#### UNIVERSITY OF TORONTO

# Faculty of Applied Science and Engineering

# Term Test III

First Year — Program 5

# MAT1854115 — Linear Algebra

Examiners: A D Rennet & G M T D'Eleuterio

18 March 2014

Student Name:			
	Last Name	First Names	
Student Number:		Tutorial Section:	TUT

### **Instructions:**

- 1. Attempt all questions.
- The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
- **3.** Write the final answers *only* in the boxed space provided for each question.
- 4. No aid is permitted.
- **5.** The duration of this test is 90 minutes.
- **6.** There are 11 pages and 5 questions in this test paper.

For Markers Only			
Question	Value	Mark	
	Α		
<b>1</b> 10			
	В		
2	10		
	С		
3	10		
4	10		
5	10		
Total	50		

## A. Definitions and Statements

Fill in the blanks.

1 (a).	The <i>column space</i> of $\mathbf{A} \in {}^m\mathbb{R}^n$ is	
		/2
1(b).	State a condition for $\mathbf{A} \in {}^m\mathbb{R}^n$ to have a <i>left inverse</i> .	
		/2
1(c).	How are the dimensions of the row space and column space of a given material related?	rix
		/2
1(d).	A basis for the row space of $\mathbf{A} \in {}^m\mathbb{R}^n$ can be determined by	, ,
		/2
1(e).	The <i>coordinates</i> of $v \in \mathcal{V}$ relative to the basis $E = \{e_1 \dots e_n\}$ are	
		/2

#### B. True or False

Determine if the following statements are true or false and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.

2(a).	The rank of a matrix is equal to the number of its nonzero rows.	

**2(b).** If 
$$\mathbf{A} \in {}^m\mathbb{R}$$
 and rank  $\mathbf{A} = n$ , then there exists  $\mathbf{B} \in {}^n\mathbb{R}^m$  such that  $\mathbf{B}\mathbf{A} = \mathbf{1}$ .

**2(c).** If 
$$c \in \mathbb{R}$$
,  $c \neq 0$  and  $\mathbf{A} \in {}^{2}\mathbb{R}^{2}$ , then rank  $(c\mathbf{A}) = \operatorname{rank} \mathbf{A}$ .

**2(d).** If 
$$\mathbf{A} \in {}^n\mathbb{R}^n$$
, then  $\dim \operatorname{im} \mathbf{A} + \dim \operatorname{null} \mathbf{A} = n$ .

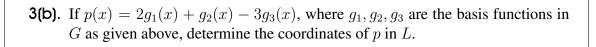
**2(e).** If 
$$\mathbf{A} \in {}^n\mathbb{R}^n$$
, then null  $\mathbf{A} \subseteq \operatorname{col} \mathbf{A}$ .

#### C. Problems

- **3.** The first three Legendre polynomials are  $L = \{1, x, \frac{1}{2}(3x^2 1)\}$  and the first three Gegenbauer polynomials are  $G = \{1, 2x, 4x^2 1\}$ . Note that L and G are bases for  $\mathbb{P}_2$ .
  - (a) Determine the transformation (transition) matrix from G to L.
  - (b) If  $p(x) = 2g_1(x) + g_2(x) 3g_3(x)$ , where  $g_1, g_2, g_3$  are the basis functions in G as given above, determine the coordinates of p in L.

<b>3(a)</b> . Determine the transformation (transition) matrix from $G$ to $L$ .	
	cont'd

3cont'd	
	/6



/4

<b>4.</b> Let $\mathbf{A} \in {}^m\mathbb{R}^n$ . Show that $\mathbf{A}\mathbf{x} = \mathbf{b}$ has no solution if and only if
$\mathrm{rank}\left[\mathbf{A} \mathbf{b}\right] = 1 + \mathrm{rank}\mathbf{A}$
cont's

4 cont'd	
/10	^
1/10	J

5.	Let $\mathcal V$ be a ve	ector space v	ith basis E	$C = \{e_1 \ldots$	$e_n$ }, let $\bullet$	$= \mathbf{Q}$	$[q_{ij}] \in$	$\mathbb{R}^n$	and	define
----	--------------------------	---------------	-------------	--------------------	------------------------	----------------	----------------	----------------	-----	--------

$$oldsymbol{f}_j = \sum_{i=1}^n q_{ij} oldsymbol{e}_i, \qquad j = 1 \dots n$$

Prove that  $F = \{f_1 \dots f_n\}$  is a basis for  $\mathcal V$  if and only if  $\mathbf Q$  is invertible.

...cont'd

5cont'd
/10
/ 10