University of Toronto

Faculty of Applied Science and Engineering

Final Examination, April, 2015

STA286S: Probability and Statistics

Examiners: B. Donmez and L. Al Labadi

Student Number:____________________________________

Family Name:______________________________ First Name:______________________________

Lecture Section: (**circle one**)

- LEC01 (Prof. Al Labadi)
- LEC02 (Prof. Donmez)

Tutorial Section: (**circle one**)

Tutorial	**Time**	**Location**	**TA**
TUT01	Mon 1-2 pm	BA2159	Hootan Habibkhani
TUT02	Mon 1-2 pm	WB144	Zhenhua Lin
TUT03	Mon 1-2 pm	BA3012	Wayne Giang
TUT04	Wed 1-2 pm	WB144	Victor Veitch
TUT05	Mon 2-3 pm	WB144	Hootan Habibkhani
TUT06	Fri 10-11 am	WB144	Wayne Giang
TUT07	Tues 1-2 pm	BA2159	Victor Veitch
TUT08	Tues 1-2 pm	BA2165	Zhenhua Lin

Instructions:

- **Time allowed:** 2 ½ hours.
- **Aids:** a non-programmable calculator and a double-sided A4 size aid sheet.
- There are **9 questions** and **16 pages** including this page. The last 4 pages contain the **probability tables and an empty page as scrap paper**. You can rip them off if you want.
- If you do not understand a question, or are having some other difficulty, do not hesitate to ask your instructor or TA for clarification.
- Points for each question are indicated in parentheses. Total points: 100.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
Max	**10**	**10**	**15**	**10**	**10**	**10**	**7**	**12**	**16**	**100**
Score										

GOOD LUCK !

Question 1: A number X is picked in the interval $(0,1)$ with the probability law described by the following probability density function (pdf)

$$f(x) = \begin{cases} 2x & if\ 0 < x < 1 \\ 0 & otherwise. \end{cases}$$

If $X = x$, a number Y is picked with the uniform distribution over the interval $(0, x)$.

(a) (5 pts) Find the joint pdf for X and Y.

We have that $f(y|x) = \frac{1}{x}$, $0 < y < x$

Thus,

$$f(x,y) = f(y|x) \cdot f(x) = \begin{cases} 2 & if\ 0<y<x<1 \\ 0 & otherwise \end{cases}$$

(b) (5 pts) Find the correlation coefficient for X and Y.

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For nonnegative integers n, k we have:

$$E(x^n y^k) = \int_0^1 \int_0^x x^n y^k \, 2 \, dy \, dx = 2\int_0^1 x^n \frac{x^{k+1}}{k+1} dx$$

$$= \frac{2}{(k+1)(n+k+2)}$$

It follows that

$$E(X) = \frac{2}{3}, \quad E(Y) = \frac{1}{3}, \quad E(X^2) = \frac{1}{2}, \quad E(Y^2) = \frac{1}{6}, \quad E(XY) = \frac{1}{4}.$$

Thus,

$$Var(X) = E(X^2) - (E(X))^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$$

$$Var(Y) = E(Y^2) - (EY)^2 = \frac{1}{6} - \left(\frac{1}{3}\right)^2 = \frac{1}{18}.$$

$$COV(X,Y) = EXY - EXEY$$

$$= \frac{1}{4} - \frac{2}{3} \cdot \frac{1}{3} = \frac{1}{36}.$$

Therefore,

$$corr(X,Y) = \frac{\frac{1}{36}}{\sqrt{\frac{1}{18} \cdot \frac{1}{18}}} = 0.5$$

Question 2: The joint density of two random variables is given by:

$$f(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)} & if\ x_1 > 0, x_2 > 0 \\ 0 & otherwise. \end{cases}$$

Let $Y_1 = \frac{X_1+X_2}{2}$ and $Y_2 = X_2$.

(a) (5 pts) Find the joint probability density function (pdf) of Y_1 and Y_2.

$x_1 = 2y_1 - y_2$ and $x_2 = y_2$. The Jacobian transformation is:

$$J = \begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = 2$$

Thus, the joint pdf of Y_1 and Y_2 is:

$g(y_1, y_2) = f(x_1, x_2)|J| = 2e^{-2y_1}$, $2y_1 > y_2 > 0$.

(b) (5 pts) Find the marginal pdf of Y_1.

The marginal pdf of Y_1 is $h(y_1) = \int_0^{2y_1} 2e^{-2y_1} d\, y_2 = 4y_1 e^{-2y_1}, y_1 > 0$.

Question 3: Suppose that X and Y are discrete independent random variables with the following moment generating functions:

$$M_X(t) = E[\exp(tX)] = \frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$$

and

$$M_Y(t) = E[\exp(tY)] = \frac{1}{10}e^{-t} + \frac{4}{10}e^{2t} + \frac{1}{2}e^{3t}$$

(a) (5 pts) Find the **mean** and **variance** of X.

$$M_X'(t) = \frac{1}{6}e^{t} + \frac{4}{6}e^{2t} + \frac{9}{6}e^{3t} \Rightarrow E(X) = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} = \frac{14}{6} = \underline{\underline{\frac{7}{3}}}$$

$$M_X''(t) = \frac{1}{6}e^{t} + \frac{8}{6}e^{2t} + \frac{27}{6}e^{3t} \Rightarrow E(X^2) = M_X''(0) = \frac{1}{6} + \frac{8}{6} + \frac{27}{6} = \frac{36}{6} = 6$$

Hence

$$Var(X) = EX^2 - (EX)^2 = 6 - \left(\frac{7}{3}\right)^2 = 6 - \frac{49}{9} = \frac{5}{9}.$$

(b) (5 pts) Find the moment generating function of $Z = X - Y$.

$$M_{X-Y}(t) = M_X(t) \cdot M_Y(-t)$$

$$= \left[\frac{1}{6}e^{t} + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}\right]\left[\frac{1}{10}e^{t} + \frac{4}{10}e^{-2t} + \frac{1}{2}e^{-3t}\right]$$

$$= \frac{1}{12}e^{-2t} + \frac{14}{60}e^{-t} + \frac{23}{60}e^{0t} + \frac{12}{60}e^{t} + \frac{1}{60}e^{2t} + \frac{2}{60}e^{3t} + \frac{3}{60}e^{4t}.$$

(c) (5 pts) Find the distribution of $Z = X - Y$. That is, compute $P(Z = z)$ for all possible values of z.

z	-2	-1	0	1	2	3	4
$P(Z=z)$	$\frac{1}{12}$	$14/60$	$\frac{23}{60}$	$\frac{12}{60}$	$\frac{1}{60}$	$2/60$	$\frac{3}{60}$

Question 4: Suppose that $X_1, \ldots, X_n$ are independent random variables from the normal distribution $N(0, \theta)$, where $\theta > 0$ is an unknown parameter.

(a) (5 pts) Find the maximum likelihood estimator $\hat{\theta}$ of θ.

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$$L(x_1, \ldots, x_n; \theta) = \prod_{i=1}^{n} f(x_i; \theta) = (2\pi)^{-n/2} \theta^{-n/2} e^{-\sum x_i^2 / 2\theta}$$

$$\ln(L) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\theta) - \frac{\sum x_i^2}{2\theta}$$

$$\frac{\partial L}{\partial \theta} = \text{zero} - \frac{n}{2} \cdot \frac{1}{\theta} + \frac{\sum x_i^2}{2\theta^2} = 0 \Rightarrow \theta = \frac{\sum_{i=1}^{n} x_i^2}{n}.$$

• Check that θ is max.: Using the 2nd derivative test:

$$\frac{\partial^2 L}{\partial \theta^2} = \frac{n}{2\theta^2} - \frac{\sum x_i^2}{\theta^3} = \frac{n\theta - 2\sum x_i^2}{2\theta^3}$$

$$\Rightarrow \left.\frac{\partial^2 L}{\partial \theta^2}\right)_{\theta = \frac{\sum x_i^2}{n}} = \frac{-\sum x_i^2}{2 \cdot \left(\sum x_i^2 / n\right)^3} < 0 \Rightarrow \text{max}.$$

$$\therefore \hat{\theta} = \frac{\sum x_i^2}{n}.$$

(b) (5 pts) Determine whether $\hat{\theta}$ is unbiased estimator of θ.

$$E(\hat{\theta}) = E\left(\frac{\sum_{i=1}^{n} x_i^2}{n}\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} E(x_i^2) \qquad ; \quad E(X_i^2) = Var(x_i) + (E X_i)^2 = \theta + 0 = \theta$$

$$= \frac{1}{n} \cdot n\theta$$

$$= \theta.$$

$\therefore \hat{\theta}$ is unbiased estimator of θ.

Question 5: Suppose we wish to estimate the average execution time of a program. The program was run 6 times with randomly chosen data sets, and the sample mean and sample standard deviation of the execution times were 228 and 14 milliseconds (ms), respectively.

(a) (5 pts) Compute a 95% confidence interval for the true mean execution time. State your assumptions.

The sample size is small and the population variance is unknown. So we need to assume that the population is normally distributed.

The confidence level 1- α = 0.95 and so $\alpha / 2 = 0.025$**.**

$t_{(n-1);\alpha/2} = t_{(5);0.025} = 2.571$

The CI is $\bar{x} \pm t_{(n-1);\alpha/2} \dfrac{s}{\sqrt{n}} = 228 \pm 2.571 \dfrac{14}{\sqrt{6}} = (213.306, 242.694)$

(b) (5 pts) It is claimed that the overall mean execution time of the program is less than 230 ms. Do the data support the claim?

Hypothesis: $H_0 : \mu = 230$ **vs** $H_a : \mu < 230$

The test statistic is: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{228 - 230}{14/\sqrt{6}} = -0.35$**, it has a** $t_{(5)}$ **distribution.**

The critical t-value is -2.015, our test statistics falls in the acceptance region. We cannot reject H_0

and conclude that the data do not provide evidence to support the claim.

Question 6: A company that supports two different types of computer networks is interested in comparing the mean down time for the two types. A random sample of 10 networks from the first type and another random sample of 9 networks from the second type are selected and the down time in hours (over a one-year period) is recorded for every network. The results are given in the table below:

	Type 1	Type 2
Sample mean	83.5	60.4
Standard deviation	34.2	22.3

(a) (5 pts) Is there sufficient evidence to conclude that the two samples are coming from two populations with different variances? Use $\alpha = 0.1$.

The hypothesis: $H_0 : \sigma_x^2 = \sigma_y^2$ **vs** $H_a : \sigma_x^2 \neq \sigma_y^2$

The test statistic is: $F_{stat} = \frac{s_x^2}{s_y^2} = \frac{(34.2)^2}{(22.3)^2} = 2.352$ **, it has a** $F_{(9,8)}$ **distribution.**

Using α = 0.1, we reject H0 if:

$$F_{stat} > F_{(9,8);0.05} = 3.39 \text{ or } F_{stat} < F_{(9,8);0.95} = \frac{1}{F_{(8,9);0.05}} = \frac{1}{3.23} = 0.309$$

Conclusion: since $F_{stat} = 2.352$ **is not in the rejection region we fail to reject H0 and conclude that there is no difference in the variability of lifetimes of batteries produced by these two companies.**

(b) (5 pts) Is there a significant difference in the mean down time between the two types of networks? Use $\alpha = 0.1$. **Hint:** use the result obtained in part (a).

The pooled estimate of the common variance is:

$$s_p^2 = \frac{(n_1 - 1)s_x^2 + (n_2 - 1)s_y^2}{n_1 + n_2 - 2} = \frac{(9)(34.2)^2 + (8)(22.3)^2}{10 + 9 - 2} = 853.24$$

$$t_{(n_1+n_2-2);\alpha/2} = t_{(17);0.05} = 1.740$$

The CI is $(\bar{x} - \bar{y}) \pm t_{(n_1+n_2-2);\alpha/2} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Substituting into the formula we get:

$$(83.5 - 60.4) \pm (1.74) \cdot \sqrt{853.24\left(\frac{1}{10} + \frac{1}{9}\right)} = (-0.253, 46.453)$$ **fail to reject since it includes 0.**

Alternatively, the t-stat is 1.721 and is in the acceptance region.

Question 7: The following are the weekly losses of work-hours due to accidents in 10 industrial plants before and after a certain safety program were put into operation:

Before	45	73	46	124	33	57	83	34	26	17
After	36	60	44	119	35	51	77	29	24	11

Assuming that the weekly losses before and after the safety program are normally distributed.

(a) (5 pts) Find a 95% confidence interval on the difference in mean weekly losses of work-hours.

Before	**45**	**73**	**46**	**124**	**33**	**57**	**83**	**34**	**26**	**17**
After	**36**	**60**	**44**	**119**	**35**	**51**	**77**	**29**	**24**	**11**
Before - After	**9**	**13**	**2**	**5**	**-2**	**6**	**6**	**5**	**2**	**6**

$\bar{d} = 5.2$ $\quad s_d = 4.077$ $\quad \bar{d} \pm t_{n-1,0.025} \frac{s_d}{\sqrt{n}}$

$$5.2 \pm 2.262 \cdot \frac{4.077}{\sqrt{10}} = (2.284, 8.116)$$

t critical is (for 9 dof) 2.262.

(b) (2 pts) Is the safety program is effective (use α=0.05)? Explain your answer.

The CI above does not include 0 so reject.

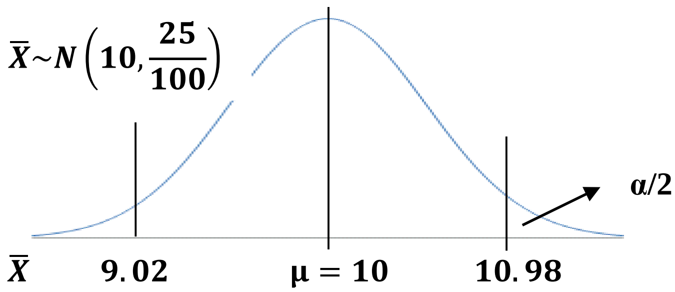
Alternatively,

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} = \frac{5.2}{4.077/\sqrt{10}} = 4.0333$$

Reject; t critical is (for 9 dof) 2.262.

Question 8: Define $X_1, \dots, X_{100}$ to be independent Normal random variables with unknown mean μ and variance 25. Suppose we test the null hypothesis $H_0: \mu = 10$ against the alternative hypothesis $H_1: \mu \neq 10$ using the test statistic $\bar{X} = \frac{1}{100}(X_1 + \cdots + X_{100})$; we reject H_0 if $\bar{X} \geq 10.98$ or if $\bar{X} \leq 9.02$.

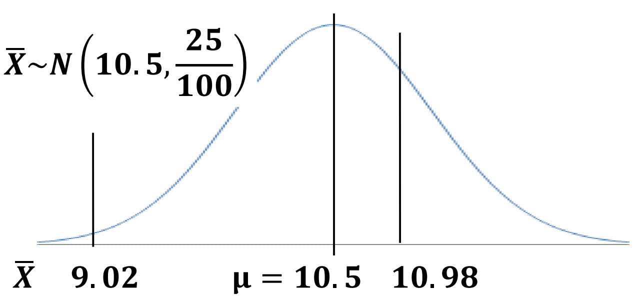
(a) (6 pts) Find the significance level (size) of this test.



$$\alpha/2 = P(\bar{X} > 10.98) = P\left(Z > \frac{10.98 - 10}{\sqrt{\frac{25}{100}}}\right) = P\left(Z > \frac{0.98}{0.5}\right) = P(Z > 1.96) = P(Z < -1.96)$$
$$= 0.025$$

$$\alpha = 0.025 * 2 = 0.05$$

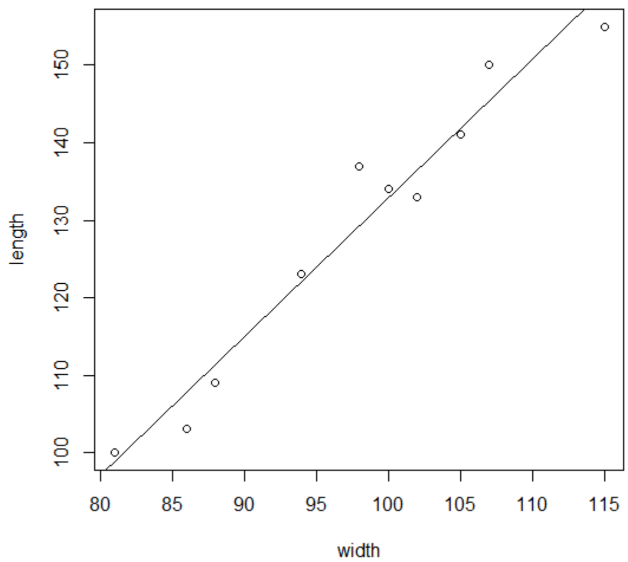
(b) (6 pts) Suppose the true value of μ is 10.5. Find the power of this test.



$$Power = P(\bar{X} > 10.98) + P(\bar{X} < 9.02) = P\left(Z > \frac{10.98 - 10.5}{\sqrt{\frac{25}{100}}}\right) + P\left(Z < \frac{9.02 - 10.5}{\sqrt{\frac{25}{100}}}\right)$$
$$= P\left(Z > \frac{10.98 - 10.5}{0.5}\right) + P\left(Z > \frac{9.02 - 10.5}{0.5}\right) = P(Z > 0.96) + P(Z < -2.96)$$
$$= P(Z < -0.96) + P(Z < -2.96) = 0.1685 + 0.0015 = 0.17$$

Question 9: The following plot and summary statistics are based on measurements on length and width (both in mm) of 10 painted female turtles (*Chrysemys picta marginta*):

$\overline{x} = 97.6,\ \overline{y} = 128.5, \qquad \sum_1^{10}(y_i - \overline{y})^2 = 3316.5$



The following is the R output for a simple linear regression fitted between length (dependent variables) and width (explanatory variable).

```
Call:
lm(formula = length ~ width)

Residuals:
    Min      1Q  Median      3Q     Max
-4.7084 -3.1131  0.0945  1.2397  7.7830

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -46.4359    13.4161  [C]      0.00855 **
width         [A]         [B]     13.107  1.09e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.295 on 8 degrees of freedom
Multiple R-squared: [D],    Adjusted R-squared: 0.9499
F-statistic: 171.8 on 1 and 8 DF,  p-value: 1.092e-06
```

(a) (8 pts) In the R output, four values (A, B, C, D) are hidden. Find A, B, C, D. Show your work.

<u>A</u> is the estimate of slope (b_1).

$$\boldsymbol{b_0 = \bar{y} - b_1\bar{x}}$$
$$\boldsymbol{-46.4359 = 128.5 - b_1 97.6}$$
$$\boldsymbol{b_1 = 1.7924}$$

<u>B</u> is the standard error for B_1.

$$\boldsymbol{t - value = \frac{estimate}{std.error}}$$

$$\boldsymbol{13.107 = \frac{1.7924}{std.error}}$$

$$\boldsymbol{B = std.error = 0.1368}$$

<u>C</u> is the t-value for B_0.

$$\boldsymbol{t - value = \frac{estimate}{std.error}}$$

$$\boldsymbol{C = \frac{-46.4359}{13.4161}}$$

$$\boldsymbol{C = -3.462}$$

<u>D</u> is the coefficient of determination.

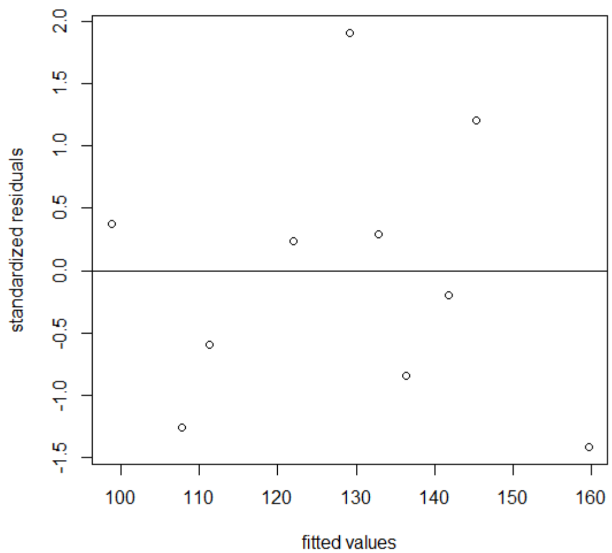
$$\boldsymbol{R^2 = 1 - \frac{SSE}{SST}}$$

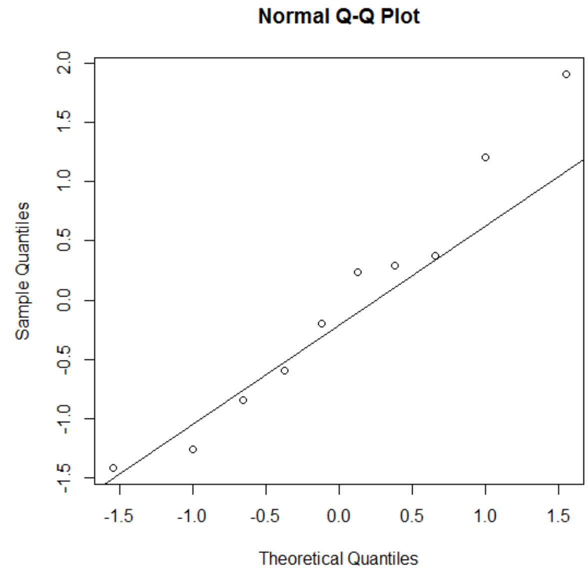
$$\boldsymbol{s^2 = (4.295)^2 = \frac{SSE}{n-2}}$$

$$\boldsymbol{SSE = 147.6}$$

$$\boldsymbol{SST = 3316.5}$$
$$\boldsymbol{R^2 = 1 - \frac{147.6}{3316.5} = 0.9555}$$

(b) (4 pts) The plot on the below left shows the standardized residuals vs. fitted values. The plot on the below right is the normal quantile quantile plot for standardized residuals. Comment on the adequacy of model fit.





The plot on the left seems to be fairly random (no systematic pattern). Further, the standardized residuals seem to fall on an approximately straight line. Although, the two large values that seem to deviate from the linear pattern may be outliers. The assumptions seem to hold.

(c) (4 pts) State the null and alternative hypothesis for the two t-tests reported in the R output. Further, comment if you should reject or not.

The first test is $\mathbf{H_0: B_0 = 0}$ vs. $\mathbf{H_1: B_0 \neq 0}$. In words, the first test evaluates if the intercept is zero or not. Reject.

The second test is $\mathbf{H_0: B_1 = 0}$ vs. $\mathbf{H_1: B_1 \neq 0}$. In words, the second test evaluates if the slope is zero or not. Reject.

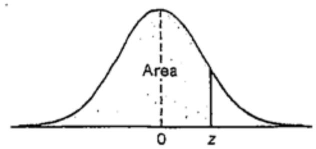


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

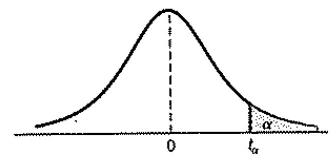


Table A.4 Critical Values of the t-Distribution

	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

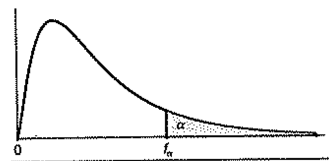


Table A.6* Critical Values of the F-Distribution

v_2	$f_{0.05}(v_1, v_2)$ 1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

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