

**UNIVERSITY OF TORONTO**  
**FACULTY OF APPLIED SCIENCE AND ENGINEERING**  
January 31<sup>st</sup> 2023 — **Duration: 90 minutes**  
First Year, MSE160 MOLECULES AND MATERIALS  
Exam Type: B - Closed Book, aid sheets provided. Calculator from Faculty approved list  
permitted.  
Examiners: J Nogami, SD Ramsay

**IMPORTANT:**

- On this page, write your name and student number clearly.
- On the computer answer form, complete all sections fully.
- Do not open this test booklet until instructed to do so.
- Numerical responses must be expressed in most appropriate units, including appropriate prefixes (ex. GPa rather than 10<sup>9</sup> Pa), and be written with appropriate significant figures.

1. (10 points) Various sources online quote the density of Vibranium, the fictional metal from Marvel Comics, as being close to  $3.9 \text{ g cm}^{-3}$ . If we assume that Vibranium has the face centred cubic crystal structure and a molar mass equal to that of titanium (Ti), what is the atomic radius of Vibranium, in pm. One pm is  $10^{-12} \text{ m}$ .

$$\rho = \frac{nA}{V_c N_A} \quad | \quad V_c = a^3 \quad | \quad a = 2\sqrt{2}R$$

$$\rho = \frac{nA}{(2\sqrt{2})^3 R^3 \cdot N_A} \rightarrow R = \left( \frac{nA}{(2\sqrt{2})^3 e \cdot N_A} \right)^{1/3} \quad | \quad n = 4 \quad | \quad A = 47.9 \quad | \quad \rho = 3.9 \frac{\text{g}}{\text{cm}^3} \quad | \quad N_A = 6.022 \cdot 10^{23}$$

$$R = \frac{1}{(2\sqrt{2})} \left( \frac{4 \cdot 47.9 \cdot 10^{30}}{3.9 \cdot 6.022 \cdot 10^{23}} \right)^{1/3} = \frac{153}{1} \frac{\text{pm}}{1}$$

work (2)



2. You are selecting a material to use in the manufacture of a light weight cylindrical tensile tie (component that supports a uniaxial tensile load). For this particular application, the tie must not fracture below a specified load, F.

- (a) (1 point) Write the equation for the mass of this cylindrical tie, having diameter D, length L and density  $\rho$ . (Hint: don't over think this.)

$$\textcircled{1} \quad m = \frac{\pi D^2}{4} \cdot L \cdot \rho \quad \text{or} \quad m = A \cdot L \cdot \rho$$

- (b) (1 point) Write the equation for the constraint, namely the stress in the tie.

$$\textcircled{1} \quad \sigma = \frac{F}{A_0} \quad \text{or} \quad \sigma = \frac{F \cdot 4}{\pi D^2}$$

- (c) (1 point) Rearrange the constraint equation so that the free variable is on the left hand side of the equality.

$$\textcircled{1} \quad A_0 = \frac{F}{\sigma} \quad \text{or} \quad D = \left( \frac{4F}{\pi \sigma} \right)^{1/2}$$

- (d) (2 points) Substitute the free variable equation above into the mass equation and arrange into functional, geometric, and materials variables.

$$\textcircled{1} \quad m = \frac{F}{\sigma} \cdot L \cdot \rho \rightarrow m = (F)(L)\left(\frac{\rho}{\sigma}\right) \textcircled{1} \quad \text{correct grouping.}$$

- (e) (1 point) Write the materials performance index for this strong light tie, in the form to be maximized.

$$\text{MPI} = \frac{\sigma}{\rho} \cdot \textcircled{1}$$

3. (5 points) You are hired by Cambridge University to build a footbridge across the 15.4m span of the river Cam, adjacent to the famous Mathematical Bridge. This bridge is to be a modern design, made entirely of chemically tempered glass having a bending strength of 300 MPa and a width of 2.0m. If the bridge must be able to support 8 people at the middle of the span, each weighing 75kg, determine what the thickness of the bridge must be. Assume the bridge is a rectangular cross-section and the railings do not support any load.

$$\textcircled{1} \quad \sigma = \frac{3FL}{2bd^2} \Rightarrow d = \left( \frac{3FL}{2b\sigma} \right)^{1/2} \quad \left| \begin{array}{l} F = 8 \cdot 75 \cdot 9.81 = 5886 \text{ N} \\ L = 15.4 \text{ m} \\ b = 2 \text{ m} \\ \sigma = 300 (10^6) \text{ Pa} \end{array} \right. \textcircled{1}$$

$$d = \left[ \frac{3 \cdot 5886 \cdot 15.4}{2 \cdot 2 \cdot 300 (10^6)} \right]^{1/2} = 1.5 (10^{-2})$$

$$\boxed{\text{thickness} = \frac{1.5}{\textcircled{1} \textcircled{1}} \text{ cm}}$$

other length units okay.

4. (10 points) A circular cross-section tensile specimen having an initial diameter of 3 cm is produced from an aluminum alloy having a yield strength (the maximum stress where the elongation is still elastic) of 325 MPa, a Poisson's ratio of 0.32, and a Young's modulus of 69 GPa. What will be the diameter of this sample while a tensile load of 125 kN is applied?



$$\epsilon_D = \frac{D_f - D_0}{D_0} \Rightarrow D_f = D_0 + D_0 \epsilon_D \quad | \quad \nu = \frac{-\epsilon_D}{\epsilon_Z} \Rightarrow \epsilon_D = -\nu \epsilon_Z$$

$$\sigma = E \cdot \epsilon_Z \Rightarrow \epsilon_Z = \frac{\sigma}{E} \quad | \quad \sigma = \frac{F}{A_0} \quad | \quad A_0 = \frac{\pi D_0^2}{4}$$

$$\sigma = \frac{4F}{\pi D_0^2} \rightarrow \epsilon_Z = \frac{4F}{\pi D_0^2 \cdot E} \rightarrow \epsilon_D = -\nu \cdot \frac{4F}{\pi D_0^2 \cdot E}$$

$$\rightarrow D_f = D_0 - D_0 \cdot \frac{4\nu F}{\pi D_0^2 \cdot E} = D_0 - \frac{4\nu F}{\pi D_0 \cdot E} \quad | \quad F = 125(10^3) \\ \nu = 0.32 \\ D_0 = 3(10^{-2}) \text{ m} \\ E = 69(10^9) \text{ Pa}$$

confirm elastic :  $\sigma = \frac{4 \cdot 125(10^3)}{\pi (3(10^{-2}))^2} = 177 \text{ MPa} < 325 \text{ MPa}$

$$\left. \begin{aligned} D_f &= 3(10^{-2}) - \frac{4 \cdot 0.32 \cdot 125(10^3)}{3.1415926 \cdot 3(10^{-2}) \cdot 69(10^9)} = 3(10^{-2}) - 2.46(10^{-2}) \\ &= 2.9975(10^{-2}) \text{ m} = 3.0(10^{-2}) \text{ m} = \frac{3.0 \text{ cm}}{\text{①}} \end{aligned} \right.$$

Other length units okay.

**Part B: Multiple Choice (1 points per question)**

1. Which of the following is the best description of a material property?
  - (a) A property of a material that is independent of microstructure.
  - (b)** A property of a material that is independent of sample size.
  - (c) A property of a material that can be measured.
  - (d) A property of a material that varies with the amount of the material.
2. Which of the following would ~~not~~ be described as having long range order?
  - (a) Methane gas in the gas phase.
  - (b)** A polycrystalline metal.
  - (c) Window glass.
  - (d) A fully amorphous polymer.
3. Which of the following can be thought of as an accurate description of the face centred cubic crystal structure?
  - (a)** One eighth of an atom situated at each corner of the unit cell and one half of an atom situated at the centre of each face.
  - (b) One quarter of an atom situated at each corner of the unit cell, one whole atom situated at the centre of each face, and one atom in the centre of the unit cell.
  - (c) One eighth of an atom situated at each corner of the unit cell, one half of an atom situated at the centre of each face, and one atom in the centre of the unit cell.
  - (d) One quarter of an atom situated at each corner of the unit cell and one whole atom situated at the centre of each face.

4. Three samples of the same metal alloy are prepared. Sample A is mechanically deformed and carefully heat treated so that its yield strength is twice that of sample B. Sample C is placed in a furnace for several hours so that its yield strength is half that of sample B. What would you expect to be true of the Young's moduli of these three samples to be?
- (a) Sample A will be slightly higher than both sample B and C, but B and C will be the same.
  - (b) Sample C will be slightly higher than sample B, which will be slightly higher than sample A.
  - (c) Sample A will be slightly higher than sample B, which will be slightly higher than sample C.
  - (d) They will all be the same.**
5. Which of the following is generally true of the mechanical behaviour of a ceramic?
- (a) Shows no permanent deformation.**
  - (b) Generally has a lower Young's modulus than polymers.
  - (c) Are typically poor in compression and strong in tension.
  - (d) Typically have mass density that is less than  $1\text{ g cm}^{-3}$
6. Describing the interatomic force-separation relationship between atoms in a solid using a mechanical spring analogy is useful for all of the following except?
- (a) Justifying that the Young's modulus is structure independent.
  - (b) Describing the limit of elastic behaviour, at large values of r.**
  - (c) Describing the state of equilibrium between attractive and repulsive interatomic forces.
  - (d) Describing the atomic nature of the Young's modulus.
7. Which of the following is the most likely consequence of attempting to test a ceramic sample in tension?
- (a) A very high fracture load that may be beyond the limit of the machine.
  - (b) Failure at low strain contributing to significant shear loading.**
  - (c) A slow failure that may occur beyond the elongation limits of the machine.
  - (d) Breaking the testing machine.

8. Which of the following is generally true of the mechanical behaviour of a polymer?

- (a) Has a Young's modulus lower than either of metals and ceramics.
- (b) Has a mass density similar to metals.
- (c) Has lower total strain to fracture than ceramics.
- (d) As a material class, has the narrowest set of Young's modulus and mass density values of all three material classes.

9. Which of the following is the best description of the difference between elastic and plastic deformation?

- (a) In plastic deformation atoms always return to their original positions and increased equilibrium interatomic spacing.
- (b) In plastic deformation atoms move to new positions, and slightly increased equilibrium interatomic spacing.
- (c) In plastic deformation atoms always return to their original positions and original equilibrium interatomic spacing.
- (d) In plastic deformation atoms move to new positions, but remain at the same equilibrium interatomic spacing.

10. An hypothetical metal alloy supports a stress of 562 MPa and experiences a total strain of ~~0.02~~<sup>0.02</sup> while this load is applied. When the load is removed the sample is observed to return to its original length. What is the Young's modulus of this metal?

- (a) 19 GPa
- (b) 43 GPa
- (c) 28 GPa
- (d) 12 GPa