ESC103F Engineering Mathematics and Computation: Tutorial #4

Question 1: Prove the following statement:

If one or both of the sides of a right angle is parallel to one of the projection planes, the orthogonal projection of the angle on that plane is also a right angle.

In your answer, let the coordinates of the three points forming the right angle be

$$A = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix},\ B = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix},\ C = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}$$

and consider the projection of angle ABC on the horizontal plane π_1 shown in the attached figure, where the plane π_1 corresponds to $z = 0$.

Question 2: Consider the points located at A(1,1,1), B(2,2,3) and C(6,1,10).

a) Find the true angle ABC with B at the vertex.

b) Find the apparent angle A'B'C' when ABC is projected orthogonally onto the x-y plane.

c) Find the apparent angle A"B"C" when ABC is projected orthogonally onto the x-z plane.

Question 3: In linear algebra, we say that three vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly dependent if one of the vectors is a linear combination of the other two. If a collection of vectors is not linearly dependent, it is said to be linearly independent.

A more precise way of stating this is as follows: the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent if and only if the only linear relationship among the vectors:

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$

is the trivial one, i.e. scalars $c_1 = c_2 = c_3 = 0$.

Which of the following sets are linearly independent:

a) $\{\vec{0}, \vec{v}, \vec{w}\}$

b) $\{\vec{v}, \vec{w}, 3\vec{v} - 4\vec{w}\}$

c) $\{\vec{v}, \vec{w}, \vec{v} \times \vec{w}\}$

assuming $\vec{v}$ and $\vec{w}$ are non-zero, non-parallel vectors? Explain your answers as fully as possible. (Note: a proof is required for part c).

Question 4: Consider the transformation $T(\vec{u}) = \begin{bmatrix} x \\ x+y \\ x+y+z \end{bmatrix}$ where $\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is a vector in R^3.

a) Is T a linear transformation? Justify your answer.

b) Develop the matrix associated with this transformation T.

c) What does the transformation T do to the unit cube shown in the attached figure? Provide a sketch of your answer directly on the figure.

Question 5: Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 3 & -1 \end{bmatrix}$.

a) Compute A^3.

b) Show that $A^3 = 9A - 8I$ where I is the identity matrix.

c) Solve for scalars a, b, c so that $A^6 = aA^2 + bA + cI$.

Question 6: Let *K* represent the transformation in R^3 associated with reflection in the x-y plane and let *J* represent the transformation in R^3 associated with reflection in the y-z plane, i.e.

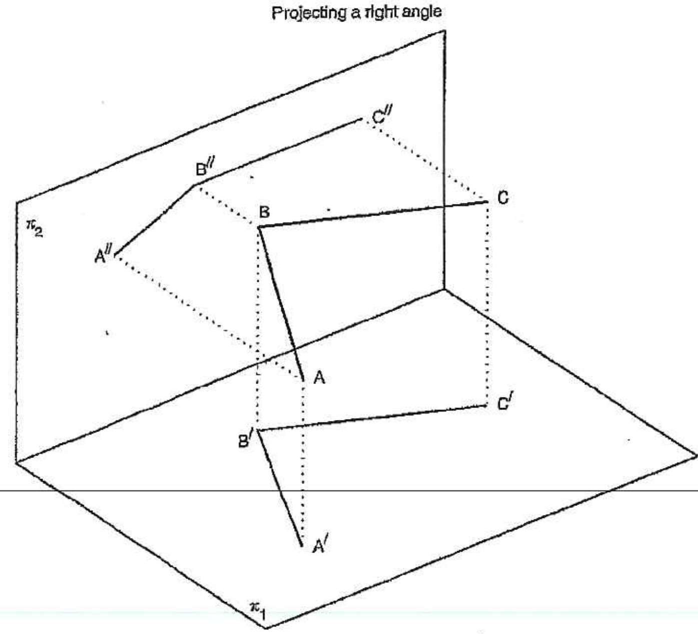
$$K\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ -z \end{bmatrix} \text{ and } J\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \\ z \end{bmatrix}.$$

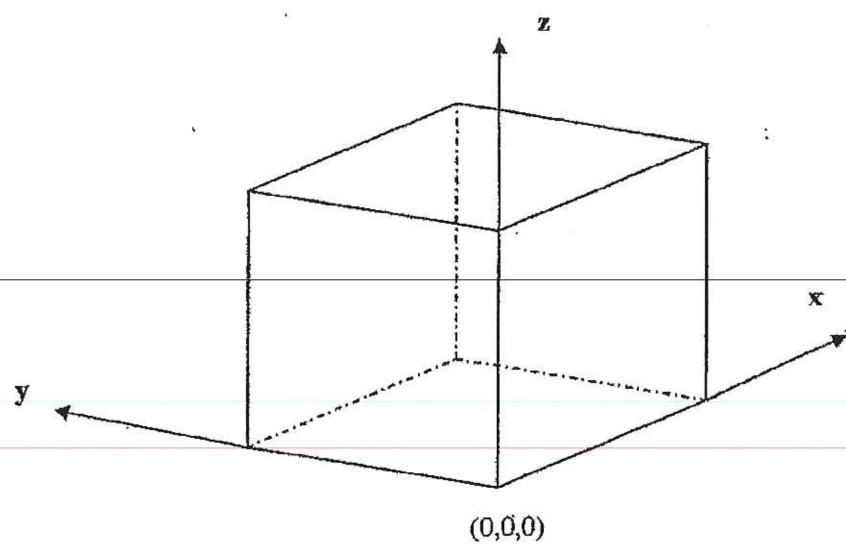
a) Find the matrices associated with these two transformations, i.e. M_K and M_J.

b) Consider the composition of the two linear transformations, with *K* followed by *J*. Find the matrix associated with this composition.

c) Prove that the composition of these two transformations is the same regardless of the order in which the transformations are performed, i.e. *K* followed by *J*, or *J* followed by *K*.

Projecting a right angle





z
x
y
(0,0,0)