1. [10 pts.] When you jump straight up as high as you can, what is the **order of magnitude of the maximum recoil speed of the earth?** In your solution, state the values you estimate for any quantities. {*Hint: The mass of the earth is about* 10^{27} *kg.*}

2. [10 pts.] Imagine a bug of mass 10.0 mg that holds onto a bicycle wheel of radius 0.350 m. If the wheel turns exactly twice per second, what is the magnitude of the **radial acceleration** and of the **tangential acceleration** experienced by the bug?

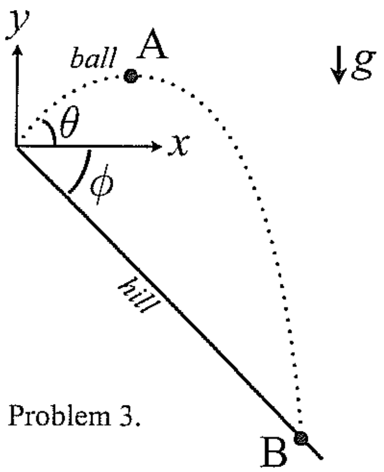
3. [40 pts.] A golf ball is hit at initial speed $v_0 = 5.00\,\mathrm{m/s}$ and angle $\theta = \pi/8$ (or 22.5 degrees) from horizontal. It flies without air resistance under gravitational acceleration $g = 9.80\,\mathrm{m/s^2}$. The ball lands down a hill that is sloped at $\phi = \pi/4$ (or 45.0 degrees) below the horizontal, as shown in the Figure below.

   (a) [15 pts.] Give the **time**, instantaneous **velocity** vector, and **acceleration** vector at the maximum height, labeled **A** in the diagram.

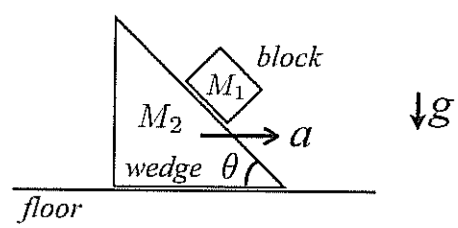
   (b) [10 pts.] Give the **average velocity** vector between launching and arriving at **A**.

   (c) [15 pts.] Give the **time** and **position** vector when the ball reaches **B**. {*Hint: Remember that* $x = -y$ *along the hill since* $\phi = \pi/4$.}

   For vector quantities, give both the **i** and **j** components. Define the initial position as $x_i = y_i = 0$ and initial time $t_i = 0$.



Problem 3.



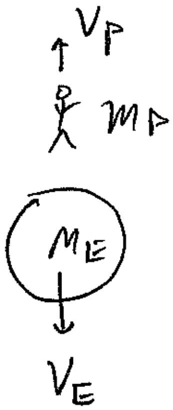
Problem 4.

4. [25 pts.] A block with mass M_1 sits on a triangular wedge with mass M_2 and angle $\theta = \pi/4$ radians (or 45.0 degrees). The ~~block~~ WEDGE accelerates at a parallel to the floor it sits on. There is static friction (coefficient μ_s) between the block and wedge, but if $a = 0$, friction is insufficient to hold up the block against gravity $\vec{g}$.

   (a) [20 pts.] What is the minimum acceleration a at which the block does not slide down? Give your answer in terms of M_1, M_2, g, and μ_s. Simplify your answer as much as possible.

   (b) [5 pts.] Assuming there is no friction between the wedge and the floor, what force F on the wedge is required to achieve the acceleration a of both block and wedge?

1)



Since total momentum conserved, $v_E = v_p \left(\frac{m_p}{M_E}\right)$

Estimate: $m_p \sim 10^2$ kg

$M_E \sim 10^{27}$ kg

$v_p \sim 10$ m/s or 1 m/s

(Check: hang time ~~[illegible]~~ ~ 0.6 s

$gt^* = 2v_p$

~~$t = \frac{2v_p}{g}$~~

$v_p \sim gt/2 \sim 3$ m/s

so 1 m/s or 10 m/s reasonable.)

STUDENTS DONT NEED TO DO THIS.

$$\to v_E \sim 3\,\text{m/s} \left(\frac{10^2\,\text{kg}}{10^{27}\,\text{kg}}\right)$$

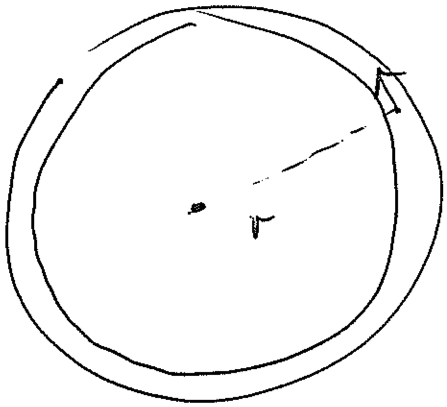
$$\sim 3 \times 10^{-25}\ \text{m/s}$$

$$\boxed{v_E \sim 10^{-24}\ \text{m/s}\ \text{OR}\ 10^{-25}\ \text{m/s}}$$

check units: $[v] =$ m/s ✓

No explicit check required

2.



$r = 0.350\ m$

$\omega = 2\pi \times \frac{2\ revs}{s}$

$= 4\pi\ (rad/s)$

$\omega = 12.57\ s^{-1}$

$\boxed{a_r = r\omega^2 = 55.3\ m/s^2}$ (within 2% ok)

$\boxed{a_t = 0}$ because $\alpha = \dot{\omega} = 0.$

Check units: $[a] = m/s^2$ ✓

not required

(3.)

Analysis model

Projectile motion under constant acceleration.

$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ & $\vec{v} = \vec{v}_0 + \vec{a}t$

We know that $\vec{r}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\hat{\imath} + 0\hat{\jmath}$

$\vec{v}_0 = v_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = v_0\cos\theta\,\hat{\imath} + v_0\sin\theta\,\hat{\jmath}$

$\vec{a} = \begin{bmatrix} 0 \\ -g \end{bmatrix} = -g\hat{\jmath}$

So: $\begin{bmatrix} x \\ y \end{bmatrix} = v_0 t \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \begin{bmatrix} 0 \\ -gt^2/2 \end{bmatrix}$, & $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = v_0 \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \Big[$

(a) When is peak height? When $v_y = 0$.

within 2% ok

Solve: $v_y = v_0\sin\theta - gt = 0 \rightarrow \boxed{t_A = \frac{v_0\sin\theta}{g}} = 0.195\,s$

check units: $\frac{m/s}{m/s^2} = s$ ✓

What is $\vec{v}$? $\boxed{\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} v_0\cos\theta \\ 0 \end{bmatrix}} = \begin{bmatrix} 4.62\,m/s \\ 0 \end{bmatrix}$

within 2% ok

What is $\vec{a}$? constant, at $\boxed{\vec{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix}}$

$= \begin{bmatrix} 0 \\ -9.80\,m/s^2 \end{bmatrix}$

(3.) cont

(b) What is average velocity at A?

$$\vec{V}_{avg} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{X}_A - \vec{X}_0}{t_A - t_0} = \frac{\vec{X}_A}{t_A}$$ since $\vec{X}_0 = 0$ here. $t_0 = 0$

What is position at A? From pt a, I know that $t_A = V_0 \sin\theta / g$.

Substitute into eqs:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} V_0 \cos\theta \\ V_0 \sin\theta \end{bmatrix} \left(\frac{V_0 \sin\theta}{g} \right) + \begin{bmatrix} 0 \\ -g/2 \end{bmatrix} \left(\frac{V_0 \sin\theta}{g} \right)^2$$

$$X = \frac{V_0^2}{g} \cos\theta \sin\theta$$

$$y = \frac{V_0^2}{g} \sin^2\theta - \frac{V_0^2}{2g} \sin^2\theta$$

$$= \frac{V_0^2}{2g} \sin^2\theta$$

$$\vec{X}_A = \begin{bmatrix} \cos\theta \\ \frac{1}{2}\sin\theta \end{bmatrix} \frac{V_0^2 \sin\theta}{g}$$

Divide: $\boxed{\vec{V}_{avg} = V_0 \begin{bmatrix} \cos\theta \\ \frac{1}{2}\sin\theta \end{bmatrix}} = \begin{bmatrix} 4.62 \text{ m/s} \\ 0.957 \text{ m/s} \end{bmatrix}$

within 2% ok.

Check units: [m/s] ✓

③, cont

(c) When does ball reach B? when $-x = y$.

Solve: $$-v_0 t\cos\theta = v_0 t\sin\theta - gt^2/2$$

$$-v_0(\cos\theta+\sin\theta) = -gt/2$$

$$\boxed{t = \frac{2v_0}{g}(\sin\theta+\cos\theta)} = 1.33\,s$$

within 2% OK

units: $\frac{m/s}{m/s^2} = s$ ✓

What is position at this time? $x = y$, so can just write down one of them.

$$x = v_0 t\cos\theta$$

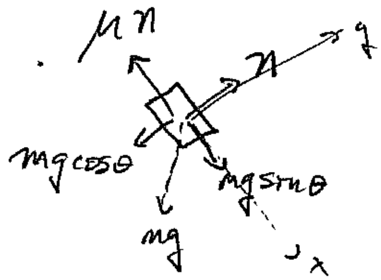
$$\boxed{x = \frac{2v_0^2}{g}(\sin\theta+\cos\theta)\overset{\cos}{\cancel{\sin}}\theta = y} = 6.16\,m$$

within 2% OK

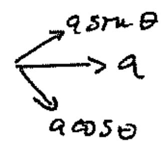
units: $\frac{(m/s)^2}{m/s^2} = m$ ✓

4.

(a) FBD of block:



Analysis model: accelerating along both coordinates as drawn:



x: $mg\sin\theta - \mu n = ma\cos\theta$

y: $n - mg\cos\theta = ma\sin\theta$ $\longrightarrow$ solve for n:

$$n = m(a\sin\theta + g\cos\theta)$$

Solve for a, after substitution for n:

$$mg\sin\theta - \mu(ma\sin\theta + mg\cos\theta) = ma\cos\theta$$

Since $\theta = \pi/4$, all $\sin\theta = \cos\theta$ & cancel.

Also all m's cancel out:

$$g - \mu a - \mu g = a$$

solve for a:

$$(1-\mu)g = a(1+\mu) \longrightarrow \boxed{a = g\,\frac{1-\mu_s}{1+\mu_s}}$$

4 cont

(b) For the total block + wedge system,

$$F = (M_{TOT})\, a$$

$$= (M_1 + M_2)\, a$$

So $\boxed{F = (M_1 + M_2)\, a}$

$\boxed{F = (M_1 + M_2)\, g \frac{1-\mu_s}{1+\mu_s}}$

either answer ok.

Also, if got wrong a in problem 4a, but substitute correctly here, give full credit for 4b.