ESC103: Lab 4

```matlab
%Clean up workspace
clc; close all; clear;

%Set up BCs
T_0 = 300;
T_10 = 400;

%Solve for three different n values
n_1 = 5; n_2 = 10; n_3 = 50;
[T_1, x_1] = solve_bvp(n_1, T_0, T_10);
[T_2, x_2] = solve_bvp(n_2, T_0, T_10);
[T_3, x_3] = solve_bvp(n_3, T_0, T_10);

%Analytic solution
x_exact = 0:0.01:10;
T_exact = 200 + 20.4671*exp(sqrt(0.05)*x_exact) + 79.5329*exp(-sqrt(0.05)*x_exact);

%Plot!
figure
plot(x_1, T_1, '--ok', 'LineWidth', 1.25);
hold on
plot(x_2, T_2, '--ob', 'LineWidth', 1.25);
plot(x_3, T_3, '-or', 'LineWidth', 1.25);
plot(x_3, T_3, '-g', 'LineWidth', 1.25);

%Annotations
title('BVP: Rod Temperature')
ylabel('Temperature (K)')
xlabel('Axial Position (m)')
legend('n=5', 'n=10', 'n=50', 'Exact', 'Location', 'NorthWest')
grid on
```
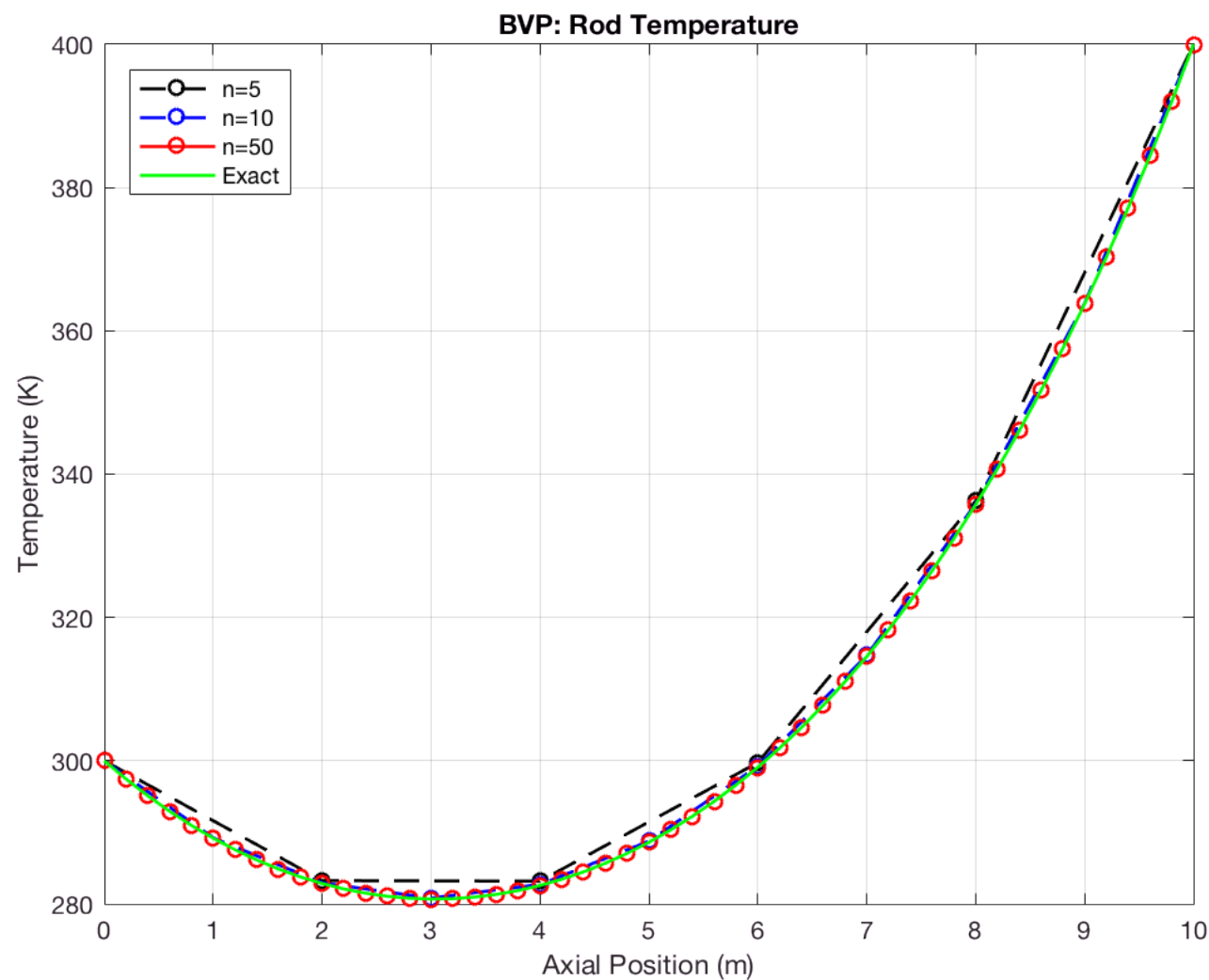
BVP: Rod Temperature

```matlab
% For this BVP, n=50 provides good accuracy,
% while being computationally efficient
```

```matlab
function [T, x] = solve_bvp(n, T_0, T_10)
    %Define constants
    h_p = 0.05;
    T_inf = 200;

    dx = 10/n;
    x = 0:dx:10;

    %k1*T(x-dx) + k2*T(x) + k3*T(x+dx)
    k1 = 1/dx^2;
    k2 = -2/dx^2 - h_p;
    k3 = 1/dx^2;

    %Compose A by exploiting its tridiagonal structure
    %For n subdivisions, there are n-1 interior points
    A = diag(k1*ones(n-2,1), -1) + diag(k2*ones(n-1,1), 0) + diag(k3*ones(n-2,1), 1);
    b = -h_p*T_inf*ones(n-1,1);

    %Boundary conditions
    b(1) = b(1) - T_0*k1;
    b(end) = b(end) - T_10*k3;

    T_interior = A\b;
    T = [T_0; T_interior; T_10]; %Include boundary conditions in the final solution
end
```