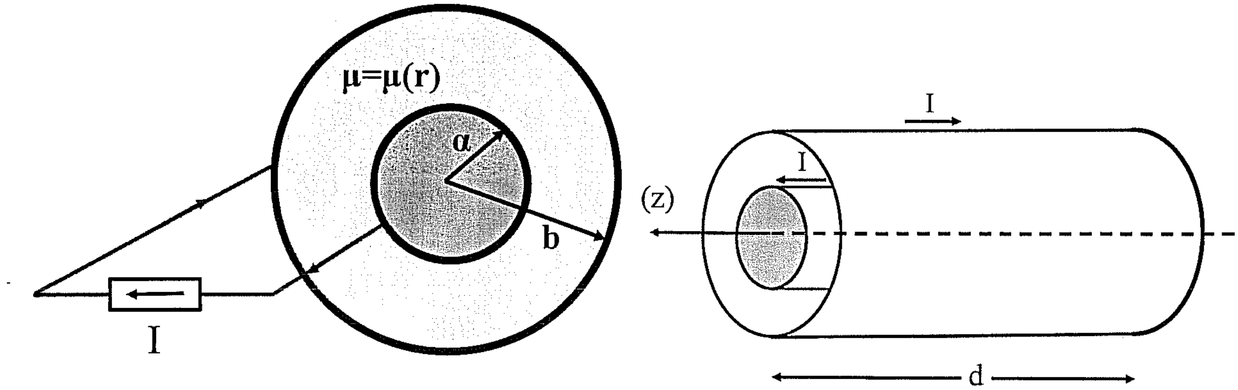
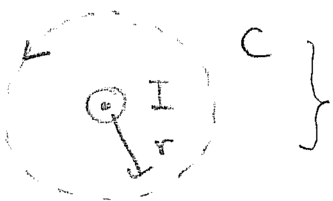
Question 2

1. Two perfectly conducting cylinders $r = \alpha$ and $r = b$ form an inductor of length d, as shown in the figure below. The total current I flowing on the surface $r = \alpha$ of the inner cylinder is returned through the outer conductor $r = b$. The medium between the two conductors has magnetic permeability $\mu = \mu_m \frac{r}{b}$.



a) Find the magnetic field intensity **H** everywhere, disregarding "edge effects" (i.e. assuming that the length d of the conductors is very large compared to α, b and hence, it can be considered as infinite). **(4 points)**

Applying Ampere's Law on circular paths, since $\underbrace{\bar{H} = H_\varphi(r)\,\bar{a}_\varphi}_{1\,pt}$ (by cylindrical sym)



1 pt

$$\underbrace{\oint_C \bar{H}\cdot d\bar{l}}_{1\,pt} = I \Rightarrow H_\varphi \cdot 2\pi r = I \Rightarrow \boxed{H_\varphi = \frac{I}{2\pi r}}$$

1 pt

b) Determine the inductance L using the following two methods and confirm that they both result in the same answer:

- Applying the definition $L = \Phi/I$, where Φ is magnetic flux through the inductor. **(5 points)**
- Finding the total magnetic energy W_m stored in the inductor and using the relation $L = 2W_m/I^2$. **(5 points)**

Method 1

$$\bar{B} = \mu_m \frac{\cancel{r}}{b} \frac{I}{2\pi \cancel{r}} \bar{a}_\varphi = \frac{\mu_m I}{2\pi b} \bar{a}_\varphi \qquad (1\,pt)$$

$$\underbrace{\Phi = \int \bar{B} \cdot d\bar{S}}_{(1pt)} = \int \frac{\mu_m I}{2\pi b} \overbrace{\bar{a}_\varphi \cdot \bar{a}_\varphi}^{=1} \underbrace{dr\, dz}_{(1pt)}$$

$$= \frac{\mu_m I}{2\pi b} \underbrace{\int_a^b dr \int_0^d dz}_{\text{Limits } (1pt)} = \left(\frac{\mu_m}{2\pi} \frac{b-a}{b} \cdot d \right) \cdot I$$

$$\Rightarrow \quad L = \Phi / I = \frac{\mu_m}{2\pi} \frac{b-a}{b} d \Big\} \; (1\,pt)$$

limits : 1.5pt

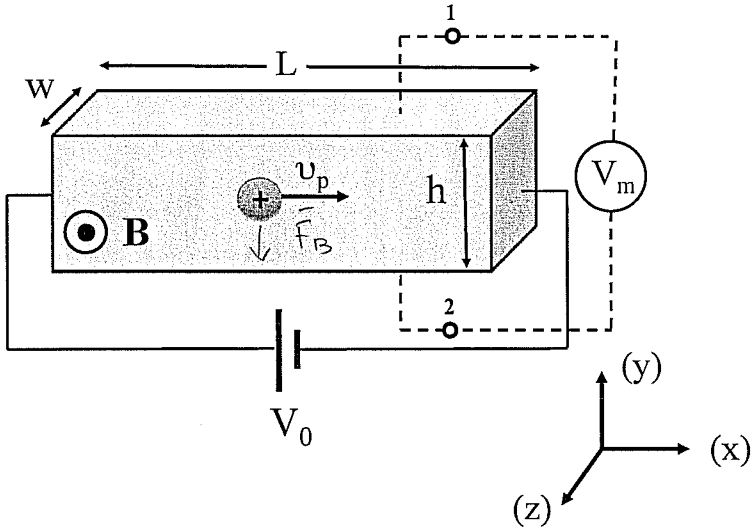
Method 2 (1pt)

$$W_m = \int \frac{1}{2} \mu |H|^2 dv = \frac{1}{2} \frac{\mu_m}{b} \int_{r=a}^{b} \int_{\varphi=0}^{2\pi} \int_{z=0}^{d} \frac{I^2 \cdot \cancel{r}}{4\pi^2 \cancel{r^2}} \underbrace{\cancel{r}\, d\varphi\, dr\, dz}_{dv\ (1.5pt}$$

$$= \frac{1}{2} \frac{\mu_m}{b} \frac{I^2}{4\pi^2} (b-a) \cdot 2\pi \cdot d \Rightarrow$$

$$L = \frac{2W_m}{I^2} = \frac{\mu_m}{2\pi} \frac{b-a}{b} \cdot d \Big\} \text{ Final result: } (1\,pt)$$

2. In a p-type semiconductor, the main charge carriers are holes of positive charge q_p. Consider a rectangular slab of such a material, with dimensions $L \times w \times h$, as shown in the figure below.



The slab is connected to a dc-voltage source V_0, which creates an electric field forcing the charge carriers to move with a velocity that is proportional to the electric field created by the source, as $\mathbf{u}_p = \mu_p \mathbf{E}$, where μ_p is the mobility of the carriers. A constant magnetic field $\mathbf{B} = B_0 \mathbf{a}_z$ is also applied as shown.

Can we use a voltmeter, connected as shown and measuring the voltage $V_m = V_1 - V_2$, to measure μ_p? If yes, give an expression of μ_p in terms of all known quantities and state any assumptions that you used. If not, why not? **(6 points)**

Assumption: electric fields develop almost uniformly due to

V_0: $\bar{E}_0 = \bar{a}_x \frac{V_0}{L}$ (1pt)

Hall effect voltage V_m: $\bar{E}_H = -\bar{a}_y \frac{V_m}{h}$. (1pt)

q_p receives force $\bar{F}_B = q u_p \bar{a}_x \times B_0 \bar{a}_z = \underbrace{-\bar{a}_y q_p u_p B_0}_{(1pt)}$

which moves positive carriers DOWNWARDS $\Rightarrow$ electric field starts developing in the $+\bar{a}_y$ direction. Steady-state:

$q\bar{E}_H + \bar{F}_B = 0 \Rightarrow \bar{E}_H = -\frac{1}{q_p}\bar{F}_B = \underbrace{+\bar{a}_y u_p B_0}_{2pts} =$

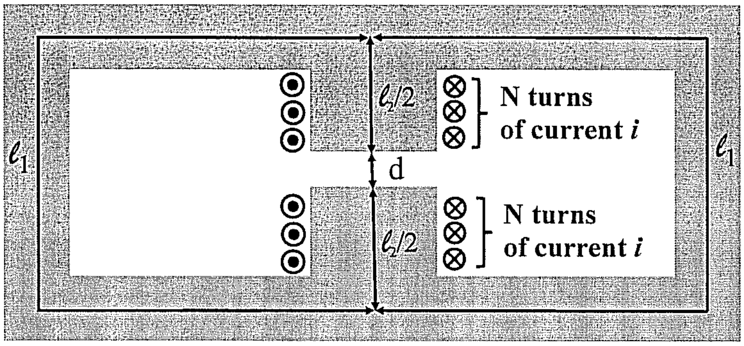
$$= \bar{a}_y \ \mu_p E_0 \cdot B_0 = \bar{a}_y \ \mu_p \cdot \frac{V_0}{L} \cdot B_0$$

This field produces a $V_m = - \mu_p \cdot \frac{V_0}{L} \cdot B_0 \cdot h$

$$\Rightarrow \boxed{\mu_p = - \frac{V_m \cdot L}{V_0 \cdot B_0 \cdot h}} \qquad \underline{(1\,pt)}$$

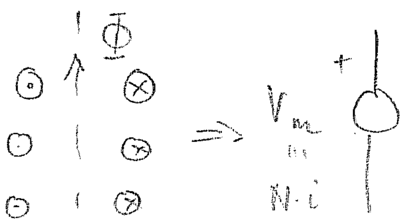
Question 3

Consider the following magnetic circuit:



The approximate lengths traced by the magnetic flux within the high-permeabilty ferromagnetic core (of permeability $\mu_0\mu_r$) and in the air gap (of permeability μ_0) are shown in the figure. All circuit segments have cross-section S, except for the middle segment, which has cross-section $2S$. Two $N-$turn coils are wrapped around the upper and the lower part of this middle segment, generating the flux that circulates in this circuit.

1. Draw the equivalent circuit and determine the magnetomotive forces (magnitude and polarity) and reluctances of this circuit. **(4 points)**



Coils produce flux pointing upwards (1 pt)

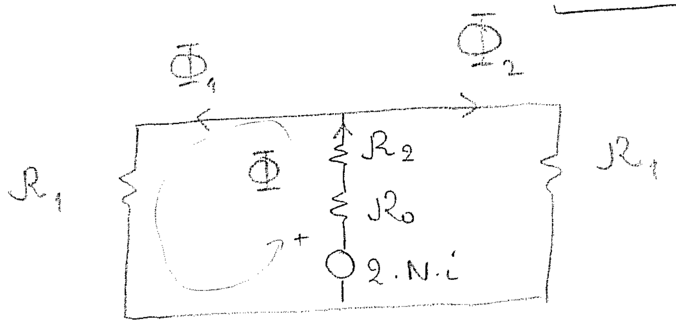
Reluctances:

1pt $\boxed{\mathcal{R}_1 = \frac{l_1}{\mu_c \cdot S}}$ for left/right branch

$\mathcal{R}_2 = \frac{l_2}{\mu_c (2S)}$ for middle branch

$\mathcal{R}_0 = \frac{d}{\mu_0 (2S)}$ for air gap

(2pts)



(1pt)

2. Show that the magnitude of the magnetic flux density B is the same throughout the circuit. **(4 points)**

(1 pt) 1 pt

Note that $\boxed{\Phi_1 = \Phi_2}$ (left/right branch have equal reluctances) and $\Phi = \Phi_1 + \Phi_2 \Rightarrow \boxed{\Phi_1 = \Phi_2 = \Phi/2}$

But: if B_{left} is the B in the left branch, B_{right} the B in the right branch B_{middle} the B in the middle:

1pt 1pt

$\Phi = B_{middle} \cdot 2S$, $\Phi_1 = \Phi_2 = B_{left} \cdot S = B_{right} \cdot S = \dfrac{B_{middle} \cdot 2S}{2}$

$\Rightarrow \boxed{B_{right} = B_{left} = B_{middle} \equiv B}$ everywhere

3. Show that the relation between the magnetic flux density B and the magnetic field intensity H inside the core is:

$$B = -\frac{\mu_0}{d}(\ell_1 + \ell_2) H + \frac{2Ni\mu_0}{d}$$

(4 points)

A) From circuit analysis: (2 pts)

KVL on left branch: $\boxed{\Phi_1 \mathcal{R}_1 - 2N \cdot i + \Phi \cdot (\mathcal{R}_0 + \mathcal{R}_2) = 0}$

$\Rightarrow B \cdot S \cdot \dfrac{\ell_1}{\mu_c S} - 2N \cdot i + B \cdot \cancel{2} \cdot \cancel{S} \dfrac{d}{\mu_0 \cdot 2 \cdot S} + B \cdot \cancel{2} \cdot \cancel{S} \cdot \dfrac{\ell_2}{\mu_c \cancel{2S}} = 0$

(Correct substitution of B, H : 2pts)

$B/\mu_c = H$

$\Rightarrow H \cdot (\ell_1 + \ell_2) - 2N \cdot i + B \cdot \dfrac{d}{\mu_0} = 0 \Rightarrow$

$B = \dfrac{\mu_0}{d}\left[2N \cdot i - H(\ell_1 + \ell_2)\right]$ 2pts

B) From Ampere Law (left branch): $\oint \bar{H} \cdot d\bar{l} = 2N \cdot i \Rightarrow$

$H_g d + \underbrace{H \cdot (\ell_1 + \ell_2)}_{} = N \cdot i \Rightarrow H \cdot d = N \cdot i - H(\ell_1 + \ell_2) \Rightarrow$

$\Rightarrow \dfrac{B}{\mu_0} d = Ni - H(\ell_1 + \ell_2)$

(1pt)

$H = \dfrac{B}{\mu_0}$ (1 pt)

4. The ferromagnetic material of the core has a hysteresis curve that can be approximated by the graph shown below. The curve indicates that the relative magnetic permeability of the core is a function of the magnetic field intensity H, i.e. $\mu_r = \mu_r(H)$.

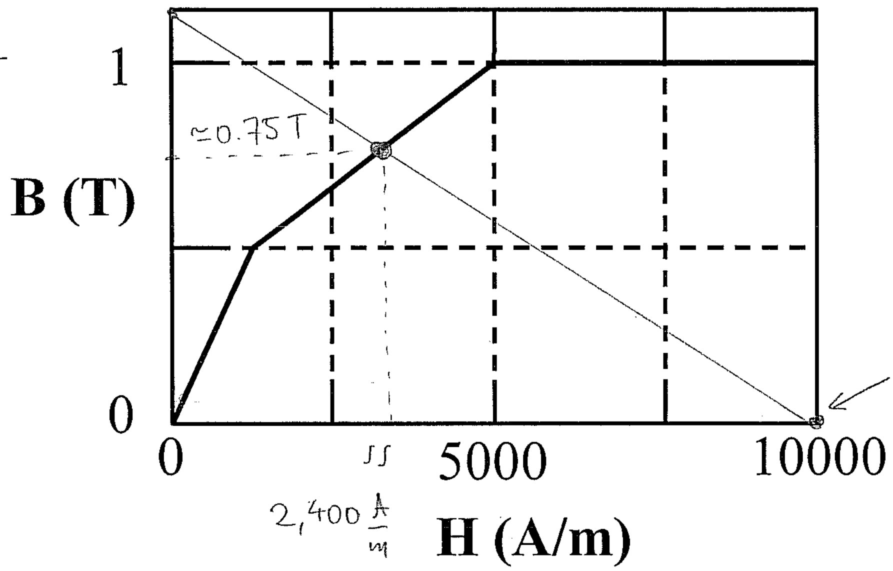
   Can you graphically determine the point on this curve that corresponds to the circuit for $N = 500$, $d = 1$ cm, $\ell_1 = 0.8$ m, $\ell_2 = 0.2$ m and $i = 10$ A? Hint: Use the $B - H$ relation of the previous question. Also, $\mu_0 = 4\pi \times 10^{-7}$ H/m. **(4 points)**

$H = 0 \Rightarrow$

$B = 0.4\pi$ T

$\simeq 1.2$ T

(1pt)



(1pt)

$B = 0$ for

$H = 10{,}000$

$B =$

Using $B = -\dfrac{\mu_0}{d}(\ell_1 + \ell_2) H + \dfrac{2 N \cdot i\, \mu_0}{d}$

$$\frac{\mu_0 (\ell_1 + \ell_2)}{d} = \frac{4\pi \times 10^{-7}}{10^{-2}\ \text{m}} \cdot 1\,\text{m} = 4\pi \times 10^{-5}$$

$$\frac{2 \cdot N \cdot i \cdot \mu_0}{d} = \frac{2 \times 500 \times 10 \times 4\pi \times 10^{-7}}{10^{-2}} = 4\pi \times 10^{-1}$$

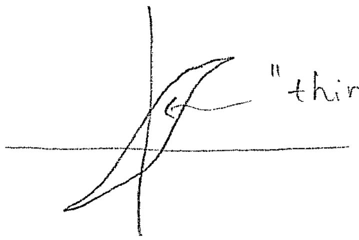
$$B = -4\pi \times 10^{-5} H + 0.4\pi = -4\pi \times 10^{-5} \left(H - 10^{4}\right)$$

$\left.\begin{array}{l}\text{The intersection of this } \underline{\underline{\text{line}}} \text{ with the B-H curve} \\ \text{gives the operating point.}\end{array}\right\}$ 2 pts.

5. If the current i is alternating (a-c), which are the main sources of energy losses in this circuit? How can these losses be mitigated ? **(4 points)**

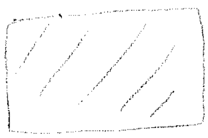
Hysteresis losses

Mitigation: choose "soft" magnetic material

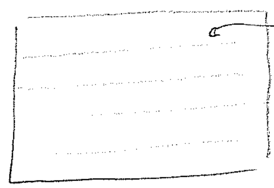


"thin" hysteresis curve

Eddy current losses ⇒ use filamentary cores of mutually insulated filaments



uniform core



insulating layer

2 pts: loss factors

2 pts: Mitigation.