



University of Toronto
Faculty of Applied Science and Engineering

First name (please write as legibly as possible within the boxes)

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Last name

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Student number

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ESC194F Calculus I
Final Exam
December 2022

No calculators or aids
There are 12 questions, each question is worth 10 marks

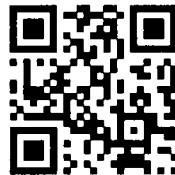
Examiners: P.C. Stangeby and J.W. Davis



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- 1) a) Suppose that $\lim_{t \rightarrow \infty} f(t) = \infty$. Can $f(t)$ ever decrease? If so, sketch a graph of an example. If not, why not?
- b) Suppose $\frac{4t-3}{t} \leq f(t) \leq \frac{4t^5+7}{t^5-3}$ when $t > 92$. Determine $\lim_{t \rightarrow \infty} f(t)$.
- c) Design a rational function for which $\lim_{t \rightarrow 4} f(t)$ is a $\frac{0}{0}$ indeterminate form, but the actual value of the limit is $\sqrt{2}$.
- d) Sketch graphs of function f and g ($\neq 0$) so that $\lim_{t \rightarrow 0} f(t)g(t)$ exists, but $\lim_{t \rightarrow 0} f(t)$ does not.
- e) Suppose $|f(x) - 7| < 10^{-3}$ whenever $0 < |x - 5| < 0.0001$. What do we know about $\lim_{x \rightarrow 5} f(x)$?



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2) Find the limits (Do not use l'Hospital's rule):

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 2x - 3}$

(b) $\lim_{x \rightarrow 1^+} \frac{x^2 - 2}{x^2 + 2x - 3}$

(c) $\lim_{x \rightarrow 2^+} \frac{2 - x}{|2 - x|}$

(d) $\lim_{x \rightarrow 0} \sin(x - 1 + \cos x)$

(e) $\lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x^2}}{x}$



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3) Calculate $\frac{dy}{dx}$ for:

(a) $y = 3x^2$

(b) $y = 3/x^2$

(c) $y = \frac{3+x^2}{2-x}$

(d) $y = \sin^2(x^3)$

(e) $x^2y^2 + x\cos y = 2.$



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- 4) Provide a rigorous proof (i.e. a $\delta - \varepsilon$ proof) that $\lim_{x \rightarrow 3} (x^2 - x - 6) = 0$.



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- 5) Find the dimensions of the rectangle of largest area that has its base on the y -axis and its other two vertices on the right side of the y -axis and lying on the parabola $x = 9 - y^2$.



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- 6) Sketch the graph of the function $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$ noting any maximum and minimum points, points of inflection, vertical tangents, and asymptotes, as well as intervals of increase and decrease, convexity and concavity.



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- 7) Let \mathfrak{R} be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 3x - 2x^2$.

Provide a sketch of the region, and calculate the following quantities:

- (i) The area of \mathfrak{R}
- (ii) The volume obtained by rotating \mathfrak{R} about the x -axis; use the washer method.
- (iii) The volume obtained by rotating \mathfrak{R} about the y -axis; use the shell method.



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8) a) Find the particular solution to the differential equation: $x^2y' = y - xy$, $y(-1) = -1$

b) Evaluate the integrating factor, and use it to solve the differential equation: $xy' + y = \sqrt{x}$



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- 9) Use the method of undetermined coefficients to find the general solution of the differential equation:

$$y'' + 6y' + 9y = 16 e^{-x} \cos 2x$$



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- 10) A clock's minute hand has length 4 and its hour hand has length 3. What is the distance between the tips of the two hands at the moment when the distance is increasing most rapidly?



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- 11) a) Given a differentiable function $f(x)$ which has a bounded derivative $|f'(x)| \leq k$, show that:

$$\left| \int_0^1 f(x) dx - \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} \right| \leq \frac{k}{2n}$$

- b) Given a function $f(x)$ with a continuous 1st derivative, and $f(0) = 0$. If $|f'(x)| \leq |f(x)|$ for all x , show that f must be a constant.



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- 12) let $f(x)$ be an increasing real-valued function defined for $x \geq 0$, for which $f(0) = 0$. Let its inverse be $f^{-1}(x)$. Given a and b in the domains of f and f^{-1} , respectively, show that:

$$\int_0^a f(x) dx + \int_0^b f^{-1}(x) dx \geq ab$$



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