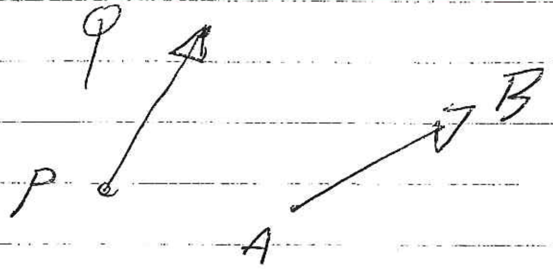
Q1:



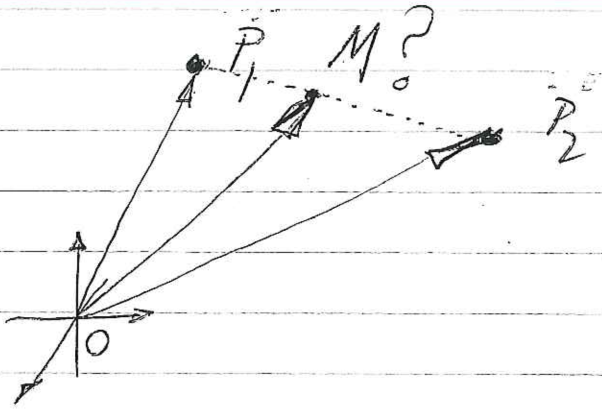
PARALLEL?

$$\vec{PQ} = \begin{bmatrix} 3-2 \\ -1-(-1) \\ 2-4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{AB} = \begin{bmatrix} 1-0 \\ 3-2 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

SINCE $\vec{AB}$ CANNOT BE EXPRESSED AS A SCALAR MULTIPLE OF $\vec{PQ}$, THEN THE TWO VECTORS ARE <u>NOT</u> PARALLEL.

Q2:



GIVEN:

$$\vec{OP_1} + \frac{1}{3}\vec{P_1P_2} = \vec{OM}$$

$$\vec{OP_1} + \frac{1}{3}\left(\vec{OP_2} - \vec{OP_1}\right) = \vec{OM}$$

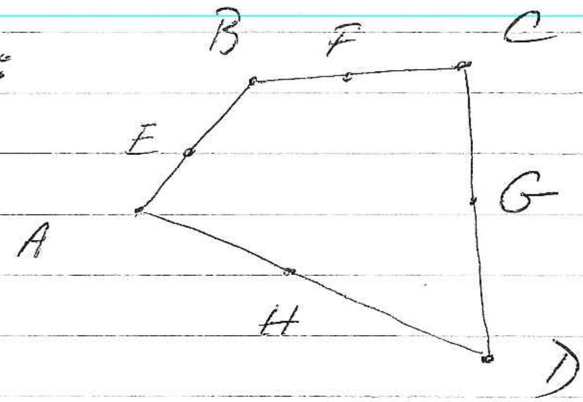
$\therefore$ $\vec{OM} = \frac{2}{3}\vec{OP_1} + \frac{1}{3}\vec{OP_2}$ (GENERAL EXPRESSION)

$P_1 = (1,2,3)$ $P_2 = (4,5,6)$

$$\vec{OM} = \frac{2}{3}\begin{bmatrix}1\\2\\3\end{bmatrix} + \frac{1}{3}\begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}2\\3\\4\end{bmatrix}$$

$\therefore$ $M = (2,3,4)$

Q3:



IS EFGH A PARALLELOGRAM?

$$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$$

$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} \quad \text{(MIDPOINTS)}$$

$$= \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC})$$

$$= \frac{1}{2}\overrightarrow{AC}$$ $\therefore$ $\overrightarrow{EF}$ AND $\overrightarrow{AC}$ ARE $\parallel$'l

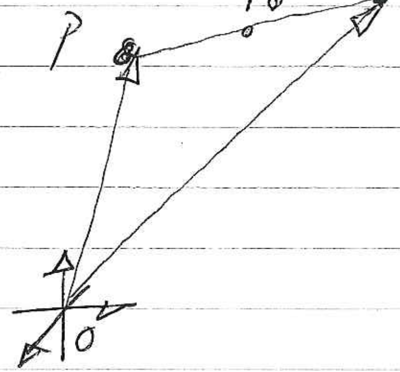
SIMILARLY WE CAN SHOW $\overrightarrow{HG}$ is $\parallel$'l TO $\overrightarrow{AC}$

$\therefore$ $\overrightarrow{EF}$ IS PARALLEL TO $\overrightarrow{HG}$

SIMILARLY WE CAN SHOW $\overrightarrow{EH}$ IS PARALLEL TO $\overrightarrow{FG}$

$\therefore$ EFGH IS A PARALLELOGRAM.

Q 4:



M: MIDPOINT

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= \begin{bmatrix} 7 \\ -4 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 3 \end{bmatrix}$$

FIND $\vec{OM}$:

$$\vec{OM} = \vec{OP} + \frac{1}{2}\vec{PQ}$$

$$= \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 5 \\ -7 \\ 3 \end{bmatrix} = \begin{bmatrix} 9/2 \\ -1/2 \\ -1/2 \end{bmatrix} \quad \therefore M = \left(\frac{9}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$$

SIMILARLY, POINT $2/3$ OF WAY FROM P TO Q IS GIVEN BY:

$$\vec{OP} + \frac{2}{3}\vec{PQ} = \begin{bmatrix} 16/3 \\ -5/3 \\ 0 \end{bmatrix} \quad \therefore \text{POINT} = \left(\frac{16}{3}, -\frac{5}{3}, 0\right)$$

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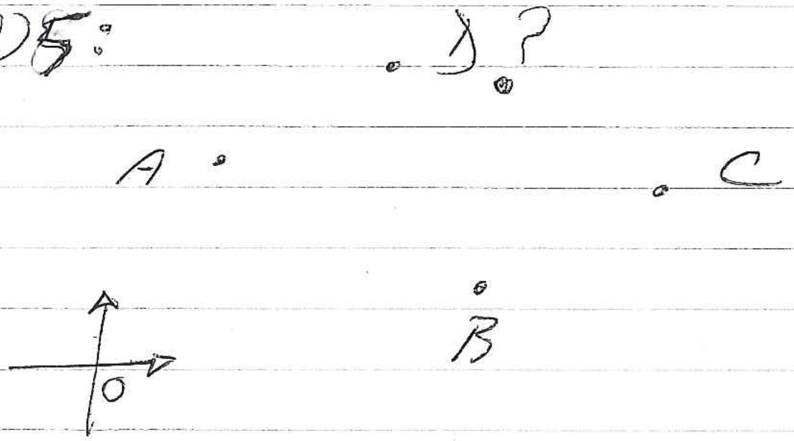
Q5:



WE WANT TO FIND D SUCH THAT:

$$\vec{BC} = \vec{AD} \quad \text{AND} \quad \vec{BA} = \vec{CD}$$

$\therefore$ WE WILL IMPOSE THE FOLLOWING:

$$\vec{BD} = \vec{BC} + \vec{CD} = \vec{BC} + \vec{BA}$$

$$= \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$\vec{OD} = \vec{OB} + \vec{BD}$$

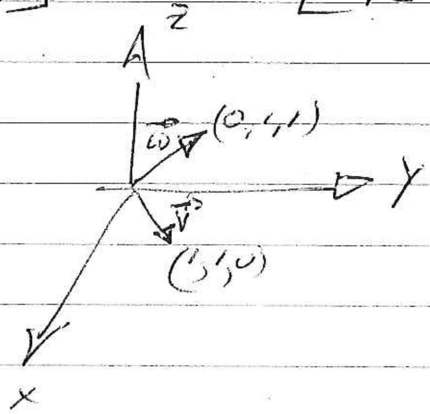
$$= \begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$\therefore$ $D = (3,5)$ NOT UNIQUE THOUGH.

Q6:

(i) $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad \vec{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$



$\vec{v}$ LIES IN THE X-Y PLANE
$\vec{w}$ LIES IN THE Y-Z PLANE

THE LINEAR COMBINATION OF $\vec{v}$ AND $\vec{w}$ FILL THE PLANE THAT CONTAINS $\vec{v}$ AND $\vec{w}$ AND GOES THROUGH THE ORIGIN.

(ii)

$$c\vec{v} + d\vec{w} = c\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c \\ c+d \\ d \end{bmatrix}$$

(iii) SECOND COMPONENT IS ALWAYS SUM OF FIRST AND THIRD COMPONENTS.

$\therefore$ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ IS NOT IN THE PLANE BECAUSE $2 \neq 1+3$

Q.7:

$$c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\7\end{bmatrix} + c_3\begin{bmatrix}1\\5\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

WRITE TWO EQUIVALENT SCALAR EQUATIONS:

$$c_1 + 2c_2 + c_3 = 0$$

$$3c_1 + 7c_2 + 5c_3 = 1$$

$$\therefore \quad c_1 = -2c_2 - c_3$$

$$3(-2c_2 - c_3) + 7c_2 + 5c_3 = 1$$

$$c_2 + 2c_3 = 1$$

$$\therefore \quad c_2 = 1 - 2c_3$$

CHOOSE $c_3 = 1 \Rightarrow c_2 = -1$ AND $c_1 = 1$

CHOOSE $c_3 = -1 \Rightarrow c_2 = 3$ AND $c_1 = -5$

NOT ALWAYS. FOR INSTANCE IF YOU CHOOSE THE 3 VECTORS TO BE PARALLEL TO EACH OTHER BUT NOT PARALLEL TO $\begin{bmatrix}0\\1\end{bmatrix}$ THEN NO LINEAR COMBINATIONS WILL PRODUCE $\vec{b}$.