1. Nancy carried out a photoelectric effect experiment similar to the one carried out by Millikan in 1915. She observed that the maximum energy of photoelectrons from aluminium is 2.3 eV for radiation of wavelength 200 nm, and 0.90 eV for radiation of wavelength 253 nm. Using this data, one can determine *experimentally* the Planck's constant and the work function of aluminum.

   (a) What is the Planck constant determined by Nancy in this experiment ? (Hint: The answer is different from the value quoted in the formula sheet.) $[7.13 \times 10^{-34}\ \mathrm{Js}]$

   (b) What is the work function of aluminum measured by Nancy in this experiment? [4.38 eV]

2. The atoms in a gas discharge tube are excited by a very brief burst of electrons at $t = 0$. The atoms subsequently fall back to the ground state, emitting visible light belonging to a single spectral line at 5500 Å. The intensity of this light falls off with time according to the law $I(t) = I_0 e^{-t/\tau}$ with $\tau = 2 \times 10^{-12}$ seconds.

   (a) What is the difference between the ground state energy and the excited state energy of this atom (in unit of eV)? [2.3 eV]

   (b) Deduce the energy uncertainty of the spectral line (in unit of eV) from the uncertainty principle. $[3.3 \times 10^{-4}\ \mathrm{eV}]$

3. A particle of mass m is in the state

$$\Psi(x,t) = Ae^{-x^2/2a^2} e^{-i\omega t},$$

   where A and a are positive real constants.

   (a) Normalize $\Psi(x,t)$ $[A = 1/\sqrt{a\sqrt{\pi}}]$

   (b) Calculate the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, and the standard deviation σ_x $[\langle x \rangle = 0,\ \langle x^2 \rangle = a^2/2,\ \sigma_x = a/\sqrt{2}]$

   (c) Calculate the expectation value of p [0].

4. Consider a particle in the infinite square well with width a. The energy eigenfunction and the energy eigenvalue can be obtained by solving the time-independent Schrödinger equation:

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right),\ E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2},$$

   where $n = 1, 2, 3, ...$ At time $t = 0$, the particle has as its initial wave function an even mixture of the first two stationary states:

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)].$$

   (a) Normalize $\Psi(x,0)$. (That is, find A. Remember that ψ_n's are orthonormal.) $[A = 1/\sqrt{2}]$

   (b) If you measure the energy of the particle at $t = 0$, what is the probability of finding that the energy is $\frac{2\pi^2\hbar^2}{ma^2}$? $[P = 1/2]$

   (c) Find $\Psi(x,t)$ at time t for this particle. Your answer should contain a, m, and $\hbar$. $[\Psi(x,t) = \sum_{n=1}^{2} \frac{1}{\sqrt{a}} \sin \frac{n\pi x}{a} e^{-\frac{n^2 \hbar \pi^2 t}{2ma^2}}]$

Useful constants:

$h = 6.626 \times 10^{-34}$ Js $= 4.14 \times 10^{-15}$eV $\cdot$ s (Planck constant)
$c = 3 \times 10^8$ m/s (speed of light)
$e = 1.602 \times 10^{-19}$ C (electron charge)
$m_e = 9.1 \times 10^{-31}$ kg (electron mass)
$M_p/m_e = 1836$ (Proton mass/ electron mass)
$k = 8.99 \times 10^9$ Nm^2/C^2 (Coulomb constant)
$k_B = 1.38 \times 10^{-23}$ J/K $= 8.6 \times 10^{-5}$ eV/K (Boltzmann constant)
$N_A = 6.02 \times 10^{23}$ (Avogadro number)
$R_H = 1.096776 \times 10^7$ m^{-1} (Rydberg constant)

Quantum Mechanics:

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1} \qquad E_{ph.el.} = hf - \phi \qquad E = hf = \hbar\omega$$

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta) \text{ (Compton)} \qquad \lambda_C = \frac{h}{mc} = 2.4263 \times 10^{-12}\text{m}$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad n\lambda = 2d\sin\theta \text{ (Bragg's law)} \qquad F_{cent} = \frac{mv^2}{r}$$

$$r_n = \frac{(n\hbar)^2}{kme^2} \qquad r_1 = 0.053\text{nm (Bohr radius)} \qquad E_n = -\frac{m(ke^2)^2}{2\hbar^2}\frac{1}{n^2}$$

$$\lambda = \frac{h}{p} \qquad \Delta x\Delta p \geq \frac{\hbar}{2} \qquad \Delta E\Delta t \geq \frac{\hbar}{2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = i\hbar\frac{\partial\Psi}{\partial t} \qquad \hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \qquad \hat{H}\psi(x) = E\psi(x)$$

$$\langle\hat{O}\rangle = \int \Psi(x,t)^*\hat{O}\Psi(x,t)dx \qquad \langle x\rangle = \sum xP(x) \text{ (discrete)} \qquad \langle x\rangle = \int xP(x)dx \text{ (continuous)}$$

$$\hat{p} = -i\hbar\frac{\partial}{\partial x} \qquad \psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \qquad E_n = \frac{\hbar^2\pi^2}{2ma^2}n^2$$

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar} \qquad c_n = \int \psi_n^*(x)\Psi(x,t)dx \qquad \langle\hat{H}\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$$

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi}}\int \phi(k)e^{ikx}dx \qquad \phi(k) = \frac{1}{\sqrt{2\pi}}\int \Psi(x,0)e^{-ikx}dx$$

Math formulae:

$$c = a + ib \qquad c^* = a - ib \qquad |c|^2 = c^*c = a^2 + b^2 \qquad e^{i\theta} = \cos\theta + i\sin\theta$$

$$\sin x = \frac{e^{+ix} - e^{-ix}}{2i} \qquad \cos x = \frac{e^{+ix} + e^{-ix}}{2} \qquad \int_{-\infty}^{\infty} e^{-\alpha^2x^2}dx = \frac{\sqrt{\pi}}{\alpha} \qquad \int_{-\infty}^{\infty} x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$

$$\sigma_y^2 = \langle y^2\rangle - \langle y\rangle^2$$

Special relativity: To be added