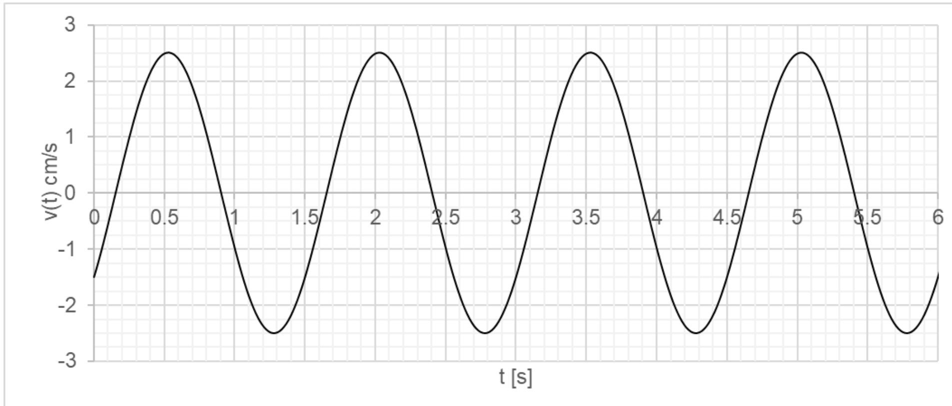
PHYS 293 – Quiz 1 Version C 4 Oct 2021

[Instructions: Solve the question below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.

You are allowed: a non-communicating calculator, a one-page formula sheet (can be annotated)].

Question

Mass m kg is attached to a spring of spring constant $k = 1.33\ \mathrm{N/m}$. The mass is oscillating, according to $x(t) = A_0\cos(\omega t + \phi_i)$, initially without any damping. The figure below shows the **velocity** of the mass as a function of time.



a. What is the value of the mass m? [1 point]

$$\frac{k}{m} = \omega_0^2 \to m = \frac{k}{\omega_o^2} = \frac{kT^2}{(2\pi)^2}$$

$$m = \frac{(1.33)(1.5^2)}{4\pi^2} = 0.0758\ \mathrm{kg}$$

NOTE: Units do not need to be substituted into the equations, final answer **requires** units for full marks.

b. What is the initial phase constant of the oscillation? [4 points]

$$x(t) = A_0\cos(\omega_0 t + \phi_i) \to v(t) = -A_0\omega_0\sin(\omega_0 t + \phi_i)$$

$$v(0) = -v_{\max} sin\phi_i$$

NOTE: Realizing equation for velocity needs to be used: 1 point

$$-1.5 = -2.5\sin(\phi_i) \to \sin\phi_i = \frac{3}{5}$$

NOTE: not realizing $A\omega$ is equal to 2.5 cm/s, missing the negative: deduction of 0.25 marks each.

$$\phi_i = +0.644\ \mathrm{rad} \text{ or } \phi_i = \pi - (+0.644)\mathrm{rad} = 2.50\ \ \mathrm{rad}$$

NOTE: Two possible solutions **HAVE TO** be considered. If only one is written without any justification, student is awarded 2/4 marks

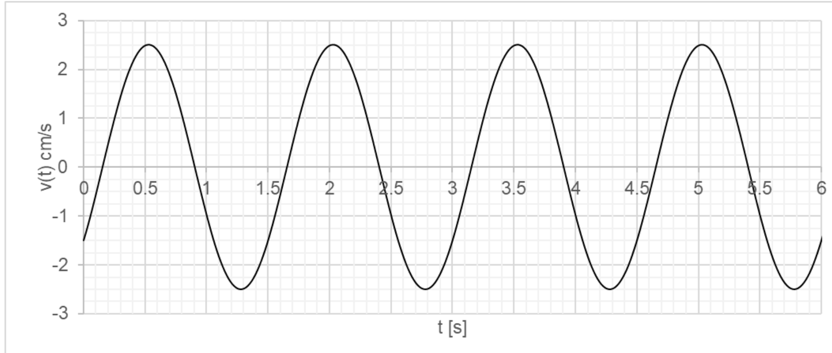
As there are two solutions, the acceleration $\frac{dv}{dt}|_{t=0}$ determines the correct one

$$\frac{dv(t)}{dt}|_{t=0s} = -A\omega_0^2(\cos\phi_i) > 0 \therefore \cos\phi_i < 0 \to \phi_i = 2.5\ rad$$

NOTE: Any check is fine (velocity at t, sign of position). If the check is completed, but a wrong conclusion is drawn, student is awarded 3/4 marks.

Check with correct conclusion but wrong math/missing units: 3.5/4

Mass m is attached to a spring of spring constant $k = 1.33\ \mathrm{N/m}$. The mass is oscillating, according to $x(t) = A_0 \cos(\omega t + \phi_i)$, initially without any damping. The figure below shows the velocity of the mass as a function of time.



c. After the first 5.00 s, at the exact time when mass is at the maximum displacement, damping is introduced. What is the value of the **drag coefficient** that would result in the critical damping of the mass? [2 points]

For critical damping $\omega_0^2 = \frac{\gamma^2}{4} = \frac{b^2}{4m^2}$

$$\frac{k}{m} = \frac{b^2}{4m^2} \rightarrow b^2 = 4mk \rightarrow b = \sqrt{4mk} = \sqrt{4 * (0.0758kg) * \left(1.33\frac{N}{m}\right)} = 0.635\ \mathrm{kg/s}$$

Solution using ω_0 is absolutely correct; solution for γ (damping coefficient) warrants a deduction of 0.5 mark (make sure you put a comment that this is a damping coefficient NOT drag coefficient).

Missing units: -0.5

d. Write the expression for the **position** of the mass as a function of time after the damping has been introduced. Start with general equation and fill in all variables you can. [3 points]

Damping, therefore the general solution

$$x(t) = Ae^{-\left(\frac{\gamma t}{2}\right)} + Bte^{-\frac{\gamma t}{2}}$$

$$\gamma = \frac{b}{m} = 8.38\frac{1}{s}\ \left(or\frac{rad}{s}\right)$$

NOTE: 0.5 point for starting with correct equation, 0.5 points for γ that is consistent with c)

0.5 points for correctly identifying initial conditions

As the damping starts at the maximum displacement, $x(t_0) = A_0$, $v(t_0) = 0$

$A = A_0 = \frac{v_{max}}{\omega_0} = 0.597\ cm$ (amplitude of the motion)

From that $\frac{dx(t)}{dt} = 0 = -\frac{\gamma}{2}Ae^{-\frac{\gamma t}{2}} + Be^{-\frac{\gamma t}{2}} + \frac{\gamma}{2}Bte^{-\frac{\gamma t}{2}}$

At t=0, $0 = -\frac{\gamma}{2}A + B \rightarrow B = \frac{A\gamma}{2} = v_0$ ($I\ mean\ props\ if\ they\ notice, but\ not\ necessary$)

NOTE: 1 point for **correctly identifying** the coefficients; any fair attempt (other than assuming they are A/B/zero) is worth 0.5 points. If any of them is correct and there is a fair attempt on the other (e.g zero but justified) award 0.75 points.

0.5 point for final equation with whatever possible substituted.

$$x(t) = (0.597\ \mathrm{cm})e^{-4.19t} + (2.5\,\frac{\mathrm{cm}}{\mathrm{s}})te^{-4.19t}$$