Q2:

Consider the equation of a plane in R^3:

$$ax + by + cz = d$$

For the purposes of this question, this equation can be rearranged as follows:

$$z = \frac{d}{c} - \frac{a}{c}x - \frac{b}{c}y = d' + a'x + b'y \text{ assuming } c \neq 0$$

a) Find the plane in R^3 that gives the least squares fit in the z-direction at the four points corresponding to the corners of a square in the x-y plane, namely:

$$\{(1,0,0), (0,1,1), (-1,0,3), (0,-1,4)\}$$

b) Show that at the centre of the square (0,0) the value of z at this point on the plane found in part (a) is equal to the average of the z values at the four points corresponding to the corners of the square.

Q2 a)

$$d' + a'x + b'y = z$$

UNKNOWNS ARE d', a', b'

$$\begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \\ 1 & x_4 & y_4 \end{bmatrix} \begin{bmatrix} d' \\ a' \\ b' \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

SUBSTITUTING IN THE 4 POINTS:

$$\underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} d' \\ a' \\ b' \end{bmatrix}}_{\vec{X}} = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}}_{\vec{b}}$$

NORMAL EQUATIONS:

$$A^T A \vec{X}_{LS} = A^T \vec{b}$$

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$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^T\vec{b} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ -3 \end{bmatrix}$$

$\therefore$ SOLVING FOR $\vec{X}_{LS}$,

$$\left[\begin{array}{ccc|c} 4 & 0 & 0 & 8 \\ 0 & 2 & 0 & -3 \\ 0 & 0 & 2 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & -1.5 \end{array}\right]$$

$\therefore$ $d' = 2$, $a' = -1.5$, $b' = -1.5$

$\therefore$ $2 - 1.5x - 1.5y = z$

OR $1.5x + 1.5y + z = 2$

b) FIND VALUE OF Z AT THE CENTRE OF THE SQUARE,

$$1.5(0) + 1.5(0) + z = z$$

$$\therefore z = z$$

COMPARE TO AVERAGE OF Z VALUES AT THE FOUR POINTS,

$$\frac{0+1+3+4}{4} = \frac{8}{4} = z$$

EQUAL AS SHOWN.