Question 1. You and your roommate are moving to a city 450km away. Your roommate drives a rental truck at a constant 100km/hr and you drive your car at 120km/hr. The two of you begin your trip at the same time. an hour after leaving, you decide to take a short break at a rest stop. If you are planning to arrive at your destination half an hour before your roommate gets there with the truck, and they don't make any stops, how long can you stay at the rest stop before resuming your trip?

First, we need to calculate how long it take your roommate to drive 450km away at 100km/hr:

$$\text{Time} = \frac{Distance}{Velocity} = \frac{450\ km}{100\ km/hour} = 4.5\ hours\ \text{(4 points)}$$

Therefore if you want to arrive 30 minutes or 0.5 hour before him, you therefore need to do the trip in $4.5 - 0.5 = 4$ hours total. How much distance did you travel after 1 hour driving at 120 km/hr?

$$\text{Distance} = Velocity \times Time = 120\ km/hour \times 1\ hour = 120\ km\ \text{(1 points)}$$

So, you have 3 hours left and 330 km to go. How long should it take you to go 330 km at 120km/hr?

$$\text{Time} = \frac{Distance}{Velocity} = \frac{330\ km}{120\ km/hour} = 2.75\ hours\ \text{(4 points)}$$

Which leaves us a difference of $3 - 2.75 = 0.25$ hours (1 point) or 15 minutes. You therefore can afford to spend 15 minutes at the next stop.

Note: The most common mistake is forgetting to subtract the 1h initial drive for the first 120km travelled, usually yielding an answer of 1h15. In that case, I would remove 2 points (1 for forgetting the 1h and another for the final answer). Usually, pure calculation/conversion errors are 1 point and extremely erroneous significant figures are also 1 point.

Takahiro Tow

2. The position of an 8.0 kg shopping cart, rolling down a ramp, is given by $x(t) = pt + qt^3$ with $p = 2.5$ m and $q = 2.50$ m/s^3.

(a) Note the forces on this cart must be something more complicated than simply gravity pulling it down the ramp. Why? (+2)

Consider the simple gravity problem. We note that the force of gravity acts in the orthogonal z-direction, and thus cannot directly influence the motion in the x-direction. But because shopping cart is constrained to move along an inclined ramp, the resultant normal force and frictional force (if any) will have an x-component. In any case, the gravity and thus normal force– and its x-projection– is constant $F_N \propto g$. The equation of motion $F = ma$ gives us:

$$a_x(t) \propto g \implies x(t) = \alpha t^2 + v_0 t + x_0$$

for some constant $\alpha \propto g$. So we see that with simple gravity, we get terms of order 0, 1, and 2 in time t. Thus, the cubic term qt^3 in the equation for $x(t)$ must be something more complicated than simple gravity!

What is the x component of the cart's average velocity?

The average velocity is defined as

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{(t_2 - t_1)}$$

(b) between $t = 2.00$ s and $t = 3.00$ s (+2)

$$x(3) = 2.5 \cdot (3) + 2.5 \cdot (3)^3 = 75 \text{ m}$$
$$x(2) = 2.5 \cdot (2) + 2.5 \cdot (2)^2 = 25 \text{ m}$$
$$v_{x,\text{avg}} = \frac{x(3) - x(2)}{(3-2)} = \frac{50 \text{ m}}{1 \text{ s}} = 50 \text{ m/s}$$

(c) between $t = 2.00$ s and $t = 2.10$ s (+1)

$$x(3) = 2.5 \cdot (2.1) + 2.5 \cdot (2.1)^3 = 28.4025 \text{ m}$$
$$v_{x,\text{avg}} = \frac{x(2.1) - x(2)}{(2.1-2)} = \frac{3.4025 \text{ m}}{0.1 \text{ s}} = 34 \text{ m/s}$$

(d) between $t = 2.00$ s and $t = 2.01$ s (+1)

$$x(3) = 2.5 \cdot (2.01) + 2.5 \cdot (2.01)^3 = 25.3265 \text{ m}$$
$$v_{x,\text{avg}} = \frac{x(2.01) - x(2)}{(2.01-2)} = \frac{0.3265 \text{ m}}{0.01 \text{ s}} = 32.65 \text{ m/s}$$

(e) Compute the limit of the velocity between $t = 2.00$ s and $t = 2.00 + \Delta t$ s as Δt approaches 0. (+2)

$$\begin{aligned}\lim_{\Delta t \to 0} v &= \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{2.5 \cdot \Delta t + 3 \cdot 2.5 \cdot (2)^2 \Delta t + 3 \cdot 2.5 \cdot (2)(\Delta t)^2 + 2.5 \cdot (\Delta t)^3}{\Delta t} \\ &= \lim_{\Delta t \to 0} \left(2.5 + 30 + 15\Delta t + 2.5(\Delta t)^2\right) \\ &= 32.5 \text{ m/s}\end{aligned}$$

(f) Show this agrees with what you get by taking the derivative of the position function. (+2)

$$v(2) = \frac{dx(t)}{dt}\bigg|_{t=2} = \left(2.5 + 7.5 \cdot t^2\right)\bigg|_{t=2} = 32.5 \text{ m/s}$$

So the limit agrees with the derivative calculation!

Matthew Pocrnic

Problem 3 Solution

PHY180H1

Problem: A ball is projected upward, with unknown velocity v_i, from an initial position 5.0m above the ground. At the same instant a block is released down a frictionless ramp, from height not necessarily equal to 5m. The two objects reach the ground at the same time, and both have a final speed of 15m/s. What is the angle of the incline?

Grading scheme: (10 points total)
Ramp acceleration equation (2 points)
Correct v_i expression (2 points) and correct v_i value (1 point)
Expression for time (2 points) and correct t value (1 point)
Correct value for θ (1 point)
Significant figures (1 point)

Solution: Defining up as positive. The ramp is frictionless, so the only force acting on it is the component of gravity along the ramp which gives:

$$\begin{aligned} F &= F_g \sin\theta \\ a &= -g\sin\theta \end{aligned} \tag{1}$$

Where the mass cancels out, g is gravitational acceleration, and θ is the angle of inclination. Now v_i for the block is zero and v_f is 15m/s, so using $a = \frac{v_f - v_i}{\Delta t}$ we can rewrite this expression:

$$\theta = \arcsin(\frac{-v_f}{gt}) \tag{2}$$

Next solving for v_i we know that on the way down when the ball is 5m above the ground it will have velocity $-v_i$ and reach v_f after traveling 5:

$$\begin{aligned} v_f^2 - v_i^2 &= 2\Delta x g \\ v_i &= \sqrt{v_f^2 - 2\Delta x g} \\ v_i &= 11.27m/s \end{aligned} \tag{3}$$

Using this we solve for the time: Sub $t = \frac{v_f - v_i}{a}$ into equation 2. This gives

$$\begin{aligned}
\theta &= \arcsin\left(\frac{-v_f}{g \cdot \frac{v_f - v_i}{a}}\right) \\
&= \arcsin\left(\frac{15m/s}{g \cdot \frac{-15m/s - 11.27m/s}{-g}}\right) \\
&= 34.8^\circ \\
&= 35^\circ
\end{aligned} \tag{4}$$

Significant digits done in the last step. Solving for t also could have been done with a quadratic equation for equal marks.

Maria Lozano

Problem 4
A 2.0 kg and a 3.0 kg cart collide on a low friction track. The 3.0 kg cart is initially moving at 1.0 m/s to the right. After the collision it is moving at 5.0 m/s to the right. After the collision the 2.0 kg cart is moving to the right at 3.0 m/s. ***Each part worth 5 points***

(a) What was the 2.0 kg cart's initial velocity?

(b) What would be the 2.0 kg cart's initial velocity, if all the other conditions given above remained the same, except the 3.0 kg cart were initially moving at 1.0 m/s to the ***left****.*

(a) Assuming friction is negligible such that momentum and kinetic energy are conserved. Following conservation of momentum and rearranging for the unknown variable $v_{2,i}$

$$\begin{aligned} m_1 v_{1,i} + m_2 v_{2,i} &= m_1 v_{1,f} + m_2 v_{2,f} \\ v_{2,i} &= \frac{m_1 v_{1,f} + m_2 v_{2,f} - m_1 v_{1,i}}{m_2}. \end{aligned}$$

Defining the rightward direction as the positive direction

$$\begin{aligned} v_{2,i} &= \frac{(3.0\text{kg})(5.0\text{m/s}) + (2.0\text{kg})(3.0\text{m/s}) - (3.0\text{kg})(1.0\text{m/s})}{2.0\text{kg}} \\ &= 9.0\text{m/s}. \end{aligned}$$

So the 2.0 kg cart is initially moving 9.0m/s to the right.

(b) Using the same equation for $v_{2,i}$ from above and taking the initial velocity of the 3.0kg to be $v_{1,i} = -1.0\text{m/s}$,

$$\begin{aligned} v_{2,i} &= \frac{m_1 v_{1,f} + m_2 v_{2,f} - m_1 v_{1,i}}{m_2} \\ &= \frac{(3.0\text{kg})(5.0\text{m/s}) + (2.0\text{kg})(3.0\text{m/s}) - (3.0\text{kg})(-1.0\text{m/s})}{2.0\text{kg}} \\ &= 12.0\text{m/s}. \end{aligned}$$

So the 2.0 kg cart is initially moving 12m/s to the right.

Problem 5

The bumper boats at your local amusement park each have an inertia of 90 *kg. In boat 1 are a man of unknown inertia, a* 45 *kg woman and a* 3.0 *kg dog. In boat 2 are an* 80 *kg father, a* 50 *kg mother and their* 30 *kg son. Boat 1 collides at* 1.5 *m/s with boat 2, which is initially at rest. Two seconds after the collision, boat 2 has moved* 2.3 *m and boat 1 has moved* 0.26 *m in the opposite direction.* ***Each part worth 2.5 points***

(a) Taking the initial direction of motion of boat 1 as positive, what is the velocity of each boat after the collision?

(b) What is the change of velocity of each boat?

(c) What is the inertia of the man in boat 1?

(d) If the collision took 0.50*s, what was each boat's average acceleration during the collision?*

(a) In a closed system with no external forces providing additional acceleration, we have

$$v_i = \frac{\Delta x_i}{\Delta t} \tag{1}$$

or

$$v_1 = \frac{2.3\text{m}}{2.000\text{s}} = 1.15\text{m/s} \tag{2}$$

$$v_2 = \frac{-0.26\text{m}}{2.000\text{s}} = -0.13\text{m/s} \tag{3}$$

$$\tag{4}$$

where to be precise, using the rule of rounding to least significant digit would get $v_1 = +1.2\text{m/s}$ and $v_2 = -0.13\text{m/s}$.

(b)

$$\Delta v_1 = v_{1,f} - v_{1,i} = 1.15 - 0.00 = 1.15\text{m/s} \tag{5}$$

$$\Delta v_2 = v_{2,f} - v_{2,i} = -0.13 - 1.5 = -1.63\text{m/s} \tag{6}$$

where again, the true precision is $v_2 = -1.6$ m/s

(c) First the inertia (mass) of each boat with its occupants can be computed as

$$m_1 = 90 + 45 + 3 + x = 138 + x \text{ kg} \tag{7}$$

and

$$m_2 = 90 + 80 + 50 + 30 = 250 \text{ kg} \tag{8}$$

By conservation of momentum we have

$$\sum_\alpha m_{\alpha,i} v_{\alpha,i} = \sum_\alpha m_{\alpha,f} v_{\alpha,f} \tag{9}$$
$$m_{1,i} v_{1,i} + 0 = m_{1,f} v_{1,f} + m_{2,f} v_{2,f} \tag{10}$$
$$(138 + x)(1.5) = (250)(1.15) + (138 + x)(-0.13) \tag{11}$$
$$207 + 1.5x = 287.5 - 17.94 - 0.13x \tag{12}$$
$$1.63x = 62.56 \tag{13}$$
$$x = 38.3804 \to 38 \text{ kg} \tag{14}$$

(d) Similar to b, by knowing the definition (or dimensional analysis)

$$a_1 = \frac{\Delta v_1 \text{m/s}}{\Delta t \text{s}} = \frac{1.15}{0.5} = 2.3\text{m/s}^2 \tag{15}$$
$$a_2 = \frac{\Delta v_2 \text{m/s}}{\Delta t \text{s}} = \frac{-1.63}{0.5} = -3.26 \to 3.3\text{m/s}^2 \tag{16}$$
$$\tag{17}$$

where a_2 would be 3.2 m/s^2 if using the sig-fig value from previously.