



University of Toronto
Faculty of Applied Science and Engineering

First name (please write as legibly as possible within the boxes)

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Last name

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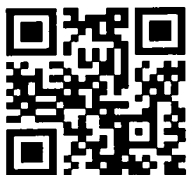
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ESC194F Calculus I
Final Exam
December 2019

No calculators or aids
There are 12 questions, each question is worth 10 marks

Examiners: P.C. Stangeby and J.W. Davis



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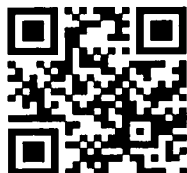


1) Evaluate the following limits:

a) $\lim_{x \rightarrow 0} (\csc x - \cot x)$

b) $\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$

c) $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$



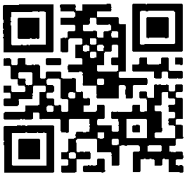
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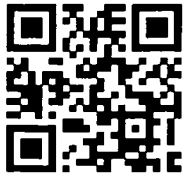
2) Find the derivative of: $3x^3$, $\cos(3x)$, $\ln(x^{1/2})$, e^{-x^2} , 3^{x^2} .



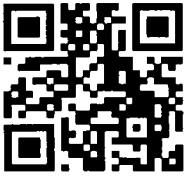
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3) Find the anti-derivative of: $3x^3$, $\cos(3x)$, xe^{-x^2} , $(9+x^2)^{-1}$, 3^x .



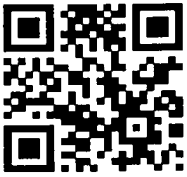
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- 4) At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 pm?



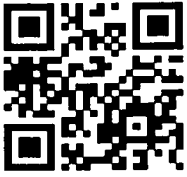
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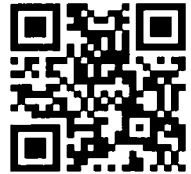
- 5) a) Find functions f and g such that each function is continuous at $x = 0$, but the composite function, $f \circ g$, is not continuous at 0.
- b) What value of b maximizes the integral: $\int_{-1}^b x^2(3 - x)dx$, $b > -1$?



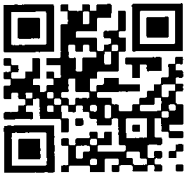
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- 6) A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.



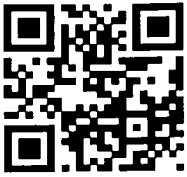
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- 7) a) [7 marks] Provide a $\delta - \varepsilon$ proof that $\lim_{x \rightarrow 3} x^2 = 9$.
- b) [3 marks] Given $f(x) = x^2$, $c = 3$, $\varepsilon = 7$, what is the largest δ that will ensure that when $0 < |x - c| < \delta$ then $|f(x) - f(c)| < \varepsilon$? Is there a smallest δ ?



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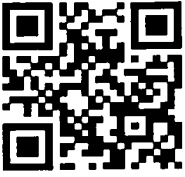
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8) Find the solution of the differential equation that satisfies the given initial condition:

a) $y' \tan x = a + y$, $y\left(\frac{\pi}{3}\right) = a$, $0 < x < \frac{\pi}{2}$

b) $(x^2 + 1)y' + 3x(y - 1) = 0$ $y(0) = 2$



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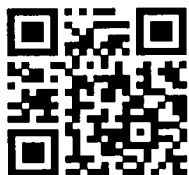
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- 9) Use the method of undetermined coefficients to find the general solution to the 2nd order DE:

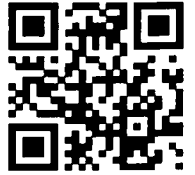
$$y'' - 3y' + 2y = \cosh x = \frac{1}{2}(e^x + e^{-x})$$



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- 10) Suppose $f'(x) < 0 < f''(x)$ for $x < a$ and $f'(x) > 0 > f''(x)$ for $x > a$. Prove that f is not differentiable at a .

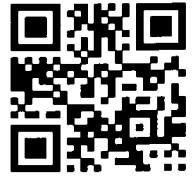
Hint: Assume that f is differentiable at a , and apply the Mean Value Theorem.



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- 11) a) Prove that $e^\pi > \pi^e$ by first finding the maximum value of $f(x) = \frac{\ln x}{x}$.
- b) Sketch a graph of $f(t) = e^t$ on an arbitrary interval $[a, b]$. Use the graph and compare areas of regions to prove that: $e^{(a+b)/2} < \frac{e^b - e^a}{b - a}$.



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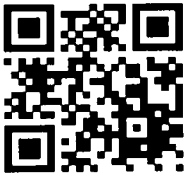
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- 12) Directly calculate the limit of a Riemann sum to evaluate the area of the region between $f(x) = \sqrt{x}$, $x \in [0, 2]$ and the x -axis.

Hint 1: Use the non-uniform partition: $x_i = i^2 \frac{2}{n^2}$

Hint 2: $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$



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