University of Toronto

Faculty of Applied Science and Engineering

ESC194F Calculus I
Final Exam
December 2019

No calculators or aids
There are 12 questions, each question is worth 10 marks

Examiners: P.C. Stangeby and J.W. Davis

1) Evaluate the following limits:

a) $\lim_{x \to 0} (\csc x - \cot x)$

b) $\lim_{x \to 0} (1 - 2x)^{1/x}$

c) $\lim_{x \to 0} \frac{sin^{-1}x}{x}$

2) Find the derivative of: $3x^3$, $cos(3x)$, $ln(x^{1/2})$, e^{-x^2}, 3^{x^2}.

3) Find the anti-derivative of: $3x^3$, $cos(3x)$, xe^{-x^2}, $(9+x^2)^{-1}$, 3^x .

4) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 pm?

5) a) Find functions f and g such that each function is continuous at $x = 0$, but the composite function, $f \circ g$, is not continuous at 0.

b) What value of b maximizes the integral: $\int_{-1}^{b} x^2(3 - x)dx, b > -1$?

6) A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.

7) a) [**7 marks**] Provide a $\delta - \varepsilon$ proof that $\lim_{x \to 3} x^2 = 9$.

b) [**3 marks**] Given $f(x) = x^2$, $c = 3$, $\varepsilon = 7$, what is the largest δ that will ensure that when $0 < |x - c| < \delta$ then $|f(x) - f(c)| < \epsilon$? Is there a smallest δ ?

8) Find the solution of the differential equation that satisfies the given initial condition:

a) $y' tanx = a + y, \quad y\left(\frac{\pi}{3}\right) = a, \quad 0 < x < \frac{\pi}{2}$

b) $(x^2 + 1)y' + 3x(y - 1) = 0 \quad y(0) = 2$

9) Use the method of undetermined coefficients to find the general solution to the 2nd order DE:

$$y'' - 3y' + 2y = coshx = \tfrac{1}{2}(e^{x}+e^{-x})$$

10) Suppose $f'(x) < 0 < f''(x)$ for $x < a$ and $f'(x) > 0 > f''(x)$ for $x > a$. Prove that *f* is not differentiable at *a*.
Hint: Assume that *f* is differentiable at *a*, and apply the Mean Value Theorem.

11) a) Prove that $e^{\pi} > \pi^{e}$ by first finding the maximum value of $f(x) = \frac{lnx}{x}$.

b) Sketch a graph of $f(t) = e^{t}$ on an arbitrary interval $[a, b]$. Use the graph and compare areas of regions to prove that: $e^{(a+b)/2} < \frac{e^{b}-e^{a}}{b-a}$.

12) Directly calculate the limit of a Riemann sum to evaluate the area of the region between $f(x) = \sqrt{x},\ x \in [0,2]$ and the x-axis.

Hint 1: Use the non-uniform partition: $x_i = i^2 \frac{2}{n^2}$

Hint 2: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$