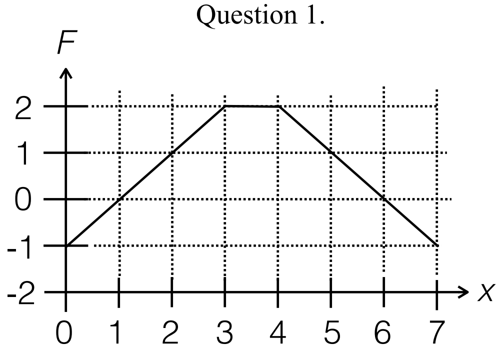
1. [20 pts.] The figure for this question (see below) shows the internal force F (in Newtons) on a particle of mass m versus its position x (shown in metres).

   (a) [5 pts.] What is the internal **work done on the particle**, when it moves from $x = 0.0\,\text{m}$ to $x = 7.0\,\text{m}$?

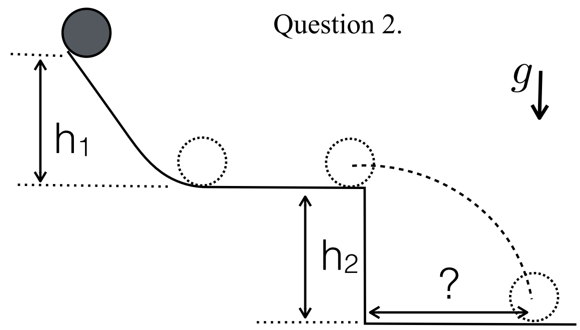
   (b) [5 pts.]Are there any **equilibrium points?** If so, state where they are, and whether they are stable or unstable.

   (c) [10 pts.] **Sketch** $U(x)$ versus x, assuming that $U(0) = 0$. Label both axes with approximate values.

Question 1.



Question 2.



2. [15 pts.] (See figure above:) A uniform cylinder of radius R and mass M starts from rest. It rolls without slipping down a ramp of height h_1, across a flat table, and then off the edge of the table. The table has height h_2. **How far away from the table does the cylinder land?**

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3. [20 pts.] A 20 g bullet moving initial at $v_i = 300\,\text{m/s}$ is fired into (and passes through) a 8.0-kg block that is 2.0 cm thick. The exit velocity of the bullet is 50 m/s.

   (a) [5 pts.] What is the **final velocity** of the block?

   (b) [10 pts.] **Estimate the time** Δt it took the bullet to pass through the block. State your assumptions, and say why they are reasonable.

   (c) [5 pts.] **Estimate the average force** of the bullet on the block, averaged across the Δt time of its passage through the block.

---

4. [10 pts.] The potential energy of a particle is $U(x) = 3.0\,x^4 + 5.0$ (in mks units.) Initially, the particle is at rest ($v_0 = 0.0\,\text{m/s}$) and located at $x = 2.0\,\text{m}$. There are no external forces or frictional forces. The particle has mass $M = 10\,\text{kg}$.

   (a) [5 pts.] What is the **initial acceleration** of the particle?

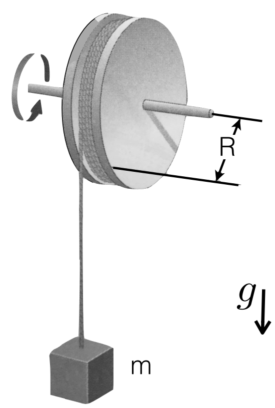
   (b) [5 pts.] What is the **kinetic energy** of the particle when it is **at** $x = 0.0\,$**m?**

5. [20 pts.] A wheel of radius $R = 0.50\,\text{m}$ and moment of inertia $I_{\text{CM}} = 24\,\text{kg}\,\text{m}^2$ is mounted on a frictionless horizontal axel, as shown in the figure. A light (massless) cord wrapped around the wheel supports a block of mass $m = 120\,\text{kg}$. At $t = 0.0\,\text{s}$, the wheel is released, the block accelerates downward, and rope is uncoiled from the wheel.

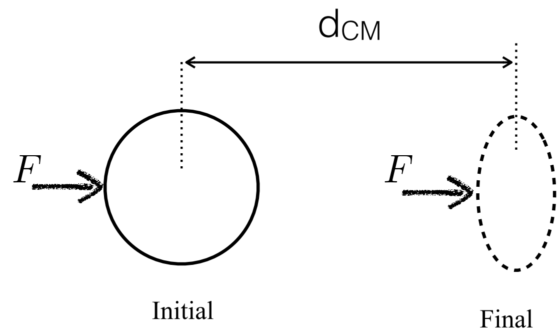
   (a) [10 pts.] What is the **acceleration** of the mass?

   (b) [5 pts.] What is the **tension** in the rope?

   (c) [5 pts.] Give an expression for the **angular velocity** versus time t.



Question 5.



Question 6.

6. [15 pts.] A pillow of mass $0.40\,\text{kg}$ is accelerated with a constant force $F = 8.0\,\text{N}$ (see figure above). The pillow is spherical at first, but after the centre of mass is moved by $d_{\text{CM}} = 30\,\text{cm}$, the pillow deforms into an ellipsoidal shape (see drawing). The distance from the centre of mass to the point of contact, initially $10.0\,\text{cm}$, is only $5.0\,\text{cm}$ at t_f.

   (a) [5 pts.] What is the **kinetic energy** of the pillow at time t_f?

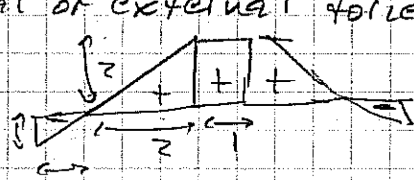
   (b) [5 pts.] How much **work was done** on the pillow by F during the time interval?

   (c) [5 pts.] Should the answer to (a) and (b) be the same? **Explain** why or why not.

END OF EXAM.

(1) (a) $W = \int F\,dx$ for either internal or external forces

Use geometry for this integral:



$W = -\frac{1}{2} + 2 + 2 + 2 - \frac{1}{2} = 7\ \text{J}$

What about sig figs? Assume all values read to 0.1 units,

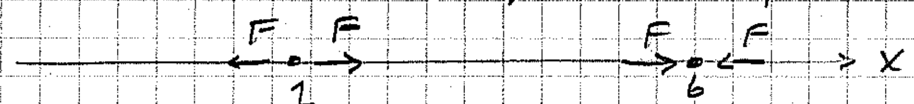
so $\boxed{W = 5.0\ \text{J}}$ (work is positive, so particle gains kinetic energy.)

(b) Equilibrium can be achieved where $F = 0$.

This is true for $\underline{x = 1.0\,m}$ and $\underline{x = 6.0\,m}$.

The first point (at $x = 1.0\,m$) is unstable, because if moved away by a little, pushed (accelerated) further.

The second point (at 6.0 m) is stable, because force pushes back towards equilibrium point.

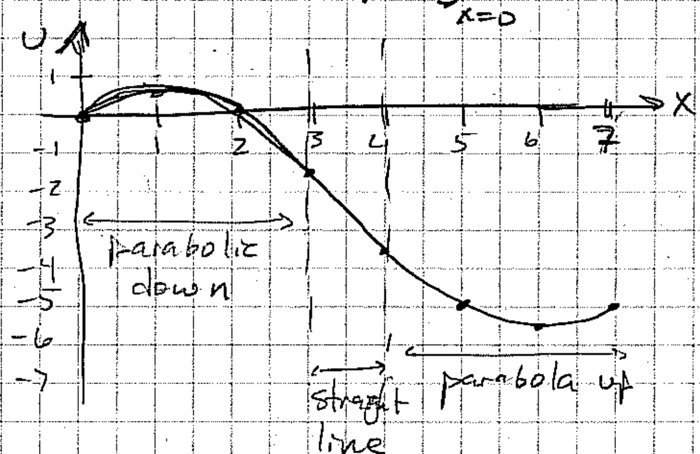


(c) We need to construct something with a $U(x)$ such that $-\frac{dU}{dx} = F$ at all points. We can also integrate $F(x)$: $U(x) = U(x=0) - \int_{x=0}^{x'} F(x')\,dx'$

So

x	U
0	0
1	+0.5
2	0
3	−1.5
4	−3.5
5	−5
6	−5.5
7	−5

Plot!



1. a) $W = \int F dx = \frac{1}{2}(-1N)(1m) + \frac{1}{2}(2N)(2m) + (2N)(1m) + \frac{1}{2}(2N)(2m) + \frac{1}{2}(-1N)(1m)$

$= 5.0\,J$

(✓✓ for saying area of graph)
(✓x for $W = Fd$)
(✓x for correct calc but not formula)

for correct answer w/ sig fig

for algebra

5/5

b) Equilibrium when $F = 0$ ✓ for explanation

$\Rightarrow x = 1.0m$ ✓ & $x = 6.0m$ ✓

for correct point   for correct point

5/5

$x = 1.0m$ is unstable
$x = 6.0m$ is stable ✓ for correct stabilities

could earn ✓x if correct idea but wrong sign for $U(x)$.

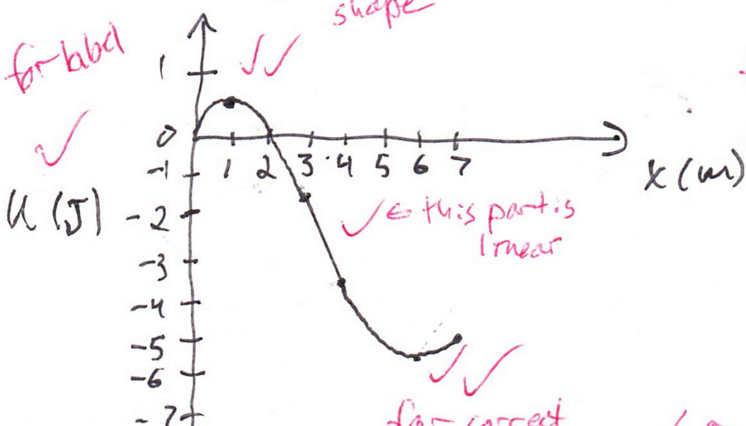
* stable because $\frac{dF}{dx}(6.0)$ is -ve
unstable because $\frac{dF}{dx}(1.0)$ is +ve

✓ for correct explanations (would also accept written physical explanations)

Note: saying to look at graph is not valid explanation

c) $U = -\int F dx$

✓✓ (also accepted a list of points)



✓ for overall sign

10/10

(2) Solve this problem in two steps:

(I) By conservation of energy, figure out V before flying off the table

(II) By kinematics, find the range of parabolic flight.

(I): $\Delta U + \Delta K = 0$, so $-Mgh_1 + K_f = 0$

For a rolling cylinder, $K_f = \frac{1}{2}MV_f^2 + \frac{1}{2}I\omega_f^2$

but for rolling without slipping, $\omega_f = V_f/R$

So $K_f = \frac{1}{2}M\left(1 + \underbrace{\frac{I}{MR^2}}_{\frac{1}{2}}\right)V_f^2 = \frac{3}{4}MV_f^2$

thus $\frac{3}{4}MV_f^2 = Mgh_1 \rightarrow V_f = \sqrt{\frac{4}{3}gh_1}$

(II): Dropping h_2 takes a time $\frac{1}{2}gt^2 = h_2$

$\longrightarrow t = \sqrt{2h_2/g}$

In that time, the cylinder flies $V_x t$,

So range $= \sqrt{\frac{2h_2}{g}}\sqrt{\frac{4}{3}gh_1}$

$= \sqrt{\frac{8}{3}h_1 h_2}$

Date

2) first, what velocity does the cylinder have when it leaves the table?

by cons. of energy for no slipping,

$Mgh_r = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv_{cm}^2$, $\omega = v_{cm}/R$

$Mgh_r = \frac{1}{2}I\frac{v_{cm}^2}{R^2} + \frac{1}{2}Mv_{cm}^2$

$Mgh_r = v_{cm}^2\left(\frac{I}{2R^2} + \frac{M}{2}\right)$

$v_{cm}^2 = 2Mgh_r / \left(I/R^2 + M\right)$ + $I = \frac{MR^2}{2}$

$v_{cm}^2 = \frac{2\cancel{M}gh_r}{\left(\frac{\cancel{M}\cancel{R^2}}{2\cancel{R^2}} + \cancel{M}\right)} = \frac{4gh_r}{3}$

$\therefore v_{cm} = \sqrt{\frac{4}{3}gh_r}$

$X = \sqrt{\frac{8}{3}h_r h_T}$

then after it leaves table:

$h_T = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h_T}{g}}$

+

$X = v_{cm}t = \sqrt{\frac{4}{3}gh_r}\sqrt{\frac{2h_T}{g}}$

2. Across h_R all potential energy is converted into rotational kinetic energy & linear kinetic energy

$$M g h_R = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 \qquad \omega = \frac{v}{R}$$

$$= \frac{1}{2} I \frac{v^2}{R^2} + \frac{1}{2} M v^2$$

$$= v^2 \left(\frac{1}{2} \frac{I}{R^2} + \frac{1}{2} M \right) \qquad \Big| \; I = \frac{1}{2} M R^2$$

$$= v^2 \left(\frac{1}{4} M + \frac{1}{2} M \right)$$

$$= \frac{3}{4} M v^2$$

$$v^2 = \frac{4}{3} g h_R$$

$$v = \sqrt{\frac{4}{3} g h_R}$$

Time it takes cylinder to fall h_T

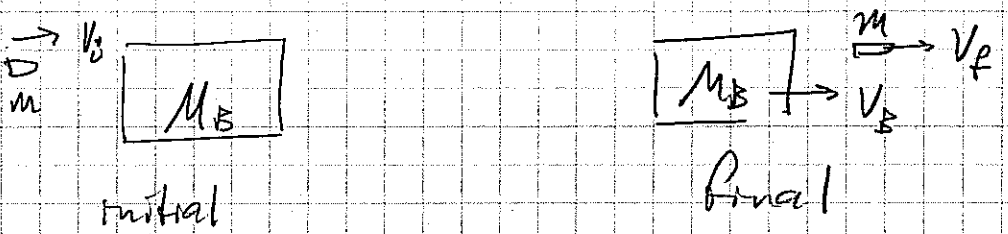
$$\frac{1}{2} g (\Delta t)^2 = h_T \; ; \quad \Delta t = \sqrt{\frac{2 h_T}{g}}$$

horizontal distance:

$$v \Delta t = \sqrt{\frac{8}{3} h_R h_T}$$

The cylinder lands $\sqrt{\frac{8}{3} h_R h_T}$ away from the table.

(3)



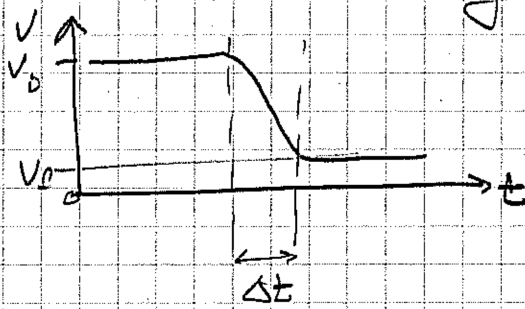
(a) Conserve momentum:

$$mV_i = mV_f + M_B V_B$$

$$\longrightarrow \quad V_B = \frac{m}{M}(V_f - V_i) = \frac{0.02\,kg}{8.0\,kg}(250\,m/s) = 0.625\,m/s$$

$$\longrightarrow \boxed{0.62\,m/s} \text{ to 2 sig figs.}$$

(b) One imagines that the velocity profile looks like

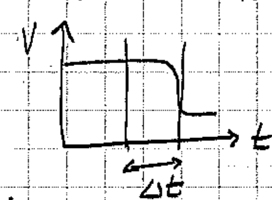


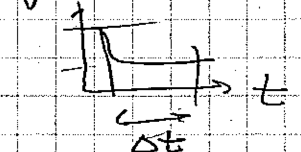
but Δt needs to be small enough that $\int V dt = \Delta X$ is only the thickness of the block. If we assume constant acceleration, then

$$\Delta X = \bar{V}\,\Delta t\,, \quad \& \text{ here } \begin{cases} \bar{V} = \frac{300+50}{2} \sim 175\,m/s \\ \Delta X = 0.02\,m \end{cases}$$

$$\longrightarrow \quad \Delta t = \frac{0.02\,m}{175\,m/s} = 1.1 \times 10^{-4}\,s$$
(or $110\,\mu s$)

What other models could one have guessed for $V(t)$?

If  then $\Delta t \sim \frac{\Delta X}{V_i} = 6.6 \times 10^{-5}\,s$

If  then $\Delta t \sim \frac{\Delta X}{V_f} = 4.0 \times 10^{-4}\,s$

} Range of reasonable

no: 3 (continuation)

b) Once the bullet is out of the block, it's speed won't change. The speed of the bullet changes while it is going through the block. On the other hand, the block starts moving while the bullet is inside of it. However, the final speed of the block, which is also its maximum speed is $\frac{0{,}005}{50} = 0{,}013$ times the lowest speed of the bullet.

We will consider displacement of the block to be negligible during the whole process, so we assume the bullet is out of the block as soon as it moves by 2 cm.

Then, we also need to account for the variation in speed of the bullet, which decreases by ≈ 84 % during the interaction.

The material will oppose the movement of the bullet with a force similar to a drag. If the material is infinitely long, the final speed will be null. The material is more dense than air and it's molecule should be harder to displace ...

$$v_b(t) = v_{ib}\, e^{-\tau t}$$

$$v_B(t) = \frac{m_b}{m_B} v_{ib}\, e^{-\tau t}$$

$$e^{-\tau t_f} = \frac{v_{b_f}}{v_{b_i}} = \frac{1}{6}$$

$$x = \int_0^t v_{ib}\, e^{-\tau t}\, dt' = v_{ib} \left[\frac{e^{-\tau t'}}{(-\tau)} \right]_0^t = v_{ib} \left[-\frac{e^{-\tau t}}{\tau} + \frac{1}{\tau} \right]$$

$$= \frac{v_{ib}}{\tau} \left[1 - e^{-\tau t} \right]$$

$$x(t_f) = 0{,}02 = \frac{v_{ib}}{\tau} \left[1 - \frac{1}{6} \right]$$

$$\tau = \frac{v_{ib}}{0{,}02} \left(\frac{5}{6} \right)$$

$$-\tau t_f = \ln \tfrac{1}{6} = -\ln 6 \Rightarrow \Delta t = \frac{1}{\tau} \ln 6 = \frac{(0{,}02)6}{5 v_{ib}} \ln 6 = 1{,}43 \cdot 10^{-4}\, s$$

For a "constant-Force" model, where we have something similar to a normal Force exerted constantly on the bullet, which is constantly seeing the same environment

$$v_{f_b}^2 = v_{i_b}^2 + 2a(0{,}02)$$

$$a = \frac{v_{f_b}^2 - v_{i_b}^2}{2(0{,}02)} = \frac{(50)^2 - (300)^2}{0{,}04} = -2{,}19 \cdot 10^6 \text{ m/s}^2$$

$$2(0{,}02)a = v_{f_b}^2 - v_{i_b}^2$$

$$x(t) = \frac{a}{2} t^2 + v_{i_b} t$$

$$\frac{a}{2} t_f^2 + v_{i_b} t - 0{,}02 = 0$$

$$t_f = \frac{-v_{ib} \pm \sqrt{v_{ib}^2 + 2a(0{,}02)}}{2(a/2)}$$

$$= \frac{-v_{ib} \pm \sqrt{v_{ib}^2 + v_{fb}^2 - v_{ib}^2}}{a}$$

$$\Delta t = \frac{v_{fb} - v_{ib}}{a} = \frac{50 - 300}{-2{,}19 \cdot 10^6} = 1{,}14 \cdot 10^{-4} \text{ s}$$

(3) (c) Now from impulse, we know that

$$I = \int F\,dt = F_{avg}\,\Delta t = \Delta p$$

Here, $\Delta p = m(V_f - V_i) = 0.020(-250) = -5.0\ \text{kg m/s}$

Need an answer from (b) to find F_{avg}:

- If used constant acceleration model,

  $F_{avg} \sim 4.4 \times 10^4\ N$

- The maximum & minimum possible Δt also give us range of reasonable F_{avg}:

  $F_{avg} \sim 1.2 \times 10^4\ N$ to $7.5 \times 10^4\ N$

(4.) $U = 3x^4 + 5$

(a) At $x = 2.0\,m$, what is F?

$$F = -\frac{dU}{dx} = -12x^3 \text{, so } a(2) = \frac{F(2)}{m} = \frac{-96N}{10kg} = \boxed{-9.6\ m/s^2}$$

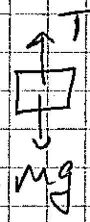
(b) If energy is conserved, $K_i + U_i = K_f + U_f$

or $\Delta K = -\Delta U$. Here $K_i = 0$, since $v_i = 0$.

$$\Delta U = U_f - U_i = 3(0)^4 + 5 - \left(3(2)^4 + 5\right) = -48\,J$$

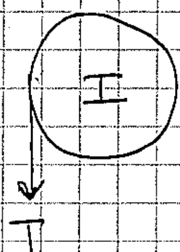
Thus $\boxed{K_f = +48\,J}$

(5.) Solve this problem with FBD:



$$\Sigma F = ma \rightarrow mg - T = ma$$

(defining positive as downward.)



$$\Sigma \tau = I\alpha \rightarrow RT = I\alpha$$

(a) eliminate T to solve for a. Also, $\alpha = a/R$, since rope does not slip around wheel.

$T = m(g-a)$ from mass

$T = \frac{I\alpha}{R} = \frac{Ia}{R^2}$ from wheel

set equal: $m(g-a) = \frac{I}{R^2} a$

solve for a: $g - a = \frac{I}{mR^2} a$

$g = \left(\frac{I}{mR^2} + 1\right) a \rightarrow a = \frac{g}{1 + I/mR^2}$

plug in #s, $a = \frac{9.8}{1 + \frac{24}{(120\,kg)(0.50)^2}} = \boxed{5.4\ m/s^2}$

(Direction is downward, as given in problem statement.)

5. (b) Now can sub back in to find T

$$T = m(g-a) = 120\,(9.8-5.4) = 523\,N$$

$\rightarrow \boxed{520\,N}$ to 2 sig figs.

(c) Acceleration is constant, so is angular acceleration.

$$\omega = \frac{d}{dt}\alpha \quad \longrightarrow \quad \omega(t) = \omega_0 + \alpha t$$

for constant α.

Here, $\omega_0 = 0$, and so $\underline{\omega = \alpha t}$.

Find α from a/R in (a): $\boxed{\omega = \left(11\ \frac{rad}{s^2}\right) t}$

(6) This problem asks about a deformable system.

(a) Under constant force, the CM has constant acceleration: $a_{cm} = \frac{F}{M} = \frac{8.0N}{0.4kg} = 50 \, m/s^2$

$V_f^2 = 2a\,\Delta x$, so $K_f = \frac{1}{2}mv^2 = m\,a_{cm}\,d_{cm}$

The problem could also be solved by noting that KE = F*d_cm, but this was not considered an obvious statement and needed further explanation or demonstration for full marks. Note that the statement W = F*d_cm is NOT correct, as explained below.

$= \boxed{2.4J}$

(b) The work done is (Force) x (displacement), but the displacement is of the POINT OF CONTACT, not the cm. Here, the point of contact moved $d_{cm} + 5cm = 0.35m$,

so $W_{ext} = (8.0N)(0.35m) = \boxed{2.8J}$

(c) These are not the same: (a) is $F \cdot d_{cm}$, but (b) is $F \cdot d_{POC}$, so differ by $F(5cm) = 0.4J$.

Energy is conserved, but this extra energy must have gone to INTERNAL ENERGY such as heat, or to some internal elasticity or other POTENTIAL ENERGY. Without more information, we don't really know which.