Name (printed legibly):________________

Student # (printed legibly):__________________

Q1:____Q2:____Q3:____Q4:____Q5:____Q6:____

Total:_____ ____

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

October 27, 2014

Instructor – W.R. Cluett

Closed book.

Allowable calculators: Casio FX-991MS or Sharp EL-520X (suffixes may differ)

All questions of equal value.

All work to be marked <u>must</u> appear on front of page. Use back of page for rough work only.

Given information:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}; \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix};\ proj_{\vec{d}}\,\vec{u} = \frac{\vec{u}\cdot\vec{d}}{\|\vec{d}\|^2}\vec{d}$$

Q1: Derive an expression for the shortest distance between the two parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ in terms of the constants a, b, c, d_1 and d_2, where $d_1 \neq d_2$.

Q1: (blank sheet)

Q2: Let the points A, B, and C form a triangle ABC. Let D and E be the midpoints of the sides AB and AC, respectively. Using a vector method approach, show that the line segment DE is parallel to BC and is half as long.

Q3: Find (if possible) conditions on 'a' such that the following system of linear algebraic equations has zero, one, or infinitely many solutions:

$$
\begin{aligned}
&x_1 + 2x_2 - 4x_3 = 4 \\
&3x_1 - x_2 + 13x_3 = 2 \\
&4x_1 + x_2 + a^2 x_3 = a + 3
\end{aligned}
$$

Q3: (blank sheet)

Q4: Suppose that P and Q are nxn matrices.

Prove that $PQ=QP$ if and only if $(P+Q)^2 = P^2 + 2PQ + Q^2$.

Q5: Recall from lecture that a linear transformation is a function that maps a vector to a vector with the following properties:

$L: R^n \to R^m$, i.e. if $\vec{u} \in R^n$, then $L(\vec{u}) \in R^m$

Property a) $L(k\vec{u}) = kL(\vec{u})$

Property b) $L(\vec{u} + \vec{v}) = L(\vec{u}) + L(\vec{v})$

where *k* is a scalar.

a) Show that the given transformation is <u>not</u> a linear transformation:

$$T: R^2 \to R^2 \text{ given by } T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 2y \\ xy \end{bmatrix}$$

b) Assume that the following are linear transformations:

$$D : R^2 \to R^3 \text{ given by } D\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ x-2y \\ x \end{bmatrix}$$

and

$$E : R^3 \to R^3 \text{ given by } E\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x+y+z \\ 2x+y+z \end{bmatrix}$$

Find the matrices associated with these transformations *D* and *E*.

c) Find the matrix associated with the indicated composition of transformations if it is defined. If it is not defined, explain why it is not defined.

$$E(D(\begin{bmatrix} x \\ y \end{bmatrix}))$$

$$E(E(\begin{bmatrix} x \\ y \\ z \end{bmatrix}))$$

Q6: Suppose that A is an *nxn* matrix with distinct (different) eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$. If k is a positive integer:

a) How many eigenvalues are there associated with the matrix A^k? Explain your answer.

b) What are the eigenvalues of A^k ? Explain your answer.