Q6:

a) Consider the matrix A where:

$$A = \begin{bmatrix} .2 & .4 & 0 \\ .4 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

One of the eigenvalues (λ) of this matrix A is equal to 0. Find a corresponding eigenvector $\vec{x}$. To receive full marks for this part, you must show that $A\vec{x} = \lambda\vec{x}$.

b) The matrix A in part (a) has another eigenvalue equal to 1 and this eigenvalue has an eigenvector that is a linear combination of two non-parallel vectors. Find an eigenvector $\vec{x}$ associated with $\lambda = 1$ that has no zero components. To receive full marks for this part, you must show that $A\vec{x} = \lambda\vec{x}$.

Q6: a)

$$\begin{bmatrix} .2 & .4 & 0 \\ .4 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

SOLVING FOR $\vec{x}$,

$$\left[\begin{array}{ccc|c} .2 & .4 & 0 & 0 \\ .4 & .8 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} .2 & .4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] R2-2R1$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R1*5 \\ R2 \rightleftarrows R3 \end{array}$$

$$x_1 + 2x_2 = 0$$
$$x_3 = 0$$

x_1, x_3 LEADING

x_2 FREE

$$x_1 = -2x_2$$
$$x_2 = x_2$$
$$x_3 = 0$$

$$\therefore \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

LET $x_2 = 1$,

$$\vec{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

CHECK,

$$\begin{bmatrix} .2 & .4 & 0 \\ .4 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad \checkmark$$

b) $$\begin{bmatrix} .2 & .4 & 0 \\ .4 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = (1) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

SOLVING FOR $\vec{x}$,

$$\left[\begin{array}{ccc|c} .2-1 & .4 & 0 & 0 \\ .4 & .8-1 & 0 & 0 \\ 0 & 0 & 1-1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} -.8 & .4 & 0 & 0 \\ .4 & -.2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} -.8 & .4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R2 + \frac{1}{2} R1$$

$$\to \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R1 * \frac{1}{(-.8)}$$

$x_1 - 0.5 x_2 = 0$

x_1 LEADING

x_2, x_3 FREE

$x_1 = 0.5 x_2$

$x_2 = x_2$

$x_3 = x_3$

$$\therefore \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 0.5 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

AS LONG AS x_3 IS CHOSEN TO BE A SCALAR NOT EQUAL TO ZERO, $\vec{x}$ WILL HAVE NO ZERO COMPONENTS.

LET $x_2 = 1$ AND $x_3 = 1$

$$\vec{x} = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix}$$

CHECK,

$$\begin{bmatrix} .2 & .4 & 0 \\ .4 & .8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix} = (1) \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix} \checkmark$$