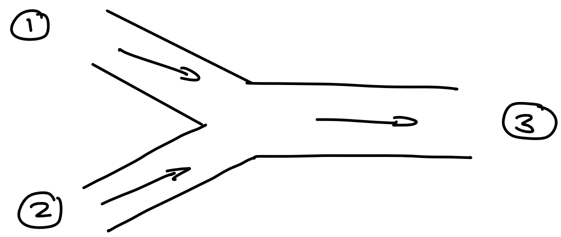
CHE 260

MIDTERM SOLUTION - 2022.

1)



$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\Rightarrow \quad \dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \quad \dot{m}_1 (h_1 - h_3) = \dot{m}_2 (h_3 - h_2)$$

$$\Rightarrow \quad \dot{m}_1 c_p (T_1 - T_3) = \dot{m}_2 c_p (T_3 - T_2)$$

$$\Rightarrow \quad T_3 (\dot{m}_2 + \dot{m}_1) = \dot{m}_1 T_1 + \dot{m}_2 T_2$$

$$\Rightarrow \quad T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_2 + \dot{m}_1}$$

$$T_3 = \frac{1\ kg/s \times 400K + 2\ kg/s \times 290K}{1\ kg/s + 2\ kg/s}$$

$$T_3 = 326.7\ K.$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 s_3 = \dot{m}_1 s_3 + \dot{m}_2 s_3$$

$$\Rightarrow \quad \dot{S}_{gen} = \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2)$$

$$P_1 = P_2 = P_3$$

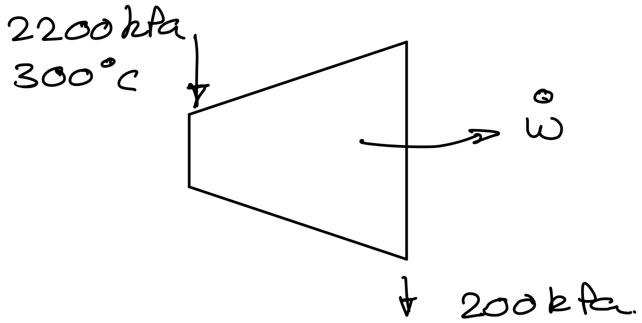
$$\Rightarrow \quad \dot{S}_{gen} = \dot{m}_1 c_p \ln T_3/T_1 + \dot{m}_2 c_p \ln T_3/T_2$$

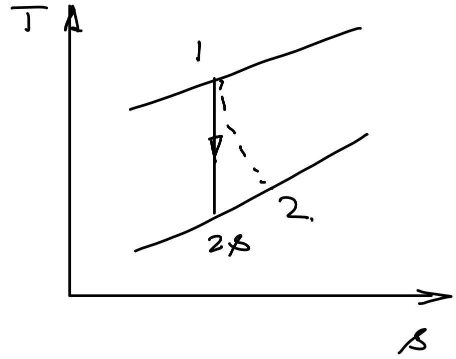
$$c_p \approx 1.004\ kJ/kg\,K.$$

$$\Rightarrow \quad \dot{S}_{gen} = 1\ kg/s \times 1.004 \frac{kJ}{kg\,K} \ln \frac{326.7K}{400K} + 2\frac{kg}{s} \times 1.004 \frac{kJ}{kg\,K} \times \frac{326.7K}{290K}$$

$$\dot{S}_{gen} = 0.036\ kW/K.$$

(2)





$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

At $T_{avg} \approx 400K$ $\quad c_p = 1.013 \frac{kJ}{kgK}$, $\gamma = 1.395$

$$\Rightarrow \quad T_{2s} = 573\ K \left(\frac{200\ kPa}{2200\ kPa}\right)^{\frac{0.395}{1.395}} = 290.6K$$

$$w_{t,s} = c_p(T_{2s} - T_1)$$

$$w_{t,s} = 1.013 \frac{kJ}{kgK}(290.6\ K - 573K) = -286.1 \frac{kJ}{kg}$$

$$w_t = \eta_t w_{t,s} = 0.85(-286.1)\frac{kJ}{kg},$$

$$w_t = -243.2\ kJ/kg$$

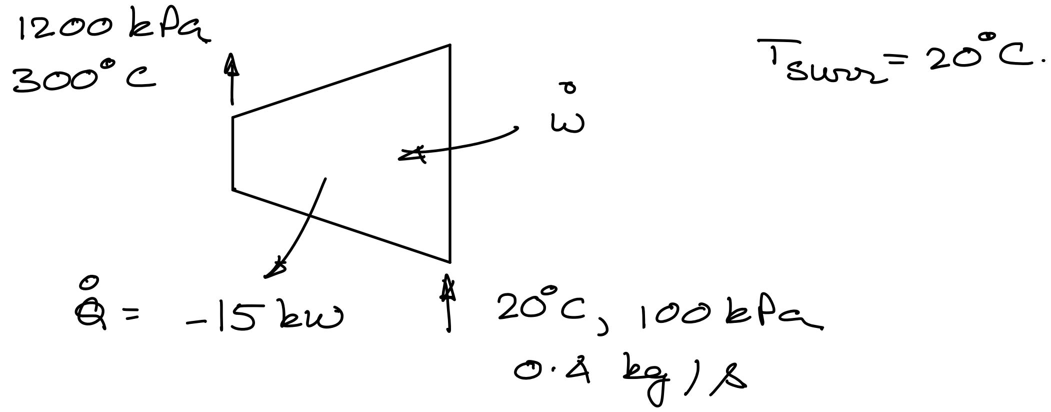
$$w_t = c_p(T_2 - T_1)$$

$$\Rightarrow \quad T_2 = T_1 + w_t/c_p$$

$$= 573\ K + \frac{(-243.2\ kJ/kg)}{1.013\ kJ/kgK}$$

$$T_2 = 332.9\ K.$$

(3)



$$\dot{W} = \dot{m} c_p (T_2 - T_1) - \dot{Q}$$

At $T_{avg} = 160°C = 433 K$, $c_p \doteq 1.018 \frac{kJ}{kg K}$

$$\dot{W} = 0.4 \, kg/s \times 1.018 \frac{kJ}{kg K} (300 - 20)°C + 15 \, kW$$

$$\dot{W} = 129.0 \, kW$$

b) Rate of entropy generation

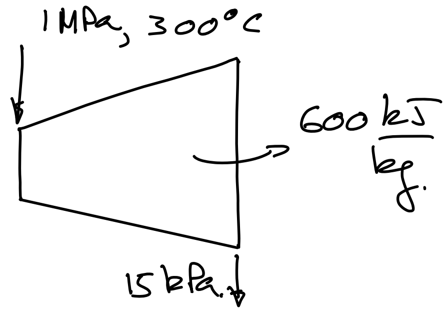
$$\dot{S}_{gen} = \Delta \dot{S}_{air} + \Delta \dot{S}_{surr}.$$

$$\dot{S}_{gen} = \dot{m}\left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}\right) + \frac{\dot{Q}}{T_{surr}}.$$

$$\dot{S}_{gen} = 0.4 \frac{kg}{s} \times \left(1.018 \frac{kJ}{kg K} \ln \frac{573 K}{293 K} - 0.287 \frac{kJ}{kg K} \ln \frac{1200 kPa}{100 kPa}\right) + \frac{15 \, kW}{293 K}.$$

$$\dot{S}_{gen} = 0.039 \, kW/K.$$

4)



$$\left.\begin{array}{l}1\,\text{MPa}\\300^\circ\text{C}\end{array}\right\}\begin{array}{l}h_1 = 3051.6\ \text{kJ/kg}\\ s_1 = 7.1246\ \text{kJ/kgK}\end{array}$$

at 15 kPa

$h_f = 225.94\ \frac{\text{kJ}}{\text{kg}}$ $h_g = 2598.3\ \frac{\text{kJ}}{\text{kg}}$

$s_f = 0.7549\ \frac{\text{kJ}}{\text{kgK}}$ $s_g = 8.0071\ \frac{\text{kJ}}{\text{kgK}}$

For isentropic expansion $s_2 = s_1$

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1246 - 0.7549}{8.0071 - 0.7549} = 0.8783$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f)$$

$$= 225.94 + 0.8783(2598.3 - 225.94)$$

$$h_{2s} = 2309.6\ \text{kJ/kg.}$$

$$w_s = h_{2s} - h_1 = 2309.6 - 3051.6 = -742.0\ \text{kJ/kg}$$

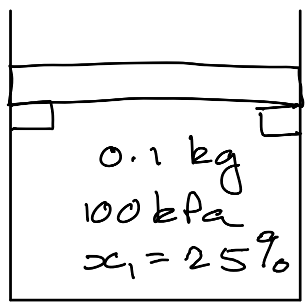
$$\eta_t = \frac{w_a}{w_s} = \frac{-600\ \text{kJ/kg}}{-742.0\ \text{kJ/kg}} = 80.9\%$$

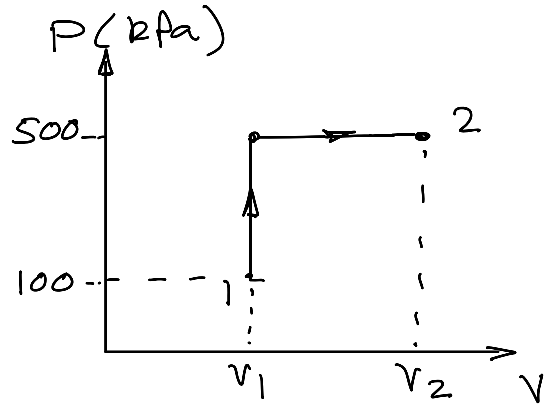
$$h_2 = h_1 + w_a = 3051.6 - 600 = 2451.6\ \text{kJ/kg.}$$

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2451.6 - 225.94}{2598.3 - 225.94}$$

$$x_2 = 0.9382$$

5)





at 100 kPa $\quad v_f = 0.001043\ m^3/kg,\ v_g = 1.6941\ m^3/kg$

$$u_f = 417.40\ kJ/kg \quad u_g = 2505.6\ kJ/kg$$

$$v_1 = v_f + x_1 (v_g - v_f)$$

$$= 0.001043\ m^3/kg + 0.25 (1.6941 - 0.001043)\ m^3/kg$$

$$v_1 = 0.42431\ m^3/kg$$

At 500 kPa, 300°C $\Rightarrow v_2 = 0.52261\ m^3/kg$

Since $v_2 > v_1 \Rightarrow$ Piston is lifted.

Constant pressure process

$$\Rightarrow W_{12} = -P (V_2 - V_1)$$

$$= -P\, m\, (v_2 - v_1)$$

$$\Rightarrow W_{12} = -500\ kPa \times 0.1\ kg\ (0.52261 - 0.42431)\ m^3/kg$$

$$W_{12} = -4.92\ kJ$$