PHY294, Winter 2017, QUIZ IV.

Answer all questions on the exam paper. Duration: 25 minutes.

Name: ___________________; **Student #**: ___________________; **Tutorial group**: ___________

I. Consider two different monatomic classical ideal gases, each of N atoms. They are at the same temperature T and initially occupy separate volumes V and $2V$, respectively. The partition between them is now removed and the two gases are allowed to fill the entire volume. What is the change of the total entropy? Would the entropy change if the gases were made of identical atoms?

Hint: The Sackur-Tetrode formula is $S = kN(\ln\left[\frac{V}{N}\left(\frac{4\pi m\, U}{3h^2 N}\right)^{\frac{3}{2}}\right] + \frac{5}{2})$.

4 points

SOLUTION:

Since the temperatures are the same U/N is the same for both gases. Since the gases are ideal, their energies do not change in the free expansion. So the only reason the entropy changes is the increase of volume: $V \to 3V$ for one of the gases and $2V \to 3V = \frac{3}{2}\,(2V)$ for the other. The entropy of the first gas increases by $kN\ln 3$ and of the second—by $kN\ln\frac{3}{2}$, bringing the total entropy increase to $kN\ln\frac{9}{2}$.

If the gases were identical, the entropy will not change. We have one gas in volume $3V$ whose entropy is the sum of the initial entropies.

II. The heat capacity of a single atomic ideal classical gas is $C_V = \frac{3Nk}{2}$. Explain why this relation can not persist at arbitrarily low temperatures.

3 points

SOLUTION:

A constant heat capacity at arbitrarily low T would imply that $S(T_2) - S(T_1) = C_V \ln\frac{T_2}{T_1}$. This change of entropies diverges as either T_2 or T_1 approach zero, implying infinite entropy at finite N, which makes no sense in view of our statistical understanding of entropy.

II. Two identical monatomic ideal gases, initially kept in separate containers of equal volumes V, have energies U_1, U_2 ($U_1 + U_2 = U$) and number of particles N_1, N_2 ($N_1 + N_2 = N$), respectively. The containers are brought together and the volumes occupied by the gases are now separated by a porous, but unmoveable, partition allowing particles to pass through. Explaining your reasoning, find the values of the energies, U_1', U_2', and numbers of particles N_1', N_2', of the two gases after equilibrium is reached.

3 points

SOLUTION:

Thermal and diffusive equilibrium imply that $\frac{U_1'}{N_1'} = \frac{U_2'}{N_2'}$ and $\frac{N_1'}{V} = \frac{N_2'}{V}$, or equivalently, that temperatures and densities (or chemical potentials are equal). The second relation, because of the equality of volumes, implies that $N_1' = N_2' = N/2$. Hence, we also have that $U_1' = U_2' = U/2$.

Total number of points: 4 + 3 + 3 = 10.