

[Instructions: Solve the question below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.]

You are allowed: a non-communicating calculator, a one-page formula sheet (can be annotated)].

A wave of amplitude A_i is coming from water ($v_{\text{water}} = 1480 \frac{\text{m}}{\text{s}}$, $\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$) is incident on a boundary with another, unknown liquid.

- a. What is the bulk modulus of the water? [2 points]

$$v = \sqrt{\frac{B}{\rho}} \rightarrow B = v^2 \rho = (1480)^2 * (1000) = 2.19 \text{ GPa}$$

1 point for referring to the equation on the sheet; 1 for calculation

- b. What is the impedance of the water? [2 points]

$$Z = \sqrt{B\rho} = \rho v$$

$$Z = 1.48 \times 10^6 \frac{\text{kg}}{\text{m}^2 \text{s}}$$

Either equation is ok as both appear on the formula sheet (ρv is hidden in the power formula)

- c. After the reflection from the boundary the reflected wave has an amplitude equal to $0.37A_i$ and the reflected and incident waves are in phase with each other. Determine the values of reflection and transfer coefficients [2 points]

$$\text{In phase: } \frac{A_r}{A_i} = +0.37$$

$$R = \frac{A_r}{A_i} = +0.37$$

$$1 + R = T = 1.37$$

- d. What percent of energy is **transferred** to the unknown liquid? [2 points]

$$R_{\text{energy}} + T_{\text{energy}} = 1$$

$$R_{\text{energy}} = R^2 = 0.37^2 = 0.1369 = 0.137$$

$$T_{\text{energy}} = 1 - R^2 = 0.8631$$

Any solution is ok, somehow energy and amplitude coefficients and energy conservation need to be included.

- e. What is the impedance of the unknown liquid? [2 points]

$$T_e = \frac{Z_2}{Z_1} T^2 \rightarrow Z_2 = \frac{T_e Z_1}{T^2} = \left(\frac{0.8431}{1.37^2} \right) 1.48 \times 10^6 = 0.681 \times 10^6 \frac{\text{kg}}{\text{m}^2 \text{s}}$$