1. [10 pts.] Two objects of equal mass m collide elastically. After the collision, one object has velocity v_{1f}, but the second object is no longer moving: $v_{2f} = 0$. What were the initial velocities before the collision? Assume the objects are constrained to move along a line. Express your answer in terms of m and v_{1f}.

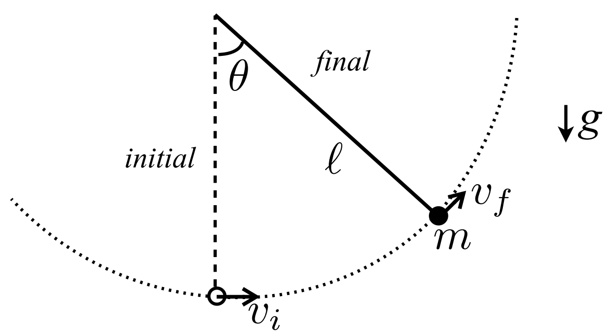
2. [20 pts.] A metallic weight with $m = 4.00\,\mathrm{kg}$ swings on a massless rod whose length is $\ell = 0.500\,\mathrm{m}$, under gravitational acceleration. At the bottom of its trajectory, the velocity is $8.00\,\mathrm{m/s}$. At some later time t_f, the mass has swung through $\pi/4$ angle (or 45.0 degrees) to a height $h = \ell(1 - 1/\sqrt{2})$.

   (a) [5 pts.] What is the speed v_f ?

   (b) [5 pts.] What is the tangential acceleration a_t at time t_f?

   (c) [5 pts.] How much work has been done by the rod on the mass, during this interval?

   (d) [5 pts.] How much work is done by gravity on the mass, during the same interval?



Problem 2.

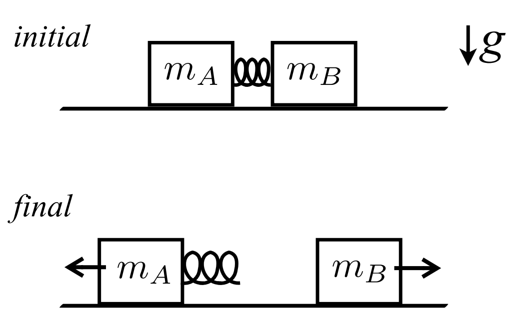
3. [30 pts.] Two blocks of masses $m_A = 2.00\,\mathrm{kg}$ and $m_B = 3.00\,\mathrm{kg}$ are placed on a horizontal frictionless surface. A nearly massless spring with $k = 3.00\,\mathrm{N/m}$ is attached to the left block, and the blocks are pushed together with the spring between them, putting $20.0\,\mathrm{J}$ of potential energy into the spring. The blocks are then released.

   (a) [5 pts.] By what amount (distance) is the spring compressed?

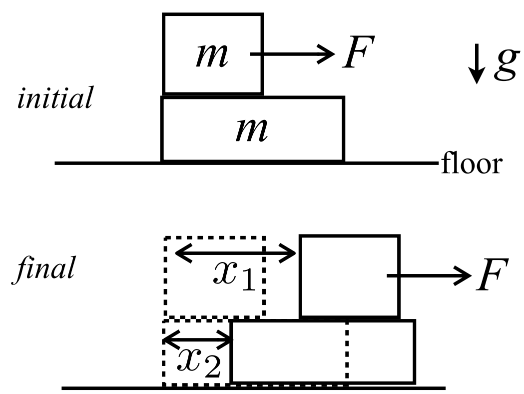
   (b) [5 pts.] What is the final kinetic energy, if the surface is frictionless?

   (c) [10 pts.] If the surface is frictionless, what is the final speed of each block?

   (d) [10 pts.] If instead the surface has a coefficient of friction $\mu_k = 0.300$, how far will each block slide before coming to rest?



Problem 3.



Problem 4.

4. [30 pts.] A stack of two blocks sits on a frictionless surface; however, between the two blocks is a static coefficient of friction μ_s and a kinetic coefficient of friction μ_k. External force F is applied to the top block. During the time the force is applied, the top block is displaced by x_1, and the bottom block is displaced by x_2. Assume enough force is applied that $x_1 > x_2$, as shown above. Express your answers in terms of m, x_1, x_2, F, g, μ_s, and μ_k:

   (a) [5 pts.] What is the magnitude of the frictional force between the two blocks?

   (b) [5 pts.] What is the final velocity of the centre of mass?

   (c) [10 pts.] What is the final kinetic energy of the system?

   (d) [10 pts.] How much energy is lost to heat?

Q1 [10pts]

We know from class that in 1D ellastic collisions,

$$(V_{rel})_i = -(V_{rel})_f .$$

$$V_{1i} - V_{2i} = -(V_{1f} - \underbrace{V_{2f}}_{=0 \text{ in this problem}})$$

so $\quad V_{1i} - V_{2i} = -V_{1f} \quad \ldots Eq(1)$

combine with conservation of momentum,

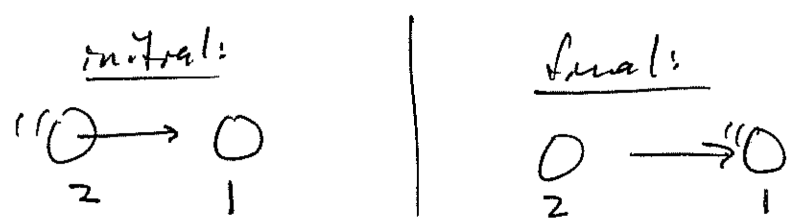
$$p_i = p_f$$

$$m V_{1i} + m V_{2i} = m V_{1f} + \cancel{m V_{2f}}^{\,0}$$

$\rightarrow \quad V_{1i} + V_{2i} = V_{1f} \quad \ldots Eq(2)$

Add these 2 eqs & find that $\boxed{V_{1i} = 0}$

in which case $\boxed{V_{2i} = V_{1f}}$.



Alternately can rederive $(V_{rel})_i = -(V_{rel})_f$ result, by conserving kinetic energy:

$$K_i = K_f$$

$$\frac{1}{2} m V_{1i}^2 + \frac{1}{2} m V_{2i}^2 = \frac{1}{2} m V_{1f}^2 + \cancel{\frac{1}{2} m V_{2f}^2} \rightarrow 0$$

cancel $m/2$, rearrange:

$$V_{1i}^2 - V_{1f}^2 = -V_{2i}^2$$

$$\underbrace{(V_{1i} - V_{1f})}_{= -V_{2i} \text{ by conservation of momentum, Eq (2) on previous page}} (V_{1i} + V_{1f}) = -V_{2i}^2$$

so

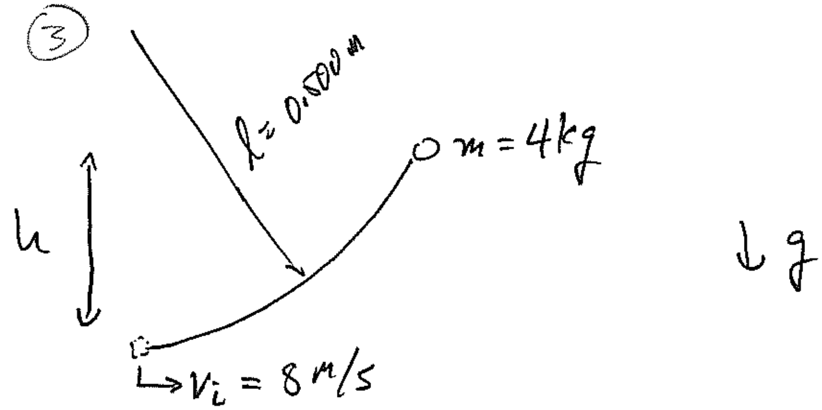
$$-V_{1i} - V_{1f} = -V_{2i}$$

or $V_{1i} - V_{2i} = -V_{1f}$, which is Eq (1) rederived.

Another initial condition that conserves p & E is that $V_{1i} = V_{1f}$, $V_{2i} = V_{2f} = 0$.... but this means no collision occured!

Q2 [20pts]

③



[5pts]

(a) Conservation of energy gives $E_i = E_f$, which we can apply in this case because the system is energetically isolated, if we include mass + earth + gravity in system.

So $K_i = \frac{1}{2} m v_i^2$, $K_f = \frac{1}{2} m v_f^2$

$U_f - U_i = \Delta U = mgh = mg\ell(1 - 1/\sqrt{2})$ as given.

thus $K_f = K_i + U_i - U_f$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 - mgh$$

$$v_f^2 = v_i^2 - 2gh$$

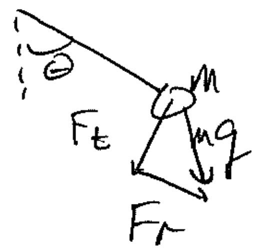
$$v_f = \sqrt{v_i^2 - 2gh} = \underline{7.82\ m/s}$$

$(8.00)^2$ 9.80

$h = \ell(1 - 1/\sqrt{2})$
$= 0.146\ m$

Q2 cont

(b) [5 pts] Tangential acceleration is provided by the tangential component of gravity.



$$F_t = -mg \sin\theta$$

$$= -4\,\text{kg} \cdot 9.8\ \text{m/s}^2 \cdot \frac{1}{\sqrt{2}}$$

$$\boxed{F_t = -27.7\ \text{N}}$$

Why negative? Restoring force, pulling in negative theta direction. Oh, wait, asking for a_t:

$$a_t = -9.80\ \text{m/s}^2 \cdot \underbrace{\sin\theta}_{1/\sqrt{2}}$$

$$\boxed{a_t = -6.93\ \text{m/s}^2}$$

Q2 cont. (3)

(c) [5 pts.] Work = $\int \vec{F} \cdot d\vec{r}$, but the rod always ~~e~~exerts a radial force, that is ~~...~~ perpendicular to $d\vec{r} = \vec{v}\,dt$.

$$\rightarrow \boxed{W = 0} \text{ for rod.}$$

(d) [5 pts.] Gravity is the force that removes kinetic energy from the moving mass.

$$\Delta K = W = -\Delta U = -mg\underbrace{h}_{= l(1-1/\sqrt{2}) = 0.146\,m}$$

$$\rightarrow \boxed{W = -5.72\,J}$$

Accept answers within 2%

Accept negative or positive sign, since question asks "how much".

Q3. [30pts]

(a) $U = \frac{1}{2}kx^2$ for a spring, where x is displacement
[5pts] from equilibrium, or compression in this case.

So $\frac{1}{2}kx_i^2 = U_i$

$$\rightarrow x_i = \sqrt{2U_i/k}$$

$$= \sqrt{2 \cdot 20J / 3\,N/m}$$

$$= \boxed{3.65\ m}$$

(b) Final kinetic energy is initial potential
[5pts] energy:

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f \quad \text{but } K_i = 0 \ \&\ U_f = 0$$

So $\boxed{K_f = U_i = 20.0\,J}$

(c) By conservation of momentum, after release
[10pts]

$$m_A V_A + m_B V_B = p_f \overset{\downarrow}{=} \underbrace{p_i = 0}_{\text{since } V_A = V_B = 0 \text{ initially}}$$

This means that $V_A = -\left(\frac{m_B}{m_A}\right)V_B$ or $\frac{V_A}{V_B} = -\frac{m_B}{m_A}$

So ratio of kinetic energies is $\frac{\frac{1}{2}m_A V_A^2}{\frac{1}{2}m_B V_B^2} = \frac{m_A}{m_B}\left(\frac{m_B}{m_A}\right)^2 = \frac{m_B}{m_A} = \frac{3}{2}$

Solve $\frac{K_A}{K_B} = \frac{3}{2}$ and $K_A + K_B = 20J$ $\rightarrow$ $K_A = 12.0\,J$
$K_B = 8.0\,J$

Q3 (c), cont.

Velocity A: $\frac{1}{2} m_A V_A^2 = K_A = 12.0 J \rightarrow \boxed{|V_A| = 3.46 \, m/s}$

B: $\frac{1}{2} M_B V_B^2 = K_B = 8.00 J \rightarrow \boxed{|V_B| = 2.31 \, m/s}$

There's probably a faster way of doing this, without finding K_A & K_B. For instance, just write down conservation of momentum & cons of energy:

$$M_A V_A + M_B V_B = 0$$

$$\frac{1}{2} M_A V_A^2 + \frac{1}{2} M_B V_B^2 = U_i$$

Solve for V_B in first eq, & sub into second:

$$V_B = -\frac{M_A}{M_B} V_A \xrightarrow{\text{sub:}} \underbrace{\left(\frac{1}{2} M_A + \frac{1}{2} M_B \left(\frac{M_A}{M_B}\right)^2\right)}_{= \frac{1}{2} M_A \left(1 + \frac{M_A}{M_B}\right) = 1.67 \, kg} V_A^2 = U_i$$

$\hookrightarrow$ gives $|V_A| = 3.46 \, m/s$

Then use $|V_B| = \left| -\frac{M_A}{M_B} V_A \right| \rightarrow |V_B| = 2.31 \, m/s$

Question asks for "speed", so answers given should be positive.

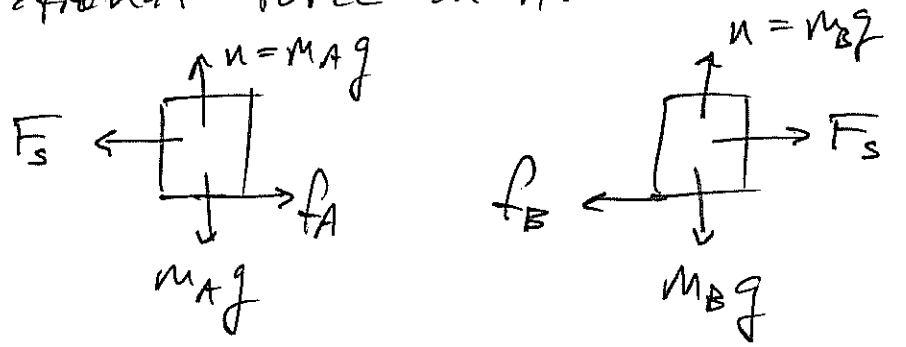
(Give full credit for answers within 2%.)

Q3(d) [10pts]

This question is too difficult to solve correctly, with analytics. Principles involved are:

(1) Energy lost from mass + spring system, because friction creates a thermal internal energy $f_k \cdot d$, where d is displacement.

(2) Momentum not conserved because frictional force on B is stronger than frictional force on A.



$f_A = \mu_k m_A g$ $\qquad$ $f_B = \mu_k m_B g$

Q3(d), cont.

We can say that spring does not expand fully. Compression from part (a) is ≈ 3.65 m. Pushing blocks apart this distance would generate at least

$$fd = \underbrace{(\mu m_A g)}_{5.68\,N}(3.65\,m) = 21.5\,J$$

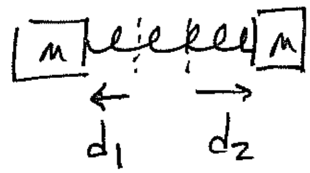
of heat,

which is more than $U_i = 20.0$ J. Thus, blocks are still touching spring at final positions.

Q3(d) – cont. (10)

If masses were equal, situation would be simpler because momentum conserved (=0).

Then: by symmetry, $d_1 = d_2 \equiv d$



In final configuration, $\Delta U = \overbrace{-f(d_1+d_2)}^{\text{energy lost to heat}} = -2fd$,
$= U_f - U_i$

and $\Delta U = -U_i + \frac{1}{2}k(x_i+2d)^2$, where $x_i = -3.65\,m$

This is a quadratic equation:

$$\cancel{-U_i + \tfrac{1}{2}kx_i^2} + 2kx_i d + 2kd^2 = -2fd$$

$$-d\,\underbrace{(2kd + 2kx_i + 2f)} = 0$$

d=0 soln says energy conserved @ initial condition

$\hookrightarrow d = -x_i - \frac{f}{k} = \underline{\underline{1.69\,m}}$

(<u>check</u>: $2d = 3.38\,m$, less than $x_i = 3.65\,m$ compression, so assumption valid.)

Lost to heat: $2fd = 19.9\,J$

Remaining in spring: $\frac{1}{2}k(x_i+2d)^2 = 0.1\,J$

|| Almost expanded, but not quite ||

Q4 [30 pts]

[5 pts]
(a) Question states that $x_1 > x_2$, so it means blocks must have slid with respect to one another. But in fact, they must *always* be sliding, since the forces are constant & accelerations constant, during the problem.

Thus $\underline{f = f_K = \mu_K m g}$

This force is equal & opposite between the blocks, but magnitude (asked for) is positive.

[5 pts]
(b) CM moves & accelerates by the net external force, which is just F here.

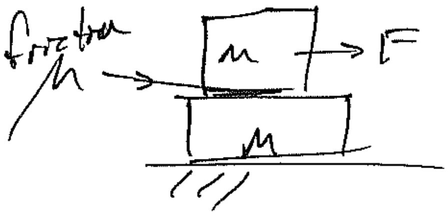
So $a_{cm} = \frac{F}{2m}$.

Final position of CM is $\frac{x_1 + x_2}{2}$, so using $V_f^2 - V_i^2 = 2 a \Delta x$, we find

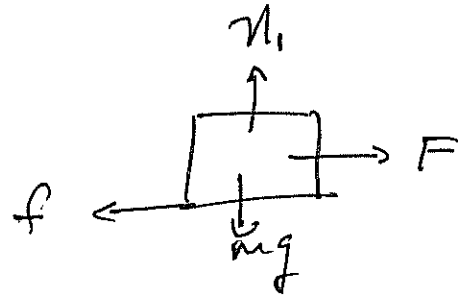
$$\boxed{(V_f)_{cm} = \sqrt{\frac{F}{2m}(x_1 + x_2)}}$$

Q4 (b) [10 pts]

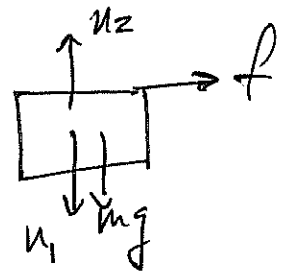
For this part the motion of the individual blocks is required.



top block:



bott block:



Vertical equilibrium gives $n_1 = mg$, so $f = \mu mg$, as said in part (a).

Horizontal degree of freedom gives

$x_1 = \frac{1}{2} a_1 t^2$ where $m a_1 = F - f$

$v_1 = a_1 t$ so $K_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (a_1 t)^2 = m x_1 a_1$

$= x_1 (F - f)$

$x_2 = \frac{1}{2} a_2 t^2$ where $m a_2 = f$

$v_2 = a_2 t$ $K_2 = \frac{1}{2} m v_2^2 = \ldots = m x_2 a_2 = x_2 f$

So final kinetic energy is

$$K_1 + K_2 = x_1 (F - f) + x_2 f$$

$$= \boxed{x_1 F - f(x_1 - x_2)} \text{ where } f = \mu mg$$

Q4 (d) [10 pts]

How much work is lost to heat?

External work = $F \cdot x_1$

Change in system energy is

$$\Delta E = \underbrace{\Delta K}_{= F x_1 - f(x_1 - x_2)} + \underbrace{\Delta U}_{=0} + \Delta E_{int}$$

ΔE_{int} must be $f(x_1 - x_2)$ to conserve energy!

$$\rightarrow \boxed{\Delta E_{int} = f(x_1 - x_2)}$$

Note that $W \neq F \cdot \Delta x_{cm}$, but rather $F \cdot x_1$, because displacement at point of application is what counts.

Q4, cont. ALTERNATE (14) SOLN TO PART (B):

$$X_1 + X_2 = \frac{1}{2} a_1 t^2 + \frac{1}{2} a_2 t^2$$

$$= \frac{1}{2} \frac{F-f}{m} t^2 + \frac{1}{2} \frac{f}{m} t^2$$

$$= \frac{F}{2m} t^2$$

So $X_{cm} = \frac{X_1 + X_2}{2} = \frac{1}{2}\left(\frac{F}{2m}\right) t^2$

$$V_{cm} = \frac{d}{dt} X_{cm} = \left(\frac{F}{2m}\right) t$$

But we were asked to write answer in terms of X_1, X_2, etc. ... not t. This can be written in several ways:

$t^2 = \frac{2X_1}{a_1} = \frac{2X_1}{F-f} m$ using $X_1(t)$

$t^2 = \frac{2X_2}{a_2} = \frac{2X_2}{f} m$ using $X_2(t)$

$t^2 = \frac{2X_{cm}}{a_{cm}} = (X_1 + X_2)\left(\frac{2m}{F}\right)$ using X_{cm}

In the answer I gave for (b), this last form was used because it showed independence of f, μ, g...

$$t = \sqrt{(X_1 + X_2) \frac{2m}{F}}$$

so $V_{cm} = \frac{F}{2m} t = \sqrt{\frac{F}{2m}(X_1 + X_2)}$ as before.

However, any correct form gets full credit.