CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

23rd NOVEMBER 2017

NAME:

STUDENT ID NUMBER:

Q1	Q2A	Q2B	Q2C	Q3	Q3 bonus	Total
15	6	5	7	17	6	50 (with bonus, 56)

INSTRUCTIONS

1. <u>This examination is closed book. Only one Letter-sized aid sheet is permitted</u>.
2. Only type 3 calculators are permissible.
3. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
4. Answers written in pencil will NOT be re-marked. This is University policy.
5. For all problems, you must present the solution process in a step by step fashion for partial marks.
6. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY. THIS WILL BE IMPLEMENTED STRICTLY THIS YEAR.**

Q.1. [15 points] HEAT LOSSES THROUGH AN INSULATED STEAM PIPE

A 12 cm OD by 9 cm ID cylindrical steam line delivers superheated steam at 500^{o}C. The line is steel wrapped with 10 cm thick cylindrical shell of asbestos, and 1 cm of cylindrical plaster shell over the asbestos. The thermal conductivities of steel, asbestos and plaster are 14 W/m-K, 0.156 W/m-K and 0.107 W/m-K, respectively. The convective heat transfer coefficients for the steam side and the air side are 3000 W/m^2-K and 10 W/m^2-K, respectively. Using the thermal resistance approach, determine the rate of heat loss per unit length of the cylindrical steam line at steady state, and the surface temperature of the plaster layer. Account for radiative losses from the surface as well, assuming a plaster emissivity of 0.9, and that the 'surrounding' surface receiving the radiation is at a temperature of 10^{o}C. The ambient air temperature is also 10^{o}C. The Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4. <u>Be sure to draw the thermal circuit for this problem.</u>

Note: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$.

Solution:

Assume a pipe length of L = 1 m.

The resistance network is shown on the right.

The various resistances are as follows:

$$R_{\text{conv}_\text{i}} = \frac{1}{h_i \pi D_i L} = \frac{1}{3000\times\pi\times9\times10^{-2}\times1} = 1.179\times10^{-3}\ ^{o}\text{C/W}.$$

$$R_{\text{steel}} = \frac{1}{2\pi k_{\text{steel}} L}\ln\left(\frac{D_o}{D_i}\right) = \frac{1}{2\pi\times14\times1}\ln\left(\frac{6\times10^{-2}}{4.5\times10^{-2}}\right) = 3.270\times10^{-3}\ ^{o}\text{C/W}.$$

$$R_{\text{asb}} = \frac{1}{2\pi k_{\text{asb}} L}\ln\left(\frac{r_o}{r_i}\right) = \frac{1}{2\pi\times0.156\times1}\ln\left(\frac{16\times10^{-2}}{6\times10^{-2}}\right) = 1.000\ ^{o}\text{C/W}.$$

$$R_{\text{plas}} = \frac{1}{2\pi k_{\text{plas}} L}\ln\left(\frac{r_o}{r_i}\right) = \frac{1}{2\pi\times0.107\times1}\ln\left(\frac{17\times10^{-2}}{16\times10^{-2}}\right) = 9.017\times10^{-2}\ ^{o}\text{C/W}.$$

The total resistance prior to the heat transfer off the surface of the plaster is

$R_{\text{eff}1} = R_{\text{conv}_\text{i}} + R_{\text{steel}} + R_{\text{asb}} + R_{\text{plas}}$

= 1.095 ^{o}C/W.

An energy balance at steady state yields

$$\dot{Q} = \frac{(T_i - T_S)}{R_{\text{eff}1}} = h_o A_o (T_S - T_\infty) + \sigma \varepsilon A_o \left[(T_S + 273.16)^4 - (T_{surr} + 273.16)^4 \right]$$

where all temperatures are in degrees Celcius.

Substituting the known values, we get

$$\dot{Q} = \frac{(500 - T_S)}{1.095}$$

$$= 10\left(2\pi \times 17 \times 10^{-2} \times 1\right)\left(T_S - 10\right) + 5.67 \times 10^{-8} \times 0.9 \times \left(2\pi \times 17 \times 10^{-2} \times 1\right)\left[\left(T_S + 273.16\right)^4 - \left(10 + 273.16\right)^4 \right]$$

Rearranging the equation and evaluating all the prefactors, we get

$$11.70(T_S - 10) + 5.969 \times 10^{-8}\left[(T_S + 273.16)^4 - 283.16^4 \right] - (500 - T_S) \quad = 0$$

or

$$12.70 T_S + 6.8093 \times 10^{-8} (T_S + 273.16)^4 - 1001 \quad = 0 .$$

The derivative of the function on the left is $12.70 + 5.969 \times 10^{-8} \times 4(T_S + 273.16)^3$.

Applying the Newton Raphson method, we get Ts =32.2^{o}C. The rate of heat transfer is

$$\dot{Q} = \frac{(500 - 32.2)}{1.095} = 427.2 \text{ W} .$$

<u>1.5 points each for Rconvi, Rsteel, Rasb, Rplas (6 points in all)</u>

<u>1 point for effective resistance prior to surface.</u>

<u>3 points for the energy balance at steady state</u>

<u>Nonlinear equation in Ts : 2 points.</u>

<u>Solution of nonlinear equation in Ts : 2 points.</u>

<u>Rate of heat transfer : 1 points.</u>

Q.2A. [6 points] COOLING OF CYLINDERS

Two copper cylinders, each at 75°C initially, are allowed to cool in a water bath containing ice. The temperature of the bath is 0°C. Cylinder 1 is twice as tall as cylinder 2 but both cylinders are the same diameter. The top and bottom of each cylinder are perfectly insulated so the only heat loss is through the sides of the cylinders. Answer the following questions:

(a) Which cylinder will melt more ice? Explain.

(b) Which cylinder will initially melt ice at a faster rate? Explain.

(c) For which cylinder does the radial temperature profile change faster? Explain.

Assume that the heat transfer coefficient is not affected by the size of the cylinder, and that there is sufficient ice to keep the bath temperature at 0^{o}C.

Solution:

(a) More ice will be melted when more heat is transferred. Since the mass of the longer cylinder is larger, it has more stored energy and will melt more ice.
<u>Answer with proper reasoning: 2 points</u>

(b) The rate of melting of ice depends on the rate of heat transfer. According to Newton's law of cooling, the rate of heat transfer is proportional to the surface area of the cylinder, which is greater for the longer cylinder. Hence the longer cylinder will show a greater rate of melting than the shorter one.
<u>Answer with proper reasoning: 2 points</u>

(c) Since the top and bottom are insulated, the temperature profile will be independent of the axial position and depend only the radial co-ordinate and time. Hence both cylinders will show the same temperature profile with respect to time.
<u>Answer with proper reasoning: 2 points</u>

Q.2B. [5 points] COFFEE BURN

Mr. X. is seated in a coffee shop, holding a cylindrical coffee mug of inner diameter 7 cm and outer diameter 8 cm. The mug does not have a handle, so Mr. X is holding the outer surface of the mug in the cup of his right palm. Mr. X has finished drinking his coffee; the mug is empty and at room temperature. While he is talking to his friend, unknown to him, the waiter comes from behind and fills the cup with boiling hot coffee, as it is a 'free-refill' day. After how much time, approximately, will he 'feel the heat'? The thermal conductivity, specific heat capacity and density of the mug are 2 W/m^{o}C, 1400 J / kg^{o}C and 2600 kg/m^3 respectively.

Solution:

The thermal diffusivity of the material is $\alpha = \dfrac{2}{1400 \times 2600} = 5.495 \times 10^{-7}$ m^2/s.

The diffusion time over the thickness of the wall of the mug is

$$t_D = \frac{L^2}{\alpha} = \frac{\left[(8-7)\times 10^{-2} / 2\right]^2}{5.495 \times 10^{-7}} \approx 45 \text{ sec.}$$

<u>Identifying that this is a thermal diffusion time problem: 3 points</u>
<u>Calculation of thermal diffusivity: 1 point</u>
<u>Calculation of diffusion time: 1 point</u>

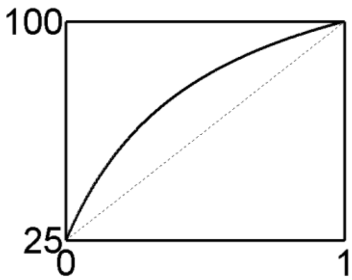
Q.2C. [7 points] STEADY STATE TEMPERATURE PROFILES

Consider a slab of thickness 1 unit maintained at a temperature of 25^{o}C on one face and 100^{o}C on the other. There are no heat sources or sinks in the slab. The thermal conductivity of the material can be a function of temperature. For subfigures (a) to (e), explain how the thermal conductivity should vary with temperature to achieve the *steady state* temperature distributions depicted in these subfigures. (The dotted curve in each subfigure is a straight line).

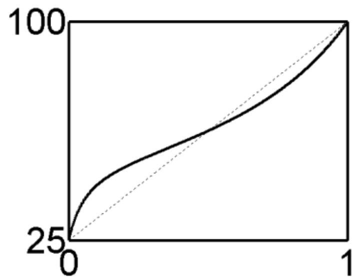
Solution:

For steady state through a slab, the heat flux is a constant at every position in the slab, i.e. the magnitude of *k* times the magnitude of *dT*/*dx* is a constant at every position of the slab.

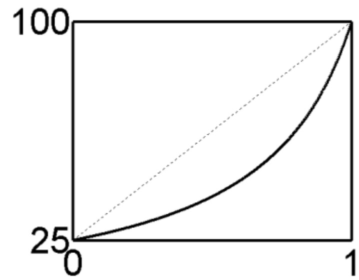
<u>Recognizing that k*abs(dT/dx) is constant for the slab geometry: 2 points</u>



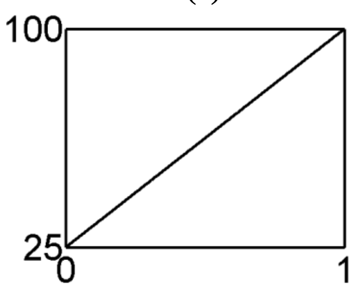
(a)



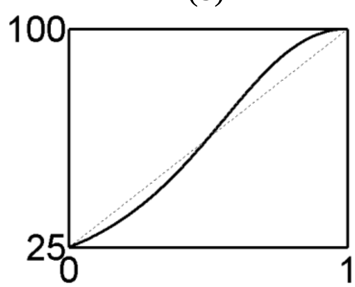
(b)



(c)



(d)



(e)

In subfigure (a), as the temperature increases from left to right, dT/dx decreases, or k is increasing. Hence *k* is an increasing function of temperature: <u>1 point</u>

In subfigure (b), as the temperature increases from left to right, dT/dx decreases and then increases, or k increases and then decreases. Thus, *k* shows a maximum with temperature: <u>1 point</u>

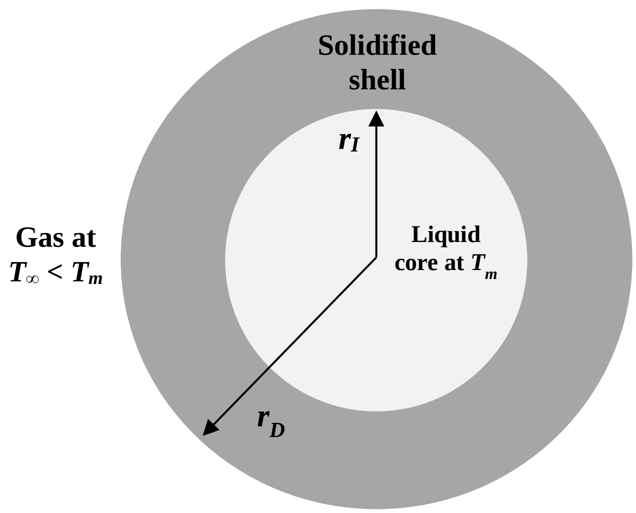
In subfigure (c), as the temperature increases from left to right, dT/dx increases, or k decreases. k is a decreasing function of temperature: <u>1 point</u>

In subfigure (d), as the temperature increases from left to right, dT/dx is constant, hence k is constant: <u>1 point</u>

In subfigure (e), as the temperature increases from left to right, dT/dx increases and then decreases, or k decreases and then increases. Thus, k shows a minimum with temperature. <u>1 point.</u>

Q.3. [17 points] SOLIDIFICATION OF A LIQUID DROP

In this problem, you will examine a highly simplified version of a mathematical model that determines the time required to completely solidify a liquid drop.



Consider a spherical liquid drop of radius r_D that is initially entirely at the melting temperature, T_m. At time t = 0, it is suspended in a flowing gas at a colder temperature $T_\infty < T_m$. As a result, the drop begins to solidify beginning from its outer surface at $r = r_D$, leading to a spherical, solidified shell or crust whose inner inner radius r_I becomes smaller and smaller with time. (see Figure above). During the inward propagation of the solidification front $r = r_I$, the latent heat released is conducted outwards from the front $r = r_I$ and through the solid spherical shell, and finally released into the gas. Eventually, when r_I is 0, the solidification process is complete.

To model this process, we assume that the solidification process (the rate at which r_I decreases with time) is slow relative to the rate of conduction of energy through the solidified shell. This is the *pseudo steady state* approximation, whereby steady state expressions for heat transfer through the spherical shell can be used. We also assume that

(a) the shape of the solidifying object remains spherical,

(b) the problem is spherically symmetric (depends on one spatial co-ordinate, r)

(c) all physical properties (density ρ, thermal conductivity k, specific heat C, etc.) of the solidifying object are constant within the object and with time,

(d) the temperature of liquid core remains constant at T_m, and

(e) the convective heat transfer resistance at the solid-gas interface is negligible.

Answer the following questions:

(a) **[7 points]** Beginning with the governing equation for energy balance in solids, use the pseudo-steady state approximation to determine the temperature distribution in the solid spherical shell $\left(r_I \le r \le r_D\right)$. Write the governing equation and boundary conditions clearly. Determine the rate of heat transfer, $\dot{Q}$ (W), through the shell. <u>Note: You will get *zero* credit if you use the resistance approach</u>. [**Hint:** See assumption (e) above.]

Solution:

At steady state, in the absence of a volumetric heat source in $r_I \le r \le r_D$, due to spherical symmetric nature of the temperature distribution i.e. $T = T\left(r\right)$, only one term survives in the governing equation:

$$\frac{k}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0.$$

<u>The correct governing equation: 2 points (1 point taken off if source term is included)</u>

The boundary conditions are $T\big|_{r=r_I} = T_m$ and $T\big|_{r=r_D} = T_\infty$.

<u>The correct boundary conditions: 1 point</u>

Integrating the governing equation twice, we get $T = \frac{c_1}{r} + c_2$.

<u>Integration: 1 point</u>

Applying the two bcs, we get $T_m = \frac{c_1}{r_I} + c_2$ and $T_\infty = \frac{c_1}{r_D} + c_2$.

Subtracting the two equations, we get c_1 as $c_1 = \frac{\left(T_m - T_\infty\right)}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}$.

Substituting this back into any one of the two b.cs gives c_2 as $c_2 = T_m - \frac{\left(T_m - T_\infty\right)}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}\frac{1}{r_I}$.

Correct constants of integration: 1 point

Substituting the constants into the temperature equation gives

$$T = T_m + \frac{\left(T_\infty - T_m\right)}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}\left(\frac{1}{r_I} - \frac{1}{r}\right).$$

Temperature profile: 1 point

The rate of heat transfer is

$$\dot{Q} = \left(-k\frac{dT}{dr}\right)4\pi r^2 \frac{\left(T_\infty - T_m\right)}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}\left(\frac{1}{r^2}\right) = \frac{4\pi k}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}\left(T_m - T_\infty\right).$$

Rate of heat transfer: 1 point

(b) **[2 points]** If the rate of change of the radius of the liquid core with time is $\frac{dr_I}{dt}$, what is the rate of change of the mass of the liquid core, $\dot{m}$, as the core shrinks, given that the density of the liquid is ρ (kg/m^3). [**Hint:** The volume of a sphere of radius r_I is $\frac{4}{3}\pi r_I^3$. Differentiate this with time.]

Solution:

$$\dot{m} = \rho\left(-\frac{dV}{dt}\right) = -\rho\frac{d}{dt}\left(\frac{4}{3}\pi r_I^3\right) = -\rho 4\pi r_I^2 \frac{dr_I}{dt}$$

1.5 points: Realizing that density * dV/dt is the rate of mass change with time

0.5 point: Final result

(c) **[2 points]** Given that the latent heat of melting of the solid is λ (J/kg), what is the rate of release of energy, $\dot{Q}$ (W), from the liquid core to the solid shell at the interface $r = r_I$?

Solution:

$$\dot{Q} = \dot{m}\lambda = -4\pi\rho\lambda r_I^2 \frac{dr_I}{dt}.$$

2 points

(d) **[2 points]** Invoke the pseudo-steady state approximation and write an energy balance at the interface $r = r_I$, equating the results in parts (a) and (c). You should get a first-order ODE for $r_I(t)$.

Solution:

$$\dot{Q} = \frac{4\pi k}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}(T_m - T_\infty) = \dot{m}\lambda = -4\pi\rho\lambda r_I^2 \frac{dr_I}{dt}.$$

1 point for equating results

The first order ODE is

$$-\rho\lambda r_I^2 \frac{dr_I}{dt} = \frac{k}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}(T_m - T_\infty).$$

1 point for the correct ODE

(e) **[4 points]** Integrate the first order ODE in part (d) to determine the total time, t_S, for complete solidification. If r_D = 1 cm, $T_m - T_\infty = 5$°C, k = 0.5 W/m°C, λ = 320 kJ/kg, ρ = 1000 kg/m^3, calculate the time t_S.

Solution:

$$-\rho\lambda r_I^2 \frac{dr_I}{dt} = \frac{k}{\left(\frac{1}{r_I} - \frac{1}{r_D}\right)}(T_m - T_\infty).$$

Separation of variables and integration gives

$$-\int_{r_D}^{0} \rho\lambda r_I^2 \left(\frac{1}{r_I} - \frac{1}{r_D}\right) dr_I = \int_0^{t_S} k(T_m - T_\infty)\,dt.$$

$$-\rho\lambda\left(\frac{r_I^2}{2} - \frac{r_I^3}{3r_D}\right)\Bigg|_{r_D}^{0} = k(T_m - T_\infty)t_S.$$

$$t_S = \frac{\rho\lambda}{k(T_m - T_\infty)}\left(\frac{r_D^2}{2} - \frac{r_D^3}{3r_D}\right), \quad \text{or} \quad t_S = \frac{\rho\lambda r_D^2}{6k(T_m - T_\infty)}.$$

Integration: 1.5 points

Application of correct limits: 1 point

Final result: 0.5 point

Given: r_D = 1 cm, $T_m - T_\infty = 5$°C, k = 0.5 W/m°C, λ = 320 kJ/kg, ρ = 1000 kg/m^3,

$$t_S = \frac{1000 \times 320000 \times \left(10^{-2}\right)^2}{6 \times 0.5 \times 5} = 2133 \text{ s or } 35 \text{ min.}$$

Calculation: 1 point

BONUS QUESTIONS [Do this only if you have time. The grading is binary.]

(a) **[3 points]** If the convective heat transfer coefficient is h, and convective heat transfer resistances at the solid-gas interface are not negligible, how is the time for complete solidification modified? You can use the resistance network approach for this part.

Solution:

$$-\rho\lambda\left(4\pi r_I^2\right)\frac{dr_I}{dt} = \frac{1}{\dfrac{1}{4\pi k}\left(\dfrac{1}{r_I} - \dfrac{1}{r_D}\right) + \dfrac{1}{h\left(4\pi r_D^2\right)}}\left(T_m - T_\infty\right).$$

$$-\int_{r_D}^{0} \rho\lambda r_I^2\left[\frac{1}{k}\left(\frac{1}{r_I} - \frac{1}{r_D}\right) + \frac{1}{h r_D^2}\right] dr_I = \int_0^{t_S}\left(T_m - T_\infty\right) dt.$$

$$t_S = \frac{\rho\lambda}{\left(T_m - T_\infty\right)}\int_0^{r_D}\left[\frac{1}{k}\left(r_I - \frac{r_I^2}{r_D}\right) + \frac{r_I^2}{h r_D^2}\right] dr_I$$

$$t_S = \frac{\rho\lambda}{\left(T_m - T_\infty\right)}\left[\frac{1}{k}\left(\frac{r_D^2}{2} - \frac{r_D^3}{3r_D}\right) + \frac{r_D^3}{3h r_D^2}\right] = \frac{\rho\lambda r_D}{6h\left(T_m - T_\infty\right)}\left(\frac{h r_D}{k} + 2\right).$$

3 points if and only if the expression time for solidification is correct.

(b) **[3 points]** Develop the mathematical criterion for the pseudo-steady state approximation to be valid, in the limits of small and large Biot numbers.

Solution:

For the pseudo-steady state approximation to be valid, the time scale for solidification needs to be much larger than the characteristic diffusion time for heat to be conducted out of the sphere. In equation form,

$$t_S >> t_D,$$

$$\frac{\rho\lambda r_D}{6h\left(T_m - T_\infty\right)}\left(\frac{hr_D}{k}+2\right) >> \frac{r_D^2}{k/\rho C},$$

$$\frac{\lambda k}{6r_D hC\left(T_m - T_\infty\right)}\left(\frac{hr_D}{k}+2\right) >> 1,$$

$$\frac{\lambda}{6\mathrm{Bi}C\left(T_m - T_\infty\right)}\left(\mathrm{Bi}+2\right) >> 1.$$

This is the mathematical criterion. For small Biot numbers, i.e. for $\frac{hr_D}{k} << 1$,

$$\frac{\lambda}{3\mathrm{Bi}C\left(T_m - T_\infty\right)} >> 1.$$

For large Biot numbers, i.e. for $\frac{hr_D}{k} >> 1$, $\frac{\lambda}{6C\left(T_m - T_\infty\right)} >> 1.$

<u>3 points if and only if the discussion and result are correct</u>