CHE 260: THERMODYNAMICS AND HEAT TRANSFER

FINAL EXAMINATION FOR HEAT TRANSFER

14th DECEMBER 2017

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3	Q4	Q5	Total
15	15	15	15	15	75

INSTRUCTIONS

1. This examination is closed textbook, closed internet, closed all communication devices.
2. One handwritten aid sheet of size 8.5” x 11” aid sheet (both sides) is permitted.
3. Any non-communicating, non-programming, non-graphing calculator is permissible.
4. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
5. Answers written in pencil will NOT be re-marked. This is University policy.
6. For all problems, you must present the solution process in a step by step fashion for partial marks.
7. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY.**

Q.1. [15 points] HOT DOG OR LAUNDRY?

On a weekend afternoon, Mr. X, lazing around in the lounge of his home, suddenly feels hungry. He goes to his refrigerator looking for food, and finds a cylindrical hot dog of diameter 2 cm. The hot dog is at a uniform temperature of 4°C. Mr. X happily takes the hot dog out and places it into his convection oven preheated to a temperature of 150°C. The heat transfer coefficient for heat exchange between the oven and the hot dog surface is 300 W/m^2 °C. 10 minutes later, Mr. X's wife yells at him saying that he has been postponing doing the laundry for a week. Afraid of overcooking the hot dog, Mr. X immediately pulls the hot dog out of the oven, places it on a plate in the lounge maintained at 25°C, and then proceeds to take the clothes to the common laundry room of his building. The convective heat transfer coefficient for heat exchange between the hot dog and the air in the lounge is 5 W/m^2 °C.

Taking the thermal conductivity, specific heat capacity and density of the hot dog to be 0.5 W/m°C, 4180 J/kg°C and 990 kg/m^3, respectively, and assuming the heating and cooling of the hot dog to be axisymmetric, invariant along the height of the hot dog, and purely due to heat exchange with surrounding air, answer the following questions:

(a) **[9 points]** What are the center and surface temperatures of the hot dog after Mr. X pulls it out of the oven?

Solution:

We begin by calculating the Biot number $Bi = \frac{hr_0}{k} = \frac{300 \times 10^{-2}}{0.5} = 6.$

Since Bi>0.1, temperature gradients within the hot dog cannot be ignored. Hence we use the eigenfunction solution. Assuming that the dimensionless time is more than 0.2, we use the one term approximation.

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 t^*\right) J_0\left(\lambda_1 x^*\right).$$

For Bi = 6, for a cylinder, $\lambda_1 = 2.0490$, $A_1 = 1.5253$.

$$\alpha = \frac{k}{\rho C} = \frac{0.5}{990 \times 4180} = 1.208 \times 10^{-7} \text{ m}^2\text{/s}.$$

$$t^* = \frac{t}{r_0^2 / \alpha} = \frac{10 \times 60}{(10^{-2})^2 / 1.208 \times 10^{-7}} = 0.7248.$$

Since $t^* > 0.2$, we can use the 1 term approximation.

At the center,

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 t^*\right) J_0\left(0\right) = 1.5253 \times \exp\left(-2.0490^2 \times 1.087\right) = 0.07274.$$

$$T_0 = T_\infty + \left(T_i - T_\infty\right) \times 0.01590 = 150 + \left(4 - 150\right) \times 0.07274 = 139.4^\circ\text{C}.$$

$$\theta_S = \frac{T_S - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 t^*\right) J_0\left(\lambda_1\right) = 1.5253 \times \exp\left(-2.0490^2 \times 1.087\right) \times 0.2239 = 0.01629.$$

$$T_S = T_\infty + \left(T_i - T_\infty\right) \times 0.003560 = 150 + \left(4 - 150\right) \times 0.01629 = 147.6^\circ\text{C}.$$

Calculation of Bi: 2 points

Taking the decision to use 1 term approximation: 1 point

Getting lambda1 and A1 from table: 1 point

Calculation of alpha: 1 point

Calculation of t* = 1 point

Calculation of th0, and hence T0: 1.5

Calculation of thS, and hence TS: 1.5

(b) **[6 points]** If Mr. X returns home from the laundry room in 60 min, what will be the center and surface temperatures of the hot dog at that time?

Solution:

The Biot number is $Bi = \dfrac{hr_0}{k} = \dfrac{5 \times 10^{-2}}{0.5} = 0.1.$ We can use the lumped system analysis.

The initial temperature will be an average of the temperature profile within the hot dog. But since the temperature at the center and surface are not different, we can assume the temperature distribution to be quadratic and compute an average temperature based on this.

$$T(r) \approx T_S + (T_0 - T_S)\left(1 - \frac{r^2}{R^2}\right)$$

The average temperature is

$$T_m = \frac{1}{\pi R^2}\int_0^R 2\pi r T(r)\,dr = 2\int_0^1 x^*\left[T_S + (T_0 - T_S)x^{*2}\right]dx^* = 2\left[\frac{1}{2}T_S + \frac{1}{4}(T_0 - T_S)\right] = \frac{(T_0 + T_S)}{2}$$

The average initial temperature for the cooling phase is (139.4+147.6)/2 = 143.5^{o}C.

The lumped system analysis says that

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA}{\rho VC}t\right) = \exp\left(-\frac{h(2\pi r_0 L)}{\rho(\pi r_0^2 L)C}t\right) = \exp\left(-\frac{2h}{\rho r_0 C}t\right) = 0.4190.$$

$$T = 25 + (143.5 - 25) \times 0.4190 = 74.7\ ^oC.$$

The center and surface temperatures are both close to 74.7^{o}C.

Calculation of Bi: 2 point

Taking the decision of using lumped system analysis (or 1 term approximation, since we are at the boundary of the decision making Bi=0.1): 1 point

Calculation of the surface and center temperatures: 3 points

Q.2A. [7 points] HEAT TRANSFER RATE FROM DRAG

A streamlined body is moving at a velocity of 1 m/s against air at an ambient temperature of 25^{o}C. The body is maintained at a temperature of 70^{o}C. The drag force acting on the object is 2.1 N. What is the rate of heat transfer between the body and fluid? For air, use a density of 1.1 kg/m^3, a specific heat capacity (at constant pressure) of 1007 J/kg^{o}C, a momentum diffusivity of 1.7×10^{-5} m^2/s, and a Prandtl number of 0.72, and assume these values to be constant over the temperature range of concern.

Solution:

Since the body is streamlined, the drag on it is dominated by skin drag as opposed to form drag. Also, since 0.6<Pr<60, we can use the Colburn analogy

$$\frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}^{1/3}} = \frac{C_f}{2}$$

$$\frac{\frac{hL}{k}}{\frac{UL}{\nu}\mathrm{Pr}^{1/3}} = \frac{1}{2}\frac{F_D}{A\frac{1}{2}\rho U^2}$$

Simplifying and rearranging the terms, we have

$$hA = \frac{k}{\nu}\mathrm{Pr}^{1/3}\frac{F_D}{\rho U} = \frac{\alpha\rho C}{\nu}\mathrm{Pr}^{1/3}\frac{F_D}{\rho U} = \frac{1}{\mathrm{Pr}^{2/3}}\frac{CF_D}{U} = \frac{1}{0.72^{2/3}}\frac{1007\times 2.1}{1} = 2632.5\frac{\mathrm{W}}{{}^{\mathrm{o}}\mathrm{C}}.$$

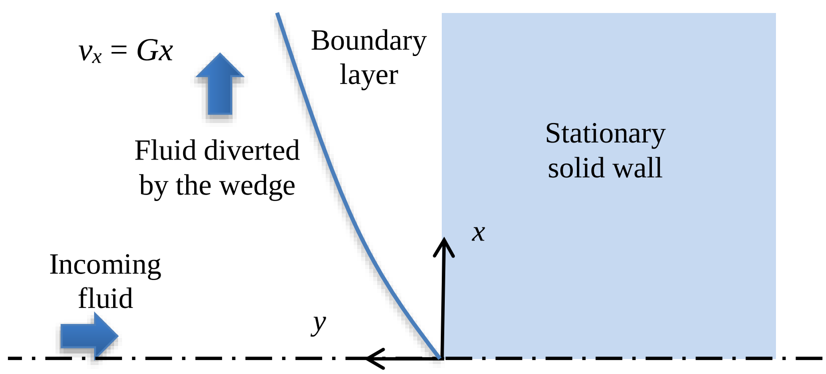
The rate of heat exchange is given by Newton's law of cooling

$$\dot{Q} = hA\left(T_S - T_\infty\right) = 2632\left(70-25\right) = 118.4\ \mathrm{kW}.$$

<u>Justification for using Colburn analogy: 3 points</u>

<u>Applying Colburn analogy formula (either in original or simplified form): 4 points</u>

Q.2B. [8 points] FLOW PAST A WALL



Consider the flow of air directly against a plane wall as shown in the adjacent figure (note: only the top half of the geometry is shown). As air hits the wall and turns around to flow past it, a steady two-dimensional laminar 'boundary layer' develops on the wall. If the velocity just outside the boundary layer is given by $v_x = Gx$, where G is a constant called the strain rate (s^{-1}), how will the momentum boundary layer thickness vary along the x –direction? If there is a temperature difference between the wall and air, how will the thermal boundary layer thickness vary along the x-direction?

Solution:

The boundary layer is the result of a balance between convection in the flow direction, and diffusion in the direction normal to the wall.

The time scale for momentum diffusion over a thickness δ_ν is $\dfrac{\delta_\nu^2}{\nu}$

The time scale for momentum to be moved by the fluid over a distance x in the flow direction is given by $\dfrac{x}{U}$.

But it is given that the velocity in the x direction increases with x as $G\,x$.

Hence $\dfrac{x}{U} \sim \dfrac{x}{Gx} \sim \dfrac{1}{G}$.

Equating the order of magnitude of the two time scales, we have

$$\frac{\delta_\nu^2}{\nu} \sim \frac{1}{G}$$

$$\delta_\nu \sim \sqrt{\frac{\nu}{G}}.$$

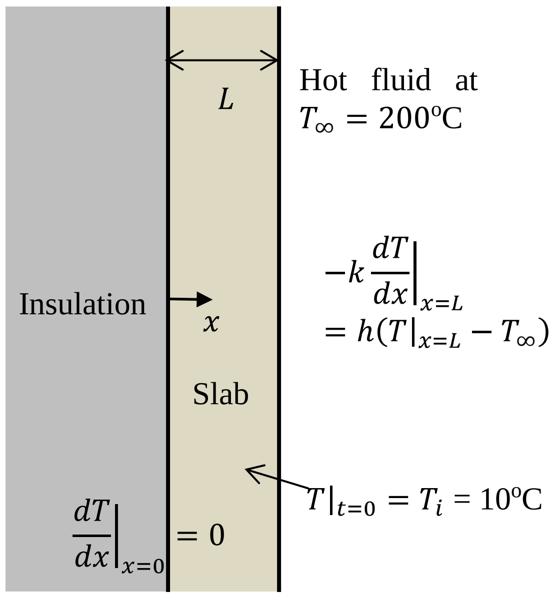
The boundary layer thickness does not change in the x direction.

Since the fluid is air, and the Prandtl number for air is close to 1, the thermal boundary layer thickness is the same as the momentum boundary layer thickness.

<u>Writing the scaling balance between convection time and diffusion time: 3 points</u>
<u>Momentum b.l. thickness : 3 points</u>
<u>Prandtl number argument : 1 point</u>
<u>Thermal b.l. thickness: 1 point</u>

Q.3. [15 points] TRANSIENT CONDUCTION

Consider a thin, plane stainless steel slab of thickness L=5 cm and surface area 4 m^2, density ρ = 8010 kg/m^3 and specific heat capacity C = 490 J/kg^{o}C. The face of the slab at x = 0 is insulated i.e. $\left.\frac{dT}{dx}\right|_{x=0} = 0$, while the other face is exposed to a hot fluid at 200^{o}C. The slab is initially at a uniform temperature of 10^{o}C. After 4 min, the temperatures at x = 0 and x = L are 45.9^{o}C and 90.4^{o}C, respectively. Answer the following questions:



(a) **[12 points]** Calculate the thermal diffusivity and thermal conductivity of the slab, and the convective heat transfer coefficient.

Solution:

The two pieces of information we have are

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{45.9 - 200}{10 - 200} = 0.8111$$

$$\theta_S = \frac{T_S - T_\infty}{T_i - T_\infty} = \frac{90.4 - 200}{10 - 200} = 0.5768.$$

Assuming that the 1 term approximation hold, we can write, for the plane wall geometry,

$\theta_0 = A_1 \exp\left(-\lambda_1^2 t^*\right)$ and $\theta_S = A_1 \exp\left(-\lambda_1^2 t^*\right)\cos\lambda_1.$

Taking the ratio of the two we get,

$$\cos\lambda_1 = \frac{\theta_S}{\theta_0} = 0.7111.$$

This gives $\lambda_1 = 0.7797$.

From the table 11-2, for a plane wall, for $\lambda_1 = 0.7797$,we pick the values of Bi = 0.775 and A_1 = 1.0991.

$$t^* = \frac{1}{\lambda_1^2}\ln\left(\frac{A_1}{\theta_0}\right) = \frac{1}{0.7797^2}\ln\left(\frac{1.0991}{0.8111}\right) = 0.4998.$$

Since t*>0.2, the one term approximation is justified.

Since $t^* = \dfrac{t}{L^2/\alpha}$, this provides the thermal diffusivity is

$$\alpha = \frac{L^2 t^*}{t} = \frac{0.05^2 \times 0.4998}{4\times 60} = 5.206\times 10^{-6}\ \text{m}^2/\text{s}.$$

The thermal conductivity is

$$k = \alpha\rho C = 5.206\times 10^{-6} \times 8010 \times 490 = 20.43\ \text{W/m}^{\circ}\text{C}.$$

To get the heat transfer coefficient we use the Biot number, $Bi = \dfrac{hL}{k}$.

$$h = 0.775\frac{k}{L} = 0.775\frac{20.43}{0.05} = 316.7\ \text{W/m}^2\,^{\circ}\text{C}.$$

<u>Selection of the correct length scale for transient analysis: 3 points</u>

<u>Application of one term approximation at x=0 and x=L: 2 points</u>

<u>Calculation of lambda1: 1 point</u>

<u>Getting Bi and A1: 2 points</u>

<u>Getting t*: 1 point</u>

<u>Getting alpha: 1 point</u>

<u>Getting k: 1 point</u>

<u>Getting h: 1 point</u>

(b) **[3 points]** What is the total heat exchanged between the wall and the fluid?

The maximum amount of heat exchangeable, Q_{max}, is $Q_{max} = \rho(LA)C(T_\infty - T_i)$

$$\frac{Q_{max}}{A} = \rho LC(T_\infty - T_i) = 37.28\ \text{MJ/m}^2.$$

The amount actually exchanged is given by

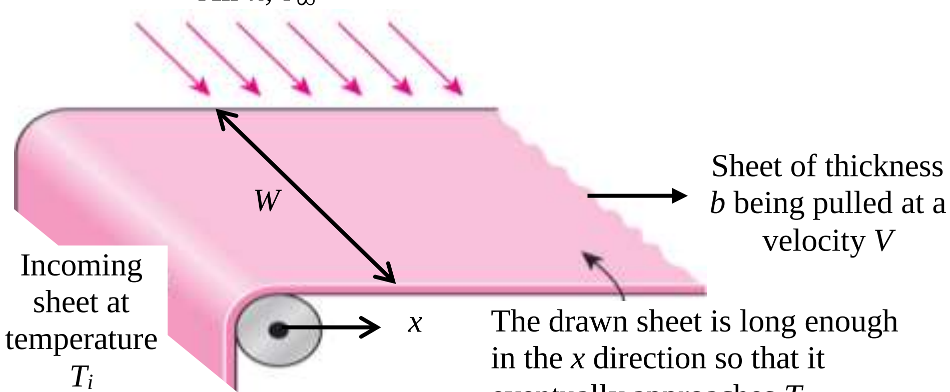
$$\frac{Q}{Q_{max}} = 1 - \theta_0 \frac{\sin\lambda_1}{\lambda_1} = 1 - 0.8111 \frac{\sin 0.7797}{0.7797} = 0.2686$$

Thus Q/A = 10.01 MJ/m^2.

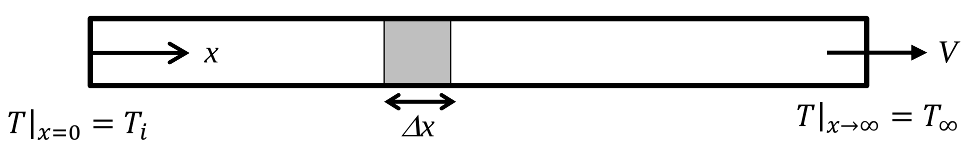
<u>Calculation of Qmax: 1.5 points</u>

<u>Calculation of Q: 1.5 points</u>

Q.4. [15 points] COOLING DOWN A SHEET METAL OR PLASTIC



(a) Schematic



(b) Section

In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment. Typically the process is a continuous one, with a sheet of thickness b and width W pulled out at a velocity V by rollers [see Fig. (a)]. The cooling is carried out by a moving airstream at a temperature T_∞. Assume that heat transfer coefficient corresponding to the heat exchange between the sheet and the air is constant at all locations on the sheet and equal to h. Also assume that the sheet being pulled is long enough that the temperature of the sheet approaches the ambient temperature T_∞ for asymptotically large *x*.

(a) **[4 points]** Perform a steady state energy balance over an element of the sheet thickness Δx [see figure (b)]. Ignore temperature variations over the thickness of the sheet. Take the limit as $\Delta x \to 0$ to obtain the following ordinary differential equation (ODE): $k\dfrac{d^2T}{dx^2} - \rho CV\dfrac{dT}{dx} - \dfrac{2h}{b}\left(T - T_\infty\right) = 0.$

Solution:

We can do a shell balance

IN = OUT

$$\dot{q}_x Wb\big|_{x=x} = \dot{q}_x Wb\big|_{x=x+\Delta x} + 2h\Delta x W\left(T - T_\infty\right).$$

Here $\dot{q}_x$ is the flux in the positive *x* direction. The factor of 2 appears because heat exchange is occurring on both the top and bottom surfaces of the sheet.

Rearranging and dividing by Δx,

$$\frac{\left(\dot{q}_x\big|_{x=x+\Delta x} - \dot{q}_x\big|_{x=x}\right)}{\Delta x} = -\frac{2h}{b}\left(T - T_\infty\right).$$

Taking the limit as Δx goes to zero, one gets

$$\frac{d\dot{q}_x}{dx} = -\frac{2h}{b}\left(T - T_\infty\right).$$

The flux comprises convective and conductive contributions:

$$\dot{q}_x = \left[U_{ref} + C\left(T - T_{ref}\right)\right]\rho V - k\frac{dT}{dx} = \left(U_{ref} - CT_{ref}\right)\rho V + \rho CVT - k\frac{dT}{dx}.$$

Here U_{ref} is the internal energy per unit mass at the reference temperature T_{ref}. Since ρ, C, V, U_{ref} and T_{ref} are constants,

$$\rho CV\frac{dT}{dx} - k\frac{d^2T}{dx^2} = -\frac{2h}{b}\left(T - T_\infty\right), \text{ or}$$

$$k\frac{d^2T}{dx^2}-\rho CV\frac{dT}{dx}-\frac{2h}{b}\left(T-T_\infty\right)=0.$$

<u>Correct shell balance: 2 points</u>

<u>Correct flux expression: 1 point</u>

<u>Simplification to ODE: 1 point</u>

(b) **[6 points]** Given the boundary conditions $T\big|_{x=0}=T_i$ and $T\big|_{x\to\infty}=T_\infty$, scale the ODE and the boundary conditions to obtain their dimensionless versions. You will get two possible choices for the length scale in the x direction. Which two terms of the governing equation are you equating when you make either choice? How many dimensionless parameters do you get, and in what limits of these dimensionless parameters are the two possible choices for the length scale valid?

<u>NOTE</u>: In normalizing the ODE, use the prefactor or scale of the term corresponding to Newton's law of cooling, as this term is always significant in determining the temperature distribution for this problem. [**Bonus:** 2 points – Why is the last statement true?]

Solution:

We define the following dimensionless variables

$$x^*=\frac{x}{L},\quad \theta=\frac{T-T_\infty}{T_i-T_\infty}.$$

Note that there is no geometric length scale available in the x direction.

$$k\frac{\left(T_i-T_\infty\right)}{L^2}\frac{d^2\theta}{dx^{*2}}-\rho CV\frac{\left(T_i-T_\infty\right)}{L}\frac{d\theta}{dx^*}-2\frac{h}{b}\left(T_i-T_\infty\right)\theta=0.$$

Dividing by the prefactor $\frac{h}{b}\left(T_i-T_\infty\right)$ of the Newton's law of cooling term, we get

$$\frac{kb}{hL^2}\frac{d^2\theta}{dx^{*2}}-\frac{\rho CVb}{hL}\frac{d\theta}{dx^*}-2\theta=0.$$

We now have two choices for L. We can either choose $\frac{kb}{hL^2}=1$, which gives $L=\sqrt{\frac{kb}{h}}$.

Substituting into the equation, we get

$$\frac{d^2\theta}{dx^{*2}}-\beta\frac{d\theta}{dx^*}-2\theta=0, \text{ where } \beta=\frac{\rho CV\sqrt{b}}{\sqrt{hk}}.$$

This is actually a Peclet number

$$\beta=\frac{\rho CV\sqrt{b}}{\sqrt{hk}}=\frac{V\sqrt{kb/h}}{\frac{k}{\rho C}}.$$

The length scale used to define the Peclet number is $\sqrt{kb/h}$.

This suggests that the length scale for the temperature to drop from Ti to T∞ is $\sqrt{\frac{kb}{h}}$, provided $\beta=\frac{\rho CV\sqrt{b}}{\sqrt{hk}}<<1$.

Let us exercise the second choice now by setting $\frac{\rho CVb}{hL}=1$, which gives

$$L=\frac{\rho CVb}{h}.$$

The prefactor of the conductive term then becomes

$$\frac{kb}{hL^2}=\frac{kb}{h\left(\frac{\rho CVb}{h}\right)^2}=\frac{1}{\beta^2}.$$

The differential equation becomes

$$\frac{1}{\beta^2}\frac{d^2\theta}{dx^{*2}}-\frac{d\theta}{dx^*}-2\theta=0.$$

This suggests that the length scale for the temperature to decrease from Ti to T∞ is $L=\frac{\rho CVb}{h}$, provided β>>1.

Thus, in either case, there is only one dimensionless parameter

$\beta = \dfrac{\rho CV\sqrt{b}}{\sqrt{hk}}$, which actually is the ratio of the two length scales we obtained $\dfrac{\rho CVb}{h}$ and $\sqrt{\dfrac{kb}{h}}$.

<u>Scaling of T and x : 1 point</u>

<u>Subbing into ODE and normalizing: 1 point</u>

<u>Option 1: 1 point</u>

<u>Option 2: 1 point</u>

<u>Dimensionless parameter: 1 point</u>

(c) **[2 points]** Consider the case when the sheet is made of aluminium (k = 237 W/m°C, ρ =2700 kg/m^3, C = 900 J/kg°C). If b = 2 mm, h = 41 W/m^2 °C, V = 10 μm/s. what is the approximate length scale in the x direction over which the temperature will fall to the ambient air temperature? Which two terms in the governing equation balance each other to produce this length scale? What would your responses change to, if V = 0.1 m/s, everything else remaining the same?

Solution:

We first calculate $\beta = \dfrac{\rho CV\sqrt{b}}{\sqrt{hk}} = \dfrac{2700\times 900\times 10\times 10^{-6}\sqrt{2\times 10^{-3}}}{\sqrt{41\times 237}} = 0.01 << 1$

The length scale will then be governed by a balance between the conductive term and Newton's law of cooling, given by $\sqrt{\dfrac{kb}{h}} = \sqrt{\dfrac{237\times 2\times 10^{-3}}{41}} = 0.11$ m.

If V=0.1 m/s , i.e. 4 orders of magnitude higher, β becomes 100>>1

The length scale will then be governed by a balance between the convective term and Newton's law of cooling, given by $\dfrac{\rho CVb}{h} = \beta\sqrt{\dfrac{kb}{h}} \approx 1100$ m.

<u>Calculation for V=10 um/s and length scale: 1 point</u>

<u>Calculation for V=0.1 m/s and length scale: 1 point</u>

(d) **[4 points]** Integrate the original dimensional governing equation and apply the boundary conditions to get the temperature distribution. Examine the solution and determine the length scale over which the temperature decreases to the ambient value (this will be the inverse of the coefficient of x in the exponent). Is this scale consistent with the conclusions in part (c)?

Solution:

$$k\frac{d^2T}{dx^2} - \rho CV\frac{dT}{dx} - \frac{2h}{b}\left(T - T_\infty\right) = 0.$$

or $$k\frac{d^2\left(T - T_\infty\right)}{dx^2} - \rho CV\frac{d\left(T - T_\infty\right)}{dx} - \frac{2h}{b}\left(T - T_\infty\right) = 0.$$

This is a homogeneous, linear, 2nd order ODE for $\left(T - T_\infty\right)$ with constant coefficients. The solutions are exponentials.

The auxiliary equation is $kr^2 - \rho CVr - \frac{2h}{b}r = 0$. Its roots are

$$r_\pm = \frac{\rho CV \pm \sqrt{\left(\rho CV\right)^2 + 4\frac{2h}{b}k}}{2k}.$$

The solution is, therefore, $T - T_\infty = c_1 \exp\left(r_+ x\right) + c_2 \exp\left(r_- x\right)$.

Note that $r_- < 0$ and $r_+ > 0$. Since the temperature is finite as $x \to \infty$, c_1 has to be set to 0.

$$T - T_\infty = c_2 \exp\left(r_- x\right).$$

Applying the condition, $T\big|_{x=0} = T_i$, we get $c_2 = T_i - T_\infty$.

$$T - T_\infty = \left(T_i - T_\infty\right)\exp\left(r_- x\right).$$

The length scale for temperature change is the inverse of the exponent

$$l = \frac{1}{-r_-} = \frac{2k}{\sqrt{\left(\rho CV\right)^2 + 4\frac{2h}{b}k} - \rho CV} = \frac{2k}{\sqrt{\beta^2\frac{hk}{b} + 4\frac{2h}{b}k} - \beta\sqrt{\frac{hk}{b}}}$$

$$= \frac{2k\sqrt{\frac{b}{hk}}}{\sqrt{\beta^2 + 8} - \beta} = \frac{2\sqrt{\frac{kb}{h}}}{\sqrt{\beta^2 + 8} - \beta}.$$

When $\beta << 1,\; l \approx \frac{2\sqrt{\frac{kb}{h}}}{\sqrt{8}} \sim \sqrt{\frac{kb}{h}}$, as predicted by the scaling analysis.

When $\beta << 1,$
$$l = \frac{2\sqrt{\frac{kb}{h}}}{\sqrt{\beta^2+8}-\beta}\frac{\left(\sqrt{\beta^2+8}+\beta\right)}{\left(\sqrt{\beta^2+8}+\beta\right)} = \frac{2\sqrt{\frac{kb}{h}}}{8}\left(\sqrt{\beta^2+8}+\beta\right)$$
$$\approx \frac{1}{4}\sqrt{\frac{kb}{h}}2\beta \sim \beta\sqrt{\frac{kb}{h}} = \frac{\rho CVb}{h},$$

which is again consistent with the scaling analysis.

<u>Solution of ODE with bc application: 2 points</u>

<u>Limit of beta<<1: 1 point</u>

<u>Limit of beta>>1: 1 point</u>

Q.5. [15 points] HEATING OF A COPPER SPHERE

A pure copper sphere of radius 5 mm and an emissivity of 0.45 is suspended in a large furnace, with walls at a uniform temperature of 500^{o}C surrounding the sphere. Air flows over the sphere at a temperature of 900^{o}C and a velocity of 8 m/s. What is the steady state temperature of the sphere?

In your calculations, determine the convective heat transfer coefficient using the following properties: μ_∞= 4.67 × 10^{-5} N·s/m^2, ν = 0.000157 m^2/s, k = 0.075 W/m·K, Pr = 0.73; Air: μ_s = 4.27 × 10^{-5} N·s/m^2. The Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4.

Note: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$.

Solution:

Since the air is at 900^{o}C, and the furnace walls are at 500^{o}C, the steady state temperature of sphere will intermediate to these temperatures. At steady state, the rate of heat gain experienced by the sphere due to the convective heat transfer with the surrounding air will be balanced exactly by the rate of heat loss due to radiation heat losses to the furnace.

In mathematical terms, the energy balance at steady state is

$$h\left(T_\infty - T_S\right) = \varepsilon\sigma\left(T_S^4 - T_{surr}^4\right).$$

To get the heat transfer coefficient, we use the correlation

$$Nu = \frac{hD}{k} = 2 + \left(0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right)\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_S}\right)^{1/4}$$

The Reynolds number is $\mathrm{Re} = \frac{UD}{\nu} = \frac{8 \times 10 \times 10^{-3}}{0.000157} = 509.6$

This provides Nu = 13.59.

$$h = 13.59\frac{k}{D} = 101.9 \text{ W/m}^2\,{}^o\text{C}.$$

Substituting the values into energy balance equation, we get

$$101.9 \times \left(900 - x\right) = 0.45 \times 5.67 \times 10^{-8}\left[\left(x + 273.16\right)^4 - \left(500 + 273.16\right)^4\right]$$

where *x* is the surface temperature in deg C.

This is a nonlinear equation in *x*. We can use the Newton iterative method to solve this.

This gives $x = 733^o$C.

The temperature of the sphere is 733^oC.

<u>Writing the energy balance at steady state: 5 points</u>

<u>Selecting the appropriate formula for Nu: 2 points</u>

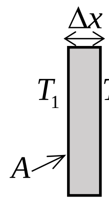
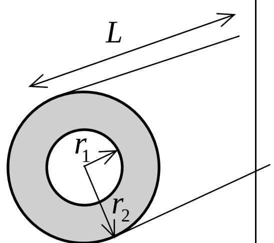
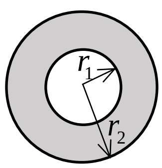
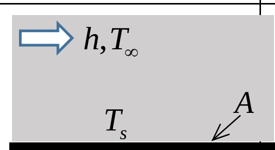
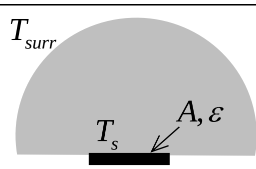
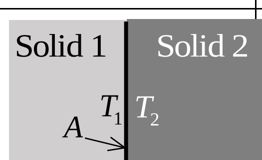
<u>Calculation of Re: 1 point</u>

<u>Calculation of Nu: 2 points</u>

<u>Calculation of h: 1 point</u>

<u>Solving for the surface temperature 4 points</u>

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\Delta x}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$
Spherical shell (Hollow sphere)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \frac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \frac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \frac{1}{\varepsilon\sigma A\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)}$
Thermal contact resistance		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{R_c}{A}$ (R_c has units of °C-m²/W)

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C\frac{\partial T}{\partial t}=\nabla\cdot\left(k\nabla T\right)+\dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_x=-k\frac{\partial T}{\partial x},\quad \dot{q}_y=-k\frac{\partial T}{\partial y},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

Constant *k*: $\rho C\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{S}$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

Constant *k*: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\dot{S}$

SPHERICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_\phi=-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$

Constant *k*: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right]+\dot{S}$

TABLE 11–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x / L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r / r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x / L)}{\lambda_n x / L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

TABLE 11–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres ($\text{Bi} = hL/k$ for a plane wall of thickness $2L$, and $\text{Bi} = hr_o/k$ for a cylinder or sphere of radius r_o)

	Plane Wall		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 11–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Total heat transferred:

$$\frac{Q}{Q_{max}} = 1-\theta_0 \frac{\sin\lambda_1}{\lambda_1} \qquad \text{Plane wall}$$

$$= 1-2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1} \qquad \text{Cylinder}$$

$$= 1-3\theta_0 \frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} \qquad \text{Sphere}$$

Here, $Q_{max} = mC(T_\infty - T_i)$, where *m* and *C* and the mass and the specific heat capacity, respectively, of the solid. θ_0 is a dimensionless center temperature.

NUSSELT NUMBER CORRELATIONS

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\mathrm{Re}_{x,\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5\times10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5\times10^5$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Pr} > 0.6$$

Turbulent: $C_{f,x} = \dfrac{0.059}{\mathrm{Re}_x^{1/5}}, \quad 5\times10^5 \le \mathrm{Re}_x \le 10^7$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}, \quad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}}, \quad \mathrm{Re}_L < 5\times10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Combined: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} - \dfrac{1742}{\mathrm{Re}_L}, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62\log\dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\mathrm{Nu} = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Re}_L < 5\times10^5$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}, \quad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}, \quad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\ \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\ \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \dfrac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$

Turbulent: $h = \dfrac{5[1-(\xi/x)^{9/10}]}{(1-\xi/L)}\,h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux,* the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.