UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING

Second Year — Engineering Science

STA286H1 S — Probability and Statistics

Term Test

Monday, March 27, 2017

POSSIBLE SOLUTIONS

Note: far more significant digits are printed here than would have been reasonable or necessary.

Also, the question numbering and sub-lettering is strange due to a document rendering error that produced the original test itself.

The Story

A gas distribution company knows that a gas meter will have a serious and dangerous defect with probability $p = 0.01$. It decides to send work crews out to inspect all gas meters to try to find the defective ones. A work crew is able to inspect 60 meters in one 8 hour shift. You are the supervisor all the work crews.

1. **(20 marks total)** Assume the defect status of one gas meter does not affect the probability that any other gas meter is defective.

Page 2 (a) **(7 marks)** What is the probability that a work crew will find at least 1 defective gas meter during one shift?

Page 2 (b) **(3 marks)** Work Crew Alpha inspects 100 gas meters without finding a single one that is defective. They phone you to complain that the whole project is a waste of time and that they should have found at least one defective meter by now. What can you say to motivate them to continue with the project?

Page 3 (a) **(3 marks)** Work Crew Alpha, the same work crew as in (b), goes two full shifts (120 gas meters) and still haven't found one that is defective. As they go out for their third shift, they tell you "We've done two shifts without finding one defective meter. Surely today our luck will change!" Is Work Crew Alpha correct in their assessment of the situation?

Page 3 (b) **(7 marks)** You decide to motivate the work crews by offering a prize to the first work crew to find a defective gas meter in 10 of its shifts. Work Crew Beta wins the prize after its 12th shift. In response, Work Crew Alpha files a union grievance claiming that there's less than a 1 in 1000 chance that Work Crew Beta could have won the prize so quickly. Assess Work Crew Alpha's complaint.

2. **(20 marks total)** Continuing with the same scenario, since a work crew can do 60 inspections in an 8 hour shift, it inspections occur at a rate of one per 8 minutes on average. This means that a work crew finds defective gas meters at a rate of 0.00125 per minute.

Page 4 (a) **(7 marks)** Calculate the probability that the 1st defective meter is found within the first 200 minutes.

Page 4 (b) **(3 marks)** By the time 200 minutes have elapsed, a work crew has found only 1 defective meter. What is the probability that the defective meter was just discovered in the previous 15 minutes?

Page 5 (a) **(7 marks)** Calculate the probability that 3 defective meters will be found within 500 minutes.

Page 5 (b) **(3 marks)** 225 minutes have elapsed since the work crew last found a defective meter. What is the expected amount of time remaining until they find their next defective meter?

3. **(10 marks total)** Still continuing with the same scenario. Six months have elapsed. Work Crew Alpha has completed 7000 gas meter inspections. Work Crew Beta has independently completed 7800 gas meter inspections. (The difference is because Alpha had to spend time on other projects from time to time.)

Give an approximation to the probability that Work Crew Alpha discovered more defective gas meters than Work Crew Beta did.

(Suggested setup: denote by X and Y the numbers of defective meters discovered and find an approximation to $P(X > Y) = P(X - Y > 0)$.)

4. **(10 marks total)** Suppose X_1, X_2, and X_3 are random variables, r_1, r_2, and r_3 are positive integers, and $0 < p < 1$. Also, the random variables have these properties:

- they are independent.
- $X_1 \sim \text{NegBin}(r_1, p)$ and $X_2 \sim \text{NegBin}(r_2, p)$
- $X_1 + X_2 + X_3 \sim \text{NegBin}(r_1 + r_2 + r_3, p)$

What is the distribution of X_3? Prove your result.

If you cannot give a rigourous mathematical proof, you may give a casual proof. (Please do not do both.)