Q2: (parts (a) and (b) are related; part (c) is separate)

a) The following (x, y) data points have been collected from an experiment:

$$\{(-2,4), (-1,2), (0,-1), (1,0), (2,0)\}$$

Without doing any calculations, give a column picture answer to the following question.

What condition must be satisfied in order that these 5 data points lie on a straight line, $y = cx + d$, where c and d are unknown constants? (Hint: your answer should involve $5x1$ column vectors.)

b) Find the least squares line of best fit $y = cx + d$ to these five data points, where c and d are unknown constants.

c) Consider the following (x, y) data points:

$$\{(0,0), (0,8), (0,8), (0,20)\}$$

The objective is to find the best horizontal line that fits these 4 data points, i.e., $y = d$, where d is an unknown constant. Find the $4x1$ matrix A and the $4x1$ vector $\vec{b}$ that are needed to solve for the least squares estimate of d (i.e., d_{LS}) in the associated normal equations $A^T A\vec{x}_{LS} = A^T\vec{b}$. Solve for d_{LS}.

Solutions:

a) $A\vec{x} = \begin{bmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ +1 & 1 \\ +2 & 1 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \vec{b} = \begin{bmatrix} +4 \\ +2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

Therefore, for these five data points to lie on a straight line, there must exist a linear combination of $\begin{bmatrix} -2 \\ -1 \\ 0 \\ +1 \\ +2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ that equals $\begin{bmatrix} +4 \\ +2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$.

b) $A^T A = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$; $A^T \vec{b} = \begin{bmatrix} -10 \\ +5 \end{bmatrix}$

$\vec{x}_{LS} = (A^T A)^{-1} A^T \vec{b} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} -10 \\ +5 \end{bmatrix} = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$

Therefore, the line of best fit is given by $y = -x + 1$.

c) $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [d] = \vec{b} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 20 \end{bmatrix}$

$A^T A = 4$; $A^T \vec{b} = 36$

$d_{LS} = \frac{36}{4} = 9$