UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Term Test II

First Year — Program 5

MAT185415 — Linear Algebra

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26 February 2013

Student Name:			
	Last Name	First Names	
Student Number:		Tutorial Section:	TUT

Instructions:

- 1. Attempt all questions.
- 2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
- **3.** Write the final answers *only* in the boxed space provided for each question.
- 4. No aid is permitted.
- **5.** The duration of this test is 90 minutes.
- **6.** There are 9 pages and 5 questions in this test paper.

For Markers Only			
Question	Value	Mark	
	Α		
1	10		
В			
2	10		
	С		
3	10		
4	10		
5	10		
Total	50		

A. Definitions and Statements

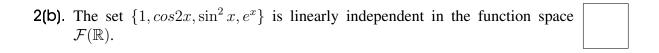
Fill in the blanks.

1(a) . The <i>span</i> of $\{oldsymbol{v}_1 \dots oldsymbol{v}_n\} \subset \mathcal{V}$, a vector space	, is defined as
	/2
1(b). The span of the empty set (span \emptyset), in a vector	or space, is
	/2
1(c). Give a mathematical test for the set of vectors dent.	s $\{oldsymbol{v}_1 \dots oldsymbol{v}_n\}$ to be linearly depen-
	/2
1(d). State the <i>contrapositive</i> of the Fundamental	Theorem of Linear Algebra.
	/2
1(e). A <i>basis</i> for the space of symmetric 2×2 ma	trices is
	/2

B. True or False

Determine if the following statements are true or false and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.

2(a).	Every vector space possesses at least two subspaces.	



2(c). The set
$$\{1+x,2-x^2,3+5x^2,7-2x\}$$
 is linearly independent in $\mathbb{P}_2(\mathbb{R})$.

2(d). If dim
$$\mathcal{V} = n$$
 and M is a finite subset of \mathcal{V} containing $k < n$ vectors, then M is linearly independent.

2(e). If
$$\mathcal{U} \cap \mathcal{W} = \mathcal{U}$$
 and $\dim \mathcal{U} = \dim \mathcal{W}$, then $\mathcal{U} = \mathcal{W}$.

C. Problems

3.	Let M	and N	be two	finite	nonempty	subsets	of a	vector	space	\mathcal{V} .
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- (a) Prove that span $(M \cap N) \subseteq \operatorname{span} M \cap \operatorname{span} N$.
- (b) Is $\operatorname{span}\left(M\cap N\right)=\operatorname{span}M\cap\operatorname{span}N?$ Justify your answer.

	3(a). Prove that span $(M \cap N) \subseteq \operatorname{span} M \cap \operatorname{span} N$.	
		/5
1		, _

3(b). Is span $(M \cap N) = \operatorname{span} M \cap \operatorname{span} N$? Justify your answer.	
	/5

4. Given that $S = \{p \in \mathbb{P}_4 \mid p(1) = p(-1) = 0\}$ is a subspace of \mathbb{P}_4 , find a basis for S and determine the dimension of S .
cont's

4 cont'd	
	/10

5. Let \mathcal{V} be a vector space and let $M \neq \emptyset$ be a subset of n vectors of \mathcal{V} . We say that M is a maximal linearly independent set if and only if
(i) M is linearly independent (ii) If $\mathbf{x} \in \mathcal{V} \setminus M$ (i.e., $\mathbf{x} \in \mathcal{V}$ but $\mathbf{x} \notin M$), then $M \cup \{\mathbf{x}\}$ is linearly dependent
Show that M is a basis for $\mathcal V$ if and only if M is a maximal linearly independent set
cont's

5cont'd	
	/10