CHE-260, MIDTERM 2014

① For helium $\gamma = 1.667$

$$\frac{W}{m} = -\int_{v_1}^{v_2} P\,dv = -\int_{v_1}^{v_2} \frac{c}{v^\gamma}\,dv$$

$$= -\left[\frac{c\,v^{1-\gamma}}{1-\gamma}\right]_{v_1}^{v_2}$$

$$c = P_1 v_1^\gamma = P_2 v_2^\gamma$$

$$\frac{W}{m} = \frac{P_2 v_2 - P_1 v_1}{\gamma - 1} = \frac{R(T_2 - T_1)}{\gamma - 1}$$

Also, for an isentropic process

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

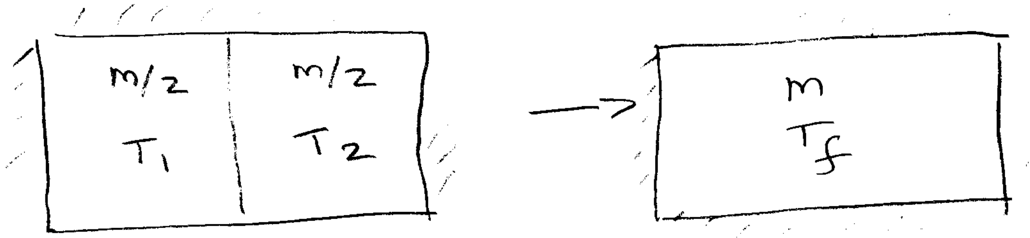
$$T_2 = 200\left(\frac{14}{1}\right)^{\frac{0.667}{1.667}} = 575\ K$$

For helium $R = 2.077$ kJ/kg K

$$\frac{W}{m} = \frac{2.077\ (575 - 200)\ K}{1.667 - 1}$$

$$= 1167.7\ \text{kJ/kg}$$

(2)



$$\underbrace{Q}_{0} + \underbrace{W}_{0} = \Delta U = 0$$

$$\Rightarrow \frac{m}{2} c \left[T_f - T_1 \right] + \frac{m}{2} c \left[T_f - T_2 \right] = 0$$

$$\Rightarrow T_f = \frac{T_1 + T_2}{2}$$

$$S_{gen} = m s_f - \left[\frac{m}{2} s_1 + \frac{m}{2} s_2 \right]$$

$$= \frac{m}{2} \left[s_f - s_1 \right] + \frac{m}{2} \left[s_f - s_2 \right]$$

For water $s_f - s_1 = c \ln \dfrac{T_f}{T_1}$

$$s_f - s_2 = c \ln \frac{T_f}{T_2}$$

$$\Rightarrow S_{gen} = \frac{m}{2} c \left[\ln \frac{T_f}{T_1} + \ln \frac{T_f}{T_2} \right]$$

$$= m c \ln \left(\frac{T_f{}^2}{T_1 T_2} \right)^{1/2}$$

$$= m c \ln \left[\frac{T_f}{\sqrt{T_1 T_2}} \right]$$

$$S_{gen} = m c \ln \left[\frac{T_1 + T_2}{2 \sqrt{T_1 T_2}} \right]$$

(3)

H_2O

$m = 2\,kg \qquad x_2 = 0$

$P_1 = 1000\ kPa$

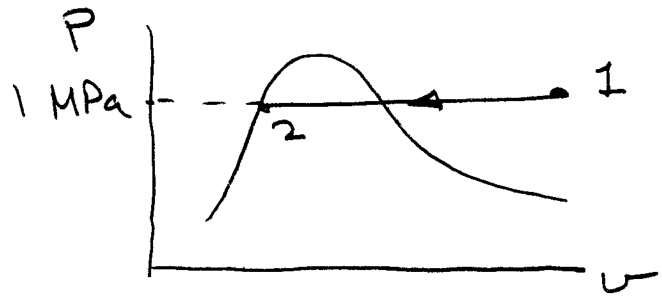
$T_1 = 250^\circ C$

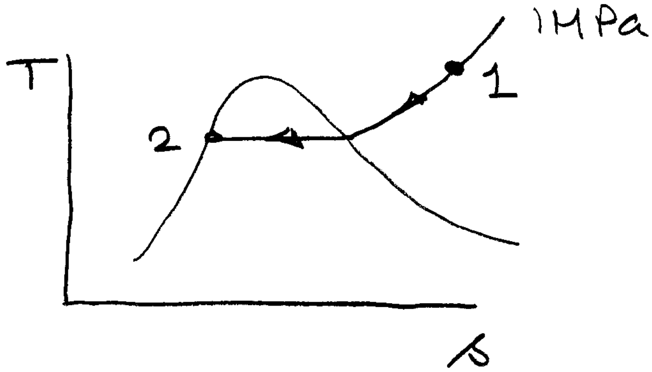
$$\left.\begin{array}{l}\text{At } 1000\ kPa \\ 250^\circ C\end{array}\right\} \begin{array}{l} v_1 = 0.2327\ m^3/kg \\ u_1 = 2709.9\ kJ/kg \end{array}$$

At 1000 kPa

$v_2 = v_f = 0.001127\ m^3/kg$

$u_2 = u_f = 761.68\ kJ/kg.$





$$W_{12} = -m\,P\,(v_2 - v_1)$$

$$= -2\ kg \times 10^6\ Pa \times (0.001127 - 0.2327)\ \frac{m^3}{kg}$$

$$= 463.1\ kJ$$

$$Q_{12} = m(u_2 - u_1) - W_{12}$$

$$= 2\ kg \times (761.68 - 2709.9)\ \frac{kJ}{kg} - 463.1\ kJ$$

$$= -4359.5\ kJ$$

(4)

$P_1 = 280\,kPa$ $\quad$ $P_2 = 85\,kPa$

$T_1 = 77^\circ C$ $\quad$ $V_2 = 320\,m/s$

$V_1 = 50\,m/s$

$q = -3.2\ kJ/kg$

Energy Balance.

$$\dot{Q} + \dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$\Rightarrow C_p(T_2 - T_1) = \frac{V_1^2 - V_2^2}{2} + q$$

For air at 350 K $\begin{cases} R = 0.2870\ kJ/kg\,K \\ C_p = 1.008\ \frac{kJ}{kg\,K} \end{cases}$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2 - V_2^2}{2C_p} + \frac{q}{C_p}$$

$$= 77^\circ C + \frac{50^2 - 320^2}{2 \times 1.008 \times 10^3} - \frac{3.2 \times 10^3}{1.008 \times 10^3}$$

$$= 24.3^\circ C = 297.5\ K.$$

$$\Delta s_{air} = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$= 1.008 \ln \frac{297.5\,K}{350.2\,K} - 0.2870 \ln \frac{85}{280}$$

$$= 0.1778\ kJ/kg\,K.$$

$$\Delta s_{surroundings} = \frac{q}{T_{surr}} = \frac{3.2\ kJ/kg}{293\,K} = 0.0109\frac{kJ}{kg\,K}$$

$$\Delta s_{TOTAL} = \Delta s_{air} + \Delta s_{surroundings} = 0.1887\ \frac{kJ}{kg\,K}$$

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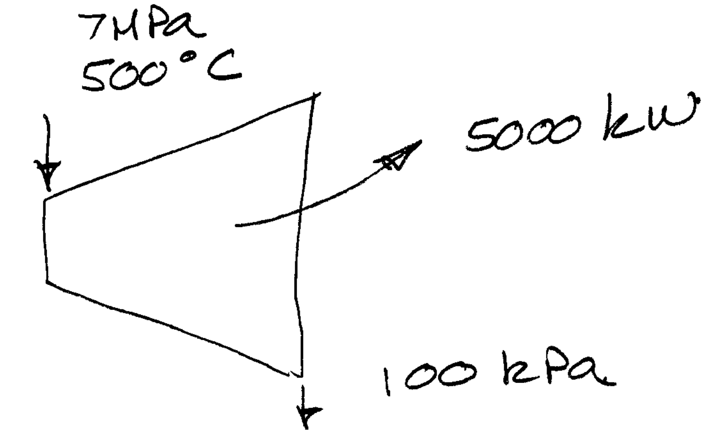

At 7 MPa, 500 °C, for steam

$$h_1 = 3410.3 \text{ kJ/kg}, \quad s_1 = 6.7975 \frac{\text{kJ}}{\text{kg K}}$$

$$P_2 = 100 \text{ kPa}$$

$$s_2 = s_1 = 6.7975 \text{ kJ/kg K}$$

At 100 kPa, $s_f = 1.3026 \text{ kJ/kg K}$, $s_g = 7.3594 \text{ kJ/kg K}$

$h_f = 417.46 \text{ kJ/kg}$, $h_g = 2675.5 \frac{\text{kJ}}{\text{kg}}$

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.7975 - 1.3026}{7.3594 - 1.3026} = 0.907$$

$$h_2 = h_f + x_2 (h_g - h_f)$$

$$= 417.46 + 0.907 (2675.5 - 417.46)$$

$$= 2465.5 \text{ kJ/kg}$$

$$\dot{W}_s = \frac{\dot{W}_a}{\eta_{turbine}} = \frac{5000 \text{ kW}}{0.77} = 6494 \text{ kW}$$

$$\dot{m} = \frac{\dot{W}_s}{h_2 - h_1} = \frac{-6494 \text{ kJ/s}}{(2465.5 - 3410.3) \text{ kJ/kg}}$$

$$= 6.87 \text{ kg/s}$$