PHY294, Winter 2023, Solutions for QUIZ III Monday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ___________

I. The methane molecule, CH_4, has the shape of a tetrahedron, with the carbon atom in the middle. Enumerate the degrees of freedom and use the equipartition theorem to find the energy and the constant-volume heat capacity of an ideal gas of N methane molecules, assuming that all degrees of freedom are thermally activated.

For the 5-atomic nonlinear molecule, there are 3 *translational,* 3 *rotational and* $3 \times 5 - 6 = 9$ *vibrational degrees of freedom (for a total of* $3 \times 5 = 15$*).*

The energy of the gas is $U = N\frac{kT}{2}(3 + 3 + 9 \times 2) = 12NkT$. *Thus* $C_V = 12Nk$.

7 points

II. Two ideal monatomic gases are placed in thermal contact. One gas has twice the number of particles than the other, $N_2 = 2N_1$, but has half the volume, i.e. $V_2 = V_1/2$. In addition, gas 1 is composed of single atoms, while gas 2 is composed of diatomic molecules. The temperature is high enough so that all degrees of freedom are in thermal equilibrium. Let the energy of the first gas be $U_1 = U - x$ and of the second $U_2 = U + x$. What values of x are found to occur in thermal equilibrium?

The volume information is not relevant for the question asked, as for ideal gases the energy does not depend on V.

We have that $U_1 = N_1\frac{3}{2}kT$ *and* $U_1 = 2N_1\frac{7}{2}kT = N_1 7kT$ *(this is because the diatomic molecule has* 2 *rotations,* 3 *translations and* 1 *vibration, for a total of* $(7/2)kT$ *energy per particle; the gases have the same* T, *of course).*

From what we are given, we have that $U_2 - U_1 = 2x$ *and we need to find* x. *From the above,* $x = \frac{U_2 - U_1}{2} = N_1kT(7 - 3/2)/2 = N_1kT(11/2)/2 = N_1kT11/4$

10 points

III. An ideal gas of nonlinear triatomic molecules is initially placed in one half of a volume, isolated from the entire world. The other half of the volume is separated by a partition and is empty. What is the work done by the gas as it fills the entire volume after the partition is quickly removed? Is this a quasistatic process?

No work is done in this adiabatic free (nonquasistatic) expansion. The gas is in equilibrium only at the beginning and the end of the process.

8 points

Total number of points: 7+10+8=25.

PHY294, Winter 2023, Solutions QUIZ III Tuesday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ____________

I. The ammonia molecule, NH_3, has the shape of a tetrahedron. Enumerate its various degrees of freedom and use the equipartition theorem to find the energy of an ideal gas of N ammonia molecules and its constant-volume heat capacity, assuming that all degrees of freedom are thermally activated.

For the 4-atomic nonlinear molecule, there are 3 *translational,* 3 *rotational and* $3 \times 4 - 6 = 6$ *vibrational degrees of freedom (for a total of* $3 \times 4 = 12$*).*

The energy of the gas is $U = N\frac{kT}{2}(3 + 3 + 6 \times 2) = 9NkT$*. Thus* $C_V = 9Nk$*.*

7 points

II. Two ideal monatomic gases are placed in thermal contact and let to reach thermal equilibrium. One gas has three times the number of particles than the other: $N_1 = N$ and $N_2 = 3N$. The gases occupy volumes V_1 and $V_2 = V_1/2$, respectively, and are thermally isolated from the rest of the world. Let the energy of the first gas be $U_1 = U - x$ and of the second $U_2 = U + x$. What values of x are found to occur in thermal equilibrium? What is the ratio of the pressures of the two gases p_1/p_2?

The energies are $U_1 = 3NkT/2$ *and* $U_2 = 9NkT/2$*. Thus, since* $U_2 - U_1 = 2x$*, we have that* $x = \frac{9-3}{2}\frac{kT}{2}$*. Thus,* $x = \frac{3kT}{2}$*.*

The pressures are $p_1 = NkT/V_1$ *and* $p_2 = 3NkT/(V_1/2) = 6NkT/V_1$*. Tus* $p_1/p_2 = 1/6$*.*

10 points

PHY294, Winter 2023, Solutions QUIZ III Wednesday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ___________________; **Student #**: ___________________; **Tutorial group**: __________

II. The water molecule, H_2O, has an equilateral triangle shape. Enumerate the degrees of freedom and use the equipartition theorem to find the energy and the constant-volume heat capacity of an ideal gas of N water molecules, assuming that all degrees of freedom are thermally activated.

This triatomic molecule has 3 *translational,* 3 *rotational and* $3 \times 3 - 6 = 3$ *vibrational degrees of freedom.*

From equipartition we have $U = N\frac{kT}{2}(3 + 3 + 3 \times 2) = 6NkT$*, giving* $C_V = 6Nk$*.*

7 points

II. Two ideal monatomic gases are placed in thermal contact and let to reach thermal equilibrium. One gas has volume V_1 and the other has volume $V_2 = 3V_1$, three times as big. The gases have the same number of molecules $N_1 = N_2 = N$. The molecules of the first gas have mass m_1 while those of the second have mass $m_2 = 3m_1$, i.e. are three times heavier (let's say, they have three times as many nucleons). Let the pressure of the first gas be p_1. What is the pressure of the second gas in terms of p_1? What is the ratio of the average thermal speeds v_i, $i = 1, 2$ of the molecules in the two gases? In other words, find $\frac{v_1}{v_2}$.

The pressure of the first gas is $p_1 = NkT/V_1$ *and of the second,* $p_2 = NkT/(3V_1) = p_1/3$*.*

Molecules at the same temperature are moving with average speeds $v = \sqrt{\frac{3kT}{m}}$ *(using the r.m.s. speed defined via the equipartition theorem; the scaling with* m *does not depend on precisely which average is used). Thus, molecules which are 3 times heavier will move* $\sqrt{3}$ *times slower,* $v_2/v_1 = \sqrt{m_1/m_2} = \frac{1}{\sqrt{3}}$*.*

10 points

PHY294, Winter 2023, Solutions Solutions QUIZ III Thursday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ___________

I. The carbon dioxide molecule, CO_2, has a linear shape $O - C - O$. Enumerate the degrees of freedom and use the equipartition theorem to find the energy and the constant-volume heat capacity of a ideal gas of N CO_2 molecules, assuming that all degrees of freedom are thermally activated.

This linear molecule has 3 *translational,* 2 *rotational and* $3\times 3-5 = 4$ *vibrational degrees of freedom (so that the total is* $3 \times 3 = 9$*).*

Thus, by equipartition , the energy is $U = N\frac{kT}{2}(3 + 2 + 4 \times 2) = N\frac{kT}{2}13$, *giving* $C_V = \frac{13Nk}{2}$.

7 points

II. Two single-atomic ideal gases are placed in two parts of a volume $2V$, separated by a rigid partition. The partition conducts heat and allows the gases to come in thermal equilibrium. The container of volume $2V$ is thermally isolated from the rest of the world. Gas 1 has N atoms and gas 2 has $4N$ atoms.

Initially, the two gases occupy equal volumes $V_1^{initial} = V_2^{initial} = V$. The partition is kept in place, say by some locking mechanism, so that the gases are allowed to reach thermal equilibrium. After this, the lock is released and the partition is now allowed to slide (like a piston, with negligible friction). The partition will slide until the pressures on its two sides equilibrate and a new thermal equilibrium state is established. In this new equilibrium state, what are the final values of the volumes V_1^{final} and V_2^{final} in terms of V? Do the energies of the gases in the final equilibrium state differ from their initial values? Is there energy flow during this process? If so, describe qualitatively the energy (heat and work) flow during this process? For your answer, does it matter if the partition slided slowly or not? (*Only a qualitative discussion of the energy flow is needed, no calculations.*)

Since the gases are initially in thermal equilibrium, they have the same temperature. Since they are ideal, their energies are determined by the temperature and the number of particles; so the total initial energy is $U^{initial} = 3/2kT(N + 4N)$. *Since the system is isolated,* $U^{final} = U^{initial}$ *and since the number of particles in each gas remains the same, it must be that the final equilibrium temperature equals the initial one.*

The initial pressures are $p_1^{initial} = NkT/V$ *and* $p_2^{initial} = 4NkT/V$. *So, clearly, after release, the partition will move so that the volume of gas* 1 *decreases and the volume of gas* 2 *increases. In the final equilibrium state the pressures have to be equal. Since the temperatures are equal as well, we have* $p_1^{final} = NkT/V_1^{final} = 4NkT/V_2^{final} = p_2^{final}$. *Thus, we have that* $V_1^{final} = V_2^{final}/4$. *Since* $V_1^{final} + V_2^{final} = 2V$, *this implies* $V_1^{final} = 2V/5$ *and* $V_2^{final} = 8V/5$.

As already discussed, the energy (and temperature) of each of the gases in the final equilibrium state is equal to that of the initial state. However, gas 2 *does work to move the partition and hence loses energy, while gas* 1 *gains energy; so there must be some heat flow from gas* 1 *to gas* 2 *during the* *process to make sure the final energies of the gases do not change. For this qualitative description, it does not matter if the partition moves slowly or not. (We could only calculate the details if the process was quasistatic, i.e. the wall moved slowly enough. This is left for another day...)*

11 points

III. In a gas containing H_2 and O_2 molecules at thermal equilibrium at some temperature T, which atoms are moving faster, on average? How much faster, i.e., 30 times, or 100 times?

Recall that each molecule has two ^1H or two ^{16}O atoms, respectively, where 1 and 16 denote the total number of protons and neutrons that the nucleus is made of, and that the mass of an atom is essentially given by the total mass of the nucleus.

The r.m.s. velocity (a measure of the average speed of the molecules in thermal equi- librium) is inversely proportional to the square root of the mass, thus lighter molecules move faster. The H_2 molecules are 16 times lighter than the O_2 ones, so they will move $\sqrt{16} = 4$ times faster.

7 points

Total number of points: 7+11+7=25.

III. An ideal monatomic gas is initially placed in one half of a volume, whose other half is separated by a partition and is empty. The gas is in thermal contact with a thermal reservoir, kept at fixed temperature T. The partition is slowly moved and the gas is let to occupy the entire doubled volume, remaining at any moment in equilibrium with the reservoir. What is the energy of the gas after the expansion is over? Does the gas do any work? Is there any heat transferred in the process? *(No equations and calculation needed, just describe in words and explain your reasoning.)*

There is work done by the gas in this isothermal expansion of the gas. However, due to the presence of the reservoir, the energy of the ideal gas remains the same, as its temperature does not change and its energy only depends on T. To balance energy, there is an amount of heat coming from the reservoir, exactly equal to the work done.

8 points

Total number of points: 7+10+8=25.

III. An ideal monatomic gas is initially placed in one half of a volume, isolated from the rest of the world. The other half of the voulme is separated by a partition and is empty. What is the work done by the gas as it fills the entire volume after the partition is quickly removed? Contrast this with the case that the partition is slowly moved to allow the gas to occupy the volume. How do the final values of the energies of the gas differ between the two cases? Which of the two processes can be considered quasistatic? (*No calculations are needed but only qualitative logical explanations.*)

In the free expansion (the quickly removed partition) there is no work done. In the case of the slow expansion, the gas does work and loses energy. The first process is a very nonequilibrium one (but if the gas is ideal, we can calculate the properties of the final equilibrium state) while the second can be quasistatic if the partition moves slowly enough.

8 points

Total number of points: 7+10+8=25.