CHE 260: THERMODYNAMICS AND HEAT TRANSFER

FINAL EXAMINATION FOR HEAT TRANSFER

13th DECEMBER 2018

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3	Q4	Q5	Total
20	20	20	20	20	100

INSTRUCTIONS

1. This examination is closed textbook, closed internet, closed all communication devices.
2. One aid sheet of size 8.5” x 11” aid sheet (both sides) is permitted.
3. Any non-communicating, non-programming, non-graphing calculator is permissible.
4. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
5. Answers written in pencil will NOT be re-marked. This is University policy.
6. For all problems, you must present the solution process in a step by step fashion for partial marks.
7. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY.**

Q.1. [20 points] WHODUNNIT?

Mrs. X returns home after work. Hungry and tired, she rushes to the freezer to lay her hands on her favorite snack – batter-coated, deep-fried potato balls. To her horror, she finds that the freezer door is open. There are two suspects: her stay-at-home naughty cat, and her absent-minded husband, Prof. X. Being a thermal engineer, Mrs. X uses a thermocouple and measures the temperatures at the surface and the center of the *vada* to be -5.7°C and -10.1°C, respectively. She also measures the temperature of the air in the open freezer to be 2.0°C, which is much greater than the originally set temperature of -18.0°C.

Assuming that

a) the *potato ball* was in thermal equilibrium with the environment of the freezer prior to the freezer door being opened;

b) the air in the freezer was at 2°C immediately after the freezer was kept open;

c) each potato ball is a sphere of diameter 5 cm;

d) the warming of the potato ball was radially symmetric; and

e) radiative heat transfer effects may be ignored,

answer the following questions to help Mrs. X figure out who opened the freezer door:

(1) **[12 points]** How long ago was the freezer door opened? Take the thermal conductivity and thermal diffusivity of the potato ball to be 0.5 W/m°C and 10^{-7} m^2/s, respectively. What is the Biot number, and the convective heat transfer coefficient?

Notes/hints:

(a) Do not assume the potato ball to have properties of water anywhere in this problem. Use only the values of properties provided in this problem.

(b) You will encounter a transcendental equation in this problem. The Newton-Raphson method fails miserably for that equation, so don't try it. You will find the adjacent table useful.

y	sin(y)/y
0.000	1
0.314	0.984
0.628	0.935
0.942	0.858
1.257	0.757
1.571	0.637
1.885	0.505
2.199	0.368
2.513	0.234
2.827	0.109
3.142	0

(2) **[2 points]** Mrs. X checks the home alarm logs and finds that Prof. X returned from work 15 min before her. Who opened the freezer door, Prof. X or the cat? Or is there insufficient information to make a deduction?

(3) **[3 points]** How much heat was received by the potato ball in the time calculated in part (1)?

(4) **[3 points]** After what *additional* time will the potato ball begin to unfreeze, assuming that the unfreezing starts when the local temperature in the potato ball exceeds 0°C?

Q.2. [20 points] FIN PROBLEM

A 0.2 cm thick, 15 cm high, and 15 cm long circuit board houses 60 closely-spaced logic chips on one side, each dissipating 0.05 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 30 W/m·°C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 30°C, with a heat transfer coefficient of 20 W/m^2·°C.

(a) Determine the temperatures on the two sides of the circuit board. Use the resistance network approach and sketch the circuit.

(b) Now, a 0.2 cm thick, 15 cm high and 15 cm long aluminium plate (k = 237 W/m-K) with 900, 2-cm-long aluminium pin fins of rectangular profile each of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02 cm thick epoxy adhesive (k = 1.8 W/m-K). Determine the new temperatures on the two sides of the circuit board. Again, use the resistance network approach and sketch the circuit. Assume that the same heat transfer coefficient is applicable in this configuration.

Q.3. [20 points] A SLAB WITH A HEAT SOURCE AND CONVECTION AT ITS SURFACES

Consider a plane wall of thickness $2L$ and a constant thermal conductivity k. A constant heat sink $\dot{S}_0$ is present in the wall. The two faces of the plane wall are exposed to fluids at the same ambient temperature T_∞. The convective heat transfer coefficient on both faces is h. In this problem, you will investigate the temperature distribution in the wall at steady state.

Answer the following questions:

(1) **[5 points]** Starting from the energy balance equation for heat conduction, write down the governing equation and the boundary conditions for this problem.

(2) **[5 points]** Scale the variables in this problem and render the governing equations and boundary conditions dimensionless. What are the temperature and length scales? You should get exactly one dimensionless parameter. What is the physical interpretation of this dimensionless parameter?

(3) **[5 points]** Without solving the governing equations, explain what temperature distributions should be expected in this problem in the limits of small and large values of the dimensionless parameter? Provide scaling estimates of the temperature difference between the center and the wall, $T|_{x=L} - T|_{x=0}$, and the center and the ambient fluid temperature, $T_\infty - T|_{x=0}$, in these two limits.

(4) **[5 points]** Solve the governing equation along with boundary conditions to get the temperature distribution. Confirm the predictions in made in part (3).

Q. 4. [20 points] HEAT LOSSES FROM A STEAM PIPE

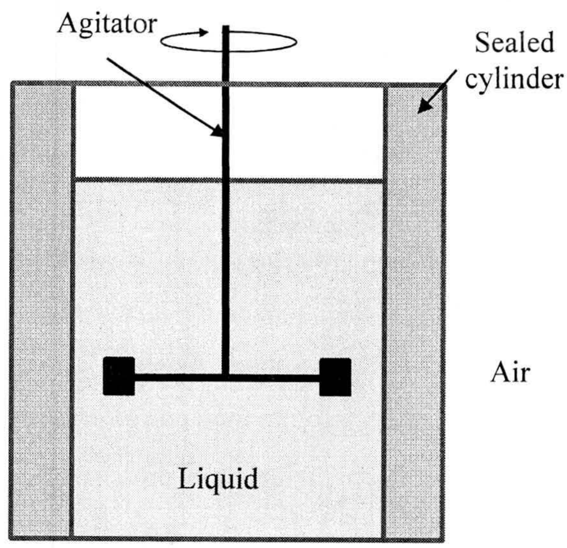
A composite steam pipe comprised of an inner stainless steel tube and a rigid urethane foam insulation is used to transport high temperature steam at 150 °C from one building to another. The thermal resistances of the convection on the steam side and that of conduction through the steel section of the composite are negligible. The stainless steel pipe has an outer diameter of 0.5 m. The pipe is insulated with a rigid urethane foam (k = 0.026 W/m°C) of 10 cm thickness and is exposed to ambient air at -10°C. The air moves in cross-flow fashion over the pipe with a velocity of 3 m/s. What is the heat loss per unit length, and what is the temperature at the outer surface of the insulation? Calculate the convective heat transfer coefficient using an appropriate correlation. Also, include radiative heat losses by accounting for a surrounding surface at a temperature of 250 K. The emissivity of the surface of the insulation is 0.95. The Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4.

Use the following properties of air: k = 0.02881 W/m-K, ρ = 1.028 kg/m^3, μ = 2.052 × 10^{-5} kg/m-s, α = 2.780 x 10^{-5} m^2/s. Ignore the film temperature considerations in the correlation.

Note: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$.

Q.5. CONCEPT QUESTIONS

Q.5A. [7 points] FINS OR NO FINS?



You are working in a chemical manufacturing facility. In one manufacturing step in this facility, a hot, viscous and corrosive liquid is cooled down in a sealed, cylindrical vessel. To protect the cylinder against corrosion, it is made of a solid plastic, with an inner diameter of 1.4 m and a wall thickness 20 cm. The thermal conductivity of the plastic is 0.5 W/m·°C. An agitator in the vessel gently mixes the viscous liquid, leading to a convective heat transfer coefficient of 40 $W/m^2 \cdot {}^oC$ between the liquid and the inner surface of the cylinder. Heat is conducted through the walls of the cylinder, and then convected away by air on the outer surface of the cylinder, with an associated heat transfer coefficient of 30 $W/m^2 \cdot {}^oC$.

Your boss is interested in speeding up the rate of cooling, and asks you to come up with suggestions to achieve this. A senior engineer suggests during a team meeting that if you installed fins on the air side, where the heat transfer coefficient is lower, you could improve the heat loss rate. Neglecting radiation, answer the following questions:

(1) Would you accept the senior engineer's suggestion? Explain why/why not.

(2) Would the installation of fins on the liquid side improve the heat transfer rate? Explain why/why not.

(3) What would be your final proposal to your boss to improve the rate of heat transfer? Why?

Q.5B. [5 points] LATENT HEAT EFFECTS: CAN THEY BE IGNORED?

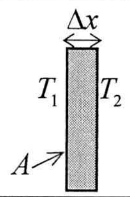
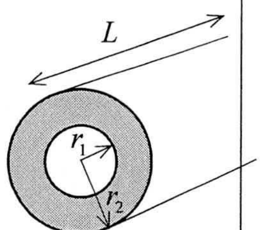
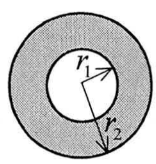
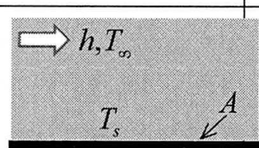
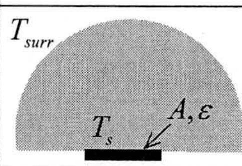
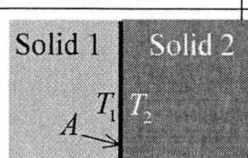
In class, there were several examples related to the concept of heat conduction, which involved a phase change (freezing of the beef, or deep frying of the ice-cream). In these examples, we ignored any latent heat effects in the calculations. Consider one such problem, e.g. the freezing of a cylindrical piece of beef, from 37°C down to -10°C. Assume the frozen and unfrozen pieces of beef to have properties identical to that of water. The latent heat of fusion of water is 33.44 kJ/kg. Was the neglect of the release of the latent heat of fusion during the freezing process appropriate in this problem?

Hint: Examine the total <u>amount</u> of energy exchanged per unit mass of beef.

Q.5C. [8 points] HEAT TRANSFER RATE FROM DRAG

A streamlined body is moving at a velocity of 3 m/s against air at an ambient temperature of 20°C. The body is maintained at a temperature of 50°C. The drag force acting on the object is 6.3 N. What is the rate of heat transfer between the body and fluid? For air, use a density of 1.1 kg/m^3, a specific heat capacity (at constant pressure) of 1007 J/kg°C, a momentum diffusivity of 1.7×10^{-5} m^2/s, and a Prandtl number of 0.72, and assume these values to be constant over the temperature range of concern.

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\Delta x}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$
Spherical shell (Hollow sphere)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \frac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \frac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \frac{1}{\varepsilon\sigma A\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)}$
Thermal contact resistance		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{R_c}{A}$ (R_c has units of °C-m^2/W)

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C\frac{\partial T}{\partial t}=\nabla\cdot\left(k\nabla T\right)+\dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_x=-k\frac{\partial T}{\partial x},\quad \dot{q}_y=-k\frac{\partial T}{\partial y},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant *k*</u>: $$\rho C\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{S}$$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant *k*</u>: $$\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\dot{S}$$

SPHERICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_\phi=-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$

<u>Constant *k*</u>: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right]+\dot{S}$

TABLE 11–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

TABLE 11–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 11–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Total heat transferred:

$$\frac{Q}{Q_{\max}} = 1-\theta_0\frac{\sin\lambda_1}{\lambda_1} \qquad \text{Plane wall}$$

$$= 1-2\theta_0\frac{J_1(\lambda_1)}{\lambda_1} \qquad \text{Cylinder}$$

$$= 1-3\theta_0\frac{\sin\lambda_1-\lambda_1\cos\lambda_1}{\lambda_1^3} \qquad \text{Sphere}$$

Here, $Q_{\max} = mC(T_\infty - T_i)$, where m and C and the mass and the specific heat capacity, respectively, of the solid. θ_0 is a dimensionless center temperature.

NUSSELT NUMBER CORRELATIONS

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\mathrm{Re}_{x,\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5\times10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5\times10^5$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Pr} > 0.6$$

Turbulent: $C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}}, \quad 5\times10^5 \le \mathrm{Re}_x \le 10^7$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}, \quad 0.6 \le \mathrm{Pr} \le 60, \quad 5\times10^5 \le \mathrm{Re}_x \le 10^7$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}}, \quad \mathrm{Re}_L < 5\times10^5$

Turbulent: $C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}}, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Combined: $C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62\log\frac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}, \quad \mathrm{Re}_L < 5\times10^5$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}, \quad 0.6 \le \mathrm{Pr} \le 60, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}, \quad 0.6 \le \mathrm{Pr} \le 60, \quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$

Turbulent: $h = \frac{5[1-(\xi/x)^{9/10}]}{(1-\xi/L)}\,h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

Efficiency and surface areas of common fin configurations

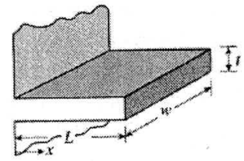
Straight rectangular fins

$m = \sqrt{2h/kt}$

$L_c = L + t/2$

$A_{fin} = 2wL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$

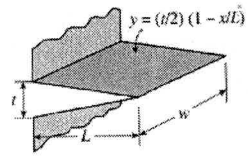


Straight triangular fins

$m = \sqrt{2h/kt}$

$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$

$$\eta_{fin} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



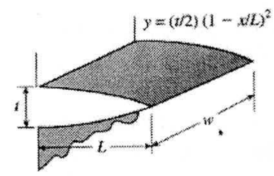
Straight parabolic fins

$m = \sqrt{2h/kt}$

$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$

$C_1 = \sqrt{1 + (t/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$



Circular fins of rectangular profile

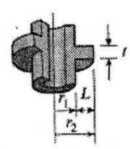
$m = \sqrt{2h/kt}$

$r_{2c} = r_2 + t/2$

$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$

$$\eta_{fin} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



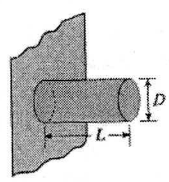
Pin fins of rectangular profile

$m = \sqrt{4h/kD}$

$L_c = L + D/4$

$A_{fin} = \pi DL_c$

$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



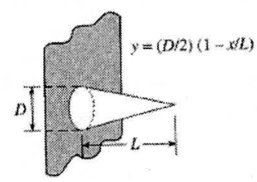
Pin fins of triangular profile

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$

$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$

$I_2(x) = I_0(x) - (2/x)I_1(x)$ where $x = 2mL$



Pin fins of parabolic profile

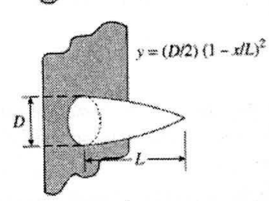
$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi L^3}{8D}[C_3C_4 - \frac{L}{2D}ln(2DC_4/L + C_3)]$

$C_3 = 1 + 2(D/L)^2$

$C_4 = \sqrt{1 + (D/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$



Pin fins of parabolic profile (blunt tip)

$m = \sqrt{4h/kD}$

$A_{fin} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$

$$\eta_{fin} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$