Solutions

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

TERM TEST 2

March 20th, 2018

70 minutes

First Year – Engineering Science

ECE 159S - ELECTRIC CIRCUIT FUNDAMENTALS

Exam Type: A

Examiners: K. Phang and N.P. Kherani

NAME: ____

Last First

STUDENT NO: Please write your student number on the top of Page 2

SECTION (circle one):

LEC101 (Phang) LEC102 (Kherani)

INSTRUCTIONS:

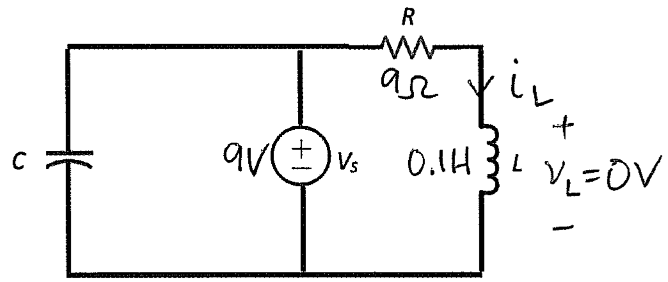
- Only non-programmable calculators are allowed. No other aids are permitted.
- The marks for each question are indicated within brackets [ ].
- When answering the questions, include all the steps of your work on these pages. For additional space, you may use the back of the preceding page and the blank page provided at the end.
- Place your final answers in the boxes where given and specify the units.
- Do not unstaple this exam booklet.

Q1	/10
Q2	/10
Q3	/5
Q4	/10
Q5	/5
Total	**/40**

QUESTION 1 [10 marks]

Answer the following four short questions. Clearly show the steps leading to the final result(s).

a) [3] Consider the following circuit where $V_S = 9$ V, $R = 9\ \Omega$, $L = 0.1$ H. Further, the charge on the capacitor C is 90 μC. Find the capacitance C, the current in the inductor i_L, and the energy w_L stored in the inductor. Assume all the circuit components are ideal and the circuit is at steady state.



$C =$ 10μF

$i_L =$ 1A

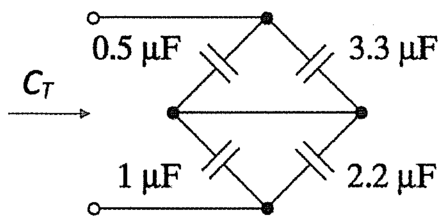
$w_L =$ 0.05 J

$$C = Q/V = \frac{90\mu C}{9V} = \underline{10\mu F}$$

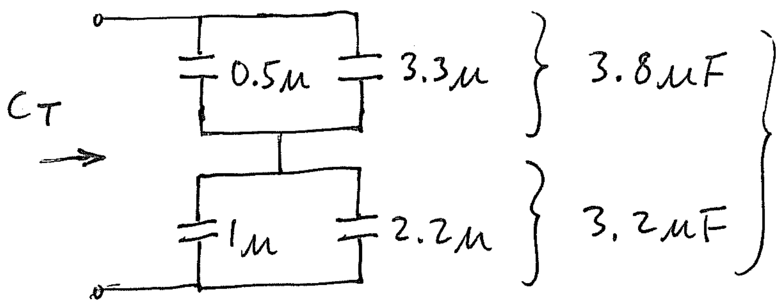
$$i_L = \frac{V_S - V_L}{R} = \frac{9V}{9\Omega} = \underline{1A}$$

$$w_L = \frac{1}{2} L i_L^2 = \frac{1}{2}(0.1)(1)^2 = 0.05\ J$$

b) [2] Find the equivalent capacitance C_T for the network shown below.



$C_T =$ 1.74 μF



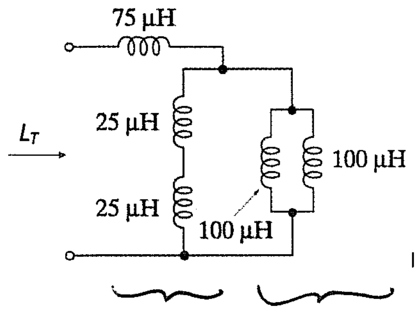
$$\frac{1}{C_T} = \frac{1}{3.8\mu} + \frac{1}{3.2\mu F}$$

$$C_T = \frac{3.8 \times 3.2}{3.8 + 3.2}\ \mu F$$

$$= 1.74\ \mu F$$

Name: ______________________ Student No.: ______________

c) [2] Find the equivalent inductance L_T for the network shown below.

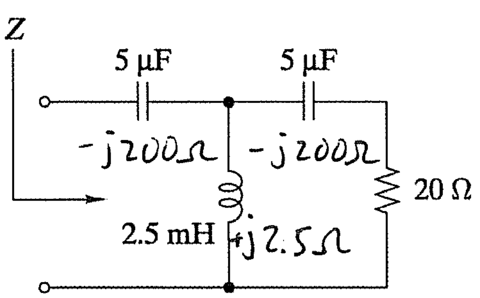


$\boxed{L_T = 100\mu H}$

$50\mu H // (100\mu H // 100\mu H)$

$$L_T = 75\mu + (25\mu + 25\mu) // 100\mu // 100\mu$$
$$= 75\mu + \left(\frac{1}{50\mu} + \frac{1}{100\mu} + \frac{1}{100\mu}\right)^{-1}$$
$$= 75\mu + 25\mu$$
$$= \underline{100\mu H}$$

d) [3] Find the equivalent impedance $\boldsymbol{Z}$ of the circuit (shown below) operating in the sinusoidal steady state at $\omega = 1000$ rad/s.



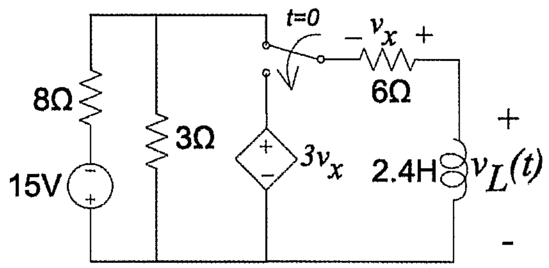
$\boxed{Z = 0.00317 - j197.5 \ \Omega \text{ or}}$

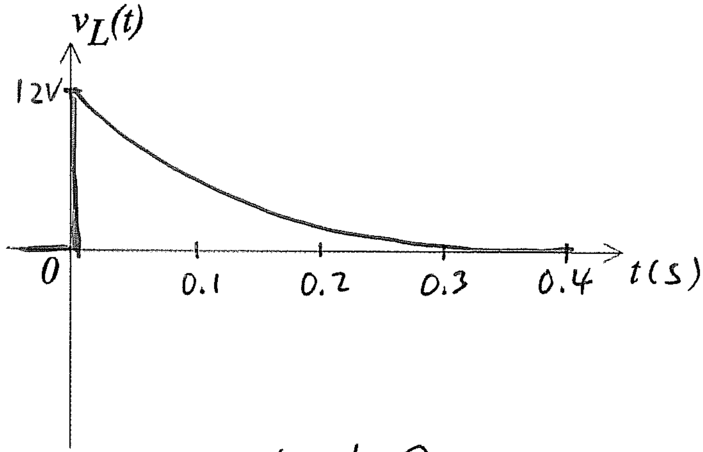
$197.5 \angle -90^\circ \ \Omega$

$$Z_{5\mu F} = \frac{-j}{(1000)(5\mu)} = \frac{-j}{0.005} = -j200\Omega$$
$$Z_{2.5mH} = j(1000)(2.5m) = j2.5\Omega$$
$$Z = -j200 + j2.5 // (-j200 + 20)$$
$$= -j200 + \frac{j2.5(-j200 + 20)}{j2.5 - j200 + 20}$$
$$= -j200 + \frac{500 + j50}{20 - j197.5}$$
$$= -j200 + 0.00317 + j2.53$$

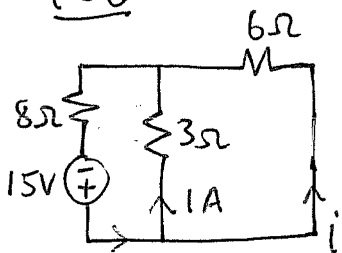
QUESTION 2 [10 marks]

For the circuit below, find the inductor voltage, $v_L(t)$, and plot the waveform on the graph below. Clearly mark the scale on the x- and y-axes and indicate the initial and final values (including time t<0).





t<0



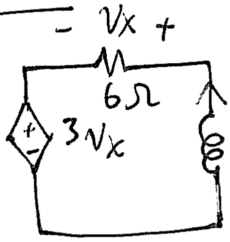
$+$
$v_L(0^-) = 0V$
$-$

$i_L(0^-) = 0.5A$

$I_s = 1.5A$

$$I_s = \frac{15V}{8+3//6} = \frac{15V}{10\Omega} = 1.5A$$

$t=0^+$



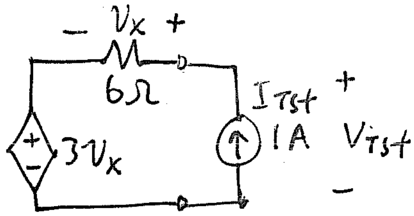
$i_L(0^+) = 0.5A$
$+$
$v_L(0^+)$
$-$

$$i_L(0^+) = i_L(0^-) = 0.5A$$

$$\text{KVL}: \quad v_L(0^+) = V_x + 3V_x$$
$$= 4V_x$$
$$= 4(6\Omega)(0.5A)$$
$$\therefore \quad v_L(0^+) = 12V$$

$t \to \infty \quad v_L(\infty) = 0V$ (short-circuit)

Time constant, τ



$I_{Tst} = 1A$ current applied

$$V_{Tst} = V_x + 3V_x$$
$$= 4(6\Omega)(1A)$$
$$= 24V$$

$$\therefore R_{Th} = \frac{V_{Tst}}{I_{Tst}} = \frac{24V}{1A} = 24\Omega$$

$$\therefore \tau = L/R_{Th} = \frac{2.4H}{24\Omega} = 0.1s$$

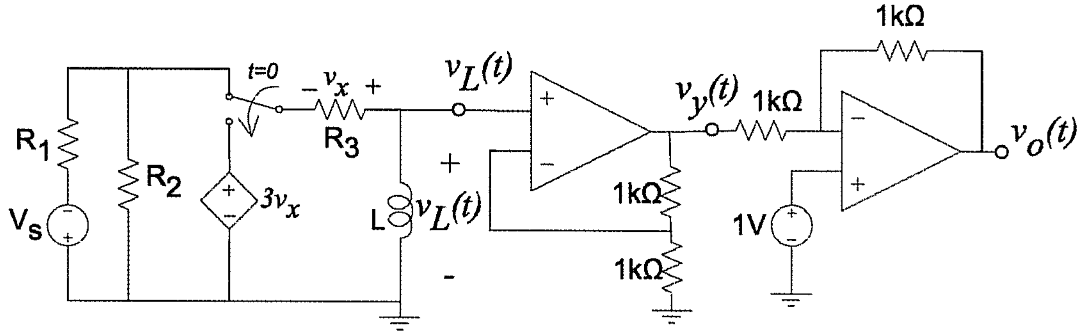
$$v_L(t) = v_L(\infty) + [v_L(0^+) - v_L(\infty)]e^{-t/\tau}$$

$$v_L(t) = 12e^{-t/0.1}\ V, \ t>0$$

Name: ______________________ Student No.: ______________

QUESTION 3 [5 marks]

The circuit from Question 2 has been modified as shown below. Component values V_s, L, R_1, R_2, R_3 have been selected to produce the new inductor voltage waveform $v_L(t)$ shown below, and additional op amp circuits have been added to amplify the signal.

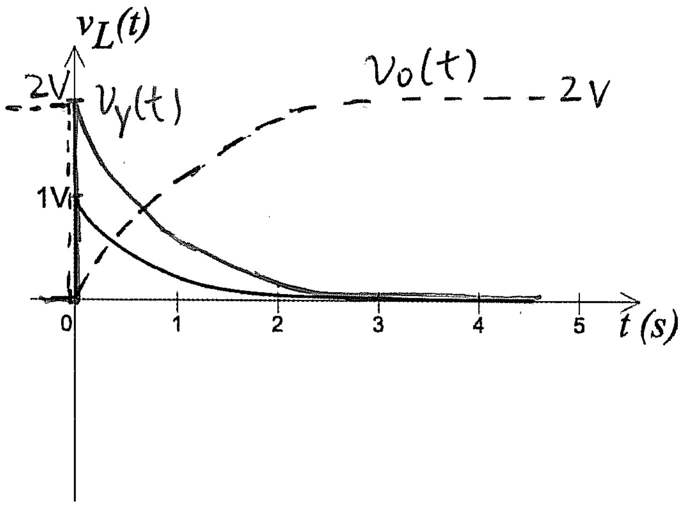


a) [2] Express outputs $v_y(t)$ and $v_o(t)$ in terms of $v_L(t)$.

$v_y(t)$ = $2 v_L(t)$

$v_o(t)$ = $-2v_L(t) + 2$

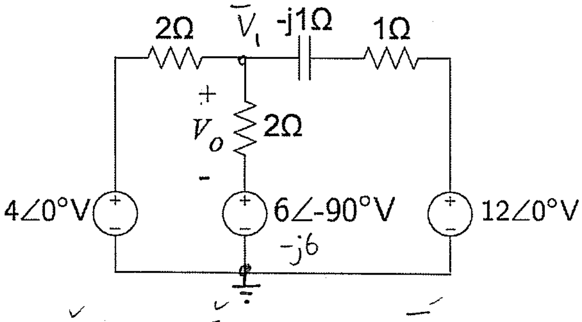
b) [3] Sketch the corresponding waveforms $v_y(t)$ and $v_o(t)$, drawing directly on the graph below of $v_L(t)$. Clearly indicate the initial and final values of each waveform (including time t<0).



Name: ______________________ Student No.: ______________

QUESTION ~~3~~ 4 [10 marks]

(a) [6] Using nodal analysis find phasor V_o for the circuit network (shown below) operating in the sinusoidal steady state.
(b) [4] Given the sinusoidal frequency of 60 Hz, draw the circuit in the time domain showing all the component values as well as the time domain voltage $v_o(t)$.



a)

KCL: $\dfrac{\bar{V}_1 - 4}{2} + \dfrac{\bar{V}_1 - (-j6)}{2} + \dfrac{\bar{V}_1 - 12}{1-j} = 0$

$\bar{V}_1\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{1-j}\right) - 2 + j3 - \frac{12}{1-j} = 0$

$\bar{V}_1\left(\frac{3}{2} + j\,{}^1\!/_2\right) - 2 + j3 - 6 - j6 = 0$

$\bar{V}_1 = \dfrac{2(8+j3)}{3+j} = \dfrac{2(8+j3)(3-j)}{10}$

$= \frac{1}{5}(27+j)$

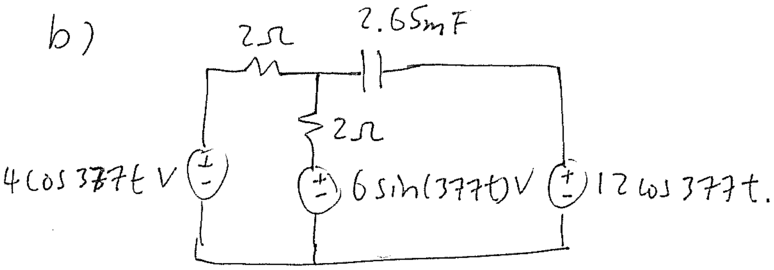
$= 5.4 + j0.2\,V$

$\bar{V}_0 = \bar{V}_1 - (-j6)\,V$

$= 5.4 + j0.2 + j6\,V$

$\bar{V}_0 = 5.4 + j6.2\,V = 8.22\underline{/48.9^\circ}\,V$

b)



$Z_C = \dfrac{-j}{\omega C}$

$-j = \dfrac{-j}{\omega C}$

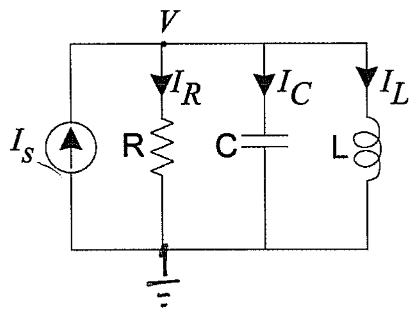
$C = \dfrac{1}{\omega} = \dfrac{1}{2\pi 60} = \underline{2.65\,mF}$

$v_o(t) = 8.22\cos(377t + 48.9^\circ)\,V$

QUESTION 5 [5 marks]
For the sinusoidal steady-state circuit shown below,

(a) [3] Draw a phasor diagram to show the relationship among all the current phasors ($\boldsymbol{I_S}$, $\boldsymbol{I_R}$, $\boldsymbol{I_C}$ and $\boldsymbol{I_L}$). For the diagram, you can assume the reference voltage phasor, $\boldsymbol{V} = V_M\angle 0^\circ$ V, and component values R=1Ω, C=1F, L=1H, and ω=1 rad/s.

(b) [2] Find the condition when the voltage phasor $\boldsymbol{V}$ is in phase with the current phasor $\boldsymbol{I_S}$. Express this condition in terms of the component values, R, C, and L, and frequency ω.

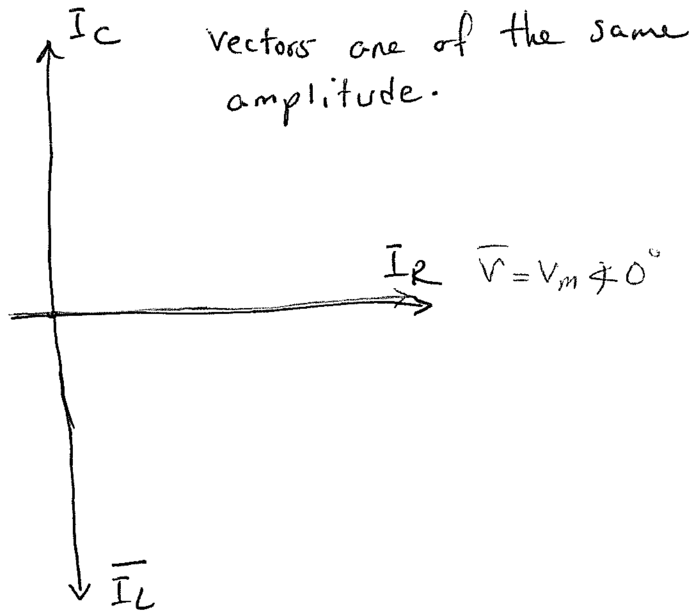


a) $\bar{I}_R = \bar{V}/R = \underline{\underline{\bar{V}}}$ [A]

$\bar{I}_C = \dfrac{\bar{V}}{1/j\omega C} = j(1)(1)\bar{V} = j\underline{\underline{\bar{V}}}$ [A]

$\bar{I}_L = \dfrac{\bar{V}}{j\omega L} = \dfrac{\bar{V}}{j(1)(1)} = -j\underline{\underline{\bar{V}}}$ [A]

Phasor Diagram:



b) Condition is valid when $\bar{I}_C$ and $\bar{I}_L$ are cancelled out!

$\longrightarrow \bar{I}_C = -\bar{I}_L \longrightarrow \dfrac{\cancel{\bar{V}}}{1/j\omega C} = -\dfrac{\cancel{\bar{V}}}{j\omega L} \longrightarrow (j\omega L)(j\omega C) = -1 \longrightarrow \omega^2 LC = 1$

$\longrightarrow \omega = \dfrac{1}{\sqrt{LC}}$

→ Resonant frequency

(Alternatively: Condition is when two impedances are identical)