CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

24th NOVEMBER 2014

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3A	Q3B	Q3C	Total
15	20	5	5	5	50

Q.1. [15 points] NUSSELT NUMBER IN THE PURE CONDUCTION LIMIT

Consider a sphere of radius a whose surface is at a uniform temperature T_S. The sphere is placed in a fluid medium that is at a constant temperature T_∞ far away from the sphere. The thermal conductivity of the fluid is k_f. The fluid is stagnant far away from the sphere (there is no forced convection). If convection effects and radiation can be ignored, conduction is the only mechanism for heat transfer from the sphere surface into the fluid. There are no heat sources in the fluid. Answer the following questions:

(a) **[10 points]** Beginning from the energy conservation equation in the spherical co-ordinate system [see last page], determine the *steady-state* temperature distribution *in the fluid*. Specify the governing equations and boundary conditions clearly. Note that the domain for the governing equation will be $a \le r < \infty$, so boundary conditions have to be applied at $r = a$ and for $r \to \infty$.

Solution:

The governing equation for the energy balance in the spherical co-ordinate system is

$$\rho C_p \frac{\partial T}{\partial t} = \frac{k_f}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{k_f}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{k_f}{r^2 \sin^2\theta}\frac{\partial}{\partial \varphi}\left(\frac{\partial T}{\partial \varphi}\right) + \dot{S}$$

At steady state, $\frac{\partial T}{\partial t} = 0$.

There is no heat source or sink in the fluid, hence $\dot{S} = 0$.

The problem is spherically symmetric. Therefore, there are no temperature variations in θ and ϕ directions, implying that $\frac{\partial T}{\partial \theta} = 0$ and $\frac{\partial T}{\partial \varphi} = 0$. Also, this means that the temperature is only a function of the radial co-ordinate.

Thus, the governing equation reduces to the following form:

$$\frac{k_f}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

<u>Arguments leading up to governing equation: 3 points</u>

The boundary conditions are $T\big|_{r=a} = T_S$, and $T\big|_{r\to\infty} = T_\infty$.

<u>Boundary conditions: 2 points</u>

Integrating once, we have

$$r^2 \frac{dT}{dr} = c_1$$

Dividing by r^2 and integrating once more, we have

$$T = -\frac{c_1}{r} + c_2$$

<u>Integration and reaching general form of temperature distribution: 2 points</u>

Applying the boundary condition at $r \to \infty$

$$T_\infty = 0 + c_2 \quad \Rightarrow \quad c_2 = T_\infty.$$

Thus,

$$T = -\frac{c_1}{r} + T_\infty$$

Applying the boundary condition at $r = a$, we have

$$T_S = -\frac{c_1}{a} + T_\infty$$

Thus, $c_1 = -a\left(T_S - T_\infty\right)$.

<u>Application of boundary conditions to get constants: 2 points</u>

The temperature distribution is, therefore,

$$T = \frac{a}{r}\left(T_S - T_\infty\right) + T_\infty$$

<u>Final temperature distribution: 1 point</u>

(b) **[3 points]** Determine the radial heat flux, $\dot{q}_r$, at the surface of the sphere, $r = a$.

Solution:

The radial flux vector is

$$\dot{q}_r = -k_f \frac{dT}{dr} = \frac{k_f a\left(T_S - T_\infty\right)}{r^2}$$

<u>Definition and expression for the surface flux: 2 points</u>

At the surface of the sphere, the flux is

$$\left.\dot{q}_r\right|_{r=a} = \frac{k_f\left(T_S - T_\infty\right)}{a}.$$

<u>Evaluation at the surface: 1 point</u>

(c) **[2 points]** Calculate the heat transfer coefficient, *h*, if it is defined as

$$h = -\frac{\left.\dot{q}_r\right|_{r=a}}{T_S - T_\infty}.$$

Hence, determine the Nusselt number for this heat transfer process, defined as

$$\mathrm{Nu} = \frac{ha}{k_f}.$$

Solution:

$$h = \frac{\dot{q}_r\big|_{r=a}}{T_S - T_\infty} = \frac{k_f\left(T_S - T_\infty\right)}{a\left(T_S - T_\infty\right)} = \frac{k_f}{a}$$

Calculation of h: 1 point

Hence the Nusselt number is

$$\mathrm{Nu} = \frac{ha}{k_f} = \frac{\left(k_f / a\right)a}{k_f} = 1.$$

Calculation of Nu: 1 point

2. [20 points] EXTENDED SURFACES

Use the resistance network approach to solve this problem.

An electronic device is in the form of a disk 20 cm in diameter and 2 mm in thickness. The electronic components on one face of this device dissipate 120W. The other face of the device is exposed to an airstream at 27°C for which the convection coefficient is 60 W/m^2-°C. The thermal conductivity of the device material is 10 W/m°C.

(a) [4 points] What are the temperatures on the two faces of the device? The steady-state device temperature on the face with the electronic components is to be kept below 85°C. Does the current configuration satisfy this condition? What is the dominant thermal resistance?

Solution:

The resistance network is as follows:

$$A = \frac{\pi D^2}{4} = \frac{\pi (0.2)^2}{4} = 3.142\times10^{-2}\ \text{m}^2.$$

$$R_{cond} = \frac{\Delta x}{kA} = \frac{0.002}{10\times3.142\times10^{-2}} = 6.366\times10^{-3}\ ^{\circ}\text{C/W}\ .$$

$$R_{conv} = \frac{1}{hA} = \frac{1}{60\times3.142\times10^{-2}} = 0.5305\ ^{\circ}\text{C/W}\ .$$

$$R_{eff} = R_{cond} + R_{conv} = \frac{1}{hA} = 6.366\times10^{-3} + 0.5305 = 0.5369\ ^{\circ}\text{C/W}\ .$$

<u>Each resistance and then effective resistance: 2 points</u>

$$\dot{Q} = \frac{(T_{bL} - T_{\infty})}{R_{eff}} \quad \Rightarrow \quad T_{bL} = T_{\infty} + \dot{Q}R_{eff} = 27 + 120 \times 0.5369 = 91.4 \ ^{o}\mathrm{C}.$$

$$T_{bR} = T_{bR} - \dot{Q}R_{cond} = 91.4 - 120 \times 6.366 \times 10^{-3} = 90.7 \ ^{o}\mathrm{C}.$$

<u>Temperature at left and right faces: 1 point total (0.5 point each)</u>

The current configuration does not satisfy the criterion since the temperature is above 85^{o}C. The dominant resistance is the convective resistance.

<u>Statement about the criterion: 0.5 point</u>

<u>Dominant resistance: 0.5 point</u>

(b) [14 points] To improve the heat transfer rate, an aluminium fin block is glued to the device. The fin block (k = 237 W/m-^{o}C) comprises 512 cylindrical pins, each of diameter 2 mm and a length of 3 cm, attached to a circular aluminium base of 20 cm diameter and 3 mm thickness. With the addition of the fin block, the convective heat transfer coefficient is reduced to 40 W/m^{2}-^{o}C. Calculate the temperatures of the two faces of the device with the fin block. Also, calculate the effectiveness of the fin.

Solution:

The new resistance network is as follows:

<u>Resistance network: 2 points</u>

Here the convective resistance now needs to include an area adjusted for the presence of the fins.

The conductive resistance of the device was already calculated. We ignore contact resistances. The conductive resistance due to base Aluminium board of the fin block is

$$R_{base} = \frac{\Delta x_{base}}{k_{base} A} = \frac{0.003}{237 \times 3.142 \times 10^{-2}} = 4.029 \times 10^{-4} \text{ °C/W} .$$

Resistance of Al base: 1 point

To calculate the convective resistance with the inclusion of the fin, we need to calculate some fin related parameters.

Corrected length $L_c = L + \frac{D_{fin}}{4} = 0.03 + \frac{0.002}{4} = 3.050 \times 10^{-2}$ m

$A_{fin,base} = A_c = 3.142 \times 10^{-6}$ m^2.

$$A_c = \frac{\pi D_{fin}^2}{4} = \frac{\pi \times (0.002)^2}{4} = 3.142 \times 10^{-6} \text{ m}^2.$$

$P = \pi D_{fin} = \pi \times 0.002 = 6.283 \times 10^{-3}$ m.

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{40 \times 6.283 \times 10^{-3}}{237 \times 3.142 \times 10^{-6}}} = 18.37 \text{ m}^{-1}.$$

Calculation of fin related parameters Lc, m, P, Ac, and Afin,base: 4 points

$$\eta_{fin} = \frac{\tanh(mL_c)}{mL_c} = 0.9070$$

$$A_{fin} = \pi D_{fin} L_c = \pi \times 0.002 \times 3.050 \times 10^{-2} = 1.904 \times 10^{-4} \text{ m}^2.$$

Calculation of fin related parameters eta_fin, Afin,: 2 points

$$R_{conv} = \frac{1}{h\left[A + n\left(\eta_{fin} A_{fin} - A_{fin,\text{base}}\right)\right]}$$

$$= \frac{1}{40\left[3.142 \times 10^{-2} + 512 \times \left(0.9070 \times 1.904 \times 10^{-4} - 3.142 \times 10^{-6}\right)\right]} = 0.2104 \text{ °C/W}.$$

Calculation of correct convective resistance with inclusion of fin: 2 points

$$R_{eff} = R_{cond} + R_{base} + R_{conv} = 6.366\times10^{-3} + 4.029\times10^{-4} + 0.2104 = 0.2168 \text{ }^{o}\text{C/W}.$$

Total resistance: 1 point

$$T_{bL} = T_{\infty} + \dot{Q}R_{eff} = 27 + 120\times0.2168 = 53.0 \text{ }^{o}\text{C}.$$

$$T_{bR} = T_{bR} - \dot{Q}R_{cond} = 53.0 - 120\times6.366\times10^{-3} = 52.3 \text{ }^{o}\text{C}.$$

T_{bL} and T_{bR} : 1 point

Fin effectiveness is $\varepsilon_{fin} = \sqrt{\dfrac{kP}{hA_c}} = 55.3 >> 1$

Fin effectiveness: 1 point

(c) [2 points] What is the maximum operating power with the attachment of the fin block, given the 85°C temperature restriction on the face of the device with the electronic components?

Solution:

$$\dot{Q}_{\max} = \frac{\left(T_{bL,\max} - T_{\infty}\right)}{R_{eff}} = \frac{85-27}{0.2168} = 268\,\text{W}$$

2 points for the correct Qmax.

3. **(a) [5 points]** Hot, spherical metal beads of size d = 100 μm and density ρ_S = 2,500 kg/m^3 are quenched in a cold fluid of density ρ_L =1,000 kg/m^3 and viscosity μ = 10^{-3} kg/m-s. The settling velocity, u_s, of the beads is given by Stokes law,

$$u_s = \frac{d^2}{18\mu}\left(\rho_S - \rho_L\right) g,$$

where g = 9.8 m/s^2 is the acceleration due to gravity. Will convection provide an important contribution to the cooling rate of the beads, if the thermal diffusivity of the fluid is 10^{-7} m^2/s?

Solution:

The settling velocity of each sphere is

$$u_s = \frac{d^2}{18\mu}\left(\rho_S - \rho_L\right) g = \frac{\left(100\times10^{-6}\right)^2 \times\left(2500-1000\right)\times 9.8}{18\times10^{-3}} = 0.008167 \text{ m/s.}$$

The Peclet number is

$$\text{Pe} = \frac{u_s d}{\alpha} = \frac{0.008167\times100\times10^{-6}}{10^{-7}} = 8.167,$$

which is comparable to unity. This means that convection is playing an important role in the rate of cooling of the spheres.

<u>3 points for realizing it is a Peclet number calculation</u>

<u>0.5 point for u_s calculation</u>

<u>0.5 point for Pe calculation</u>

<u>1 point for comment</u>

(b) [5 points] Mr. X loves a specific type of soft cookie that is in the form of a cylindrical disc of 10 cm diameter and 2 cm thickness. He purchases the cookies from a departmental store, brings them home, and puts them in an oven at 250^{o}C for 1 min. This produces a crust of thickness 5 mm on the top and bottom surfaces of each cookie, while maintaining the soft interior of the cookie. One day, he goes to the departmental store, only to find that the cookie company now supplies the same cookies, but at *half* the size: 5 cm diameter and 1 cm thickness. Reluctantly, he buys a batch and brings them home, and subjects these smaller cookies to a baking routine at the same temperature but for half the time- 0.5 min instead of 1 min. Will he get the same ratio of crunchy to soft layers in the cookie, if thermal diffusion controls the rate of crust formation? What should be the duration of the bake to get the same ratio?

Solution:

It is specified that the rate of crust formation is controlled by thermal diffusion. This means the thickness of the crust, L, is simply a thermal diffusion length, and this will depend on the time of bake, t_B, via the relationship, $L \propto \sqrt{\alpha t_B}$, where α is the thermal diffusivity of the cookie material.   The diffusion lengths in the two cases are, therefore, related to the times of bake as follows:

$$\frac{L_2}{L_1} = \sqrt{\frac{t_{B_2}}{t_{B_1}}} \qquad \text{(A)}$$

The subscripts 1 and 2 are for the small and large cookies, respectively.   It is given that $t_{B_1} = 1$ min, $t_{B_2} = 0.5$ min, $L_1 = 5$ mm. Therefore, L_2 is

$$L_2 = 5\sqrt{\frac{0.5}{1}} = 3.5 \text{ mm.}$$

Thus the bake time for the small cookie is sufficient to product total crust thickness (top+bottom) of about 3.5×2 = 7 mm. Since the total thickness of the small cookie is 1 cm, the soft interior is only 3 mm thick.   The ratio of the crunchy to soft portions in the large cookie is $5 \times 2/10 = 1$. The ratio of the crunchy to soft portions in the small cookie is $7/3 = 2.3$. The small cookie will turn out to be much crunchier than the large one.

To get the same crunchy to soft ratio, the diffusion length for the small cookie needs to be 2.5 mm, and not 3.5 mm. To achieve this, the bake time has to be reduced according to Eq. (A).

$$t_{B_2} = t_{B_1}\frac{L_2^2}{L_1^2} = 1\times\frac{2.5^2}{5^2} = 0.25 \text{ min}, \text{ or 15 seconds.}$$

3 points for stating the diffusion length/ time scale correctly.

1 point for crunchy to soft calculation

1 point for modified bake time.

(c) **[5 points]** In the textbook, it is emphasized that the fins are employed to increase the rate of heat transfer when the heat transfer coefficients are small, which is typically true for forced convection of air past a surface. Describe a situation where fins could increase the rate of heat transfer for convective heat transfer with liquids, where the heat transfer coefficients are typically much higher than for air.

Solution:

The purpose of fins is to reduce the convective contribution, R_{conv}, to the total heat transfer resistance, R_{eff}, in a thermal circuit. Since R_{conv} is inversely related to the interfacial area between the surface and the fluid, by providing additional area for heat transfer via the attachment of *effective* fins, R_{conv} can be reduced signficantly.

An underlying assumption here is R_{conv} provides a dominant contribution to the total resistance R_{eff}. If this is not the case, then there is no need to add fins! Thus, in any example where R_{conv} dominates R_{eff}, the addition of *effective* fins will lead to an increase in the rate of heat transfer, irrespective of the nature of the fluid (air, water or other heat transfer liquids).

An example that you have seen in class and in tutorials is convection past a solid slab at steady state. The dimensionless number that characterizes the ratio of conductive to convective heat transfer resistance is the Biot number (Bi). When Bi<<1, then the conductive resistance is much smaller than the convective resistance, and the total resistance is dominated by the convective component. For such cases, the addition of fins at the fluid-solid interface will lead to an enhancement in the rate of heat transfer.

3 points for realizing that convective resistance needs to be the dominant resistance
2 points for any example that describes a situation where convective resistance can be high.