

[Instructions: Solve the question below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.]

You are allowed: a non-communicating calculator, a one-page formula sheet (can be annotated)].

A wave of amplitude A_i is coming from mineral oil ($B_{\text{mineral oil}} = 1.80 \times 10^9 \frac{\text{N}}{\text{m}^2}$, $\rho_{\text{mineral oil}} = 857 \frac{\text{kg}}{\text{m}^3}$) is incident on a boundary with another, unknown liquid.

- a. What is the speed of the mechanical wave in the mineral oil? [2 points]

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.8 \times 10^9 \frac{\text{N}}{\text{m}^2}}{857 \frac{\text{kg}}{\text{m}^3}}} = 1450 \text{ m/s}$$

1 point for referring to the equation on the sheet; 1 for calculation

- b. What is the impedance of the mineral oil? [2 points]

$$Z_a = \sqrt{B\rho} = \rho v = \sqrt{\left(1.8 \times 10^9 \frac{\text{N}}{\text{m}^2}\right) * 857 \frac{\text{kg}}{\text{m}^3}} = 1.24 \times 10^6 \frac{\text{kg}}{\text{s} \cdot \text{m}^2}$$

Either equation is ok as both appear on the formula sheet (ρv is hidden in the power formula)

- c. After the reflection from the boundary the reflected wave carries exactly 10% of the energy of the original and the reflected and incident waves are **out of phase** with each other. Assuming there are no losses, determine the values of the transmission of reflection coefficients for the **energy** of the wave. [2 points]

$$R_e = \frac{P_r}{P_i} = 0.10$$

$$R_e + T_e = 1 \rightarrow T_e = 1 - R_e = 0.90$$

Any solution is ok, somehow energy/power coefficients and energy/power conservation need to be included.

- d. What are the transmission and reflection coefficients for the amplitude of the wave? [2 points]

$$R_e = R^2 \rightarrow R = \pm\sqrt{R_e} = -\sqrt{R_e} \text{ (out of phase amplitudes)}$$

$$R = -0.316$$

$$1 + R = T \rightarrow R = 0.684$$

One point each; two signs need to be considered for R (as it could be positive or negative); if negative is missed, 0.5 mark is deducted

- e. Determine the impedance of the unknown liquid [2 points]

$$T_e = \frac{Z_2}{Z_1} T^2 \rightarrow Z_2 = \frac{T_e Z_1}{T^2} = \left(\frac{0.90}{0.684^2}\right) 1.24 \times 10^6 \frac{\text{kg}}{\text{m}^2 \text{s}} = 2.39 \times 10^6 \frac{\text{kg}}{\text{m}^2 \text{s}}$$

