UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

Quiz 2, November 21, 2022

DURATION: 1.0 hours

Second Year – Engineering Science

CHE260H1 – Thermodynamics and Heat Transfer

Calculator Type: 1 (Any, non-communicating)

Exam Type: A (Closed Book)

Examiner: J. Werber

Last Name: Solutions

First Name: ______________________________

Email: ______________________________

$\dot{Q}_{conduction} = -kA\frac{dT}{dx}$

$\dot{Q}_{convection} = hA(T_s - T_\infty)$

$\dot{Q}_{radiation} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$

$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4

$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$

$R_{convection} = \frac{1}{hA}$

$R_{wall} = \frac{L}{kA}$

$R_{radiation} = \frac{1}{h_{rad}A}$

$h_{rad} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr})$

$R_{cylinder} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$

$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$

$R_c = \frac{\Delta T_{interface}}{\dot{Q}/A}$

For fluid temp change: $Q = mC_p\Delta T$

For vaporization: $Q = m\Delta\hat{H}_{\mathrm{vap}}$

$a = \sqrt{\frac{hP}{kA_c}}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

For infinitely long fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-ax)$$

$$\dot{Q}_{fin,long} = \sqrt{hPkA_c}(T_b - T_\infty)$$

$$\eta_{fin,long} = \frac{1}{aL}$$

For insulated tip fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh\,[a(L - x)]}{\cosh\,(aL)}$$

$$\dot{Q}_{fin,ins} = \sqrt{hPkA_c}(T_b - T_\infty)\tanh(aL)$$

$$\eta_{fin,ins} = \frac{\tanh\,(aL)}{aL}$$

x	tanh(x)	x	tanh(x)
0	0	0.64	0.5649
0.04	0.04	0.68	0.5915
0.08	0.0798	0.72	0.6169
0.12	0.1194	0.76	0.6411
0.16	0.1586	0.8	0.664
0.2	0.1974	0.84	0.6858
0.24	0.2355	0.88	0.7064
0.28	0.2729	0.92	0.7259
0.32	0.3095	0.96	0.7443
0.36	0.3452	1	0.7616
0.4	0.3799	2	0.964
0.44	0.4136	3	0.9951
0.48	0.4462	4	0.9993
0.52	0.4777	5	0.9999
0.56	0.508	10	1
0.6	0.537		

Multiple choice questions (4 pts each)

(Note: these are first only so you don't miss them! Feel free to do them after the problems, if desired)

Q1. In problem 1, which of the following would help to decrease the rate of N_2 evaporation? Choose all that apply. There's no need to do any calculations.

- [x] Double the thickness of the foam insulation
- [ ] Use a fan to blow air at the tank
- [x] Decrease the room temperature to 10 °C
- [x] Coat the insulation surface with a highly reflective coating

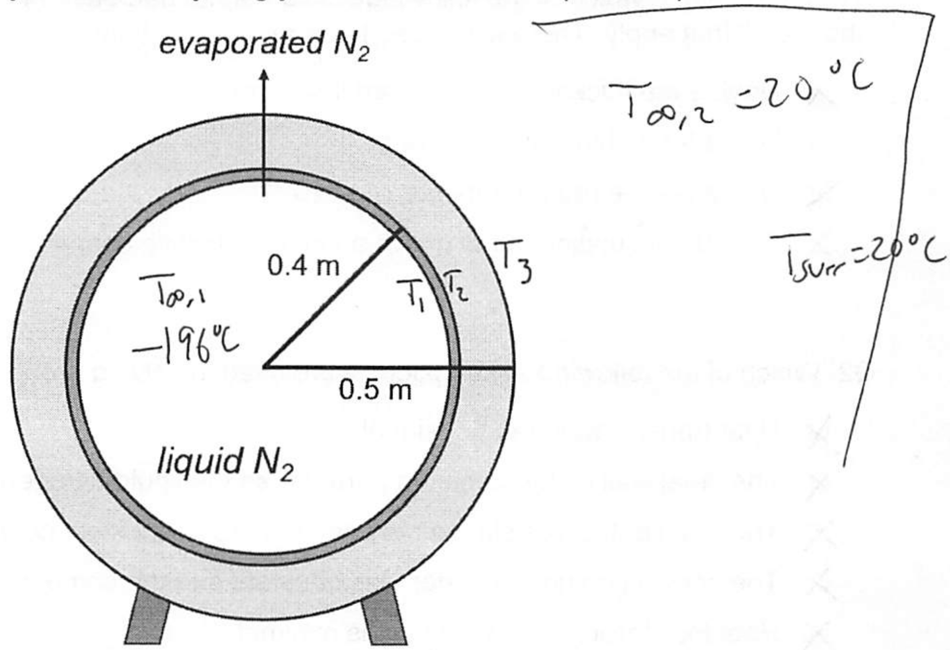
Q2. Which of the following assumptions were used in solving problem 1? Choose all that apply.

- [x] Heat transfer was 1-dimensional
- [x] The steel wall is the same temperature as the liquid nitrogen
- [x] Thermal contact resistance between the steel and foam could be neglected
- [x] The convection heat transfer was constant and the same around the tank
- [x] Heat loss through the vent hole is minimal

Q3. In problem 3, which of the following would increase heat flux through the fin (i.e., total heat transfer divided by fin cross-sectional area)? Choose all that apply.

- [x] Increasing k
- [x] Increasing h
- [ ] Increasing the horizontal dimensions
- [x] Increasing fin length, L
- [ ] Increasing the fluid temperature
- [x] Increasing the base temperature

Problem 1 (34 pts). A spherical insulated steel tank contains liquid nitrogen at its boiling point of –196 °C. The steel tank has an outer diameter of 0.8 m. The 0.5-cm thick steel wall is surrounded by 10 cm of foam insulation with a thermal conductivity of 0.05 W m^{-1} K^{-1}. The tank is in a room at 20 °C with a convection heat transfer coefficient of 5 W m^{-2} K^{-1}. The walls of the room are also at 20 °C. The emissivity of the outside of the foam wall is 0.7. A small hole in the top of the tank allows gaseous nitrogen to vent. The heat of vaporization ($\Delta\widehat{H}_{vap}$) of liquid nitrogen is 198 kJ/kg. (Note: you have everything you need to solve the problem.)



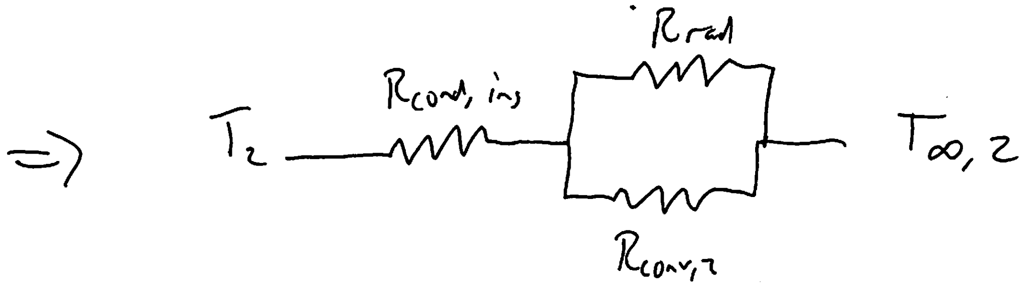
a. What is the rate of heat transfer (in W) from the tank to the surroundings?
b. What is the rate of evaporation (in kg/s) from the tank?

Use the resistance approach.

Main Assumptions:

1) 1-D radial heat flow
2) Resistance in conv, 1 & cond, 1 are negligible $\Rightarrow T_2 = -196\,^{\circ}C$
liquid   thin metal

Also, $h_2 = 5 \frac{W}{m^2 K}$. Quite low. Radiation must be considered.



$$R_{cond,ins} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{0.5 - 0.4\text{ m}}{4\pi (0.05 \frac{W}{mK})(0.5\text{m})(0.4\text{m})} = 0.79577 \frac{K}{W}$$

$$R_{conv,2} = \frac{1}{hA} = \frac{1}{(5 \frac{W}{m^2K})(3.14159 m^2)} = 0.06366 \frac{K}{W}$$

$$A = \frac{4}{\cancel{3}} \pi r_2^2 = 3.14159 \text{ m}^2$$

For radiation, we need to guess T_3.

- Foam is thick, but large driving force. ⇒ 10°C as guess.

$$h_{rad} = \varepsilon \sigma (T_3^2 + T_{surr}^2)(T_3 + T_{surr})$$

$$= (0.7)(5.67 \times 10^{-8} \frac{W}{m^2K^4})(283^2 + 293^2)(283 + 293)$$

$$= 3.7936 \frac{W}{m^2K} \quad \text{(note: similar to } h_2\text{)}$$

$$R_{comb} = \frac{1}{(h_2 + h_{rad})A} = \frac{1}{(8.7936)(3.14159)} = \underline{\cancel{0.029977}}\ \frac{K}{W}$$
$$0.036198$$

$$R_{TOTAL} = R_{cond,ins} + R_{comb}$$

$$= 0.79577 + \cancel{0.029977} = \cancel{0.82575} \frac{K}{W}$$
$$\qquad 0.036198 \qquad\qquad 0.83197$$

$$\dot{Q} = \frac{T_2 - T_{\infty,2}}{R_{TOTAL}} = \frac{-196°C - 20°C}{\cancel{0.82575} \frac{K}{W}} = \boxed{-260 \text{ W}}$$
$$0.83197$$

Check T_3 assumption

$$\dot{Q} = hA\,(T_3 - T_\infty)$$

$$-259.6\,W = 8.7936\,\frac{W}{m^2K} \cdot 3.14159\,m^2 \left(\overset{T_3}{\cancel{10°C}} - 20°C \right)$$

$$\Rightarrow T_3 - 20°C = -9.398\,°C \quad \Rightarrow T_3 = 10.6°C$$

close enough!

Note: any guess for T_3 between 5 – 15°C is acceptable.

$T_3 = 5°C$:

$$h_{rad} = 3.697\,\frac{W}{m^2K}$$

$$R_{Total} = 0.83237\,\frac{K}{W}$$

$$\dot{Q} = -259.5\,W$$

(−259.5 W. Very close to when $T_3 = 10°C$ is used)

$T_3 = 15°C$:

$$h_{rad} = 3.8923\,\frac{W}{m^2K}$$

$$R_{total} = 0.83157\,\frac{K}{W}$$

$$\dot{Q} = -259.8\,W$$

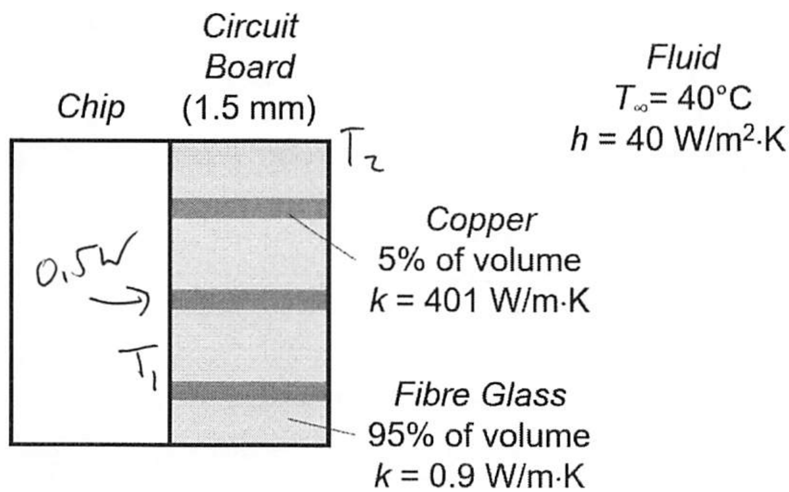
For evaporation, $\underbrace{\dot{Q}}_{\text{heat added to solution @ boiling pt}} = \dot{m}\,\Delta\hat{H}_{vap}$

$$\Rightarrow +259.6\,W = \dot{m}\left(198\,\frac{kJ}{kg}\right)\left(\frac{10^3\,J}{1\,kJ}\right) = +259.6\,\frac{J}{s}$$

$$\Rightarrow \boxed{\dot{m} = 0.00131\,\frac{kg}{s}} \quad \text{or} \quad 113\,\frac{kg}{day}.$$

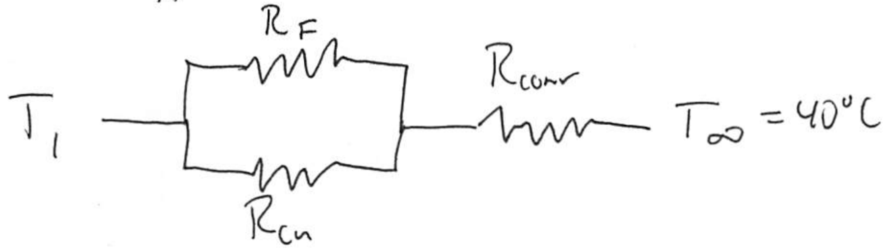
← a lot! Liq N_2 dewars use double wall vacuum to further limit evaporation.

Problem 2 (27 pts) Engineers are putting an electronic chip with 1 cm^2 area that consumes 0.5 W of power onto a circuit board with 1.5-mm thickness. The board is mostly fiber glass (k = 0.9 W/m·K), but 5% of its volume comprises copper vias (k = 401 W/m·K) that span the thickness of the board. Heat dissipates into air at 40 °C with a convection heat transfer coefficient of 40 W/m^2·K. The engineers chose not to use a heat sink.



What would the temperature of the chip/circuit board interface be (without a heat sink)?

Use resistance approach.



$A = 10^{-4}\ m^2$ $\quad A_F = 0.95\times10^{-4}\ m^2$ $\quad A_{cu} = 0.05\times10^{-4}\ m^2$

$L = 0.0015\ m$

$$R_F = \frac{0.0015}{(0.9)(0.95\times10^{-4})} = 17.544\ \frac{K}{W}$$

$$R_{cu} = \frac{0.0015}{(401)(0.05\times10^{-4})} = 0.7481\ \frac{K}{W}$$

$$\frac{1}{R_{cond}} = \frac{1}{17.544} + \frac{1}{0.7481} = 1.3937\frac{W}{K} \Rightarrow R_{cond} = 0.7175\ \frac{K}{W}$$

$$\frac{1}{hA} = \frac{1}{(40)(10^{-4})} = 250\ \frac{K}{W}$$

$$R_{Total} = R_{cond} + R_{conv}$$

$$= 0.7175\ \frac{K}{W} + 250\ \frac{K}{W}$$

$$= 250.7175\ \frac{K}{W}$$

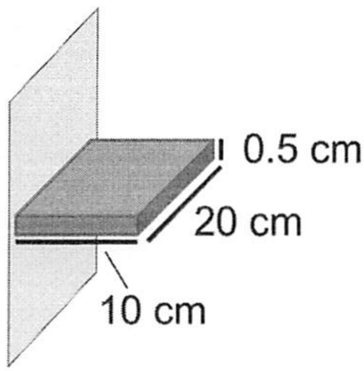
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{Tot}} \Rightarrow T_1 = \dot{Q} R_{Tot} + T_\infty$$

$$\Rightarrow T_1 = (0.5\,W)(250.7175\ \tfrac{K}{W}) + 40^\circ C$$

$$= \boxed{165^\circ C}$$

☆ Convective resistance was high! Should have used a heat sink...

Problem 3 (27 pts) A rectangular aluminum fin (k = 237 W/m·K) is 10 cm long and has horizontal dimensions of 20 cm and 0.5 cm. The convective heat transfer coefficient is 11 W/m²·K. The base temperature is 85 °C. The fluid temperature is 25 °C.



Determine the percent error in the rate of heat transfer from the fin when the infinitely long assumption is used instead of the (more accurate) assumption of adiabatic tip with corrected length.

We are looking for $\dfrac{\dot{Q}_{Fin,\,long} - \dot{Q}_{Fin,\,adiab\ corr}}{\dot{Q}_{Fin,\,adiab\ corr}}$

$$= \frac{\dot{Q}_{Fin,\,long}}{\dot{Q}_{adiab,\,corr}} - 1$$

$$= \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{\sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(aL} - 1$$

$$= \frac{1}{\tanh(aL)} - 1$$

$$P = (20\,cm)\times 2 + (0.5\,cm)\,2 = 41\,cm = 0.41\,m$$

$$a = \sqrt{\frac{hP}{kA_c}}$$

$$A_c = (20\,cm)(0.5\,cm) = 10\,cm^2 = 10^{-3}\,m^2$$

$$= \left(\frac{11 \frac{W}{m^2K} \cdot 0.41\,m}{237 \frac{W}{mK} \cdot 10^{-3}\,m^2} \right)^{1/2}$$

$$= 4.3623\ m^{-1}$$

$$\tanh(aL) = \tanh(0.4362) = 0.4105$$

$$\Rightarrow \frac{1}{\tanh(aL)} - 1 = \frac{1}{0.4105} - 1 = 1.436$$

$\Rightarrow$ 144% error when using inf. long assumption

Note: $$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh[a(L-x)]}{\cosh(aL)}$$

Consider $x = L$. $\cosh(0) = 1$

$\cosh(aL) = \cosh(0.4362) = 1.097$

$$\Rightarrow \frac{T(x) - T_\infty}{T_b - T_\infty} = 0.91$$

tip is still quite close to T_b.