UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Term Test III

First Year — Program 5

MAT185H1S — Linear Algebra

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29 March 2012

Student Name:

Last Name First Names

Student Number: Tutorial Section: TUT

Instructions:

1. Attempt *all* questions.
2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question.
4. *No* aid is permitted.
5. The duration of this test is 90 minutes.
6. There are 9 pages and 5 questions in this test paper.

For Markers Only		
Question	**Value**	**Mark**
A		
1	10	
B		
2	10	
C		
3	10	
4	10	
5	10	
Total	50	

A. Definitions and Statements

Fill in the blanks.

1(a). The *rank* of $\mathbf{A} \in {}^m\mathbb{R}^n$ is

/2

1(b). State the *dimension formula*.

/2

1(c). What is a *transformation* matrix?

/2

1(d). State the *Laplace expansion* for determinants.

/2

1(e). State the *transpose theorem* for determinants.

/2

B. True or False

Determine if the following statements are true *or* false *and indicate by "$\mathcal{T}$" (for true) and "$\mathcal{F}$" (for false) in the box beside the question. The value of each question is 2 marks.*

2(a). If $\mathcal{S}$ is subspace of $\mathbb{R}^6$ with $\dim \mathcal{S} = 5$ then every basis for $\mathbb{R}^6$ can be reduced to a basis for $\mathcal{S}$ by removing one vector.

2(b). If $\mathbf{A} \in {}^6\mathbb{R}^8$ then the columns of $\mathbf{A}$ are linearly dependent.

2(c). If the columns of $\mathbf{A} \in {}^m\mathbb{R}^n$ are linearly independent then $\mathbf{Ax} = \mathbf{b}$ has at least one solution for every $\mathbf{b} \in {}^m\mathbb{R}$.

2(d). The matrix

$$\mathbf{P} = \begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix}$$

is a transformation matrix between two bases for ${}^2\mathbb{R}$.

2(e). If $\mathbf{A}(\mathbf{B} + \mathbf{C}) \in {}^n\mathbb{R}^n$ and $\det \mathbf{A}(\mathbf{B} + \mathbf{C}) \neq 0$ then $\mathbf{A}$ is invertible.

C. Problems

3. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } \mathbf{U} = \mathbf{E}(-1;3,1)\mathbf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) Find bases for $\mathcal{C}(\mathbf{A})$ (*i.e.*, col $\mathbf{A}$) and $\mathcal{C}(\mathbf{U})$ (col $\mathbf{U}$).

(b) Find bases for $\mathcal{R}(\mathbf{A})$ (*i.e.*, row $\mathbf{A}$) and $\mathcal{R}(\mathbf{U})$ (row $\mathbf{U}$).

(c) Find bases for $\mathcal{S}_{\mathbf{A}}$ (*i.e.*, null $\mathbf{A}$) and $\mathcal{S}_{\mathbf{U}}$ (null $\mathbf{U}$).

3(a). Find bases for $\mathcal{C}(\mathbf{A})$ (*i.e.*, col $\mathbf{A}$) and $\mathcal{C}(\mathbf{U})$ (col $\mathbf{U}$).

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3(b). Find bases for $\mathcal{R}(\mathbf{A})$ (*i.e.*, row $\mathbf{A}$) and $\mathcal{R}(\mathbf{U})$ (row $\mathbf{U}$).

/3

3(c). Find bases for $\mathcal{S}_{\mathbf{A}}$ (*i.e.*, null $\mathbf{A}$) and $\mathcal{S}_{\mathbf{U}}$ (null $\mathbf{U}$).

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4. Let $\mathbf{A} \in {}^n\mathbb{R}^{n-1}$ with rank $\mathbf{A} = n - 1$ and $\mathbf{b} \in {}^n\mathbb{R}$. Note that the augmented matrix is $[\mathbf{A} \,|\, \mathbf{b}] \in {}^n\mathbb{R}^n$.

(*a*) Prove that $\mathbf{Ax} = \mathbf{b}$ has no solution if and only if $\det [\mathbf{A} \,|\, \mathbf{b}] \neq 0$.

(*b*) Prove that $\mathbf{Ax} = \mathbf{b}$ has a solution if and only if $\det [\mathbf{A} \,|\, \mathbf{b}] = 0$.

4(a). Prove that $\mathbf{Ax} = \mathbf{b}$ has no solution if and only if $\det [\mathbf{A} \,|\, \mathbf{b}] \neq 0$.

. . . *cont'd*

4(a). . . . *cont'd*

/6

4(b). Prove that $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a solution if and only if $\det[\mathbf{A} \mid \mathbf{b}] = 0$.

/4

5. (a) Prove the *Maclaurin-Cramer rule*.

(b) Let $\mathbf{A} \in {}^n\mathbb{R}^n$ be invertible wherein the last column is $\mathbf{b} \in {}^n\mathbb{R}$. Find the unique solution to $\mathbf{Ax} = \mathbf{b}$.

5(a). Prove the *Maclaurin-Cramer rule*.

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5(b). Let $\mathbf{A} \in {}^n\mathbb{R}^n$ be invertible wherein the last column is $\mathbf{b} \in {}^n\mathbb{R}$. Find the unique solution to $\mathbf{A}\mathbf{x} = \mathbf{b}$.

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