MAT194F Calculus
Midterm Test
9:00 – 10:45, 23 November 2015
105 minutes
No calculators or aids
Each question is worth 10 marks

1. Evaluate f′ for:

   (a) $f(x) = x^3 \ln x$ (b) $f(x) = \ln|2 - x - 5x^2|$ (c) $f(x) = 2e^{2x^2}$

   (d) $f(x) = x \log_4 (\sin x)$ (e) $f(x) = x^{\sin x}$.

2. Evaluate:

   (a) $\int e^x \sqrt{e^x + 5}\, dx$ (b) $\int_{-2}^{0} \sqrt{4 - x^2}\, dx$ (c) $\int_1^e \frac{\ln x}{x}\, dx$

   (d) $\int_0^{\pi/4} \frac{\cos x}{1 - \sin x}\, dx$ (e) $\int \frac{x + \sqrt{x}}{3x^2} dx$

3. Sketch, indicating all important features, the curve $y = xe^{1-x}$.

4. Let R be the region above the x-axis and under the graph of $f(x) = 1/\sqrt{x}$ for $1 \le x \le 4$.

   (a) Find the area of R.

   (b) There is a choice of $k \in [1,4]$ such that the vertical line $x = k$ divides R into two regions R_1 and R_2 with the property that the solids obtained by revolving R_1 and R_2 about the x-axis have the same volume. Find this k.

5. Let R be the region above $y = (x - 1)^2$ and below $y = x + 1$. Find the volume of the solid obtained by revolving R about the line $y = -1$.

6. Let $f(x) = 2kx$ and $g(x) = kx^2$, where k is a constant. Is there a value of $k > 1$ (strict inequality) for which the average values of $f(x)$ and $g(x)$ for $1 \leq x \leq k$ are equal?

7. $f(x)$ is continuous, positive, real and is such that $f(n) = 1$ for all integers n.
   Let $F(x) = \int_4^{x^2} f(t)dt$, $x \geq 2$.

   (a) Find $\lim_{x\to 2} F(x)$ and $\lim_{x\to 2} F'(x)$.

   (b) Explain why F is 1-1 on $[2, \infty)$.

   (c) Find $(F^{-1})(0)$ and $(F^{-1})'(0)$.

8. What is the area of the largest triangle in the first quadrant with two sides on the axes and the third side tangent to the curve $y = e^{-x}$?

9. A leaky 10 kg bucket is lifted from the ground to a height of 12 m at a very slow, constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36 kg of water but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12-m level. How much work was done?

10. For what values of constants b and c does the curve $y = cx^3 + e^{bx}$ have (i) zero, (ii) exactly one, (iii) exactly two, (iv) more than 2, inflections points?