LAST NAME **as on student card**	First Name(s) **as on student card**	Student Number	Tutorial Group

University of Toronto
Faculty of Applied Science and Engineering

Final Examination: Friday, Apr 28, 2017
Duration: 150 minutes

Second Year - Engineering Science
PHY294H1 - Quantum and Thermal Physics

Calculator Type:	**4 - No electronic or mechanical computing devices permitted.**
Exam Type:	**C - Allowed aid includes one page of notes, hand-written on both sides of paper (8.5"x11").**
Examiner:	**John Wei and Erich Poppitz**
Pages:	**3**

Before starting, please **PRINT IN BLOCK LETTERS your name, student number, and tutorial group code** at the top of this page **and** on the exam booklet.

There are two parts of the exam, one for Quantum Physics with four questions, and the other for Thermal Physics with three questions. Each part is worth 40 marks, making the exam 80 marks in total. For full marks, you must clearly show all of your work and reasoning, including any assumptions you make and steps of your calculations.

Good luck!

QUANTUM PHYSICS PART (40 marks in total)

Question 1 (10 marks)

(a) A material is zapped with a pulsed laser beam, liberating a wave packet of *relativistic* electrons. The laser pulse has frequency spread $\Delta\nu \sim 10^9$ Hz, and the material's work function is negligible. Use the uncertainty principle to estimate the minimum spatial extent Δx of this wave packet. [Note: *relativistic* here means the energy-momentum relation is $E \approx pc$, where the speed of light $c = 3 \times 10^8$ m/s.]

(b) An electron is in the ground state of a 3D infinite-potential well, defined as follows: the potential is zero inside the well ($-a < x < a$, $-a < y < a$, $0 < z < 4a$) and infinite everywhere else. Does this electron have well-defined angular momentum about the z-axis? [Hint: $\hat{L}_z = -i\hbar \frac{\partial}{\partial\varphi} = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$]

Question 2 (10 marks) A 1D quantum harmonic oscillator of mass m and spring constant κ is initially ($t = t_0$) in the ground state. Suddenly the spring stiffens, such that κ is quadrupled from its initial value.

(a) At a later time $t_1 > t_0$, what is the probability of finding the oscillator in the ground state?
(b) After finding the oscillator in the ground state at t_1, the spring suddenly softens such that the initial κ is restored. At a time $t_2 > t_1$, what is the probability of finding the oscillator in the 1st excited state?
Note: for a 1D quantum oscillator, the ground-state and 1st excited-state wave functions are:

$$\psi_0(x) = \pi^{-1/4} b^{1/2} e^{-b^2x^2/2} \text{ and } \psi_1(x) = 2^{1/2}\pi^{-1/4} b^{3/2} x e^{-b^2x^2/2}, \text{ where } b \equiv \left(m\kappa / \hbar^2\right)^{1/4}.$$

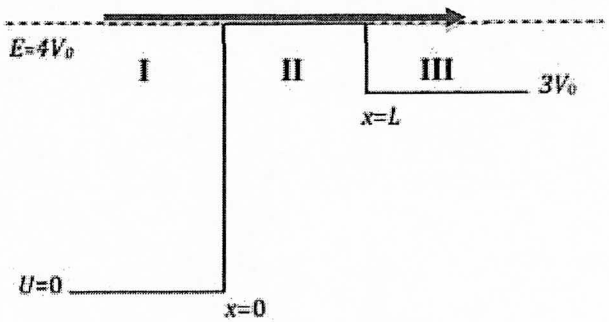
Hint: $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\pi / \alpha}$ and $\int_{-\infty}^{\infty} x e^{-\alpha(x-\beta)^2} dx = \beta\sqrt{\pi/\alpha}$, where $\alpha > 0$.

Question 3 (10 marks) Consider a hydrogen atom in an equally-weighted superposition of 10 of its eigenstates ψ_{n,l,m_l,m_s} at $t = 0$, and assume that the expectation value of energy for this superposition state is minimized.

(a) Prior to any measurement, what is the expectation value of energy? [Give your answer in eV].
(b) A measurement at $t > 0$ finds the energy to be -3.4eV and the angular momentum to be $\sqrt{2}\hbar$ in magnitude. At this time, what is the probability of finding the atom to have $\hbar$ angular momentum along the $+z$ axis? After this time, what is the probability of finding it to have $\hbar$ angular momentum along the $+x$ axis?

Question 4 (10 marks) Consider an electron beam of energy $E = 4V_0$ skimming a potential step $U(x)$ as shown below.

(a) Determine the unnormalized wavefunction in each region (I, II, III), and specify boundary conditions.
(b) Derive an exact expression for the transmission probability T in terms of V_0, L, $\hbar$ and m_e.



THERMAL PHYSICS PART (40 marks in total)

Problem 5 (14 marks) The particles, of mass m, of an ideal single-atomic gas at temperature T are constrained to move in the x-y plane, making the gas effectively two-dimensional. Use a heuristic argument to determine the Maxwell speed distribution of this two dimensional gas. In other words, find the probability density $p(v)$. $p(v)$ determines the probability $p(v)dv$ that a particle has speed between v and $v + dv$ or, equivalently, the fraction of particles in the gas that have speed between v and $v + dv$. Normalize the distribution and determine the most likely speed as well as the *r.m.s* speed. Do your findings agree with your intuition from the equipartition theorem?
You may find these integrals useful: $\int_0^\infty dy\, e^{-y} = 1\,, \int_0^\infty dy\, ye^{-y} = 1$.

Problem 6 (12 marks) Consider a paramagnet made of N spin-1/2 magnetic moments μ in thermal equilibrium at temperature T. Let the energy of a single magnetic moment be $E = -B\mu s$, where $s = \pm 1$ denotes the value of the spin and B is the strength of the applied magnetic field. Determine the average magnetization $\langle M \rangle$ and the average of the square of the magnetization $\langle M^2 \rangle$. Compute the standard deviation $\sigma_M^2 = \langle M^2 \rangle - \langle M \rangle^2$ and compute $\sqrt{\sigma_M^2}/\langle M \rangle$. How does this ratio behave in the large-N limit? Explain the significance of your finding.

Problem 7 (14 marks) The partition function of a simple harmonic oscillator in equilibrium with a thermal bath of temperature T is $Z = \sum_0^\infty e^{-\frac{n\hbar\omega}{kT}} = \left(1 - e^{-\frac{\hbar\omega}{kT}}\right)^{-1}$. Use this result to find the partition function $Z(N, T)$, and then the entropy $S(N, T)$ of an Einstein solid of N oscillators, all of frequency ω. For simplicity, consider only the high-temperature limit. Then, express the entropy as a function of the average energy E of the solid, i.e. find $S(N, E)$. Compare with the entropy obtained as a logarithm of the multiplicity function $\ln\Omega(N, q) = \frac{(N-1+q)!}{(N-1)!q!}\,|_{q \gg N} \approx N \ln\frac{qe}{N}$, where q is the total number of quanta of the Einstein solid. Are you surprised by the result? Explain!