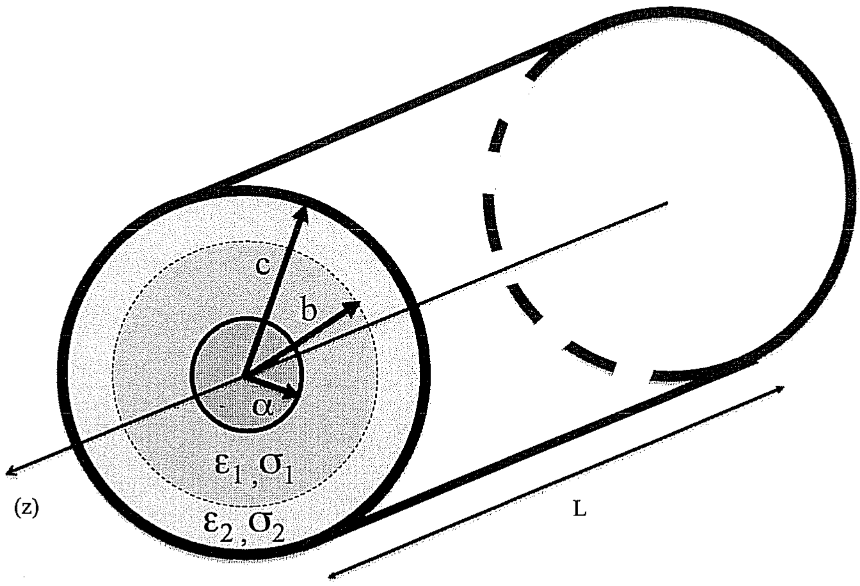
Question 1

Consider the lossy coaxial capacitor shown in the figure below. The capacitor consists of two perfect conductors at $r = \alpha$ and $r = c$ and two lossy dielectric media with dielectric permittivities and conductivities ε_i, σ_i, $i = 1, 2$. The interface between the two media is at $r = b$. The capacitor has finite length L in the z–direction, however, its electric field can be approximated by the field of a capacitor with $L \to \infty$. The voltage difference between the inner and outer perfect conductor is $V(r = \alpha) - V(r = c) = V_0$.



1. Using the Laplace equation, show that the general form of the electric field in the two regions is $\mathbf{E}_i = \dfrac{A_i}{r}\mathbf{a}_r$, $i = 1, 2$. **(4 pts)**

2. Using boundary conditions for the volume current density $\mathbf{J}$, show that $\frac{A_1}{A_2} = \frac{\sigma_2}{\sigma_1}$. **(2 pts)**

3. Find the resistance R of the resistor. **(8 pts)**

R =

Derivation:

4. Is there a surface charge density ρ_s at the interface between the two lossy dielectrics? If yes, calculate it (you can use A_1, A_2 in this calculation). If not, why not ? **(4 pts)**

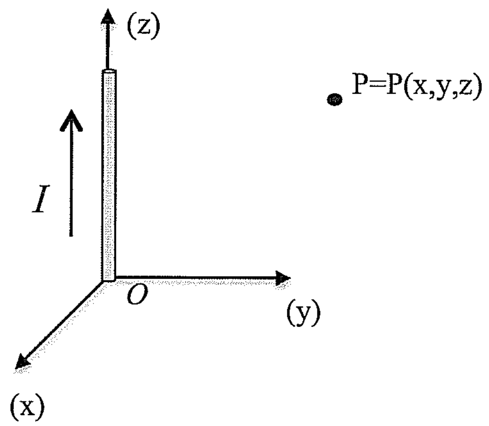
5. Is there a volume charge density ρ_v within the capacitor? If yes, why; if not, why not ? **(2 pts)**

Question 2

1. A thin wire of length L carries current I along the z−axis for $0 \leq z \leq L$. Using the Biot-Savart law, the magnetic field that this wire produces at an arbitrary observation point $P(x, y, z)$ can be expressed as follows:

$$\mathbf{B}(x,y,z) = \frac{\mu_0 I}{4\pi} \int_{z'=0}^{z'=L} \frac{?}{(x^2+y^2+(z-z')^2)^{3/2}}$$

Derive the term that is missing in this expression. **(10 pts)**



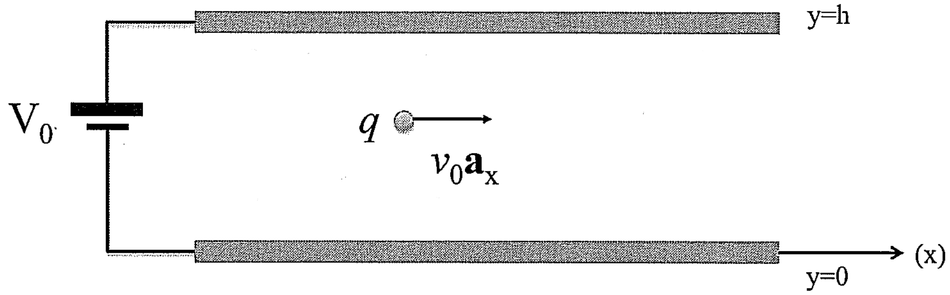
Answer:

$? =$

Derivation:

(1 per component)

2. The charge q shown in the figure moves with constant velocity within the electric field of a parallel plate capacitor with voltage V_o and plate separation h, due to a constant magnetic field within the capacitor. Find the magnitude and direction of the magnetic flux density $\mathbf{B}$ of this magnetic field. **(4 pts)**



Answer:

$\mathbf{B} =$

Derivation:

3. The two circular coils shown in the figure, centered at A_1 and A_2, support co-directional currents $I_1 = I_2 = I$. Let the magnetic field densities generated by each of the two coils alone be $\mathbf{B_1}$, $\mathbf{B_2}$. The total magnetic field density $\mathbf{B}$ at point P on the axis is:

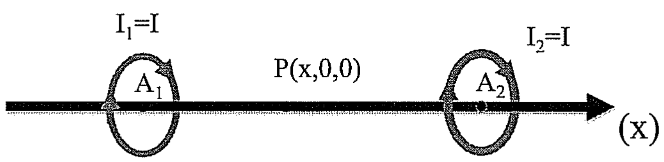
   a) In the positive x direction and has magnitude smaller than the magnitude of $\mathbf{B_1}$ at P.

   b) In the positive x direction and has magnitude smaller than the magnitude of $\mathbf{B_2}$ at P.

   c) In the negative x direction and has magnitude greater than the magnitude of $\mathbf{B_2}$ at P. (0.5 pt)

   d) In the negative x direction and has magnitude greater than the magnitude of $\mathbf{B_1}$ at P. (0.5 pt)

   Choose all answers that apply and briefly explain. **(4 pts)**



4. Which of the following expressions can represent a magnetic flux density $\mathbf{B}$? Choose all answers that apply and briefly explain. **(2 pts)**

   a) $B_0\mathbf{a}_x$, where B_0 is a constant.

   b) x y $\mathbf{a}_x$.

   c) $\frac{B_0}{r}\mathbf{a}_r$, where B_0 is a constant.

   d) $\frac{B_0}{r}\mathbf{a}_\phi$, where B_0 is a constant.