The Edward S. Rogers Sr. Department
of Electrical & Computer Engineering
UNIVERSITY OF TORONTO

ECE259: Electromagnetism

Final exam - Wednesday April 25, 2018
Instructors: Profs. Micah Stickel and Piero Triverio

Last name: Solutions / Marking

First name: ....

Student number: ....

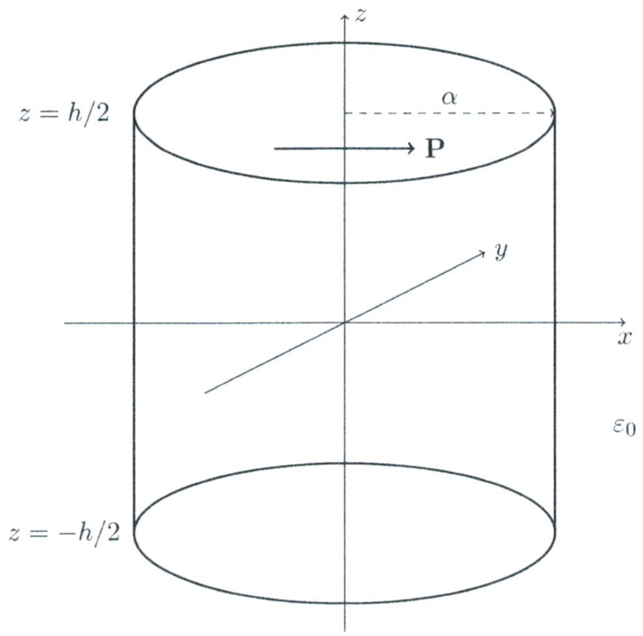
Instructions

- Duration: 2 hour 30 minutes (14:00 to 16:30)
- Exam Paper Type: A. Closed book. Only the aid sheet provided at the end of this booklet is permitted.
- Calculator Type: 2. All non-programmable electronic calculators are allowed.
- **Only answers that are fully justified will be given full credit!**

Marks:

Q1: /20	Q2: /20	Q3: /20	Q4: /20	TOTAL: /80

Question 1



The cylinder in the figure has radius α, height h and lies along the z axis with the origin in the middle. The cylinder is made by a perfect dielectric material and is polarized. The polarization vector is $\mathbf{P} = P_0\mathbf{a}_x$ with $P_0 > 0$.

(a) Find the density of **all** polarization charge distributions that may exist within or on the cylinder. [4 points]

$\rho_{p,v} = -\nabla\cdot\bar{P} = 0$ since $\bar{P}$ uniform] 1

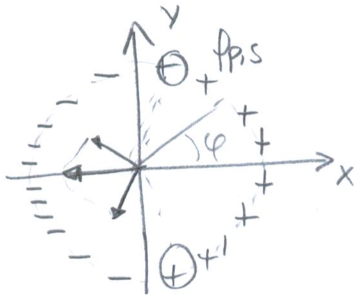
on top face: $\rho_{p,s} = -\bar{a}_n\cdot(\bar{P}_2 - \bar{P}_1) = -\bar{a}_z\cdot(0-\bar{P}) = \bar{a}_z\cdot\bar{P} = 0$ since $\bar{P}\perp\bar{a}_z$] 1

on bottom face: $\rho_{p,s} = 0$ since also on this face $\bar{P}\perp\bar{a}_z$] 1

on side: $\rho_{p,s} = -\bar{a}_r\cdot(0-\bar{P}) = P_0\,\bar{a}_r\cdot\bar{a}_x = P_0\cos\varphi$] 1

(b) Without doing calculations, determine the direction of the electric field **E** at the origin. [2 points]

$\bar{E}$ has no $\bar{a}_z$ component since the distribution of charge is symmetric with respect to the x-y plane



$\bar{E}$ has no $\bar{a}_y$ component because of symmetry with respect to x-z plane

$\bar{E}$ points in the direction of $-\bar{a}_x$

2

(c) Find the electric field **E** at the origin. [14 points]

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int_S \rho_{s}\, dS \, \frac{\bar{R}-\bar{R}'}{|\bar{R}-\bar{R}'|^3}$$

S: $r' = \alpha$ $\quad \varphi' \in [0, 2\pi]$ $\quad z' \in [-\frac{h}{2}, \frac{h}{2}]$

$dS = \alpha\, d\varphi'\, dz'$

2 integr. limits & dS

$\bar{R} = 0$

$\bar{R}' = \alpha\,\bar{a}_{r'} + z'\bar{a}_z = \alpha\cos\varphi'\,\bar{a}_x + \alpha\sin\varphi'\,\bar{a}_y + z'\,\bar{a}_z$

$|\bar{R}-\bar{R}'| = \sqrt{\alpha^2 + (z')^2}$

$\bar{R}-\bar{R}' = -\alpha\cos\varphi'\,\bar{a}_x - \alpha\sin\varphi'\,\bar{a}_y - z'\,\bar{a}_z$

4 position & distance vectors

$$\bar{E} = \frac{1}{4\pi\varepsilon_0}\int_{\varphi'=0}^{2\pi}\int_{z'=-\frac{h}{2}}^{+\frac{h}{2}} P_0 \cos\varphi' \alpha\, d\varphi' dz' \cdot \frac{-\alpha\cos\varphi'\bar{a}_x - \alpha\sin\varphi'\bar{a}_y - z'\bar{a}_z}{\left[\alpha^2 + (z')^2\right]^{3/2}}$$

$\bar{a}_y$ term integrates to zero since contains $\cos\varphi'\sin\varphi'$
$\bar{a}_z$ term integrates to zero since contains $\cos\varphi'$ 2

$$\bar{E} = -\frac{P_0\alpha^2\bar{a}_x}{4\pi\varepsilon_0}\int_{\varphi'=0}^{2\pi}\int_{z'=-\frac{h}{2}}^{\frac{h}{2}} \frac{\cos^2\varphi'\, d\varphi'\, dz'}{\left[\alpha^2+(z')^2\right]^{3/2}} =$$

$$= -\frac{P_0\alpha^2\bar{a}_x}{4\pi\varepsilon_0}\underbrace{\int_{\varphi'=0}^{2\pi}\cos^2\varphi' d\varphi'}_{\pi}\ \underbrace{\int_{z'=-\frac{h}{2}}^{\frac{h}{2}}\frac{dz'}{\left[\alpha^2+(z')^2\right]^{3/2}}}_{\left[\frac{z'}{\alpha^2\sqrt{\alpha^2+(z')^2}}\right]_{-\frac{h}{2}}^{\frac{h}{2}}} =$$

$$= -\frac{P_0\alpha^2\bar{a}_x}{4\varepsilon_0}\left[\frac{h}{2\alpha^2\sqrt{\alpha^2+h^2/4}} + \frac{h}{2\alpha^2\sqrt{\alpha^2+h^2/4}}\right] =$$

4

$$= -\frac{P_0\cancel{\alpha^2}\bar{a}_x}{4\varepsilon_0}\ \frac{h}{\cancel{\alpha^2}\sqrt{\alpha^2+h^2/4}} = \boxed{-\frac{P_0 h}{4\varepsilon_0\sqrt{\alpha^2+h^2/4}}\bar{a}_x}$$

Correct final answer: 2

Question 2

General Marking: −0.5 for minor mathematical or copy errors.

A very long wire with radius a lies along the z-axis and has a current density given by $\mathbf{J}_{inner} = J_0 r \mathbf{a}_z$. Coaxial to this wire is situated a very thin cylinder with radius b. The outer cylinder carries a total current that is equal and opposite to the inner conductor. You may assume that for these conductors, $\mu_r = 1$.

(a) Determine the magnetic field intensity, $\mathbf{H}$, everywhere. [10 points]

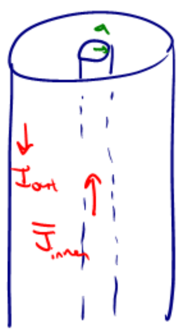
2 marks
Correct assumption for $\bar{H} = H_\phi \hat{a}_\phi$

5 marks $r<a$

2 - LHS $(H_\phi)(2\pi r)$

2 - RHS I_{enc}

1 - Final Statement



* Use Ampere's Law for the three regions $r<a$, $a \le r < b$, and $r > b$.

⇒ Assume that $\bar{H} = H_\phi \hat{a}_\phi$ 2

For $r<a$:

$$\oint_C \bar{H} \cdot \bar{dl} = I_{enc} = \iint_S \bar{J}_{inner} \cdot \bar{dS}$$

$$(H_\phi)(\cancel{2\pi} r) = \int_0^r \int_0^{2\pi} (J_0 u \hat{a}_z) \cdot (u \, d\phi \, du \, \hat{a}_z)$$

2   1 limits $u = r$ for integration

$$= J_0 (\cancel{2\pi}) \left(\frac{r^3}{3}\right)$$ 1 result

$$\therefore \bar{H} = \frac{J_0 r^2}{3} \hat{a}_\phi \left[\frac{A}{m}\right] \quad r<a$$ 1

2 marks $a \le r < b$

1 - LHS

1 - RHS

For $a \le r < b$

$$\oint_C \bar{H} \cdot \bar{dl} = \iint_S \bar{J}_{inner} \cdot \bar{dS}$$

$$H_\phi (\cancel{2\pi} r) = \int_0^a \int_0^{2\pi} (J_0 r)(r \, d\phi \, dr)$$ 1

$$= \cancel{2\pi} J_0 \frac{a^3}{3}$$ 1

$$\therefore \bar{H} = \frac{J_0 a^3}{3r} \hat{a}_\phi \left(\frac{A}{m}\right) \quad a \le r < b$$

1 mark $r>b$

1 - $I_{enc} = 0$ $\therefore \bar{H} = 0$

For $r>b$

$$\oint_C \bar{H} \cdot \bar{dl} = I_{enc} = I_0 - I_0 = 0$$

$$\therefore \bar{H} = 0 \left[\frac{A}{m}\right] \quad r>b$$

(b) Determine the stored energy per unit length of this coaxial system. [5 points]

3 Marks
Problem set-up
↳ $\mu_0|\bar{H}|^2$
↳ int. limits over 2 regions with correct $\bar{H}$

$$W_m' = \frac{1}{2}\iiint_{\text{unit length}} \bar{B}\cdot\bar{H}\,dv = \frac{1}{2}\iiint \mu_0|\bar{H}|^2\,dv \quad ①$$

$(\bar{B}=\mu_0\bar{H},\ \mu_r=1)$

$$= \frac{1}{2}\int_0^1\int_0^a\int_0^{2\pi} \mu_0\left(\frac{J_0 r^2}{3}\right)^2 r\,d\phi\,dr\,dz \ ① + \frac{1}{2}\int_0^1\int_a^b\int_0^{2\pi}\mu_0\left(\frac{J_0 a^3}{3r}\right)^2 r\,d\phi\,dr\,dz \ ①$$

2 marks
Evaluation & simplification

$$= \left(\frac{\mu_0}{\cancel{2}}\right)(\cancel{2}\pi)(1)\left(\frac{J_0^2}{9}\right)\left[\frac{r^6}{6}\Big|_0^a\right] + \left(\frac{\mu_0}{\cancel{2}}\right)(\cancel{2}\pi)(1)\left(\frac{J_0^2 a^6}{9}\right)\ln(b/a)$$

$$= \frac{\mu_0\pi J_0^2 a^6}{54} + \frac{\mu_0\pi J_0^2 a^6}{9}\ln(b/a) \quad ①$$

$$= \underline{\underline{\frac{\mu_0\pi J_0^2 a^6}{9}\left[\frac{1}{6}+\ln(b/a)\right]}} \ ① \quad [J/m]$$

Note: If they approach this by first finding L' from flux the result will be $L' = \frac{\mu_0}{2\pi}\left[\frac{1}{3}+\ln(b/a)\right]$ if they chose the simple flux formation.

⇒ Thus, if someone does this take <u>1 mark off</u> for part (c) <u>only</u> and give <u>full</u> marks for part (b) if process is correct (i.e. $W_m' = \frac{1}{2}L'I_0^2$ w/ correct I_0).

(c) Determine the inductance per unit length of this coaxial system. [5 points]

3 marks
Problem Set-up

$$* \text{ Using } L' = \frac{2W_m'}{I_0^2} \quad ②$$

In this case ① $I_0 = \iint \bar{J}_{inner}\cdot d\bar{s} = \int_0^a\int_0^{2\pi}(J_0 r)(r\,d\phi\,dr) = J_0(2\pi)\frac{a^3}{3}$

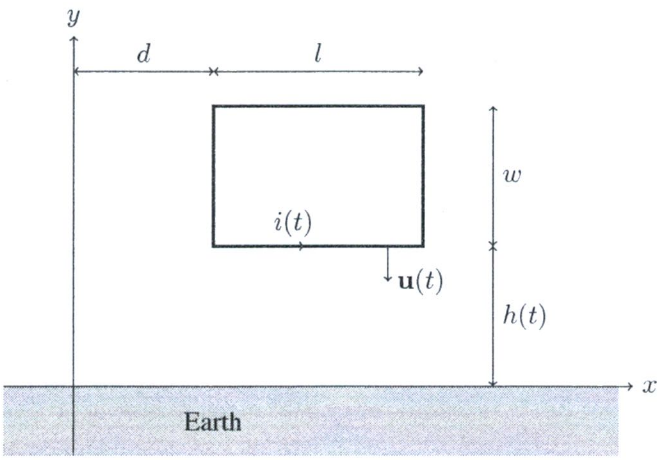
2 marks
Evaluation

$$\therefore L' = \frac{\cancel{2}}{\cancel{J_0^2}\,\frac{4\pi^2}{2}\,\frac{\cancel{a^6}}{\cancel{9}}}\left\{\frac{\mu_0\cancel{\pi}\cancel{J_0^2}\cancel{a^6}}{\cancel{9}}\left[\frac{1}{6}+\ln(b/a)\right]\right\} \quad ②$$

$$= \underline{\underline{\frac{\mu_0}{2\pi}\left[\frac{1}{6}+\ln(b/a)\right]}} \quad [H/m]$$

Q2 (c) (continued)

Question 3



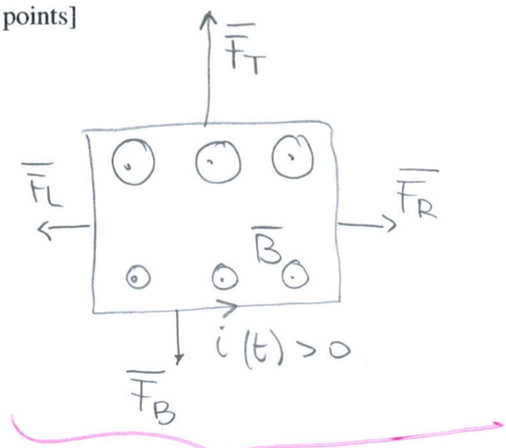
Consider the rectangular metallic frame shown in the figure. The frame is falling under the effect of gravity with velocity $\mathbf{u}(t) = -u_y(t)\mathbf{a}_y$ where $u_y(t) > 0$. The frame is rigid, has total resistance R and negligible inductance. A magnetic field $\mathbf{B} = B_0 y \mathbf{a}_z$ with $B_0 > 0$ is present in the region $y > 0$.

(a) Using Lenz's law, determine the sign of current $i(t)$. [2 points]

As frame falls, magnetic flux decreases $\Rightarrow i(t) > 0$ so it produces a positive contribution to flux that tries to oppose to the change (Lenz's law)

2

(b) Sketch the direction of the magnetic force acting on each edge of the frame. Briefly justify your answer. [2 points]



1

$$\underbrace{d\bar{F} = I\, d\bar{l} \times \bar{B}}_{\text{justification}}$$

1

(c) Do magnetic forces increase the frame velocity, decrease it, or leave it unchanged? Briefly justify your answer. [2 points]

$\overline{F_R} + \overline{F_L} = 0$

$|\overline{F_T}| > |\overline{F_B}|$ since $\overline{B}$ higher on top edge

$\Rightarrow$ decrease velocity

2

(d) Determine $i(t)$ using Faraday's law in the form $V_{emf} = -\frac{\partial}{\partial t}\Phi(t)$. Express $i(t)$ in terms of $u_y(t)$. [4 points]

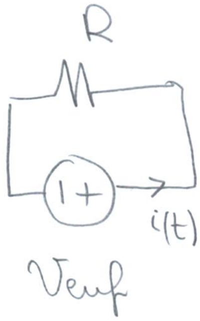
$$\Phi(t) = \int_{y=h(t)}^{h(t)+w} \int_{x=d}^{d+\ell} B_0 y \, \overline{a_z} \cdot \overline{a_z} \, dx\,dy = B_0 \ell \int_{h(t)}^{h(t)+w} y\,dy =$$

$$= B_0 \ell \frac{[h(t)+w]^2 - h^2(t)}{2} = \frac{B_0 \ell}{2}\left[2h(t)w + w^2\right]$$

2

$$V_{emf} = -\frac{B_0 \ell}{\cancel{2}} \cancel{2} w \underbrace{\frac{\partial h}{\partial t}}_{= -u_y(t)} = B_0 \ell w u_y(t)$$

1



$$i(t) = \frac{V_{emf}}{R} = \frac{B_0 \ell w}{R} u_y(t) > 0 \text{ as expected}$$

1

(e) Determine $i(t)$ using the alternative form of Faraday's law $V_{emf} = \underbrace{-\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot \mathbf{dS}}_{=0 \text{ since } B \text{ const.}} + \int_c (\mathbf{u} \times \mathbf{B}) \cdot \mathbf{dl}$. Express $i(t)$ in terms of $u_y(t)$. [4 points]

$$V_{emf} = \oint_C (\bar{u} \times \bar{B}) \cdot d\bar{\ell}$$

only top and bottom edge contribute to emf
left & right edge do not "cut" $\bar{B}$ lines

$$V_{top} = -\int_{x=d}^{d+\ell} (-u_y(t)\,\bar{a}_y \times B_0 y\,\bar{a}_z) \cdot \bar{a}_x\,dx = u_y(t)\,B_0\,\ell\,[h(t)+w]$$

$$V_{bot} = \int_{x=d}^{d+\ell} [-u_y(t)\,\bar{a}_y \times B_0 h(t)\,\bar{a}_z) \cdot \bar{a}_x\,dx = -u_y(t)\,B_0\,\ell\,h(t)$$

(2)

$$V_{emf} = V_{top} + V_{bot} = u_y(t)\,B_0\,\ell\,w$$ (1)

$$i(t) = \frac{V_{emf}}{R} = \frac{u_y(t)\,B_0\,\ell\,w}{R}$$ (1)

(f) Find the net magnetic force $\mathbf{F}_m$ acting on the frame, and express it in terms of $u_y(t)$. [6 points]

$$\bar{F}_m = \oint_C i(t)\,d\bar{\ell} \times \bar{B}$$

Forces on left and right edge cancel out

$$\bar{F}_{top} = -\int_{x=d}^{d+\ell} i(t)\,dx\,\bar{a}_x \times B_0[h(t)+w]\,\bar{a}_z = i(t)\,\ell\,B_0\,[h(t)+w]\,(+\bar{a}_y)$$

$$\bar{F}_{bot} = \int_{x=d}^{d+\ell} i(t)\,dx\,\bar{a}_x \times B_0 h(t)\,\bar{a}_z = i(t)\,\ell\,(-\bar{a}_y)\,B_0 h(t)$$

(4)

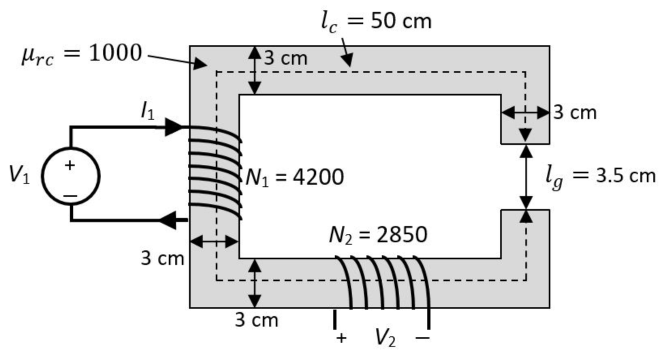
$$\bar{F}_m = \bar{F}_{top} + \bar{F}_{bot} = i(t)\,\ell\,w\,B_0\,\bar{a}_y = \frac{B_0^2\,\ell^2\,w^2}{R}\,u_y(t)\,\bar{a}_y$$ (2)

Pointing up as expected

Question 4.1

General Guideline -0.5 for minor mathematical, copy, or simple conceptual or concluding error.

For the magnetic circuit shown below, answer the following True/False questions. For this problem you can ignore the effects of fringing fields, and you can assume the core has a square cross-section (i.e., it extends 3 cm into the page). Both coils are tightly wound around the core. Briefly justify each of your answers with appropriate descriptions and/or calculations. [5 points]



(a) (True / False) The magnitude of the magnetic field intensity, H, is larger in the air gap than in the magnetic core.

2 marks

$\Rightarrow$ At the interface $B_{n1} = B_{n2} \Rightarrow B_{core} = B_{air}$ ①

$\therefore B_{core} = 1000\mu_0 H_{core} = \mu_0 H_{air}$

$\Rightarrow \underline{H_{air} = 1000 H_{core}}$ ①

(b) (True / False) The reluctance of the air gap is smaller than that of the magnetic core.

1 mark

$$R_g = \frac{l_g}{\mu_0 S}, \quad R_c = \frac{l_c}{1000\mu_0 S}$$

$$\therefore \frac{R_g}{R_c} = \frac{1000 l_g}{l_c} = \frac{1000(50\text{ cm})}{3.5\text{cm}} \underline{> 1}$$ ①

(c) (True / False) The self-inductance of the second coil at the bottom is $L_{22} = 259$ mH (rounded to the nearest mH).

2 marks

With coil #1 turned off and coil #2 turned on we have

$$\Phi_{22} = \frac{N_2 I_2}{R_c + R_g} \text{ ① } \Rightarrow L_{22} = \frac{N_2 \Phi_{22}}{I_2} = \frac{N_2^2}{\frac{l_c}{1000\mu_0 S} + \frac{l_g}{\mu_0 S}} \text{ ①}$$

$$\therefore L_{22} = \frac{(2850)^2}{\frac{0.50}{1000\mu_0(0.03)^2} + \frac{0.035}{\mu_0(0.03)^2}} = \underline{259 \text{ mH}}$$

Question 4.2

Marking: 3 marks correct graph shapes (1 each); 3 marks correct values: 0.5 – $E, D = 0$ @ $R = 0$; 0.5 – D @ $R = 4$ cm; 1 – E @ 4, 6 cm; 1 – V_0, V_1

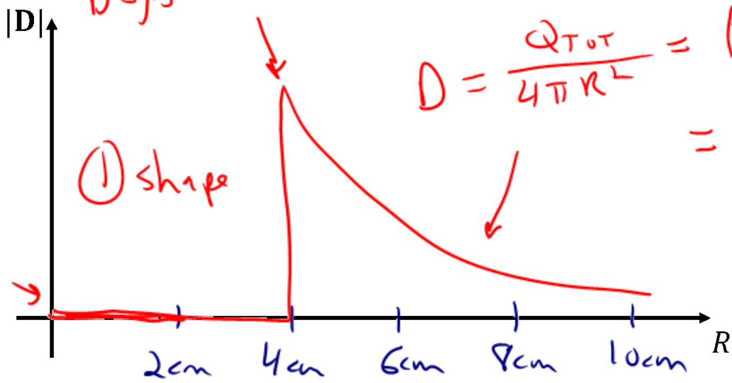
A solid perfectly conducting sphere of radius $R = 4$ cm is centered on the origin and has a charge density of $\rho_S = 2$ mC/m^2 on its surface. It is surrounded by a spherical dielectric shell $\varepsilon_r = 5$ that extends from $R = 4$ cm to $R = 6$ cm. On the axes below, sketch the variation of the magnitudes of the electric field intensity, electric flux density, and the electric scalar potential (with $V(R = \infty) = 0$). Your plots should include values at key points (i.e., $R = 0$ cm, $R = 4$ cm, and $R = 6$ cm) and should extend from $R = 0$ cm to $R = 10$ cm. [6 points]

0.5

$D = \rho_s = 2$ mC/m^2

$$D = \frac{Q_{TOT}}{4\pi R^2} = \frac{\rho_s (4\pi)(4\text{cm})^2}{4\pi R^2} = \frac{3.2\times10^{-6}}{R^2}\ \text{C/m}^2$$

D = 0



0.5

0.5

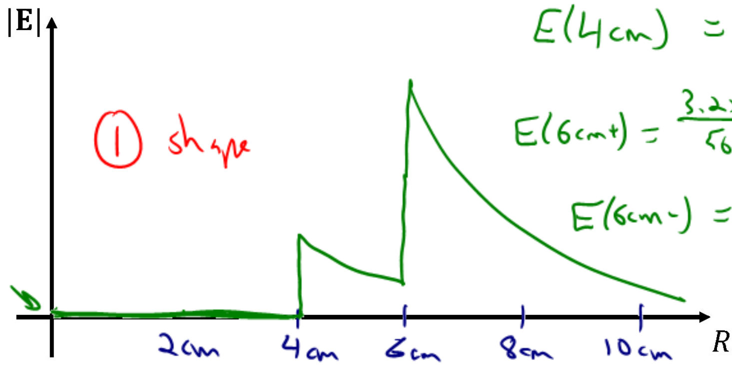
$$E(4\text{cm}) = \frac{D}{5\varepsilon_0} = \frac{\rho_s}{5\varepsilon_0} = \frac{4\times10^{-4}}{\varepsilon_0} = 4.52\times10^{7}\ \text{V/m}$$

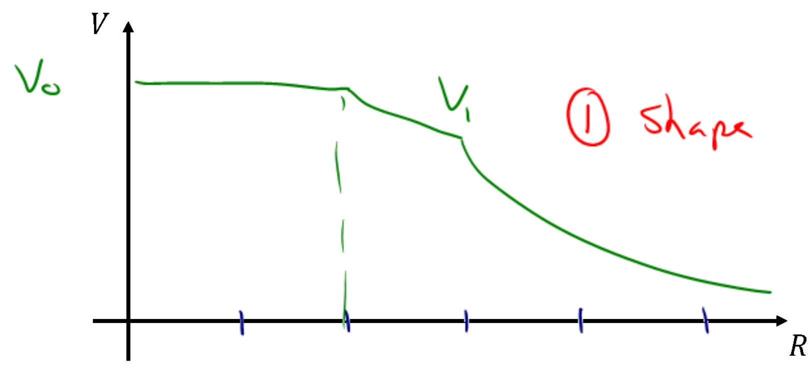
0.5

$$E(6\text{cm}+) = \frac{3.2\times10^{-6}}{\varepsilon_0(6\text{cm})^2} = \frac{8.89\times10^{-4}}{\varepsilon_0} = 10.04\times10^{7}\ \text{V/m}$$

$$E(6\text{cm}-) = \frac{3.2\times10^{-6}}{5\varepsilon_0(6\text{cm})^2} = \frac{1.78\times10^{-4}}{\varepsilon_0} = 2.01\times10^{7}\ \text{V/m}$$

E = 0





0.5

$$V_1 = -\int_{\infty}^{6\text{cm}} \frac{3.2\times10^{-6}}{\varepsilon_0 R^2}\,dR = \frac{+3.2\times10^{-6}}{\varepsilon_0}\left[+\frac{1}{R}\Big|_{\infty}^{6\text{cm}}\right] = \frac{3.2\times10^{-6}}{\varepsilon_0(6\text{cm})} = 6.02\times10^{6}\ \text{V}$$

$$V_0 = V_1 - \int_{6\text{cm}}^{4\text{cm}} \frac{3.2\times10^{-6}}{5\varepsilon_0 R^2}\,dR = V_1 + \frac{3.2\times10^{-6}}{5\varepsilon_0}\left[+\frac{1}{R}\Big|_{6\text{cm}}^{4\text{cm}}\right] = 6.02\times10^{6} + 6.02\times10^{5} = 6.62\times10^{6}\ \text{V}$$

0.5

Question 4.3

General Guideline: −0.5 for minor mathematical, copy, or simple conceptual or concluding error.

A sphere of radius a that is made of a conductive dielectric ($\sigma = \sigma_0$ and $\varepsilon = \varepsilon_r\varepsilon_0$) is centered about the origin. The sphere is charged at $t = 0$ s with a uniform charge density given by $\rho_v(t = 0) = \rho_0$ for all $R \leq a$, where ρ_0 is a positive constant.

(a) Starting from the continuity equation, $\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$, prove that the charge density within the dielectric sphere varies according to $\rho_v(t) = \rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}$. [3 points]

1 - $\bar{J} = \sigma\bar{E}$

1 - Gauss's Law

1 - SolD

Since $\bar{J} = \sigma\bar{E} \rightarrow \bar{\nabla}\cdot(\sigma\bar{E}) = -\frac{\partial\rho_v}{\partial t} \rightarrow \bar{\nabla}\cdot\bar{E} = -\frac{1}{\sigma}\frac{\partial\rho_v}{\partial t}$ (1)

since σ does not depend on position.

From Gauss's Law: $\bar{\nabla}\cdot\bar{E} = \frac{\rho_v}{\varepsilon_r\varepsilon_0}$ (1)

$$\frac{\rho_v}{\varepsilon_r\varepsilon_0} = -\frac{1}{\sigma_0}\frac{\partial\rho_v}{\partial t} \rightarrow \frac{\partial\rho_v}{\partial t} + \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_v = 0. \quad (1)$$

$\Rightarrow$ The solution to this type of DE has the form $\rho_v(t) = \rho_0 e^{-mt}$

$$\therefore \cancel{\rho_0}(-m)\cancel{e^{-mt}} + \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\cancel{\rho_0}\cancel{e^{-mt}} = 0 \qquad \therefore m = -\frac{\sigma_0}{\varepsilon_r\varepsilon_0}$$

$$\therefore \rho_v(t) = \rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}$$

(b) If it is known that at $t = 0$ s the conduction current density within the sphere is given by $\mathbf{J}(R, t = 0) = \frac{\rho_0\sigma_0}{3\varepsilon_r\varepsilon_0}R\mathbf{a}_R$, determine the expression for the conduction current density for $t \geq 0$ s. Hint: Assume this current density is only a function of R. [3 points]

1 - Simplified divergence equation

1 - Integration

1 - Conclusion $C = ?$

Assume $\bar{J} = J_R\hat{a}_R$

$$\therefore \underbrace{\bar{\nabla}\cdot\bar{J} = \frac{1}{R^2}\frac{d(R^2 J_R)}{dR} = -\frac{\partial\rho_v}{\partial t} = -\frac{\partial}{\partial t}\left[\rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}\right] = \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}}_{(1)}$$

$$\int d(R^2 J_R) = \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}\int R^2\,dR$$

$$R^2 J_R = \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}\left[\frac{R^3}{3} + C\right] \quad (1)$$

$$J_R = \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_0 e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}\left[\frac{R}{3} + \frac{C}{R^2}\right]$$

For $t=0$, $J_R(R, t=0) = \frac{\sigma_0}{\varepsilon_r\varepsilon_0}\rho_0\left[\frac{R}{3} + \frac{C}{R^2}\right] = \frac{\sigma_0\rho_0 R}{3\varepsilon_r\varepsilon_0} \rightarrow \therefore \underline{C = 0}$ (1)

$$\therefore \bar{J} = J_R\hat{a}_R = \underline{\frac{\sigma_0\rho_0 R}{3\varepsilon_r\varepsilon_0}e^{-\frac{\sigma_0 t}{\varepsilon_r\varepsilon_0}}\hat{a}_R}$$

(c) Find the ratio of the magnitude of the conduction current density relative to the magnitude of the displacement current density for $t \geq 0$ s. [3 points]

From part (b) $\bar{J}_{cond} = \frac{\sigma_0 \rho_0 R}{3\varepsilon_r \varepsilon_0} e^{-\frac{\sigma_0 t}{\varepsilon_r \varepsilon_0}} \hat{a}_R$

1 - $\bar{E}$ from $\bar{J}_{cond}$

But $\bar{E} = \frac{\bar{J}_{cond}}{\sigma} = \frac{\rho_0 R}{3\varepsilon_r \varepsilon_0} e^{-\frac{\sigma_0 t}{\varepsilon_r \varepsilon_0}} \hat{a}_R$ 1

1 - Displacement current

The displacement current is 1

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \varepsilon_r \varepsilon_0 \frac{\partial \bar{E}}{\partial t} = \cancel{\varepsilon_r \varepsilon_0} \left(\frac{\rho_0 R}{3\cancel{\varepsilon_r \varepsilon_0}}\right)\left(-\frac{\sigma_0}{\varepsilon_r \varepsilon_0}\right) e^{-\frac{\sigma_0 t}{\varepsilon_r \varepsilon_0}} \hat{a}_R$$

$$= -\frac{\sigma_0 \rho_0 R}{3\varepsilon_r \varepsilon_0} e^{-\frac{\sigma_0 t}{\varepsilon_r \varepsilon_0}} \hat{a}_R$$

1 - ratio

$$\therefore \frac{|\bar{J}_{cond}|}{|\bar{J}_d|} = \underline{\underline{1}}$$ 1