MAT185 Linear Algebra
Term Test 1

Instructions:

Please read the Term Test 1 Information document for details on submission policies, permitted resources, how to ask a question, test announcements, and more. You were expected to read them in detail in advance of the test.

1. **Submissions are only accepted by** Gradescope. Do not send anything by email. Late submissions will be assessed with a penalty of 2% per minute. Remember you can resubmit anytime before your 2 hour time limit expires.
2. **Submit your solutions using only this template PDF**. Your submission should be a single PDF with your full written solutions for each question. If your solution is not written using this template PDF (scanned print or digital) then your submission will not be assessed. Organize your work neatly in the space provided.
3. **Show your work and justify your steps** on every question unless otherwise indicated. Put your final answer in the box provided, if necessary.
4. **You must fill out and sign the academic integrity statement below**; otherwise, you will receive zero for the test.

Academic Integrity Statement:

Full Name: ______________________________

Student number: ______________________________

I confirm that:

- I have not communicated with any person about the test other than a MAT185 teaching team member.
- I have not used any resources other than those that are listed as permitted in the Term Test 1 Information document at any point during the test.
- I have not participated in or enabled any MAT185 group chat during the test.
- I have not viewed the answers, solutions, term work, or notes of anyone.
- I have read and followed all of the rules described in the Term Test 1 Information document.
- I understand the consequences of violating the University's academic integrity policies as outlined in the Code of Behaviour on Academic Matters. I have not violated any of them while writing this assessment.

By signing this document, I agree that all of the statements above are true.

Signature: ______________________________

1. Decide if the following statement is True or False:

The subset $W = \{(x, y, z) \in \mathbb{R}^3 \mid x - |y| + z = 0\}$ is a subspace of $\mathbb{R}^3$ with respect to the usual vector addition and scalar multiplication.

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●) and justify your choice with a proof or counter-example. [4 marks]

○ True

○ False

2. Decide if the following statement is True or False:

Let $W_1 = \text{span}\{(1, 1, 2), (1, 0, 1)\}$, and $W_2 = \text{span}\{(0, k, 1), (1, -2, -1)\}$. There is no value of $k \in \mathbb{R}$ such that $W_1 = W_2$.

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●) and justify your choice with a proof or counter-example. [4 marks]

○ True

○ False

3. Decide if the following statement is True or False:

Let $f_j : \mathbb{R} \to \mathbb{R}$ be defined by $f_j(x) = e^{r_j x}$ where $r_j \in \mathbb{R}$ for $j = 1, 2, 3$, and let $W = \operatorname{span}\{f_1, f_2, f_3\}$. If f_1, f_2, f_3 is a basis for W, then r_1, r_2, r_3 must be distinct.

Indicate your final answer by **filling in exactly one circle** below (unfilled ○ filled ●) and justify your choice with a proof or counter-example. [4 marks]

○ True

○ False

4. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be linearly independent vectors in a vector space V. Suppose that $V \neq \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and let $\mathbf{x} \in V$ but $\mathbf{x} \notin \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4(a) Prove that $\mathbf{v}_1 + \mathbf{x}, \mathbf{v}_2 + \mathbf{x}, \mathbf{v}_3 + \mathbf{x}$ is linearly independent. [4 marks]

4. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be linearly independent list of vectors in a vector space V. Suppose that $V \neq \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, and let $\mathbf{x} \in V$ but $\mathbf{x} \notin \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4(b) Prove that $V \neq \text{span}\{\mathbf{v}_1 + \mathbf{x}, \mathbf{v}_2 + \mathbf{x}, \mathbf{v}_3 + \mathbf{x}\}$. [4 marks]