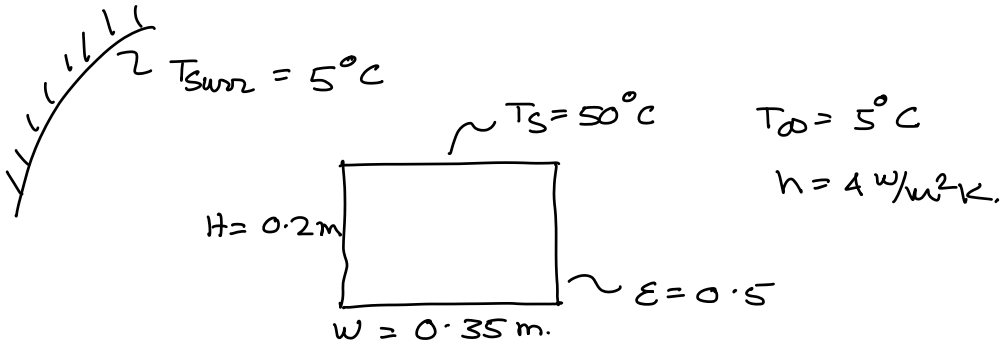
CHE 260 - QUIZ 2 SOLUTION
2021.



a) $\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s^4 - T_{surr}^4)$

$A_s = L(2W + 2H) = 15\ m.\ (2 \times 0.35 + 2 \times 0.2)m$

$A_s = 16.5\ m^2$

$\dot{Q}_{conv} = 4 \frac{W}{m^2K} \times 16.5\ m^2\ (50-5)°C = 2970\ W$

$\dot{Q}_{rad} = 0.5 \times 16.5\ m^2 \times 5.67 \times 10^{-8} \frac{W}{m^2K^4} (323^4 - 278^4) K^4 = 2298 W$

$\dot{Q}_{total} = 2970\ W + 2298 W = 5268\ W.$

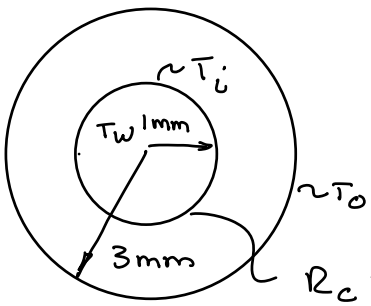
b) $\dot{Q} = \dot{m}(h_o - h_i) = \dot{m} c_p (T_o - T_i)$

$\dot{m} = \rho V A_c = 1.10 \frac{kg}{m^3} \times 4 \frac{m}{s} \times (0.35m \times 0.2m) = 0.308 \frac{kg}{s}$

$T_o = T_i + \frac{\dot{Q}}{\dot{m} c_p} = 58°C + \frac{(-5268 W)}{0.308\ kg/s \times 1008\ J/kgK}$

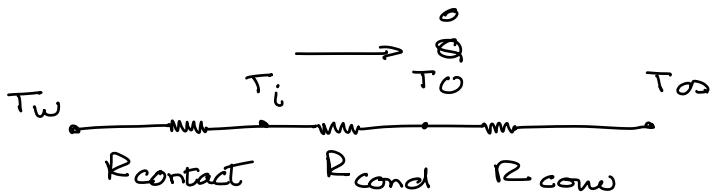
$T_o = 41°C.$

2)



$T_\infty = 20^\circ C$

$R_c = 3 \times 10^{-4}\ m^2K/W$



$$R_{cond} = \frac{\ln(r_o/r_i)}{2\pi k L} = \frac{\ln(3/1)}{2\pi \times 0.13 \frac{W}{mK} \times 1m} = 1.345 \frac{K}{W}$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{2\pi (.003m) \times 1m \times 10 \frac{W}{m^2K}} = 5.305 \frac{K}{W}$$

$$\dot{Q} = \frac{T_i - T_\infty}{R_{cond} + R_{conv}} = \frac{50^\circ C - 20^\circ C}{1.345 \frac{K}{W} + 5.305 \frac{K}{W}} = 4.51\ W$$

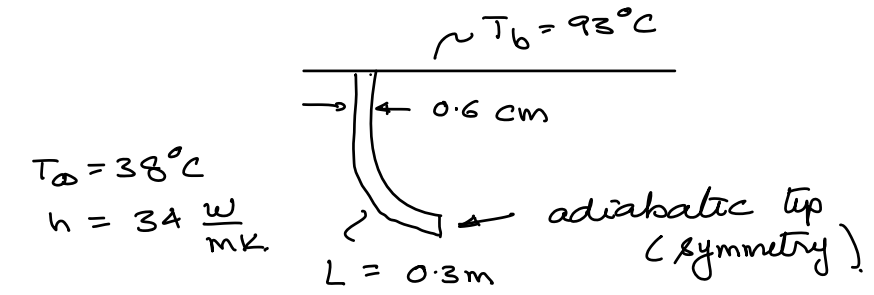
$$R_{contact} = \frac{R_c}{A} = \frac{\left(3 \times 10^{-4} \frac{m^2K}{W}\right)}{\pi (0.002m) \times 1m} = 0.0477 \frac{K}{W}$$

$$\dot{Q} = \frac{T_w - T_i}{R_{contact}} \Rightarrow T_w = T_i + \dot{Q} R_{contact}$$

$$T_w = 50^\circ C + 4.51\ W \times 0.0477 \frac{K}{W}$$

$$T_w = 50.2^\circ C$$

(3)



For adiabatic tip fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L-x)}{\cosh aL}$$

$$a = \sqrt{\frac{hP}{kA_c}} = \left(\frac{h\,\pi D}{k\,\pi D^2/4}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4 \times 34 \frac{W}{m^2K}}{396 \frac{W}{mK} \times 0.006m}\right)^{1/2} = 7.566\ m^{-1}$$

$$\frac{T(L) - 38}{93 - 38} = \frac{\cosh(0)}{\cosh(7.566 \times 0.3)}$$

$$\Rightarrow \quad T(L) = 49.2°C.$$

$$\dot{Q}_{fin,insulated} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh(aL)$$

$$= \sqrt{h(\pi D)k\frac{\pi D^2}{4}}\,(T_b - T_\infty)\tanh(aL)$$

$$= \frac{\pi}{2}(hkD^3)^{1/2}(T_b - T_\infty)\tanh(aL)$$

$$= \frac{\pi}{2}\left(34 \frac{W}{m^2K} \times 396 \frac{W}{mK} \times (.006m)^3\right)^{1/2} (93-38)°C \tanh(7.57 \times 0.3m)$$

$$= 4.56\ W$$

Total heat transfer $= 2\dot{Q}_{fin,insulated} = 9.12\ W$