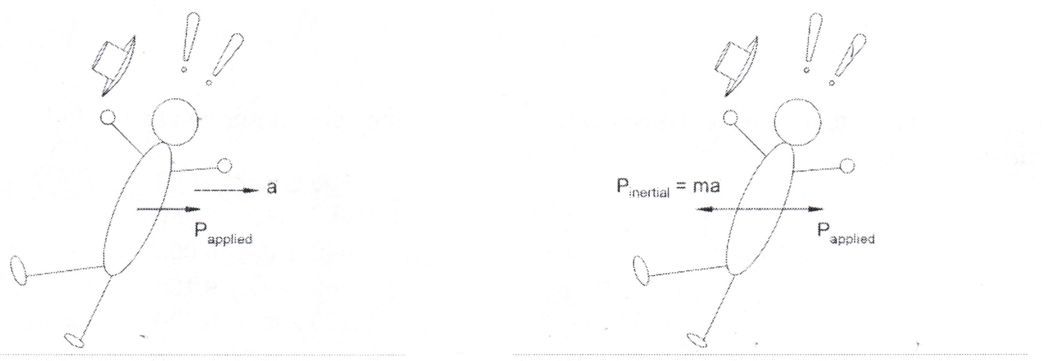
CIV102F Problem Set # 3 – September 26 and 27, 2019

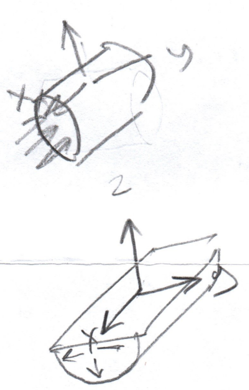
1. D'Alembert's principle permits us to analyze a dynamic system (i.e. a system with accelerating bodies) by converting it to an equivalent static system where the accelerating body has a fictitious inertial force equal to $F_{inertial} = ma$, acting in the direction opposite to the acceleration.

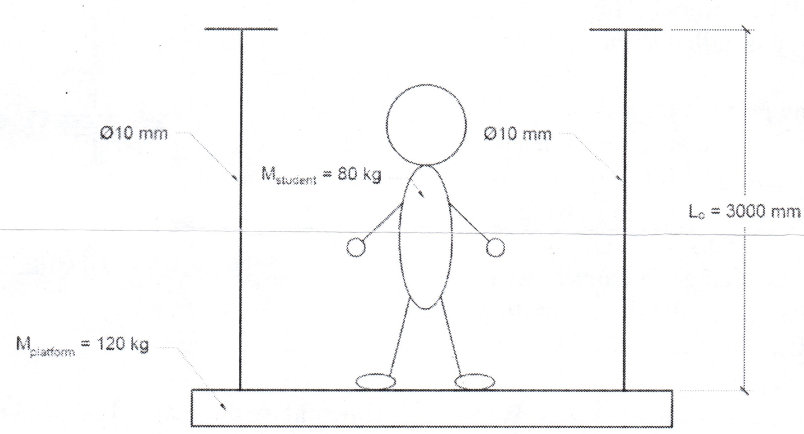


Dynamic system, not in equilibrium. $\sum F \neq 0$ *Equivalent static system in equilibrium.* $\sum F = 0$

A student weighing 80 kg jumps directly onto the centre of a platform which is supported by two ropes. Each rope is 10 mm in diameter and is made from a material with E = 10,000 MPa and σ_{ult} = 60 MPa. When the student lands onto the platform, the maximum length of the rope is measured to be 3015 mm.

a) Using a series of free body diagrams and D'Alembert's principle, calculate the maximum force that the platform exerts onto the student.
b) Calculate the amount of energy absorbed by the ropes. What is the factor of safety?
c) Calculate the natural frequency of the system containing the student standing on top of the platform.

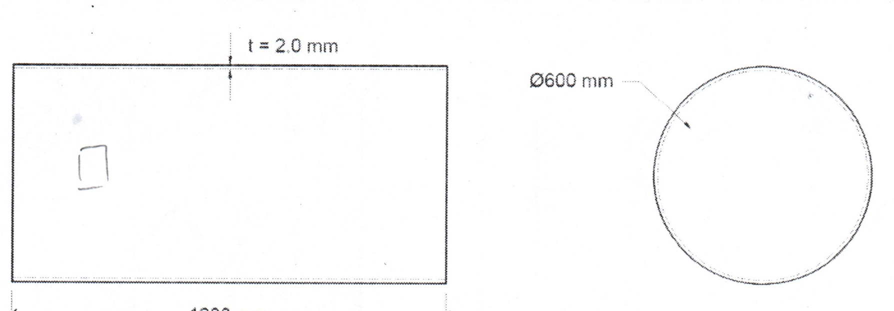




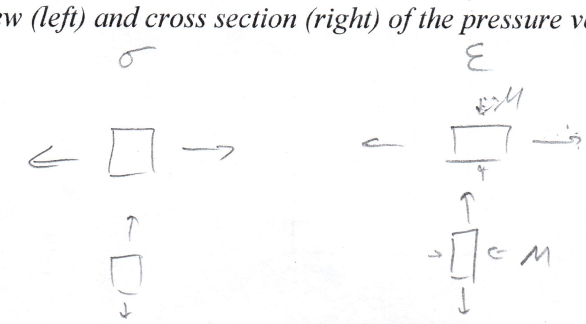
$\varepsilon_c = -\mu \varepsilon_L$

2. Consider a cylindrical thin-walled pressure vessel built with a material with a Young's modulus E and a Poisson's ratio μ. In the lab experiment you just did, you learned that the Poisson's ratio related the transverse strains to the longitudinal strains (i.e. $\varepsilon_y = -\mu\varepsilon_x$, and $\varepsilon_z = -\mu\varepsilon_x$ in x-y-z coordinates, where ε_x is the longitudinal strain and ε_y and ε_z are the transverse strains). For a thin-walled cylindrical pressure vessel, the Poisson's ratio can also be used in a similar way to relate the longitudinal and circumferential strains.

a) Using the equations presented in Lecture 9, derive the equations for the longitudinal and circumferential strains in a thin-walled cylinder which consider Poisson effects.
b) Consider the thin-walled pressure vessel shown below, which is made of aluminum alloy where μ = 0.35. An electronic gauge records that the longitudinal strain is 0.37×10^{-3} mm/mm. What is the gage pressure inside the cylinder?
c) Calculate the factor of safety against yielding of the aluminum alloy.



Elevation view (left) and cross section (right) of the pressure vessel in question 2b and 2c.



$\varepsilon_{LT} = \varepsilon_L - \mu \varepsilon_C$

$= \frac{DP}{4tE} - \mu \frac{DP}{2tE}$

$\varepsilon_{LT} = \frac{DP}{2tE}\left(\frac{1}{2} - \mu\right)$

$\sigma_L = \frac{DP}{4t}$

$E\varepsilon_L = \frac{DP}{4t}$

$\therefore \varepsilon_L = \frac{DP}{4tE}$

$\varepsilon_{CT} = \varepsilon_C - \mu \varepsilon_L$

$= \frac{DP}{2tE}\left(1 - \mu\frac{1}{2}\right)$

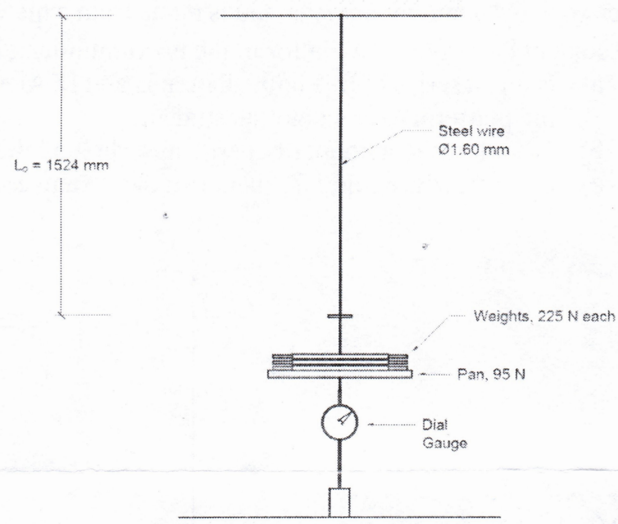
3. Suppose that the steel wire shown below has a stress-strain curve described by the following set of equations:

$$\sigma_s = \begin{cases} E_s\varepsilon_s & 0.000 \le \varepsilon_s \le 0.002 \\ \sigma_{yield} & 0.002 \le \varepsilon_s \le 0.040 \\ 4000(\varepsilon_s - 0.040) + \sigma_{yield} & 0.040 \le \varepsilon_s \le 0.065 \\ \sigma_{ult} & 0.065 \le \varepsilon_s \le 0.100 \\ -1500(\varepsilon_s - 0.100) + \sigma_{ult} & 0.100 \le \varepsilon_s \le 0.200 \end{cases}$$

Where E_s = 200,0000 MPa, σ_{yield} = 400 MPa and σ_{ult} = 500 MPa. For the following question, assume that a wire with the geometry described in the figure below is being tested to determine its stress-strain behaviour.

a) Plot the relationship between the wire tension P and the elongation Δ if the wire was tested by by gradually stretching the wire in small increments of Δ and measuring the tension in the wire. The graph does not need to be to scale along the ε axis but should highlight all key points. This methodology is called *displacement control*.

b) On the same graph, plot the relationship between the wire tension P and the elongation Δ if the wire was tested done by placing 225 N weights onto the 95 N loading pan and then measuring the change in length (i.e. in the same manner as the demonstration in tutorial 2). This methodology is called *force control*.



c) Explain the differences in results. Which experimental method would be preferred if one was trying to measure the "true" stress-strain curve of the material?

4. A 2 m long wire made of mild steel is attached to a ceiling and has a rigid catch-plate securely attached to the other end. The diameter of the wire is 16 mm for the top 1 m, and 6 mm for the bottom 1 m. Ignore the self-weight of the wire and the flange and neglect strain hardening.

A 4 kg weight is dropped onto the flange:

a) Calculate what height above the bottom flange can the weight be dropped so there is no plastic deformation in the wire.

The 4 kg weight is now dropped from a height of 1.9 m above the flange:

b) Calculate the maximum extension of the wire due to the weight landing on the flange.

c) The weight is dropped another nine times onto the flange from the same height. The weight is then removed from the experimental setup. What is the final length of the wire?