University of Toronto
Faculty of Applied Science and Engineering

Term Test II

March 11, 2014
Duration: 90 minutes

ECE159 - Electric Circuit Fundamentals
Examiners: Ali Sheikholeslami and Li Qian

ANSWER QUESTIONS ON THESE SHEETS, USING THE BACKS IF NECESSARY.

WRITE IN PEN ONLY (NO PENCIL)!

1. Calculator type is restricted (no programmable calculators).
2. Weight for each question is indicated in []. Attempt all questions, since a blank sheet will certainly get a zero.

maximum grade = 30

Last Name: Solutions

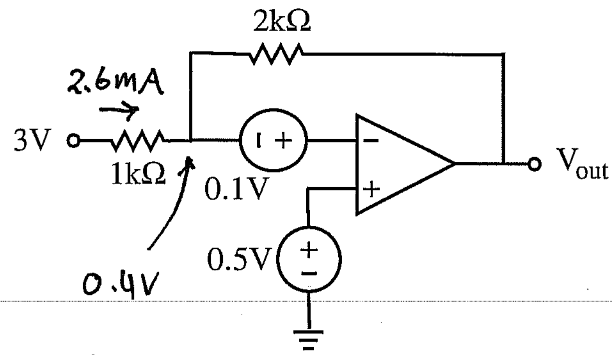
First Name:

Student Number:

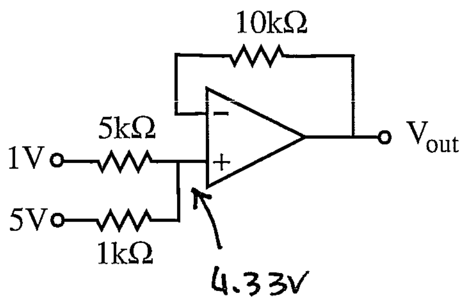
Tutorial Section:

Question	Mark
1	........../6
2	........../6
3	........../6
4	........../6
5	........../6
Total	........../30

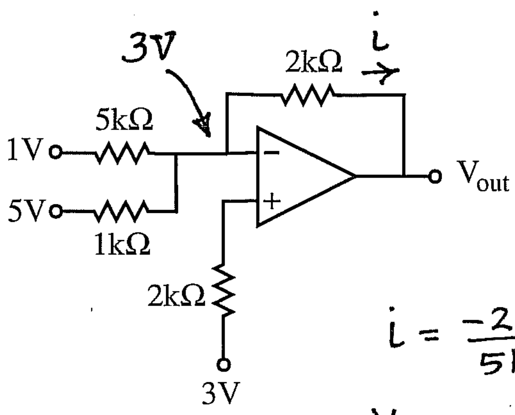
Q1. [6 marks] In the circuits shown below, assume the op-amps are ideal and operate in linear region (i.e. not saturated). Fill out the output voltage values in the boxes provided.



$V_{out} = -4.8V$



$V_{out} = 4.33V$

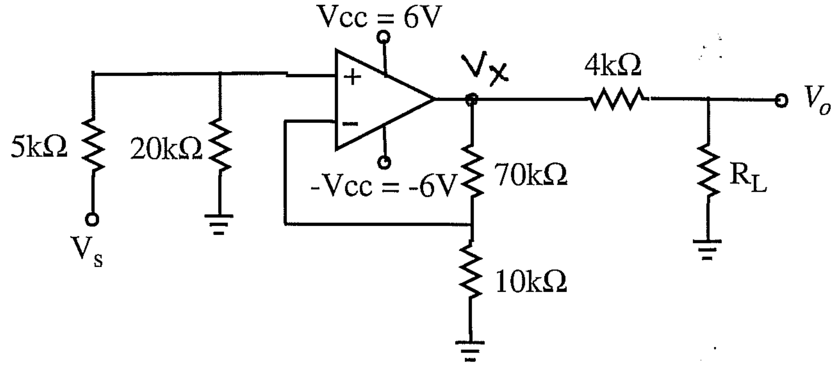


$V_{out} = -0.2V$

$i = \frac{-2}{5K} + \frac{2}{1K} = 1.6mA$

$V_{out} = 3 - 3.2 = -0.2V$

Q2. [6 marks] For the circuit shown below, assume the op-amp is ideal.



(a) [2 marks] Assume R_L = 4kΩ. Determine the voltage gain (V_o/V_s)

$$V_s \frac{20K}{25K} = V_x \frac{1}{8} \Rightarrow V_x = \frac{160K}{25K} V_s$$

$$V_o = \frac{4K}{8K} V_x = \frac{80}{25} V_s \Rightarrow \boxed{\frac{V_o}{V_s} = 3.2 \ V/V}$$

(b) [2 marks] Assume R_L = 4kΩ. Specify the range of V_s for which the op-amp operates in linear region.

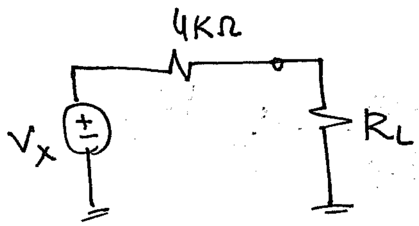
$$-6V \leq V_x \leq +6V$$

$$\Rightarrow -6V \leq \frac{160K}{25K} V_s \leq +6V \Rightarrow \frac{-6}{6.4} V \leq V_s \leq \frac{6}{6.4} V$$

$$\Rightarrow \boxed{-0.94V < V_s < 0.94V}$$

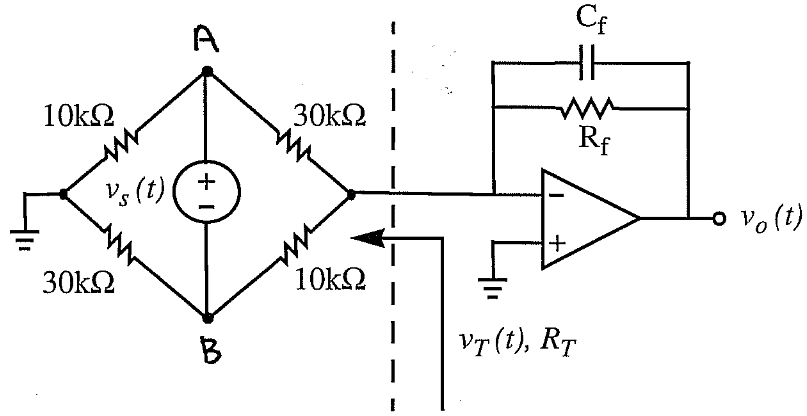
(c) [2 marks] Determine the value of R_L that will transfer the maximum power to R_L.

Output resistance of an ideal op-amp is zero!



$$\Rightarrow \boxed{R_L = 4K\Omega}$$

Q3 [6 marks] For the circuit shown below, assume the op-amp is ideal.



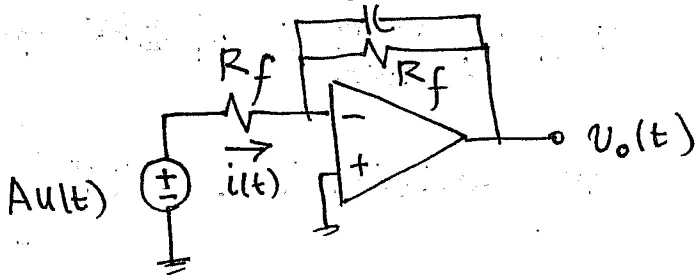
(a) [2 marks] Determine the Thevenin equivalent circuit of the bridge as indicated on the figure.

$$V_A = \frac{10K}{40K} v_s = \frac{v_s}{4} \Rightarrow v_T(t) = v_A - v_s(t)\frac{30K}{40K}$$

$$= \frac{v_s}{4} - v_s\frac{3}{4} = \boxed{-0.5\, v_s(t)}$$

$$R_T = 2\,(10K \parallel 30K) = 2\,(7.5K) = \boxed{15K\Omega}$$

(b) [4 marks] Assuming $v_T(t) = A\,u(t)$ (in Volts), where u(t) is a step function, and $R_T = R_f$, derive an expression for the output voltage of the op-amp as a function of t; $v_o(t)$, in terms of A, R_f, and C_f. Assume the initial voltage of the op-amp is 0V at time 0, i.e. $v_o(0) = 0$.



$$i(t) = \frac{A}{R_f} u(t)$$

$$= \frac{-v_o(t)}{R_f} - C_f\frac{dv_o}{dt}$$

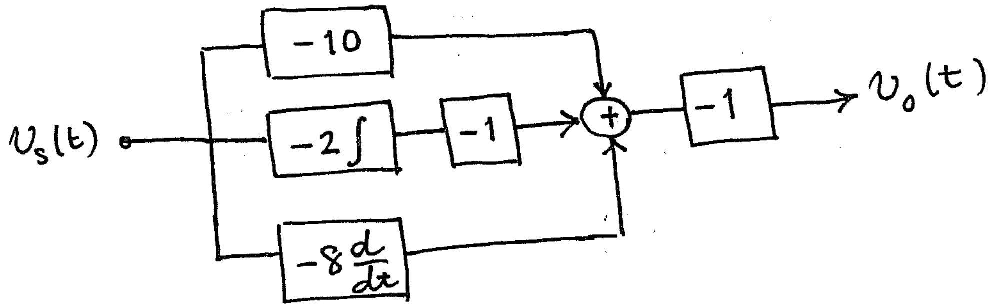
$$\Rightarrow R_f C_f \frac{dv_o}{dt} + v_o(t) = -A\,u(t)$$

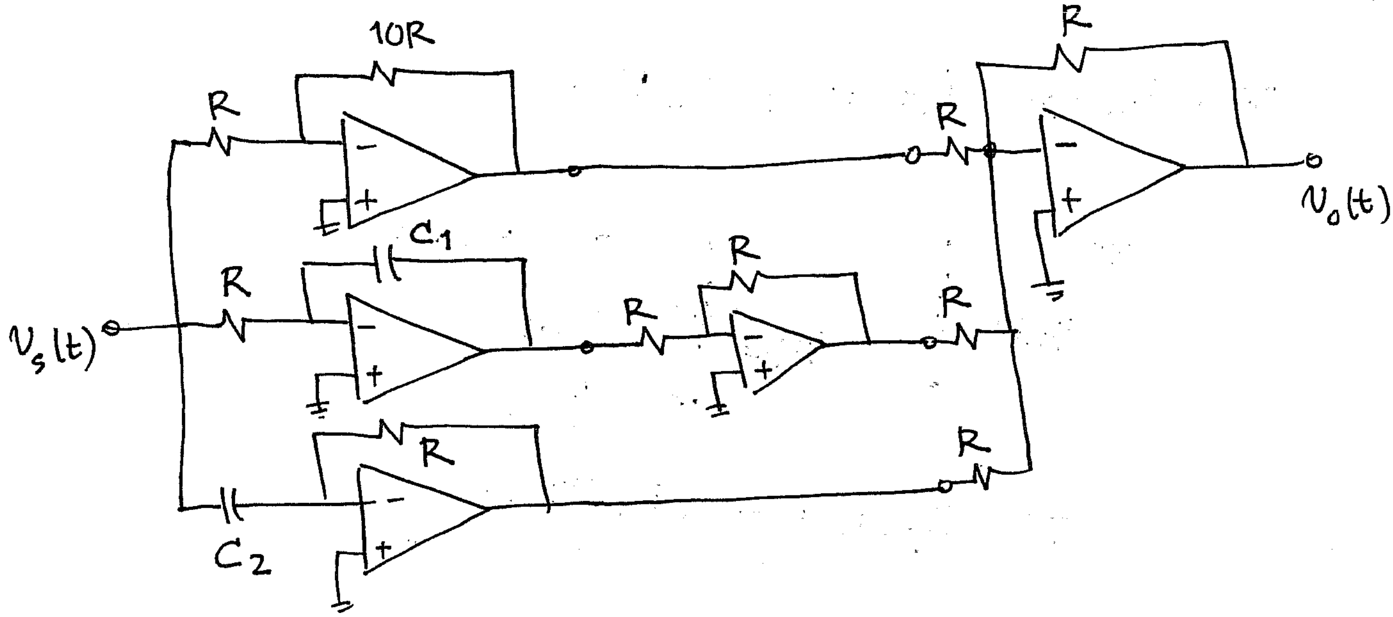
$$\Rightarrow v_o(t) = -A\,u(t)\left(1 - e^{-\frac{t}{R_f C_f}}\right)$$

Q4. [6 marks]

(a) [3 marks] Using resistors, capacitors, and ideal op-amps only, draw a circuit diagram that receives $v_s(t)$ as its input voltage and produces $v_o(t)$ as its an output voltage according to the following equation:

$$v_o(t) = 10.v_s(t) - 2\int_0^t v_s(t)dt + 8\frac{d}{dt}v_s(t)$$

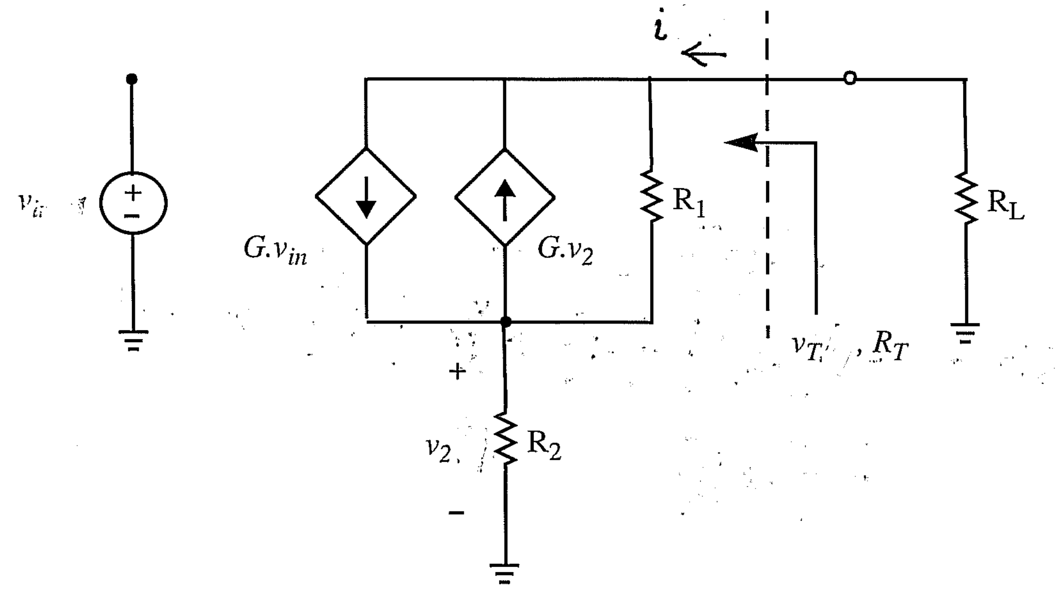




$$\frac{1}{RC_1} = 2 \Rightarrow C_1 = \frac{1}{2R}$$

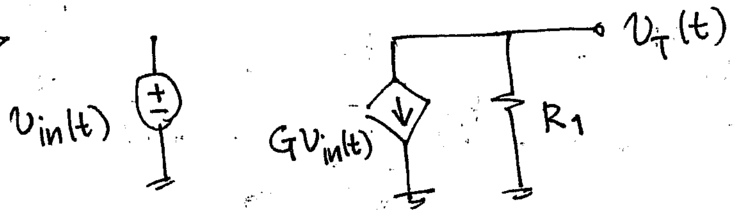
$$RC_2 = 8 \Rightarrow C_2 = \frac{8}{R}$$

Q4(b) [3 marks] Find the Thevenin equivalent of the following circuit



Open $R_L \Rightarrow \quad i(t)=0 \Rightarrow v_2(t)=0$

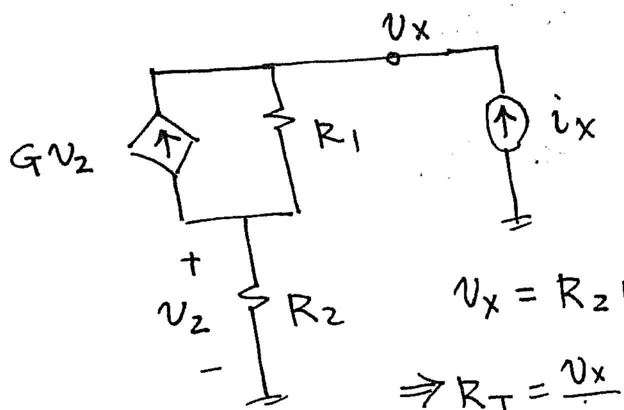
$\Rightarrow$



$$\Rightarrow \boxed{v_T(t) = -GR_1 v_{in}(t)}$$

To find R_T, short v_{in}, Apply i_x

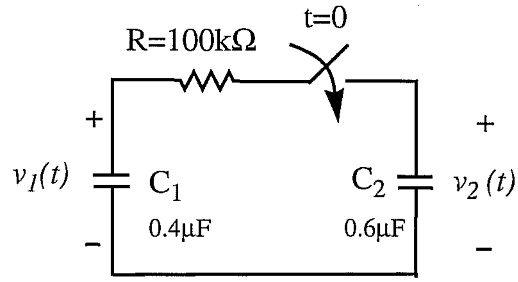
$\Rightarrow$



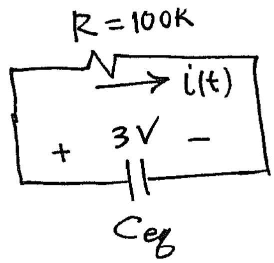
$$v_x = R_2 i_x + R_1(i_x + GR_2 i_x)$$

$$\Rightarrow R_T = \frac{v_x}{i_x} = R_2 + R_1 + GR_1R_2$$

Q5 [6 marks]
In the circuit shown below, $C_1 = 0.4\mu F$, $C_2 = 0.6\mu F$, and $R = 100k\Omega$. C_1 and C_2 are charged initially (at t=0) to 5V and 2V, respectively. At time 0, we close the switch.



(a) [3 marks] Write an expression for $v_2(t)$ for t > 0.



Caps are in series $\Rightarrow C_{eq} = 0.24\mu F$

voltage across C_{eq} : $5-2=3V$

$$\Rightarrow i(t) = \frac{3}{100K} e^{-t/RC_{eq}}$$

where $RC_{eq} = 100K \times 0.24\mu = 24m\ sec$

$$\Rightarrow i(t) = 30 e^{-t/24m} \mu A$$

$$v_2(t) = v_2(0) + \frac{1}{C_2}\int_0^t i(t)dt = \boxed{3.2 - 1.2 e^{-t/24m}}$$

(b) [3 marks] Calculate the total energy lost in the resistor from time 0 to infinity.

From part (a), $V_1(\infty) = V_2(\infty) = 3.2V$

Energy consumed in resistor = Energy lost in capacitors

$$\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1+C_2)V_\infty^2$$

$$= \frac{1}{2}0.4\mu(5)^2 + \frac{1}{2}(0.6\mu)(2)^2 - \frac{1}{2}(1\mu)(3.2)^2$$

$$= 6.2\mu J - 5.12\mu J = \boxed{1.08\mu J}$$

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