UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Term Test III

First Year — Program 5

MAT185H1S — Linear Algebra

Examiners: A D Rennet & G M T D'Eleuterio

18 March 2014

Student Name: ____________________

Last Name First Names

Student Number: ____________________ Tutorial Section: TUT

Instructions:

1. Attempt *all* questions.
2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question.
4. *No* aid is permitted.
5. The duration of this test is 90 minutes.
6. There are 11 pages and 5 questions in this test paper.

For Markers Only		
Question	**Value**	**Mark**
A		
1	10	
B		
2	10	
C		
3	10	
4	10	
5	10	
Total	50	

A. Definitions and Statements

Fill in the blanks.

1(a). The *column space* of $\mathbf{A} \in {}^m\mathbb{R}^n$ is

/2

1(b). State a condition for $\mathbf{A} \in {}^m\mathbb{R}^n$ to have a *left inverse*.

/2

1(c). How are the dimensions of the row space and column space of a given matrix related?

/2

1(d). A basis for the row space of $\mathbf{A} \in {}^m\mathbb{R}^n$ can be determined by

/2

1(e). The *coordinates* of $\boldsymbol{v} \in \mathcal{V}$ relative to the basis $E = \{\boldsymbol{e}_1 \dots \boldsymbol{e}_n\}$ are

/2

B. True or False

Determine if the following statements are true *or* false *and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.*

2(a). The rank of a matrix is equal to the number of its nonzero rows.

2(b). If $\mathbf{A} \in {}^m\mathbb{R}$ and rank $\mathbf{A} = n$, then there exists $\mathbf{B} \in {}^n\mathbb{R}^m$ such that $\mathbf{BA} = \mathbf{1}$.

2(c). If $c \in \mathbb{R}$, $c \neq 0$ and $\mathbf{A} \in {}^2\mathbb{R}^2$, then rank $(c\mathbf{A}) =$ rank $\mathbf{A}$.

2(d). If $\mathbf{A} \in {}^n\mathbb{R}^n$, then $\dim$ im $\mathbf{A} + \dim$ null $\mathbf{A} = n$.

2(e). If $\mathbf{A} \in {}^n\mathbb{R}^n$, then null $\mathbf{A} \subseteq$ col $\mathbf{A}$.

C. Problems

3. The first three Legendre polynomials are $L = \{1, x, \frac{1}{2}(3x^2 - 1)\}$ and the first three Gegenbauer polynomials are $G = \{1, 2x, 4x^2 - 1\}$. Note that L and G are bases for $\mathbb{P}_2$.

(a) Determine the transformation (transition) matrix from G to L.

(b) If $p(x) = 2g_1(x) + g_2(x) - 3g_3(x)$, where g_1, g_2, g_3 are the basis functions in G as given above, determine the coordinates of p in L.

3(a). Determine the transformation (transition) matrix from G to L.

. . . cont'd

3. . . . cont'd

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3(b). If $p(x) = 2g_1(x) + g_2(x) - 3g_3(x)$, where g_1, g_2, g_3 are the basis functions in G as given above, determine the coordinates of p in L.

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4. Let $\mathbf{A} \in {}^m\mathbb{R}^n$. Show that $\mathbf{Ax} = \mathbf{b}$ has no solution if and only if

$$\operatorname{rank}\,[\mathbf{A} \mid \mathbf{b}] = 1 + \operatorname{rank}\mathbf{A}$$

. . . cont'd

4. . . . cont'd

/10

5. Let $\mathcal{V}$ be a vector space with basis $E = \{\boldsymbol{e}_1 \ldots \boldsymbol{e}_n\}$, let $\mathbf{Q} = [q_{ij}] \in {}^n\mathbb{R}^n$ and define

$$\boldsymbol{f}_j = \sum_{i=1}^{n} q_{ij} \boldsymbol{e}_i, \qquad j = 1 \ldots n$$

Prove that $F = \{\boldsymbol{f}_1 \ldots \boldsymbol{f}_n\}$ is a basis for $\mathcal{V}$ if and only if $\mathbf{Q}$ is invertible.

. . . cont'd

5. . . . cont'd

/10