UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, DECEMBER 2020

CIV102F – Structures and Materials – An Introduction to Engineering Design
Exam Version A

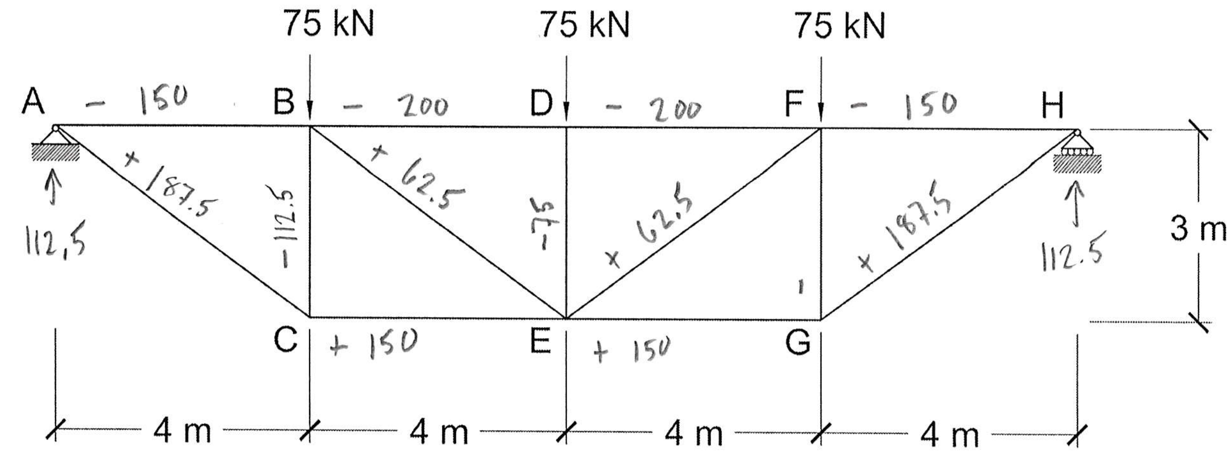
Examiner --- M.P.Collins and A.Kuan

Permissible Aids: Course Notebook, calculator and class notes.

IMPORTANT INSTRUCTIONS

Question No.	Score	Possible Points	time
1		30	19:26
2		20	24:12
3		35	22,20
4		24	14:48
Total		109	1:20:46

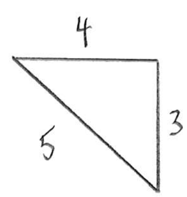
1. The truss shown below supports a pedestrian bridge and is made from steel hollow structural sections with a yield stress of 350 MPa. The truss spans 16 m, and when the bridge is crowded with people, supports the three 75 kN loads as shown.



1(a). Calculate the axial force in each member of the truss due to the 75 kN loads. Produce a neat drawing summarizing the results of your calculations and indicate the calculated forces above the appropriate members. Use the convention +ve for tension and -ve for compression. **(10 marks)**

Intermediate or sample calculations must be provided for full marks.

V = 112.5 37.5



1(b). Each of the members in compression are HSS 127x127x6.4, and each of the members in tension are HSS 102x102x4.8. Is the truss **safe** under the 100 kN loads? Yes or no? At what value of the loads will be on the boundary between safe and unsafe? **(6 marks)**

HSS 127x127x6.4 $A = 2960\ mm^2$

$I = 7.05 \times 10^6\ mm^4$

HSS 102x102x4.8 $A = 1790\ mm^2$

$I = 2.75 \times 10^6\ mm^4$

Tension members $P_{max} = 187.5\ kN$

$P_{safe} = \frac{350 \times 1790}{2} = 313.25\ kN$ ✓

$\left.\right\}\ FOS = \frac{313.25}{187.5} = \underline{\underline{1.671}}$

Compression members $P_{max} = 200\ kN$

$l = 4m$

crush → $P_{safe} = \frac{350 \times 2960}{2} = 518\ kN \rightarrow FOS = \frac{518}{200} = \underline{\underline{2.59}}$

buckle → $P_{safe} = \frac{\pi^2 \times 200{,}000 \times 7.05 \times 10^6}{3 \times 4000^2} = 290\ kN \rightarrow FOS = \frac{290}{200} = \underline{\underline{1.45}}$

truss is <u>safe</u> on boundary when $P = 75 \times 1.45 = \underline{\underline{108.8\ kN\ \text{each}}}$

1(c). Using the Method of Virtual Work, calculate the vertical deflection of Joint E due to the application of the 75 kN loads. Create, and fill in a table like the one shown below. Note that the table lists the members for only one half of the truss. **(10 marks)**

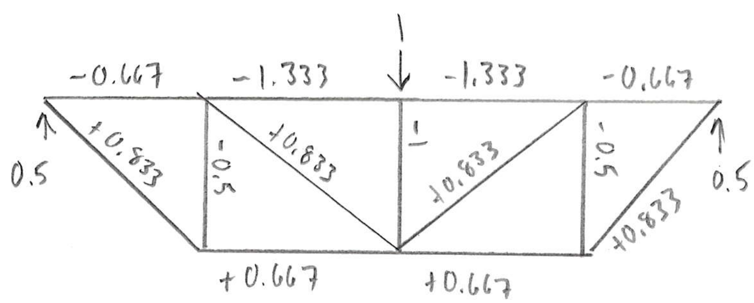
Member	Member Force, P (kN)	Strain, ε (mm/m)	Length, L (m)	Δl (mm)	Virtual Force, P* (kN)	Work (J)
AB	−150	−0.253	4	−1.014	−0.667	0.676
BD	−200	−0.338	4	−1.351	−1.333	1.801
AC	+187.5	+0.524	5	+2.619	+0.833	2.182
BE	+62.5	+0.175	5	+0.873	+0.833	0.727
BC	−112.5	−0.190	3	−0.570	−0.5	0.285
DE	−75	−0.127	3	−0.380	−0.5	0.19
CE	+150	+0.419	4	+1.676	+0.667	1.118

move here

Sum = 6.979 J

$A_{comp} = 2960\ mm^2$

$A_{ten} = 1790\ mm^2$



Total work = 2 × 6.979 = 13.96

$$\boxed{\Delta_E = 13.96\ mm\ \text{down}}$$

1(d). Calculate the natural frequency of the bridge when it is crowded with people, and comment on the stiffness of the bridge. **(4 marks)**

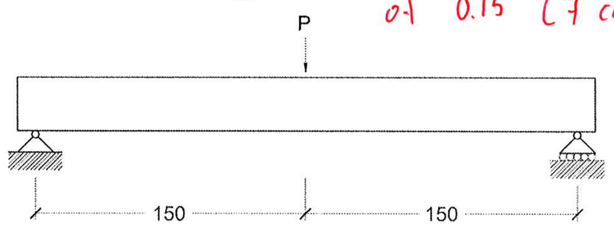
$$f_n = \frac{17.76}{\sqrt{13.96}} = \underline{\underline{4.75\ hz}}$$

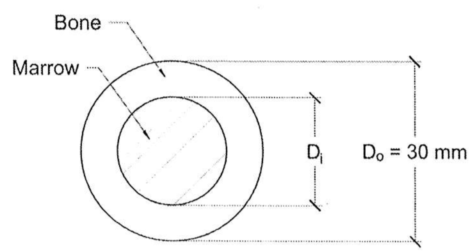
$$\Delta \approx \frac{L}{1146} \quad \therefore \text{very stiff}$$

2. Prof. Alexander of Leeds University has observed that for the bones of both birders and mammals, "bending is the predominant form of applied stress". It is because of this that many bones resemble circular tubes. In mammals, the space in the centre of the bone is usually filled with fatty yellow marrow. This marrow performs no structural function but does weigh about 8 kN/m^3 and hence increases the weight of the bone, W. Bone breaks at about 180 MPa and weighs 20 kN/m^3.

Consider a bone 300 mm long, supported at each end and loaded with a point load, P, at midspan. The bone has an external diameter, D_o, of 30 mm. Prepare a plot showing how the breaking load-to-weight ratio, P/W, of the bone beam, changes as the ratio D_i/D_o changes from 0 to 0.9. Comment on design considerations if it is desired to maximize P/W. **(20 marks)** Add reference to I in course notes

Suggest finding in increments of 0.15 (7 calcs)





When performing your calculations, the bending moments due to the self-weight should be neglected.

$$\boxed{A_{bone} = \frac{\pi D_o^2}{4} - \frac{\pi D_{in}^2}{4} = \frac{\pi D_o^2}{4}(1-r^2)}$$, $r = D_i/D_o$

$$\boxed{I = \frac{\pi D_o^4}{64} - \frac{\pi D_{in}^4}{64} = \frac{\pi D_o^4}{64}(1-r^4)}$$

$$M = \frac{PL}{4} = \frac{P \times 300}{4} = 75P \quad (Nmm)$$

$$y = 15\,mm$$

$$\boxed{A_{marrow} = \frac{\pi D_i^2}{4} = \frac{\pi D_o^2}{4} r^2}$$

$$\gamma_{marrow} = 8\ kN/m^3 = 8 \times 10^{-6}\ N/mm^3$$

$$\gamma_{bone} = 20\ kN/m^3 = 20 \times 10^{-6}\ N/mm^3$$

$$\boxed{W = L \times (A_{bone} \times \gamma_{bone} + A_{marrow} \times \gamma_{marrow})}$$

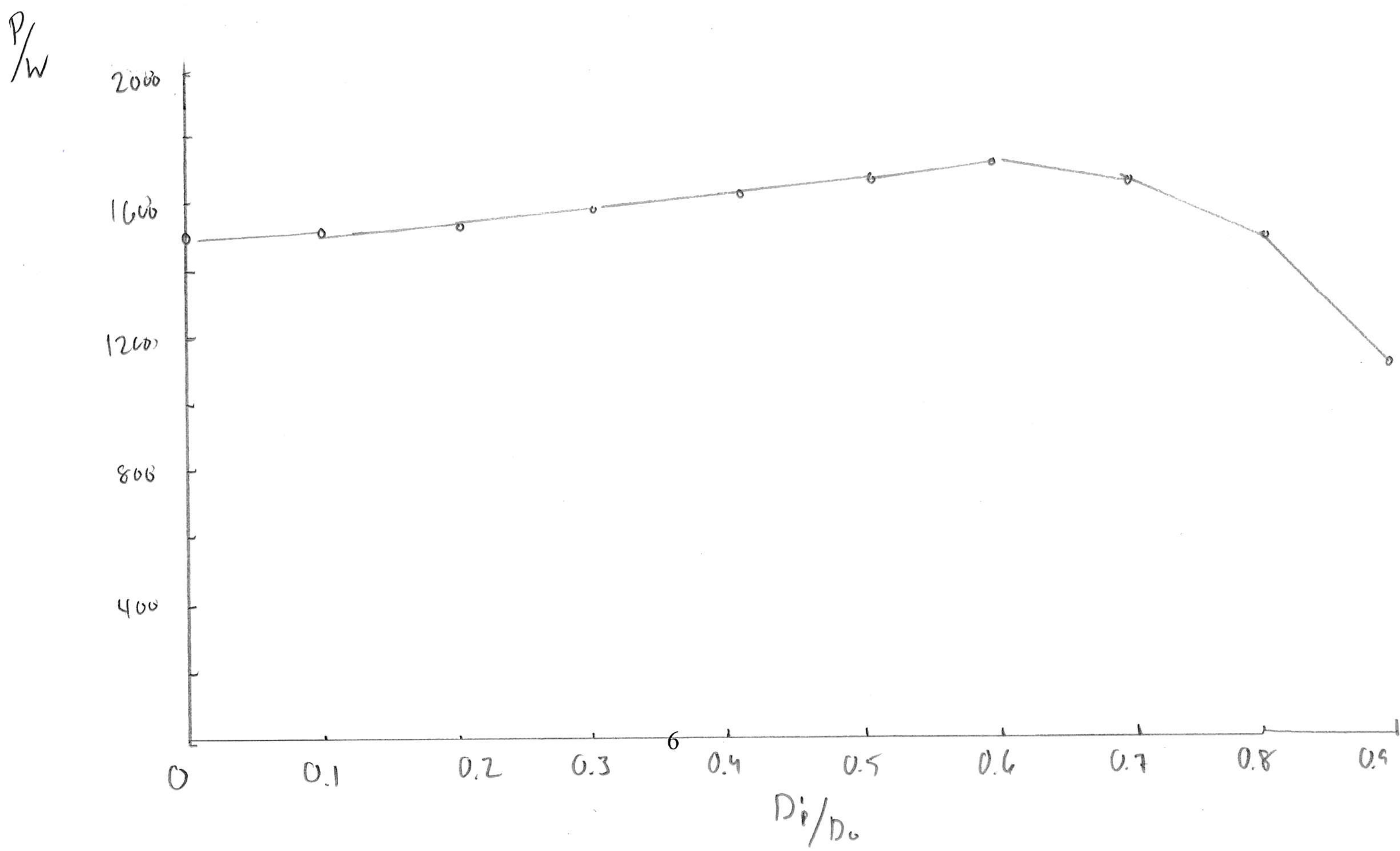
$$\sigma_{ult} = \frac{My}{I} \Rightarrow 75P = \frac{\sigma_{ult} I}{y}$$

$$\boxed{P = \frac{\sigma_{ult} \times I}{75y}}$$

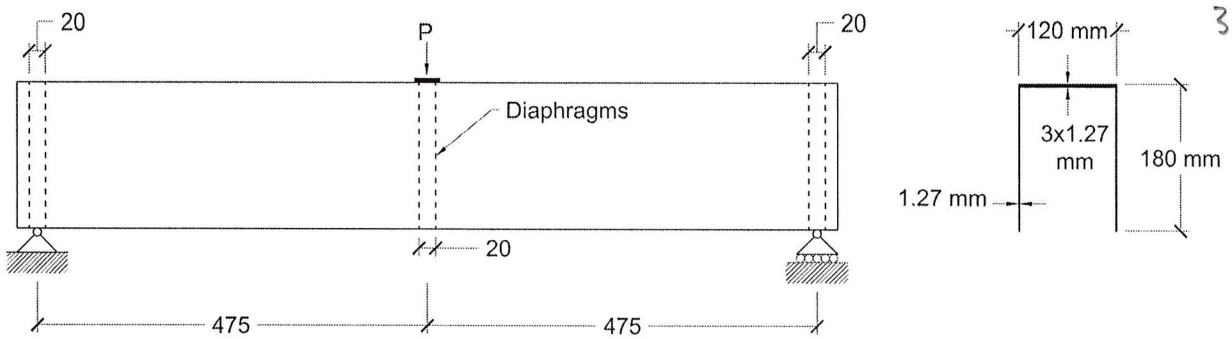
r	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
A_{bone}	707	700	679	643	594	530	452	360	254	134
A_{marrow}	0	7.07	28.3	63.6	113.1	176.7	254.5	346.4	452	573
I	39,761	39,757	39,697	39,439	38,743	37,276	34,608	30,214	23,475	13,674
P_{fail}	6362	6361	6352	6310	6199	5964	5537	4834	3756	2188
W	4.242	4.217	4.142	4.011	3.835	3.604	3.323	2.991	2.609	2.18
P/W	1500	1508	1534	1573	1616	1655	1666	1616	1440	1004

highest strength/weight ratio when

$D_i/D_o \doteq 0.6$



3. A design-build competition challenges your team to build the strongest possible beam to span 950 mm and carry a point load at midspan using a 1015 mm x 812 mm x 1.27 mm thick sheet of matboard. The matboard has a tensile strength of 30 MPa, a compressive strength of 6 MPa, a shear strength of 4 MPa, a Poisson's ratio of 0.2 and a modulus of elasticity of 4000 MPa. One member of your team has suggested the simple design shown below. It consists of a top flange made by gluing together three pieces of matboard and two vertical webs each made from one piece of matboard. The remaining matboard is used to make diaphragms at midspan and at the supports.



$3 \times 1.27 = 3.81$

3(a). Calculate the location of the centroid and the value of I for the proposed section of the bridge. **(7 marks)**

Note: the 120 mm and 180 mm measurements are based on the outside dimensions of the cross section.

$A_1 = 120 \times 3 \times 1.27 = 457.2\ mm^2$ $\qquad y_1 = 180 - 1.5 \times 1.27 = 178.1\ mm$

$A_2 = \underbrace{(180 - 3.81)}_{176.2} \times 2.54 = 447.5\ mm^2$ $\qquad y_2 = 88.1\ mm$

$I_{o1} = \dfrac{120 \times 3.81^3}{12} = 553.1\ mm^4$

$I_{o2} = \dfrac{2.54 \times 176.2^3}{12} = 1.158 \times 10^6\ mm^4$

$\bar{y} = \dfrac{457.2 \times 178.1 + 447.5 \times 88.1}{457.2 + 447.5}$

$\bar{y} = 133.6\ mm$

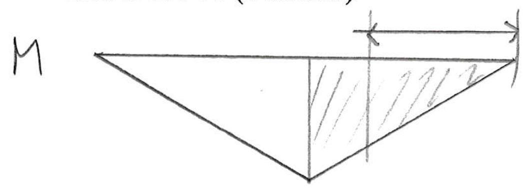
$d_1 = 178.1 - 133.6 = 44.5\ mm$

$d_2 = 133.6 - 88.1 = 45.5\ mm$

$I = 553.1 + 457.2 \times 44.5^2 + 1.158 \times 10^6 + 447.5 \times 45.5^2$

$I = 2.99 \times 10^6\ mm^4$

3(b). Use Moment Area Theorem no. 2 to calculate the mid-span deflection of the beam when the point load is 200 N. **(6 marks)**



$\Delta_{mid} = \delta_{sup-mid} = \left[\frac{1}{2} \times 475 \times 3.972 \times 10^{-6}\right] \times \left[\frac{2}{3} \times 475\right]$

$\frac{PL}{4} = \frac{200 \times 950}{4} = 47{,}500N$

$\Delta_{mid} = \underline{\underline{0.299\,mm}}$

$\phi_{max} = \frac{47500}{4000 \times 2.99 \times 10^{6}} = 3.972 \times 10^{-6}$

3(c). Use Navier's equation for flexural stress to determine the value of P which will cause a tensile stress of 30 MPa at mid-span, and then determine the value of P which will cause a compressive stress of 6 MPa at mid-span. **(6 marks)**

$M = \frac{PL}{4} = \underline{\underline{237.5P}}$

tension: $30 = \frac{237.5P \times 133.6}{2.99 \times 10^{+6}}$

$y_{b.t} = 133.6\,mm$

$\boxed{P = 2827\,N}$

Compression:

$y_{top} = 180 - 133.6 = 46.4$

$6 = \frac{237.5P \times 46.4}{2.99 \times 10^{6}}$

$\boxed{P = 1628\,N}$

3(d). Use Jourawski's equation for shear stress to determine the value of P which will cause a shear stress of 4 MPa. **(4 marks)**

$V = \frac{P}{2}$

$4 = \frac{0.5P \times 22668}{2.99 \times 10^6 \times 2.54}$

$I = 2.99 \times 10^6 \, mm^4$

$\boxed{P = 2680 \, N}$

$b = 2 \times 1.27 = 2.54 \, mm$

$Q = 133.6 \times 2.54 \times \frac{1}{2} \times 133.6$

$Q = 22{,}668 \, mm^3$

3(e). Based on the results in parts (c) and (d), what would be the predicted failure load for the proposed bridge? Will plate buckling reduce this failure load and if so by how much? **(6 marks)**

Note: When you are checking for plate buckling, consider the thicknesses when calculating your values of b in the equations.

flange buckling

$t = 3.81$

$b = 120 - 2.54 = 117.5 \, mm$

$$\sigma_{crit} = \frac{4 \times \pi^2 \times 4000}{12(1 - 0.2^2)} \left(\frac{3.81}{117.5}\right)^2 = 14.41 \, MPa$$

web buckling

$t = 1.27 \, mm$

$b = 46.4 - 3.81 = 42.6 \, mm$

$$\sigma_{crit} = \frac{6 \times \pi^2 \times 4000}{12(1 - 0.2^2)} \left(\frac{1.27}{42.6}\right)^2 = 18.27 \, MPa$$

shear buckling

$t = 1.27 \, mm$

$h = 176.2 \, mm$

$a = 475 - 20 = 455 \, mm$

$$\tau_{crit} = \frac{5 \times \pi^2 \times 4000}{12(1 - 0.2^2)} \left[\left(\frac{1.27}{176.2}\right)^2 + \left(\frac{1.27}{455}\right)^2\right] = 1.023 \, MPa$$

Check shear buckling P: $\frac{1.023}{4} \times 2680 = 685 \, N$

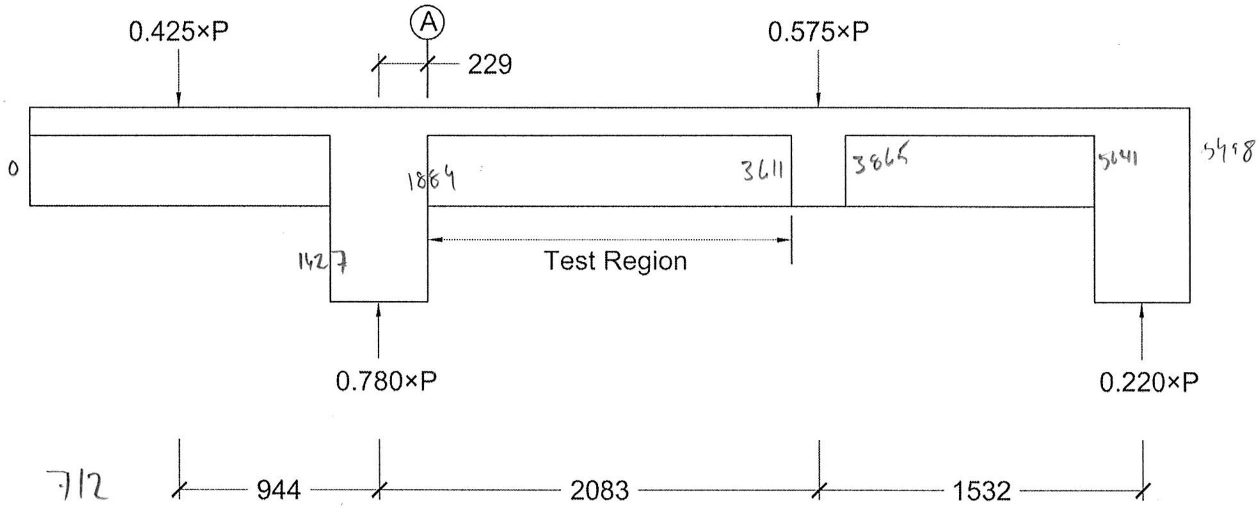
P_{fail} with no plate buckling: 1628 N

P_{fail} with plate buckling: 685 N

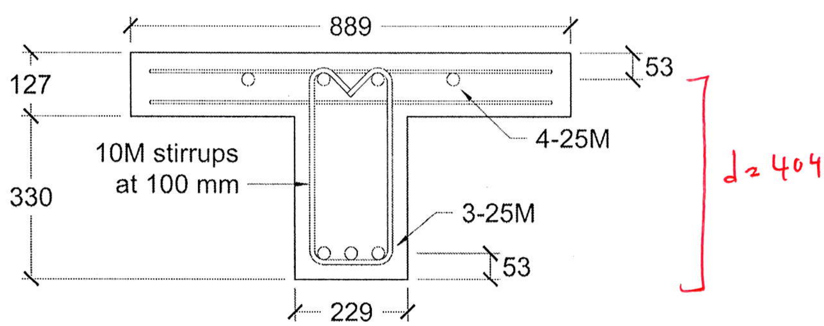
3(f). Suggest at least two ways the proposed design can be improved so that the failure load will be increased. **(6 marks)**

- add a diaphragm/ stiffener between load and supports to reduce a
- add another web to reduce τ.

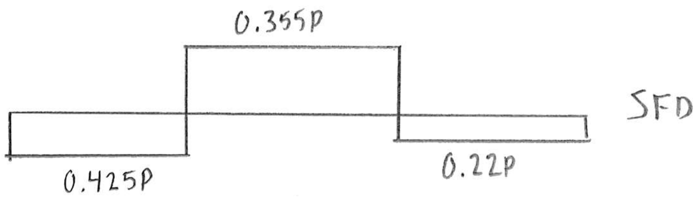
4. The reinforced concrete beam described in the figures shown below is one of the eight specimens which will be loaded to failure under the Baldwin testing machine as part of the doctoral research of Allan Kuan. It is loaded at two locations from above and supported from below at two locations, resulting in the reaction forces shown in the figure below.



The specimen is designed so that it will fail within the test region, which has the cross section shown in the figure below. The beam is longitudinally reinforced using seven 25M bars (four on top and three on the bottom), which have a yield strength of 480 MPa. 10M stirrups (which have a yield strength of 440 MPa) spaced apart by 100 mm are used as shear reinforcement. The concrete has a compressive strength, f_c', of 45 MPa, and has a modulus of elasticity, E_c, of 29,000 MPa.

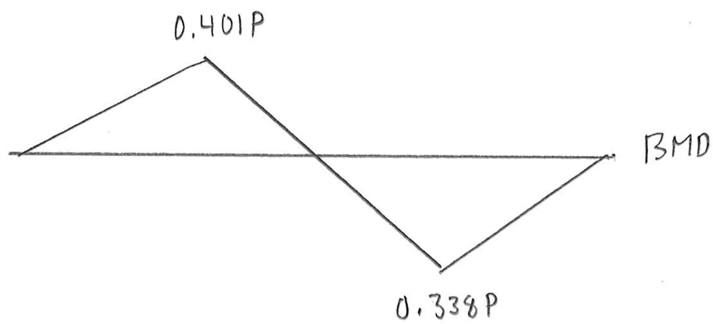


4(a). Draw the bending moment and shear force diagrams in terms of P, showing important values. In particular, calculate the bending moment at point A, which is 229 mm to the right of the left support. Neglect the self-weight of the structure when performing your calculations, and express P in units of kN, and the bending moments in units of kNm. **(4 marks)**



$M_A = 0.401P - 0.355P \times 0.229$

$M_A = \underline{\underline{0.320P}}$



4(b). Calculate the highest value of the bending moment, and corresponding value of P, which can be **safely** resisted by the beam at point A so that the tensile stress in the longitudinal reinforcement does not exceed $0.6 \times f_y = 288$ MPa and the compressive stress in the concrete does not exceed $0.5 \times f_c' = 22.5$ MPa. Neglect the longitudinal steel in compression when performing your calculations. **(8 marks)**

$A_s = 4 \times 500 = 2000 \text{ mm}^2$

$b = 229$

$d = 330 + 127 - 53 = 404 \text{mm}$

$n = \dfrac{200{,}000}{29{,}000} = 6.90$

$\rho = \dfrac{2000}{229 \times 404} = 0.0216$

$n\rho = 0.1492$

$k = \sqrt{0.1492^2 + 2 \times 0.1492} - 0.1492$

$k = \underline{\underline{0.417}}$

$j = 1 - \frac{1}{3} \times 0.417 = \underline{\underline{0.861}}$

Steel

$288 = \dfrac{M_A}{2000 \times 0.861 \times 404} \Rightarrow M_A = 200 \text{ kNm}$

Concrete

$22.5 = \dfrac{0.417}{1 - 0.417} \times \dfrac{M_A}{6.90 \times 2000 \times 0.861 \times 404}$

$M_A = \underline{\underline{151 \text{ kNm}}}$

$P = \dfrac{151}{0.32}$

$\boxed{P = 472 \text{ kN}}$

4(c). Calculate the highest shear force, and corresponding value of P, which can be **safely** resisted by the beam in the test region. When performing your calculations, use the value of jd which was computed in question 4(b). **(6 marks)**

$$\frac{A_v f_y}{b_w s} = \frac{2 \times 100 \times 440}{229 \times 100} = 3.84 \text{ MPa} > 0.06\sqrt{45} = 0.40 \text{ MPa}$$

$$jd = 0.861 \times 404 = 348 \text{ mm}$$

$$V_r = 0.5 \times 0.18\sqrt{45} \times 229 \times 348 + \frac{0.6 \times 2 \times 100 \times 440 \times 348}{100} \cot 35^\circ$$

$$V_r = 310.5 \text{ kN}$$

$$P = \frac{310.5}{0.355} = \underline{\underline{875 \text{ kN}}}$$

4(d). The simplified procedures for the moment and shear design of reinforced concrete incorporate safety factors by reducing the concrete contributions by a factor of 0.50 (corresponding to a factor of safety of 2.0) and by reducing the steel contributions by a factor of 0.60 (corresponding to a factor of safety of 1.67).

Using your calculations from questions 4(b) and 4(c) and the above information on the factors of safety to predict what was value of P which caused failure of the beam. ~~Did~~ Does the beam fail in flexure or shear? **(6 marks)**

note - only consider yielding for a flexural failure, not crushing

$$M_{flex} = \frac{200}{0.6} = 333 \text{ kNm} \rightarrow P = \underline{\underline{1041 \text{ kN}}}$$

$$V_r = 96.23 + 437.36 = 533.5 \text{ kN} \quad P = \underline{\underline{1503 \text{ kN}}}$$

fails in flexure