Student Number: ______________ Name: Solution

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, DECEMBER 2014

CIV102H1F – Structures and Materials-
An Introduction to Engineering Design

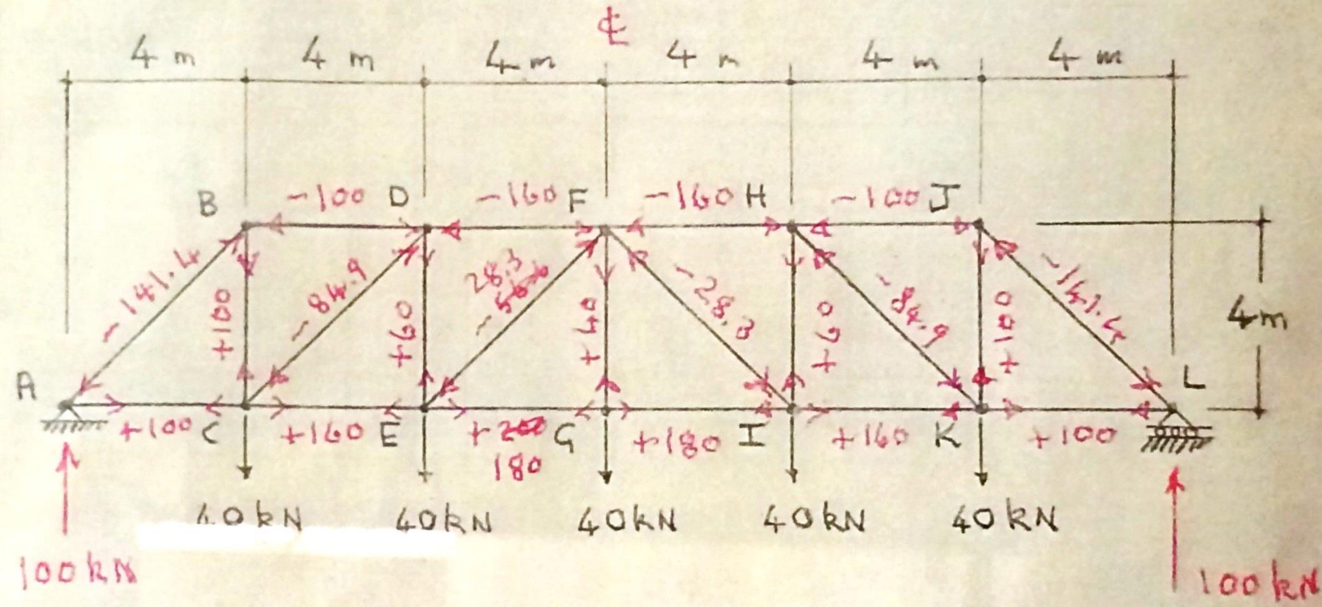
Examiner --- M.P.Collins

1	30
2	28
3	25
4	22
Total	105

Permissible Aids: Notebook, calculator and set-square.

1. The truss shown below supports a pedestrian bridge and is made from steel hollow structural sections with a yield stress of 350 MPa. The truss spans 24 m and when the bridge is crowded with people supports the five 40 kN loads shown.

1(a). Calculate the axial force in each member of the truss due to the 40 kN loads. Neatly write your calculated forces in the table on page 2. Use the convention +ve for tension and –ve for compression. (10 marks)



$\sigma_{max} = 137.4$ MPa for EG $\quad \sigma_{safe} = 350/2 = 175$ MPa ✓

AB $\quad P_{cr} = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2 \times 200000 \times 8.35\times10^6}{5657^2} = 515$ kN$/3.0$

$P_{safe} = 171.7$ kN

DF $\quad P_{cr} = 1030$ kN $\quad P_{safe} = 343$ kN

$A = 3620$ mm^2 $\quad I = 8.35\times10^6$ mm^4

1(b). All the compression members of the truss consist of HSS 127x127x8.0 while all the tension members of the truss consist of HSS 76x76x4.8. Write your calculated stresses for the truss members in the table on page 2. Is the truss safe under the 40kN loads? Yes or no? At what value of the loads will the truss be on the boundary between safe and unsafe? (6 marks)

$A = 1310$ mm^2

Yes truss is safe.

EG $\quad 137.4/175 = 78.5\%$

AB $\quad 141.4/171.7 = 82.4\%$

Truss will be on boundary between safe and unsafe if loads P are increased to

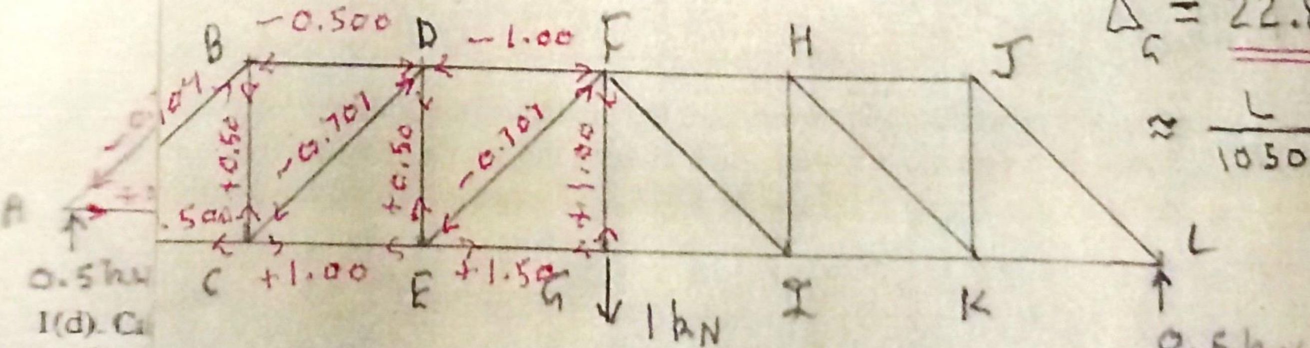
$P = 40/0.824 \approx \underline{48.5\ kN}$

1(c). Using the method of virtual work calculate the vertical deflection of joint G due to the application of the 40 kN loads. Fill in the table below. Note that the table lists the members for only the left half of the truss. (10 marks)

Member	P (kN)	A (mm²)	σ (MPa)	ε (mm/m)	L (m)	Δ (mm)	P• (kN)	Work(J)
BD	−100	3620	−27.6	−0.138	4	−0.552	−0.500	0.276
DF	−160	3620	−44.2	−0.221	4	−0.884	−1.000	0.884
AC	+100	1310	76.3	0.382	4	1.527	+0.500	0.764
CE	+160	1310	122.1	0.611	4	2.44	+1.000	2.440
EG	+180	1310	137.4	0.687	4	2.75	+1.500	4.122
AB	−141.4	3620	−39.1	−0.195	5.66	−1.105	−0.707	0.781
CD	−84.9	3620	−23.5	−0.117	5.66	−0.664	−0.707	0.469
EF	−28.3	3620	−7.82	−0.039	5.66	−0.221	−0.707	0.156
BC	+100	1310	76.3	0.382	4	1.527	+0.500	0.764
DE	+60	1310	45.8	0.229	4	0.916	+0.500	0.458
FG	+40	1310	30.5	0.153	4	0.611	+1.00	0.611

Ext Work $= 1 \times \Delta_G$ J   11.72 J

Int Work $= 2 \times 11.72 - 0.611 = \underline{22.8\ J}$



$\Delta_G = \underline{22.8\ mm}$

$\approx \dfrac{L}{1050}$

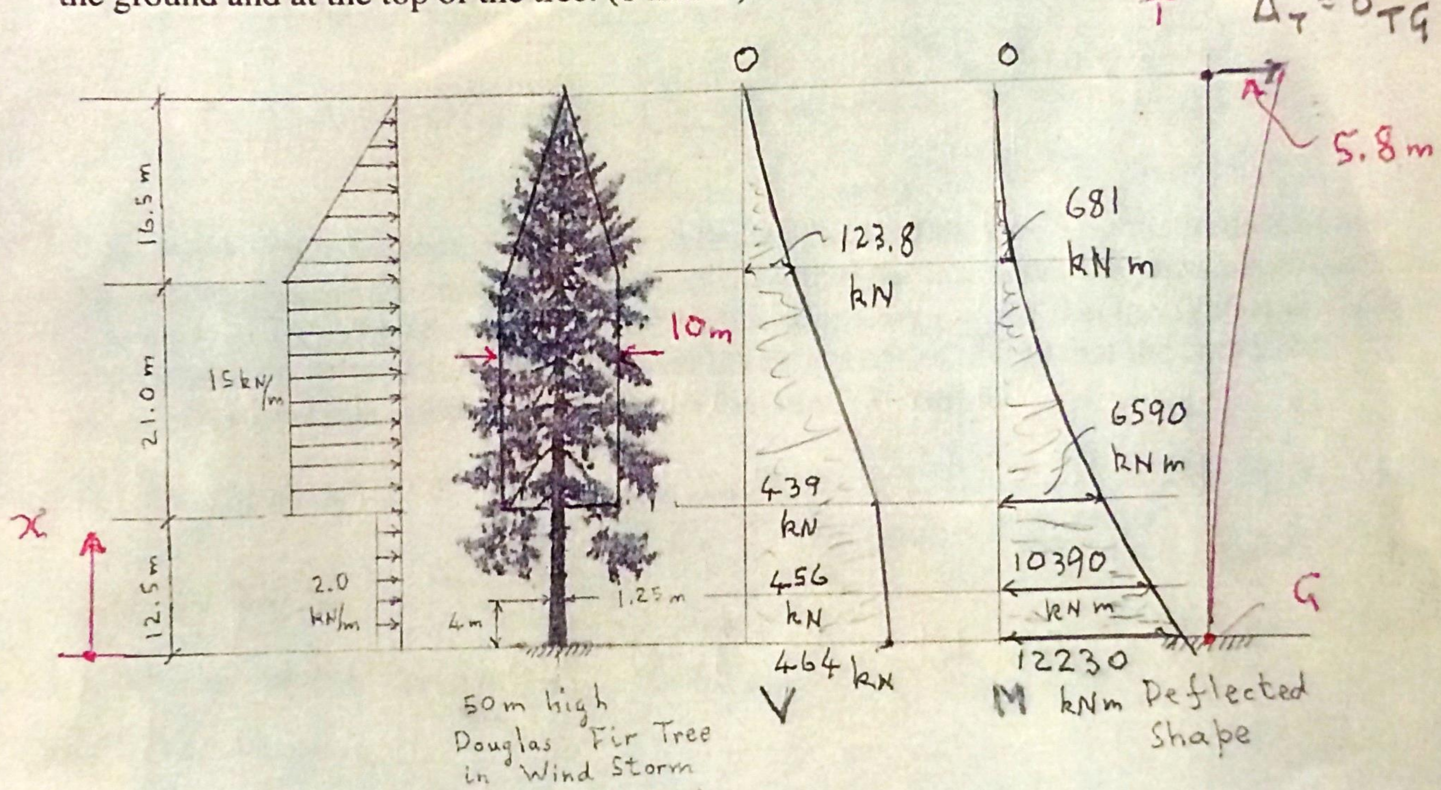
1(d). Calculate the natural frequency of the bridge when crowded with people and comment on the stiffness of the bridge. (4 marks)

$f = \dfrac{17.75}{\sqrt{22.8}} \approx \underline{3.72\ Hz}$

For bridge crowded with people this is a high frequency.

2. During an extreme wind storm a 50 m high Douglas Fir tree is subjected to a horizontal wind pressure of about 1.5 kN/m^2 on the frontal area of the tree. The resulting horizontal loads which must be resisted by the trunk of the tree and carried down to the ground can be approximated as shown in the diagram below. Note that for the forces shown the tree bends to the right causing tensile stresses on the left half of the trunk and compressive stresses on the right.

2(a). Draw the shear force and bending moment diagrams for this vertical cantilever beam (called "baum" or tree). Calculate and show the values of V and M at the ground (assumed to be the fixed support), at 4 m above the ground, at 12.5 m above the ground, at 33.5 m above the ground and at the top of the tree. (6 marks)



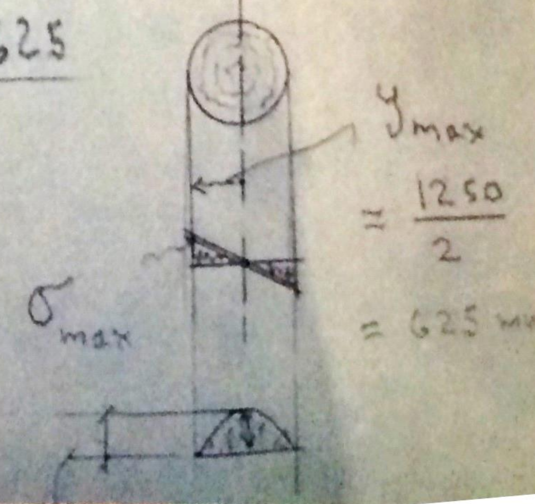
x (m)	50	33.5	12.5	4	0
V (kN)	0	$\frac{1}{2}\times 15\times 16.5 = 123.8$	$123.8 + 15\times 21 = 439$	$439 + 2\times 8.5 = 456$	$456 + 2\times 4 = 464$
M (kN m)	0	$\frac{1}{3}\times 123.8\times 16.5 = 681$ kN m	$681 + 21.0\times \frac{1}{2}(123.8+439) = 6590$ kNm	$6590 + 8.5\times \frac{1}{2}(439+456) = 10394$ kNm	$10394 + 4\times \frac{1}{2}(456+464) = 12234$ kNm

2(b). Use Navier's equation to calculate the maximum longitudinal stress, σ, in the wood of the tree at the section 4 m above the ground. The section is circular with a diameter of 1250 mm. For circular sections I is equal to $\pi D^4/64$. (4 marks)

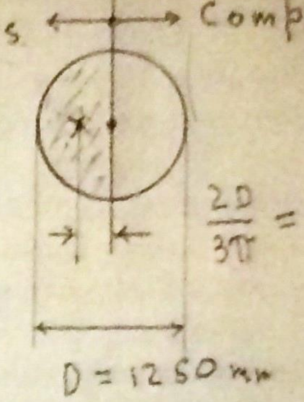
$$\sigma_{max} = \frac{M y_{max}}{I} = \frac{10390\times 10^6 \times 625}{\pi\times 1250^4/64}$$

$$= 54.2 \text{ MPa}$$

$$I = \frac{\pi\times 1250^4}{64} = 119.8\times 10^9 \text{ mm}^4$$



2 (c). Use Jourawski's equation to calculate the maximum shear stress, τ, that occurs in the wood of the tree at the section 4 m above the ground. This maximum occurs at the centroidal axis of the section, which is on a plane halfway between the left face of the trunk and the right face of the trunk. In finding Q note that the centroid of a semicircle is at 2D/(3π) from the centre of the circle. (4 marks)



$$\tau_{max} = \frac{VQ}{Ib}$$

$$= \frac{456\times10^3 \times 162.8\times10^6}{119.8\times10^9 \times 1250}$$

$$= \underline{0.495\ \text{MPa}}$$

$$Q = \frac{1}{2}\cdot\frac{\pi D^2}{4}\cdot\frac{2D}{3\pi} = \frac{D^3}{12} = \underline{162.8\times10^6\ \text{mm}^3}$$

Note: $\sigma_{max}/\tau_{max} = 54.2/0.495 = \underline{109.5}$

2 (d). In the figure on page 3 sketch the deflected shape of the tree. Use the moment-area theorem to estimate the lateral deflection of the top of the tree under the pressure of the wind. The value of E can be taken as 11000 MPa. For simplicity assume that the 1250 mm diameter at 4 m from the ground stays constant over the height of the tree so that the curvature diagram has the same shape as the moment diagram. (8 marks)

At ground $\phi = \frac{M}{EI} = \frac{12230\times10^6}{11000\times119.8\times10^9} = 9.28\times10^{-6}$ rad/mm

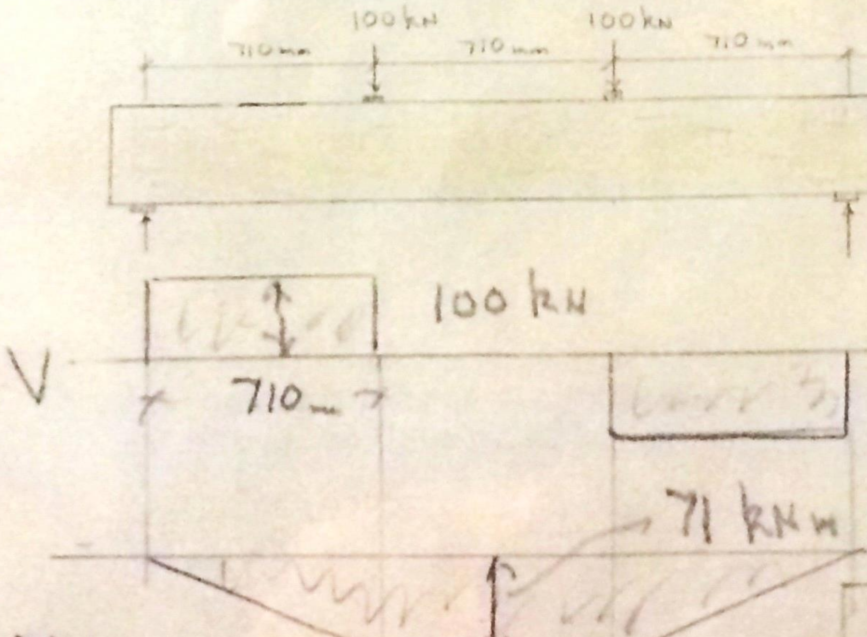
$EI = 1318\ \text{MN m}^2$

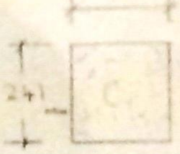
$= \underline{9.28\times10^{-3}\ \text{rad/m}}$

Approximate moment diagram as parabolic

$$\Delta_{top} = \frac{1}{3}\times 9.28\times10^{-3}\times 50\times\frac{3}{4}\times 50 = \underline{5.80\ \text{m}}$$

2(e). In extreme wind storms trees often fail in bending but almost never fail in shear. However when we tested four large Douglas Fir beams under the Baldwin in the CIV102 classes two beams failed in flexure and two failed in shear. The beams failed at a total load of about 200 kN. Why was shear so much more critical for the beams in the laboratory than it is for the trees in the forest? (6 marks)





$$I = \frac{241^4}{12} = 281\times10^6\ \text{mm}^4$$

$$Q = \frac{241^3}{8} = 1.750\times10^6$$

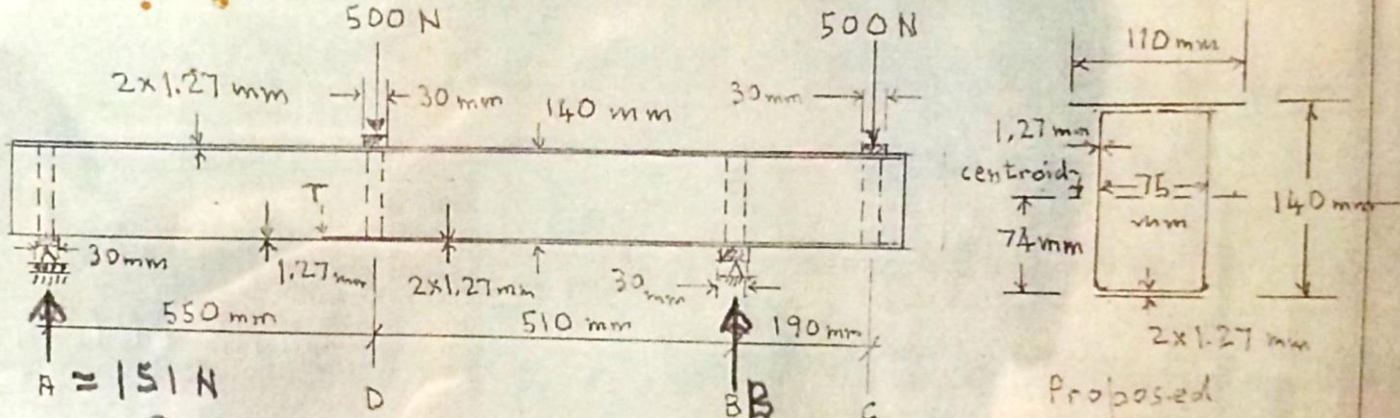
$$\tau_{max} = \frac{VQ}{Ib} = \underline{2.58\ \text{MPa}}$$

$$\sigma_{max} = \frac{My_{max}}{I} = \frac{71\times10^6\times120}{281\times10^6} = 30.4\ \text{MPa}$$

	Tree	Beam
$\frac{\sigma_{max}}{\tau_{max}}$	109.5	11.78

Ratio of flexural stress to shear stress is nearly ten times higher in tree than beam.

3. A design-build competition challenges your team to build the strongest possible model bridge using a 813 mm x 1016 mm x 1.27 mm thick sheet of cardboard. The cardboard has a tensile strength of 16 MPa, a compressive strength of 6 MPa, a shear strength of 4 MPa, a modulus of elasticity of 4000 MPa and a Poisson's ratio of 0.2. One member of your team has suggested the design described in the figure below which she believes will enable your team to join the "KiloNewton Club". It consists of a rectangular box (made by folding one piece of cardboard) with one additional sheet 110 mm wide glued to the top of the box for the total length of the box and a second sheet 78 mm wide glued to the bottom of the box over most of the length. The remaining cardboard is used for the 8 diaphragms shown.

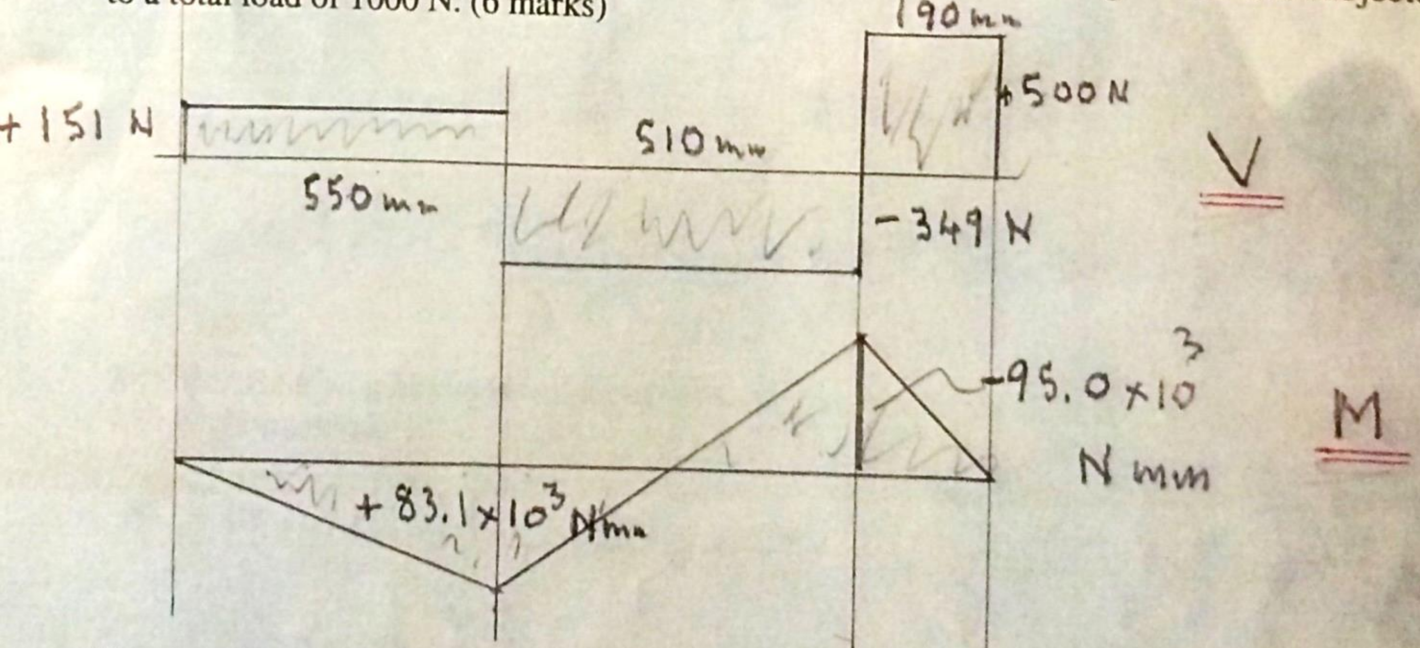


$\Sigma M_A = 0$ ↻ $500 \times 550 + 500 \times (550 + 510 + 190)$

$- B \times (550 + 510) = 0 \therefore B = \underline{\underline{849\,N}}$

$I = 2563 \times 10^3\ mm^4$

3(a). Draw the shear force and bending moment diagrams for the bridge when it is subjected to a total load of 1000 N. (6 marks)



3(b). Use Navier's equation to calculate the maximum flexural compressive stress which occurs at support B under the two 500 N loads. Note that the location of the centroidal axis and the value of I for the proposed section are given in the figure. (3 marks)

$$\sigma_{max} = \frac{95 \times 10^3 \times 74}{2563 \times 10^3} = \underline{\underline{2.74\ MPa}}\ comp$$

Note: $\frac{\pi^2 E}{12(1-\mu^2)} = \frac{\pi^2 4000}{12(1-0.2^2)} = 3430 \text{ MPa}$

3(c). Calculate the maximum flexural compressive stress that can be resisted before the thin webs of the box buckle in compression. (4 marks)

$$\sigma_{max} = \sigma_{cr} = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = 6 \times 3430 \times \left(\frac{1.27}{74-2\times1.27}\right)^2$$

$$= \underline{6.49 \text{ MPa}} > 6.0 \text{ MPa compressive strength}$$

3(d). Use Jourawski's equation to calculate the maximum shear stress in the web of the box beam under the two 500 N loads. (4 marks)

$$\tau_{max} = \frac{V_{max} Q}{I b} = \frac{500 \times 20810}{2563\times10^3 \times 2\times1.27} = \underline{1.598 \text{ MPa}}$$

$$Q = 2\times1.27\times75\times(74-1.27) + 2\times1.27\times74\times74/2 = 13855 + 6955 = \underline{20810 \text{ mm}^3}$$

$$\text{In BD } \tau = 1.598 \times \frac{349}{500} = \underline{1.115 \text{ MPa}}$$

3(e). Calculate the maximum shear stress that can be resisted before the thin webs of the box buckle in shear. (4 marks)

Region BD

$$\tau_{cr} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right] = 5\times3430\left[\left(\frac{1.27}{140-4\times1.27}\right)^2 + \left(\frac{1.27}{510-30}\right)^2\right]$$

$$= 5\times3430\left[88.6\times10^{-6} + 7.00\times10^{-6}\right]$$

$$= \underline{1.64 \text{ MPa}} < 4.0 \text{ MPa shear strength hence will govern.}$$

3(f). Based on your above calculations what do you predict the total failure load of the bridge will be. (4 marks)

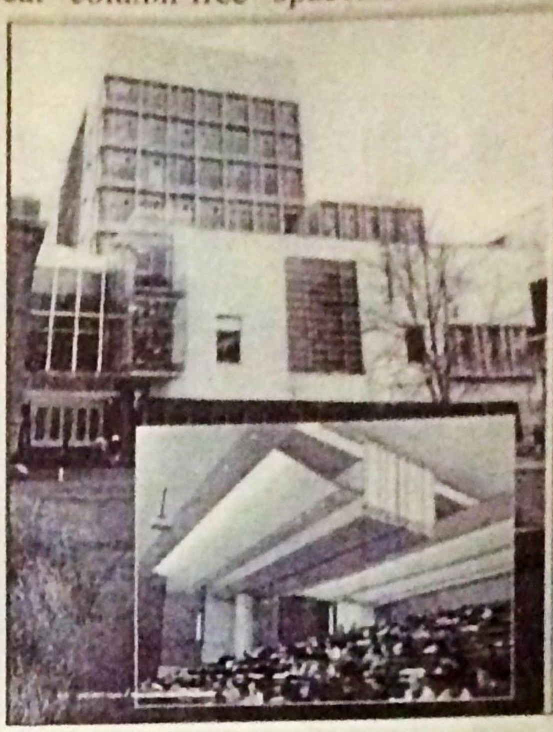
- Flexural failure from (b) and (c) when $\sigma_{comp} = 6.0$ MPa

$$1000 \times \frac{6.0}{2.74} = \underline{2190 \text{ N}}$$

- Shear failure from (d) and (e) when $\tau = 1.64$ MPa

$$1000 \times \frac{1.64}{1.115} = \underline{1470 \text{ N}}$$

4. In the Adel Sedra lecture room in the Bahen Centre large, wide reinforced concrete beams carry loads from the eight upper levels across the 273 seat "column-free" space. Because the beams were designed with the Canadian Concrete Code (CSA) they contain large amounts of shear reinforcement. See cut-away drawing. However if the beams had been designed with the American Concrete Code (ACI) the structural engineer could have chosen to use somewhat wider beams, with somewhat stronger concrete and then the expensive shear reinforcement could have been omitted. To demonstrate that such an approach is unsafe the large, wide reinforced concrete beam described in the figure below was tested under the Baldwin .

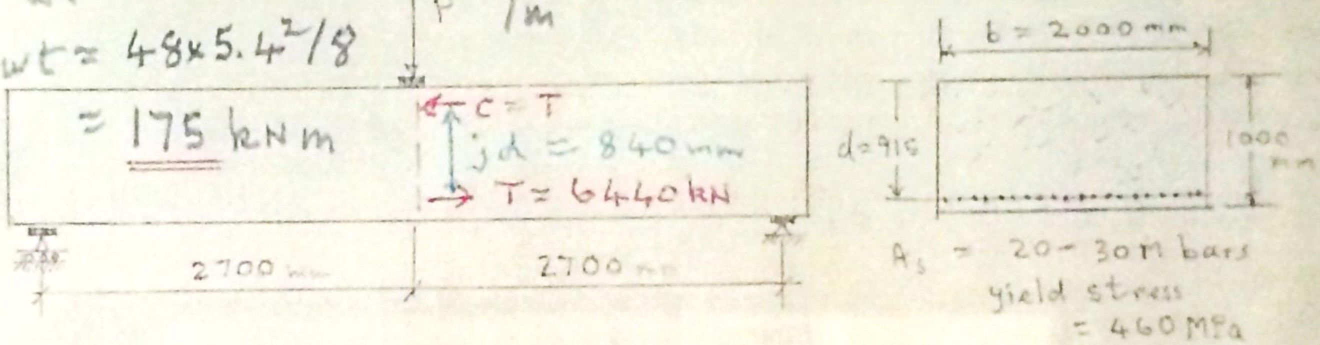


4 (a). Calculate the moment required to cause the stress in the reinforcing bars to reach their yield stress of 460 MPa. Allowing for the moment caused by the self-weight of the large beam, what will be the magnitude of the load, P, applied by the Baldwin which will cause yielding of the reinforcing bars? (6 marks)

$w_{self\ wt} = 1 \times 2 \times 24 = 48\ \mathrm{kN/m}$ $\quad E_c = 4730\sqrt{64} = 37\,840\ \mathrm{MPa}$

$M_{self\ wt} = 48 \times 5.4^2/8$

$= \underline{175}\ \mathrm{kNm}$



$\rho = \frac{A_s}{bd} = 0.00765$ $\quad$ concrete crushing strength $f_c' = 64\ \mathrm{MPa}$

$n = \frac{E_s}{E_c} = \frac{200\,000}{37840} = 5.285 \quad n\rho = 0.0404$

$k = \sqrt{0.0404^2 + 2 \times 0.0404} - 0.0404 = \underline{0.247}$

$j = 1 - k/3 = 0.918 \quad jd = 0.918 \times 915 = \underline{840}\ \mathrm{mm}$

$M_{yield} = 20 \times 700 \times 460 \times 840 = \underline{5410}\ \mathrm{kN\,m}$

$P_{yield} \times 5.4/4 = 5410 - 175 \quad P_{yield} = \underline{3877}\ \mathrm{kN}$

4 (b). The Simplified Working Stress Design handout states that shear reinforcement is not required if the shear stress at working loads does not exceed $0.50 \times 230\sqrt{f_c'}/(1000 + 0.9d)$ in which the 0.5 accounts for a factor of safety of 2.0. Thus failure is predicted when the shear stress reaches $230\sqrt{f_c'}/(1000 + 0.9d)$. This is the CSA simplified expression. Based on this calculate the shear required to cause a shear failure of the tested beam. Allowing for the shear caused by the self-weight of the beam calculate the magnitude of the load, P, applied by the Baldwin predicted to cause a shear failure of the beam. Is the beam predicted to fail in bending or in shear? (6 marks).

$\tau_{fail} = \frac{230 \times 8}{1000 + 0.9 \times 915} = 1.009\ \mathrm{MPa}$

$V_{fail} = \tau_{fail}\, b_w\, jd = 1.009 \times 2000 \times 840 = \underline{1695}\ \mathrm{kN}$

at section d from ℄ of support shear

$$V = \frac{P}{2} + (2.700 - 0.915)\,48 = \frac{P}{2} + 86 \text{ kN}$$

$$\frac{P}{2} + 86 = V_{fail} = 1695 \text{ kN} \quad P_{shear} = \underline{3218 \text{ kN}}$$

As $P_{shear} = \underline{3218}$ kN while $P_{flex} = \underline{3877}$ kN

beam is predicted to fail in shear.

4. (c). The American Concrete Code (ACI) expression for the shear strength of beams without shear reinforcement is $0.166\sqrt{f'_c}\,b_w d$. What is the ACI predicted shear strength for the tested beam? What is the ratio of V_{ACI} / V_{CSA}? The experimental beam was a half-scale model of the ACI alternative design for the beams in the Sedra lecture room. If all dimensions are doubled (ρ remains the same) what will be the new values of V_{ACI}, V_{CSA} and the ratio V_{ACI} / V_{CSA}? Comment on the influence of increasing member size on the structural safety of reinforced concrete members designed with traditional ACI procedures.(10 marks)

$$V_{ACI} = 0.166\sqrt{64} \times 2000 \times 915 = \underline{2430} \text{ kN}$$

$$V_{ACI} / V_{CSA} = 2430/1695 = \underline{1.43}$$

Dimensions all doubled ρ same n same hence k and j have same values ∴ $d \rightarrow 915 \times 2 = 1830$ mm

$b_w \rightarrow 4000$ mm

CSA

$$\tau_{fail} = \frac{230 \times 8}{1000 + 0.9 \times 1830} = 0.695 \text{ MPa}$$

$$V_{fail} = 0.695 \times 4000 \times 0.918 \times 1830 = \underline{4670} \text{ kN}$$

ACI

$$V_{fail} = 0.166\sqrt{64} \times 4000 \times 1830 = \underline{9720} \text{ kN}$$

$$V_{ACI} / V_{CSA} = 9720/4670 = \underline{2.08}$$

If CSA procedures are correct the large member designed by ACI could fail under service loads.