CHE 260

SOLUTIONS

QUIZ 1 – 2015

1) $P = cV^{1/3} = \left(100 \frac{kPa}{m}\right) V^{1/3}$.

$P_1 = \left(100 \frac{kPa}{m}\right)(1\ m^3)^{1/3} = 100\ kPa$

$P_2 = \left(100 \frac{kPa}{m}\right)(3\ m^3)^{1/3} = 144.22\ kPa.$

$$W_{12} = -\int_{V_1}^{V_2} P\,dV$$

$$= -\int_{V_1}^{V_2} cV^{1/3}\,dV = -\frac{3}{4} cV^{4/3} \Big|_{V_1}^{V_2}$$

$$c = P_1 V_1^{-1/3} = P_2 V_2^{-1/3}$$

$$\Rightarrow W_{12} = -\frac{3}{4}\left[cV_2^{4/3} - cV_1^{4/3}\right]$$

$$= -\frac{3}{4}\left[P_2V_2 - P_1V_1\right]$$

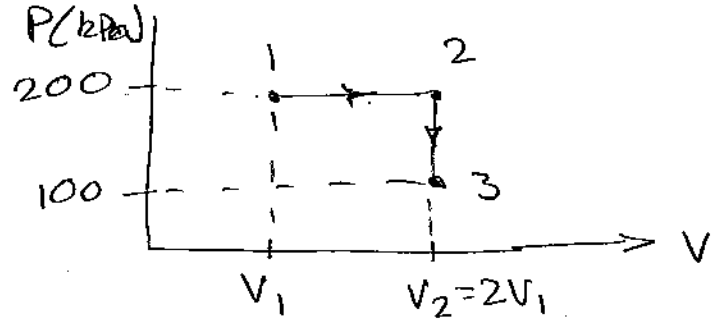
$$= -\frac{3}{4}\left[144.22\ kPa \times 3\ m^3 - 100\ kPa \times 1\ m^3\right]$$

$$= -249.5\ kJ$$

$$m_2 = \frac{P_2V_2}{RT_2} = \frac{144.22\ kPa \times 3\ m^3}{0.2870\ kJ/kg\,K \times 298\ K}$$

$$= 5.06\ kg$$

(2) on a P-V diagram



$P_1 = 200\ kPa \qquad T_1 = 600\ K.$

$P_2 = P_1 = 200\ kPa.$

For const P, $T_2 = T_1 \frac{V_2}{V_1} = 600\ K \times 2 = 1200\ K.$

$T_3 = T_1 = 600\ K$

For const V, $P_3 = P_2 \frac{T_3}{T_2} = 200\ kPa \times \frac{600\ K}{1200\ K}$

$P_3 = 100\ kPa.$

From tables at 600 K

$h_1 = h_3 = 607.02\ kJ/kg$; $u_1 = u_3 = 434.76\ \frac{kJ}{kg}$

At 1200 K, $h_2 = 1277.79\ kJ/kg$; $u_2 = 933.33\ \frac{kJ}{kg}$

For process 1-2. (const P)

$q_{12} = h_2 - h_1 = 1277.79 - 607.02 = 670.77\ kJ/kg$

$w_{12} = (u_2 - u_1) - q_{12} = (933.33 - 434.76) - 670.77$

$w_{12} = -172.22\ kJ/kg.$

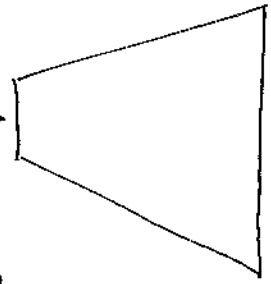
For process 2-3 (const V)

$w_{23} = 0$

$q_{23} = u_3 - u_2 = 434.73 - 933.33$

$q_{23} = -498.53\ kJ/kg.$

(3) Diffuser



$P_1 = 100$ kPa
$T_1 = 300$ K
$V_1 = 200$ m/s
$A_1 = 100$ mm^2

$V_2 = 20$ m/s
$A_2 = 860$ mm^2

Energy balance $h_1 + \frac{V_1^2}{2} + g z_1 = h_2 + \frac{V_2^2}{2} + g z_2$

$\Rightarrow h_2 - h_1 = c_p (T_2 - T_1) = \frac{V_1^2 - V_2^2}{2}$

$\Rightarrow T_2 = T_1 + \frac{V_1^2 - V_2^2}{2 c_p}$

For air $c_p = 1.004$ kJ/kg K.

$\Rightarrow T_2 = 300\text{ K} + \frac{(200\text{ m/s})^2 - (20\text{ m/s})^2}{2 \times 1.004 \times 10^3\text{ J/kg K}} = 319.72\text{ K}$

$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{A_1 V_1}{v_1}$

$\Rightarrow v_2 = v_1 \left(\frac{A_2 V_2}{A_1 V_1}\right) = \frac{R T_1}{P_1}\left(\frac{A_2 V_2}{A_1 V_1}\right) = \frac{R T_2}{P_2}$

$\Rightarrow P_2 = P_1 \left(\frac{T_2}{T_1}\right)\left(\frac{A_1 V_1}{A_2 V_2}\right)$

$= 100\text{ kPa} \left(\frac{319.72\text{ K}}{300\text{ K}}\right)\left(\frac{100\text{ mm}^2 \times 200\text{ m/s}}{860\text{ mm}^2 \times 20\text{ m/s}}\right)$

$P_2 = 123.92$ kPa.