Soln

PHY294 Quiz #2 2016

Name (Last, First):
Signature:
Student ID:
Tutorial Section:

- 4 questions, 25 minutes.
- Closed book, closed notes and no calculators.
- Please write only in the Quiz paper (double-sided).

1. What is the phase velocity of the deBroglie wave for a proton moving at 7/8 the speed of light? [Justify your answer using wave-particle duality and relativistic expressions for energy and momentum.]

$$E = \hbar\omega = \gamma m c^2 \qquad \gamma \equiv \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$$

$$p = \hbar k = \gamma m v \qquad v = \frac{7}{8}c$$

$$v_\varphi = \frac{\omega}{k} = \frac{E}{p} = \frac{c^2}{v} = \frac{8}{7}c$$

2. An electron is in an infinite-potential spherical well of radius b. When the orbital kinetic energy is zero, $R(r) = B\sin(kr)/r$ satisfies the radial Schrodinger equation. Find the allowed values for k, and calculate B.

Boundary Condition: $R(b) = 0 = \sin(kb)$

$$\Downarrow$$

$$kb = n\pi \qquad \therefore \; k = \frac{n\pi}{b} \quad n = 1, 2, 3 \ldots$$

Normalization: $1 = \int_0^\infty dr\, r^2 R^2(r) = B^2 \underbrace{\int_0^b \sin^2\left(\frac{n\pi r}{b}\right) dr}_{b/2}$

$$\therefore \; B = \sqrt{\frac{2}{b}}$$

3. A hydrogen atom is in a superposition state described by: $\psi = \frac{1}{\sqrt{2}}\psi_{3,2,0} - \frac{1}{2}\psi_{3,2,2} + \frac{1}{2}\psi_{3,2,-2}$
What is the expectation value $<L_z>$ for this state? [Subscripts represent n, l, m_l respectively.]

$$\langle L_z \rangle = \int_{\text{All Space}} d\upsilon\, \psi^* L_z \psi$$

Note: $L_z \psi_{n l m_l} = m_l \hbar \psi_{n l m_l}$

- Component eigenfunctions are orthonormal

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \int d\upsilon\, \psi^*_{320} L_z \psi_{320} + \left(\frac{-1}{2}\right)^2 \int d\upsilon\, \psi^*_{322} L_z \psi_{322} + \left(\frac{1}{2}\right)^2 \int d\upsilon\, \psi^*_{32-2} L_z \psi_{32-2}$$

$$= \frac{1}{2}(0)\hbar\,[1] + \frac{1}{4}(2)\hbar\,(1] + \frac{1}{4}(-2)\hbar\,(1]$$

$$= 0$$

4. Consider 11 non-interacting neutrons in a 2D infinite-potential square well of dimensions d x d.
What is the minimum energy of this system? [Neutrons are charge neutral and have 1/2 spin.]

$$E_{n_x n_y} = \frac{h^2}{8md^2}(n_x^2 + n_y^2)$$

$n_x = 1, 2, 3 \ldots$
$n_y = 1, 2, 3 \ldots$

$E\left(\frac{h^2}{8md^2}\right)$	n_x	n_y	spin degeneracy
2	1	1	2
5	1	2	2
	2	1	2
8	2	2	2
10	1	3	2
	3	1	2

For 11 Fermions (spin $= \frac{1}{2}$)
Pauli Exclusion applies

$$E_{tot} = \frac{h^2}{8md^2}\left[2 \times 2 + 4 \times 5 + 2 \times 8 + 3 \times 10\right]$$

$$= \frac{70\, h^2}{8\, md^2}$$