(1.) Dimensional Analysis
[13 pts.]

Units of $[F] = kg\, m/s^2$

$h = kg\, m^2/s$

$A = 1/s$

$L = m$

Now $F = h^b A^c L^d$

Matching units:

$$kg\, m/s^2 = (kg\, m^2/s)^b\, (1/s)^c\, (m)^d$$

Three equations for b, c, d:

kg: $1 = b$

m: $1 = 2b + d$

s: $-2 = -b - c$

this idea: [8 pts]

Solve: $b = 1$
$c = 1$
$d = -1$

Rest of math: [5 pts.]

& thus $\boxed{F = \frac{hA}{L}}$

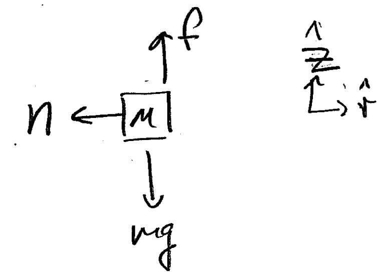
[What if guessed $F = hA/L$, & then showed units work, but did not solve 3 equations for b, c, d? Full credit.]

② [12 pts.]

Two ways to solve this problem: (I) In rest frame, or (II) in accelerating frame of block.

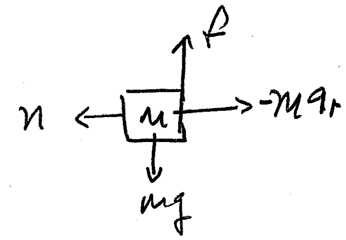
[3 pts: FBD]

(I) Rest frame:



$\sum F = m a_r$

(II) Block frame: with an inertial force



$\sum F = 0$

[3 pts.]: use $a = r\omega^2$

In either approach, find that radial equation gives $n = -m a_r$, and $a_r = -r\omega^2 \rightarrow \underline{n = m r \omega^2}$

{Can also write this $n = m v^2/r$, & eventually use $v = r\omega$, and $T = 2\pi/\omega$.}

In the z direction, force balance gives $\underline{f = mg}$.

[3 pts: / static friction upper bound]

Static friction, however, can only be as much as $\mu_s n$, so in this case, $\underline{f \le \mu_s m r \omega^2}$

[3 pts:] rest of algebra

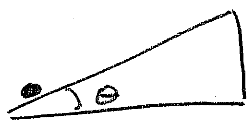
Combining with $f = mg$, we have $g \le \mu_s r \omega^2$ or $\omega \ge \sqrt{\dfrac{g}{\mu_s r}}$. This means $T = \dfrac{2\pi}{\omega} \le 2\pi\sqrt{\dfrac{\mu_s r}{g}}$.

Plug in #s: $T_{max} = 2\pi\sqrt{\dfrac{(0.40)(2.9\,m)}{(9.80\,m/s^2)}} = \boxed{2.2\,s}$

maximum period.

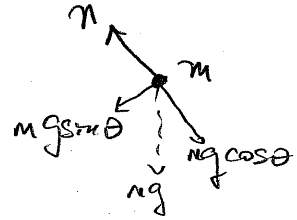
3. (P#6)

side view



$\downarrow g$

FBD



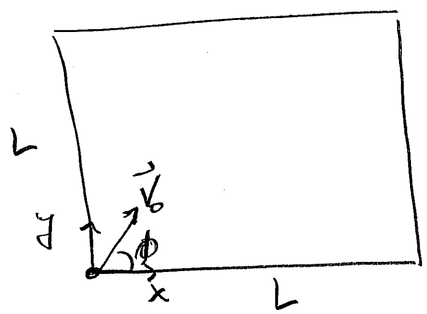
F=ma

$n = mg\cos\theta$

$a = g\sin\theta$

Setup of problem: [5pts.]

Define coordinates on surface of the board,



where $x_0 = y_0 = 0$

$V_{ox} = V_0\cos\phi \quad V_{oy} = V_0\sin\phi$

$a_y = -g\sin\theta \quad a_x = 0$

Now address the two parts of question:

Logic: [4pts]

(a) Maximum time?

Trajectories on the surface of the board are ballistic, with constant $a_y = -g\sin\theta$. The longest "time-of-flight" is any trajectory with the maximum height, $y = L$. The x component doesn't matter, so long as the particle doesn't slide off the left or right edge.

Intuitively, the max time is twice the time it would take to roll from the top edge down, $-L = \frac{1}{2}a_y t_{half}^2 \rightarrow t_{half} = \sqrt{2L/g\sin\theta}$

so that max time is $\boxed{\sqrt{8L/g\sin\theta}}$.

Plug in #s: $\boxed{t \simeq 2.6s}$

[(a) cont.]

[max V_{yo}: 3 pts]

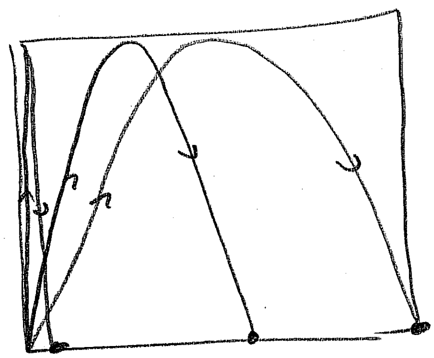
More directly, we solve this in two steps: first, find max V_{oy}; then, find the associated t_f for $y_f = 0$.

• Max V_{oy}: $V_f^2 = V_{oy}^2 + 2 a_y L = 0$ using $V_f^2 = V_i^2 + 2a_x(x_f - x_i)$

so $V_{oy}\Big|_{max} = \sqrt{2 g \sin\theta \, L}$ check units: $\sqrt{m/s^2 \cdot m} = m/s$ ✓

[finding time: 3 pts]

• Now what is t_f for any trajectory that goes up to $y = L$ & back to $y = 0$?



Ballistic flight: $V_{yf} = -V_{yo}$, so

$$V(t) = V_i + a_y t$$

$$-V_{yo} = V_{yo} - g\sin\theta \, t \rightarrow t = \frac{2V_{yo}}{g\sin\theta}$$

Substitute:

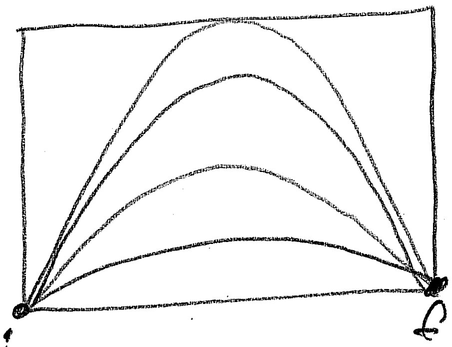
$$t\Big|_{max} = \frac{2}{g\sin\theta}\sqrt{2g\sin\theta \, L} = \sqrt{\frac{8L}{g\sin\theta}} = 2.6\,s$$

[no units? −1 pt.]

[wrong number, right approach? −1 pt.]

36. [12 pts.]

The trajectories that end up in other corner are



parabolic

this idea: [5pts.]

with $x_f = x_0 + V_{0x} t \Rightarrow L = V_{0x} t \rightarrow V_{0x} = \dfrac{L}{t_f}$

Now use vertical component to determine t_f:

$y_f = y_0 + V_{0y} t + \frac{1}{2} a_y t^2$ (or $V_f = -V_{0y}$)

$0 = 0 + V_{0y} t + \frac{1}{2}(-g \sin\theta) t$

$= t\left(V_{0y} - \frac{g t}{2} \sin\theta\right) \longrightarrow t_f = \dfrac{2 V_{0y}}{g \sin\theta}$

This idea, not constrained to max time: [3pts.]

Together, $V_{0x} = \dfrac{L}{t_f} = \dfrac{L g \sin\theta}{2 V_{0y}}$

Solve for V_0: [4pts.]

Solve for V_0: use $V_{0x} = V_0 \cos\phi$, $V_{0y} = V_0 \sin\phi$

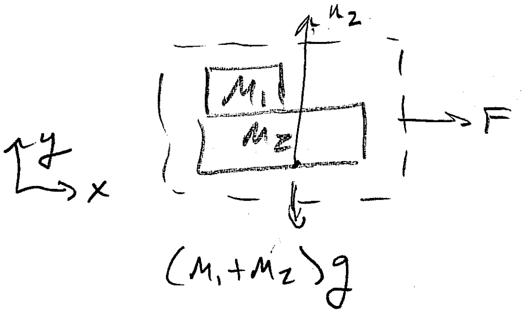
$\underbrace{2 V_{0x} V_{0y}}_{V_0^2 \sin 2\phi} = L g \sin\theta \rightarrow V_0^2 = \dfrac{L g \sin\theta}{\sin 2\phi}$

Plug in #s: $\boxed{V_0 = \dfrac{4.4 \text{ m/s}}{\sqrt{\sin 2\phi}}}$

or, if left as $\sin\phi\cos\phi$, $\boxed{V_0 = \dfrac{3.1 \text{ m/s}}{\sqrt{\sin\phi \cos\phi}}}$

(4.) a [6 pts.]

Internal forces don't matter if the blocks move together, so just need one FBD



$n_2 = (M_1+M_2)g$ since $a_y = 0$

$F = (M_1+M_2)a_x$

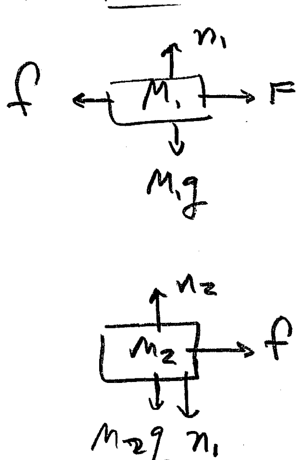
$\longrightarrow a_x = \dfrac{F}{M_1+M_2}$

[10 pts.]

(b) Now we need to consider the internal force, f = friction between blocks.

[4 pts. for FBD]

FBD's



F = ma

$n_1 = M_1 g$ since $a_y = 0$

$F - f = M_1 a_x$

$n_2 = M_2 g + n_1 \stackrel{\checkmark}{=} (M_1+M_2)g$ as in (a)

$f = M_2 a_x$

Notice that f accelerates block 2. The a_x here is the same as in (a).

[4pts.]

However the maximum f is $\mu_s n_1 = \mu_s M_1 g$

Since $f = M_2 a_x$, then $\underline{a_x \le \mu_s \frac{M_1}{M_2} g}$.

This means $\underline{F \le \mu_s \frac{M_1}{M_2}(M_1+M_2)g}$. [2pts. for rest of math.]

[5pts.]

(c.) If m_2 & m_1 don't move together, then the friction is kinetic, and $f = \mu_K n_1$. [2pts.]

[2pts.] We can still use the same FBD as in (b), which gave for block 1 that

$$F - f = m_1 a_x \quad \& \quad n_1 = m_1 g$$

$$\rightarrow a_x = \frac{F - \mu_K m_1 g}{m_1} = \underline{\frac{F}{m_1} - \mu_K g}$$

[1pt.] for rest of math.

(5.) The plot shows an acceleration $a = 1 - t/2$.
{You can solve (a) - (c) without this equation, but starting with (d) you need it.}

[4pts.]
(a) Looking at plot, $\int a\,dt = 0$, because area between curve & t axis sums to 0. We can also do the integral, $\int_0^4 (1-t/2)\,dt$

$$= \left[t - \frac{1}{4}t^2\right]_0^4 = \left(4 - \frac{1}{4}4^2\right) = 0.$$ A third approach is to remember $a_{avg} = \frac{\Delta V}{\Delta t}$, and find ΔV.

$$\Delta V = \int_0^t a\,dt' = \int_0^t (1 - t'/2)\,dt' = t - \frac{1}{4}t^2.$$ As before, $\Delta V = 0$.

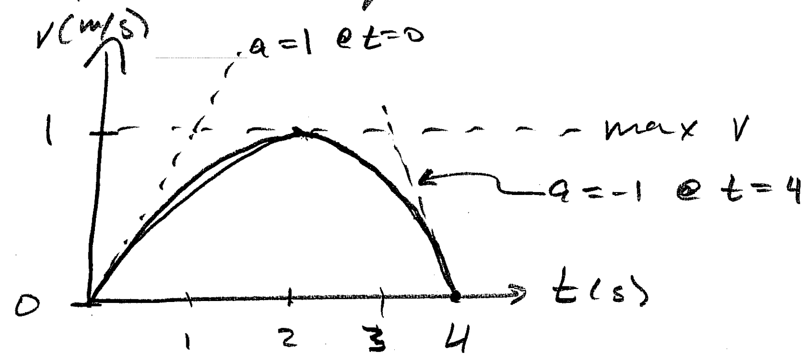
[5pts.]
(b) Using initial condition $V(0) = 0$, we can write

$V = \int a\,dt = t - \frac{1}{4}t^2 + C$, but $C = 0$ from i.c.

The maximum in $V(t)$ is when $\frac{dV}{dt} = 0$, which we could have found from the plot: $\boxed{t = 2.0s}$

Plug this into $V(t)$: $V(2.0s) = 2 - \frac{1}{4}(2)^2 = \boxed{1.0\,m/s}$

[5pts.]
(c) Since $a(t)$ linear, $V(t)$ is quadratic in time: a parabola.



[7pts.]

5(d) Integrate one more time to get $\Delta x = \int_0^t v\,dt$

$$\Delta x(t) = \int_0^t \left(t' - \tfrac{1}{4}t'^2\right) dt'$$

$$\boxed{\Delta x = \tfrac{1}{2}t^2 - \tfrac{1}{12}t^3}$$

Could write with "$\frac{1}{2.0}$" instead of $\frac{1}{2}$, or "0.50"; also $\frac{1}{12}$ can be "0.085".

[6pts.]

5(e) Recalling $v_{avg} = \frac{\Delta x}{\Delta t}$, just need to evaluate $\frac{\Delta x(4s)}{\Delta t}$: $\frac{\frac{1}{2}4^2 - \frac{1}{12}4^3}{4} = 2 - \frac{4}{3} = \frac{2}{3}$ m/s

or 0.67 m/s

No longer asked ↓

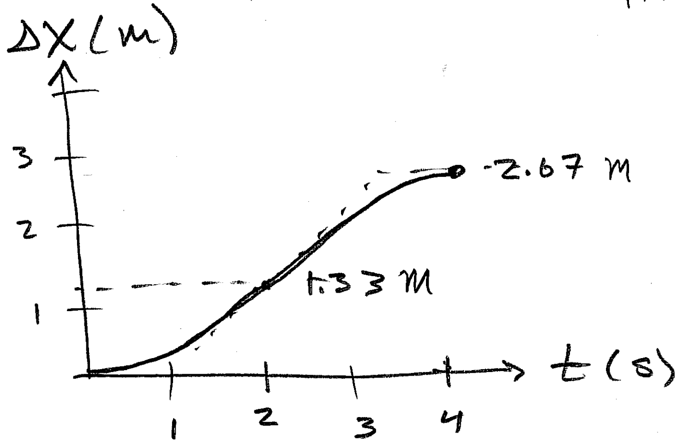
5(f) This is the area under the curve drawn in 5c. Slope should be 0 @ $t=0$

1 @ $t = 2s$

0 @ $t = 4s$

& values should be 0 @ $t=0$, $4/3$ @ $t=2s$, $8/3$ @ $t=4s$

1.33 2.67



6. Bonus [4pts.]

One can use the results of Q3 to find the angle that both has the maximum t_f and arrives in the far corner: $t_f = \sqrt{8L/g\sin\theta}$ from $V_{oy} = \sqrt{2g\sin\theta\, L}$ } math [1pt.]

$V_{ox} t = L \rightarrow V_{ox} = \frac{L}{t_f} = \sqrt{\frac{1}{8} g \sin\theta\, L}$

Thus $\tan\phi = \frac{V_{oy}}{V_{ox}} = 4$ independent of $L, g,$ or θ.

This angle is $\boxed{\phi = \tan^{-1} 4 = \underbrace{1.33 \text{ radians}}_{\text{(could write "1.3")}} = 76 \text{ degrees}}$ [1pt.] for correct angle

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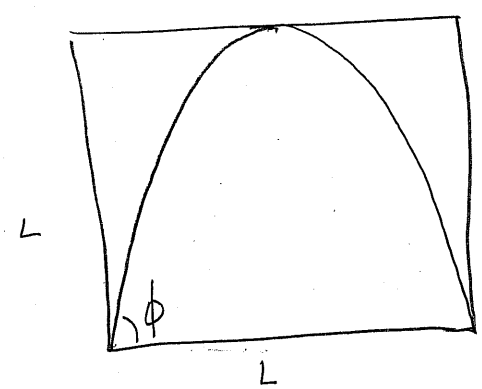
☆ But why? The trajectory is a parabola fit into a square:



$\tan\phi = \frac{dy}{dx} = \frac{V_{oy}}{V_{ox}}$

but the slope of this parabola does not depend on how quickly the particle goes through the path. (Slope = ratio of velocities.)

Trajectory can be described by $y = -4(x-\frac{1}{2})^2 + 4$ in dimensionless units. $\frac{dy}{dx} = -8(x-\frac{1}{2}) = 4$ at $x=0$.

[2pts] for an argument from geometry.