AER210F VECTOR CALCULUS AND FLUID MECHANICS

Quiz 2

20 October 2014 8:45 am - 9:45 am

Closed Book, No aid sheets, No calculators

Instructor: J. W. Davis

Last Name: ______________________

Given Name: ______________________

Student #: ______________________

Tutorial/TA: ______________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	10	
2	12	
3	8	
4	8	
5	14	
TOTAL	52	/ 48

Note: The following integrals may be useful.

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C; \qquad \int \sin^2\theta \, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dR; \qquad \oiint_S \vec{F}\cdot\hat{n}\,dS = \iiint_V \nabla\cdot\vec{F}\,dV; \qquad \oint_C \vec{F}\cdot d\vec{r} = \iint_S \nabla\times\vec{F}\cdot d\vec{S}$$

1) Use the coordinate transformation: $x = \frac{u}{v}\cos\theta,\ y = \frac{u}{v}\sin\theta,\ z = u^2$, to evaluate the triple integral $I = \int_V \frac{dV}{x^2 + y^2}$, where V is the volume that lies between the parabaloids $z = x^2 + y^2$, $z = 4(x^2 + y^2)$ and between the planes $z = 1$, $z = 4$. Provide a sketch of the volume.

Hint: While the limits for θ and u are easily found, the bounds for v are not so obvious. Consider the traces of the parabaloids in the $z = 1$ or $z = 4$ planes to help determine the limits on v.

(10 marks)

2) Evaluate the line integrals:

a) $\int_C x^2\,dx + y^2\,dy + z^2\,dz$, where C consists of the line segment from $(1,2,-1)$ to $(3,2,0)$.

b) $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = \sin x\,\hat{i} + \cos y\,\hat{j} + xz\,\hat{k}$ and $C: \bar{r}(t) = t^3\,\hat{i} - t^2\,\hat{j} + t\,\hat{k}$, $0 \le t \le 1$.

c) $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = e^y\,\hat{i} + xe^y\,\hat{j} + (z+1)e^z\,\hat{k}$ and $C: \bar{r}(t) = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$, $0 \le t \le 1$.

(12 marks)

3) Given a surface defined parametrically by $\bar{r}(u,v) = x(u,v)\,\hat{\imath} + y(u,v)\,\hat{\jmath} + z(u,v)\,\hat{k}$, show that the surface area can be found from: $\int_S dS = \iint \|\bar{r}_u \times \bar{r}_v\| \, du\, dv$.

(8 marks)

4) Use a parametric representation of the surface to find the surface area of the cylinder $x^2 + y^2 = 16$ between the planes $z = 0$ and $z = 16 - 2x$. Provide a sketch of the area.

(8 marks)

5) Verify the divergence theorem for the vector field $\bar{F}(x,y,z) = xyz\hat{i} + x^2y\hat{j} + x^2z\hat{k}$ and S is the surface formed by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 1$.

(14 marks)