Problem: Given the incompressible flow $\vec{V} = 3y\hat{i} + 2x\hat{j}$. Does this flow satisfy the continuity?

If so, find the stream function $\psi(x,y)$ and plot a few streamlines, with arrows.

$\vec{V} = 3y\hat{i} + 2x\hat{j} \Rightarrow u = 3y \quad v = 2x$

Continuity for incompressible flow requires $\nabla \cdot \vec{V} = 0$. Let's check:

$$\vec{\nabla} \cdot \vec{V} = \underbrace{\frac{\partial u}{\partial x}}_{0} + \underbrace{\frac{\partial v}{\partial y}}_{0} = 0 \Rightarrow \quad \frac{\partial u}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 0 \Rightarrow \vec{\nabla} \cdot \vec{V} = 0$$

Continuity is satisfied.

$$u = \frac{\partial \psi}{\partial y} \quad \& \quad v = -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial \psi(x,y)}{\partial y} \Rightarrow \int d\psi(x,y) = \int \overset{3y}{u}\, dy \Rightarrow \psi(x,y) = \frac{3y^2}{2} + f_1(x) \quad (1)$$

$$v = -\frac{\partial \psi(x,y)}{\partial x} \Rightarrow \int d\psi(x,y) = -\int \underset{2x}{v}\, dx \Rightarrow \psi(x,y) = -x^2 + f_2(y) \quad (2)$$

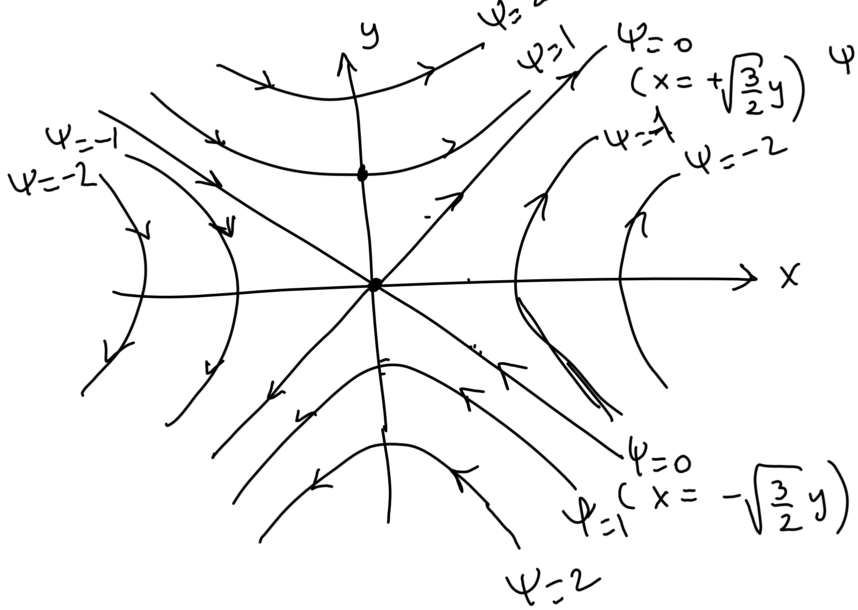
Combining eqns (1) & (2)

$$\psi(x,y) = \frac{3y^2}{2} - x^2 + \underset{\text{constant}}{C}$$

Since the velocities are related to the derivatives of $\psi(x,y)$, the constant's value is of no significance and hence for simplicity $C$ can be set equal to zero. ($C = 0$)

$$\boxed{\psi(x,y) = \frac{3y^2}{2} - x^2}$$

Set $\psi = 0, \mp 1, \mp 2$, etc. and plot some streamlines.




$\psi = 0 \Rightarrow \frac{3y^2}{2} - x^2 = 0$

$x^2 = \frac{3}{2} y^2$

$x = \mp \sqrt{\frac{3}{2}}\, y$

$\psi = \frac{3y^2}{2} - x^2 \Rightarrow \frac{3y^2}{2\psi} - \frac{x^2}{\psi} = 1$

(hyperbolas)

To define directions of arrows: $u = 3y \Rightarrow \begin{cases} \text{if } y > 0 \Rightarrow u > 0 \\ \text{if } y < 0 \Rightarrow u < 0 \end{cases}$

$v = 2x \Rightarrow \begin{cases} \text{if } x > 0 \Rightarrow v > 0 \\ \text{if } x < 0 \Rightarrow v < 0 \end{cases}$