University of Toronto
Faculty of Applied Science and Engineering

Term Test I

Feb. 9, 2015
14:15 – 15:45
Duration: 90 minutes

ECE159 - Electric Circuit Fundamentals
Examiners: Prof. Li Qian

ANSWER QUESTIONS ON THE FRONT PAGES, USE THE BACK PAGES FOR ROUGH WORK ONLY. THERE ARE EXTRA PAGES IN BETWEEN QUESTIONS. WRITE IN PEN ONLY (NO PENCIL)!

1. Calculator type is restricted (no programmable calculators).
2. Weight for each question is indicated in [].

LAST NAME: Solutions

FIRST NAME: ______________

STUDENT NUMBER: ______________

TUTORIAL SECTION: ______________

Questions	Mark
Q1	/6
Q2	/4
Q3	/7
Q4	/7
Q5	/6

Q1. [6 marks] Answer the following questions.

(a) [2 marks] Circle TRUE or FALSE for the following statements:

1. When the passive sign convention is followed, if the product of current and voltage associated with a circuit element is positive, the electric field does positive work. TRUE / FALSE

2. Electric current can flow only from a high voltage node to a low voltage node in a circuit. TRUE / FALSE

3. Kirchhoff's voltage law is a result of energy conservation. TRUE / FALSE

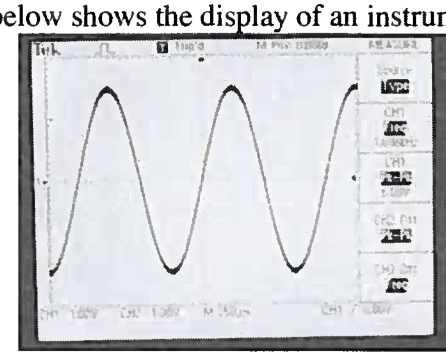
4. For a linear circuit with multiple ideal independent voltage sources, if the voltage output of one of these sources is doubled, the current going through any circuit element is doubled. TRUE / FALSE

(b) [2 marks] Write your answers in the boxes provided:

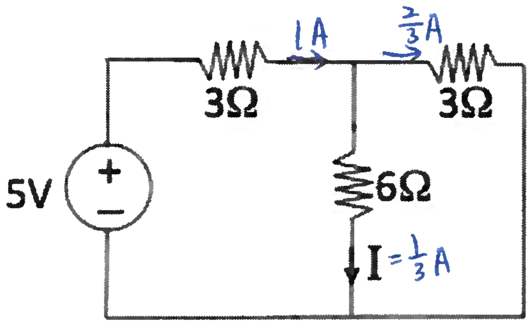
1. The resistance value of the resistor shown below is 560 kΩ

COLOR	VALUE OF RESISTANCE			COLOR	TOLERANCE
Black		0	$10^0 = 1$	Red	2 %
Brown	1	1	$10^1 = 10$	Gold	5 %
Red	2	2	$10^2 = 100$	Silver	10 %
Orange	3	3	$10^3 = 1{,}000$	None	20 %
Yellow	4	4	$10^4 = 10{,}000$		
Green	5	5	$10^5 = 100{,}000$		
Blue	6	6	$10^6 = 1{,}000{,}000$		
Violet	7	7	$10^7 = 10{,}000{,}000$		
Gray	8	8	$10^8 = 100{,}000{,}000$		
White	9	9	$10^9 = 1{,}000{,}000{,}000$		
	1st band	2nd band	3rd band	4th band	

2. The picture below shows the display of an instrument called an oscilloscope

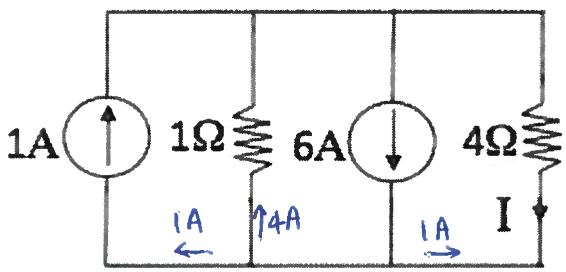


(c) [2 marks] Write your answers in the boxes provided

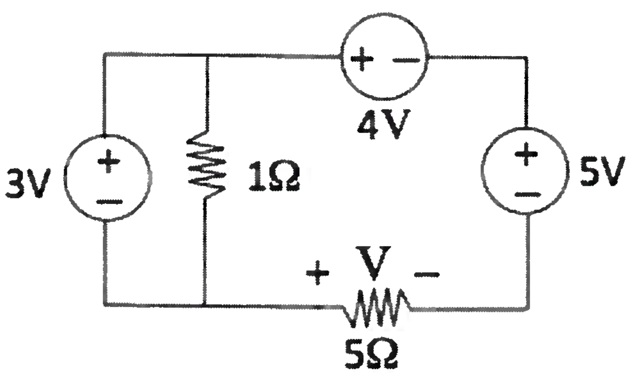


$I = \frac{1}{3}$ A

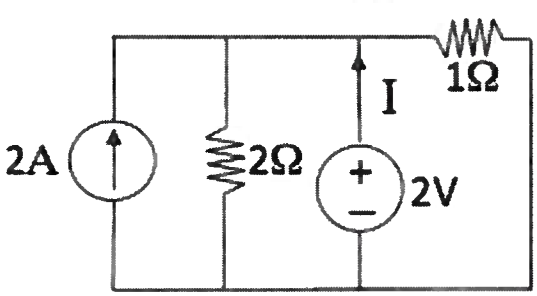
or 0.33 A



I= −1 A

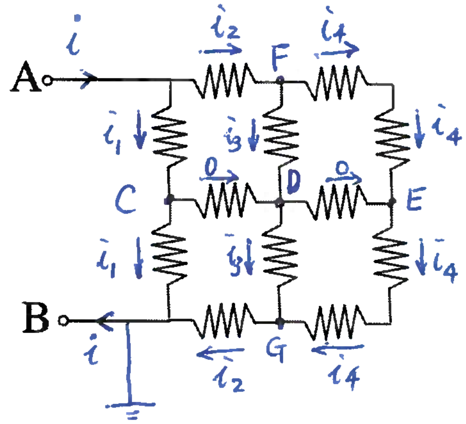


V= 6 V



I= 1 A

Q2. [4 marks] All resistors in the diagram below have the same resistance R. Find the equivalent resistance (in terms of R) evaluated at terminals A and B:



Use symmetry, we know that

$V_C = V_D = V_E = \frac{1}{2} V_A$

Therefore, $i_{CD} = 0$ for the branch between C & D, and $i_{DE} = 0$ for the branch between D & E.

The rest of the currents are labelled as shown.

Note the symmetrical labelling.

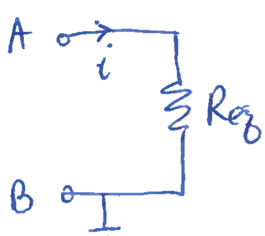
$V_{FD} = V_{FE} \Rightarrow i_3 R = i_4 (R+R) \Rightarrow i_3 = 2 i_4$

$KCL @ F \Rightarrow i_2 = i_3 + i_4 = 3 i_4$

$V_{AC} = V_{AD} = V_{AF} + V_{FD} \Rightarrow i_1 R = i_2 R + i_3 R \Rightarrow i_1 = 5 i_4$

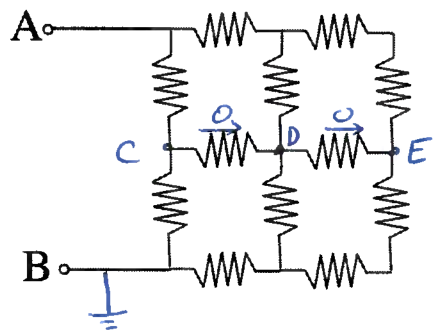
$KCL @ A \Rightarrow i = i_1 + i_2 = 8 i_4$

Finally, $R_{eq} = \frac{V_A}{i} = \frac{V_{AC} + V_C}{i} = \frac{i_1 R + i_1 R}{i} = \frac{10 i_4}{8 i_4} R$



$\Rightarrow R_{eq} = \frac{5}{4} R$

Q2. [4 marks] All resistors in the diagram below have the same resistance R. Find the equivalent resistance (in terms of R) evaluated at terminals A and B:



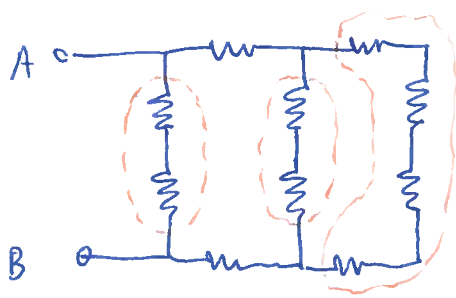
Alternative solution:

Use symmetry, we know that

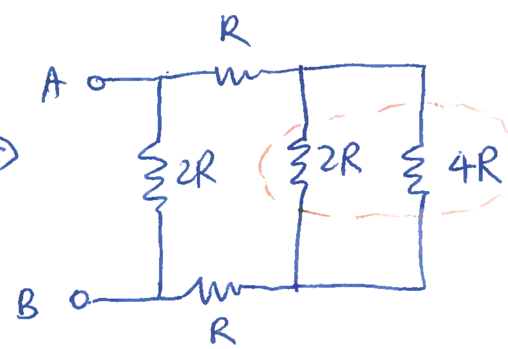
$V_C = V_D = V_E = \frac{1}{2} V_A$

Therefore $i_{CD} = 0$ for the branch between C & D; and $i_{DE} = 0$ for the branch between D & E.

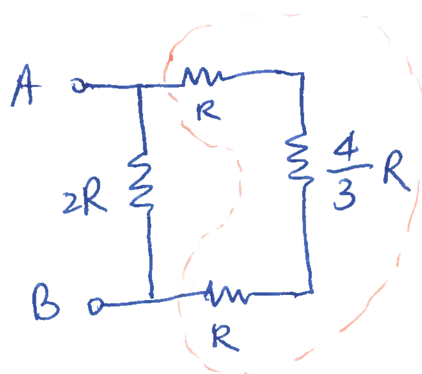
The circuit is then simplified to:



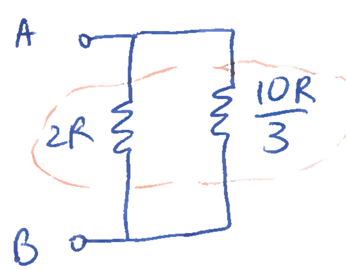
$\Leftrightarrow$



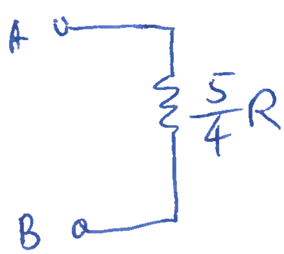
$\Leftrightarrow$



$\Leftrightarrow$



$\Leftrightarrow$



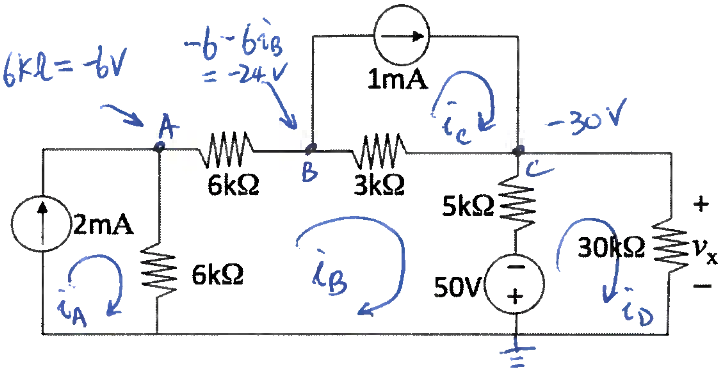
Q3. [7 marks] For the circuit shown below,

(a) [4 marks] Use *mesh* analysis to find v_x. Indicate mesh currents on the diagram, write down KVL equations, and carry out the calculations.

(b) [2 marks] Find the power supplied or consumed by the 1mA current source.

(c) [1 mark] Indicate whether the 1mA current source supply or consume power.

$(i_A - i_B)6k\Omega = -6V$



(a) $i_A = 2mA \qquad i_C = 1mA$

KVL for mesh B: $6i_B + 3(i_B - i_C) + 5(i_B - i_D) - 50 + 6(i_B - i_A) = 0$

$\Rightarrow 20i_B - 5i_D = 50 + 3i_C + 6i_A = 65\,mA$ ①

KVL for mesh D: $30i_D + 50 + 5(i_D - i_B) = 0$

$\Rightarrow -5i_B + 35i_D = -50\,mA$ ②

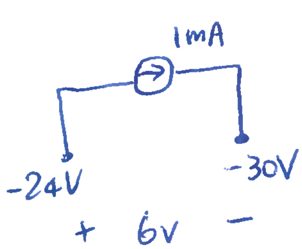
$\begin{cases} 20i_B - 5i_D = 65 \\ -5i_B + 35i_D = -50 \end{cases} \Rightarrow \begin{cases} i_B = 3\,mA \\ i_D = -1\,mA \end{cases}$

$v_x = 30k\Omega \cdot i_D = \underline{-30\,V}$

(b) $V_A = (i_A - i_B)\cdot 6k\Omega = (2-3)\cdot 6 = -6\,V$

$V_B = V_A - 6k\Omega \cdot i_B = -6 - 6\cdot 3 = -24V$

$V_C = -30V$



$\therefore P_{\ominus} = (1\,mA)(-24-(-30))V = 6mW > 0$

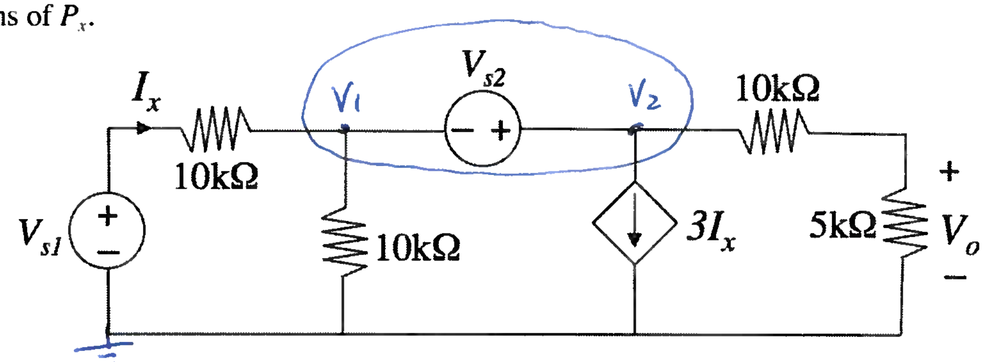
(c) the 1mA current source <u>consumes</u> 6mW power.

Q4. [7 marks] For the circuit below,

(a) [4 marks] Use *nodal* analysis to find an expression for V_o as a function of V_{s1} and V_{s2}.

(b) [1 mark] Find the power consumption by the 5kΩ resistor if $V_{s1} = 2\text{V}$, $V_{s2} = 6\text{V}$

(c) [2 marks] If the *power* supplied by the dependent source is P_x, what will the *power* supplied by the dependent source be if *both* V_{s1} and V_{s2} are *doubled*? Write your answer in terms of P_x.



(a) Use supernode as shown: $V_2 - V_1 = V_{s2}$ ①

KCL @ the supernode: $\underbrace{\dfrac{V_{s1}-V_1}{10k\Omega}}_{=I_x} - \dfrac{V_1}{10k\Omega} - 3\underbrace{\dfrac{V_{s1}-V_1}{10k\Omega}}_{=3I_x} - \dfrac{V_2}{10k\Omega + 5k\Omega} = 0$ ②

$$\Rightarrow \left(-\frac{1}{10} - \frac{1}{10} + \frac{3}{10}\right)V_1 - \left(\frac{1}{15}\right)V_2 = \left(\frac{3}{10} - \frac{1}{10}\right)V_{s1}$$

$$\Rightarrow \frac{V_1}{10} - \frac{V_2}{15} = \frac{V_{s1}}{5} \quad ③$$

Substitute ① into ③ $\Rightarrow \dfrac{V_2 - V_{s2}}{10} - \dfrac{V_2}{15} = \dfrac{V_{s1}}{5}$

$$\Rightarrow V_2 = 6V_{s1} + 3V_{s2}$$

Use voltage division: $V_o = \dfrac{5}{5+10} V_2 = 2V_{s1} + V_{s2}$

(b) $V_o = 2V_{s1} + V_{s2} = 2 \cdot 2 + 6 = 10\text{V}$

$P_{5k\Omega} = \dfrac{V_o^2}{5k\Omega} = \dfrac{100\text{V}^2}{5k\Omega} = \underline{\underline{20\text{mW}}}$

(c) $P_x = (-V_2)(3I_x)$

use linearity, if both V_{s1} & V_{s2} double, then V_2 doubles.

if " " " " ", then $3I_x$ doubles.

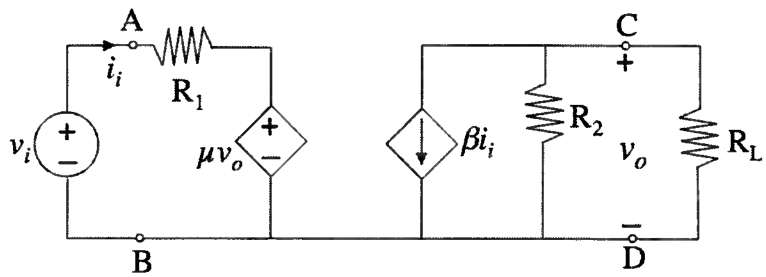
Therefore, if both V_{s1} & V_{s2} double, then $P_x' = \underline{\underline{4P_x}}$

power quadruples.

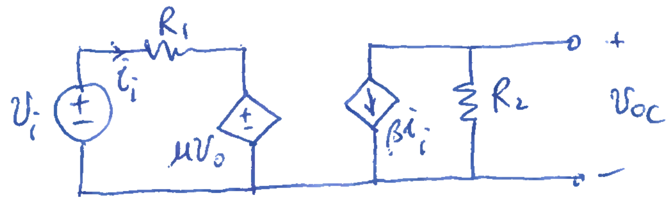
Q5. [6 marks] For the circuit shown below,

(a) [4 marks] Find the Thévenin equivalent circuit seen by the load R_L (i.e., looking into terminals C and D, with source connected, and the load disconnected).

(b) [2 marks] Find the equivalent circuit seen by the source v_i (i.e., looking into terminals A and B, with load connected and the source disconnected.)



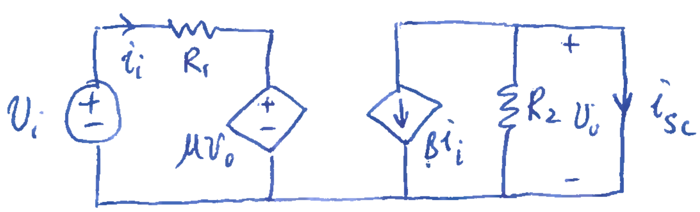
(a) Find v_{oc}



$$v_{oc} = -\beta i_i R_2 = -\beta R_2 \underbrace{\left(\frac{v_i - \mu v_{oc}}{R_1} \right)}_{i_i}$$

$$\Rightarrow \left(1 - \frac{\mu\beta R_2}{R_1}\right) v_{oc} = -\beta \frac{R_2}{R_1} v_i \Rightarrow v_{oc} = \frac{-\beta R_2}{R_1 - \mu\beta R_2} v_i$$

Find i_{sc}



$i_{sc} = -\beta i_i$

Note $v_o = 0 \Rightarrow i_i = \dfrac{v_i}{R_1}$

$$\therefore\ i_{sc} = -\beta i_i = -\frac{\beta}{R_1} v_i$$

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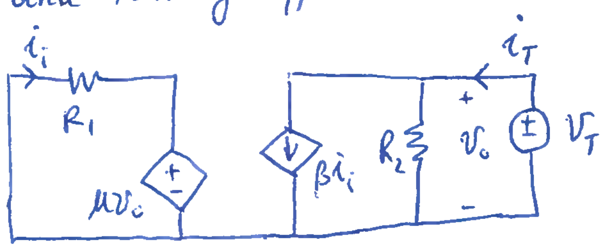
$$\therefore\ R_{eq} = \frac{v_{oc}}{i_{sc}} = \frac{-\beta R_2 \cancel{v_i}}{R_1 - \mu\beta R_2} \Big/ \left(-\frac{\cancel{\beta}}{R_1}\right)\cancel{v_i} = \underline{\underline{\frac{R_1 R_2}{R_1 - \mu\beta R_2}}}$$

Alternatively, R_{eq} can also be found directly using a test voltage v_T and turning off v_i:



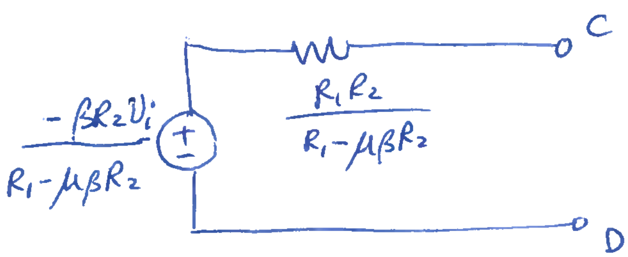
$$i_T = \beta i_i + \frac{v_T}{R_2}$$

$$i_i = -\frac{\mu v_o}{R_1} = -\frac{\mu v_T}{R_1}$$

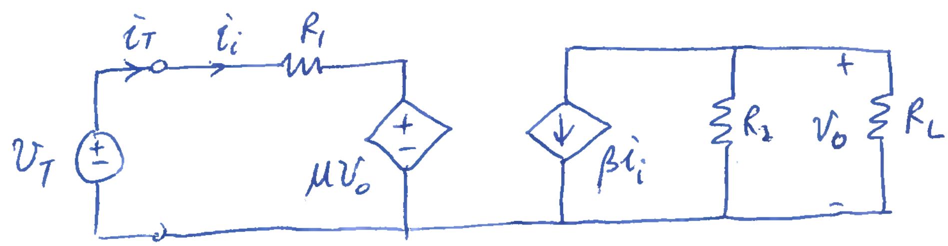
$$\Rightarrow i_T = -\frac{\mu\beta v_T}{R_1} + \frac{v_T}{R_2}$$

$$\therefore\ R_{eq} = \frac{v_T}{i_T} = \frac{v_T}{-\frac{\mu\beta v_T}{R_1} + \frac{v_T}{R_2}} = \underline{\underline{\frac{R_1 R_2}{R_1 - \mu\beta R_2}}}$$

$\therefore$ Equivalent circuit seen by the load (Thévenin form) is:



(b) Use test voltage source to find R_{eq}' seen by the source.



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$$v_o = -\beta i_i \, (R_2 \parallel R_L) = -\beta i_i \cdot \frac{R_2 R_L}{R_2 + R_L}$$

$$i_i = i_T = \frac{V_T - \mu V_o}{R_1} = \frac{V_T}{R_1} + \frac{\mu \beta i_i \, (R_2 R_L)}{R_1 \, (R_2 + R_L)}$$

$$\Rightarrow \quad \left(1 - \frac{\mu \beta R_2 R_L}{R_1 (R_2 + R_L)}\right) i_i = \frac{V_T}{R_1}$$

$$\Rightarrow \quad i_i = \left[R_1 \left(1 - \frac{\mu \beta R_2 R_L}{R_1 (R_2 + R_L)}\right)\right]^{-1} V_T$$

$$\therefore \quad R_{eq}' = \frac{V_T}{i_T} = \frac{V_T}{i_i} = R_1 \left(1 - \frac{\mu \beta R_2 R_L}{R_1 (R_2 + R_L)}\right)$$

$$= R_1 - \frac{\mu \beta R_2 R_L}{R_2 + R_L}$$

$\therefore$ The equivalent circuit seen by the source is: