ECE159H1: Fundamentals of Electric Circuits

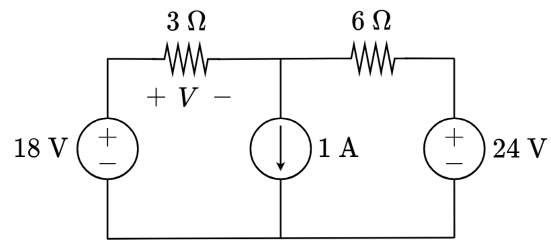
Midterm – Thursday March 2, 2023

$\boxed{\text{Solutions}}$

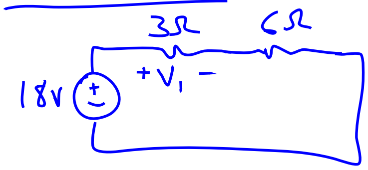
- Make sure to ***accurately*** enter your first name, last name, and student number above.
- The Midterm is worth 80 marks and has four questions. Each question is worth 20 marks.
- Show all of your work, and the final page is left blank which you can use for rough work or for extra space for your answers.
- Take a deep breath and relax 😊.

Question #1 (20 marks)

(*8 marks*) 1. (a) For the circuit shown, find the voltage V using superposition and the voltage/current division rules.

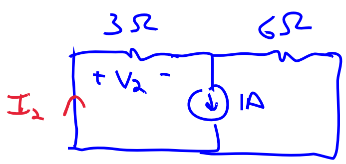


Source #1:



$$V_1 = 18\left(\frac{3}{3+6}\right) = \frac{18}{3} = \underline{6\,V}$$

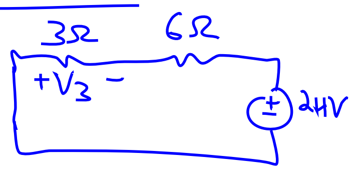
Source #2:



$$I_2 = (1)\left(\frac{6}{3+6}\right) = \frac{6}{9} = \frac{2}{3}\,A$$

$$\therefore\ V_2 = (3)\left(\tfrac{2}{3}\right) = \underline{2V}$$

Source #3:



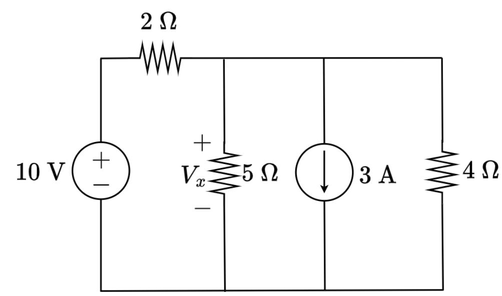
$$V_3 = -24\left(\frac{3}{6+3}\right) = \frac{-24}{3} = \underline{-8\,V}$$

negative due to the opposite polarity of the 24V source and how V is defined

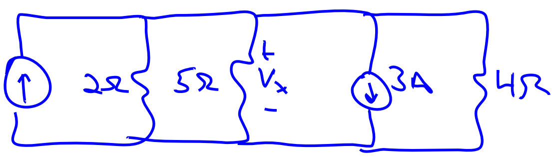
Total V: $V = V_1 + V_2 + V_3 = 6 + 2 - 8 = \underline{\underline{0V}}$

Question #1 (cont'd)

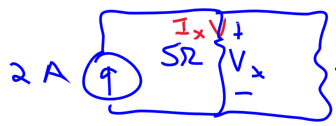
(*6 marks*) 1. (b) In the circuit shown to the right, find the voltage V_x using source transformation.



$\frac{10}{2} = 5A$



Combining sources and simplifying leads to:



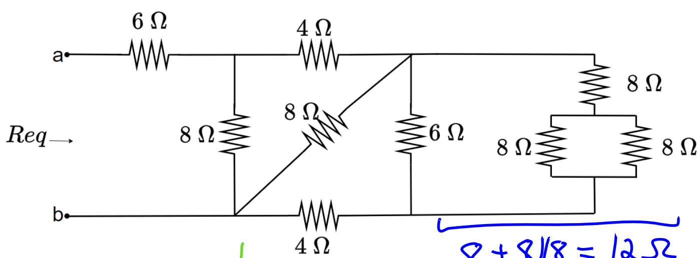
$2 \| 4 = \left(\frac{1}{2} + \frac{1}{4}\right)^{-1} = \frac{4}{3}\ \Omega$

$$I_x = 2\left(\frac{4/3}{4/3 + 5}\right) = \frac{8}{19}\ A$$

$$\therefore\ V_x = 5 I_x = \underline{\underline{\frac{40}{19}\ A}} = \underline{\underline{2.105\ A}}$$

Question #1 (cont'd)

(*6 marks*) 1. (c) For the combination of resistors shown below, find the equivalent resistance (R_{eq}) between terminals a-b. Clearly indicate the steps you take to find this equivalent resistance.



$8 + 8 \| 8 = 12\,\Omega$

$4 + 6 \| 12 = 8\,\Omega$

$8 \| (4 + 8 \| 8) = 4\,\Omega$

$\therefore R_{eq} = 6\,\Omega + 4\,\Omega$

$= \underline{\underline{10\,\Omega}}$

Question #2 (20 marks)

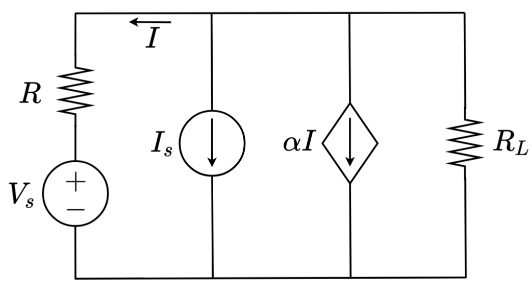
2. In the circuit given below,

(*12 marks*) (a) Find the Thévenin equivalent circuit seen by the resistor R_L.

(*2 marks*) (b) Find R_L such that the maximum power is delivered to R_L.

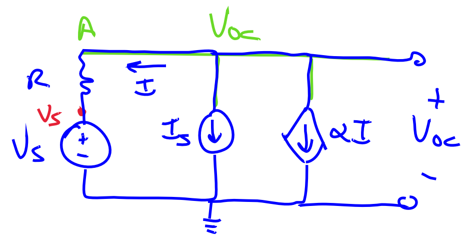
(*6 marks*) (c) Derive an expression for the maximum transferrable power to R_L in terms of R, V_S, I_S, and α.

You may assume that α is a positive constant.



(a)

For V_{oc}:



KVL @ A: $\frac{V_{oc} - V_S}{R} + I_S + \alpha I = 0$

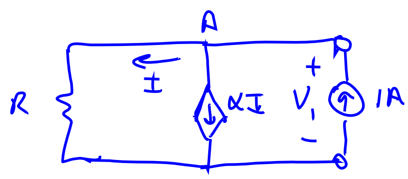
But $I = \frac{V_{oc} - V_S}{R}$

$\therefore (1+\alpha)\left(\frac{V_{oc} - V_S}{R}\right) + I_S = 0$

$V_{oc}\left(\frac{1+\alpha}{R}\right) = -I_S + V_S\left(\frac{1+\alpha}{R}\right)$

$V_{oc} = V_S - \left(\frac{R}{1+\alpha}\right) I_S$

For R_{Th}:



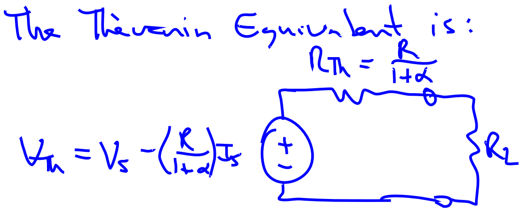
$R_{Th} = \frac{V_1}{1A}$ → need to find V_1

KCL @ A: $1 = \alpha I + I = (\alpha + 1) I$ → $I = \frac{1}{1+\alpha}$

$V_1 = IR = \frac{R}{1+\alpha}$

$\therefore R_{Th} = \frac{V_1}{1} = \frac{R}{1+\alpha}$

$\therefore$ The Thévenin Equivalent is:



Question #2 (cont'd)

(b) For maximum power transfer $R_L = R_{Th} = \underline{\underline{\frac{R}{1+\alpha}}}$

(c) $$P_{max} = \frac{V_{oc}^2}{4R_{Th}} = \frac{\left[V_s - \left(\frac{R}{1+\alpha}\right)I_s\right]^2}{4\left(\frac{R}{1+\alpha}\right)}$$

$$= \frac{V_s^2 - 2V_sI_s\left(\frac{R}{1+\alpha}\right) + \left(\frac{R}{1+\alpha}\right)^2 I_s^2}{4\left(\frac{R}{1+\alpha}\right)}$$

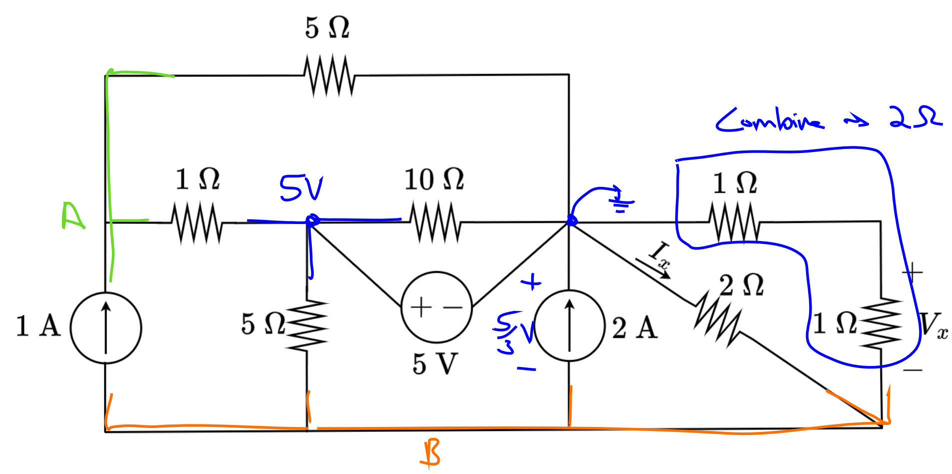
$$= \underline{\underline{\frac{V_s^2}{4}\left(\frac{1+\alpha}{R}\right) - \frac{V_sI_s}{2} + \frac{I_s^2}{4}\left(\frac{R}{1+\alpha}\right)^2}}$$

Question #3 (20 marks)

3. In the circuit below, use Nodal Analysis or Mesh Analysis to find:

(*15 marks*) (a) The voltage V_x and the current I_x

(*5 marks*) (b) The power supplied by the 2 A current source



(a) Using Nodal Analysis: 2 unknowns, V_A & V_B

↳ However, we only need to find V_B to find V_x & I_x

KCL@B: $10\left(1 + \frac{V_B - 5}{5} + 2 + \frac{V_B}{2} + \frac{V_B}{2}\right) = (0)10$

$10 + 2V_B - 10 + 20 + 5V_B + 5V_B = 0 \rightarrow V_B = \frac{-20}{12} = -\frac{5}{3}\,V$

$\therefore I_x = \frac{-V_B}{2} = \frac{5}{6}\,A \qquad V_x = -V_B\left(\frac{1}{1+1}\right) = \left(\frac{5}{3}\right)\left(\frac{1}{2}\right) = \frac{5}{6}\,V$

(b) Since the 2A source leaves the positive terminal of the voltage across this current source → the source is supplying power.

$P_{2A} = -\left(\frac{5}{3}\right)(2) = -\frac{10}{3}\,W$ (supplying)

↳ This is negative since it is supplying power.

Question #4 (20 marks)

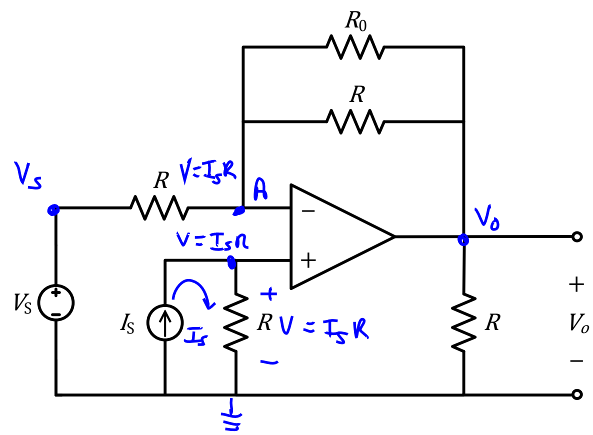
4. For the op-amp circuit shown below:

(*12 marks*) (a) Determine the values of R and R_0 that will produce an output voltage given by:

$$V_o = 160 I_S - \frac{1}{9} V_S$$

(*8 marks*) (b) Calculate the percentage of the power dissipated in the circuit's 5 resistors that is provided by the two sources V_S and I_S. Briefly justify your answer. For this part (b), you can assume that $R = 150\ \Omega$ and $R_0 = 50\ \Omega$, and that the sources have the values of $V_S = 18$ V and $I_S = 90$ mA.

You can use the ideal model for the op-amp for both parts of this question.



Sol^{n} #1

(a) KVL @ A:

$$RR_o\left(\frac{V_s - I_sR}{R}\right) = \left(\frac{I_sR - V_o}{R} + \frac{I_sR - V_o}{R_o}\right)RR_o$$

$$R_oV_s - I_sRR_o = I_sRR_o - R_oV_o + I_sR^2 - RV_o$$

$$V_o(R+R_o) = (R^2 + 2RR_o)I_s - R_oV_s$$

$$\therefore V_o = \frac{R(R+2R_o)}{R+R_o} I_s - \frac{R_o}{R+R_o} V_s$$

For $V_o = 160 I_s - \frac{1}{9} V_s$

$$\frac{R(R+2R_o)}{R+R_o} = 160 \text{ ①} \quad \text{and} \quad \frac{R_o}{R+R_o} = \frac{1}{9} \text{ ②}$$

$$R + R_o = 9R_o$$

$$R = 8R_o \text{ ③}$$

③ → ① $\frac{8\cancel{R_o}(8R_o + 2R_o)}{8\cancel{R_o} + \cancel{R_o}} = 160$

$$80R_o = 160(9) = 1440$$

$$R_o = \frac{1440}{80} = \underline{18\ \Omega} \text{ ④}$$

④ → ③ $R = 8(18) = \underline{144\ \Omega}$

(a) Sol^{n} #2: Inv. & Non-Inv. configurations and superposition

Inv. Output

$$V_{o1} = -\left(\frac{R_o \| R}{R}\right) V_s = -\left(\frac{R_o}{R+R_o}\right) V_s$$

Non-Inv. Output   Input voltage at the terminal

$$V_{o2} = \left(1 + \frac{R \| R_o}{R}\right) I_sR = \left(R + \frac{RR_o}{R+R_o}\right) I_s$$

$$= \left[\frac{R(R+R_o) + RR_o}{R+R_o}\right] I_s$$

$$= \frac{R(R+2R_o)}{R+R_o} I_s$$

$$\therefore V_o = V_{o1} + V_{o2}$$

$$= \frac{R(R+2R_o)}{R+R_o} I_s - \frac{R_o}{R+R_o} V_s$$

↳ for $V_o = 160 I_s - \frac{1}{9} V_s$ we can solve and find that $R = \underline{144\ \Omega}$ & $R_o = \underline{18\ \Omega}$

Question #4 (cont'd)

(b) With $V_s = 18V$, $I_s = 90\,mA$, $R = 150\Omega$, $R_o = 50\Omega$:

$$V_o = \frac{R(R+2R_o)}{R+R_o} I_s - \frac{R_o}{R+R_o} V_s = 187.5(0.09) - \frac{1}{4}(18)$$

$$= \underline{12.375\ V}$$

The power dissipated by the circuits 5 resistors is:

$$P_{diss} = I_s^2 R + \frac{(V_s - I_s R)^2}{R} + \frac{(I_s R - V_o)^2}{R} + \frac{(I_s R - V_o)^2}{R_o} + \frac{V_o^2}{R}$$

$$= 1.215 + 0.135 + 0.00844 + 0.0253125 + 1.0209375$$

$$= \underline{2.405\ W}$$

The power supplied by the sources:

$$P_{supp} = V_s\left(\frac{V_s - I_s R}{R}\right) + I_s(I_s R)$$

$$= 0.54 + 1.215 = \underline{1.755\ W}$$

$\therefore$ The percentage of dissipated power supplied by the sources is $\frac{\text{Power supplied}}{\text{Power dissipated}} = \frac{1.755W}{2.405W} = 73\%$ $\left(\frac{P_{diss}}{P_{supp}} = 137\% \text{ is also acceptable}\right)$

This makes sense as the op-amp DC sources will provide the other 27% of this dissipated power.