NAME:______________________________

STUDENT NUMBER:________________________________

TUTORIAL GROUP:________________________________

Time: 150 minutes

This is a closed-book exam worth a total of 120 points. Please answer all questions.

ONLY THE FOLLOWING ITEMS ARE ALLOWED ON YOUR DESK DURING THE EXAM:

1. **THIS EXAM BOOK –** It contains this cover page, six question pages and a two-sided formula sheet (which may be torn off). Make sure you start by putting your **NAME, ID NUMBER,** and **TUTORIAL GROUP** on the front (cover) page of the exam. The entire exam book (minus the formula sheet) **will be handed in** at the end of the exam and marked.
    a. **FORMULA SHEET**, annotated on the printed page. The formula sheet has to be printed from Quercus.
2. **A CALCULATOR**, which can be anything that does NOT connect to wifi and does NOT communicate with other devices. ACCEPTABLE calculators include programmable and graphing calculators, scientific calculators, etc. UNACCEPTABLE calculators include: cell phones, tablets, laptops, etc.
3. **A PEN OR/AND A PENCIL.**
4. **YOUR STUDENT ID CARD**, used to check your identity.

If you are missing anything from the above items, raise your hand and ask an exam supervisor to supply what is missing. If you are missing an item that should have been brought by you (e.g., calculator, pen/pencil) there is a limited supply of extras and are on a first come, first served basis.

COMPLETE SOLUTION INCLUDES:

a) A sketch with a coordinate system, where appropriate. Where a diagram is provided, additional markings can be added to it.
b) Clear and logical workings, including quoting the equations that were used, proper substitutions of numbers (when necessary) with all steps clearly indicated.
c) The final answer with units and three significant figures.

FOR OFFICE USE ONLY							
QUESTION	**I**	**II**	**III**	**IV**	**V**	**VI**	**TOTAL**
MARK							
MAXIMUM	**20**	**20**	**20**	**20**	**20**	**20**	**120**

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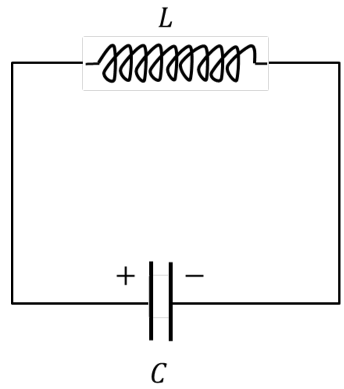
Question I

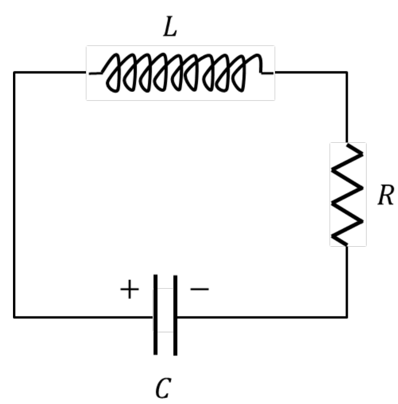
A $2.50\ \mu\mathrm{C}$ capacitor is connected in series with a inductor with inductance $L = 6.40\ \mathrm{mF}$. There is no battery in the circuit and when the measurement of the charge across the capacitor starts the capacitor has just started discharging and there is still 87% of the initial charge stored on the plates.



1. Determine the **angular frequency** of the oscillation. [2 points]

2. Assuming that the charge across the capacitor can be expressed using equation $q(t) = q_0 \cos(\omega_0 t + \phi_i)$, what is the value of the phase constant ϕ_i? [8 points]

3. When the capacitor is fully charged ($q(t) = q_0$, a resistor $R = 8.10\ \Omega$ is added in series with the capacitor and inductor. If Kirchhoff's Loop Law for the system can be written as
$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{q(t)}{C} = 0$$
what is the damping factor γ of the system? [4 points]

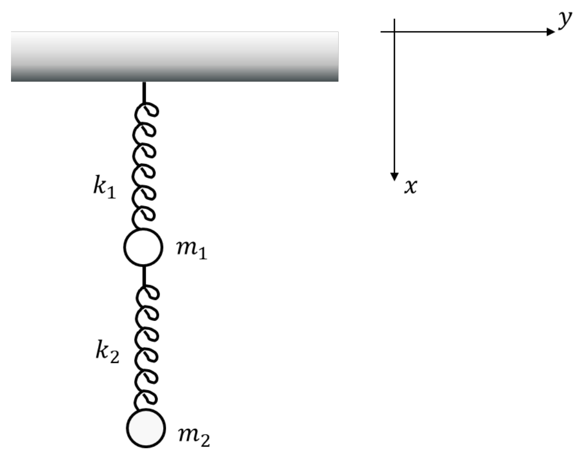


4. Determine the quality factor for the circuit [2 points]

5. Determine the value of the resistor for which this circuit would be critically damped. [4 points]

Question II

Consider two masses m_1 and m_2 hanging vertically. Mass m_1 is attached directly to the rigid ceiling with spring k_1 and mass m_2 is attached to mass m_1 with a spring k_2.



1. Assuming both masses are displaced downwards, draw arrows that clearly indicate the direction of forces on each mass **due to each spring** attached to it. Clearly label each force. If you cannot draw a picture, describe what forces would be acting on each mass and in which direction. [4 points]

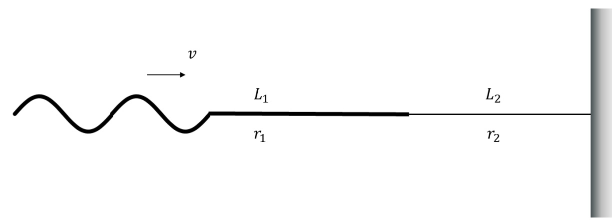
2. Write an equation of motion for each mass. [6 points]

3. Determine the coefficient matrix for the system. [4 points]

4. Show that for $m_2 = 0.75m_1$ where $m_1 = m$ and $k_2 = 1.5k_1$ where $k_1 = k$, the normal frequencies of the oscillation are $\omega = \sqrt{\frac{k}{2m}}$ and $\omega = \sqrt{\frac{4k}{m}}$ [6 points]

Question III

A wave of amplitude $A(r)$ and frequency f is sent down a taut string of radius r_1 made of material with density ρ that is under tension F_T. After traveling distance L_1 along the string, the wave reaches a boundary with another string of radius $r_2 < r_1$ and length L_2 made of the same material and held under the same tension.



1. Based on the information provided, estimate the possible range of amplitude reflection and transmission coefficients at the boundary. Justify your answer either through reasoning or mathematically [4 points]

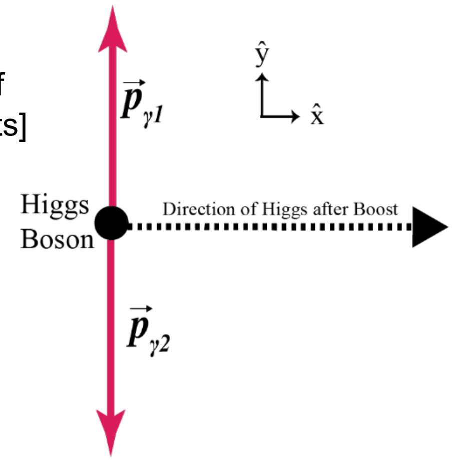
2. If $r_2 = 0.400r_1$, determine coefficients of reflection and transmission for amplitude and power. Clearly label the coefficients. [8 points]

3. String L_2 is fixed to the rigid support. The transmitted wave creates a standing wave pattern with the third harmonic observed. Determine the expression for the frequency f as a function of the properties of the second wire (length, radius, tension, density) and known constants. Clearly state the assumptions you make about the boundary conditions. [4 points]

4. Before reaching the boundary, the wave travelled distance $L_1 = 5.0 \text{ m}$ along the first string. If the attenuation coefficient along that string $\alpha = 0.006 \text{ m}^{-1}$, what was the **amplitude** of the wave at the beginning of the string? Express your answer as a percentage of the amplitude at the boundary. [4 points]

Question IV

A Higgs boson decays into two photons.



1. In the rest frame of the Higgs boson, each photon has an energy of $62.5\ \mathrm{GeV}$. Calculate the invariant mass of the Higgs boson. [5 points]

2. The Higgs boson is boosted in the x direction so that its energy is $200\ \mathrm{GeV}$, with the orientation of the resulting Higgs boson in the figure. What is the relativistic γ-factor for the boost and the speed $\beta = v/c$ of the Higgs boson relative to its rest frame? [5 points]

3. The decay takes place at $\Delta t = 2.0 \times 10^{-23}$ seconds in the Higgs boson rest frame. What is the distance the Higgs boson travels in the boosted frame? [4 points]

4. What are the 4-momenta of the photons in the boosted frame? [6 points]

Question V

A proton in a nucleus has a typical energy of $10\ \mathrm{MeV}$.

1. What is the velocity of the proton? Is it relativistic ? [7 points]

2. What is the de Broglie wavelength of the proton in the nucleus? How does the proton's wavelength compare with the size of the nucleus, which is approximately $5 \times 10^{-15}\ \mathrm{m}$? [6 points]

3. High-energy photons are used to probe the interior of nucleus. If their wavelength has to be 2% of the size of the nucleus, what is their energy in eV ? [7 points]

Question VI

Methane (CH_4) is another important greenhouse gas. It absorbs infrared energy at a wave number $k = 3156\ \mathrm{cm}^{-1}$. The vibration of the molecule associated with this frequency is one where the carbon atom is vibrating between the four motionless hydrogen atoms. The mass of a carbon atom is $m_C = 1.99 \times 10^{-26}\ \mathrm{kg}$.

1. What are the angular frequency ω and energy (in eV) of the photons absorbed by CH_4? [3 points]

2. What is the zero-point energy (in eV) of the vibrations of the CH_4 molecule? [3 points]

3. The wave function of the CH_4 molecule in its ground state is

$$\psi(x) = A_0 \exp\left(-\frac{x^2}{2b^2}\right)$$

where A_0 is a normalization constant, x is the displacement of the carbon atom from equilibrium and $b = \sqrt{\hbar/(m_C\omega)}$.

Determine A_0 in terms of b (see the formula sheet for the definite integral you need!). Calculate the length b for this wave function. [8 points]

4. Sketch the probability density for the displacement x. The parameter b is the amplitude of oscillation if this were a classical system. Indicate the area under the curve where the displacement is outside the classically allowed region. [6 points]

OSCILLATIONS

$\omega = 2\pi f = \frac{2\pi}{T}$	$\omega_0 = \sqrt{\frac{k}{m}}$	$\omega_0 = \sqrt{\frac{mgd}{I}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
$x(t) = A\cos(\omega t + \phi_i)$	$x(t) = A_0 \exp\left(-\frac{\gamma t}{2}\right)\cos(\omega t + \phi_i))$	$x(t) = A(\omega)\cos(\omega t - \delta)$	
$x(t) = A\exp\left(-\frac{\gamma t}{2}\right) + Bt\exp\left(-\frac{\gamma t}{2}\right)$	$x(t) = A\exp\left(\left(-\frac{\gamma}{2} + \left(\left\lvert\omega_0^2 - \frac{\gamma^2}{4}\right\rvert\right)^{\frac{1}{2}}\right)t\right) + B\exp\left(\left(-\frac{\gamma}{2} - \left(\left\lvert\omega_0^2 - \frac{\gamma^2}{4}\right\rvert\right)^{\frac{1}{2}}\right)t\right)$		
$q_0(\omega) = \frac{\varepsilon_0}{\omega Z}$	$q(t) = q_0(\omega)\cos(\omega t - \delta)$	$Z = \sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}$	$i = \frac{dq}{dt}$
$V_R = i(t)R$	$V_C = \frac{q}{C}$	$V_L = L\frac{di}{dt}$	
$K = \frac{1}{2}mv^2$	$U = \frac{1}{2}kx^2$	$E(t) = E_0\exp(-\gamma t)$	$P = \frac{dE}{dt} = Fv$
$Q = \frac{\omega_0}{\gamma}$	$\omega^2 = \omega_0^2 - \frac{\gamma^2}{4}$		
$A(\omega) = \frac{a\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$	$\tan\delta = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$		
$\bar{P}(\omega) = \frac{b[v_0(\omega)]^2}{2}$	$\bar{P}_{max} = \frac{F_0^2}{2m\gamma}$	$\bar{P}(\omega) = \frac{F_0^2}{2m\gamma\left[\frac{4(\Delta\omega)^2}{\gamma^2} + 1\right]}$	

WAVES

$v = \lambda f$	$y(x,t) = f(x \pm vt)$	$y(x,t) = A\cos(kx \pm \omega t + \phi_i)$				
$k = \frac{2\pi}{\lambda}$	$y(x,t) = (A\sin(kx) + B\cos(kx))\cos(\omega t)$					
$v = \sqrt{\frac{F_T}{\mu}}$	$v = \sqrt{\frac{B}{\rho}}$	$v = \sqrt{\frac{Y}{\rho}}$	$v = \sqrt{\frac{\gamma RT}{M}}$	$\frac{\partial y^2}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$	$\omega = \frac{2\pi}{T}$	$f = \frac{1}{T}$
$\omega_n = \frac{n\pi v}{L}$	$\omega_n = \frac{n\pi v}{2L}$	$f_n = nf_1$	$E = \frac{1}{4}\mu\omega_n^2 A_n^2 L$			
$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$	$P_{ave} = \frac{1}{2}ZA^2\omega^2$	$I_{ave} = \frac{1}{2}Z_a A^2\omega^2$	$P = \mu v A^2\omega^2\sin^2(kx - \omega t + \phi_0)$			
$Z = \sqrt{\mu\tau}$	$Z_a = \sqrt{Y\rho}$	$Z_a = \sqrt{B\rho}$	$R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$	$T = \frac{2Z_1}{Z_1 + Z_2}$		
$I(r) = I(r_0)\left[e^{-\alpha(r - r_0)}\right]\left(\frac{r_0}{r}\right)^{N-1}$	$v_g = \frac{d\omega}{dk}\Big\vert_{k=k_0}$	$v_g = v - \lambda\frac{dv}{d\lambda}$	$R^2 + \frac{Z_2}{Z_1}T^2 = 1$			
$c = (\mu_0\varepsilon_0)^{-\frac{1}{2}}$	$v = \frac{c}{n}$		$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$			

MATHEMATICAL FORMULAE

$\cos\alpha + \cos\beta = 2\cos\left[\frac{\alpha + \beta}{2}\right]\cos\left[\frac{\alpha - \beta}{2}\right]$	$\cos\alpha - \cos\beta = -2\sin\left[\frac{\alpha + \beta}{2}\right]\sin\left[\frac{\alpha - \beta}{2}\right]$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$
$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$	$\tan^{-1}(x) = \{\theta, \theta + \pi\} + 2\pi n$ $\cos^{-1}(x) = \pm\theta + 2\pi n$
$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$	$\sin^{-1}(x) = \{\theta, \pi - \theta\} + 2\pi n$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$	$\tilde{A} = Ae^{j\theta} = A(\cos\theta + j\sin\theta)$
$\int \sin(ax)dx = -\frac{1}{a}\cos ax$	$\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$

CONSTANTS

$c = 3.00 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T}\cdot\frac{\mathrm{m}}{\mathrm{A}}$	$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{C}^2}$	$g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}}$
$v_{sound\ at\ 20^\circ\mathrm{C}} = 343\,\frac{\mathrm{m}}{\mathrm{s}}$	$T_K = T_{^\circ\mathrm{C}} + 273.15^\circ\mathrm{C}$		

SPECIAL RELATIVITY			
$\beta = \frac{v}{c}$	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$	$\Delta t' = \gamma \Delta t$	$\frac{f_s}{f_r} = \sqrt{\frac{1+\beta}{1-\beta}}$
Lorentz Transformations: $\tilde{x} = (ct, x, y, z)$ $\tilde{p} = (E/c, p_x, p_y, p_z)$	$ct' = \gamma ct - \gamma\beta x$ $x' = \gamma x - \gamma\beta ct$ $y' = y$ $z' = z$	$E'/c = \gamma E/c - \gamma\beta p_x$ $p'_x = \gamma p_x - \gamma\beta E/c$ $p'_y = p_y$ $p'_z = p_z$	
Lorentz boost : $B = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$(ct', x', y', z') = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$		
Minkowski tensor : $\eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	$\tilde{x}^{\mathrm{T}}\eta\tilde{x} = (c\tau)^2 = (ct)^2 - x^2 - y^2 - z^2$ $\tilde{p}^{\mathrm{T}}\eta\tilde{p} = (mc)^2 = E^2/c^2 - p_x^2 - p_y^2 - p_z^2$		

QUANTUM MECHANICS			
Photoelectric Effect:	$E = h\nu = \hbar\omega$	$E_{pe} = h\nu - W$	
Compton Scattering:	$\hbar\omega' = \dfrac{\hbar\omega}{1 + \frac{\hbar\omega}{m_e c^2}(1-\cos\theta)}$	Compton wavelength of electron:	$\lambda = \frac{2\pi\hbar}{m_e c}$ $= 2.4263 \times 10^{-12}$ m
Rydberg Formula:	$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$	Diffraction angle (d = slit spacing)	λ/d
de Broglie Wavelength:	$\lambda = \frac{h}{p}$	Bohr radius:	$a_0 = \frac{\hbar^2}{me^2} = 0.053$ nm
Heisenberg Uncertainty:	$\Delta x\,\Delta p \geq \frac{\hbar}{2}$	$\Delta E\,\Delta t \geq \frac{\hbar}{2}$	
Schrödinger Equation:	$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$	Time-independent Schrödinger Equation:	$E\psi(x) = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x)$
Integrals:	$\int_{-\infty}^{+\infty} \exp(-\alpha^2 x^2)dx = \frac{\sqrt{\pi}}{\alpha}$	Harmonic oscillator energy states:	$E_n = \hbar\omega\left(\frac{1}{2} + n\right)$
Hermite polynomials:	$H_0(y) = 1 \quad H_1(y) = y$ $H_2(y) = y^2 - 1 \quad H_3(y) = y^3 - 3y$	Infinite square well (width $2a$) energy states:	$E_n = \frac{\pi^2\hbar^2 n^2}{8ma^2} = \frac{\hbar^2 k^2}{2m}$

MODERN PHYSICS CONSTANTS			
$c = 3.00 \times 10^8$ m/s	$h = 6.626 \times 10^{-34}$ m^{2}kg/s	$\hbar = 1.055 \times 10^{-34}$ m^{2}kg/s $= 6.582 \times 10^{-16}$ eV · s	1 eV $= 1.602 \times 10^{-19}$ J
$R_H = 1.096776 \times 10^7 \mathrm{m}^{-1}$	$m_e = 9.1094 \times 10^{-31}$ kg $m_p = 1.6726 \times 10^{-27}$ kg $m_n = 1.6749 \times 10^{-27}$ kg	1 eV/c$^2 = 1.7827 \times 10^{-36}$ kg 1 eV/c $= 5.344 \times 10^{-28}$ kg · m/s	