$\boxed{1}$

$$\Delta p \Delta x \gtrsim \frac{\hbar}{2} \Rightarrow p \cdot x \sim \hbar$$

$$E = \frac{p^2}{2m} + a\frac{\hbar^4}{p^4}$$

$$0 = \frac{\partial E}{\partial p} = \frac{2p}{2m} - \frac{4a\hbar^4}{p^5} \Rightarrow p_0 = [4a\hbar^4 m]^{1/6}$$

$$\Rightarrow E_{min} = \frac{[4a\hbar^4 m]^{1/3}}{2m} + \frac{a\hbar^4}{[4a\hbar^4 m]^{2/3}}$$

$$= a^{1/3}\hbar^{4/3}m^{-2/3}2^{-4/3}[2^1+1]$$

$\boxed{2}$

Note: $\int_{-\infty}^{\infty}|\psi|^2 = 1$

$$\langle x \rangle = (x_2-x_1)^{-1}\int_{x_1}^{x_2} e^{-ik_0x}\, x\, e^{ik_0x}dx$$

$$= (x_2-x_1)^{-1}\int_{x_1}^{x_2} x\, dx = \frac{x_2^2-x_1^2}{2(x_2-x_1)} = \frac{x_2+x_1}{2}$$

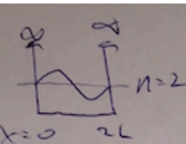
$$\langle p \rangle = (x_2-x_1)^{-1}\int_{x_1}^{x_2} e^{-ik_0x}\left(-i\hbar\frac{\partial}{\partial x}\right)e^{ik_0x}dx$$

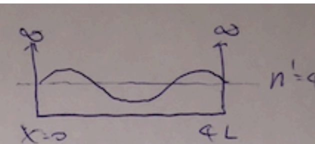
$$= \frac{-i\hbar(ik_0)}{(x_2-x_1)}\int_{x_1}^{x_2}dx = \hbar k_0$$

$$\frac{\langle p^2 \rangle}{2m} = \frac{(x_2-x_1)^{-1}}{2m}\int_{x_1}^{x_2} e^{-ik_0x}\left(-\hbar^2\frac{\partial^2}{\partial x^2}\right)e^{ik_0x}dx$$

$$= -\frac{\hbar^2(ik_0)^2}{2m(x_2-x_1)}\int_{x_1}^{x_2}dx = \frac{\hbar^2k_0^2}{2m}$$

3





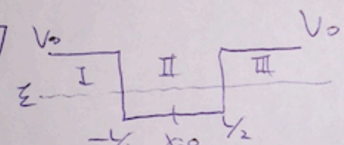
$$\psi_{n=2} = \sqrt{\frac{2}{2L}} \sin\frac{2\pi x}{2L} \qquad \psi_{n=2} = \sum_{n'=1}^{\infty} b_{n'} \psi_{n'} \qquad \psi_{n'=4} = \sqrt{\frac{2}{4L}} \sin\frac{4\pi x}{4L}$$

$$b_{n'=4} = \int_0^{2L} \psi^*_{n'=4} \psi_{n=2}\, dx$$

$$= \frac{2}{2\sqrt{2}L} \underbrace{\int_0^{2L} \sin^2\frac{\pi x}{L}\, dx}_{L} = \frac{1}{\sqrt{2}}$$

$$\text{Prob} = |b_{n'=4}|^2 = \frac{1}{2}$$

4



$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U(x)\psi = E\psi$$

I, III : $\psi'' = \alpha^2 \psi$ $\qquad \alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

$\psi \sim \{e^{\alpha x}, e^{-\alpha x}\}$ must not diverge at $x \to \pm\infty$

II : $\psi'' = -k^2 \psi$ $\qquad k = \frac{\sqrt{2mE}}{\hbar}$

$\psi \sim \{\cos kx, \sin kx\}$ alternates in parity starting from "even" ground state

"Even"	"Odd"
$\psi_I = Ae^{\alpha x}$	$\psi_I = Ce^{\alpha x}$
$\psi_{II} = B\cos kx$	$\psi_{II} = D\sin kx$
$\psi_{III} = Ae^{-\alpha x}$	$\psi_{III} = -Ce^{-\alpha x}$

$$P_{op}\psi_{II} = -i\hbar\frac{\partial}{\partial x}\psi_{II} \neq \text{constant} \times \psi_{II}$$

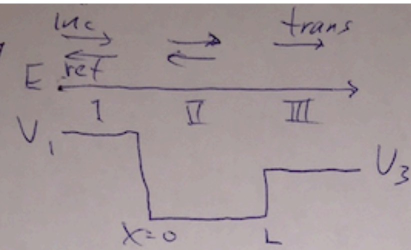
$\therefore$ Not momentum eigenstate

In III $\quad \psi'_{III} = -\alpha\psi_{III}, \quad \psi^{*\prime}_{III} = -\alpha\psi^*_{III}$

$$\Rightarrow j(x) = \frac{-i\hbar}{2m}\left(-\alpha|\psi_{III}|^2 + \alpha|\psi_{III}|^2\right) = 0$$

Similarly in Region I

$\boxed{5}$



Use Steady State approach

$$\psi_I(x) = A e^{ik_1x} + B e^{-ik_1x} \qquad k_1 = \sqrt{2m(E-V_1)}/\hbar$$

$$\psi_{II}(x) = C e^{ik_2x} + D e^{-ik_2x} \qquad k_2 = \sqrt{2mE}/\hbar$$

$$\psi_{III}(x) = F e^{ik_3x} \qquad k_3 = \sqrt{2m(E-V_3)}/\hbar$$

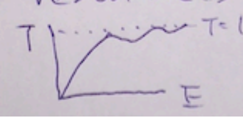
B.C.'s: $\psi_I(0) = \psi_{II}(0)$, $\psi_I'(0) = \psi_{II}'(0)$

$\psi_{II}(L) = \psi_{III}(L)$, $\psi_{II}'(L) = \psi_{III}'(L)$

$$T = \frac{|\psi_{trans}|^2 v_{III}}{|\psi_{inc}|^2 v_I} = \frac{|F|^2 \cancel{\hbar} k_3 \cancel{/m}}{|A|^2 \cancel{\hbar} k_1 \cancel{/m}} , \qquad \frac{k_3}{k_1} = \frac{\sqrt{E-V_3}}{\sqrt{E-V_1}}$$

If $V_1 = V_3 \Rightarrow T = \frac{|F|^2}{|A|^2}$

But resonances at $k_2 L = n\pi$, n = nonzero integer



$\therefore \frac{\partial T}{\partial E}$ can be < 0

$\boxed{EC}$ $[\hat{x},\hat{p}]\Psi = \hat{x}\hat{p}\Psi - \hat{p}\hat{x}\Psi$ $\quad \Psi(x,t)$ is any wavefunction

$$= i\hbar\left(\frac{\partial}{\partial p}p\Psi - p\frac{\partial}{\partial p}\Psi\right)$$

$$= i\hbar\left(\Psi + p\frac{\partial\Psi}{\partial p} - p\frac{\partial\Psi}{\partial p}\right) \quad \therefore [\hat{x},\hat{p}] = i\hbar$$

$$[\hat{x},\hat{p}^2]\Psi = \hat{x}\hat{p}^2\Psi - \hat{p}^2\hat{x}\Psi$$

$$= i\hbar\left(\frac{\partial}{\partial p}p^2\Psi - p^2\frac{\partial}{\partial p}\Psi\right)$$

$$= i\hbar\left(2p\Psi + p^2\frac{\partial\Psi}{\partial p} - p^2\frac{\partial\Psi}{\partial p}\right) \quad \therefore [\hat{x},\hat{p}^2] = i\hbar\, 2\hat{p}$$

$(\Delta A)^2(\Delta B)^2 \geq \frac{1}{4}\langle i[\hat{A},\hat{B}]\rangle^2$, $\quad \hat{A},\hat{B}$ are any two operators

where $(\Delta A)^2 = \langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2$
$(\Delta B)^2 = \langle\hat{B}^2\rangle - \langle\hat{B}\rangle^2$

$$\Rightarrow \quad (\Delta p)^2(\Delta x)^2 \geq \frac{1}{4}\langle i\underbrace{[\hat{p},\hat{x}]}_{-i\hbar}\rangle^2 \quad \therefore \Delta p\Delta x \geq \frac{\hbar}{2}$$

Same as for x-space representation