CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

21st NOVEMBER 2019

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q2 bonus	Q3A	Q3B	Q3C	Total
16	19	4	5	6	4	50 (54 with bonus)

INSTRUCTIONS

1. This examination is closed book. Only one Letter-sized aid sheet is permitted. It can be computer printed, and both sides of the sheet can be used.
2. Only type 3 calculators are permissible.
3. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
4. Answers written in pencil will NOT be re-marked. This is University policy.
5. For all problems, you must present the solution process in a step by step fashion for partial marks.
6. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY. THIS WILL BE IMPLEMENTED STRICTLY.**

Q.1. [16 points] A SPHERE CARRYING RADIOACTIVE WASTE

A radioactive material of thermal conductivity k is cast as a solid sphere of radius r_0, and placed in a liquid bath at a temperature of T_∞. Heat is uniformly generated in the solid at a volumetric rate of $\dot{S} = \dot{S}_0 \ \left(\mathrm{W/m^3}\right)$, where $\dot{S}_0 \ \left(\mathrm{W/m^3}\right)$ is a constant. The energy is carried away from the surface of the sphere by convection, and the associated convective heat transfer coefficient is h. Beginning with the governing equation for energy balance in solids in the appropriate co-ordinate system, find

(a) the temperature distribution in the sphere, and

(b) the temperature at the center of the sphere.

<u>Specify the governing equation and boundary conditions clearly at the beginning</u>. Classify the boundary conditions into the types that you have learned in class.

Solution:

To solve this problem, we use the spherical co-ordinate system. The governing equation for energy balance in a solid for constant thermal conductivity is

$$\rho C \frac{\partial T}{\partial t} = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right] + \dot{S}$$

At steady state, assuming only a radial temperature dependence, we get

$$\frac{k}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\dot{S} = -\dot{S}_0$$

<u>Coming up with the correct governing equation with appropriate arguments: 3 points</u>

The boundary conditions for temperature are that the temperature is finite at the center, and that there is convective heat transfer off the cylinder's surface.

$$-k\left.\frac{dT}{dr}\right|_{r=r_0} = h\left(\left.T\right|_{r=r_0} - T_\infty\right)$$

<u>The boundary conditions: 2 points</u>

This is a Robin or mixed boundary condition

In this problem, the domain of solution is $r \in (0, r_0]$, as the governing equation is not defined at r=0. However, the temperature is expected to be finite at the center

$T\big|_{r\to 0}$ is finite.

This condition does not fall into any of the categories discussed in class.

<u>Classification of bc: 1 point</u>

Rearranging the governing equation, we get

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = -\frac{\dot{S}_0}{k} r^2.$$

Integrating the above equation once, we get

$$r^2 \frac{dT}{dr} = -\frac{\dot{S}_0}{k}\frac{r^3}{3} + \hat{c}_1.$$

Dividing by r,

$$\frac{dT}{dr} = -\frac{\dot{S}_0}{k}\frac{r}{3} + \frac{c_1}{r^2}.$$

Integrating the above equation, we get

$$T = -\frac{\dot{S}_0}{k}\frac{r^2}{6} - \frac{c_1}{r} + c_2.$$

<u>Two integrations to get the general solution of temperature: 4 points</u>

Since the temperature is finite at the center, we get $c_1 = 0$.

$$T = -\frac{\dot{S}_0}{k}\frac{r^2}{6} + c_2.$$

At r=r0,

$$-k\frac{dT}{dr}\bigg|_{r=r_0} = h\left(T\big|_{r=r_0} - T_\infty\right)$$

$$\dot{S}_0 \frac{r_0}{3} = h\left(-\frac{\dot{S}_0}{k}\frac{r_0^2}{6} + c_2 - T_\infty\right)$$

This gives c_2 as

$$c_2 = T_\infty + \frac{\dot{S}_0 r_0}{3h} + \frac{\dot{S}_0 r_0^2}{6k}.$$

Using the finiteness of temperature at the center: 1 point

Application of the convection boundary condition: 3 points

The temperature distribution is

$$T = T_\infty + \frac{\dot{S}_0 r_0}{3h} + \frac{\dot{S}_0 r_0^2}{6k} - \frac{\dot{S}_0}{k}\frac{r^2}{6}$$
$$= T_\infty + \frac{\dot{S}_0 r_0}{3h} + \frac{\dot{S}_0 r_0^2}{6k}\left(1 - \frac{r^2}{r_0^2}\right).$$

Correct temperature distribution: 1 point

The temperature at the center is

$$T_c = T_\infty + \frac{\dot{S}_0 r_0}{3h} + \frac{\dot{S}_0 r_0^2}{6k}.$$

Center temperature : 1 point

Q.2. [19 points] WHY THIN WIRES ARE INSULATED

Consider a straight 40 gauge nichrome wire of diameter 0.122 mm and length 30 cm. The specific electric resistance of nichrome is 1.5×10^{-6} ohm-m, its thermal conductivity is 11.3 W/(m°C) and its melting point is 1400°C.

(a) [5 points] What is the maximum current the bare wire can carry before it melts? The ambient air is at 25°C and the convective heat transfer transfer coefficient for heat exchange between the wire and air is 30 $W/m^{2o}C$. The ambient air is at 25°C. Also consider radiative heat losses from the wire surface, considering that nichrome has an emissivity of 0.75, and that the wire is exchanging energy radiatively with an enclosing surface at 25°C. Neglect the thermal resistance of the wire in this calculation. Assume that the specific resistance and other properties are independent of temperature.

[**Hints :** Electric resistance (ohm) = Specific resistance (ohm-m) × Length of wire / Cross-sectional area of wire, power = current^2 × electrical resistance].

Solution:

For a wire temperature equal to the melting point (1400ºC), the rate of heat transfer at steady state, ignoring the thermal resistance of wire, is equal to the sum of convective and radiative heat loss rates.

$$\dot{Q} = h\left(2\pi r_0 L\right)\left(T_m - T_\infty\right) + \varepsilon\sigma\left(2\pi r_0 L\right)\left(T_m^4 - T_{surr}^4\right)$$
$$= 30\left(2\pi \times 61\times10^{-6} \times 30\times10^{-2}\right)\left(1400-25\right)$$
$$+0.75\times5.67\times10^{-8}\left(2\pi\times61\times10^{-6}\times30\times10^{-2}\right)\left[\left(1400+273\right)^4-\left(25+273\right)^4\right]$$
$$= 4.743 + 38.27$$
$$= 43.01 \text{ W}$$

<u>Formula for the convective and radiative heat losses: 2 points</u>

<u>Calculation of the heat loss rate: 1 point</u>

The rate of heat loss

The power generated due to a current flow of I amps in a wire of radius r0 and length L is $I^2 \dfrac{\rho L}{\pi r_0^2}$, and at the steady state this should be equal to the rate of heat transfer from the wire to the ambient.

$$I^2 \frac{\rho L}{\pi r_0^2} = \dot{Q}$$

Therefore the current that the wire can carry close to its fusion temperature is

$$I = \sqrt{\frac{\dot{Q}\pi r_0^2}{\rho L}} = \sqrt{\frac{43.01\times\pi\times\left(61\times10^{-6}\right)^2}{1.5\ \times\ 10^{-6}\times30\times10^{-2}}} = 1.057 \text{ A},$$

or about 0.75 amps.

<u>Formula for current and calculation: 2 points</u>

(b) [3 points] The wire is to be insulated with a material of thermal conductivity 0.03 W/(m°C). What is the critical insulation radius? Comment on how the addition of insulation to the bare wire is expected to influence the overall thermal resistance of the composite. To calculate the critical insulation radius, use combined heat transfer coefficient, $h_{eff} = \dot{Q}/(A\Delta T)$, to include the effects of both radiation and convection. Here, $\dot{Q}$ is the rate of heat transfer from part (a), *A* is the surface area of the wire, and ΔT is the temperature difference in part (a).

Solution:

The combined heat transfer coefficient is

$$h_{eff} = \frac{\dot{Q}}{(2\pi r_0 L \Delta T)} = \frac{43.01}{\left[2\pi \times 61\times 10^{-6} \times 30\times 10^{-2} \times (1400-25)\right]} = 272.0 \ \frac{\mathrm{W}}{\mathrm{m}^2\,{}^{\circ}\mathrm{C}}.$$

The critical insulation radius for a cylinder is given by

$$r_c = \frac{k_I}{h_{eff}} = \frac{0.03}{272.0} = 2.206\times 10^{-4}\ \mathrm{m} = 110.3\ \mu\mathrm{m}$$

Since the wire radius is smaller than the critical radius, the initial addition of insulation is expected to *decrease* the thermal resistance of the insulated wire.

<u>Calculation of h_{eff}: 1 point</u>

<u>Calculation of r_c : 1 point</u>

<u>Comment: 1 point</u>

(c) [9 points] If the thickness of the insulation is selected to be 2 mm, what is the maximum current that the insulated wire can carry? What is the percentage change in the maximum current compared to the bare wire? Account for convective heat losses with the same parameters as in part (a).

Account for radiative heat losses from the insulation surface, considering that the insulation has an emissivity of **0.9** (different from nichrome), and that the surface is exchanging energy radiatively with an enclosing surface at 25°C. Again, ignore the thermal resistance of the wire, and assume all physical properties to be independent of temperature.

Note that, in this case, you do not know the surface temperature of the insulation *a priori*. You will have to solve a non-linear equation to get this temperature.

Note: The Newton iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - f(x_i) / f'(x_i)$.

Solution:

The outer radius of the composite after adding 2 mm thick insulation will be 2+0.061 = 2.061 mm.

At steady state, the rate of heat transfer through the 1 mm thick insulation should be equal to the convective and radiative heat loss rates from the surface of the insulation

$$\dot{Q} = \frac{2\pi k_I L\left(T_m - T_S\right)}{\ln\left(\dfrac{r_1}{r_0}\right)} = h\left(2\pi r_1 L\right)\left(T_S - T_\infty\right) + \varepsilon\sigma\left(2\pi r_1 L\right)\left(T_S^4 - T_{surr}^4\right)$$

<u>Overall energy balance equation : 3 points</u>

Simplification yields

$$\frac{k_I\left(T_m - T_S\right)}{\ln\left(\dfrac{r_1}{r_0}\right)} = hr_1\left(T_S - T_\infty\right) + \varepsilon\sigma r_1\left(T_S^4 - T_{surr}^4\right)$$

If x is the temperature of the insulation surface in deg C, then

$$\frac{0.03\left(1400 - x\right)}{\ln\left(\dfrac{2.061\times10^{-3}}{0.061\times10^{-3}}\right)} = 30\times2.061\times10^{-3}\left(x - 25\right)$$
$$+0.9\times5.67\times10^{-8}\times2.061\times10^{-3}\left[\left(273 + x\right)^4 - \left(25 + 273\right)^4\right]$$

Substitution gives

$$11.93-0.008523x=0.06183x-1.546+1.052\times10^{-10}\left(273+x\right)^4-0.8294$$

This simplifies to

$$1.052\times10^{-10}\left(273+x\right)^4+0.07035x-14.31=0$$

We can use the Newton method to solve for the root iteratively. Setting

$f(x)=1.052\times10^{-10}\left(273+x\right)^4+0.07035x-14.31,$ we get

$f'(x)=1.052\times10^{-10}\times4\times\left(273+x\right)^3+0.07035.$

With a guess of 700°C, using the Newton iterative method, we get $x = 153.7$.

The temperature of the surface is 153.7°C.

<u>Calculation of surface temperature: 3 points</u>

Knowing the surface temperature, we can calculate $\dot{Q}$.

$$\dot{Q}=\frac{2\pi k_I L\left(T_m-T_S\right)}{\ln\left(\dfrac{r_1}{r_0}\right)}=\frac{2\pi\times0.03\times30\times10^{-2}\left(1400-153.7\right)}{\ln\left(\dfrac{2.061\times10^{-3}}{0.061\times10^{-3}}\right)}=20.02\text{ W.}$$

<u>Calculation of Qdot: 1 point</u>

From the energy balance written in part (a),

$$I=\sqrt{\frac{\dot{Q}\pi r_0^2}{\rho L}}=\sqrt{\frac{31.76\times\pi\times\left(61\times10^{-6}\right)^2}{1.5\ \times\ 10^{-6}\times30\times10^{-2}}}=0.7212\text{ A,}$$

or about 0.91 A. This is about a 31% *decrease* over the bare wire.

<u>Calculation of I : 1 point</u>

<u>Comparison with case (a) : 1 point</u>

(d) [2 points] Based on (c), comment, from a heat transfer perspective, on why one would consider insulating 'thin' wires. How thin does the wire need to be?

Solution:

The expectation from part (a) is that if the bare wire has a radius less than the critical insulation radius, then the addition of insulation should lower the thermal resistance of the composite. A lower thermal resistance implies that larger rates of heat transfer could be sustained for the same temperature difference $T_m - T_{inf}$, where T_m is the wire melting temperature, and T_{inf} is the ambient temperature. This means that the wire would be able to carry more current with insulation.

However, as seen in class, there is a limit to which the thermal resistance can be reduced, which is the critical insulation radius. Beyond this insulation radius, the thermal resistance begins to increase, and beyond a second critical value, the overall thermal resistance can be above that of the bare wire. According to part (b), the critical insulation radius is estimated to be about 110 microns. The 2 mm insulation added leads to a radius of 2.061 mm, and this radius could be beyond the second critical value, which could be a reason for observing a heat loss rate in part (c) that is smaller that for the bare wire in part (a).

So, it would make sense to add insulation only up to the critical insulation radius in order to have the maximum current carrying capacity.

<u>2 points for recognizing that adding insulation up to the critical insulation radius would help to increase the current carrying capacity.</u>

But the above comments, as well as the discussion in class, were on the basis of neglecting radiation effects. Part (a) showed that radiation was the dominant heat loss mechanism. Recall that radiation is a non-linear thermal resistance, and increases sharply as the temperatures involved are lowered. In this problem, adding the insulation lead to a drop in the surface temperature from 1400 ºC for the bare wire to ~153ºC with 2 mm of insulation. Such a temperature drop leads to a strong increase in the radiative thermal resistance and therefore the overall thermal resistance. *Thus, when radiation effects are strong, insulation can increase the overall thermal resistance of the wire irrespective of the added insulation thickness, and decrease its ability to carry current.*

For wires made out of low melting point metals (E.g. Aluminium, mp = 660ºC), the addition of insulation could indeed lead to an increase in the current carrying capacity. This would be the situation in which the addition of insulation would be beneficial.

2 BONUS points for recognizing the effect of radiation.

Bonus [4 points]: In part (a), you ignored the thermal resistance of the wire. Derive the mathematical criterion for this assumption to be valid, by considering the general case of a bare wire of radius r_0, thermal conductivity k_w, and specific resistance ρ, carrying a current I, and cooled purely convectively (no radiation) with a heat transfer coefficient h.

Solution:

For this question, we simply solve the equation the conduction equation for a cylinder with a constant heat source in W/m^3 of $\dot{S}_0 = \dfrac{I^2 \dfrac{\rho L}{\pi r_0^2}}{\pi r_0^2 L} = \dfrac{\rho I^2}{\pi^2 r_0^4}$

The governing equations can be solved for a cylindrical rod , and the temperature at the surface of the wire and the center of the wire are

$$T_S = T_\infty + \frac{\dot{S}_0 r_0}{2h}, \quad \text{and} \quad T_c = T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k_w}.$$

The rate of heat transfer per unit length of the wire is

$$\dot{Q} = \dot{S}_0 \pi r_0^2 L$$

The thermal resistance of the wire is, therefore,

$$R_{wire} = \frac{T_c - T_S}{\dot{Q}} = \frac{\dfrac{\dot{S}_0 r_0^2}{4k_w}}{\dot{S}_0 \pi r_0^2 L} = \frac{1}{4\pi k_w L}$$

The convective thermal resistance is

$$R_{conv} = \frac{1}{h2\pi r_0 L}$$

Their ratio is

$$\frac{R_{wire}}{R_{conv}} = \left(\frac{1}{4\pi k_w L}\right) / \left(\frac{1}{h2\pi r_0 L}\right) = \frac{hr_0}{2k_w}$$

which is a biot number based on the length scale $r_0/2$. For the parameters in this problem, this ratio turns out to be 10^{-3}, showing that the wire's thermal resistance is negligible.

Alternatively, the temperature at the surface of the wire and the center of the wire are

$$T_S = T_\infty + \frac{\dot{S}_0 r_0}{2h}, \text{ and } T_c = T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k_w}.$$

The temperature drop in the rod relative to the temperature drop on the air side is

$$\frac{T_c - T_S}{T_S - T_\infty} = \frac{\left(T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k_w}\right) - \left(T_\infty + \frac{\dot{S}_0 r_0}{2h}\right)}{\left(T_\infty + \frac{\dot{S}_0 r_0}{2h}\right) - T_\infty} = \frac{\frac{\dot{S}_0 r_0^2}{4k_w}}{\frac{\dot{S}_0 r_0}{2h}} = \frac{hr_0}{2k_w}$$

which is the same ratio as the ratio of the thermal resistances. Similar comments follow. <u>Full points if and only if the ratio of the resistances or the ratio of the temperature drops is derived from scratch.</u>

Q3A. [5 points] TURKEY - HOT OR NOT?

A turkey, initially entirely at room temperature (25ºC), is placed in an oven preheated to 170ºC. What will be temperature reading at the center of the turkey after 10 min, given that the cooking of the turkey is a thermal diffusion process? Assume that the turkey is a solid sphere of radius 20 cm with a thermal diffusivity 10^{-7} m^2/s.

Solution:

Since the cooking of the turkey is a thermal diffusion process, in 10 min (600 s), the distance from the turkey surface that heat reaches into the turkey is the thermal diffusion length,

$$\sqrt{\alpha t} = \sqrt{10^{-7} \times 600} = 0.0077 \text{ m}$$

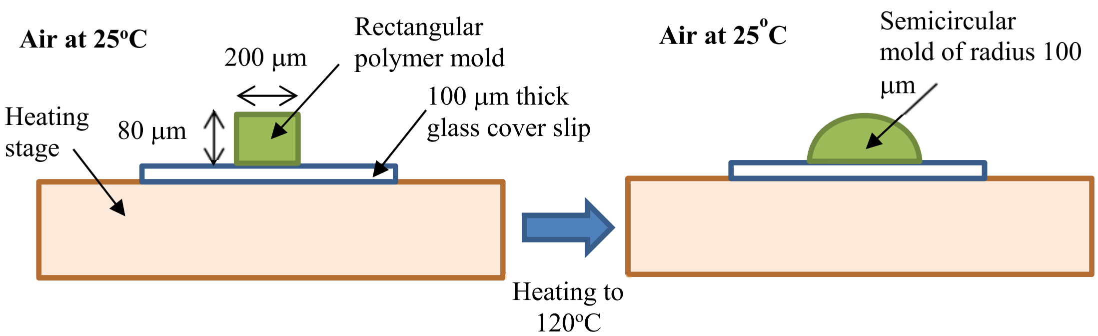
which is on the order of a cm. Since the radius of the sphere is 20 cm, which is significantly higher, the heated front has not had a chance to reach the center. The center temperature is, therefore, just the room temperature.

Alternatively, you can calculate the thermal diffusion time for 20 cm, and show that 10 min is a lot shorter than that.

<u>Recognizing that this is a thermal diffusion time/length calculation: 3 points</u>
<u>Length/ time calculation: 1 point</u>
<u>Comparison with turkey radius/waiting time) and comment: 1 point</u>

Q. 3B. [6 points] THE REFLOW TECHNIQUE

Dr. Y creates a rectangular polymer micromold of cross-section 80 μm × 200 μm (see figure below) on a glass cover slip of thickness 100 microns. Dr. Y. wishes to heat the polymer by placing it on a heating stage maintained at 120ºC, which is greater than the softening point of the polymer. The purpose is to make the polymer flow like a liquid, so that the shape of the polymer cross-section changes from a rectangle to a semicircular arc of radius 100 μm due to surface tension effects (again, see figure below). However, Dr. Y. is worried that, since the ambient air is at 25ºC, temperature gradients set up in the polymer would cause only the bottom part of the rectangular polymer



mold to flow, and not the top part which would be closer to room temperature and therefore still be a rigid solid. Is Dr. Y's concern justified? Explain why/why not.

You *may* find the following properties useful. At 120ºC, the density of the polymer is 1000 kg/m^3, viscosity of the polymer is 10,000 Pa-s, the thermal conductivity of the polymer is 0.5 W/mºC, the thermal conductivity of the air ~~in the oven~~ is 0.025 W/mºC, the heat transfer coefficient for heat exchange with air ~~in the gas phase in the oven~~ is 15 W/m^2 ºC, and the thermal conductivity of the glass cover slip is 1.0 W/mºC.

Solution:

There are two resistances in series here: conduction through the glass and polymer layers, and then convection past the polymer surface. This is a question of whether the dominant temperature drop occurs in the polymer, or in the air phase. To get a measure of this, we can calculate the Biot number. The glass cover slip has a higher thermal conductivity than the polymer, so we can get an upper bound on the Biot number by considering a single material of thermal conductivity equal to that of the polymer. The total thickness over which heat has to diffuse is 100+80 = 180 μm. The Biot number based on this length scale is $\mathrm{Bi} = \frac{hL}{k} = \frac{15 \times 180 \times 10^{-6}}{0.5} = 0.0054$, which is significantly smaller than unity. Hence the the conductive resistances are weak, and convective resistance is dominant. The temperature drop going from the surface of the polymer to the air phase is sginficiantly higher than the temperature through the solids. The polymer is essentially at a constant temperature, and the temperature drop is very approximately, 0.0054*(180-25) = 0.837 ºC.

<u>Realizing that this is a Biot number problem: 4 points</u>

<u>Calculation of approximate Biot number: 1 point</u>

<u>Comment: 1 point</u>

<u>Alternative approaches that calculate the resistances individually will also receive credit.</u>

Q. 3C. [4 points] WHICH CYLINDER COOLS FASTER?

Two copper cylinders, each at 75°C, are allowed to cool in a room where the air temperature is maintained at 25°C. Cylinder 1 is twice as tall as cylinder 2 but both cylinders are the same diameter. The top and bottom of each cylinder are perfectly insulated so the only heat loss is through the sides of the cylinders.

(a) [2 points] Which cylinder loses more energy at the end of the cooling process? Why?

Solution:

The cylinder that is taller has more mass and therefore has more capacity to store energy. Since the initial and final temperatures of the two cylinders are the same, and $Q = mC\Delta T$, the taller cylinder loses more energy on account of its greater mass.

Realizing that this is a $Q = mC\Delta T$ question: 1 point

Comparison of the two cylinders with comment: 1 point

(b) [2 points] For which cylinder does the radial temperature profile change faster? Why?

Solution:

Since the bottom and top of the cylinders are insulated, the heat transfer is only in the radial direction of the cylindrical co-ordinate system for both cylinders; the height of the cylinder does not affect the temperature profile.

Since the radius, initial temperature and fluid conditions are identically for the two cylinders, the temperature profile will change in exactly the same way for the two cylinders.

Realizing that the temperature distribution is a function of only the radial spatial co-ordinate and not z: 1 point

Comparison and comment: 1 point

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C\frac{\partial T}{\partial t}=\nabla\cdot\left(k\nabla T\right)+\dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_x=-k\frac{\partial T}{\partial x},\quad \dot{q}_y=-k\frac{\partial T}{\partial y},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant k</u>: $$\rho C\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{S}$$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(k\frac{\partial T}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial\theta},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant k</u>: $$\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\dot{S}$$

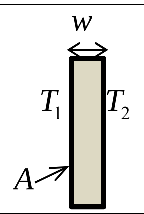
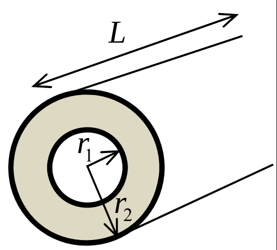
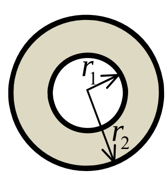
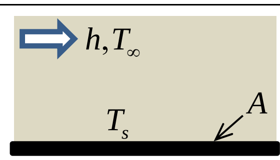
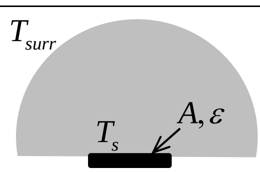
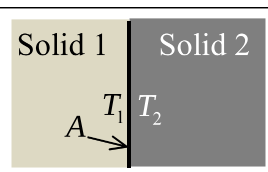
SPHERICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial\theta},\quad \dot{q}_\phi=-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi}$

<u>Constant k</u>: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2}\right]+\dot{S}$

TABLE OF THERMAL RESISTANCES

Geometry / Situation	**Schematic**	**Heat transferred (W)**	**Resistance (ºC/W)**
Slab (plane wall)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{w}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$
Spherical shell (Hollow sphere)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \frac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \frac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \frac{1}{\varepsilon\sigma A\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)}$
Thermal contact resistance		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{R_c}{A}$ (R_c has units of °C-m²/W)