Question 1 [20 pts]

A. Consider a coaxial capacitor consisting of two concentric, perfectly conducting cylindrical surfaces, one of radius α, and another of radius b, as shown in Fig. 1 below. The dielectric space between the two cylinders is divided equally in two semi-cylindrical sections, with dielectric permittivities ε_1 and ε_2, respectively. A voltage source is used to set $V(r=\alpha)=V_0$, $V(r=b)=0$. In this capacitor, the electric flux density in the two regions (disregarding edge effects) is: $\overline{D}_1 = \frac{A_1}{r}\hat{r}$, $\overline{D}_2 = \frac{A_2}{r}\hat{r}$, where A_1, A_2 are unknown constants.

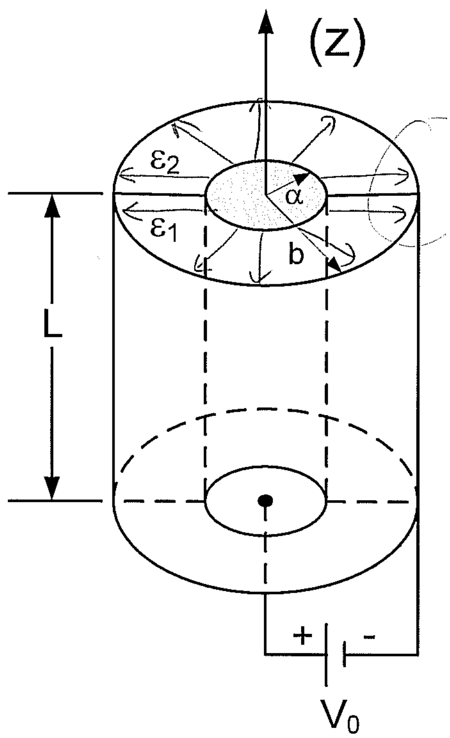


Fig. 1

1. Use boundary conditions to show that: $\frac{A_1}{\varepsilon_1} = \frac{A_2}{\varepsilon_2}$ **[2 pts]**

2. Show that the capacitance of the coaxial capacitor of Fig. 1 is

$$C = \frac{\pi(\varepsilon_1 + \varepsilon_2)L}{ln\frac{b}{a}}$$

[5 pts]

3. Calculate the electrostatic energy stored in this capacitor and show that it can be expressed as

$$W_e = \frac{1}{2} C V_0^2$$

[5 pts]

B. The two plates of an empty (air-filled) parallel plate capacitor are charged to +/-100 nC, respectively, by a voltage source. The voltage source stays connected to the capacitor. When a dielectric is inserted between the plates, the charge on the two plates becomes +/-500 nC.

1. Does the electric field intensity in the capacitor increase (and if so by what factor), decrease (and if so by what factor) or stay the same when the dielectric is inserted? Explain. **[2 pts]**

2. Does the electric flux density inside the capacitor increase (and if so by what factor), decrease (and if so by what factor), or stay the same when the dielectric is inserted? **[2 pts]**

C. The two plates of an empty (air-filled) parallel plate capacitor are charged until the electric field strength inside is 10,000 V/m. Then, the capacitor is disconnected from the voltage source. When a dielectric is inserted between the plates, the electric field strength reduces to 2000 V/m.

1. Does the amount of charge on the capacitor plates increase (and if so by what factor), decrease (and if so by what factor), or stay the same? **[2 pts]**

2. Does the potential difference across the capacitor increase (and if so by what factor), decrease (and if so by what factor) or stay the same when the dielectric is inserted? Explain. **[2 pts]**

■ End of question 1

Question 2 [20 pts]

A. The coaxial resistor of Fig. 2.1 is made of two concentric, perfectly conducting cylindrical surfaces, one of radius α, and another of radius b, respectively. The space in between is filled with two lossy dielectric media with dielectric permittivity and conductivity ε_α, σ_α and ε_b, σ_b, respectively. The first medium covers the space $0<\varphi<\varphi_0$; the rest is occupied by the second medium. A voltage source is used to set $V(r=\alpha)=V_0$, $V(r=b)=0$. In the following calculations, you can disregard edge effects.

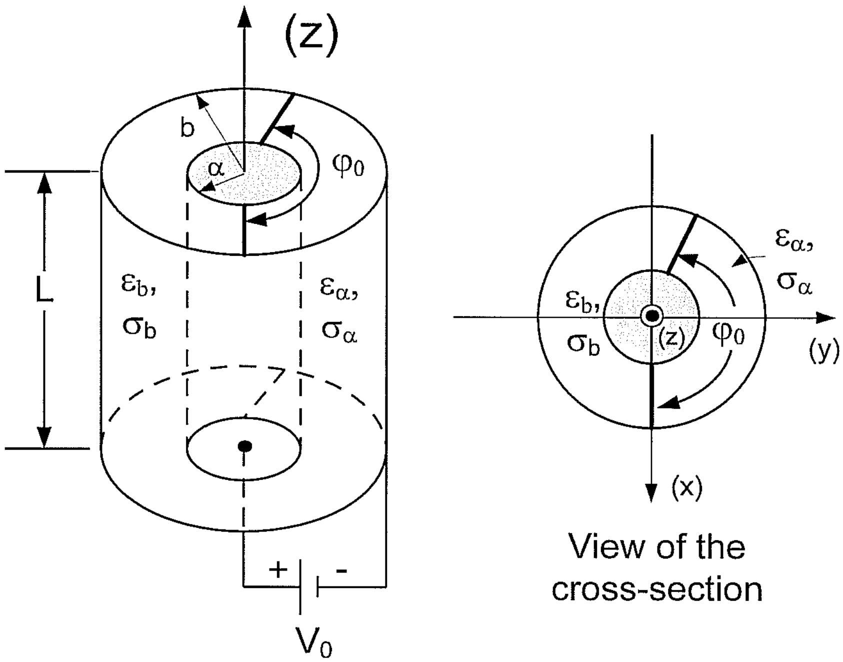


Fig. 2.1

1. State the boundary conditions for the electric current density (**J**) and the electric flux density (**D**) at the interfaces between the two media. **[2 pts]**

2. Find the resistance of this resistor. **[6 pts]**

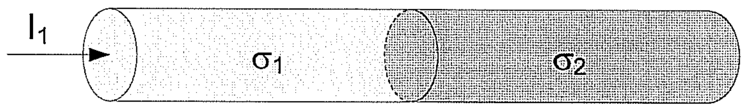
R=

[1pt]

3. Calculate the power lost in the resistor. **[6 pts]**

P=

B. A wire consists of two equal diameter segments. Their conductivities are σ_1, σ_2 with $\sigma_1 < \sigma_2$ and the electron densities are N_1, N_2 with $N_1 < N_2$. The current in segment 1 is I_1.



1. Compare the steady-state current densities J_1, J_2, in the two segments. **[2 pts]**

2. Compare the electric fields E_1, E_2, in the two segments. **[2 pts]**

3. Compare the electron velocities v_1, v_2 in the two segments. **[2 pts]**

■ End of question 2

Question 3 [20 pts]

A. The washer shaped coil in Fig. 3.1 has a thickness Δ that is much less that the inner radius b and outer radius α. It supports a volume current density $\bar{J} = J_0\hat{\varphi}$, where J_0 is a constant.

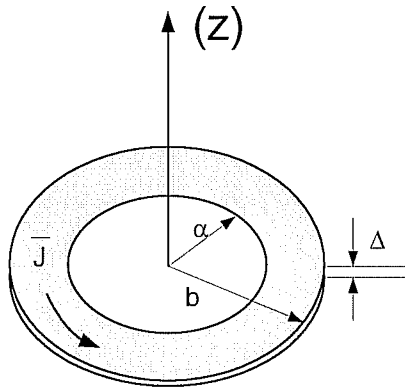


Fig. 3.1

1. As the thickness of the coil is given to be very small compared to its other dimensions, the current flow in the coil can be expressed in terms of an equivalent <u>surface current density</u> $\bar{J}_s$. Show that this surface current density is:

$$\bar{J}_s = J_0\Delta\hat{\varphi}, \text{ at } \alpha < r < b, z = 0$$ **[2 pts]**

Based on that, find the total current I_{coil} flowing through the coil. **[3 pts]**

$I_{coil}=$

2. Use the Biot-Savart law to determine the magnetic flux density vector $\overline{B}$ at a point (0,0,z) on the z-axis. Express the result in terms of a one-dimensional integral (which you do not need to evaluate) and show that $\overline{B}$ has only one component in cylindrical coordinates. You can use the result of the previous question. **[10 pts]**

$\overline{B} =$

B. A long straight wire, perpendicular to the page, passes through a uniform magnetic flux density $\overline{B}_0$. Dashed line vectors indicate this uniform magnetic flux density (i.e. **not** the flux of the wire). The *net* magnetic flux density at point 3 is zero.

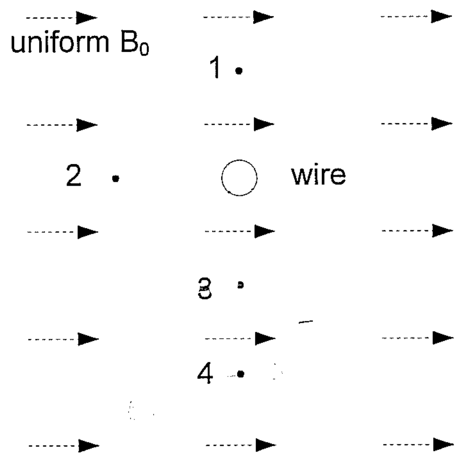


Fig. 3.2

1. On the figure, show the direction of the current in the wire ($\odot$:out of the page or $\otimes$:into the page). Explain your choice. **[1 pt]**

2. Points 1, 2 are at the same distance from the wire as point 3. Point 4 is twice as distant from the wire as points 1, 2, 3. Construct vector diagrams to determine the *net* $\overline{B}$ at points 1, 2, 4. **[4 pts]**