Midterm Test: PHY 294

[3 problems, 15 points, 60 mins] (11 Feb 2019, 9:30am-10:30am)

Useful formulae

Units, Constants, Identities

$$1eV = 1.6 \times 10^{-19} J$$

$$\hbar = 1.05 \times 10^{-34} kg \ m^2/s$$

$$e^{ix} = \cos x + i \sin x$$
(1)

Harmonic oscillator frequency

$$\Omega = \sqrt{\frac{K}{m}}$$
 $(K = \text{spring constant})$

Time-dependent Schrodinger equation in 1D

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x)\psi(x,t)$$
 (2)

Time-independent Schrodinger equation in 1D

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$$
(3)

Harmonic oscillator

$$E_n = (n + \frac{1}{2})\hbar\Omega \qquad (n = 0, 1, 2...)$$

$$\psi_0(x) = \left(\frac{m\Omega}{\pi\hbar}\right)^{1/4} \exp\left[-\frac{m\Omega x^2}{2\hbar}\right]$$

$$\psi_1(x) = \left(\frac{m\Omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{m\Omega}{2\hbar}} \ 2x \exp\left[-\frac{m\Omega x^2}{2\hbar}\right]$$

Particle in a 1D box: 0 < x < L

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \qquad (n = 1, 2 \dots)$$

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Fourier transforms

$$\tilde{\psi}(k) = \int_{-\infty}^{+\infty} \frac{dx}{\sqrt{2\pi}} e^{-ikx} \psi(x)$$

$$\psi(x) = \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\pi}} e^{+ikx} \tilde{\psi}(k)$$

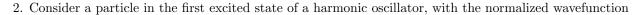
Integrals

$$\int_{-\infty}^{\infty} dx \, e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} \, e^{-\alpha x^2} \, e^{-ikx} = \sqrt{\frac{1}{2\alpha}} \, e^{-\frac{k^2}{4\alpha}}$$

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} \, x e^{-\alpha x^2} \, e^{-ikx} = -\sqrt{\frac{1}{2\alpha}} \, \frac{ik}{2\alpha} \, e^{-\frac{k^2}{4\alpha}}$$

- 1. Consider a particle in a box, confined to a region -L < x < L, with infinitely high walls. (5 points)
 - (i) Find the wavefunction for the second excited state (i.e., n = 3).
 - (ii) What is the corresponding energy?
 - (iii) What would be the frequency of a photon emitted if the particle jumps to the state n = 1?



$$\psi(x) = \left(\frac{m\Omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{m\Omega}{2\hbar}} \ 2x \ \exp\left[-\frac{m\Omega x^2}{2\hbar}\right]$$



Determine the momentum wavefunction $\tilde{\psi}(k)$ for this particle, and the most probable outcomes when you measure its momentum. (5 points)

3. Consider a potential step at x = 0, where the potential energy drops as (5 points)

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ -V_0 & \text{for } x > 0 \end{cases}$$

where $V_0 > 0$. Sketch the potential energy as a function of x. For a particle incident from the left with energy E > 0, the most general wavefunction for x < 0 is of the form $A_1 e^{ik_1x} + B_1 e^{-ik_1x}$, and the most general wavefunction for x > 0 is $A_2 e^{ik_2x}$. What are k_1 and k_2 in terms of E, V_0, m, \hbar ? Using boundary conditions, and solving, determine the ratio $\frac{B_1}{A_1}$. For $V_0 = 4E$, determine the reflection coefficient $\frac{|B_1|^2}{|A_1|^2}$.

