I. 2015 Modern Physics Mid-Term (PHY 293)

Q.1. (10 marks) Correct answer = (b)

We're told that both the source frequency f_{source} and the observed frequency f_{obs} are the same and we're also told the speed of the source $v = 0.4c$. Using the formula for the relativistic Doppler effect given on the exam we can find the angle θ at which the source is moving.

$$\begin{aligned} f_{obs} &= f_{source}\frac{\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v}{c}\cos\theta}, \\ 1+0.4\cos\theta &= \sqrt{1-0.4^2}, \\ \cos\theta &= \left(\sqrt{1-0.4^2}-1\right)/0.4, \\ \therefore \theta &= 102^\circ. \end{aligned} \tag{1}$$

Thus the source is getting closer to the observer.

Q.2. (10 marks) Correct answer = (a)

Using the photoelectric effect formula $(K.E.)_{max} = hf - \phi$ and the expression for kinetic energy $K.E. = \frac{1}{2}m_e v^2$ we can isolate for the maximum speed of the ejected photoelectrons $v = \sqrt{\frac{2}{m_e}\left(\frac{hc}{\lambda} - \phi\right)}$. Subbing in the given values for $\lambda = 250\,nm$ and $\phi = 4.3\,eV$ as well as $hc = 1240\,eV.nm$, $m_e = 511 \times 10^3\,\frac{eV}{c^2}$ and $c = 3 \times 10^8\,m/s$ we get $v = 482,000\,m/s$.

Q.4. Before the collision we have two particles with mass m_1, $v_1 = 0.8c$ and mass m_2, $v_2 = -0.6c$. The γ factor for each particle is easy to work out for the particular speeds given, $\gamma_1 = 5/3$, $\gamma_2 = 5/4$. After the collision we have a single particle with mass M, $v_f = 0$ (hence $\gamma_f = 0$).

(a) (10 marks) We can use momentum conservation to isolate for m_2.

$$\begin{aligned} \gamma_1 m_1 v_1 + \gamma_2 m_2 v_2 &= \gamma_f M v_f, \\ \frac{5}{3}m_1\frac{4}{5}c &= \frac{5}{4}m_2\frac{3}{5}, \\ \therefore m_2 &= \frac{16}{9}m_1. \end{aligned} \tag{2}$$

(b) (10 marks) Now that we know m_2 we can use energy conservation to isolate for M.

$$\begin{aligned} \gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 &= \gamma_f M c^2, \\ \frac{5}{3}m_1 + \frac{5}{4}m_2 &= M, \\ \frac{5}{3}m_1 + \frac{5}{4}\frac{16}{9}m_1 &= M, \\ \therefore M &= \frac{35}{9}m_1. \end{aligned} \tag{3}$$

(c) (10 marks) The change in kinetic energy is $\Delta K.E. = K.E.^f - K.E.^i$ with

$K.E. = (\gamma - 1)mc^2$, but since there is only s single particle at rest in the final state $K.E.^f = 0$ (since $\gamma_f = 1$). The change in kinetic energy is thus:

$$\Delta K.E. = 0 - (\gamma_1 - 1)m_1c^2 - (\gamma_2 - 1)m_2c^2,$$
$$\Delta K.E. = -(\frac{5}{3} - 1)m_1c^2 - (\frac{5}{4} - 1)\frac{16}{9}m_1c^2,$$
$$\therefore \Delta K.E. = -\frac{10}{9}m_1c^2. \qquad (4)$$

(d) (10 marks) You're asked for the invariant mass of the initial state. However we know that the invariant mass before and after the collision is the same. The invariant mass of the final state is M as we have just a single particle at rest (this can be seen either from $m_{inv}^2c^2 = (E/c)^2 - p_x^2 = (E/c)^2 = (\gamma_f Mc^2/c)^2 = Mc^2$ or by the definition of invariant mass as the mass of a particle as observed in its own rest frame). Since the invariant mass of the final state is M it means that the invariant mass of the initial state is also $M = \frac{35}{9}m_1$.

If you like doing calculations you can also calculate the invariant mass of the initial state directly using $m_{inv}^2c^2 = (E_1/c + E_2/c)^2 - (p_1^x + p_2^x)^2$. Recognising that the total initial momentum is zero (the final state momentum is zero so, by momentum conservation, so must the total initial momentum, see part (a)) we're left with $m_{inv}c = (E_1/c + E_2/c) = Mc$, as in part (b).

Q3

a)

25 points

The 4 equally spaced clocks divide each ship into 3 equal sections. In Bob's rest frame S, Anna's ship appears to be $\frac{2}{3}$ the length of Bob's and therefore $\frac{2}{3}$ the length of Anna's ship in her rest frame S'. The origin of S is the tail of Bob's ship, and the origin of S' is the front of Anna's ship. "Right" in the diagram is the $+\hat{x}$ direction.

$$
\begin{aligned}
L_{Anna} &= \frac{2}{3}L_{Bob} \\
L_{Anna} &= \frac{2L'_{Anna}}{3} \\
L_{Anna} &= \frac{L'_{Anna}}{\gamma} \\
\gamma &= \frac{3}{2} \text{ 4 points} \\
\gamma &= \frac{1}{\sqrt{1-\beta^2}} \\
|\beta| &= \sqrt{1-\frac{1}{\gamma^2}} \\
|\beta| &= \frac{\sqrt{5}}{3} \approx 0.745 \text{ 3 points}
\end{aligned}
$$

The locations of Anna's clocks (labelled 1, 2, 3, and 4 from the front to the rear of Anna's ship) in Bob's reference frame are: $x_1 = 0$, $x_2 = \frac{L_{Anna}}{3} = \frac{2L}{9}$, $x_3 = \frac{2L_{Anna}}{3} = \frac{4L}{9}$, and $x_4 = L_{Anna} = \frac{2L}{3}$. The times on the clocks can be found by using the Lorentz transformation from Bob s frame S to Anna's frame S' which is moving with velocity $-|\beta|c\hat{x}$ (Anna's ship is moving in the

$-\hat{x}$ direction in frame S):

$$\begin{aligned}
t_4' &= \gamma\left(t_4 - \frac{(-|\beta|)}{c}x_4\right) \\
&= \gamma\left(0 + \frac{|\beta|x_4}{c}\right) \\
&= \frac{|\beta|L}{c} = \frac{\sqrt{5}L}{3c} \approx 0.745\frac{L}{c} \approx 2.48L \text{ ns/m } \textcolor{red}{\text{12 points}} \\
t_3' &= \frac{\gamma|\beta|x_3}{c} \\
&= \frac{2}{3}t_4' = \frac{2\sqrt{5}L}{9c} \approx 0.497\frac{L}{c} \approx 1.66L \text{ ns/m } \textcolor{red}{\text{3 points}} \\
t_2' &= \frac{\gamma|\beta|x_2}{c} \\
&= \frac{1}{3}t_4' = \frac{\sqrt{5}L}{9c} \approx 0.248\frac{L}{c} \approx 0.828L \text{ ns/m } \textcolor{red}{\text{3 points}}
\end{aligned}$$

We can also use the locations of Anna's clocks in Anna's reference frame $x_1' = 0$, $x_2' = \frac{L}{3}$, $x_3' = \frac{2L}{3}$, and $x_4' = L$ to get the same answers:

$$\begin{aligned}
x_4 &= \gamma(x_4' + (-|\beta|)ct_4') \\
t_4' &= -\frac{(\frac{x_4}{\gamma} - x_4')}{|\beta|c} \\
&= -\frac{(\frac{4L}{9} - L)}{\frac{\sqrt{5}}{3}c} = \frac{\sqrt{5}L}{3}
\end{aligned}$$

A way to confirm that t_4' should be positive is to realize that Anna's tail has already passed the front of Bob's ship (in Bob's rest frame). If the front of Anna's ship was aligned with the tail of Bob's at time 0 in Anna's rest frame, the front of Bob's ship would be aligned with Anna's third clock.

b)

15 points

In Bob's frame, Anna's ship has to move a distance of $L_{Anna} = \frac{2L}{3}$ until the tails are aligned. This takes $\Delta t = \frac{L_{Anna}}{|\beta|c} = \frac{2L}{3|\beta|c}$ 5 points. The time in Bob's frame when the tails are aligned is $t_b = t_0 + \Delta t = 0 + \frac{2L}{3|\beta|c} = \frac{2L}{3|\beta|c}$. The location of the tail of Bob's ship in Bob's rest frame is 0. Using the Lorentz transformation we can find the time in Anna's frame 5 points:

$$t'_b = \gamma\left(t_b - \frac{-|\beta|x_b}{c}\right) = \gamma(t_b - 0) = \frac{3}{2}\frac{2L}{3|\beta|c} = \frac{L}{|\beta|c} = \frac{3L}{\sqrt{5}c} = 1.34\frac{L}{c} \approx 4.47L \text{ ns/m} \text{ 5 points}$$

We can also look from the point of view of the clock at Anna's tail. This clock is initially aligned with Bob's third clock (counting from the tail) so $d = \frac{2L}{3}$ of Bob's ship (in his rest frame) has to pass Anna's tail. In Anna's frame, Bob's ship is moving and this distance is $\frac{d}{\gamma} = \frac{4L}{9}$. So the time it takes for Bob's tail to reach Anna's is $\Delta t' = \frac{d}{\gamma|\beta|c}$. The time on the clock at Anna's tail will read:

$$\begin{aligned} t'_b &= t'_4 + \Delta t' \\ &= \frac{|\beta|L}{c} + \frac{d}{\gamma|\beta|c} = \frac{\sqrt{5}L}{3c} + \frac{4L}{3\sqrt{5}c} = \frac{3L}{\sqrt{5}c} \end{aligned}$$

Alternatively, we can take Δt from the first method and realize that Bob sees Anna's clock running more slowly and $\gamma\Delta t' = \Delta t$. We can then use the second method to find the time when the tails are aligned.

Another Solution

A way to solve the problem, without thinking about Lorentz transformations, is to consider Bob as moving from Anna's point of view. Below are diagrams of the initial event when the front of her ship is aligned with the tail of Bob's and when all of Anna's clocks read 0, the event when Bob's second clock (from the front) is aligned with her tail (question a), and the event when the tails are aligned (question b). To get to (a), Bob has to travel $L - \frac{4}{9}L = \frac{5}{9}L$, this takes $\Delta t = \frac{5L}{9}\frac{1}{|\beta|c} = \frac{\sqrt{5}L}{3c}$. To get to (b) Bob has to travel L, this takes $\Delta t = \frac{L}{|\beta|c} = \frac{3L}{\sqrt{5}c}$.

In (a), Anna's 2nd and 3rd clocks are aligned with $2L/9$ and $4L/9$ from the tail of Bob's ship in Bob's frame, but these distances are contracted in

Anna's frame so Bob will have travelled $\frac{L}{3} - \frac{4L}{9\gamma} = \frac{5L}{27}$ to be at the same alignment with Anna's second clock and $\frac{2L}{3} - \frac{4L}{9\gamma} = \frac{10L}{27}$ to be at the same alignment with Anna's third clock.