Name: ______________________________ Student No.: __________________

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, April 25, 2019
DURATION: 2.5 hours

Second Year – Engineering Science

ECE286H1 S – PROBABILITY AND APPLICATIONS

Calculabor Type: 2

Exam Type: C

Examiner – Deepa Kundur

- Answer **all** questions in the space provided in this question booklet. You may use the backs of pages if needed, but please specify the question you are answering clearly. There are **8 pages in total** to this exam.
- Allowable aids: (1) a 8.5"x11" double-sided *handwritten or typed* aid sheet and (2) Type 2 (nonprogrammable) calculator. Please note that important tables are given at the end of this exam.
- If a particular question seems unclear, please explicitly state any reasonable assumptions and proceed with the problem.
- Please properly label all points of interest on sketches and graphs that you are requested to draw, so that there is no ambiguity.
- **For full marks, show all steps and present results clearly.**

Question	1	2	3	4	5	Total
Value of Ques.	10	10	10	10	10	**50**
Earned						

1. **(10 marks)**

(a) **(5 marks)** Impurities in a batch of final product of a manufacturing process often reflect a serious problem. From considerable manufacturing data gathered, it is known that the proportion Y of impurities in a batch has a density function given by:

$$f(y) = \begin{cases} c(1-y)^9 & 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}.$$

i. Find the constant c in the above density function.

ii. A batch is considered not sellable and then not acceptable if the percentage of impurities exceeds 60%. With the current quality of the process, what is the percentage of batches that are not acceptable?

(b) **(5 marks)** The length of time, in minutes, for an airplane to obtain clearance for takeoff at a certain airport is a random variable $Y = 3X - 2$, where X has the density function:

$$f(x) = \begin{cases} \frac{1}{4}e^{-x/4} & x > 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Find the mean and variance of the random variable Y.

2. **(10 marks)**

   (a) **(5 marks)** You are a participant in a game show. Denote A, B and C to be the events that a grand prize is behind doors A, B and C respectively. Suppose you picked door A. The game show host opened door B to and showed no prize was behind it. Now the host offers you the option of either staying at the door that you picked (A) or switching to the remaining door (C). Use probability to explain whether you should switch or not. Assume that the host knows where the prize is, and as such will never at first open the door that you pick or the door with the prize behind it.

   (b) **(5 marks)** Suppose you need to toss a fair coin to determine the serve order in a badminton game, but you only have two *biased* coins. The first coin has a probability of heads $p_1 = 1/3$ and the other coin has probability of heads $p_2 = 1/4$. Further suppose you **cannot** distinguish the two coins. How could you generate the equivalent of a fair coin flip? Please assume that the two coins are independent.

3. **(10 marks)**

   (a) **(5 marks)** Find the probability of observing 3 red cards in 5 draws from an ordinary deck of 52 playing cards:

      i. without replacement of cards after each draw.

      ii. with replacement of cards after each draw.

   (b) **(5 marks)** A pair of dice is rolled 180 times. What is the probability that a total of 7 occurs:

      i. at least 25 times?

      ii. between 33 and 41 times inclusive?

   Hint: Consider approximating the actual distribution.

4. **(10 marks)**

   (a) **(5 marks)** A random sample size 25 is taken from a normal population having a mean of 80 and a standard deviation of 5. A second random sample of size 36 is taken from a different normal population having a mean of 75 and a standard deviation of 3. Find the probability that the sample mean computed from the 25 measurements will exceed the sample mean computed from the 36 measurements by at least 3.4, but less than 5.9. Assume the difference of the means to be measured to the nearest tenth.

   (b) **(5 marks)** A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per millilitre of raw material. To check this claim he samples 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with his claim.

      i. What conclusion should he draw from a sample that has mean $\overline{X} = 518$ grams per millilitre and a sample standard deviation $s = 40$ grams. Assume the distribution of yields to be approximately normal.

      ii. What is a satisfiable claim for the population mean yield of the batch process that the engineer can make?

5. **(10 marks)**

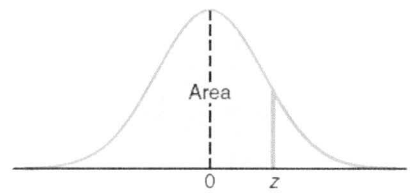
   (a) **(6 marks)** A random sample of 100 automobile owners in Toronto shows that an automobile is driven on average $23,500$ kilometres per year with a standard deviation of 3900 kilometres. Assume the distribution of measurements to be approximately normal.

      i. Construct a 99% confidence interval for the average number of kilometres an automobile is driven annually in Toronto.

      ii. What can we assert with 99% confidence about the possible size of our error if we estimate the average number of kilometres driven by car owners in Toronto to be $23,500$ kilometres per year?

   (b) **(4 marks)** An efficiency expert wishes to determine the average time that it takes to drill three holes in a certain concrete structure. How large a sample will she need to be 95% confident that her sample mean will be within 15 seconds of the true mean? Assume that it is known from previous studies that $\sigma = 40$ seconds.

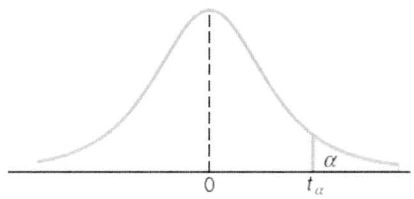
From Table A.3:



Areas under the standard normal curve, for positive z-values:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

From Table A.4:



Critical values of the t-distribution, for degrees of freedom v and significance level α:

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960