PHY 180: final exam

Dec 14, 2018
Duration: **150 minutes**
Exam Type: C
100 pts total.

1. *(20 pts.)* A particle of mass m moves in one dimension in the potential $V(x) = a\frac{x^2}{2} + b\frac{x^4}{4}$ $(b > 0)$.

   Identify all the (stable and unstable) equilibrium points, and calculate the angular frequency of small oscillations around each stable equilibrium point, for the following cases:

   (a) When $a > 0$.

   (b) When $a < 0$.

2. *(20 pts.)* A block of mass m is placed on a ramp that makes an angle θ to the horizontal, as shown in Fig. 1. The ramp moves with a constant acceleration $\vec{a} = a\,\hat{\imath}$. The acceleration due to gravity is $\vec{g} = -g\,\hat{\jmath}$. The coefficient of static friction between the block and the ramp is μ. Assume that $a < g$, and $\mu < 1$.

   What is the largest possible value of θ so that the block does not slide down the ramp (i.e., the block is stationary, as observed in a frame fixed to the ramp) ?

3. *(20 pts.)* A uniform spherical ball with mass m and radius R moves down a ramp as shown in Fig. 2. When the ball is initially released from rest, its centre of mass is $h + R$ above the surface of the table. Assume that the ball rolls without slipping down the ramp. The acceleration due to gravity is $\vec{g} = -g\hat{\jmath}$.

   When it reaches the point labeled A, what are the values of the momentum of the centre of mass $\vec{p}$ of the ball, and the angular momentum $\vec{L}$ of the ball (around an axis through its centre of mass)?

4. *(10 pts.)* Given that the angular momentum of the earth due to its orbit around the sun has magnitude L_0, what is the magnitude of the orbital angular momentum of Jupiter around the sun?

   (You can ignore the rotation of Jupiter and the earth around their own axes. You may assume that the orbits of Jupiter and the earth can be approximated as circles.)

   The mass of Jupiter is 320 times the mass of the earth, and its orbital period is T = 12 earth years.

5. *(10 pts.)* A large cylinder is oriented vertically and rotates around its axis (defined as the z axis) with angular velocity $\vec{\Omega} = 3\,\hat{k}$ rad/s. The acceleration due to gravity is $\vec{g} = -9.8\,\hat{k}$ m/s^2. A person standing inside the cylinder drops a 2 kg object from the location $\vec{r} = 1.5\,\hat{k}$ m. With what speed does the object hit the floor (defined as the plane $z = 0$) ?

6. *(20 pts.)* Two objects, A and B, with identical density have similar shapes but different sizes, as shown by the shaded portions in Fig. 3. You can assume that the objects are essentially two dimensional, with negligible thickness compared to R.

   (a) If the frequency of small angular oscillations around equilibrium for object A is ω_0, what is the frequency of small angular oscillations around equilibrium for object B?

   (b) If the moment of inertia about an axis passing through its centre of mass (and perpendicular to the plane of the drawing) for object A is I_0, what is it for object B?

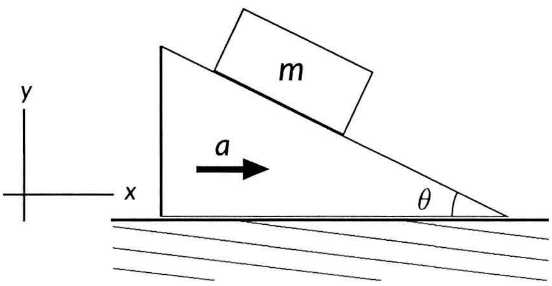

FIG. 1

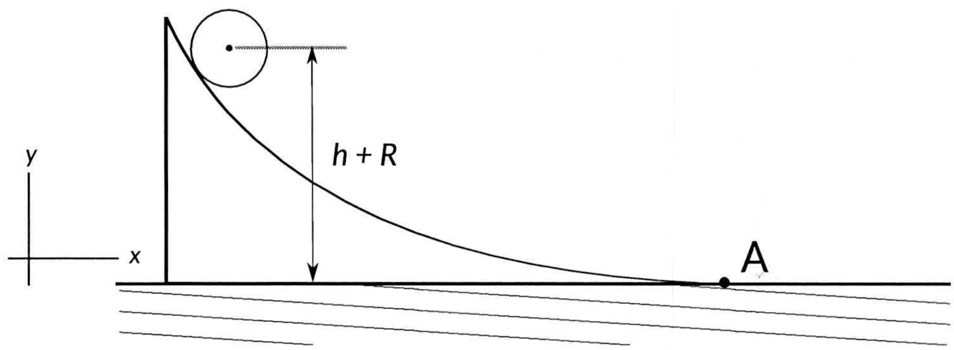

FIG. 2

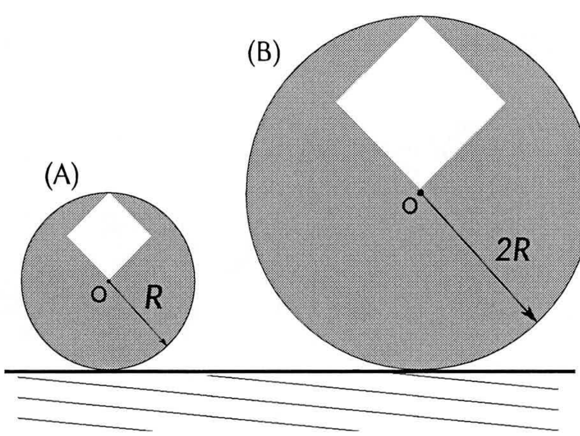

FIG. 3