CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

23rd NOVEMBER 2015

Q.1. [17 points] HOT SPOT IN A HEATED FLUID

Consider a sphere of radius *a* whose surface is maintained at a uniform temperature T_0. The sphere is placed in a fluid medium that is at a constant temperature T_0 far away from the sphere. The thermal conductivity of the fluid is k_f. The fluid is stagnant everywhere. If convection effects and radiation can be ignored, conduction is the only mechanism for heat transfer from the sphere surface into the fluid. Now, present in the fluid is a heat source that releases heat according to the relationship, $\dot{S} = \dot{S}_0 \dfrac{a^4}{r^4}$, where $\dot{S}_0$ has units of W/m^3. As can be seen, the heat source term depends on the radial position. It is largest at the sphere surface, and decays away rapidly far away from the sphere according to a $1/r^4$ relationship.

Answer the following questions:

(a) **[10 points]** Beginning from the energy conservation equation in the spherical co-ordinate system [see last page], determine the *steady-state* temperature distribution *in the fluid*. Specify the governing equations and boundary conditions clearly. Note that the domain for the governing equation will be $a \le r < \infty$, so boundary conditions have to be applied at $r = a$ and for $r \to \infty$.

(b) **[3 points]** Determine the radial heat flux, $\dot{q}_r$, at the surface of the sphere, $r = a$. Is heat being lost to the ambient or being gained from the ambient?

(c) **[4 points]** Determine if there can be a hot spot in the fluid, i.e., if there is a radial position where the temperature profile shows a maximum. Find the radial location of the hot spot. What is the temperature at the hot spot?

2. [15 points] HEAT LOSSES FROM A STEAM PIPE

A 10 cm OD by 6 cm ID *cylindrical* steam line delivers superheated steam at 1000^{o}C. The line is steel wrapped with 10 cm thick cylindrical shell of asbestos, and 1 cm of cylindrical plaster shell over the asbestos. The thermal conductivities of steel, asbestos and plaster are 14 W/m-K, 0.156 W/m-K and 0.107 W/m-K, respectively. The convective heat transfer coefficients for the steam side and the air side are 2500 W/m^2 and 7 W/m^2. Using the thermal resistance approach, determine the rate of heat loss per unit length of the cylindrical steam line at steady state. Account for radiative losses from the surface as well, assuming a plaster emissivity of 0.85, and that the 'surrounding' surface receiving the radiation to be at a temperature of 30^{o}C. The ambient air temperature is also 30^{o}C. The Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4.

Note: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

3. [18 points] COMPOSITE MATERIAL

<u>Use the resistance network approach to solve this problem (see table on page 6)</u>.

In class, you have always consider isotropic materials with a constant thermal conductivity, irrespective of the direction in which temperature gradient is applied. However, this is not always true, particularly for composite materials. Consider a slab of a composite material made of alternating planar layers of two different materials A and B in the x-direction of thicknesses W_A and W_B, and thermal conductivities k_A and k_B, respectively (see figure to the left). The total width of the material in the x direction is W [with $W = n\ (W_A+W_B)$, where n is the number of pairs of the A-B layers], and the height in the y-direction is H. The depth into the plane of the paper is D.

Answer the following questions:

(a) [6 points] If a temperature difference is imposed across the material in the x direction, with the face at $x = 0$ maintained at a temperature T_1, and the face at $x = W$ maintained at a temperature T_2, determine the total rate of heat transfer by conduction in the x-direction through the slab. Assume that there is no temperature variation in the y direction.

(b) [2 points] Equate the rate of heat transfer obtained in part (a) to a slab of effective thermal conductivity k_x in the x-direction. Hence determine k_x.

(c) [6 points] If a temperature difference is imposed across the material in the y direction, with the face at $y = 0$ maintained at a temperature T_1, and the face at $x = H$ maintained at a temperature T_2, determine the total rate of heat transfer by conduction in the y-direction through the slab. Assume that there is no temperature variation in the x direction.

(d) [2 points] Equate the rate of heat transfer obtained in part (c) to a slab of effective thermal conductivity k_y in the y-direction. Hence determine k_y.

(e) [2 points] If $W_A = W_B$, which of k_x or k_y is greater?

<u>Bonus</u> [3 points]: For isotropic materials, Fourier's first law of heat conduction is written as $\vec{q} = -k\vec{\nabla}T$, and therefore, the conductive heat flux is always parallel to the direction of the temperature gradient. How should Fourier's law of heat conduction be written for this composite slab? Will the direction of heat transfer always be parallel to the direction of the gradient?