CHE 260 MIDTERM (2013)

1

For a polytropic process

$$P_1 V_1^n = P_2 V_2^n$$

Assuming ideal gas $PV = mRT$

$$W_{12} = \frac{P_2 V_2 - P_1 V_1}{n-1}$$

$$= mR \frac{(T_2 - T_1)}{n-1}$$

$$T_2 = T_1 \left(\frac{P_1}{P_2}\right)^{\frac{1-n}{n}} = (303\text{ K}) \left[\frac{120\text{ kPa}}{1200\text{ kPa}}\right]^{\frac{-0.2}{1.2}}$$

$$= 444.7\text{ K}$$

$$w_{12} = \frac{W_{12}}{m} = R \frac{(T_2 - T_1)}{n-1}$$

$$= 0.2081 \frac{\text{kJ}}{\text{kg K}} \left[\frac{444.7\text{ K} - 303\text{ K}}{0.2}\right]$$

$$= 147.5\text{ kJ/kg}$$

Energy Balance

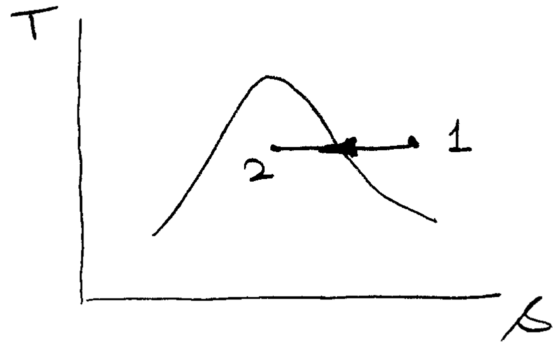
$$q_{12} + w_{12} = \Delta u_{12} = c_v (T_2 - T_1)$$

$$\Rightarrow q_{12} = c_v (T_2 - T_1) - w_{12}$$

$$= 0.312 \frac{\text{kJ}}{\text{kg K}} (444.7 - 303)\text{K} - 147.5 \frac{\text{kJ}}{\text{kg}}$$

$$= -103.3\text{ kJ/kg}.$$

(2)



$$P_1 = 240\,\text{kPa},\quad T_1 = 20^\circ\text{C} \quad \Big\} \quad u_1 = 244.30\ \text{kJ/kg},\quad s_1 = 1.0034\ \text{kJ/kgK}.$$

At $20^\circ C$ $\quad u_f = 76.80\ \text{kJ/kg} \quad u_g = 237.91\ \text{kJ/kg}$

$s_f = 0.2924\ \text{kJ/kgK}, \quad s_g = 0.9102\ \text{kJ/kgK}$

$$u_2 = u_f + x_2(u_g - u_f)$$
$$= 76.80\ \text{kJ/kg} + 0.2\,(237.91 - 76.80)\ \text{kJ/kg}$$
$$= 109.02\ \text{kJ/kg}.$$

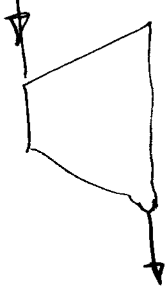
$$s_2 = s_f + x_2(s_g - s_f)$$
$$= 0.2924\ \frac{\text{kJ}}{\text{kgK}} + 0.2\,(0.9102 - 0.2924)\ \text{kJ/kgK}$$
$$= 0.4160\ \text{kJ/kgK}.$$

Heat transfer $\quad q_{12} = \int_1^2 T\,ds = T_1(s_2 - s_1)$

$$= 293\,\text{K}\,(0.4160 - 1.0034)\ \text{kJ/kgK}$$
$$= -172.1\ \text{kJ/kg}$$

$$w_{12} = (u_2 - u_1) - q_{12}$$
$$= (109.02 - 244.30) + 172.1$$
$$= 36.82\ \text{kJ/kg}.$$

(3)



1 MPa, 300°C: $h_1 = 3051.2$ kJ/kg, $s_1 = 7.1229$ kJ/kg K

at 15 kPa

$h_f = 225.94$ kJ/kg $h_g = 2599.1$ kJ/kg

$s_f = 0.7549$ kJ/kg K $s_g = 8.0085$ kJ/kg K

For an isentropic process $s_2 = s_1$

$$x_{2,s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1229 - 0.7549}{8.0085 - 0.7549} = 0.8779$$

$$h_{2s} = h_f + x_{2,s}(h_g - h_f)$$
$$= 225.94 + 0.8779(2599.1 - 225.94)$$
$$= 2309.3 \text{ kJ/kg}$$

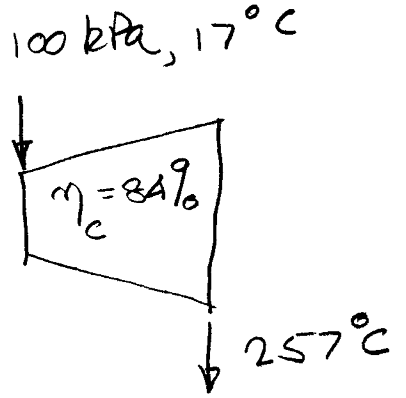
$$w_s = h_1 - h_{2s} = 3051.2 - 2309.3$$
$$= 741.9 \text{ kJ/kg}$$

$$\eta_t = \frac{w_a}{w_s} = \frac{600 \text{ kJ/kg}}{741.9} = 80.9\%$$

$$h_2 = h_1 - w_a = 3051.2 - 600$$
$$= 2451.2 \text{ kJ/kg}$$

$$x'_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2451.2 - 225.94}{2599.1 - 225.94}$$
$$= 0.9377$$

4)



$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$\frac{P_{2s}}{P_1} = \left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$T_2 = 257\,°C = 530\text{ K}$

$T_{2s} = T_1 + \eta_c (T_2 - T_1)$

$= 290\text{K} + 0.84\,(530 - 290) = 491.6\text{ K}$

$P_{2s} = P_1 \left(\frac{T_{2s}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

$= 100\text{ kPa} \left(\frac{491.6\text{ K}}{290\text{ K}}\right)^{\frac{1.4}{0.4}}$

$= 634\text{ kPa}.$

Density of air $\rho = \frac{P}{RT} = \frac{100\text{ kPa}}{0.287\frac{\text{kJ}}{\text{kg K}} \times 290\text{K}}$

$= 1.20\text{ kg/m}^3$

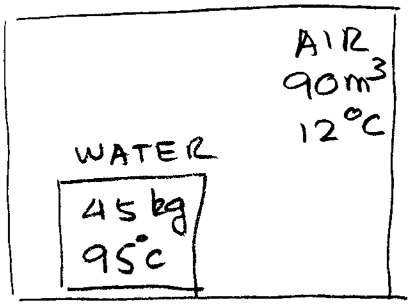
$\dot{m} = \rho \dot{V} = 1.20\frac{\text{kg}}{\text{m}^3} \times 2.4\frac{\text{m}^3}{\text{s}} = 2.88\frac{\text{kg}}{\text{s}}$

$\dot{W} = \dot{m}(h_2 - h_1) = \dot{m}\, c_p (T_2 - T_1)$

$= 2.88\frac{\text{kg}}{\text{s}} \times 1.004\frac{\text{kJ}}{\text{kg K}} (530 - 290)\text{K}$

$= 694.0\text{ kW}.$

(5)



Assume $P_{air} = 1$ atm

$$m_a = \frac{PV}{RT}$$

$$= \frac{101.3 \text{ kPa} \times 90 \text{m}^3}{0.287 \text{ kJ/kg K} \times 285 \text{K}}$$

$$= 111.5 \text{ kg}$$

Isolated system $\Rightarrow \Delta U = 0$

$$m_w c_w (T_2 - T_{w,1}) + m_a c_{v,a}(T_2 - T_{a,1}) = 0$$

$$45 \times 4.18 (T_2 - 95) + 111.5 \times 0.717 (T_2 - 12) = 0$$

$$\Rightarrow T_2 = 70.2°C$$

Heat transfer to air $Q = m_a c_{v,a}(T_2 - T_{a,1})$

$$\Rightarrow Q = 111.5 \text{ kg} \times 0.717 \text{ kJ/kg K} (70.2 - 12) = 4660 \text{kJ}$$

$$\Delta S_w = m_w c_w \ln \frac{T_2}{T_{w1}} = 45 \times 4.18 \times \ln \frac{343.2}{368}$$

$$= -13.12 \text{ kJ/kg}.$$

$$P_2 = \frac{m_a R T_2}{V} = \frac{111.5 \text{ kg} \times 0.287 \text{ kJ/kg K} \times 343.2 \text{K}}{90 \text{m}^3}$$

$$= 122.0 \text{ kPa}$$

$$\Delta S_a = m_a \left[c_{p,a} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= 111.5 \text{ kg} \left[1.004 \ln \frac{343.2 \text{K}}{285 \text{ K}} - 0.287 \ln \frac{122.0}{101.3} \right]$$

$$= 14.85 \text{ kJ/K}$$

$$\Delta S_{TOTAL} = \Delta S_w + \Delta S_a = -13.11 + 14.85$$

$$= 1.74 \text{ kJ/K}.$$