PHY294 Quantum Term Test #1 (February 8th 2016)

- 75 minutes (closed book, no calculator, one single-sided page of hand-written notes is allowed)
- All the questions are equally weighted (except Extra Credit question #6)
- Note the helpful identities and integrals on the back page

1. Consider a particle of mass m bound by an unusual spring, described by: $E = p^2/2m + ax^4$; a is a constant. Use the Uncertainty Principle to estimate the minimum energy $E_{\min}$, in terms of m, a, $\hbar$.

2. Consider a slow-moving 1D electron wave packet, represented at $t = 0$ by:

$$\psi(x) = \frac{1}{\sqrt{x_2 - x_1}} e^{ik_0 x} \qquad \text{for } x_1 \le x \le x_2\,;$$

$$\psi(x) = 0 \qquad \text{for } x < x_1 \text{ and } x > x_2 \qquad [x_1, x_2, k_0 \text{ are constants}].$$

Calculate the expectation values for position, momentum and kinetic energy.

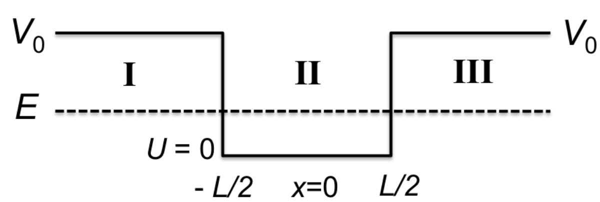
3. Consider a particle of mass m in a 1D infinite-potential square well of width $2L$, centered at $x=L$. Initially the particle is in the 1st excited state. The well width is suddenly expanded to $4L$, centered at $x=2L$. What is the probability for finding the particle in the 3rd excited state of the widened well?

4. A particle of mass m is bound by the 1D finite-potential square well shown below [Note coordinates].

(a) Using the Schrodinger equation, determine the proper form of $\psi(x)$ in each region (I, II, III).

(b) Can $\psi(x)$ in region II be momentum eigenstates? Is the probability current $j(x)$ zero in regions I and III?

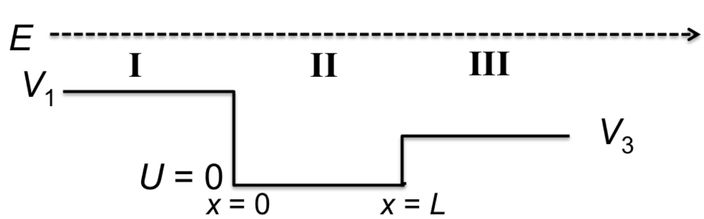
Note: $j(x) = \frac{-i\hbar}{2m}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$



5. A flux of electrons with energy E is incident upon a dip potential as shown below.

(a) Write down the proper form of the wave function for each region (I, II, III), in terms of: $E, V_1, V_3, \hbar, L, m_e$.

(b) State the boundary conditions, and express (need not solve) the transmission probability T in terms of: E, V_1, V_3 and the prefactors in the wave functions. Now if we let $V_1 = V_3$, would dT/dE always be positive?



6. Extra Credit (half-weight):

In momentum-space representation, the 1D position and momentum operators are: $\hat{x} = i\hbar\frac{\partial}{\partial p}$ and $\hat{p} = p$.

Calculate $[\hat{x}, \hat{p}]$ and $[\hat{x}, \hat{p}^2]$. Is there still an uncertainty relation between position and momentum?

Identities and Integrals:

$$\sin 2\theta = 2\cdot\sin\theta\cdot\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$2\cos\theta = e^{i\theta} + e^{-i\theta}$$
$$2i\sin\theta = e^{i\theta} - e^{-i\theta}$$

$$\int_0^\pi \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad m, n \text{ integers}$$

$$\int_0^\pi \cos mx \cos nx dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{\pi}{2} & \text{if } m = n \end{cases} \quad m, n \text{ integers}$$

$$\int_0^\pi \sin mx \cos nx \, dx = \begin{cases} 0 & \text{if } m + n \text{ even} \\ \frac{2m}{m^2 - n^2} & \text{if } m + n \text{ odd} \end{cases}$$