AER210F VECTOR CALCULUS AND FLUID MECHANICS

Quiz 2

21 October 2013 9:15 am - 10:15 am

Closed Book, No aid sheets, No calculators

Instructor: J. W. Davis

Last Name: ____________________

Given Name: ____________________

Student #: ____________________

Tutorial/TA: ____________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	10	
2	7	
3	10	
4	8	
5	8	
6	12	
TOTAL	55	/ 50

Note: The following integrals may be useful.

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C; \qquad \int \sin^2\theta\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$$

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dR; \qquad \oiint_S \vec{F} \cdot \hat{n}\, dS = \iiint_V \nabla \cdot \vec{F}\, dV; \qquad \oint_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot d\vec{S}$$

1) Use the coordinate transformation: $x = 4u\cos v$, $y = 2u\sin v$, $z = w$, to evaluate the triple integral $I = \int_V z\,dV$, where V is the volume bounded by the paraboloid: $z = 16 - x^2 - 4y^2$, and the x-y plane. Provide a sketch of the volume.

(10 marks)

2) a) Find the work done by the force $\vec{F}(x,y,z) = x^2\,\hat{i} + xy\,\hat{j} + z^2\,\hat{k}$ applied to an object that moves along the circular helix $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k},\ \ 0 \le t \le 2\pi$.

(3 marks)

b) Let $\vec{F} = \nabla f$, where $f(x,y) = \sin(x - 2y)$. Find curves C_1 and C_2 that are not closed and satisfy the equations:

i) $\int_{C_1} \vec{F} \cdot d\vec{r} = 0$ ii) $\int_{C_2} \vec{F} \cdot d\vec{r} = 1$

(4 marks)

3) Find a parametric representation of the surface, and use this to find the surface area of the portion of the paraboloid $z = 9 - x^2 - y^2$ that lies inside the cylinder $x^2 + y^2 = 16$. Provide a sketch of the surface.

(10 marks)

4) Let S be the surface given in cylindrical coordinates by $z = f(r, \theta)$, where $(r, \theta) \in \Omega$. Show that if f is continuously differentiable then the surface area of S is given by:

$$S = \iint_{\Omega} \sqrt{r^2\left(\frac{\partial f}{\partial r}\right)^2 + \left(\frac{\partial f}{\partial \theta}\right)^2 + r^2}\; dr d\theta$$

(8 marks)

5) Calculate the flux of the vector field $\vec{F} = e^{-y}\,\hat{i} + 2z\,\hat{j} + xy\,\hat{k}$ across the curved sides of the surface: $z = \cos y,\ 0 \le x \le 4,\ 0 \le y \le \pi/2$, where the normal vectors point upward.

(8 marks)

6) Verify Stokes' Theorem for $\vec{F} = -3y\hat{i} + 3x\hat{j} + z^4\hat{k}$ taking S as the portion of the ellipsoid $2x^2 + 2y^2 + z^2 = 1$ that lies above the plane $z = 1/\sqrt{2}$. Provide a sketch of the region.

(12 marks)