Student Number: ____________________ Name: Solution

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, DECEMBER 2016

CIV102H1F – Structures and Materials-
An Introduction to Engineering Design

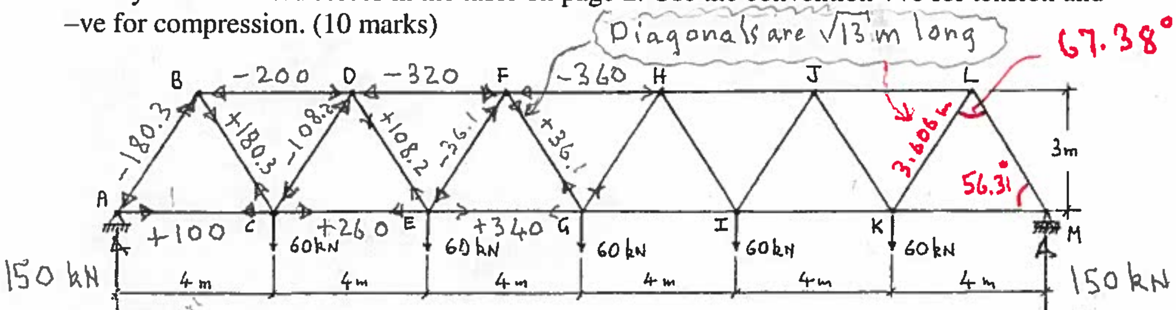
Examiner --- M.P. Collins

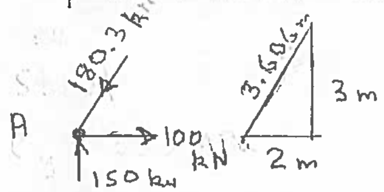
Permissible Aids: Notebook, calculator and set-square.

1	30
2	27
3	22
4	26
Total	105

1. The steel truss shown below supports a pedestrian bridge with a span of 24 metres. When a crowd of people are on the bridge, joints C, E, G, I and K are each subjected to a downwards force of 60 kN due to the weight of the people and the self-weight of the bridge.

1(a). Calculate the axial force in the members of the truss due to the five 60 kN loads. Neatly write your calculated forces in the table on page 2. Use the convention +ve for tension and –ve for compression. (10 marks)





Yield Stress = 350 MPa

1(b). The top horizontal members are HSS 152 x 152 x 4.8, the bottom horizontal members are HSS 102 x 102 x 6.4 while the diagonal members are HSS 102 x 102 x 8.0. Are the truss members safe under the 60 kN loads? At what value of the five equal point loads will the truss be on the boundary between safe and unsafe? (6 marks)

• FH $I = 9.93\times10^6\ mm^4$ $P_{safe} = \dfrac{\pi^2 EI}{3.0\times L^2} = \dfrac{\pi^2\times 200000\times 9.93\times10^6}{3.0\times 4000^2}$

1.134 $= 408.4\ kN > 360\ kN$ ✓

• AB $I = 3.98\times10^6\ mm^4$

$P_{safe} = \dfrac{\pi^2\times 200000\times 3.98\times10^6}{3.0\times 3606^2}$

Least safe. $\dfrac{Safe}{Load} = 1.117$

$= 201.4\ kN > 180.3\ kN$ ✓

• EG $A = 2320\ mm^2$

1.194 $T_{safe} = \dfrac{2320\times 350}{2.0} = 406\ kN > 340\ kN$ ✓

Yes truss members safe with 60 kN loads.

$1.117\times 60 = 67.02$

Member AB most critical

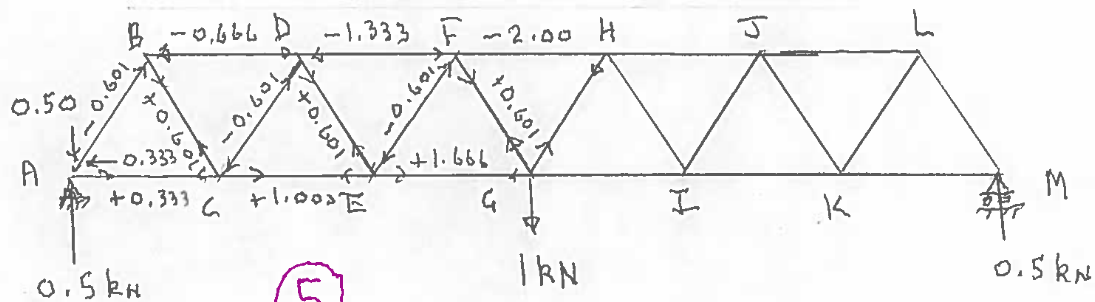
At 67 kN point loads truss on boundary.

1(c). Calculate the vertical deflection of joint G due to the application of the five 60 kN loads. Use the method of virtual work. Fill in the table below. Note that the table lists the members for only the left half of the truss. What is the natural frequency of the bridge with the crowd of people on it? (14 marks)

~~4~~ 3

Member	P (kN)	A (mm^2)	σ (MPa)	ε (mm/m)	L (m)	Δ (mm)	P* (kN)	Work(J)
BD	−200	2760	−72.5	−0.363	4.00	−1.45	−0.666	0.966
DF	−320	2760	−115.9	−0.580	4.00	−2.32	−1.333	3.093
FH	−360	2760	−130.4	−0.652	2.00	−1.30	−2.000	2.600
AC	+100	2320	+43.1	+0.216	4.00	0.86	+0.333	0.287
CE	+260	2320	+112.1	+0.560	4.00	2.24	+1.000	2.240
EG	+340	2320	+146.6	+0.733	4.00	2.93	+1.666	4.881
AB	−180.3	2820	−63.9	−0.319	3.61	−1.15	−0.601	0.691
CD	−108.2	2820	−38.4	−0.192	3.61	−0.69	−0.601	0.415
EF	−36.1	2820	−12.8	−0.064	3.61	−0.23	−0.601	0.138
BC	+180.3	2820	+63.9	+0.319	3.61	+1.15	+0.601	0.691
DE	+108.2	2820	+38.4	+0.192	3.61	+0.69	+0.601	0.415
FG	+36.1	2820	+12.8	+0.064	3.61	+0.23	+0.601	0.138

16.56 J



5

$$1 \times \Delta_G = 2 \times 16.56 = 33.1 \text{ mm}$$

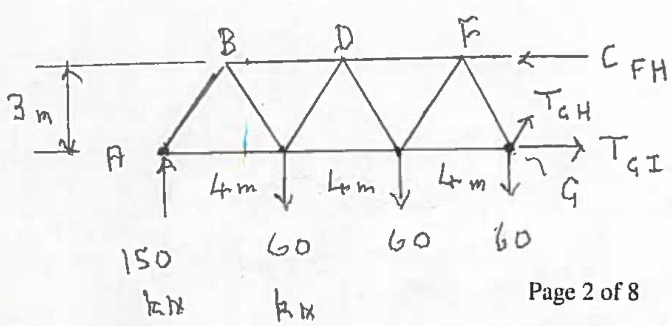
$$\Delta_G = \underline{\underline{33.1 \text{ mm}}}$$

~~5~~ 3

2

$$f_o = \frac{17.75}{\sqrt{33.1}} = \underline{\underline{3.08}} \text{ Hz}$$

Quick check of statics for 1.(a).



$\Sigma M_G = 0$

$150 \times 12 - 60 \times 8 - 60 \times 4 - C_{FH} \times 3 = 0$

$\therefore C_{FH} = \underline{\underline{360}}$ kN

2. The Douglas Fir, No.1 Grade, 241 x 241 sawn timber beam shown below spans 3.50 m and supports an off-centre point load with a magnitude of 35 kN which includes an allowance for the self-weight of the beam.

(a) Draw the shear force and bending moment diagrams for the beam. Calculate and show important values. (5 marks)

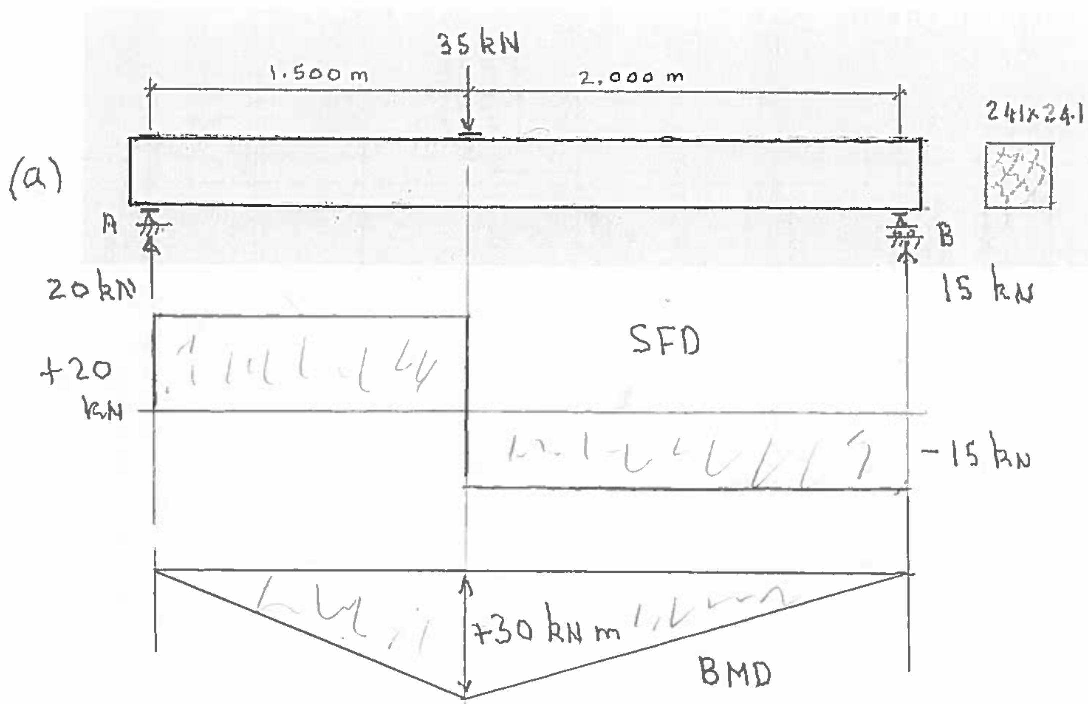
(b) Use Navier's equation to find the maximum flexural stress, σ, in the beam. (3 marks)

(c) Use Jourawski's equation to find the maximum shear stress, τ, in the beam. (3 marks)

(d) Are the stresses safe? (4 marks)

(e) What will be the deviation of support A from a tangent draw at support B and hence what is the anti-clockwise rotation of joint B caused by the 35 kN load. (6 marks)

(f) What is the deflection at mid-span of the beam caused by the 35 kN load. (6 marks)



(b) $$\sigma = \frac{My}{I} = \frac{30\times10^6 \times 241/2}{\underbrace{241\times241^3/12}_{}} = \underline{\underline{12.86\ \text{MPa}}}$$

1.750x10⁶

(c) $$\tau = \frac{VQ}{Ib} = \frac{20000 \times 241\times0.5\times241 \times 0.25\times241}{241 \times 241\times 241^3/12}$$

$$\approx \underline{\underline{0.517\ \text{MPa}}}$$

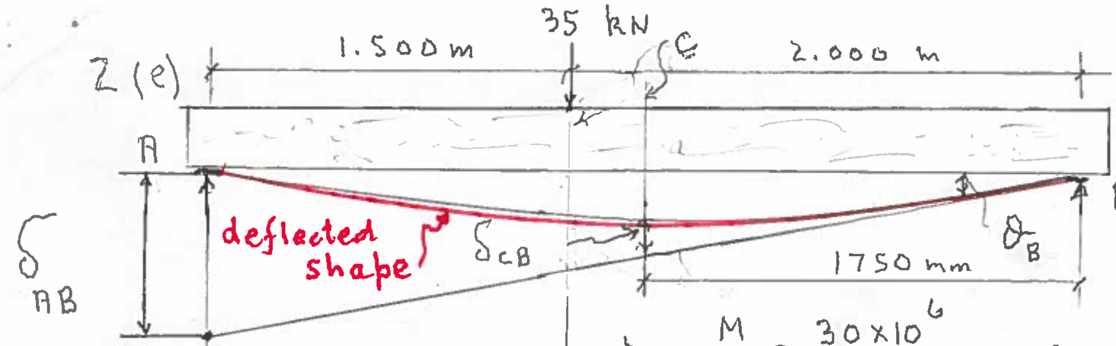
(d) $\sigma_{safe} \approx 20.0/1.5 = \underline{13.33\ \text{MPa}} > 12.86\ \text{MPa}$ ✓

$\tau_{safe} = 1.1/1.5 \approx 0.733\ \text{MPa} > 0.517\ \text{MPa}$ ✓

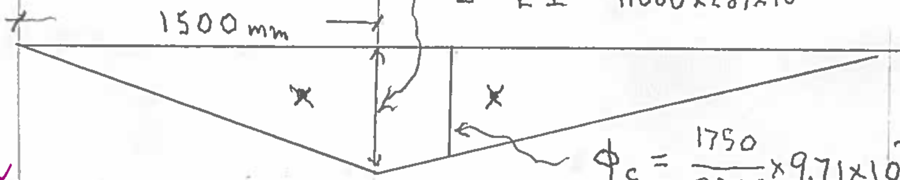
From Table 9-11-2(b) handout!

<u>Yes... stresses safe,</u>

2 (e)



$\phi_L = \dfrac{M}{EI} = \dfrac{30\times10^6}{11000\times281\times10^6} = 9.71\times10^{-6}$ rad/mm



$\phi_c = \dfrac{1750}{2000}\times 9.71\times10^{-6}$

$= 8.50\times10^{-6}$ rad/mm

(6)

if $E = 200000$ MPa, $\phi_c = 0.534\times10^{-6}$ rad/mm

$$\delta_{AB} = \tfrac{1}{2}\times 9.71\times10^{-6}\times 2000\times\left(1500+\tfrac{1}{3}\times2000\right)$$

$$+\tfrac{1}{2}\times 9.71\times10^{-6}\times 1500\times\tfrac{2}{3}\times1500$$

$$= 21.0 + 7.28 = \underline{\underline{28.3}}\ \text{mm}$$

$$\theta_B = \frac{28.3}{3500} = 0.00809 \text{ radians}$$

$$= \underline{\underline{0.463^\circ}}$$

2 (f) $\delta_{CB} = \tfrac{1}{2}\times 8.50\times10^{-6}\times 1750\times\tfrac{1}{3}\times1750$

(6)

$= 4.39$ mm

$$\Delta_c = \frac{1.750}{3.500}\times 28.3 - 4.39$$

$$= \underline{\underline{9.76}}\ \text{mm}$$

$$\frac{\Delta}{L} = \frac{1}{359} \quad \ldots \text{ not visible}$$

3. A professor is considering making a small machine to take to lectures to demonstrate the behaviour of thin walled members under compression. As shown in the figure it would have a wooden base with a wooden post on one side which would support an aluminum lever. By placing a weight, W Newtons, at the end of the lever a compression force would be applied to the 600 mm long thin walled compression member. The square hollow member would be made from 0.35 mm thick cardboard (i.e. the wall thickness would be 0.35 mm) with a compressive strength of 6 MPa, a modulus of elasticity of 4000 MPa and a Poisson's ratio of 0.25. The figure on the right shows how the cross-sectional area, A, of the specimen and it's second moment of area, I, can be calculated from the external dimension b which is in mm.

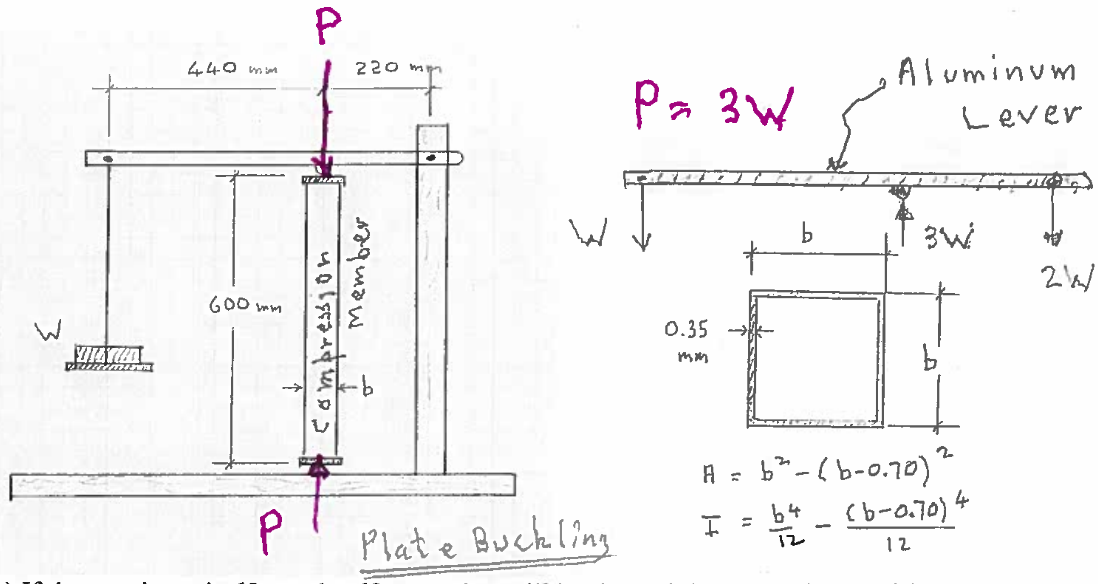


Plate Buckling

(a) If the specimen is 60 mm by 60 mm what will be the weight W required to fail the specimen and what will be the mode of failure? (6 marks)

$A = 60^2 - (60-0.70)^2 = 83.5\ mm^2$ $\quad I = \frac{(60^4 - 59.3^4)}{12} = 49.5\times10^3\ mm^4$

- $P_{crush} = 83.5 \times 6 = 501\ N$ $\quad W = 167 N$
- $\sigma_{cr} = \frac{4\pi^2 \times 4000}{12(1-0.25^2)}\left(\frac{0.35}{59.3}\right)^2 = 0.489 MPa$
- $P_{Euler} = \frac{\pi^2 \times 4000 \times 49.5\times10^3}{600^2} = 5430\ N$ $\quad W = 1810\ N$

$P_{cr} = 0.489 \times 83.5 = 40.8\ N$

$W = 13.6\ N$

(b) If the specimen is 15 mm by 15 mm what will be the weight W required to fail the specimen and what will be the mode of failure? (6 marks)

$A = 15^2 - 14.3^2 = 20.5\ mm^2$ $\quad I = \frac{15^4 - 14.3^4}{12} = 734\ mm^4$

$W = 41\ N$

- $P_{crush} = 20.5 \times 6 = 123\ N$

Mode of Failure = Euler Buckling

- $P_{Euler} = \frac{\pi^2 \times 4000 \times 734}{600^2} \simeq 80.5\ N$ ← governs
- $\sigma_{cr} = \frac{4\pi^2 \times 4000}{12(1-0.25^2)}\left(\frac{0.35}{14.3}\right)^2 = 8.41\ MPa$ $\quad P_{cr} = 172.4\ N$

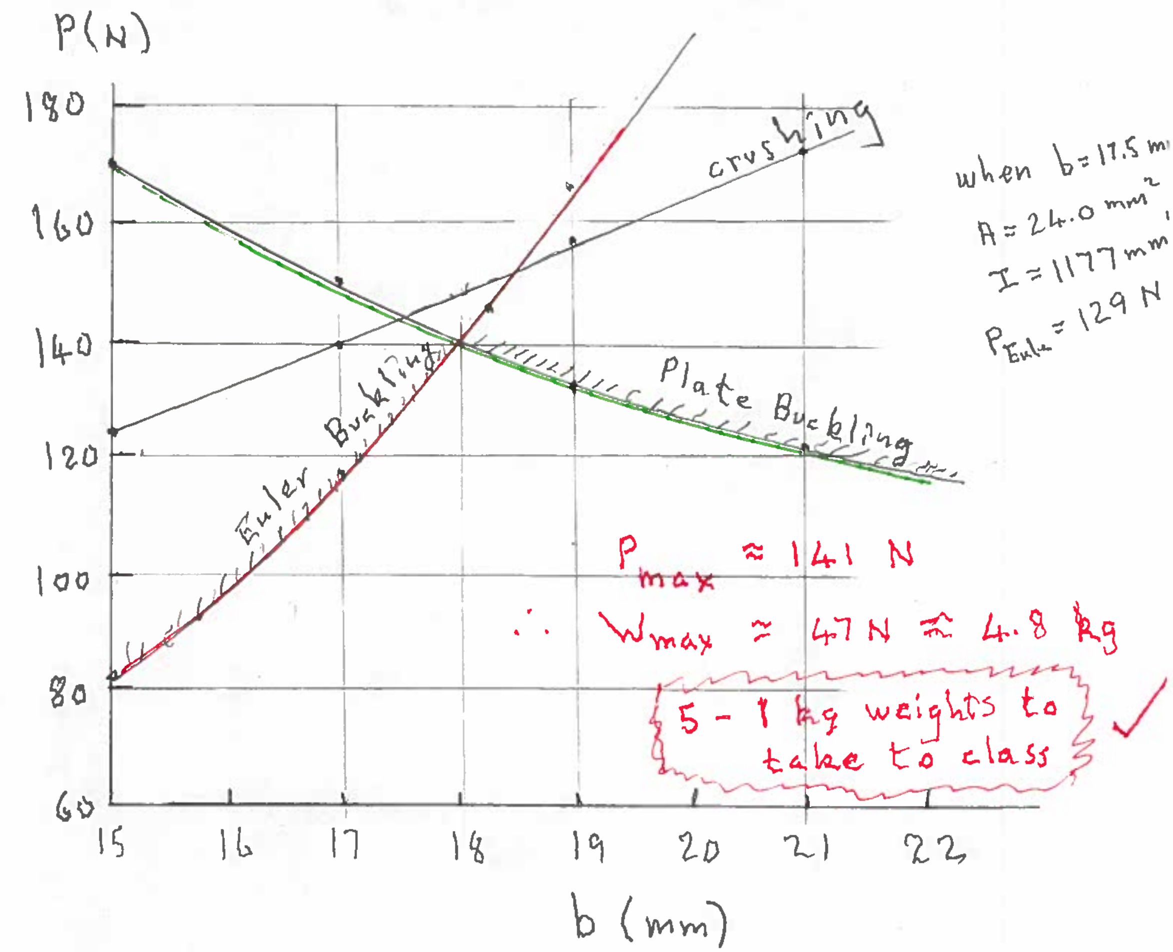
$W = 57.5\ N$

$P_{fail} = 80.5\ N$

$W_{fail} \approx \frac{80.5}{3} = 26.8\ N$

3(c) Draw a plot showing how the required failure value of the weight W changes as the value b of the external width of the specimen changes. What value of b gives the highest value of W? (10 marks)

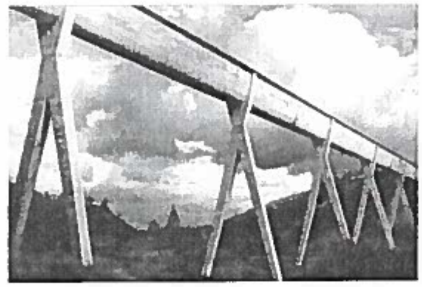
b (mm)	15	16	17	18	19	20	21
A (mm^2)	20.5		23.3	24.7	26.1		28.9
I (mm^4)	734		1077	1283	1514		2055
Crush A×6 (N)	123		140	148	157		173
Euler I×0.1097 (N)	80.5 ✓		118 ✓	141 ✓	166		225
Plate $1720A/(b-0.70)^2$	172		151	142	134 ✓		121 ✓
P_{fail} (N)	80.5		118	141	134		121
W_{fail} (N)	26.8		39.3	47.0	44.7		40.3



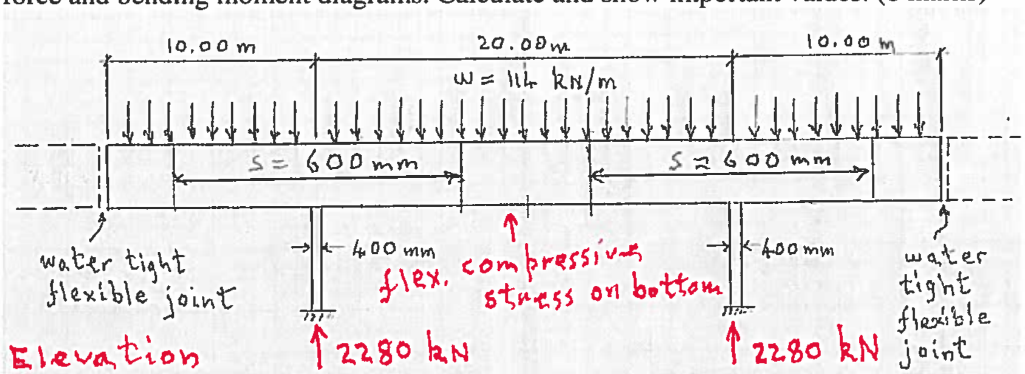
$P_{max} = 141$ N when $b \simeq 18$ mm

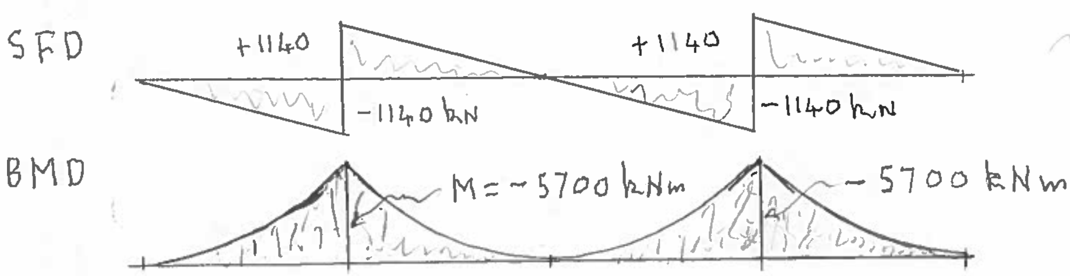
$W_{max} = 141/3 = \underline{\underline{47 \text{ N}}}$

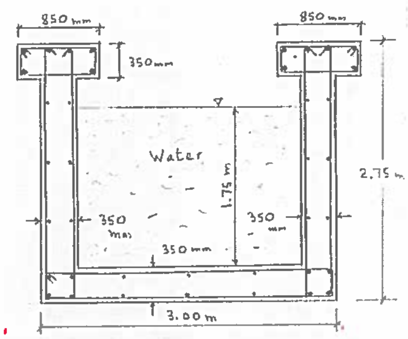
4. The prestressed concrete Alloz Aqueduct in Spain designed by Eduardo Torroja is an elegant structure. Every second span of the structure contains a water-proof, flexible joint at mid-span which does not transmit load making the structure statically determinate. You are to perform some calculations on a simplified reinforced concrete version of this structure with 40 MPa concrete and 400 MPa reinforcing bars.



4(a) The load the aqueduct must carry is the weight of the water, 40 kN/m, and the self-weight of the reinforced concrete structure, 74 kN/m. For the structure between two flexible joints draw the shear force and bending moment diagrams. Calculate and show important values. (6 marks)







4(b) The concrete cross-sectional dimensions and the preliminary choice of longitudinal reinforcing bars is shown in the drawing. In the top flanges there are twelve 25M bars whose centroid is 88 mm from the top face, while in the bottom "flange" there are fourteen 15M bars whose centroid is 175 mm from the bottom face. The twelve 15M bars in the webs can be ignored. The depth of concrete in flexural compression can be taken as 250 mm. ... to simlify calculations.

$I_{gross} = 2.79 \times 10^{12}\ mm^4 \quad y_b \approx 1198\ mm$

Flexural tension bars are the 12 - 25M bars

$A_s = 12 \times 500 = 6000\ mm^2$

$d = 2750 - 88 = \underline{\underline{2662}}\ mm \qquad kd = 250\ mm$

$jd = 2662 - \frac{1}{3} 250 = \underline{\underline{2579}}\ mm$

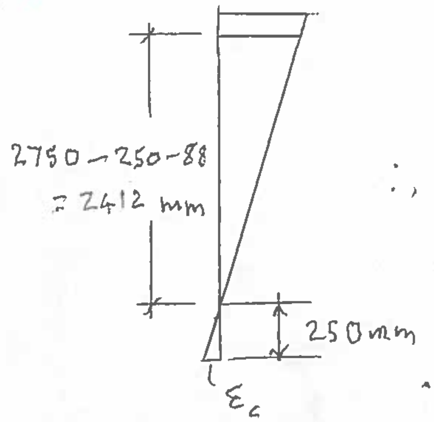
$\sigma_s = \frac{M}{A_s jd} = \frac{5700 \times 10^6}{6000 \times 2579} \approx \underline{\underline{368}}\ MPa > 240\ MPa$

Must increase area of top steel. Use 8 - 35M bars to replace 8 - 25M bars in top row $A_s = 10000\ mm^2$

$\sigma_s = 221\ MPa$ ✓

4(b) (Continued) Check if the tensile stress in the longitudinal bars in the longitudinal bars is below the allowable stress of 240 MPa. Also check that the compressive stress in the concrete does not exceed the allowable stress of 20 MPa. (10 marks)

$$\varepsilon_s = 221/200000 \approx 1.105 \times 10^{-3}$$



$$\varepsilon_c = -\frac{250}{2412} \times 1.105 \times 10^{-3}$$

$$= -0.1145 \times 10^{-3}$$

$$\therefore \sigma_c = -30000 \times 0.1145 \times 10^{-3}$$

$$= \underline{-3.44 \text{ MPa}} \checkmark$$

$\therefore$ Section flexural stresses now safe.

With 12-25 M

$\varepsilon_s = 1.84 \times 10^{-3}$

$\varepsilon_c = -0.1907 \times 10^{-3}$

$\sigma_c = -5.72$ MP

4(c) On your shear force diagram indicate the regions of the aqueduct that will require shear reinforcement. Further indicate those regions where more than the minimum amount of shear reinforcement is required. Finally calculate the smallest spacing of the 10M stirrups that will be required along the aqueduct. Note that A_v is 4 x 100 = 400 mm^2. (10 Marks)

- Shear reinforcement not required where shear stress, τ, is less than;

$$\tau \leq 0.50 \times 230\sqrt{40}/(1000 + 0.9 \times 2662) = \underline{0.214 \text{ MPa}}$$

$$\therefore V \leq 0.214 \times 700 \times 2579 = 387 \text{ kN}$$

- Minimum shear reinforcement

$$\frac{A_v \sigma_y}{b_w s} = \frac{400 \times 400}{700 \times s} \geq 0.06\sqrt{40} = \underline{0.380 \text{ MPa}}$$

$$\therefore s \leq 602 \text{ mm} \quad \text{use } \underline{s = 600 \text{ mm}}$$

- With minimum shear reinforcement

$$\tau \leq 0.09\sqrt{40} + 0.60 \cot 35^\circ \times 0.380 = \underline{0.895 \text{ MPa}}$$

$$V \leq 0.895 \times 700 \times 2579 = \underline{1615 \text{ kN}}$$

Because V is much less than 1615 kN, more than minimum shear reinforcement is not required. Regions needing minimum shown on elevation.