UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

October 29, 2018

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Closed book.

All questions are of equal value.

Permitted calculators:

- Sharp EL-520X
- Sharp EL-520W
- Casio FX-991
- Casio FX-991EX
- Casio FX-991ES Plus
- Casio FX-991MS

This test contains 20 pages including this page and the cover page, printed two-sided. Do not tear any page from this test.

Present complete solutions in the space provided. Page 20 is for rough work. If you want anything on page 20 to be marked, you must indicate in the relevant previous question that the solution continues on page 20.

Given information:

$$cos\,\theta = \frac{\vec{u}\cdot\vec{v}}{||\vec{u}||\,||\vec{v}||}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

$$proj_{\vec{d}}\,\vec{u} = \frac{\vec{u}\cdot\vec{d}}{||\,\vec{d}\,||^2}\vec{d}$$

$$det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Q1:

a) Find four orthogonal unit vectors, meaning all pairs are orthogonal, with each component equal to $\frac{1}{2}$ or $-\frac{1}{2}$.

b) If $\begin{bmatrix} a \\ b \end{bmatrix}$ is parallel to $\begin{bmatrix} c \\ d \end{bmatrix}$, with $abcd \neq 0$, show that $\begin{bmatrix} a \\ c \end{bmatrix}$ is parallel to $\begin{bmatrix} b \\ d \end{bmatrix}$.

c) Can 3 vectors in the $x - y$ plane have $u \cdot v < 0$ and $v \cdot w < 0$ and $w \cdot u < 0$? Explain your answer.

Q2:

a) Find what linear combination of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is closest to $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

b) Find what linear combination of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is closest to $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

More space for part b)

Q3:

Consider the following transformation:

$T\left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ except that $T\left(\begin{bmatrix} 0 \\ v_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

a) Apply the transformation to $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$.

b) Show that this transformation satisfies $T(cv) = cT(v)$ for scalar c, but does not satisfy $T(v + w) = T(v) + T(w)$.

Q4:

a) When $a + b = c + d$ for matrix A:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

show that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector. What is the eigenvalue associated with this eigenvector?

b) Find the other eigenvalue of matrix A.

c) Find the eigenvector associated with the eigenvalue found in part (b).

Q5:

Consider the following system of linear equations:

$2x + 5y + z = 0$

$4x + dy + z = 2$

$0x + y - z = 3$

a) Which value for d forces a row exchange when applying Gaussian elimination?

b) Using the value for d from part (a), find the reduced normal form of the augmented system and the solution to this system of equations, assuming a solution exists.

c) What value of d makes the system inconsistent? Show work to support your answer.

Q6:

a) Determine the rank of matrix A:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 4 \\ 1 & 1 & 8 \end{bmatrix}$$

b) Express matrix A as the sum of two rank one matrices.

c) It turns out that every mxn matrix of rank r can be expressed as a product of an mxr matrix times an rxn matrix where the mxr matrix contains the columns in the original matrix A associated with the leading 1's in the reduced normal form of the matrix, and the rxn matrix contains the first r rows of the reduced normal form of the matrix. Use this fact to express matrix A in part (a) as the product of an mxr matrix times an rxn matrix.

This page is for rough work or for extra space to finish a previous question. This page will not be marked unless you have indicated in a previous question to look at this page.