University of Toronto
Faculty of Applied Science and Engineering

Term Test I

Feb. 6, 2014
Duration: 90 minutes

ECE159 - Electric Circuit Fundamentals
Examiners: Ali Sheikholeslami and Li Qian

ANSWER QUESTIONS ON THESE SHEETS, USING THE BACKS IF NECESSARY.

WRITE IN PEN ONLY (NO PENCIL)!

1. Calculator type is restricted (no programmable calculators).
2. Weight for each question is indicated in [ ]. Attempt all questions, since a blank sheet will certainly get a zero.

Solution

maximum grade = 30

Last Name: ____________

First Name: ____________

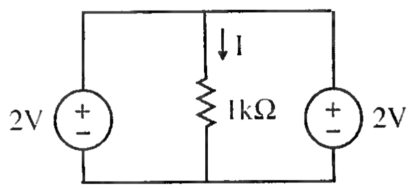
Student Number: ____________

Tutorial Section: ____________

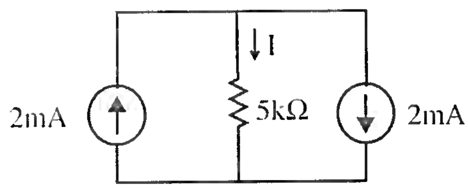
Question	Mark
1	........../6
2	........../6
3	........../6
4	........../8
5	........../4
Total	........../30

Q1. [6 marks]

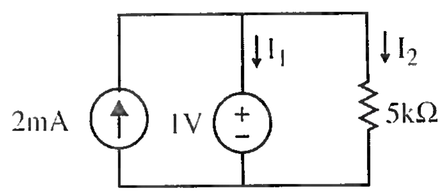
(a) [4 marks] Fill out the current values in the boxes below:



I = 2mA



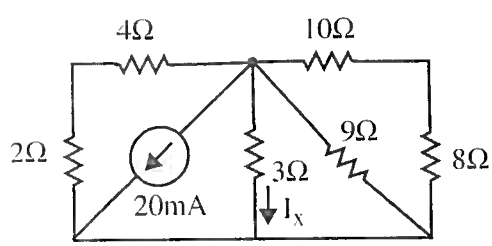
I = 0



I_1 = 1.8mA

I_2 = 0.2mA

(b) [2 marksFind current I_x as indicated below.



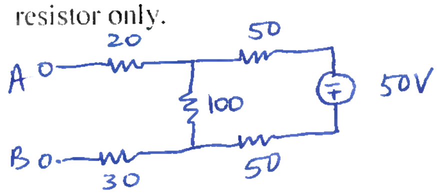
I_x = −10mA

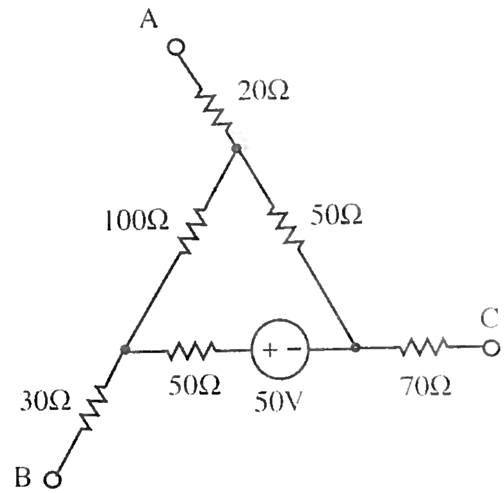
use current division

$$I_x = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{9}+\frac{1}{18}}(-20) = -10\text{ mA}$$

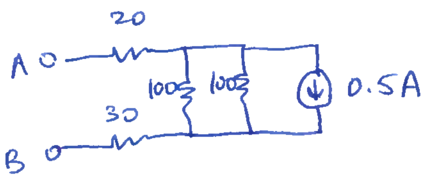
Q2 [6 marks]

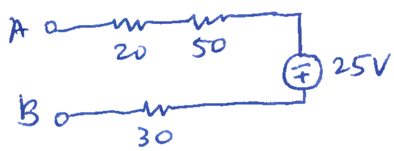
(a) [3 marks] For the circuit shown below, find an equivalent circuit between terminals A and B while terminal C is open. The equivalent circuit should consist of one ideal source and one resistor only.

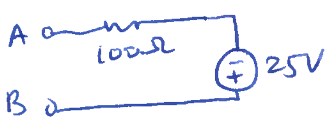




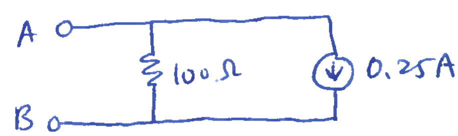
use source transformation:



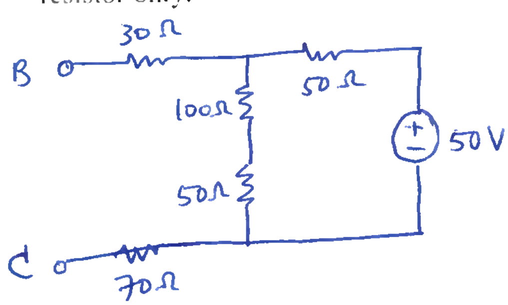




or



(b) [3 marks] For the circuit shown above, find an equivalent circuit between terminals B and C while terminal A is open. The equivalent circuit should consist of one ideal source and one resistor only.



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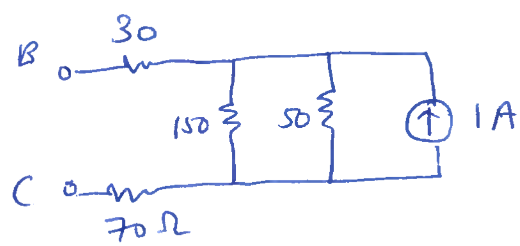

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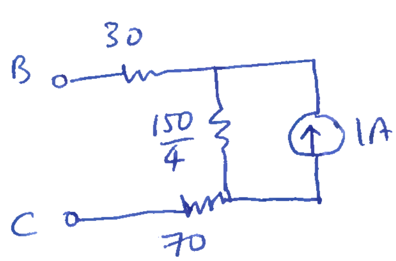

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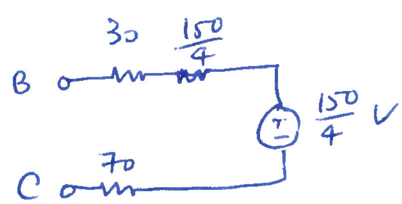

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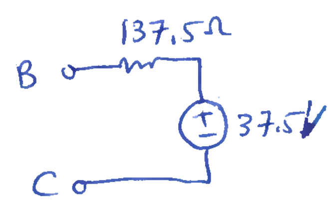

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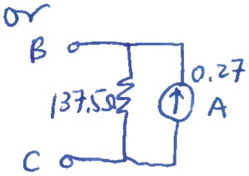
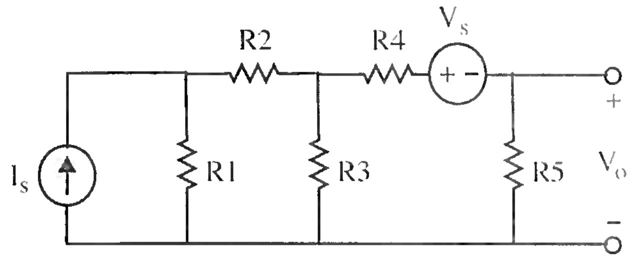


Q3 [6 marks]

(a) [3 marks] In the circuit shown below, the following measurements are obtained:



For $I_s = 2mA$ & $V_s = 3V$, $V_o = 7V$

For $I_s = 4mA$ & $V_s = 9V$, $V_o = 20V$

Find V_o when $I_s = 1mA$ and $V_s = 6V$

using linearity : $V_o = \alpha I_s + \beta V_s$

$$\Rightarrow \begin{cases} \alpha \cdot 2mA + \beta \cdot 3V = 7V \\ \alpha \cdot 4mA + \beta \cdot 9V = 20V \end{cases}$$

$$\Rightarrow \begin{cases} \alpha = 0.5k\Omega \\ \beta = 2 \end{cases}$$

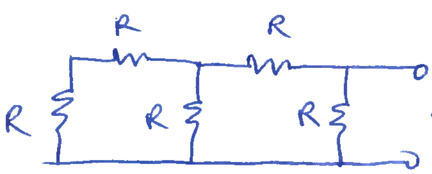
$$\Rightarrow V_o = 0.5k\Omega \cdot 1mA + 2 \cdot 6V = 12.5V$$

$V_o =$ 12.5V

(b) [3 marks] As in part (a), assume with $I_s = 2mA$ and $V_s = 3V$, we measure V_o=7V. If we now short the V_o terminals together, we will measure a short-circuit current of 10mA. Assuming all resistors (R1 to R5) are equal, i.e. R1 = R2 = R3 = R4 = R5 = R, calculate R.

Solution # 1 : Given open-circuit voltage $V_{oc} = 7V$, short-circuit current $I_{sc} = 10mA$ $\Big\}$ $R_{eq} = \dfrac{V_{oc}}{I_{sc}} = 0.7k\Omega$

With I_s & V_s turned off .

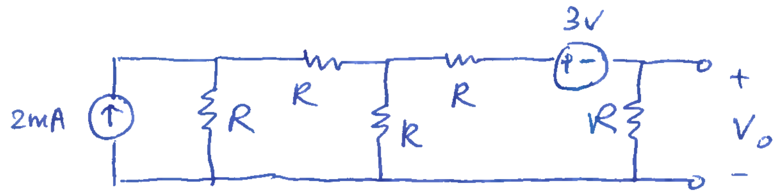


$\Leftarrow R_{eq} = (2R \parallel R + R) \parallel R = \frac{5}{3} R \parallel R = \frac{5}{8} R$

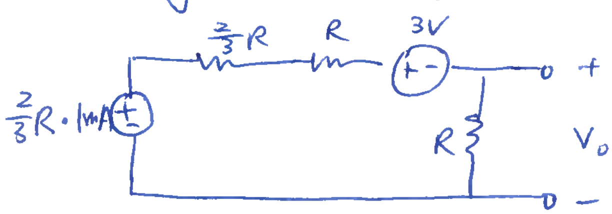
$$\therefore \frac{5}{8} R = 0.7k\Omega \Rightarrow R = \frac{5.6}{5} k\Omega = 1.12k\Omega$$

R = 1.12 kΩ

Solution #2. If one assumes all resistors have equal value R, and $I_s = 2mA$, $V_s = 3V$, Let's find R in order to produce $V_o = 7V$



Using source transformation, one can simplify the circuit to:



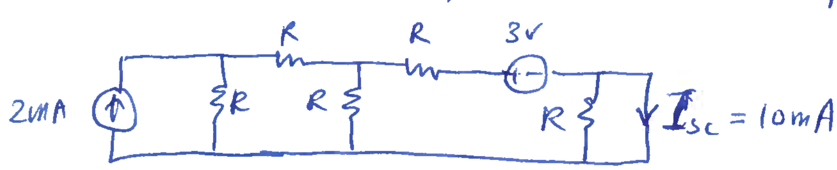
Then, use voltage division, we have

$$V_o = \frac{R}{R+\frac{2}{3}R+R}\left(\frac{2}{3}R\cdot 1mA - 3V\right) = 7V$$

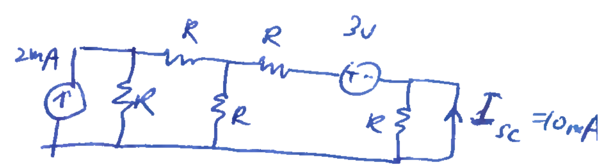
Solving for R, one gets $R = \underline{\underline{32.5\,k\Omega}}$

(Note here the I_{sc} is <u>not</u> 10mA)

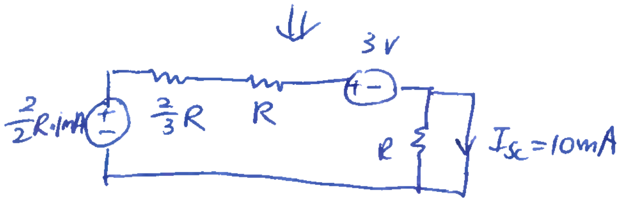
Solution #3: If one assumes all R's equal, and $I_s = 2mA$, $V_s = 3V$, let's find R in order to produce $I_{sc} = 10mA$



or



$\Downarrow$



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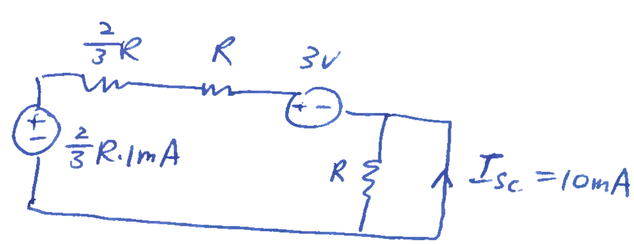


$R < 0$ imposible

$R = 0.17\,k\Omega$

(Note here the V_{oc} cannot be 7V)

Any one of the above solutions will get full marks for the question

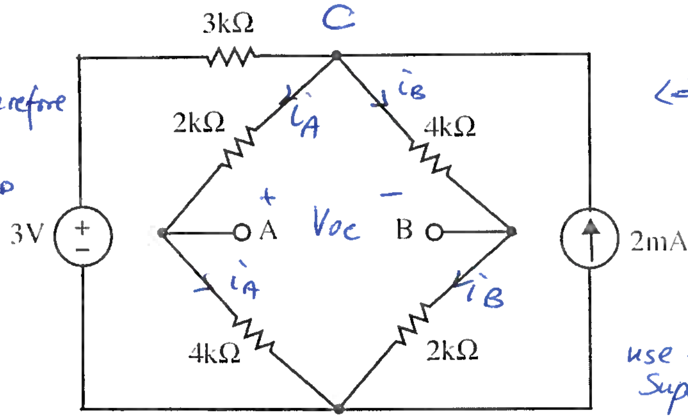
Q4. [8 marks] In the circuit shown below:

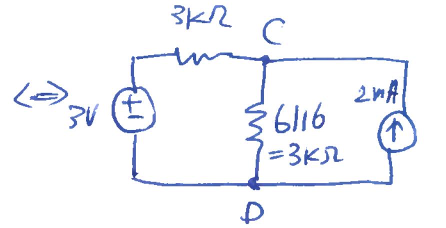
Note: i_A & i_B are the same. Therefore

$V_{AB} = V_{oc} = \frac{4}{6} V_{CD} - \frac{2}{6} V_{CD}$

$= \frac{1}{3} V_{CD}$

$= \frac{1}{3} \cdot 4.5V = 1.5V$





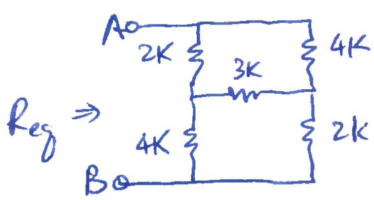
$V_{CD} = \frac{1}{2} \cdot 3V + 1mA \cdot 3k\Omega$

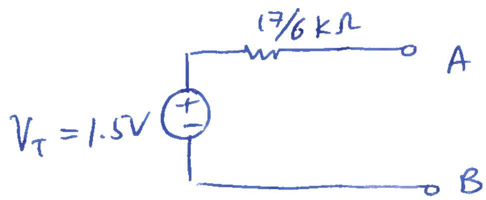
use Superposition

$= 1.5 + 3 = 4.5V$

(a) [4 marks] Find the Thevenin equivalent circuit (V_T and R_T) between terminals A and B.

$V_{oc} = 1.5V$





$Req = \frac{17}{6} k\Omega$

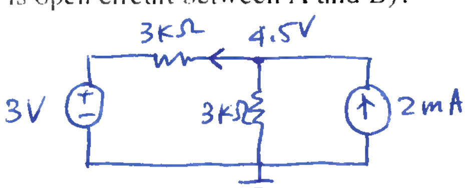
V_T = $1.5V$	R_T = $\frac{17}{6} k\Omega$

(b) [2 marks] If a variable load resistor is connected between terminals A and B, what is the maximum power that can be transferred to this load?

$$P_{max} = \frac{V_T^2}{4R_T} = \frac{1.5^2}{4 \cdot \frac{17}{6}} = 0.199\,mW \approx 0.2\,mW$$

P_{max} = $0.2\,mW$

(c) [2 marks] How much power is generated by each source when there is no load (i.e. when there is open circuit between A and B)?



$P_1 = 3V \cdot 0.5mA = 1.5mW$

$P_2 = 4.5(-2mA) = -9mW$

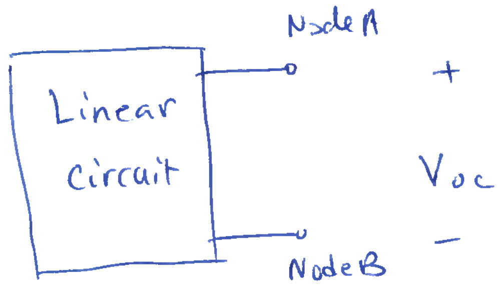
$i = \frac{4.5 - 3}{3} = 0.5mA$

p (by 3V Supply) = $1.5mW$	p (by 2mA Supply) = $-9mW$

+ve means power consumed

−ve means power supplied.

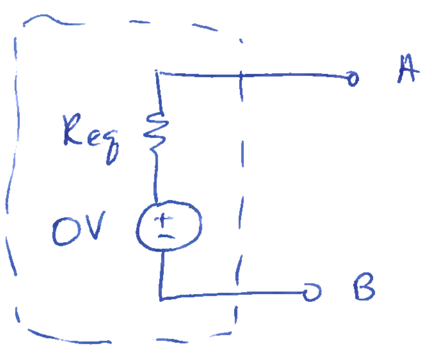
Q5 [4 marks] In a linear circuit that includes resistors and independent voltage and current sources only, we measure the voltages of two distinct nodes (A & B) to be identical. If we connect these two nodes using a short circuit, prove that the current in the short circuit will be zero. (Hint: Use Thevenin Equivalent Circuit Theorem).



$V_A = V_B$

$\therefore V_{oc} = 0$

$\therefore$ The equivalent circuit for the above is



Assume $R_{eq} \neq 0$

Therefore, no current will flow if A & B are short-circuited because $V_T = 0\,V$