UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

October 31, 2019

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Closed book.

All questions are of equal value.

Permitted calculators (with ANY suffixes):

- Sharp EL-520
- Casio FX-991

This test contains 20 pages including this page and the cover page, printed two-sided. Do <u>not</u> tear any pages from this test. Present complete solutions in the space provided.

Given information:

$$cos\,\theta = \frac{\vec{u}\cdot\vec{v}}{||\vec{u}||\,||\vec{v}||}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

$$proj_{\vec{d}}\,\vec{u} = \frac{\vec{u}\cdot\vec{d}}{||\,\vec{d}\,||^2}\,\vec{d}$$

$$det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

The inverse of a 2x2 matrix given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is equal to:

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Q1:

a) Find the solution to the system of equations given below:

$$x - 2y = 0$$

$$x + y = 6$$

b) Draw the row picture of the two equations in part (a), indicate row 1 and row 2 in your picture, and show the solution in this row picture as a point of intersection of row 1 and row 2.

c) Draw the column picture of the two equations in part (a), indicate column 1 and column 2 in your picture, and show the solution in this column picture as a linear combination of column 1 and column 2.

Q2:

a) Find a unit vector $\vec{u}$ that has the same direction as $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

b) Find a unit vector $\vec{v}$ that is orthogonal to the unit vector $\vec{u}$ in part (a).

c) How many possibilities are there for $\vec{v}$ in part (b)? Give a geometric explanation of your answer.

Q3:

a) Project the vector $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$ onto the line that passes through the origin and the point $P_1 = (2,2,1)$.

b) Project the vector $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$ onto the plane that passes through the origin and contains both the point P_1 from part (a) and $P_2 = (1,0,0)$.

c) Express the vector $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$ as the sum of two orthogonal vectors, where one of the vectors is parallel to the plane in part (b).

Q4:

a) The basic equation associated with the eigenvalue/eigenvector problem in linear algebra is given by $A\vec{\omega} = \lambda\vec{\omega}$, where A is a square matrix and λ is a scalar. What is the relationship between the eigenvalues of A and A^2?

b) Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$. Find the eigenvalues for both A and A^2 and use the result to verify the relationship developed in part (a).

Blank page for the continuation of answer to Q4 part (b).

Q5:

a) Find the $4x4$ matrix P that transforms the vector $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$ to $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$.

b) Calculate the matrix P^2.

c) By inspection of the transformation in part (a), determine the exponent n where P^n is the inverse of matrix P. Using your value for n, show that P^n is the inverse of P.

Q6:

a) Consider two planes in R^3 with the following scalar equations:

$$x + y + 3z = 6$$

$$x - y + z = 4$$

Put this system of linear equations in its reduced normal form.

b) From your answer in part (a), write the solution to the system in vector form.

c) What is the geometric interpretation of the solution in part (b)?

d) Does the solution in part (b) go through a point in R^3 with $y = 2$? If so, what are the corresponding values of x and z?