UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

# Term Test II

First Year — Program 5

## MAT185S — Linear Algebra

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8 March 2011

Student Name: ______________________

Last Name First Names

Student Number: ______________ Tutorial Section: ________

## Instructions:

1. Attempt *all* questions.
2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question.
4. *No* aid is permitted.
5. The duration of this test is 90 minutes.
6. There are 9 pages and 5 questions in this test paper.

| For Markers Only | | |
|---|---|---|
| **Question** | **Value** | **Mark** |
| **A** | | |
| **1** | 10 | |
| **B** | | |
| **2** | 10 | |
| **C** | | |
| **3** | 10 | |
| **4** | 10 | |
| **5** | 10 | |
| **Total** | 50 | |

## A. Definitions and Statements

*Fill in the blanks.*

**1(a).** State the *Fundamental Theorem of Linear Algebra*.

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**1(b).** A *basis* of a finite-dimensional vector space $\mathcal{V}$ is defined as

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/2

**1(c).** The *rank* of $\mathbf{A} \in {}^m\mathbb{R}^n$ is defined as

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/2

**1(d).** Given that $\{\boldsymbol{v}_1 \ldots \boldsymbol{v}_n\} \subset \mathcal{V}$ is linearly independent and $\boldsymbol{v} \in \mathcal{V}$, then $\{\boldsymbol{v}, \boldsymbol{v}_1 \ldots \boldsymbol{v}_n\}$ is linearly independent if and only if

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**1(e).** The relationship between $\mathcal{C}(\mathbf{A}) = \operatorname{col} \mathbf{A}$ and $\mathcal{C}(\mathbf{AB}) = \operatorname{col} \mathbf{AB}$, where $\mathbf{A} \in {}^m\mathbb{R}^n$ and $\mathbf{B} \in {}^n\mathbb{R}^n$, is

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## B. True or False

*Determine if the following statements are* true *or* false *and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.*

**2(a).** If the dimension of $\mathcal{V}$ is 100, then any set of 99 vectors from $\mathcal{V}$ is linearly independent.

**2(b).** Let $\boldsymbol{0}$ be the zero of a vector space $\mathcal{V}$. Then $\{\boldsymbol{0}\} = \text{span}\{\boldsymbol{0}\}$.

**2(c).** If $\mathcal{S}$ is a subspace of $\mathcal{V}$ then any basis for $\mathcal{V}$ contains a basis for $\mathcal{S}$.

**2(d).** The dimension of the row space of $\mathbf{A}^T$ is equal to the dimension of the column space of $\mathbf{A}$.

**2(e).** The dimension of the null (solution) space of a $5 \times 19$ matrix must be less than or equal to 13.

## C. Problems

**3.** If $\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$ is a basis for $\mathcal{V}$, show that $\{\boldsymbol{u}+\boldsymbol{v}, \boldsymbol{w}+\boldsymbol{u}, \boldsymbol{v}+\boldsymbol{w}\}$ is also a basis for $\mathcal{V}$.

*. . . cont'd*

3. . . . cont'd

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4. Consider the subspace of $\mathbb{P}_4$,

$$\mathcal{W} = \{p \in \mathbb{P}_4 \mid p(1) + p(-1) = 0 \text{ and } p(2) + p(-2) = 0\}$$

where $\mathbb{P}_4$ is the vector space of polynomials of degree 4 or less. Determine the dimension of $\mathcal{W}$.

*. . . cont'd*

4. . . . cont'd

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5. Let $\mathbf{A} \in {}^m\mathbb{R}^n$. Show that $m = n$ and $\mathbf{A}$ is invertible if and only if

(*i*) There exists $\mathbf{b}_0 \in {}^m\mathbb{R}$ such that $\mathbf{A}\mathbf{x} = \mathbf{b}_0$ has a unique solution for $\mathbf{x} \in {}^n\mathbb{R}$; and

(*ii*) There exists $\mathbf{c}_0 \in {}^n\mathbb{R}$ such that $\mathbf{A}^T\mathbf{x} = \mathbf{c}_0$ has a unique solution for $\mathbf{x} \in {}^m\mathbb{R}$.

*. . . cont'd*

5. . . . cont'd

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