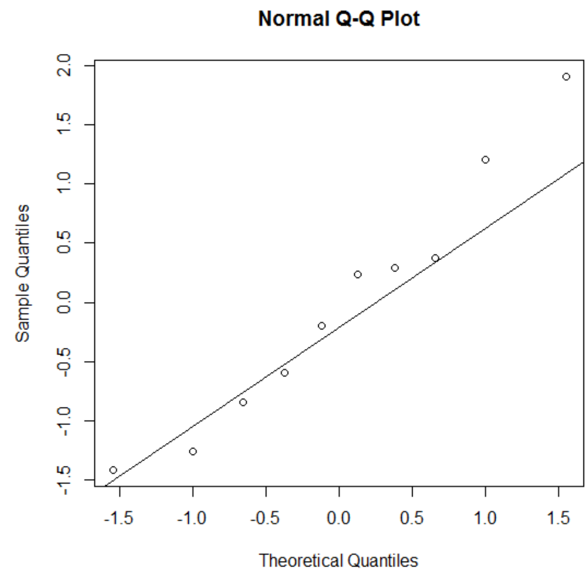
$$\boldsymbol{b_1 \pm t_{\frac{\alpha}{2}, n-2} \frac{s}{\sqrt{S_{xx}}}}$$

$$\boldsymbol{1.7924 \pm 2.306 \frac{4.294}{\sqrt{986.4}} = [1.4771, 2.1077]}$$

Yes the data support the claim as the confidence interval excludes 0.

(a) (4 pts) The plot on the below left shows the standardized residuals vs. fitted values. The plot on the below right is the normal quantile quantile plot for standardized residuals. Comment on the adequacy of model fit.





The plot on the left seems to be fairly random (no systematic pattern). Further, the standardized residuals seem to fall on an approximately straight line. Although, the two large values that seem to deviate from the linear pattern may be outliers. The assumptions seem to hold.

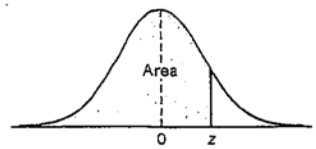


Table **A.3** Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

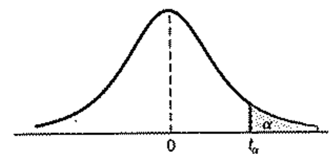


Table A.4 Critical Values of the t-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

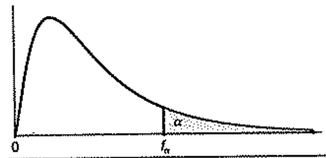


Table A.6* Critical Values of the F-Distribution

$f_{0.05}(v_1, v_2)$

v_2	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

*Reproduced from Table 18 of *Biometrika Tables for Statisticians*, Vol. I, by permission of E.S. Pearson and the Biometrika Trustees.

END!