#227

1 of 26



University of Toronto Faculty of Applied Science and Engineering FINAL EXAMINATION – April, 2018

FIRST YEAR - ENGINEERING SCIENCE

MAT195S CALCULUS II

Examiners: F. Al Faisal and J. W. Davis

					ŀ					ŀ						
	l				L	L	<u> </u>		<u> </u>	<u> </u>			<u> </u>	<u> </u>	<u> </u>	<u> </u>
T																
Last	name		f			,					,	,		,	· · · · · ·	ı
	l						<u></u>				<u> </u>			l .		
Stude	ent n	.umbe	<u>:</u>													
				Γ	i	·	T	Ι	T	r –	I	Ι	<u> </u>	Ι	Ι	l
	ŀ			ļ												
				l.	I			1				l		Į.		

Instructions:

- (1) Closed book examination; no calculators, no aids are permitted
- (2) Answer as many questions as you can. Parts of questions may be answered.
- (3) Do not separate or remove any pages from this exam booklet.

FOR MARKER USE ONLY											
Question	Marks	Earned	Question	Marks	Earned						
1	13		7	10							
2	8		8	12							
3	9		9	8							
4	10		10	10							
5	10		11	12							
6	8		12	10							
	\searrow	\searrow	Total	120							



F4DADA5B-866C-476E-A823-920B458886C0

 $\verb|crowdmark-assessment-7ad9e| \\$

#227 2 of 26

#227

3 of 26



- 1) Evaluate the integrals:
- a) $\int \frac{dx}{\sqrt{2x-x^2}}$
- b) $\int \tan^5 x \, dx$
- $c) \int \frac{4x}{x^3 + x^2 + x + 1} dx$

(13 marks)



EEFCF691-D70D-4949-9C11-45D2D6BEF174

crowdmark-assessment-7ad9e

#227

90BB7368-3E29-414C-A48D-3B24D12A62A4

crowdmark-assessment-7ad9e

227 5 of 26



2) Find the area of region outside $r = \cos(2\theta)$ and inside r = 1/2. Provide a sketch of the region. (8 marks)



0F4F9D17-9B65-48A6-9B85-355DB4417590

crowdmark-assessment-7ad9e

#227

C5F6490A-9641-4D35-8461-158FCDF3E2A6

crowdmark-assessment-7ad9e

#227 7 of 26



3) Sketch the parametric curve: $x = t^3 - 3t$, $y = t^2$ (9 marks)



DCB81424-B112-4357-993B-25F4D6945019

crowdmark-assessment-7ad9e

#227

#227

9 of 26



- 4) a) Determine whether the sequence converges or diverges. If it converges, find its limit.

 - i) $a_n = n^2 e^{-n}$ ii) $a_n = \ln(n+1) \ln(n)$

(5 marks)

b) Determine the radius and interval of convergence for the series: $\sum_{k=1}^{\infty} \frac{(5x-4)^k}{k^3}$

(5 marks)



2F1A1B45-036D-4A0E-B50F-36265CFABB4A

crowdmark-assessment-7ad9e

#227

#227 11 of 26



5) Prove the Alternating Series Test for series convergence:

Let
$$\{a_n\}$$
 be a sequence of positive numbers. If $a_{k+1} < a_k$, and $a_k \to 0$ as $k \to \infty$, then
$$\sum_{k=1}^{\infty} (-1)^{k-1} a_k$$
 converges. (A diagram is most helpful in formulating this proof.)

(10 marks)



75A9FB07-F478-4904-9B38-83A1DC7E88EF

crowdmark-assessment-7ad9e

#227

#227

13 of 26



6) Let $a_n = \sum_{k=1}^n \frac{1}{k^2}$. Show that $\sum_{n=2}^{\infty} \frac{1}{n^2 a_n a_{n-1}}$ converges, and find its sum.

Hint:
$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$$

(8 marks)



1B0BAC6C-F893-4EB6-8E48-643BE21F85DD

crowdmark-assessment-7ad9e

#227

. 14 of 26

#227 15 of 26



A bee and two trains: One day a bee was frightened by a train (train A) travelling at 20 km/hr eastward along a straight stretch of track. The bee flies away from the train, going east at 30 km/hr. At that exact moment, a westbound train (train B) is exactly 10 km away, travelling toward train A at 20 km/hr. The bee flies east until it meets train B, at which point it turns around and flies westward (again at 30 km/hr) back towards train A. Each time the bee meets a train, it turns around and flies the other way. Eventually, the trains meet, and the bee falls down dead from exhaustion. How far has the bee flown? The simple answer is found by finding how long it takes for the trains to meet, and noting that the bee is always flying at 30 km/hr. (There are no marks for this solution.) The harder way is to derive and sum the infinite series formed by the distance travelled by the bee on each of the eastbound and westbound legs of its journey. Find this series and its sum.

(10 marks)



76A8F60E-BE9A-4837-A534-17794FAF3856

crowdmark-assessment-7ad9e

#227 16 of 26

#227

17 of 26



8) Find the unit tangent vector, the principal normal vector and an equation in x, y, z for the osculating plane at the point t = 1 on the curve: $\vec{r}(t) = t \hat{i} + 2t \hat{j} + t^2 \hat{k}$

(12 marks)



17D58842-8E27-4442-A626-EFFC4BDA4137

crowdmark-assessment-7ad9e

#227

#227

19 of 26



9) Suppose that z = f(x, y) has continuous second order partial derivatives. Suppose also that $x = s^2 - t^2$ and y = 2st. Show that $\frac{d^2z}{ds^2} + \frac{d^2z}{dt^2} = g(s,t) \left(\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2} \right)$ for some function g(s,t) – and determine this function explicitly.

(8 marks)



3281D82C-B041-4193-B126-B1265708BAC6

crowdmark-assessment-7ad9e

#227

#227 21 of 26



10) Let
$$f(x,y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$$

- a) Find $f_x(a,b)$ for all (a,b) in \mathbb{R}^2 .
- b) Show that f_x is continuous at (0,0).
- c) Without doing any extra work, find $f_y(a,b)$ and explain why Find f_y is continuous at (0,0).
- d) Hence, or otherwise, find the directional derivative $D_u f(0,0)$ where \hat{u} is an arbitrary unit vector.

(10 marks)



4FE43B15-107B-4797-B787-B8FAF2F70569

crowdmark-assessment-7ad9e

#227

38281A83-D39D-444A-B21E-F1A96709C3E6

crowdmark-assessment-7ad9e

#227 23 of 26



11) Find the absolute max/min values of $f(x, y) = y^2 - 2y - 3x^2y$ on the closed and bounded set enclosed by the curve $x = \sqrt{y}$ and the lines x = 1 and y = 0. Provide a sketch of the region, and indicate the locations of the maximum and minimum.

(12 marks)



082BAC36-3D25-47F6-B9AA-CC0540CEB1D9

crowdmark-assessment-7ad9e

#227 24 of 26

#227 25 of 26



12) The plane x + 2y + 3z = 30 intersects the paraboloid $z = (x^2 + y^2)/3$ in an ellipse. Use Lagrange multipliers to find the points on the ellipse that are furthest away from and closest to the origin.

(10 marks)



158F501E-47E3-45E4-9580-7D559794FCC7

crowdmark-assessment-7ad9e

#227