This test has **three** questions. Do **all** of them. One page of formulae is provided. A non-programmable calculator without text storage is permitted. No other aids are allowed. *Show all of your work and explain all of your answers*. The three questions are of equal value. All of your work should be done in the exam booklet. Test papers must be handed in with your booklets.

Question 1 [20 points]

An object of mass $3m$ traveling at $\vec{v} = 0.8c\,\hat{x}$ collides head on with an object of mass $8m$ traveling at $\vec{v} = -0.7c\,\hat{x}$ to form a single object in the final state. [All masses are rest masses].

a) What are the mass and velocity of the final-state object? [15 points]

b) What is the change in kinetic energy of the system, going from the initial state to the final state? [5 points]

Question 2 [20 points]

a) In frame *S* two events occur with the space-time coordinates given below:

Event 1: $x_1 = a, y_1 = 0, z_1 = 0, t_1 = a/c$ Event 2: $x_2 = 3a, y_2 = 0, z_2 = 0, t_2 = a/(3c)$

There is a reference frame S' in which these two events are simultaneous (assume the origins of S and S' coincide at $t = t' = 0$). Find

(i) the velocity of the S' frame with respect to frame *S*. [5 points]

(ii) the time at which these events occur in S'. [5 points]

b) Bob (at large negative x) runs towards the origin of reference frame S at speed $0.7c$, while Anna (at large positive x) runs towards the origin at $0.5c$. If Bob carries a pole of length 4m (in his rest frame) oriented in the direction in which he runs, what length would the rod have as viewed by Anna? [10 points]

Question 3 [20 points]

A light source that emits a wavelength of 550 nm moves directly towards a metal photo-cathode. If the photo-cathode has a work-function of 2.0 eV and the electrons emitted from it have a maximum velocity of $0.002c$, answer the following questions:

a) How fast is the source moving? [15 points]

b) What would the maximum velocity of the electrons be if the source were moving in the opposite direction, at the same speed? [5 points]

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Equations / Constants: (note that you will not need all of these)

Lorentz transformations (from frame S to a frame S' with relative velocity $v\hat{x}$):

$$x' = \gamma\left(x - \beta ct\right) \quad t' = \gamma\left(t - \frac{\beta}{c}x\right) \quad y' = y \quad z' = z \quad u'_x = \frac{u_x - v}{1-(v/c^2)u_x} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \frac{v}{c}$$

Photoelectric Effect: $\left(K.E.\right)_{\max} = h\nu - \phi$

Relativistic Doppler Effect: $f_{obs} = f_{source} \cdot \dfrac{\sqrt{1-\dfrac{v^2}{c^2}}}{1+\dfrac{v}{c}\cos\theta}$

Photons: $E = h\nu \quad c = \nu\lambda$

Energy and Momentum:

Non-relativistic: Kinetic energy (K.E.) = $\frac{1}{2}mu^2 \quad \vec{p} = m\vec{u}$

Relativistic: $E = \gamma mc^2 \quad \vec{p} = \gamma m\vec{u} \quad E^2 = p^2c^2 + m^2c^4$

Four-vectors: Position-time: $\left(x, y, z, ct\right)$ Energy-momentum: $\left(p_x, p_y, p_z, \frac{E}{c}\right)$

Lorentz Invariants: for any four-vector $A = \left(A_x, A_y, A_z, A_t\right)$

$A \cdot A = A^2 \equiv A_t^2 - A_x^2 - A_y^2 - A_z^2$ is a Lorentz invariant

$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

$$\left(\frac{E}{c}\right)^2 - p_x^2 - p_y^2 - p_z^2 = m^2c^2$$

Constants/conversions

$m_e = 9.1\times10^{-31}\text{kg} = 511\times10^3\,\dfrac{\text{eV}}{c^2} \qquad h = 6.626\times10^{-34}\,\text{J}\cdot\text{s} = 4.14\times10^{-15}\,\text{eV}\cdot\text{s}$

$c = 3\times10^8\,\text{m/s} \qquad 1\text{eV} = 1.6\times10^{-19}\,\text{J} \qquad hc = 1240\ \text{eV}\cdot\text{nm}$