[15] 1. *Definitions and terms* — Define each of the following terms or concepts, as used in this course, in one or two sentences written in your test booklet. Then explain the significance or importance, in thermal physics, as well, in one or two sentences. Provide equations if directly relevant.

a) multiplicity function of a binary model system

b) ensemble average

c) Helmholtz Free Energy

[20] 2. *Multiple Choice* — In each part below, select the *best* single choice, and mark your answer on the special sheet provided for multiple choice answers. No explanations are required; part-marks may be awarded for partially correct answers. All parts (i)–(iv) below have equal marking weight.

i) Given 1000 spins of magnetic moment m that can be aligned with a magnetic field B ('up' state), or aligned against a magnetic field B ('down' state), and a very high fundamental temperature $\tau \gg mB$, what is the probability that 600 spins are 'up' and 400 are 'down'?

A. 5×10^{-11}
B. 0.40
C. 0.60
D. 10^{-157}
E. 0
F. none of A–E given are correct

ii) The nitric oxide molecule NO (nitrogen monoxide) has a binding potential which for low-lying states can be approximated by a simple harmonic oscillator potential, with a force constant k = 1860 $\mathrm{Nm^{-1}}$. Thus for its lower-energy states, NO vibrates as a quantum-mechanical simple harmonic oscillator, and it can change quantum levels from $n = 1$ to $n = 0$ by emitting a photon at frequency $\nu = 6.42 \times 10^{13}$ Hz.

Does this happen? At room temperature (300 K), what is the approximate ratio of NO molecules found in the $n = 1$ state compared to being in the ground state?

A. 0.5
B. 3.5×10^{-5}
C. 4.3×10^{-7}
D. 2.2×10^{-10}
E. 2.8×10^{-14}
F. none of A–E given are correct

iii) A system consists of three different identical quantum simple harmonic oscillators (QMSHOs), each of frequency ω_0. We choose the zero of energy such that the ground-state energy of each individual QMSHO is $E_0 = \frac{1}{2}\hbar\omega_0$. What is the multiplicity of the *system macrostate* defined as having energy $4.5\hbar\omega_0$?

A. 3
B. 4.5
C. 9
D. 10
E. 27
F. none of A–E given is correct

(Question #2 continues…)

iv) A system consists of two very large mirrors, parallel to each other, which reflect any frequency of light. Thus it can support electromagnetic normal modes with frequencies ω_0, $2\omega_0$, $3\omega_0$… Consider only a single polarization. Starting from the system in its ground state, with no photons, an energy $3\hbar\omega_0$ is added to the system. What is the multiplicity of the system macrostate having this energy?

A. 3
B. 4.5
C. 9
D. 10
E. 27
F. none of A–E given is correct

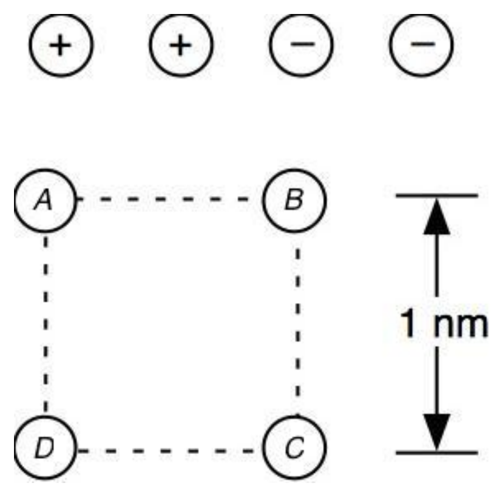
[20] 3. *Most probable configuration* — Consider two binary model spin systems $\mathcal{S}_1$ and $\mathcal{S}_2$, which have particle numbers N_1 and N_2 respectively, where N_1 and N_2 are each very large numbers (greater than 10^{20}, let's say). All spins are identical, their magnetic moment is m, and do not interact with each other – only with the constant external magnetic field B. These two systems are initially each closed, and then are brought into thermal contact with each other, but the combined system $\mathcal{S}$ is still closed.

[5] a) What is the multiplicity function $g_1(N_1, s_1)$ for the subsystem $\mathcal{S}_1$ when it has energy U_1? What then is the multiplicity function $g_2(N_2, s_2)$ for $\mathcal{S}_2$? From these, give the multiplicity function for the combined system $\mathcal{S}$ *before* thermal contact is permitted?

[5] b) What is the multiplicity function for the combined system $\mathcal{S}$ *after* thermal contact between $\mathcal{S}_1$ and $\mathcal{S}_2$? Show that when they come into thermal contact, thus relaxing the constraint that systems $\mathcal{S}_1$ and $\mathcal{S}_2$ are each closed, the result is an increase in the entropy.

[10] c) Derive the condition shown in class for the energy sharing between $\mathcal{S}_1$ and $\mathcal{S}_2$ which gives the *most probable configuration* of this spin system $\mathcal{S}$.

[20] 4. *Crystal of charges* — Consider a system made up of only four ions, Na^+, K^+, Cl^- and F^- placed at distinct fixed locations on a flat surface. Exactly one ion must be placed at each corner *A, B, C, D* of a square 1 nm on a side. Any ion may be placed at any corner.



[5] a) How many *microstates* are there of this system?

[5] b) With respect to system energy ε, how many *macrostates* are there? Give their energies, and their multiplicities.

[10] c) In thermal equilibrium at 300K (room temperature), what is the probability of the system being in its lowest-energy state? Its highest-energy state? Repeat for 3000K.

(END OF TEST)

[75] TOTAL MARKS