

Last name:

First name:

ID number:

ECE 286

Final exam

April 18, 2023
9:30 am - 12:00 pm

Guidelines:

- Write your answer in the space provided for each question. Show your work and use the back side of sheets as needed.
- The exam is Type D. You may use a non-programmable calculator and a two-sided, 8.5 by 11 handwritten note sheet (prepared by you).
- **You will find tables for the normal distribution in the back of the exam. You may remove them.**

Problem	Score
1	/9
2	/9
3	/5
4	/7
5	/14
Total	/44

1. You flip a fair coin n times. The probability of heads is $P(H) = 0.5$. Let X be a random variable representing the number of heads. Answer questions (a) - (f) below.

(a) (1 point) Write down the probability mass function of X .

(b) (1 point) If $n = 4$, what is the probability of 3 heads, $P(X = 3)$?

- (c) (2 points) Write down the cumulative distribution function for the number of heads if $n = 4$.

(d) (1 point) What is the mean of X ?

(e) (1 point) What is the variance of X ?

- (f) (3 points) What is the probability of **not** having two or more consecutive heads?
(E.g., THTH does not have two consecutive heads, and THHT does).

2. X and Y are random variables with joint distribution

$$f(x, y) = \begin{cases} 2 & \text{if } x \geq 0, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Answer parts (a)-(e) below. Justify your answers.

(a) (2 points) Find the marginal distribution of X .

(b) (2 points) Use the marginal distribution to find the mean of X .

(c) (2 points) Find the variance of X .

(d) (2 points) Let $u(X, Y) = 2X - Y$. Find the expectation of $u(X, Y)$.

(e) (1 point) Are X and Y independent? Explain your reasoning.

3. You take a sample from a population. The sample size is $n = 36$, the sample variance is $s^2 = 4$, and the sample mean is $\bar{x} = 3$.
- (a) (3 points) Compute the 95% confidence interval for the mean.

(b) Suppose you obtain a new sample with 20 observations. The sample variance of the 20 new observations is $s_{\text{new}}^2 = 0.01$. You combine the new observations with the existing sample, so the overall sample size is $n = 56$. You compute a new 95% confidence interval. Answer the following two questions qualitatively—explain your reasoning, but do not try to compute numerical solutions.

i. (1 point) If the sample mean of the new data is $\bar{x}_{\text{new}} = 3$, will your new confidence interval be bigger or smaller than the confidence interval in part (a)?

ii. (1 point) If the sample mean of the new data is $\bar{x}_{\text{new}} = 400$, will your new confidence interval be bigger or smaller than the confidence interval in part (a)?

4. A shoe store sells only sandals and boots. The events are:

- S : sale of sandals.
- B : sale of boots.
- R : after a sale, an item is returned.

There are no other events. We know the following probabilities:

- 40% of sales are boots, i.e., $P(B) = 0.4$.
- 20% of sales are later returned, i.e., $P(R) = 0.2$.
- 30% of boots are returned, i.e., $P(R | B) = 0.3$.

Answer parts (a)-(c) below.

(a) (1 point) What is the probability of selling sandals?

(b) (3 points) If something is returned, what is the probability it is boots?

(c) (3 points) If a customer buys sandals, what is the probability they will later be returned?

5. Suppose X is a random variable for the roll of a biased die. The probability of a particular roll is given by $P(X = x) = (7 - x)/21$ for $x = 1, \dots, 6$. Answer parts (a)-(c) below. Justify your answers.

(a) (2 points) What is the probability that $X \leq 4$?

(b) You roll the die four times. Answer parts i-iii below.

i. (2 points) What is the probability of rolling 6, 5, 4, 3, in this order?

- ii. (2 points) What is the probability of rolling 6, 5, 4, 3, in any order?

iii. (2 points) What is the probability of rolling two 1's and two 4's, in any order?

(c) Now suppose Y is a random variable for the roll of a fair die, i.e., $P(Y = y) = 1/6$ for $y = 1, \dots, 6$. Let $Z = X + Y$. Note that X and Y are independent. Answer parts i-ii below.

i. (3 points) Find $P(Z = 5)$.

- ii. (3 points) Find the covariance of X and Z , σ_{XZ} .

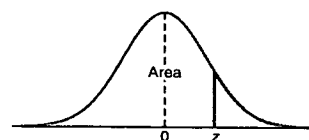


Table A.3 Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

