AER210 VECTOR CALCULUS and FLUID MECHANICS

Quiz 3

Duration: 70 minutes

8 November 2018

Closed Book, no aid sheets

Non-programmable calculators allowed

Instructor: Prof. Alis Ekmekci

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Given Name: Alis

Student #: Prof.

TA Name/Tutorial #: ______________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	12	
2	9	
3	9	
4	10	
5	10	
TOTAL	50	/50

Hints: $E_V = -\dfrac{dP}{dV\!\!\!/ \,/ V\!\!\!/}$ $\qquad \tau = \mu \dfrac{du}{dy}$ $\qquad -\nabla p + \rho \vec{g} = \rho \vec{a}$

(Gravitational acceleration: g = 10 m/s^2)

 Name: ___________________

1) (a) [**2 marks**] Define the following terms with a sentence: streamline, streakline.

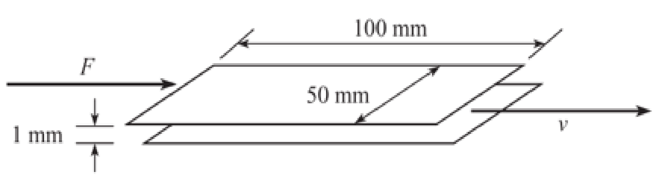
streamline: a line drawn everywhere tangent to the velocity vectors

streakline: a line that connects all the particles that passed through the same point in space at a previous time.

(b) [**7 marks**] Indicate whether the statement is TRUE (T) or FALSE (F):

__F__ Boundary layers and wakes are regions where viscous effects are negligible.
__T__ With an increase in temperature, the viscosity of liquids decreases.
__T__ A totally submerged bottom-heavy object is always in stable condition.
__F__ Buoyant force depends on the density of the solid body.
__T__ Body forces are forces that develop without physical contact.
__F__ Surface forces acting on hydrostatic fluid particles come from two sources: pressure forces and shear forces.
__T__ In Newtonian fluids, shear stress is linearly proportional with the deformation rate.

c) [**3 marks**] The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume linear velocity distribution.



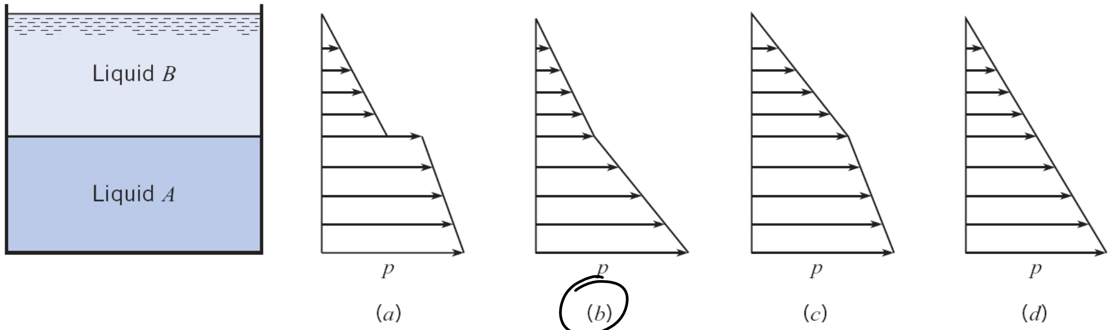
Shear force:

$$\tau = \frac{\text{Force}}{\text{Area}} = \frac{3\text{N}}{(0.05\text{m})(0.1\text{m})} = 600 \frac{\text{N}}{\text{m}^2}$$

From $\tau = \mu \frac{du}{dy} \Rightarrow$ viscosity: $\mu = \frac{\tau}{(du/dy)} = \frac{\tau}{\frac{\Delta v}{\Delta y}} = \frac{600}{\frac{10}{0.001}} = 0.06 \frac{\text{N.s}}{\text{m}^2}$

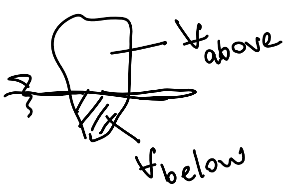
 Name: ____________________

2) (a) [**2 mark**] The reservoir in the figure contains two immiscible liquids of density ρ_A and ρ_B, respectively, one above the other. $\rho_A > \rho_B$. Circle the graph that depicts the correct distribution of gage pressure along a vertical line through the liquids?



2) (b) [**3 marks**] Consider an iceberg floating in seawater. Ice has a density of 920 kg/m^3 (it is made up of fresh water), and seawater has a density of 1025 kg/m^3. Find the fraction of the volume of the iceberg that shows above the sea surface.

$\rho_{ice} = 920\ kg/m^3$; $\rho_{water} = 1{,}025\ kg/m^3$



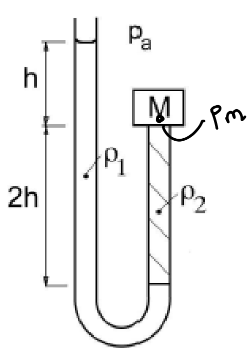
In equilibrium: $F_B = W$

$\rho_{water}\, \cancel{g}\, V_{below} = \rho_{ice}\, \cancel{g}\, V_{total}$

$\frac{V_{below}}{V_{total}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{920}{1025} = 0.897$ is submerged

$V_{above} + V_{below} = V_{total} \Rightarrow \frac{V_{above}}{V_{total}} = 1 - \frac{V_{below}}{V_{total}} = 1 - 0.897 = 0.102 \Rightarrow$ Only %10.2 of the ice is above the sea surface

2) (c) [**4 marks**] A manometer tube is filled with two fluids with densities ρ_1 = 1000 kg/m^3, and ρ_2 = 800 kg/m^3, as shown in the figure below. The tube has a diameter of 0.01 m. One end is open to the atmospheric pressure, and the other end is blocked by a block of steel of mass m = 0.1 kg. Find the height h where the steel block is about to be dislodged.

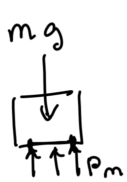


$P_m + \rho_2 g (2h) - \rho_1 g (3h) = 0$ ← atmospheric pressure

$P_m = 3\rho_1 gh - 2\rho_2 gh$

$= 3 \cdot 1000 \cdot gh - 2 \cdot 800 \cdot gh$

$P_m = 1400\, gh$

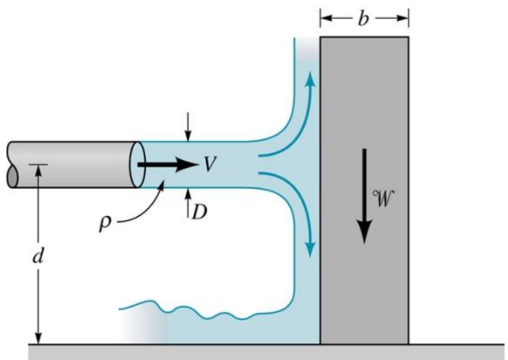


$\underset{1400\,gh}{P_m} \cdot \frac{\pi d^2}{4} = mg \Rightarrow 1400\, \cancel{g} h \frac{\pi d^2}{4} = m \cancel{g}$

d = 0.01 m. m = 0.1 kg

$\boxed{h = 0.909\ m}$

3) [**9 marks**] A jet of liquid directed against a block, as shown in the figure, can tip over the block. Assume that the minimum velocity, *V*, needed to tip over the block is a function of the fluid density, *ρ*, the diameter of the jet, *D*, the weight of the block, *W*, the width of the block, *b*, and the distance, *d*, between the jet and the bottom of the block. Using the method of repeating variables, determine a set of dimensionless parameters for this problem, and re-write the original dimensional relationship in dimensionless form.



$V = f(\rho, D, W, b, d)$

$[V] = \frac{L}{T}$ | $[D] = L$ | $[b] = L$

$[\rho] = \frac{M}{L^3}$ | $[W] = M\frac{L}{T^2}$ | $[d] = L$

From Buckingham Pi theorem:

(# of Π terms) = (# of variables) − (min # of reference dimensions)

$= 6 - 3 = 3$

If we choose d, W, ρ as repeating variables:

$\Pi_1 = V d^a W^b \rho^c$

$M^0 L^0 T^0 = \left(\frac{L}{T}\right) L^a M^b \frac{L^b}{T^{2b}} \frac{M^c}{L^{3c}}$

$M^0 L^0 T^0 = M^{b+c} L^{a+1+b-3c} T^{-1-2b}$

$-1-2b = 0 \Rightarrow \boxed{b = -1/2}$

$b + c = 0 \Rightarrow \boxed{c = +1/2}$

$a + 1 + b - 3c = 0 \Rightarrow \boxed{a = 1}$

$\boxed{\Pi_1 = \frac{V d \sqrt{\rho}}{\sqrt{W}}}$

$\Pi_2 = D d^a W^b \rho^c$

$M^0 L^0 T^0 = L L^a M^b \frac{L^b}{T^{2b}} \frac{M^c}{L^{3c}}$

$M^0 L^0 T^0 = L^{1+a+b-3c} M^{b+c} T^{-2b}$

$-2b = 0 \Rightarrow \boxed{b = 0}$

$b + c = 0 \Rightarrow \boxed{c = 0}$

$1 + a + \cancel{b} - 3\cancel{c} = 0 \Rightarrow \boxed{a = -1}$

$\boxed{\Pi_2 = \frac{D}{d}}$

$\Pi_3 = b d^a W^b \rho^c$

$M^0 L^0 T^0 = L L^a \frac{M^b L^b}{T^{3b}} \frac{M^c}{L^{3c}}$

Solving a, b, c:

$\boxed{\Pi_3 = \frac{b}{d}}$

$\Pi_1 = f_2(\Pi_2, \Pi_3) \Rightarrow \boxed{\frac{V d \sqrt{\rho}}{\sqrt{W}} = f_2\left(\frac{D}{d}, \frac{b}{d}\right)}$

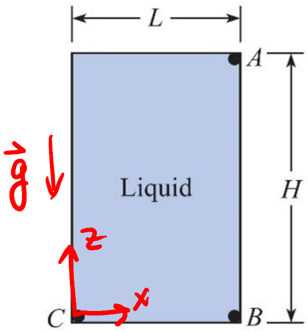
NOTE:

There can be other possible solutions, depending on the selection of repeating variables:

- If W, ρ, D are selected as repeating var. $\Rightarrow \Pi_1 = \frac{V D \sqrt{\rho}}{\sqrt{W}}$, $\Pi_2 = \frac{b}{D}$, $\Pi_3 = \frac{d}{D}$
- If W, ρ, b " " " " " $\Rightarrow \Pi_1 = \frac{V b \sqrt{\rho}}{\sqrt{W}}$, $\Pi_2 = \frac{b}{D}$, $\Pi_3 = \frac{D}{b}$
- If W, ρ, d " " " " " $\Rightarrow \Pi_1 = V d \sqrt{\rho}/\sqrt{W}$, $\Pi_2 = D/d$, $\Pi_3 = b/d$

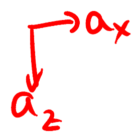
 Name: ____________________

4) [**10 marks**] The closed tank shown, which is full of liquid, undergoes a linear motion with constant acceleration. The components of the acceleration are 1g to the right and 2/3g downward. Here g is the gravitational acceleration, which can be taken as 10 m/s^2. L = 2.5 m, H = 3 m, and the density of the liquid is 1300 kg/m^3. Points A, B and C are marked on the figure. Determine p_C-p_A and p_B-p_A.



Hint: The governing equation for fluids in rigid body motion is given by:

$$-\nabla p + \rho\vec{g} = \rho\vec{a}$$



$$\vec{a} = a_x\vec{i} + a_z\vec{k} = 1g\vec{i} - \frac{2}{3}g\vec{k}$$

$$-\vec{\nabla}p + \rho\vec{g} = \rho\vec{a}$$

$$-\frac{\partial p}{\partial x}\vec{i} - \frac{\partial p}{\partial y}\vec{j} - \frac{\partial p}{\partial z}\vec{k} - \rho g\vec{k} = \rho\left(1g\vec{i} - \frac{2}{3}g\vec{k}\right)$$

$$\vec{i}: \frac{\partial p}{\partial x} = -\rho g$$

$$\vec{j}: \frac{\partial p}{\partial y} = 0$$

$$\vec{k}: \frac{\partial p}{\partial z} = -\rho g + \frac{2}{3}\rho g$$

$$p = p(x, z)$$

$$dp = \underbrace{\frac{\partial p}{\partial x}}_{-\rho g}dx + \underbrace{\frac{\partial p}{\partial z}}_{-\rho g + \frac{2}{3}\rho g = -\frac{1}{3}\rho g}dz$$

$$\boxed{p = -\rho g x - \frac{1}{3}\rho g z + C_1}$$

Based on the coordinate system I selected: $\begin{cases} x_A = L, \; z_A = H \\ x_B = L, \; z_B = 0 \\ x_C = 0, \; z_C = 0 \end{cases}$

$$p_A = -\rho g x_A - \frac{1}{3}\rho g z_A + C_1 \Rightarrow \boxed{p_A = -\rho g L - \frac{1}{3}\rho g H + C_1}$$

$$p_B = -\rho g x_B - \frac{1}{3}\rho g z_B + C_1 \Rightarrow \boxed{p_B = -\rho g L - \cancel{\frac{1}{3}\rho g(0)} + C_1}$$

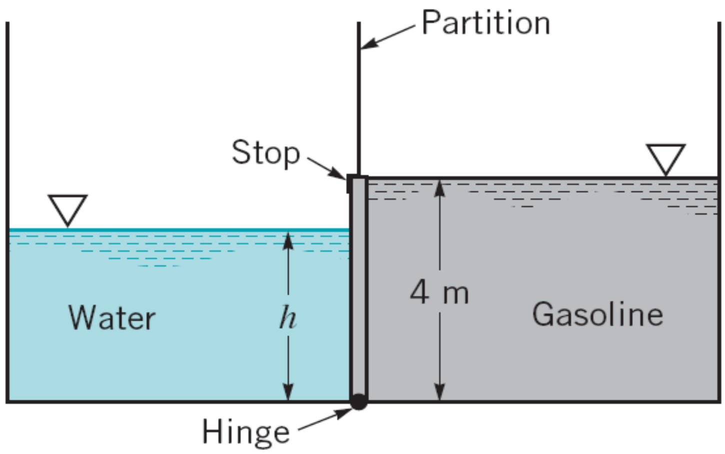
$$p_C = -\rho g x_C - \frac{1}{3}\rho g z_C + C_1 \Rightarrow \boxed{p_C = C_1}$$

$$p_C - p_A = \rho g\left(L + \frac{1}{3}H\right) = (1300)(10)\left(2.5 + \frac{1}{3}\cdot 3\right) = \underline{45500\,\text{Pa}}.$$

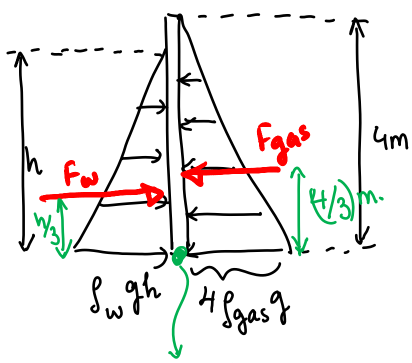
$$p_B - p_A = \frac{1}{3}\rho g H = \frac{1}{3}(1300)(10)\cdot(3) = \underline{13000\ \text{Pa}}$$

 Name: ____________________

5) [**10 marks**] An open tank has a vertical partition and on one side contains gasoline with density $\rho_{gas} = 700\ kg/m^3$ at a depth of 4 m, as shown in the figure below. A rectangular gate that is 4 m high and 2 m wide and hinged at its end is located in the partition. There is a stopper at the top end of the gate to prevent it from opening by the effect of the gasoline. Water is slowly added to the empty side of the tank. At what depth h, will the gate start to open? (Take the density of the water as $\rho_{water} = 1000\ kg/m^3$)
Hint: At the time when the gate starts to open, the stopper will not be in effect any more, and hence, at that time, no reaction force will be coming from the stopper.



Method 1: Pressure Prism:



In equilibrium moment caused by the gasoline about an axis passing through the hinge should be equal to the moment caused by the water.

$\Sigma M = 0$

$F_w = \frac{(\rho_w g h)\, h\, (2)}{2}$; $F_{gas} = \frac{(4\rho_{gas} g)(4)(2)}{2}$

$$\Sigma M = 0 \Rightarrow \underbrace{\frac{(\rho_w \cancel{g} h)(h)(\cancel{2})}{\cancel{2}}\frac{h}{\cancel{3}}}_{\text{Moment caused by water}} = \underbrace{\frac{4\rho_{gas}\cancel{g}(4)(\cancel{2})}{\cancel{2}}\frac{4}{\cancel{3}}}_{\text{moment caused by gasoline}}$$

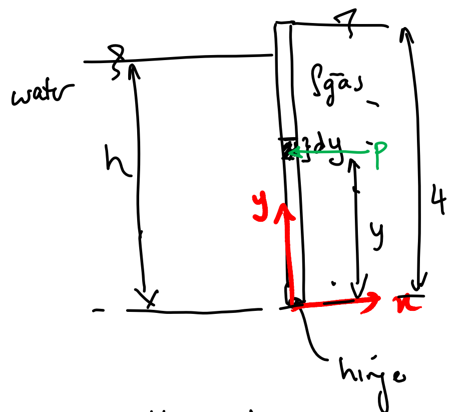
$$\rho_w h^3 = 4^3 \rho_{gas}$$

$$h^3 = 4^3 \frac{\rho_{gas}}{\rho_w} = (64)\frac{(700)}{1000} \Rightarrow \boxed{h = 3.55m.}$$

 Name: ____________________

EXTRA PAGE

Method 2: Integration



Moment of gasoline $= \iint_{Area} y\, p\, \underbrace{dA}_{dA = w\,dy}$

$p = \rho_{gas}\, g(4-y)$

$$= \int_{y=0}^{4} y\, \rho_{gas}\, g(4-y)\, w\, dy$$

$$= \rho_{gas}\, g w \int_{y=0}^{4} (4y - y^2)\, dy$$

$$= \rho_{gas}\, g w \left(\frac{4y^2}{2} - \frac{y^3}{3}\right)\Big|_0^4$$

$$= \rho_{gas}\, g w \left(\frac{(4) \cdot 4^2}{2} - \frac{4^3}{3}\right)$$

$$\boxed{M_{gasoline} = \rho_{gas}\, g w\, 4^3\, \frac{1}{6}}$$

Moment of water $= \iint_{Area} y\, p\, dA$, $dA = w\,dy$

$p = \rho_w\, g(h-y)$

$$= \int_{y=0}^{h} y\, \rho_w\, g(h-y)\, w\, dy$$

$$= \rho_w\, g w \int_0^h (hy - y^2)\, dy$$

$$= \rho_w\, g w \left(\frac{hy^2}{2} - \frac{y^3}{3}\right)\Big|_0^h$$

$$= \rho_w\, g w \left(\frac{h^3}{2} - \frac{h^3}{3}\right)$$

$$\boxed{M_{water} = \rho_w\, g w\, h^3 \left(\frac{1}{6}\right)}$$

$M_{gasoline} = M_{water} \Rightarrow$ $\rho_{gas}\, \cancel{g}\, \cancel{w}\, 4^3\, \cancel{\frac{1}{6}} = \rho_w\, \cancel{g}\, \cancel{w}\, h^3 \cancel{\left(\frac{1}{6}\right)}$

$$h^3 = 4^3 \frac{\rho_{gas}}{\rho_w} = \frac{64 \times 700}{1000} \Rightarrow \boxed{h = 3.55\,m}$$