PHY294 Quiz #1 (January 29th 2016)

Name (last, first): Solution
Student ID:
Signature:

- **4 questions, 25 minutes.**
- **Closed book, closed notes and no calculators.**
- **Please write only in the Quiz paper (double-sided).**

1. What is the deBroglie wavelength of a proton (mass m_p) moving at 4/5 the speed of light? [Give your answer in terms of m_p, c and h.]

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m_p v}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = \frac{5}{3}$$

$$= \frac{h}{\frac{5}{3} m_p \frac{4}{5} c} = \frac{3}{4}\frac{h}{m_p c}$$

2. Use the Uncertainty Principle to estimate the minimum energy of the electron in a hydrogen atom. [Note: the orbital energy is given by $E = p^2/2m_e - \alpha e^2/r$, where p is the momentum, r the radius, m_e the electron mass, α the Coulomb constant, and e the electron charge (magnitude).]

$$\Delta p \Delta x \gtrsim \frac{\hbar}{2} \Rightarrow p \cdot r \sim \hbar$$

$$E = \frac{\hbar^2}{2mr^2} - \frac{\alpha e^2}{r}$$

$$0 = \left.\frac{\partial E}{\partial r}\right|_{r_0} = -\frac{2\hbar^2}{2mr^3} + \frac{\alpha e^2}{r^2} \Rightarrow r_0 = \frac{\hbar^2}{m\alpha e^2}$$

$$\therefore E_0 = \frac{\hbar^2}{2m}\frac{m^2\alpha^2 e^4}{\hbar^4} - \frac{\alpha e^2 m \alpha e^2}{\hbar^2} = -\frac{\alpha^2 m e^4}{2\hbar^2}$$

3. The ground-state wave function of a 1D harmonic oscillator is: $\psi(x) = Ae^{-b^2x^2/2}$, A and b are constants.
(a) Does this quantum state have well-defined kinetic energy? [Give reason]
(b) Write down an integral expression for the expectation value of its potential energy. [Do not solve]

A $p\psi = -i\hbar\left[\psi\left(-\frac{2}{2}b^2x\right)\right]$

$p^2\psi = -b^2\hbar^2\left[-\psi-(-\psi b^2x)x\right] \neq \text{constant}\cdot\psi$

$\therefore$ $KE = \frac{p^2}{2m}$ is not well defined.
i.e. ψ is not an eigenstate of p^2.

B $\langle PE\rangle = \frac{1}{2}k\langle x^2\rangle = \frac{1}{2}kA^2\int_{-\infty}^{\infty} e^{\frac{-b^2x^2}{2}}x^2e^{\frac{b^2x^2}{2}}dx$

spring constant

4. Consider an electron in a 1D infinite-potential square well of width L centered at $x=L/2$.
Let it be in an equally-mixed superposition of the ground (n=1) and 2nd-excited (n=3) states.
(a) Prior to any measurements, does the expectation value $\langle p^2\rangle$ depend on time? [Give reason]
(b) After a measurement that finds the electron in the n=3 state, what is $\Psi(x, t)$ [in term of m_e, L, h] ?

A No $\because$ $\langle p^2\rangle = 2m_e\langle E\rangle$ and $\langle E\rangle = \frac{E_1+E_3}{2}$

Note: $\Psi = \frac{1}{\sqrt{2}}\Psi_{n=1} + \frac{1}{\sqrt{2}}\Psi_{n=3}$, $\Psi_n = \psi_n e^{-i\frac{E_n}{\hbar}t}$

$\langle E\rangle = \int_0^L \Psi^* E_{op}\Psi\, dx = \left(\frac{1}{\sqrt{2}}\right)^2 E_1 \underbrace{\int_0^L \psi_1^*\psi_1\, dx}_{1} + \left(\frac{1}{\sqrt{2}}\right)^2 E_3 \underbrace{\int_0^L \psi_3^*\psi_3\, dx}_{1}$

$E_{op}\Psi_n = E_n\Psi_n$

B Measurement collapses Ψ into $\Psi_{n=3}$

$\therefore$ $\Psi_{after} = \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi x}{L}\right)e^{-i\frac{E_3 t}{\hbar}}$

$E_3 = \frac{3^2h^2}{8m_eL^2}$