**PHY294 Quantum Quiz #2 (2017)**

**Name (Last, First):** Soln (ver. 2)
**Signature:**
**Student ID:**
**Tutorial Section:**

- **4 questions, 25 minutes.**
- **Closed book, closed notes and no calculators.**
- **Please write only in the Quiz paper (double-sided).**

**1.** Consider a wave packet of non-relativistic free electrons in $k$-space at $t=0$: $A(k) = C\, e^{-(k-k_0)^2 \varepsilon^2} e^{-ikx_0}$
$C$, $\varepsilon$, $k_0$, $x_0$ are constants. Without Fourier transforming $A(k)$, describe this wave packet in $x$-space at $t \geq 0$.

- Gaussian transforms into Gaussian, with inverse width
- An offset transforms into a phase factor, and vice-versa

$\Rightarrow \psi(x)$ is a Gaussian of width $\sim\varepsilon$, initially centered at $x_0$ and moves with momentum $+\hbar k_0$ for $t>0$, spreading as it moves $\because k \sim \omega^2$ ($E \sim p^2$)

i.e. component plane waves disperse.

**2.** A particle of mass $m$ is in an infinite-potential spherical well of radius $r_0$. Assume the kinetic energy to be all radial, such that the radial Schrodinger equation has solutions of the form $R(r) \sim \sin(kr)/r$. Determine the allowed values of $k$ to find the energy eigenvalues, and show how to normalize $R(r)$.

$u(r) \equiv r R(r) = A \sin kr$

$A$ ← constant

B.C.: $u(r_0) = 0 \Rightarrow kr_0 = n\pi$, $n = 1, 2, 3 \ldots$

Radial Equation: $-\frac{\hbar^2}{2m}\frac{d^2 u(r)}{dr^2} = E\,u(r) \Rightarrow E = \frac{\hbar^2 k^2}{2m} = \frac{n^2 h^2}{8 m r_0^2}$

for $r \leq r_0$

Normalization: $1 = \int_0^{r_0} \underbrace{A^2 r^2 \frac{\sin^2 kr}{r^2}}_{|u|^2 = |rR|^2} dr \Rightarrow A = \sqrt{\frac{2}{r_0}}$

**3.** A hydrogen atom is in a superposition state: $\psi = \frac{1}{\sqrt{2}}\psi_{3,2,0} + \frac{1}{\sqrt{3}}\psi_{3,2,-1} - \frac{1}{\sqrt{6}}\psi_{3,2,2}$ [subscripts denote $n, l, m_l$]. Calculate the expectation value for the $z$-component of angular momentum $\langle L_z \rangle$.

$$\langle L_z \rangle = \int_{\text{All space}} dV\, \psi^* \hat{L}_z \psi \qquad \hat{L}_z \psi = m_\ell \hbar \psi$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 \left[0\cdot\hbar \underbrace{\int |\psi_{320}|^2 dV}_{=1}\right] + \left(\frac{1}{\sqrt{3}}\right)^2 \left[-1\cdot\hbar \underbrace{\int |\psi_{321}|^2 dV}_{=1}\right] + \left(\frac{1}{\sqrt{6}}\right)^2 \left[2\cdot\hbar \underbrace{\int |\psi_{322}|^2 dV}_{=1}\right]$$

All other terms vanish ∵ of orthogonality

$$= \left(\frac{0}{2} - \frac{1}{3} + \frac{2}{6}\right)\hbar = 0$$

**4.** Imagine a novel type of particles called *tripleons* (mass = $m_t$), up to three of which can have the same wave function. If 16 *tripleons* are in a 3D infinite-potential cubic well of edge length $d$, what is the minimum total energy of this system? [Note: neglect spin and any interactions.]

$$E = \frac{h^2}{8md^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

| $E\left[\frac{h^2}{8md^2}\right]$ | $n_x$ | $n_y$ | $n_z$ | Degeneracy | # particle |
|---|---|---|---|---|---|
| 3 | 1 | 1 | 1 | 3 | 3 |
| 6 | 1 | 1 | 2 | 3 | 9 |
| | 1 | 2 | 1 | 3 | |
| | 2 | 1 | 1 | 3 | |
| 9 | 1 | 2 | 2 | 3 | any 4 |
| | 2 | 1 | 2 | 3 | |
| | 2 | 2 | 1 | 3 | |

$$\therefore E_{min} = \frac{h^2}{8md^2}\underbrace{\left(3\cdot 3 + 9\cdot 6 + 4\cdot 9\right)}_{99}$$