UNIVERSITY OF TORONTO
Engineering Science

PHY293, Part A: Waves and Oscillations

Term Test 1, 16 October 2017

Duration: 60 minutes

- *Write your name, student number and tutorial group on top of **all** examination booklets and test pages used.*
- *Aids allowed: only calculators, from a list of approved calculators as issued by the Faculty Registrar are allowed. Not other aid (notes, textbook, dictionary) is allowed. **Communication devices are strictly forbidden. Turn them off and make sure they are in plain sight to the invigilators.***
- *Answer **all** questions. For each question, the mark breakdown for each subsection is listed in square brackets at the beginning of the question.*
- *There are four questions in this mid-term, with two being standard question-and-answer (QA) questions and two being multiple-choice (MC) questions.*
- *For MC questions, only the final answers will be marked by a machine.*
- *For QA questions, partial credit will be given for partially correct answers. So, please show any intermediate calculations that you do and write down, in a clear fashion, any relevant assumptions you are making along the way.*
- *Do not separate the stapled sheets of the question paper. Hand in the question and rough work sheets together with your exam booklet at the end of the test.*
- *This test has 4 pages, and the total number of marks is 100.*

Some possibly (but not necessarily!) useful equations.

	Amplitude	Velocity	Dissipated Power
Peak freq.	$\omega_{max} = \omega_0\sqrt{1-1/(2Q^2)}$	$\omega_{max} = \omega_0$	$\omega_{max} = \omega_0$
Peak value	$A_{max} = \dfrac{QA_f}{\sqrt{1-1/(4Q^2)}}$	$V_{max} = \omega_0 Q A_f$	$P_{max} = \dfrac{mA_f^2\omega_0^3 Q}{2}$
Miscellaneous	$A(\omega) = \dfrac{\omega_0^2 A_f}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}$ $\tan\delta = \dfrac{\omega\gamma}{\omega_0^2-\omega^2}$	$V(\omega) = \omega A(\omega)$	$\overline{P}(\omega) = \dfrac{m\gamma V^2(\omega)}{2}$ $\approx \dfrac{P_{max}}{1+\frac{4(\omega_0-\omega)^2}{\gamma^2}}$ $(Q \gg 1)$

Q1 [*15 marks*] Consider a simple, damped harmonic oscillator. Consider the following statements:

I. If there is no forcing but light damping, the oscillator will oscillate at the natural frequency of oscillation ω_0.

II. If harmonic forcing is applied to the oscillator, the steady-state response has the same frequency as that of the harmonic force, but there may be a phase shift between the two.

III. If there is no forcing, the larger the damping is, the faster the return to equilibrium is.

IV. If there is no forcing but light damping, the time it takes for the mass position to go from one maximum to the next is longer that if there was no damping.

Which one of the options below is the correct one?

A. Only IV is always correct.

B. Only I and IV are always correct.

C. Only II and IV are always correct.

D. Only I is always correct.

E. Only I and III are always correct.

F. Only I, II and IV are always correct.

Write down your answer in the **multiple-choice answer sheet** provided.

Q2 [*15 marks*] Consider a system of two pendulums coupled by a spring, like the one we saw in class. We ignore the possibility of damping or forcing the system. Consider the following statements:

I. Any initial condition will eventually lead to a motion of both masses.

II. Any initial condition will eventually lead to an activation of both modes.

III. There are three degrees of freedom: one for each pendulum, and one for the spring.

IV. In general, knowing the physical characteristics of the coupled system (masses, lengths, stiffness of the spring and gravity acceleration) and the initial positions of the masses is all we need to predict the future evolution of the positions of the masses.

Which one of the options below is the correct one?

A. Only IV is always correct.

B. Only I and IV are always correct.

C. Only II and IV are always correct.

D. Only I is always correct.

E. Only I and III are always correct.

F. Only I, II and IV are always correct.

Write down your answer in the **multiple-choice answer sheet** provided.

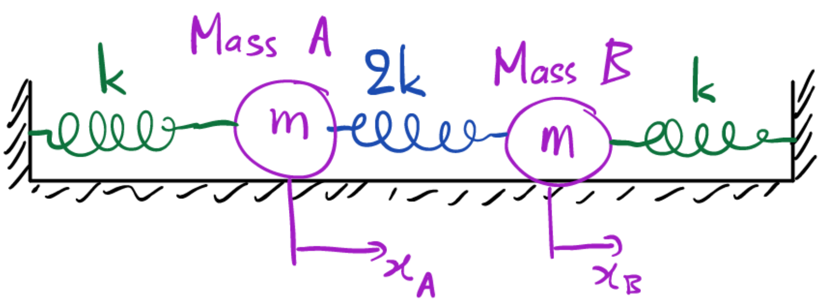
Q3 [*30 marks*] The (time-)average power $\overline{P}$ is absorbed by a driven oscillator and its resonance curve is symmetric. $\overline{P}$ is a maximum at $\nu = 470$ Hz and it half-maximum occurs at 485 Hz.

(a) [*10 marks*] What is the value of the full width at half height ω_{fwhh}?

(b) [*10 marks*] What is the value of Q?

(c) [*10 marks*] If the force is suddenly removed, after how many cycles will the energy of the oscillations be 1/e of its initial value?

Q4 [*40 marks*] Consider the coupled system of oscillating masses, pictured here. The masses, m, are identical. Both of them are attached to each other by a spring of stiffness $2k$. Mass A, the one on the left, is attached to a wall on its left via a spring of stiffness k, while mass B on the right is also attached to the wall on the right via a spring of stiffness k as well. x_A and x_B are the positions of mass A and B, with respect to their respective rest positions.



(a) [*15 marks*] Show that the equations of motion can be written in matrix form as

$$\ddot{\vec{X}} + P\vec{X} = 0, \quad \text{with} \quad \vec{X} = \begin{bmatrix} x_A \\ x_B \end{bmatrix}, \quad P = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \omega_s^2 \quad \text{and} \quad \omega_s^2 = \frac{k}{m}.$$

(b) [*15 marks*] Compute the normal frequencies for $m = 10$ kg and $k = 50$ N.m^{-1}

(c) [*10 marks*] Describe the corresponding normal modes.

THIS IS THE END OF THE TEST.