University of Toronto

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAM

April 21st, 2022

150 minutes

First Year – Engineering Science

ECE159S – ELECTRIC CIRCUIT FUNDAMENTALS

Exam Type: A

Calculator Type: 2

Examiners: A. Hooshyar, R. Paranjape

Name: [redacted] [redacted]

First **Last**

Please write your name and student number on Page 2 as well.

SECTION (circle one):

LEC101 (Paranjape) LEC102 (Hooshyar)

INSTRUCTIONS:

- This is a Type A examination, i.e., no aids except for non-programmable calculators are allowed.
- This exam has five questions and 28 pages.
- All work is to be done on the pages of this booklet.
- When answering the questions, include all the steps of your work on these pages. In addition, place the answers in the provided boxes that are located next to each question.
- Do not unstaple this exam booklet.

Q1	10 /15
Q2	17.5 /25
Q3	17 /20
Q4	20 /20
Q5	18 /20
Total	82.5 /100

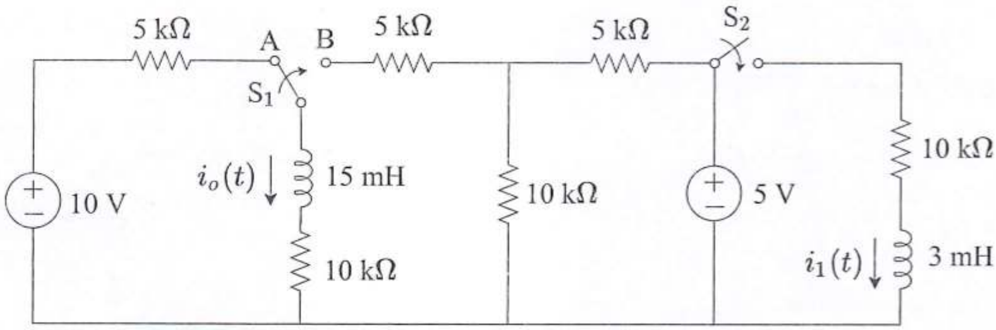
[redacted]

1. In the circuit below, for $t < 0$, switch S_1 has been at position A, and switch S_2 has been open. At $t = 0$ s, switch S_1 is moved from position A to position B. At $t = 0.1$ s, switch S_2 is closed. Find $i_o(t)$ and $i_1(t)$ for $t > 0$.

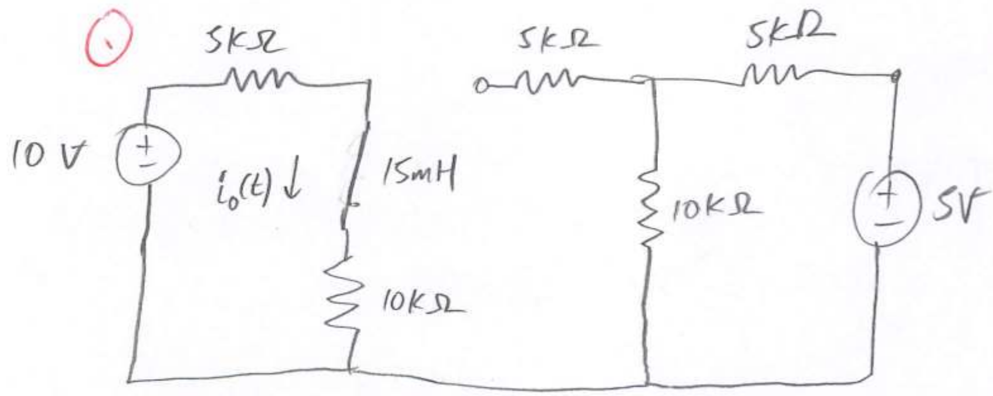
$v(t) = v_o + (v(\infty) - v(0))\, e^{-t/R_{th}C}$

$i(t) = i_o + (i(\infty) - i(0))\, e^{-t/(R/L)}$

$0 < t < 0.1$	$i_o(t) = 6.67\times10^{-4} + (1.15\times10^{-4})\, e^{-\frac{t}{1.22\times10^{6}}}$ A
$t > 0.1$	$i_o(t) = 7.82\times10^{-4} - 5.6\times10^{-4}\, e^{-\frac{(t-0.1)}{1.22\times10^{6}}}$ A.
$0 < t < 0.1$	$i_1(t) = 0$ A
$t > 0.1$	$i_1(t) = \frac{1}{2000}\, e^{-(t-0.1)/3.33\times10^{6}}$ A



for $t < 0$; steady state DC cond; open cap
short ind.



(1)

$i_1(t) = 0$
since open.

$\therefore i_1(0) = 0.$

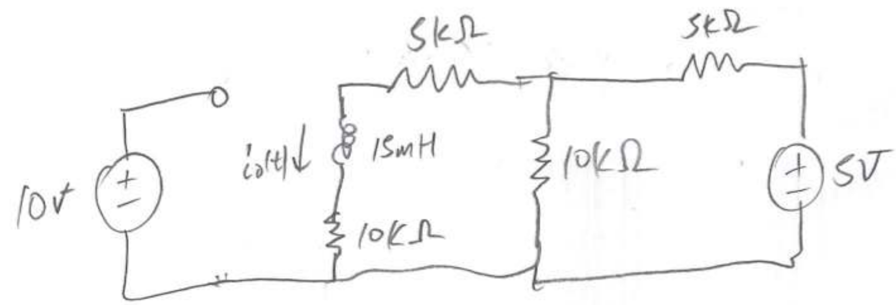
$i_o(t)$; KVL; $-10 + 5000 i_o + 10000 i_o = 0.$

$i_o = 6.67\times10^{-4}$ A.

(1)

$\therefore i_o(0) = 6.67\times10^{-4}$ A. $= 0.667$ mA.

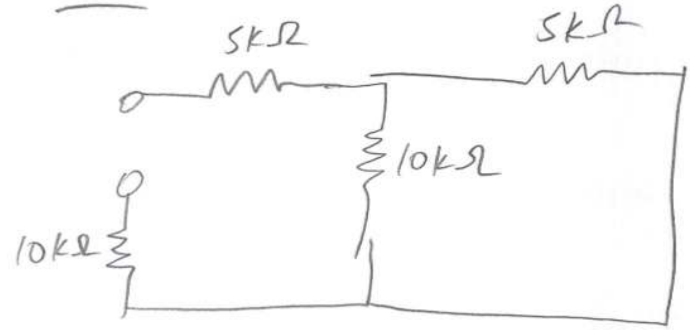
for $t>0,\ t<0.1$:



$i_1(t)=0$ (open).

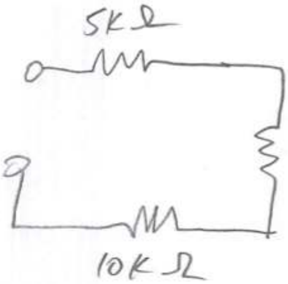
find R_{th} & $i_0(\infty)$;

R_{th}: method 1



$\Rightarrow$

5//10;

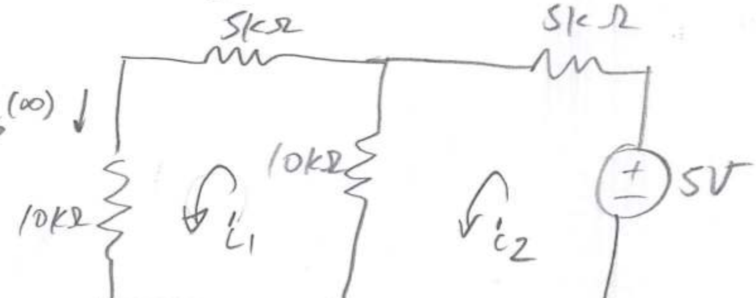


$\frac{10\times5}{10+5}=\frac{10}{3}k\Omega.$

$\Rightarrow R_{th}=5+\frac{10}{3}+10=\frac{55}{3}k\Omega.$

2

$i_0(\infty)$: Steady state DC:



loop ①;

$10i_1+10(i_1-i_2)+5i_1=0$

loop ②;

$-5+5i_2+10(i_2-i_1)=0$

$\Rightarrow \quad 25i_1-10i_2=0$

$-10i_1+15i_2=5$

$i_1=\frac{2}{11}mA \Rightarrow i_0(\infty)=\frac{2}{11}mA=\frac{2}{11}\times10^{-3}A.$

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$\therefore$ for this interval:

$$i_o(t) = 6.67\times10^{-4} + \left(\tfrac{2}{11}\times10^{-3} - 1.67\times10^{-4}\right) e^{-t/\left(\frac{55}{3}\times10^{3}/15\times10^{-3}\right)} \ A$$

$$= 6.67\times10^{-4} + (1.1515\times10^{-4}) e^{-\frac{t}{1.222\times10^{6}}} \ A.$$

$$i_1(t) = 0.$$

-1

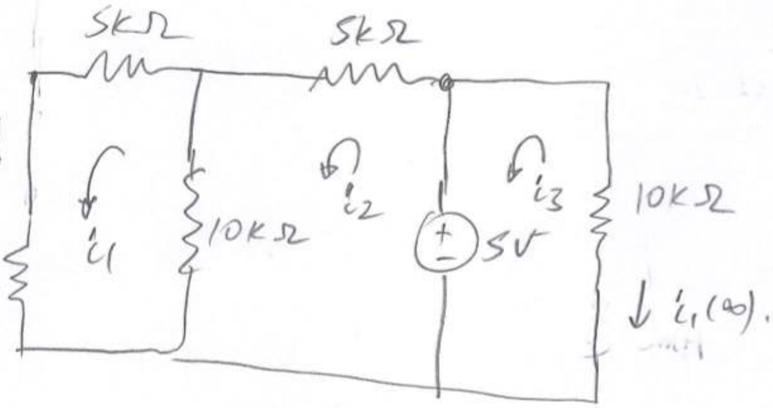
$\Rightarrow$ init: $i_o(0.1) = 7.8215\times10^{-4}$ A.

$i_1(0.1) = 0$

$t > 0.1$:

find R_{th_0}, R_{th_1}, $i_1(\infty)$, $i_o(\infty)$

i_o, i_1 @ ∞:



loop 1: $5i_1 + 10i_1 + 10(i_1 - i_2) = 0$

loop 2: $10(i_2 - i_1) - 5 + 5(i_2) = 0$ $\Rightarrow$

$(5+10+10)i_1 - 10i_2 = 0$

$(-10)i_1 + (10+5)i_2 = 5$

loop 3: $+5 + 10i_3 = 0 \rightarrow i_3 = -\frac{1}{2}$mA.

$i_1 = \frac{5}{23}$mA

$i_2 = \frac{11}{23}$mA.

$i_o(\infty) = \frac{5}{23}$mA

$i_1(\infty) = \frac{1}{2}$mA. 1

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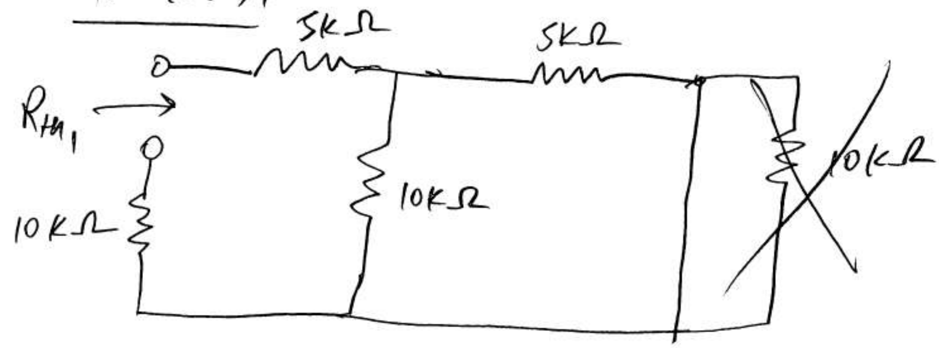
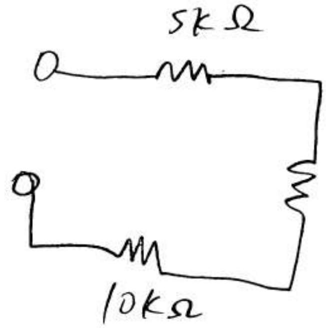
R_{th_1} ; R_{th_2} ;

for $i_o(t)$; method 1 @ DC behaviour;

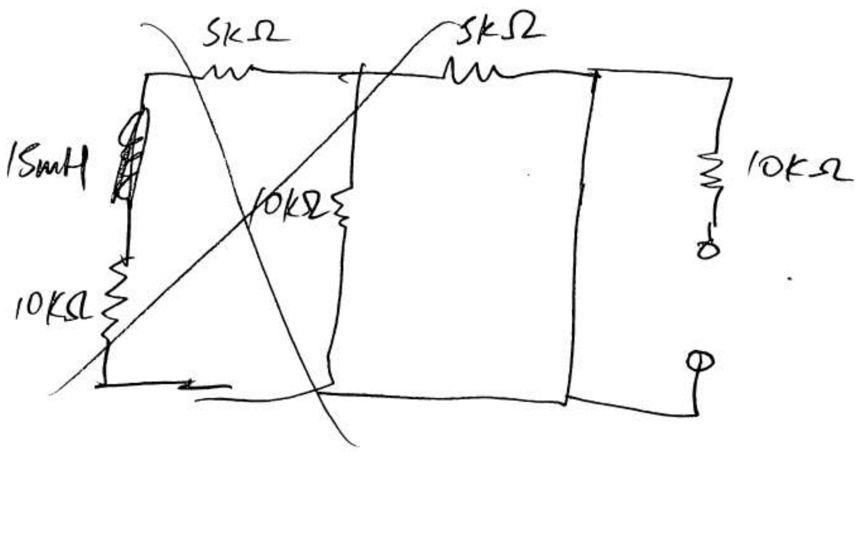




$5 // 10 = \frac{5 \times 10}{15} = \frac{10}{3} k\Omega$.

$\Rightarrow R_{th_1} = \frac{55}{3} k\Omega$.

for $i_\phi(t)$: method 1; DC;



$\Rightarrow R_{th_2} = 10k\Omega$.

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$\Rightarrow \quad i_o(0.1) = 7.8215\times10^{-4}\ A = 0.782\,mA.$

$i_1(0.1) = 0.$

$R_{th_1} = \frac{55}{3}\ k\Omega.$

$R_{th_2} = 10\,k\Omega.$

$i_o(\infty) = \frac{5}{23}\ mA$

$i_1(\infty) = \frac{1}{2}\ mA.$

So, $i_o(t) = 7.8215\times10^{-4} + \left(\frac{5}{23}\times10^{-3} - 7.8215\times10^{-4}\right) e^{\frac{-(t-0.1)}{\frac{55}{3}\times10^3/15\times10^{-3}}}$

$i_1(t) = 0 + \left(\frac{1}{2}\times10^{-3} - 0\right) e^{\frac{-(t-0.1)}{10000/3\times10^{-3}}}$

$\Rightarrow i_o(t) = 7.8215\times10^{-4} - 5.647\times10^{-4}\, e^{\frac{-(t-0.1)}{1.22\times10^6}}\ A.$

$i_1(t) = \frac{1}{2000}\, e^{\frac{-(t-0.1)}{3.33\times10^6}}\ A.$

(-1) τ

(-1) R_{Th}

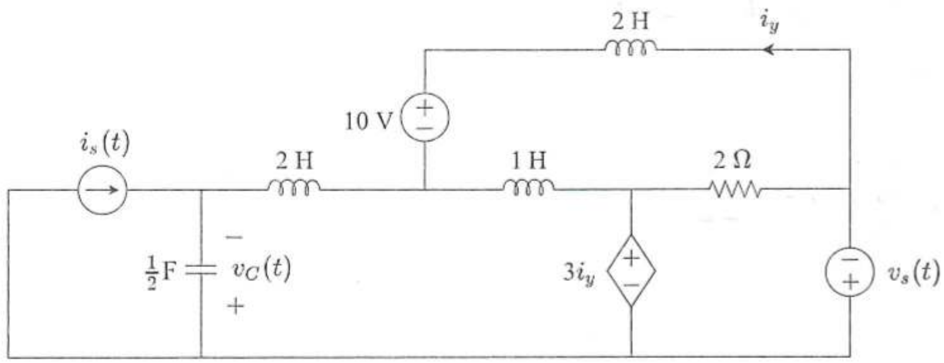
(-3) $i_1(t)$

N[redacted]

Super.

2. In the circuit below, $i_s(t) = 2\cos t$ A and $v_s(t) = 5\sin(2t + 15°)$ V. Find the capacitor voltage, $v_C(t)$.

$$\boxed{v_c(t) = 10\cos(t+143.13°) - 15.81\cos(2t-146.6°) + 10 \text{ V}}$$

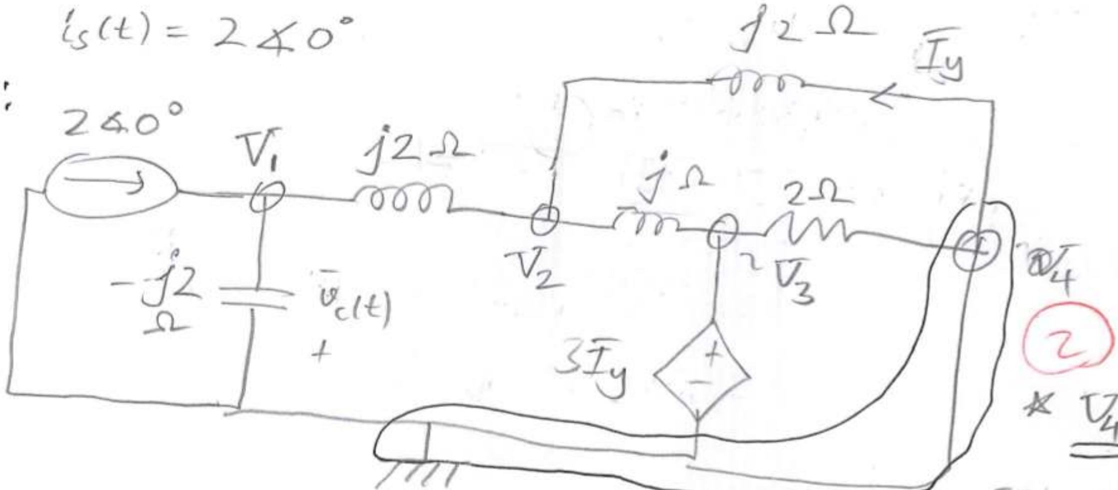


must use superposition since sources are not the same ω.

only $i_s(t)$: $\omega = 1$.

$i_s(t) = 2\angle 0°$

nodal:



2

$V_4 = 0$.

$V_3 = 3I_y = +j\frac{3}{2}V_2$ 2

$I_y = \frac{0 - V_2}{j2} = -\frac{V_2}{j2}$

KCL ①:

$$-2\angle 0° + \frac{V_1}{-j2} + \frac{V_1 - V_2}{j2} = 0$$ 2

KCL ②:

$$\frac{V_2 - V_1}{j2} + \frac{V_2 - \cancel{V_4}^{\,0}}{j2} + \frac{V_2 - V_3}{j} = 0$$

$$-\frac{V_3}{j} = -\frac{j\frac{3}{2}V_2}{j} = -\frac{3}{2}V_2$$

$$-\frac{V_3}{j} = -\frac{3I_y}{j} = +\frac{3\frac{V_2}{j2}}{j} = +\frac{3V_2}{2}$$

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$$\Rightarrow \quad \cancelto{0}{V_1\left(-\frac{1}{j2}+\frac{1}{j2}\right)} + V_2\left(-\frac{1}{j2}\right) = 2\angle 0^\circ. \quad (1)$$

$$\Rightarrow \quad V_1\left(-\frac{1}{j2}\right) + V_2\left(\frac{1}{j2}+\frac{1}{j2}+\frac{1}{j}-\frac{3}{2}\right) = 0. \quad (2)$$

$$(1) \Rightarrow \quad V_2 = \frac{2\angle 0^\circ}{-\frac{1}{j2}} = -4j \checkmark$$

$$\Rightarrow \quad V_1 = -V_2\left(-\frac{3}{2}-2j\right) = 4j\left(-\frac{3}{2}-2j\right)$$

$$= 8-j6$$

(0.5)

$$\therefore \quad v_c'(t) = -V_1 = -8+j6$$

$$= 10\cos(t+143.13^\circ)$$

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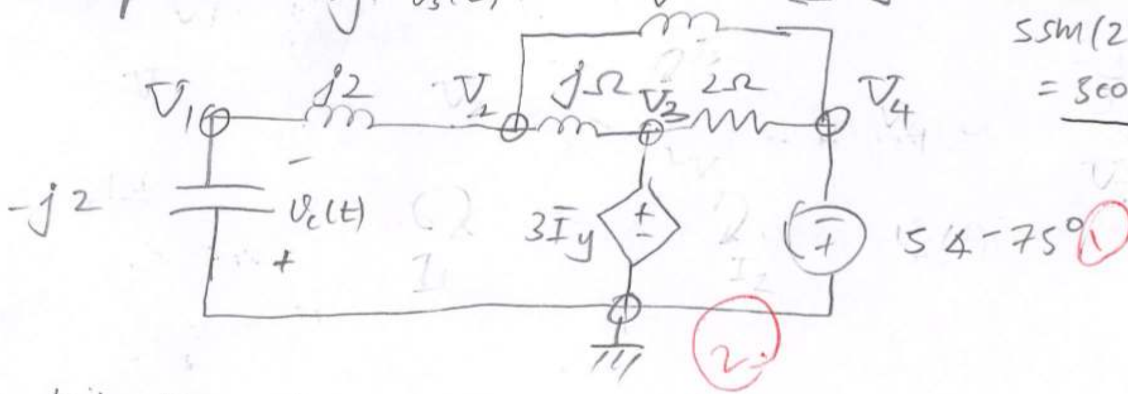
second part: only $v_s(t)$:

$\omega = 2$

$5\sin(2t+15^\circ)$

$= 5\cos(2t-75^\circ)$



(1)

(2)

KCL 1; $\quad \dfrac{V_1}{-j2} + \dfrac{V_1-V_2}{j2} = 0$

KCL 2; $\quad \dfrac{V_2-V_4}{j2} + \dfrac{V_2-V_1}{j2} + \dfrac{V_2-V_3}{j} = 0$ (2)

KCL 4; $\quad V_4 = -5\angle -75^\circ$

KCL 3; $\quad V_3 = 3I_y = 3\cdot\dfrac{V_4-V_2}{j2}$ (2)

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$\Rightarrow$ ①: $V_1\cancel{\left(-\frac{1}{j2}+\frac{1}{j2}\right)}+V_2\left(-\frac{1}{j2}\right)=0$

②: $V_1\left(-\frac{1}{j2}\right)+V_2\left(\frac{1}{j2}+\frac{1}{j2}+\frac{1}{j}\right)-\frac{V_3}{j}-\frac{V_4}{j2}=0$

$\ast \begin{cases} V_4 = -5\angle -75° \\ V_3 = \frac{3}{j2}\left(V_4-V_2\right) \end{cases}$

$\Rightarrow$ ①: $V_2\cdot\frac{-1}{j2}=0 \Rightarrow V_2=0.$

$\Rightarrow$

②: $V_1\left(-\frac{1}{j2}\right)+\frac{3}{2}\left(-5\angle -75°\right)+\frac{5\angle -75°}{j2}=0.$

$V_1\left(-\frac{1}{j2}\right)+1.9411+7.244j+-2.4148-0.647j=0$

$V_1\left(-\frac{1}{j2}\right)=4.3559-6.597j$

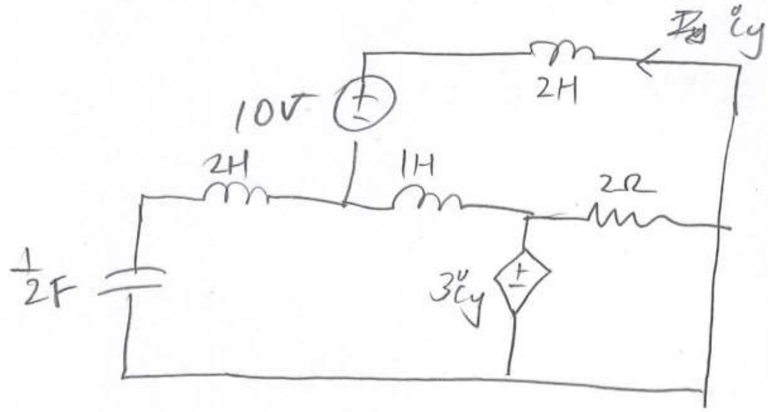
$=-13.194-8.7118j$

$v_C(t)=-V_1=-15.81\angle -146.6°.$

$=-15.81\cos\left(2t-146.6°\right)$

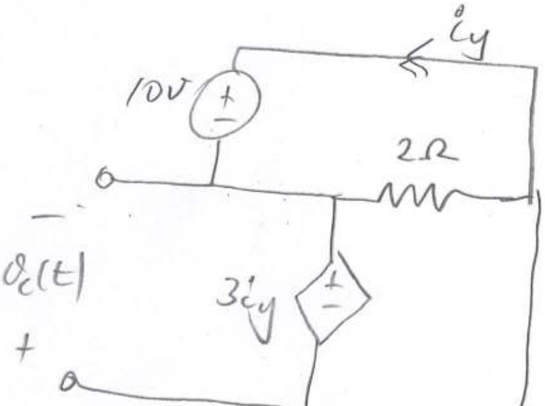
Na[redacted]

final part: just DC:

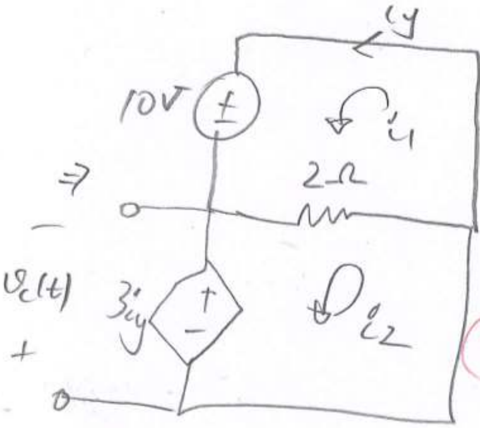


⇒ DC cond.
short ind
open cap.

⇒



⇒



3

①; $+10 + 2(i_1 - i_2) = 0 \Rightarrow \quad 2i_1 - 2i_2 = -10$

②; $+3i_1 + 2(i_2 - i_1) = 0 \Rightarrow \quad i_1 + 2i_2 = 0$

$i_1 = -\frac{10}{3}$ A

$i_2 = \frac{5}{3}$ A.

$v_c(t) = -3i_y = -3i_1 = -3\left(-\frac{10}{3}\right) = 10$ V. 1

All together:

$v_c(t) = 10 - 15.81\cos(2t - 146.6°) + 10\cos(t + 143.13°)$

1

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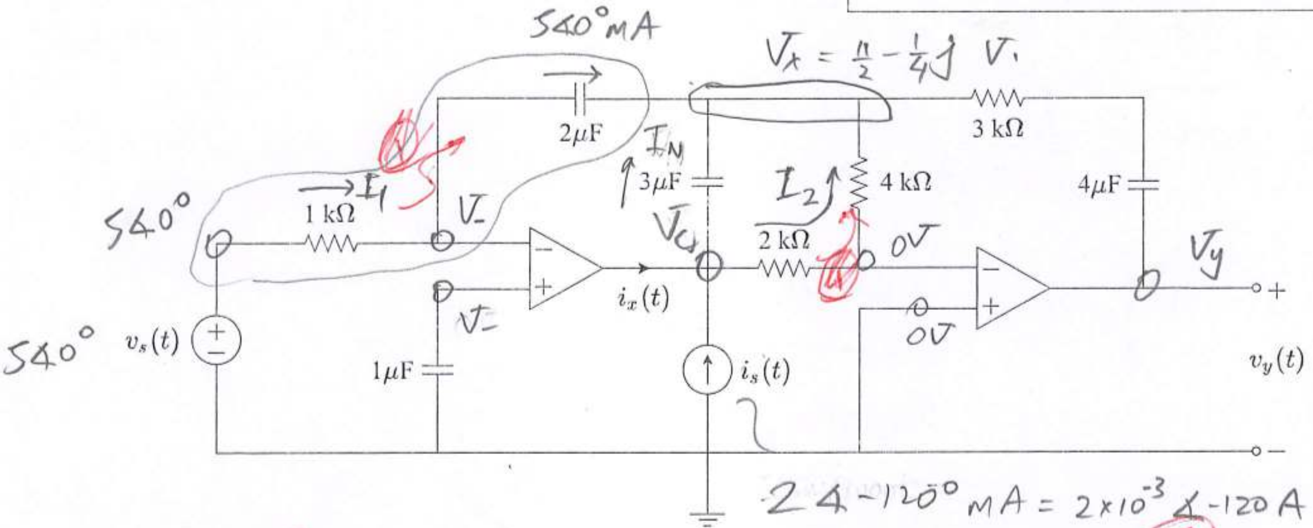
3. In the circuit below, $v_s(t) = 5\cos(1000t)$ V and $i_s(t) = 2\sin(1000t - 30°)$ mA. The circuit is in the steady state condition.

(a) Find $i_x(t)$.

(b) Find $v_y(t)$.

$i_x(t) = 0.023\cos(1000t - 93.7°)$ A
$v_y(t) = 22\cos(1000t + 4.11°)$ V



$2\angle -120°$ mA $= 2\times10^{-3}\angle -120$ A

$$I_1 = \frac{5\angle 0°}{1k\Omega} = 5\angle 0° \text{ mA} = 5\times10^{-3}\angle 0° \text{ A}.$$

$$\frac{V_x - 5\angle 0° \text{ V}}{\frac{-j}{1000\times2\times10^{-6}} + 1000} = 5\times10^{-4}\angle 0° \text{ A}, \qquad V_x = \frac{11}{2} - \frac{1}{4}j \text{ V}.$$

$$I_2 = \frac{0 - V_x}{4000} = 1.376\times10^{-3}\angle 177.4° \text{ A}.$$

$$\frac{V_c - 0}{2000} = I_2, \quad V_c = -\frac{11}{4} + j\frac{1}{8} \text{ V}.$$

$$I_N = \frac{V_c - V_x}{\frac{-j}{3\times1000\times10^{-6}}} = 0.0248\angle -92.6° \text{ A}.$$

KCL @ centre: $2\times10^{-3}\angle -120° + i_x(t) = 1.376\times10^{-3}\angle 177.4° + 0.0248\angle -92.6°$

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solving: $i_x(t) = -1.5\times10^{-3} \angle -0.023$

$= 0.023 \cos(1000t - 93.7^\circ)$

$$\frac{V_y - V_x}{3000 + -\frac{j}{3\times10^{-6}\cdot 1000}} = I_1 + I_2$$

-1

$$\Rightarrow V_y - V_x = \frac{50000 + 6.242j}{3000 + \ldots}$$

-1

$= 16.463 + 1.83j$

$V_y = 22 + 1.58j$

$= 22.02 \angle 4.11$

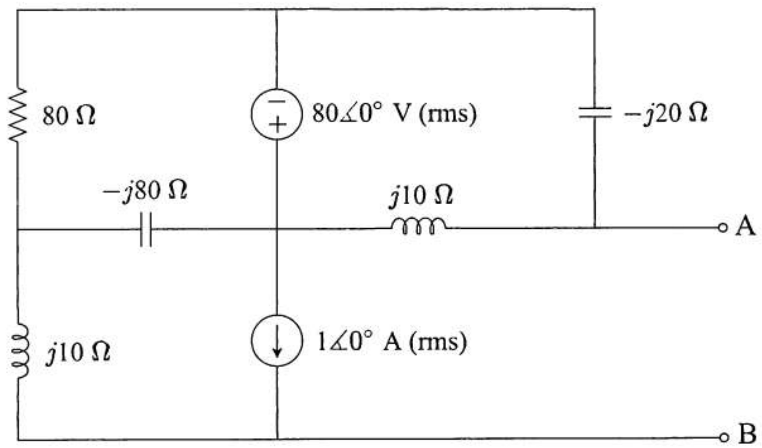
$v_y(t) = 22.02 \cos(1000t + 4.11^\circ)$.

[redacted]

4. In the circuit below. the frequency of the sources is 1000 rad/s.

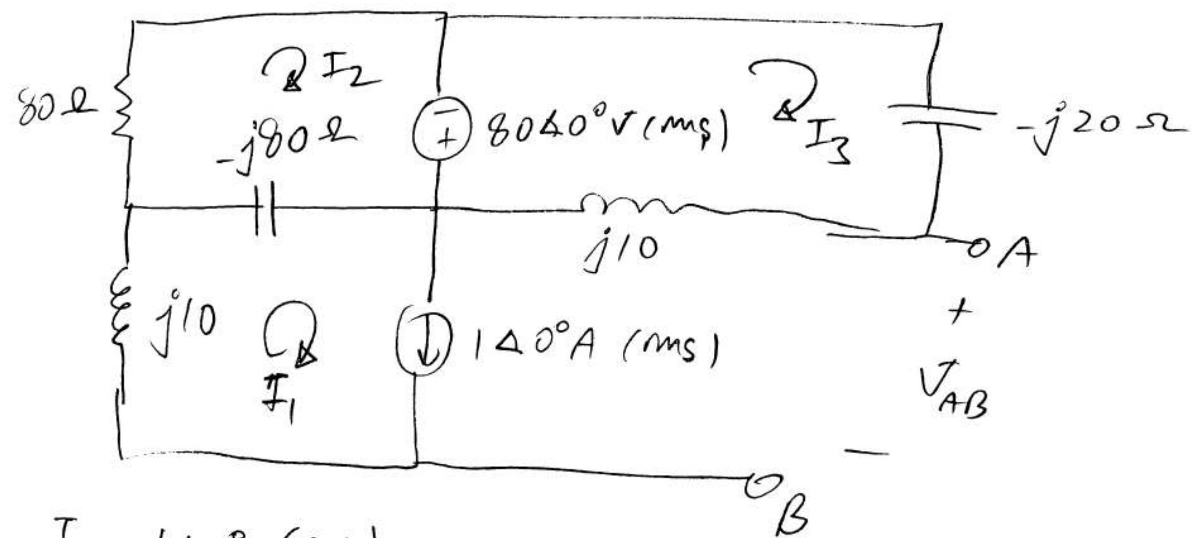
Fix RMS! (a) Find the Thevenin equivalent circuit (V_{th} and Z_{th}) seen between terminals A and B.

(b) A load is to be connected between terminals A and B. This load is composed of a resistor, R_o, and an inductor, L_o, connected in series. R_o and L_o can be changed within [0,30] Ω and [5,15] mH, respectively. Find the maximum average power, P_o, that can be delivered to R_o.



$V_{th} =$	$80 - j10$ V (rms)
$Z_{th} =$	$40 - j10\ \Omega$ (rms)
$P_o =$	39.8 W

a) V_{th}:



$\underline{\underline{I_1 = 1\angle 0° \text{ (rms)}}}$

loop 2: $I_2(80) - 80\angle 0° V(rms) + (-j80)(I_2 - I_1) = 0$

loop 3: $80\angle 0° V(rms) + (-j20)I_3 + (j10)(I_3) = 0.$

$\hookrightarrow I_3 = \dfrac{-80\angle 0°}{j10 - j20} = \underline{\underline{-8j}} \text{ (rms)}$

[redacted]

$$I_2(80) - 80\angle 0^\circ + (-j80)(I_2 - 1\angle 0^\circ) = 0$$

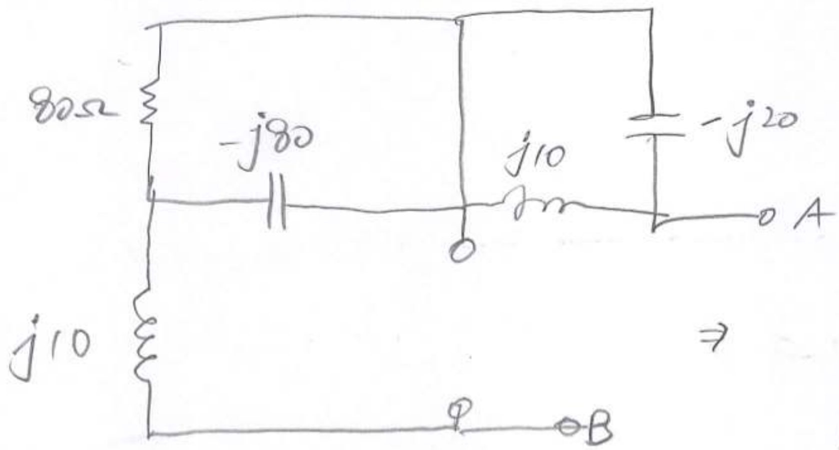
$$(80 - j80)\, I_2 = 80\angle 0^\circ - (1\angle 0^\circ)(j80)$$

$$\underline{\underline{I_2 = 1\angle 0^\circ}}.$$

KVL: $+V_{AB} + j10 I_1 + 80 I_2 - j20 I_3 = 0$

$$V_{AB} = j20(-8j) - 80(1) - j10(1) = 80 - 10j \text{ (rms)}$$

<u>Z_{th}</u>: <u>first method</u>:

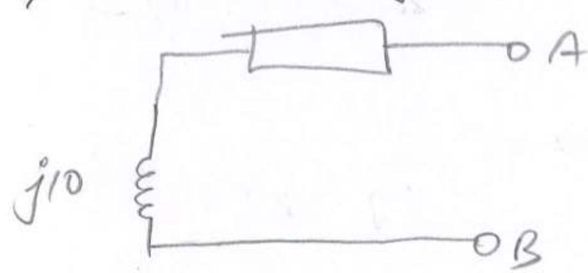


$(80 /\!/ -j80) + (j10 /\!/ -j20)$.

$$\frac{(80)(-j80)}{80-j80} + \frac{(j10)(-j20)}{j10 - j20}$$

$$= 40 - 20j$$

$\Rightarrow$



$$\Rightarrow Z_{th} = 40 - j10\ \Omega \text{ (rms)}$$

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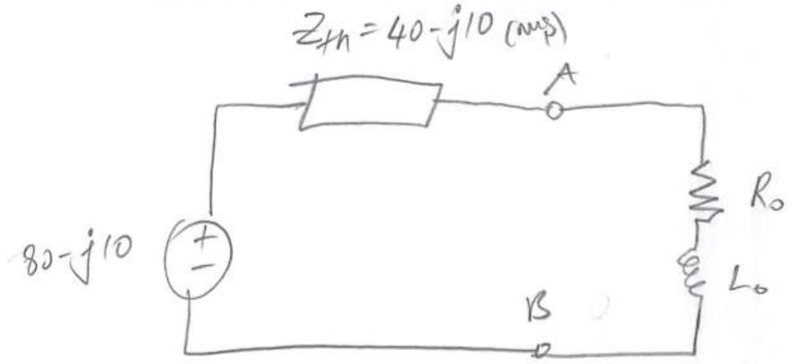
b)

$0 \leq R_o \leq 30\ \Omega$.

$5 \leq L_o \leq 75\text{ mH}$

[redacted]



Under no restrictions:

$Z_L = R_o + jX_L$

$X_L = -X_{th} = 10.$

$R_L = \sqrt{(40)^2 + (\cancel{X_L + X_{th}})^2}$ 0

$= 40\ \Omega.$

So, choose closest $R_o = 30\ \Omega$. ✓

For L_o:

$5\times10^{-3} < L_o < 15\times10^{-3}$ H $\qquad \omega = 1000$ rad/s

$5 < \omega L_o < 15$

$5 < X_L < 15 \Rightarrow$ want ideal case of $10 \rightarrow$ possible!

$\therefore Z_L = 30 + j10\ \Omega$. ✓

Max power: $P_{avg} = \frac{1}{2} R_L |I|^2$

KVL: $80 - j10 + j10 I + 30 I + I(40 - j10) = 0$

$I = \dfrac{j10 - 80}{j10 + 30 + 40 - j10} = -\frac{8}{7} + \frac{1}{7}j$ (rms)

$= P_{avg,rms} = \frac{1}{2}(30)\left(\frac{\sqrt{65}}{7}\right)^2 = 19.898$ W

$\Rightarrow P_{avg} = 2 \cdot 19.898 = 39.8$ W ✓

M

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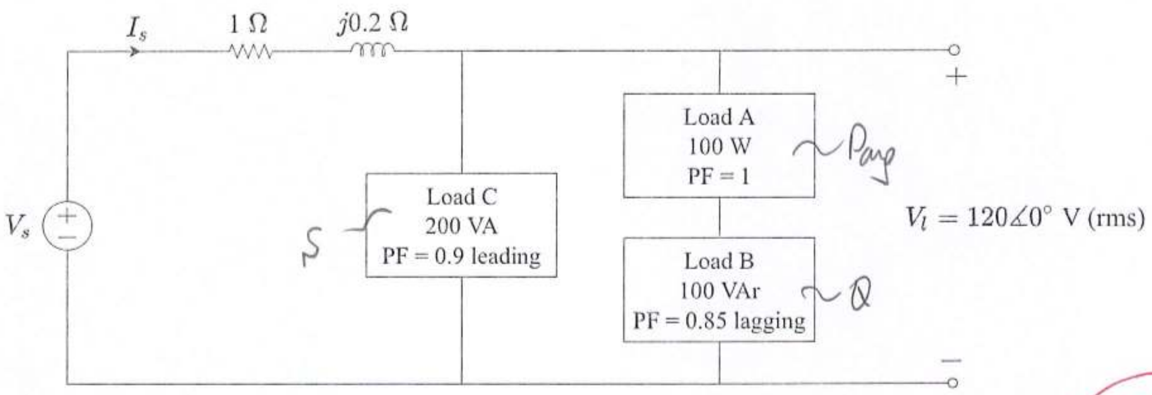
5. In the circuit below, the voltage at the terminal of the loads is $120\angle 0°$ V (rms). Find

(a) the source voltage, V_s,

(b) the current drawn from the source, I_s, and

(c) the total complex power delivered by the source, S_s.

$V_s =$	$174.9 \angle 0.2914°$
$I_s =$	$5.204 \angle -1.66°$
$S_s =$	$624.16 + j\,18.14$



b)

A: $P_{avg} = 100\,W = S \cdot PF. \quad \therefore S = 100\,W. \Rightarrow \bar{S} = 100 \angle 0°$ (rms) $\rightarrow$ since PF = 1.

3

B: $Q = 100\,VAr = S \sin\theta, \quad \theta = \cos^{-1}(0.85) = \pm 31.79 \rightarrow lag \rightarrow \theta = 31.79°.$

4

$$\Rightarrow S = \frac{100}{\sin\theta} = 189.82\,VA \Rightarrow \bar{S} = 189.82 \angle 31.79°.\ (rms).$$

3

C: $S = 200\,VA \rightarrow \theta = \cos^{-1}(0.9) = -25.842$ (lead). $\Rightarrow \bar{S} = 200 \angle -25.84°.$

$$\bar{S}_{tot} = 100\angle 0° + 189.82 \angle 31.79° + 200 \angle -25.84° = 441.347 + j\,12.83.$$

(rms) 3

$$S_{tot} = V_l I_s^*, \quad I_{s\,rms} = \frac{S_{tot}^*}{V_l^*} = \frac{441.347 - j\,12.83}{120\angle 0°} = 3.68 \angle -1.66° A$$

3 (rms).

$$\therefore I_s = \sqrt{2} I_{rms} = 5.204 \angle -1.66°$$

a) KVL:

$$V_l + I_s(1) + I_s(j0.2) = V_s = \sqrt{2}\,120\angle 0° + (1 + j0.2) I_s$$

$$= 174.9 \angle 0.2914°.$$

2

S_s ?