CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

22nd NOVEMBER 2018

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3A	Q3B	Q3C	Q3D	Total
14	18	3	3	6	6	50

INSTRUCTIONS

1. <u>This examination is closed book. Only one Letter-sized aid sheet is permitted</u>.
2. Only type 3 calculators are permissible.
3. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
4. Answers written in pencil will NOT be re-marked. This is University policy.
5. For all problems, you must present the solution process in a step by step fashion for partial marks.
6. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY. THIS WILL BE IMPLEMENTED STRICTLY THIS YEAR.**

Q.1. [14 points] FROZEN BIRDMAN?

Birdman is a lame superhero character created in 1960s. Birdman derives his superpowers from the sun, and if the sun is not visible (as would be the case on a cloudy day), he becomes powerless in a short time. In a clash with a stooge of his arch enemy 'Number one', the sky becomes overcast suddenly. Birdman loses his powers, and falls through the sky. If the heat loss due to metabolic processes from the surface of Birdman's body is 200 W at steady state, is any part of Birdman frozen?

Account for convective and radiative heat losses, and calculate the individual contributions as well. The convective heat transfer coefficient is 17.2 W/m^{2}K. The air around Birdman is at $-5^\circ\mathrm{C}$, and the sky exchanging thermal radiation with Birdman is at -15°C. The Stefan-Boltzmann constant is $\sigma = 5.67\times10^{-8}$ W/m$^2\cdot$K^4. The emissivity of Birdman's suit is 0.95. For modeling purposes, assume that Birdman is a cylinder of diameter 40 cm and height 1.8 m. Ignore the heat exchange at the flat circular surfaces of the cylinder. **Note**: The Newton-Raphson iterative formula for finding the root x^* of a function $f(x)$, such that $f(x^*) = 0$, is $x_{i+1} = x_i - f(x_i)/f'(x_i)$.

Solution:

$h = 17.20$ W/m$^{2\,\mathrm{o}}$C.

A heat balance gives

$$200 = hA\left(T_S - T_\infty\right) + \sigma\varepsilon A\left(\left(T_S + 273\right)^4 - \left(T_{surr} + 273\right)^4\right)$$

<u>7 points for writing down the correct heat balance</u>

$A = \pi DL = 2.262$ m^2.

$$200 = 17.2\times2.262\times\left(T_S + 5\right) + 5.67\times10^{-8}\times0.95\times2.262\times\left(\left(T_S + 273\right)^4 - \left(-15 + 273\right)^4\right)$$

One can use the Newton Raphson method to solve this equation. The surface temperature turns out to be -2.7°C.

<u>Setting up Newton Raphson: 3 points</u>

<u>Solution of the nonlinear equation: 2 points</u>

The individual contributions of convection and radiation are 89 W and 111 W respectively.

Individual contributions: 1 point each

Q.2. [18 points] A CYLINDER CARRYING RADIOACTIVE WASTE

A radioactive material of thermal conductivity *k* is cast as a solid cylinder of radius r_0 and length *L*, and placed in a liquid bath at a temperature of T_∞. Heat is uniformly generated in the solid at a volumetric rate of $\dot{S} = \dot{S}_0 \; \left(\text{W/m}^3\right)$, where $\dot{S}_0 \; \left(\text{W/m}^3\right)$ is a constant. The energy is carried away from the surface of the cylinder by convection, and the associated convective heat transfer coefficient is *h*. Beginning with the governing equation for energy balance in solids in the appropriate co-ordinate system, find

(a) the temperature distribution in the cylinder, and

(b) the temperature at the center of the cylinder.

Specify the governing equation and boundary conditions clearly at the beginning. Assume that the thermal conductivity is independent of temperature, and that $L >> r_0$, so that end effects on temperature can be ignored.

Hint: The governing equations diverge at $r = 0$, where *r* is the radial co-ordinate measured from the axis of the cylinder. Hence, you will need to solve the equations in the interval $(0, r_0]$. As the center of the cylinder is approached, the temperature has to be finite. Use this as one of the boundary conditions for solving the ODE in this problem).

Solution:

To solve this problem, we use the cylindrical co-ordinate system. The governing equation for energy balance in a solid for constant thermal conductivity is

$$\rho C \frac{\partial T}{\partial t} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{S}$$

At steady state, assuming only a radial temperature dependence, we get

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\dot{S}=-\dot{S}_0$$

<u>Coming up with the correct governing equation with appropriate arguments: 5 points</u>

The boundary conditions for temperature are that the temperature is finite at the center, and that there is convective heat transfer off the cylinder's surface.

$$-k\left.\frac{dT}{dr}\right|_{r=r_0}=h\left(T\big|_{r=r_0}-T_\infty\right)$$

<u>The boundary conditions: 2 points</u>

Rearranging the governing equation, we get

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{S}_0}{k}r.$$

Integrating the above equation once, we get

$$r\frac{dT}{dr}=-\frac{\dot{S}_0}{k}\frac{r^2}{2}+c_1.$$

Dividing by r,

$$\frac{dT}{dr}=-\frac{\dot{S}_0}{k}\frac{r}{2}+\frac{c_1}{r}.$$

Integrating the above equation, we get

$$T=-\frac{\dot{S}_0}{k}\frac{r^2}{4}+c_1\ln r+c_2.$$

<u>Two integrations to get the general solution of temperature: 4 points</u>

Since the temperature is finite at the center, we get $c_1 = 0$.

$$T=-\frac{\dot{S}_0}{k}\frac{r^2}{4}+c_2.$$

At r=r0,

$$-k\left.\frac{dT}{dr}\right|_{r=r_0}=h\left(T\big|_{r=r_0}-T_\infty\right)$$

$$\dot{S}_0\frac{r_0}{2}=h\left(-\frac{\dot{S}_0}{k}\frac{r_0^2}{4}+c_2-T_\infty\right)$$

This gives c_2 as

$$c_2 = T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k}.$$

<u>Using the finiteness of temperature at the center: 1 point</u>

<u>Application of the convection boundary condition: 4 points</u>

The temperature distribution is

$$\begin{aligned} T &= T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k} - \frac{\dot{S}_0}{k}\frac{r^2}{4} \\ &= T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k}\left(1 - \frac{r^2}{r_0^2}\right). \end{aligned}$$

<u>Correct temperature distribution: 1 point</u>

The temperature at the center is

$$T_c = T_\infty + \frac{\dot{S}_0 r_0}{2h} + \frac{\dot{S}_0 r_0^2}{4k}.$$

<u>Center temperature : 1 point</u>

Q.3. CONCEPT QUESTIONS

Q.3A. [3 points] TILE COLDER THAN CARPET?

(The grading on this question is binary: 0 or 3 points.)

An engineering student walking barefoot (without shoes or socks) from a tile floor onto a carpeted floor notices that the tile feels cooler than the carpet. Which of the following explanations is the most plausible way to explain this observation?

1. The carpet has a slightly higher temperature because air trapped in the carpet retains energy from the room better
2. The carpet has more surface area in contact with the student's foot than the tile does, so the carpet is heated faster and feels hotter
3. The tile conducts energy better than the carpet, so energy moves away from the student's foot faster on tile than carpet

4. The rate of heat transfer into the room by convection (air movement) is different for tile and carpet surfaces
5. The carpet has a slightly higher temperature because air trapped in the carpet slows down the rate of energy transfer through the carpet into the floor.

Solution:

Option 3

Q.3B. [3 points] DEEP-FRIED ICE-CREAM

Mr. X went to a Thai restaurant, where, on the dessert menu, was an item called deep-fried ice-cream. Mr. X ordered it and dissected it, only to discover that deep-fried ice cream is essentially a spherical scoop of ice-cream coated with a thick layer of batter, and deep fried in hot oil. Mr. X. then decided to make some deep-fried ice-cream at home. The ice-cream scoop he took was a sphere of 2.0 cm radius, and he coated it with a 2 mm thick layer of batter. In order to cook only the batter and to not melt the ice-cream, how long should Mr. X have immersed the batter-coated ice-cream in hot oil? <u>Provide an estimate of the time only</u>. Assume that thermal diffusion through the batter is the rate limiting step.

Solution:

This is a thermal diffusion time problem. The properties of batter are the same as water, so $\alpha = 10^{-7}$ m^2/s.

$$t_D = \frac{L^2}{\alpha} = \frac{\left(2\times10^{-3}\right)^2}{10^{-7}} = 40 \text{ s.}$$

He should deep fry the batter coated ice-cream for 40 s.

<u>Realizing that the diffusion time is to be calculated: 1.5 points</u>

<u>Using the correct length scale: 1 point</u>

<u>Final answer: 0.5 point</u>

Q.3C. [6 points] CONVECTIVE HEAT TRANSFER PAST A SOLID PLANE WALL

Consider a metal plane slab of thickness of w = 12 cm and thermal conductivity k = 70 W/m°C. The wall is maintained at a temperature of T_1 = 100°C on one face. A fluid at an ambient temperature of $T_\infty = 20^\circ\mathrm{C}$ flows over the other face, and carries away heat by convection. The associated convective heat transfer coefficient, h, depends on the velocity, V, of the fluid, according to the relationship, $h = cV^{0.5}$, where the constant c is 1555 SI units. If the fluid is moving at a velocity of V = 1 mm/s, answer the following questions, assuming steady state conditions:

(a) [3 points] Show that the convective barrier is the rate limiting step of the heat transfer process. Calculate the total heat flux, and compare it with the heat flux assuming the entire temperature drop to occur across the convective thermal resistance.

Solution:

To demonstrate this, we calculate the Biot number

$$\mathrm{Bi} = \frac{hw}{k}$$

Realizing that this is a Biot number problem: 1 point

The heat transfer coefficient is $h = cV^{0.5} = 1555 \times \left(10^{-3}\right)^{0.5} = 49.17\ \mathrm{W/m^2 \cdot {}^\circ C}$.

The Biot number is

$$\mathrm{Bi} = \frac{hw}{k} = \frac{49.17 \times 0.12}{70} = 0.08429$$

This is significantly smaller than 1, implying that the limiting resistance is convection past the surface of the wall.

Calculation of Bi and correct conclusion based on magnitude of Bi: 1 point

The total heat flux is

$$\dot{q} = \frac{T_1 - T_\infty}{\left(\frac{w}{k} + \frac{1}{h}\right)} = \frac{100 - 20}{\left(\frac{0.12}{70} + \frac{1}{49.17}\right)} = 3629 \text{ W/m}^2.$$

The total heat flux assuming the entire temperature drop to occur across the convective resistance is

$$\dot{q} = \frac{T_1 - T_\infty}{\frac{1}{h}} = \frac{100 - 20}{\frac{1}{49.17}} = 3934 \text{ W/m}^2.$$

<u>Calculation of the fluxes: 0.5 point each</u>

(b) [3 points] As the velocity is increased beyond 1 mm/s, the rate of heat transfer will increase, but this trend will stop beyond a velocity. Explain this behavior and estimate the order of magnitude of this critical velocity. Also, estimate the heat flux for velocities much larger than the critical velocity.

Solution:

As the velocity is increased, the heat transfer coefficient increases, and consequently the limiting resistance decreases. Eventually, however, the velocity will become large enough, so that the convective resistance will drop to the level of the conductive resistance. Beyond this velocity, the limiting resistance will be conductive resistance.

The critical velocity can be obtained by setting the Biot number to 1.

$$\text{Bi} = \frac{cV_c^{0.5} w}{k} = 1.$$

$$V_c = \left(\frac{k}{cw}\right)^2 = \left(\frac{70}{1555 \times 0.12}\right)^2 = 0.12 \text{ m/s}.$$

<u>Realizing that this is again a Biot number problem: 1.5 points</u>

<u>Correct calculation of critical velocity: 1 point</u>

The heat flux at velocities much larger than this velocity will be just the conductive flux across the wall with a temperature difference of 80C.

$$\dot{q} = \frac{T_1 - T_\infty}{\frac{w}{k}} = \frac{100 - 20}{\frac{0.12}{70}} = 46.67 \text{ kW/m}^2.$$

Heat flux calculation: 0.5 point

Q.3D. [6 points] HOT GOLD NANOPARTICLES FOR DISRUPTING DRUG DELIVERY VEHICLES

When a solution containing gold nanoparticles (AuNPs) is irradiated with near infrared radiation (NIR), the AuNPs heat up, conduct heat to the solution and produce local maxima in temperature, called 'hot spots'. Since NIR is benign to the body, and can be transmitted through centimeters of tissue, a researcher, Dr. Y, wishes to use the NIR-induced hot spots idea to disrupt drug delivery vehicles to release their payload in the body. Dr. Y examines 500 nm diameter spherical Au-NPs in a trial experiment, but finds that the AuNPs become so hot that they destroy the surrounding cells. Dr. Y's colleague, who is an expert at making polymer films, offers to make a polymer coating on the surface of the AuNPs that will serve as an insulation and reduce the temperature of the nanoparticles.

(a) [3 points] Should Dr. Y agree to this offer, given that the thermal conductivity of the polymer is 0.1 W/m°C and the heat transfer coefficient is 2×10^5 W/m$^2\cdot$°C (both parameters are constants)? The strength of the NIR source is fixed, and leads to a fixed amount of energy per unit time released from the surface of each AuNP.

(b) [3 points] How large does the insulation thickness need to be to achieve a reduction in temperature?

Solution:

To see this we need to check the size of the AuNPs relative to the critical insulation radius r_c. For a sphere, r_c is given by $\dfrac{2k_I}{h} = \dfrac{2\times0.1}{2\times10^5} = 10^{-6}$ m.

Since the critical radius of insulation, 1 μm, is greater than the radius of the nanoparticle (250 nm), the addition of the polymer layer will lead to a decrease in the overall thermal resistance and an increase in the heat transfer, which is the opposite of the desired result. Dr. Y should disagree with his colleague.

Realizing that this is a critical insulation radius problem: 1.5 points
Calculation of critical insulation radius: 0.5 point
Comparison with AuNP radius and conclusion: 1 point

The effective thermal resistance with the insulation is

$$R_{eff} = \frac{1}{4\pi k_I}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{h\left(4\pi r_2^2\right)}$$

When the sphere is bare, the effective resistance is

$$R_{eff} = \frac{1}{h\left(4\pi r_1^2\right)}$$

For very large thickness of insulation, r2>>r1, we get

$$R_{eff} \approx \frac{1}{4\pi k_I r_1}$$

The ratio of the thermal resistances of the bare AuNP and thick insulation limits is

$$\frac{\dfrac{1}{h\left(4\pi r_1^2\right)}}{\dfrac{1}{4\pi k_I r_1}} = \frac{k_I}{h r_1} = \frac{0.1}{2\times 10^5 \times 250\times 10^{-9}} = 2$$

This means that even if the insulation is made extremely thick, the thermal resistance for this insulation will not be larger than that of the bare AuNP. The polymer film, therefore, will not help reduce the rate of heat transfer at all.

<u>Comparison of the effective resistance for very large insulation thicknesses with that for the bare AuNP: 2 points</u>
<u>Comment / conclusion based on the comparison: 1 point</u>

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C\frac{\partial T}{\partial t}=\nabla\cdot\left(k\nabla T\right)+\dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_x=-k\frac{\partial T}{\partial x},\quad \dot{q}_y=-k\frac{\partial T}{\partial y},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

Constant *k*: $$\rho C\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{S}$$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(k\frac{\partial T}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial\theta},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

Constant *k*: $$\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\dot{S}$$

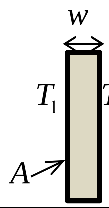
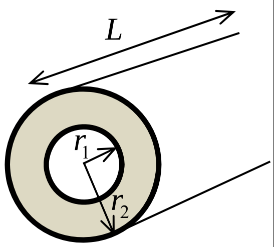
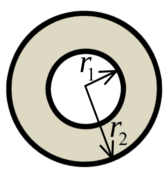
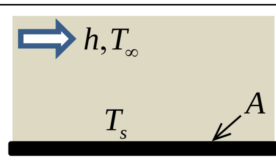
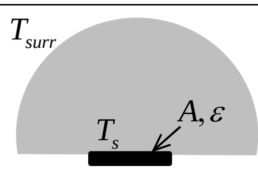
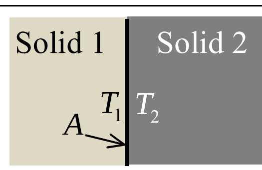
SPHERICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\varphi}\left(k\frac{\partial T}{\partial\varphi}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial\theta},\quad \dot{q}_\phi=-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial\phi}$

Constant *k*: $$\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2}\right]+\dot{S}$$

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{w}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$
Spherical shell (Hollow sphere)		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \frac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \frac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \frac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \frac{1}{\varepsilon\sigma A\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)}$
Thermal contact resistance		$\dot{Q} = \frac{T_1 - T_2}{R}$	$R = \frac{R_c}{A}$ (R_c has units of °C-m²/W)