1. [13 pts.] (4 pts. for) approach of $\frac{\Delta L}{I_E} = \Delta\omega_E$, as noted below.)

The population of the earth is 7×10^9, with each person ~50kg, so a total mass of 4×10^{10} kg. At R_E, their collective $I = MR_E^2$, and walking at V, $\omega = V/R_E$, so $L = MR_E V$. Assuming the initial L of people ≈0, because they are walking in random directions, this gives us $\Delta L = MR_E V$.

1pt: M people

1pt: $\Delta L = MR_E V$

What is V for someone walking? About 3km/hr which is ~3000 m/3600 s ~ 1 m/s.

1pt: V est.

Now, the planet is a sphere of R_E, M_E; it's not uniform but let's estimate it as if it were. $I_E \approx \frac{2}{5}M_E R_E^2$, and $\omega = \frac{2\pi}{24h\cdot3600s} \sim 7\times10^{-5}\,s^{-1}$

1pt: I_E

1pt: ω_E

Now we're ready to answer the question.

Since angular momentum is conserved,

$L_i = L_f$, or $I_E\omega_i = I_E\omega_f + \Delta L$

so $\Delta\omega = \frac{-\Delta L}{I_E} = \frac{-M_p R_E V}{2/5\, M_E R_E^2}$

4 pts for approach (cons. of L)

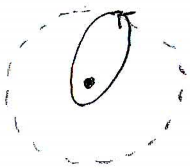
$$\sim \frac{-(4\times10^{10}\,kg)(1\,m/s)}{0.4\,(5.97\times10^{24}\,kg)(6.37\times10^6\,m)} = 2.6\times10^{-21}\,s^{-1}$$

and thus $\frac{\Delta\omega}{\omega_E} = 3.6\times10^{-17}$ or $\boxed{\sim 10^{-16}}$

Accept $10^{-15} \to 10^{-17}$ with full marks.

4 pts for algebra & answer

(2.) [20pts.]



Approach: conserve L, E and find matching orbit.

[4pts] 2pts for basic idea of conservation of L

(a) L is the same as it was before earth vanished.

$$L = m R_E^2 \omega_E = (70\,kg)(6.37\times10^6\,m)^2\,(7.27\times10^{-5}\,s^{-1})$$

because $\omega_E = 2\pi/(24h \cdot 3600s)$ 2 pts for algebra

$\rightarrow$ $\boxed{L = 2.1\times10^{11}\ kg\,m^2/s}$

[4pts] 1pt for idea

(b) E also the same as before. There is both potential & kinetic energy, which are

$$U = -\frac{G M_E m}{R_E} \quad \text{and} \quad K = \frac{1}{2} m (\underbrace{R_E \omega_E}_{=463\,m/s})^2$$

$$= -\frac{(6.67\times10^{-11})(5.97\times10^{24})(70kg)}{(6.37\times10^6 m)} \quad \Big| \quad = \frac{1}{2}(70kg)(463\,m/s)^2$$

1 pt for calc. of U $= -4.38\times10^9\,J$ $\quad = 7.51\times10^6\,J$

1 pt. for calc of K or statement that it's small.

So notice K is negligible! And $E_i \approx U_i$ to 0.1% precision.

$\rightarrow$ $\boxed{E = -4.4\times10^9\,J}$ 1 pt for algebra

(c) Figure out a using energy equation for an orbit: [4pts]

$$E = -\frac{G M_E m}{2a} = E_i$$

2 pts for approach ↳ but this $= -\dfrac{G M_E m}{R_E}$

So $a = R_E/2$ 2 pts for algebra.

$\rightarrow$ $\boxed{a = 3.2\times10^6\,m}$ [Full credit if (b) wrong but used wrong E_i correctly here.]

2 pts for approach: 1 for conservation,...

(d) Now match $L_i = L_f$ to find b, using

[4pts.]

$$L^2 = GMm^2b^2/a \quad \rightarrow \quad b^2 = a\frac{L^2}{GMm^2}$$

... 1pt. for correct formula

Last page we wrote $L_i = m R_E^2 \omega_E$, so

$$L^2/GMm^2 = \frac{R_E^4 \omega_E^2}{GM}, \quad \text{also} \quad a = \frac{R_E}{2}, \quad \$$$

together $$b^2 = \frac{R_E}{2}\frac{R_E^4\omega_E^2}{GM_E}$$

2pts for algebra. [Full credit if approach right but used wrong L from (a).]

$$= \frac{(6.37\times10^6\,m)^5\,(7.27\times10^{-5}\,s^{-1})^2}{2(6.67\times10^{-11}kg^{-1}m^3s^{-2})(5.97\times10^{24}kg)}$$

$$\boxed{b = 2.6\times10^5\,m}$$

(e) Orbital period is determined by a, using

[4pts.] 2 pts for

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3 = \frac{4\pi^2\,(3.2\times10^6\,m)^3}{(6.67\times10^{-11})(5.97\times10^{24})}$$

$\rightarrow$ $\boxed{T = 1800\,s}$ which is half an hour, much shorter than a day.

1pt for answer in s.

1 pt for statement of comparison to 24h day.

What if someone used eqs for circular motion instead? (a) and (b) would be unchanged. Give up to 3 pts on (c) if found a but called it r. No points for (d), since $b=a$ for a circular orbit. Up to 3 pts. on (e).

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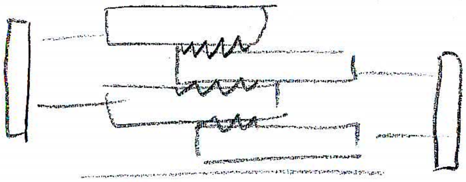
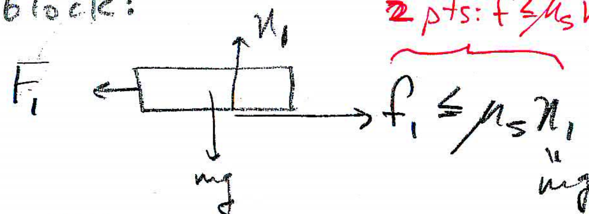
3. [12 pts.] ---- (first part: 9 pts total.)



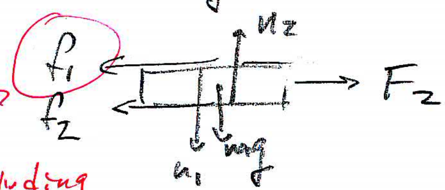
consider FBD for each block: ← 1 pt.



2 pts: $f \leq \mu_s n$ for static friction

block 2: $f_1 \leq \mu_s n_1 = \mu_s mg$

$f_2 \leq \mu_s n_2 = 2\mu_s mg$



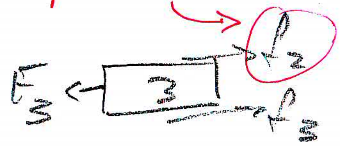
3 pts: including static friction on top of blocks

$n_2 = 2mg$

block 3: same pattern,

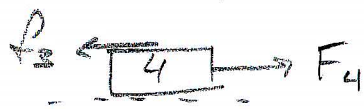
$f_2 \leq 2\mu_s mg$

$f_3 \leq 3\mu_s mg$



block 4: only friction on the top,

$f_3 \leq 3\mu_s mg$



3 pts: Algebra & correct answer

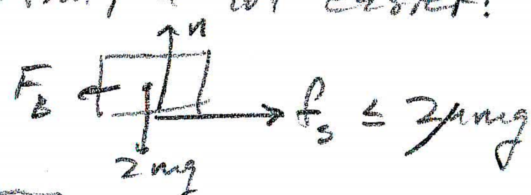
Together, $F_1 + F_3 = f_1 + f_2 + f_3 \leq (1+2+3)\mu_s mg$

and $F_2 + F_4 = f_1 + f_2 + f_3 \leq (1+2+3)\mu_s mg$

for static equilibrium. This condition is broken for $\boxed{F_A \geq 6\mu_s mg}$ ← Since asked for minimum force, can give answer as an equality.

---- (second part: 3 pts total)

For second configuration, a lot easier: only one frictional force,



& broken when

$\boxed{F_B = 2\mu_s mg}$ or higher.

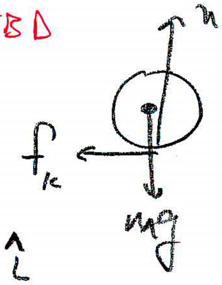
[20 pts.]

4. Ball starts at V_i, but $\omega = 0$. Apply

(a) [12 pts.] $F = ma$ and $\tau = I\alpha$ to consider dynamics during acceleration.

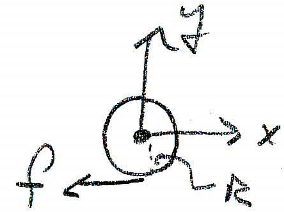
1 pt: Correct FBD

FBD:



Torque bal:

1 pt: correct $\vec{\tau}$ diagram



$\sum \vec{F} = -f_k \hat{i}$

$\sum \vec{\tau} = -f_k R \hat{k}$

and $f_k = \mu_k n = \mu_k mg$

~~[12 pts]~~ ~~(a)~~

So now use constant-a and constant-α formulae:

$$V(t) = V_i + at = V_i - \frac{\mu_k g \cancel{m}}{\cancel{m}} t$$

1 pt: constant a

and 1 pt: constant α

$$\omega(t) = \cancelto{0}{\omega_i} + \alpha t = -\frac{f_k R}{I} t$$

1 pt: correct α

here $I = \frac{2}{5} MR^2$, so $\alpha = -\frac{\mu_k g MR}{2/5 MR^2} = -\frac{5g}{2R}$

1 pt: correct I

Basic idea 3 pts

This continues until t_1, when the no-slip condition can be met: $|V| = R|\omega|$, so here

$$V_i - \mu_k g t_1 = R \frac{5g\mu_k}{2R} t_1$$

Solve: $V_i = \underbrace{\left(1 + \frac{5}{2}\right)}_{7/2} \mu_k g t_1 \rightarrow \boxed{t_1 = \frac{2V_i}{7\mu_k g}}$

3 pts for algebra & right answer.

check units:

$\frac{m/s}{m/s^2} = s$ ✓

[8 pts.]

(6) Mechanical energy lost:

- calculate final & initial K, or
- calculate work done by friction

2 pts: strategy

Choose first strategy:

$$\Delta K = K_f - K_i$$

1 pt: correct approach to calculate K_f

1 pt: K_i

$$= \left(\tfrac{1}{2} M v_f^2 + \tfrac{1}{2} I \omega_f^2\right) - \left(\tfrac{1}{2} m v_i^2\right)$$

What is v_f? $v(t_1) = v_i - \mu_k g \left(\frac{2 v_i}{7 \mu_k g}\right)$

$$= \frac{5}{7} v_i$$

What is ω_f? $\omega_f = v_f / R = \frac{5}{7} \frac{v_i}{R}$

2 pts v_f & ω_f

$$\rightarrow \quad \Delta K = \frac{1}{2} m \left(\frac{5}{7}\right)^2 v_i^2 + \frac{1}{2}\left(\frac{2}{5} M R^2\right)\left(\frac{5}{7}\right)^2\left(\frac{v_i}{R}\right)^2 - \frac{1}{2} m v_i^2$$

Everything proportional to $m v_i^2$, just find the fractions:

$$\Delta K = \left(\frac{1}{2}\left(\frac{5}{7}\right)^2 + \frac{1}{\cancel{2}}\left(\frac{\cancel{2} \cdot 5^{\cancel{2}}}{\cancel{5} \cdot 7^2}\right) - \frac{1}{2}\right) m v_i^2 = \boxed{-\frac{1}{7} M v_i^2}$$

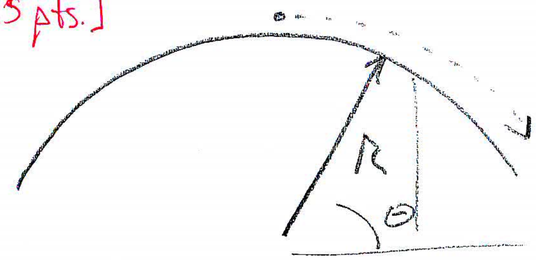
or $\boxed{-0.14\, m v_i^2}$

2 pts for algebra & answer.

Note that K_f can also be calculated with $\frac{1}{2} I \omega_f^2$ alone if $I = I_{cm} + M R^2$ using parallel axis th.

This does not work for K_i because wheel is not initially spinning.

5. [15 pts.]



case of circular motion, so long as gravity can provide sufficient radial acceleration

Strategy: ① conserve energy to find v at any h.
② find $\vec{a}$ provided by gravity + normal force
③ "Launch" when $n=0$, since $n<0$ impossible.

(5 pts if gave correct strategy but did no math.)

1. Conservation of E: $E_i = \frac{1}{2}mv_i^2 + mgh_i$ (h_i "R")

including $E_f = \frac{1}{2}mv_f^2 + mgh_f$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mg(R-h_f)$$

2pts: idea
3pts: math

$$v_f^2 = v_i^2 + 2g(R-h_f)$$

5 pts for using cons of E to find v as a function of h

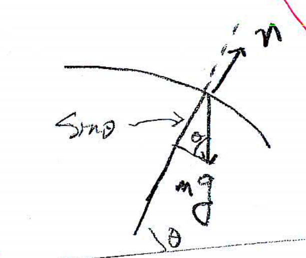
2. To stay in circular motion, need $a_r = -\frac{v^2}{R}$

this part = 6 pts:

Idea of circular motion = 2 pts

Gravity + Normal force provide

$$m a_t = -mg\cos\theta$$
$$m a_r = -mg\sin\theta + n$$



2 pts: $a_r = -v^2/R$

3. @ $n=0$, $a_r = -g\sin\theta = -g\frac{h}{R}$

2 pts: launch occurs when $n \to 0$

Solve with above equations:

$$-\frac{v^2}{R} = -g\frac{h}{R} \rightarrow -(v_i^2 + 2g(R-h)) = -gh$$
$$v_i^2 + 2gR - 2gh = gh$$
$$v_i^2 + 2gR = 3gh$$

$$\boxed{h = \frac{2}{3}R + \frac{v_i^2}{3g}}$$

4 pts for algebra & right answer

[20pts.]

(6.) Approach: conserve $\vec{p}$ and $\vec{L}$.

$$\vec{p} = \sum m\vec{v} \quad , \quad \vec{L} = m\vec{R}_1 \times \vec{v}_1 + I_z\vec{\omega}_1 + m\vec{r}_2 \times \vec{v}_2 + I_{cm\,z}\vec{\omega}_2$$

pts for this should be included in part b.

Choose axis for $\vec{L}$ to simplify calculation, eliminating cm terms if possible.

(a) By conservation of momentum,

[5pts.]

$$(M_{tot}\vec{V}_{cm})_i = (M_{tot}\vec{V}_{cm})_f$$

← idea 2pts

so just calculate $\vec{V}_{cm,i}$.

$$M_{tot}\vec{V}_{cm} = M_1\vec{V}_{cm,1} + M_2\underbrace{\vec{V}_{cm,2}}_{=0}$$

$$\rightarrow \vec{V}_{cm} = \underbrace{\frac{m_1}{M_{TOT}}}_{=\frac{3}{3+12}}\left(V_0\sqrt{\frac{3}{2}}\,\hat{\imath} + \frac{V_0}{2}\,\hat{\jmath}\right), \quad V_0 = 20$$

math: 3pts.

$$= \frac{3}{15}\cdot 20\cdot\sqrt{\frac{3}{4}}\,\hat{\imath} + \frac{3}{15}\cdot\frac{20}{2}\,\hat{\jmath}$$

$$= \boxed{3.5\,\hat{\imath} + 2.0\,\hat{\jmath}} \text{ in mks, m/s}$$

[15 pts]

(6) First, find $\vec{V}_{2f}$:

3 pts: finding $\vec{V}_{2f}$

$$m_1 \vec{V}_{1i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f}$$

(1 pt: conservation idea
1 pt: vector math
1 pt: algebra.)

$$\frac{m_1}{m_2}\left(\underbrace{\vec{V}_{1i} - \vec{V}_{1f}}\right) = V_{2f}$$

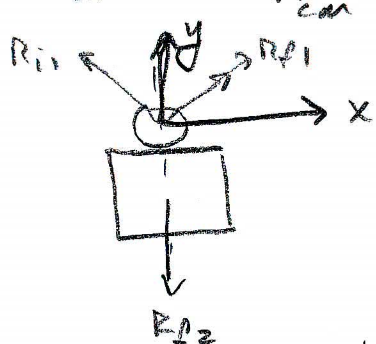
$$\frac{3}{12} \uparrow \quad \left(V_0\sqrt{\tfrac{3}{4}}\,\hat{i} - V_0\tfrac{1}{2}\,\hat{j}\right) - \left(V_0\sqrt{\tfrac{3}{4}}\,\hat{i} + V_0\tfrac{1}{2}\,\hat{j}\right) = -V_0\,\hat{j}$$

$$\longrightarrow V_{2f} = -\frac{3}{12}V_0\,\hat{j} = \underline{-5.0\ \text{m/s}\ \hat{j}}$$

3 pts: Any approach that conserves $\vec{L}$.

This means that if we centre our axes at the position of the sphere at the instant of the collision, then the cm terms in $\vec{L}$ will be zero, since $\vec{R}_{cm}$ & $\vec{V}_{cm}$ parallel for all cases.

1 pt: specify axis for calc of $\vec{L}$.



4 pts: Correct calculation of $\vec{L}$ about any specified axis

Then: $\vec{L}_i = 0$

$$\vec{L}_f = I_{cm,1}\,\vec{\omega}_{1f} + I_{cm,2}\,\vec{\omega}_{2f}$$

Using $\vec{\omega}_{1f} = -20\,\hat{k}$,

1 pt: $I_{cm,1}$ 1 pt: $I_{cm,2}$

$$I_{cm,1} = \frac{2}{5}(3.0)(0.50\,\text{m})^2 = 0.3\ \text{kg m}^2, \quad I_{cm,2} = \frac{1}{12}(1.5^2+1.5^2)\,12 = 4.5\ \text{kg m}^2,$$

we have $W_{2f} = -\frac{0.3}{4.5}\cdot(-20) = \boxed{-1.3\ \text{s}^{-1}}$ ← 1 pt: direction

1 pt: algebra here.

ie, counter-clockwise.