PHY294, Winter 2017, QUIZ III.

Answer all questions on the exam paper. Duration: 25 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ___________

I. Acetylene molecules are linear chains that look like $H - C - C - H$. An ideal gas of N acetylene molecules is adiabatically expanding from an initial pressure p to a final pressure equal to one-half the initial one, $p_{fin.} = p/2$. Find the ratio of the final to initial volume, $V_{fin.}/V_{in.}$, explaining your reasoning.

You don't need to use calculators to find the number, an answer of the form, e.g. $20^{\frac{133}{17}+5}$ *is acceptable. None of the degrees of freedom are assumed to be "frozen out."*

SOLUTION: The adiabate equation says that $pV^\gamma = const.$, where γ is the adiabate exponent $\gamma = 1 + \frac{2}{f} = \frac{f+2}{f}$. Thus, we have $pV_{in.}^\gamma = \frac{p}{2} V_{fin.}^\gamma$, or $\frac{V_{fin.}}{V_{in}} = 2^{\frac{1}{\gamma}} = 2^{\frac{f}{2+f}}$. For a linear four-atomic molecule, we have, $f = 3 + 2 + (3 \times 4 - 5) \times 2 = 19$, including the 3 translations, 2 rotations, and 7 vibrations of this molecule. Thus $\frac{V_{fin.}}{V_{in}} = 2^{\frac{19}{2+19}} = 2^{\frac{19}{21}}$.

3 points

II. An ideal gas in thermal equilibrium is a mixture of H_2 (hydrogen) molecules and He (helium) atoms (atoms of hydrogen have one proton and of helium—two protons and two neutrons). What is the ratio of r.m.s. speeds of helium to hydrogen?

SOLUTION: The r.m.s. speed is $v = \sqrt{\frac{3kT}{m}}$, as follows from equipartition. The helium atoms are approximately two times heavier than the hydrogen molecules, thus they are moving $\sqrt{2}$ times *slower*, in a gas at the same temperature.

2 points

Turn over please ⟶

III. An ideal monatomic gas is initially placed in one half of an isolated volume, whose other half is separated by a partition and is empty. What is the work done by the gas as it fills the entire volume after the partition is quickly removed?

SOLUTION: The gas does zero work: there is no partition to push at and the molecules have no way of losing energy while spreading out (diffusing) to fill the entire volume. *2 points*

IV. The adiabate equation for the ideal gas is $pV^\gamma = const$, in $p - V$ "coordinates". Find the adiabate equation in $U - V$-coordinates.

SOLUTION: To find the adiabate equation in $U-V$ instead of $p-V$ coordinates, we have to replace p by U. We can do that in two steps. First, using the ideal gas law $pV = NkT$ and starting from the adiabate $pV^\gamma = const.$, we rewrite it as $pVV^{\gamma-1} = const$ or $NkTV^{\gamma-1} = const.$. Absorbing the Nk factor into the constant, we have $TV^{\gamma-1} = const.$. Finally we replace T by U using $U = \frac{f}{2}NkT$. Since $\frac{fNk}{2}$ is constant, by redefining the constant once more, we have $UV^{\gamma-1} = const.$ or $UV^{\frac{2}{f}} = const.$

3 points

Total number of points: 3 + 2 + 2 + 3 = 10.