Q4: (all parts are related)

Suppose you have a matrix A that is $3x4$ and you know the solution to:

$$A\vec{x} = A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \vec{0}$$

is given by:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$$

where x_3 is a free variable.

a) What is the rank of matrix A? You must explain your answer.

b) What is the reduced normal form of matrix A? You must show how you arrive at your answer.

c) How do you know that $A\vec{x} = \vec{b}$ can be solved for all values of $\vec{b}$?

Solutions:

a) There are four variables in total and one free variable in the solution. Therefore, the rank of matrix A is $4 - 1 = 3$ (the # of leading variables).

b) The three equations associated with the leading variables are given by:

$x_1 - 2x_3 = 0$

$x_2 - 3x_3 = 0$

$x_4 = 0$

Therefore, the RNF of matrix A is given by:

$$R = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) $\left[A|\vec{b}\right] \rightarrow \left[R|\vec{d}\right]$

Regardless of the value of $\vec{b}$, the RNF of the augmented matrix will be full rank (3) with one free variable, i.e., infinite solutions. Therefore, $A\vec{x} = \vec{b}$ can be solved for all values of $\vec{b}$.