Family name, given name (please print) Student number Tutorial group

UNIVERSITY OF TORONTO
Faculty of Applied Science and Engineering

Term Test 2. November 2019

PHY293: Modern Physics
Instructor: Professor N. Krasnopolskaia

Duration: 60 minutes

Exam Type A: Closed Book
Calculator Type 3: Non-programmable calculators without text storage

- Print in BLOCK LETTERS your name, student number and tutorial group on top of all examination booklets and test papers used. If you use more than one booklet, enter the BOOK NO and TOTAL NUMBER OF BOOKS USED at the upper right corner of the booklet.

- Place your student ID on the desk.

- Aids allowed: only calculators, from a list of approved calculators as issued by the Faculty Registrar are allowed. No other aid (notes, textbook, dictionary) is allowed.

- Turn off any communication device (phone, pager, PDA, iPod, etc.) you may have and place it far from where you are sitting.

- This test consists of 3 questions. Answer all questions. Show all important steps.

- Part marks will be given for partially correct answers.

- Do not separate the stapled sheets of the question paper. Hand in the questions with your exam booklet at the end of the exam.

- The total number of marks is 100.

- Questions are on page 3.

Possibly Useful Constants and Formulae

$$c = 3.00 \cdot 10^{8} \frac{m}{s}; \quad h = 6.63 \cdot 10^{-34} J \cdot s = 4.14 \cdot 10^{-15} eV \cdot s; \quad h = 2\pi\hbar$$

$$e = 1.60 \cdot 10^{-19} C; \quad m_e = 9.11 \cdot 10^{-31} kg = 0.511 MeV / c^2; \quad \lambda_C = 2.43 \cdot 10^{-12} m \ \ (electron);$$

$$\beta = \frac{u}{c}; \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}; \quad x = \gamma(x' + ut'); \quad x' = \gamma(x - ut); \quad t = \gamma\left(t' + \frac{ux'}{c^2}\right); \quad t' = \gamma\left(t - \frac{ux}{c^2}\right);$$

$$\Delta t = \gamma \Delta t'; \quad L = \frac{L'}{\gamma}; \quad u_x = \frac{u_x^{'} + v}{1 + \frac{u_x^{'} v}{c^2}}; \quad u_x^{'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \quad f = f_s \frac{\sqrt{1-\beta^2}}{1 + \beta \cos\theta}; \quad \lambda = \lambda_s \frac{\sqrt{1 + \frac{u}{c}}}{\sqrt{1 - \frac{u}{c}}};$$

$$p = \gamma_u m u; \quad E_0 = m_0 c^2; \quad E^2 = p^2 c^2 + \left(m_0 c^2\right)^2; \quad K = (\gamma - 1) E_0; \quad E = hf; \quad hf = K_{\max} + \phi;$$

$$\lambda' - \lambda = \lambda_C (1 - \cos\theta); \quad \lambda_C = \frac{h}{m_e c} (electron); \ p = \frac{h}{\lambda} (photon);$$

$$R_H = 1.10 \cdot 10^{7} m^{-1}; \quad R_{EH} = 13.6 eV; \quad \frac{1}{\lambda_{nm}} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2}\right); \quad hf = R_{EH} \left(\frac{1}{n^2} - \frac{1}{m^2}\right);$$

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1. [40 marks].
In the frame S, a stationary particle of rest mass m_0 explodes into two identical particles of equal rest masses moving in opposite directions at 0.6c.
In the frame of reference S′, associated with the mass moving to the right, find:

(a) total initial momentum of the system;
(b) velocity of the fragment moving to the left;
(c) rest mass of the fragment moving to the left;
(d) kinetic energy of the fragment moving to the left;
(e) total energy of the system in the frame S′ before and after the explosion.

Solution

(a) In the frame S', the initial momentum of the system is the momentum of the mass $\gamma_{0.6}$ m_0. The momentum is

$$P_{\text{initial}} = \gamma_{0.6}\, m_0(-0.6c) = -1.25m_0 0.6c = -0.75\, m_0 c$$

(b) The velocity of the fragment moving to the left is

$$u' = \frac{u-v}{1-\frac{uv}{c^2}} = \frac{-0.6c-0.6c}{1-\frac{(-0.6c)(0.6c)}{c^2}} = -0.882c.$$

(c) From the law of conservation of momentum – 0.75 $m_0c = \gamma_{0.882}\, mu'$. $m = 0.4\, m_0$.

(d) $K = (\gamma_{0.882} - 1)\, mc^2 = 1.125\, mc^2 = 0.450\, m_0c^2$.

(e) After the explosion, $E = \gamma_{0.882}\, mc^2 + mc^2 = (0.85 + 0.4)\, m_0c^2 = 1.25\, m_0c^2$.
Before the explosion, $E_i = \gamma_{0.6}\, m_0c^2 = 1.25\, m_0c^2$.

2. [24 marks].
A photon of energy 100 keV scatters at 180^o in a Compton collision with a free electron initially being at rest.

(a) What is the wavelength of the incident photon?
(b) What is the energy of the backscattered photon?
(c) What is the linear momentum of the electron after the collision?

Solution

(a) $\lambda = (hc)/E = 1.24\text{x}10^{-11}$ m.

(b) $\lambda' - \lambda = \lambda_C\,(1 - \cos 180^o) = 2\,\lambda_C$ (λ_C is given in the formula sheet for an electron)
$\lambda' = \lambda + 2\,\lambda_C = (1.24\text{x}10^{-11} + 2\text{x}\ 2.43\text{x}10^{-12})$ m = $1.726\text{x}10^{-11}$ m =17.26 pm
$E' = (hc)/\lambda' = 1.15\text{x}10^{-14}$ J = 72.0 keV.

(c) Law of conservation of momentum gives $\text{h}/\lambda = -\text{h}/\lambda' + p_e$
$p_e = h\,(1/\lambda + 1/\lambda')$
$= 6.63\text{x}10^{-34}$ J s $(8.0645\text{x}10^{10}\ \text{m}^{-1} + 5.7937\text{x}10^{10}\ \text{m}^{-1})$
$= (6.63\text{x}10^{-34}$ J s) x $(1.3858\text{x}10^{11}\ \text{m}^{-1})$
$= 9.19\text{x}10^{-23}$ kg m/s

3. [36 marks].

Suppose that a detector in the Hubble Space Telescope was capable of detecting visible light in the wavelength range of 400 to 700 nm. A hydrogen atom emits four lines in this range.

(a) Find all the wavelengths for the hydrogen atom that are in this range.
(b) The detector measures visible wavelengths of 537.5 nm, 480.1 nm, and 453.4 nm that researchers believe are due to the hydrogen atom. Why are all the known visible hydrogen lines not detected?
(c) Use these data to calculate the speed of the stellar object that emitted the spectra. Assume that the object is not rotating.
(d) Why might rotation be an issue?

Solution

(a) $1/\lambda_n = R_H\,(1/n^2 - 1/m^2) = 1.097\text{x}10^7\ \text{m}^{-1}\,(1/4 - 1/m^2)$, $m = 3, 4, 5, 6$.

$m = 3$, $\lambda_n = 656$ nm;
$m = 4$, $\lambda_n = 486$ nm;
$m = 5$, $\lambda_n = 434$ nm;
$m = 6$, $\lambda_n = 410$ nm.

(b) Line with $\lambda_n = 656$ nm was not detected due to the Doppler shift.

(c) All lines are red shifted, which means that the stellar object is receding from the telescope. The Doppler shift for the wavelengths looks as follows (is given in the formula sheet):

$\lambda = \lambda_0\,(1+u/c)^{1/2}/(1-u/c)^{1/2}$

$u/c = [(\lambda/\lambda_0)^2 - 1] / [(\lambda/\lambda_0)^2 + 1]$

For all shifted lines $\lambda/\lambda_0 = 1.106$.

u = c x 0.223 / 2.223 = 0.100 c.