AER210 VECTOR CALCULUS and FLUID MECHANICS

Midterm Test # 1

Duration: 1 hour, 50 minutes

28 October 2021

Closed Book, no aid sheets, no calculators

Instructor: Prof. Alis Ekmekci

Family Name: ____

Given Name: ____

Student #: ____

TA Name/Tutorial #: ____________________________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	17	
2	8	
3	10	
4	13	
5	10	
6	12	
7	12	
8	18	
TOTAL	100	

Note the following integrals may be useful:

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C; \quad \int \sin^2\theta\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C; \quad sin\frac{\pi}{4} = cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\oint_C Pdx + Qdy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA \qquad \oint_C \vec{F}\cdot d\vec{r} = \iint_S \left(\vec{\nabla}\times\vec{F}\right)\cdot\vec{n}\, dS$$

$$\oiint_S \vec{F}\cdot\vec{n}dS = \iiint_V \nabla\cdot\vec{F}dV$$

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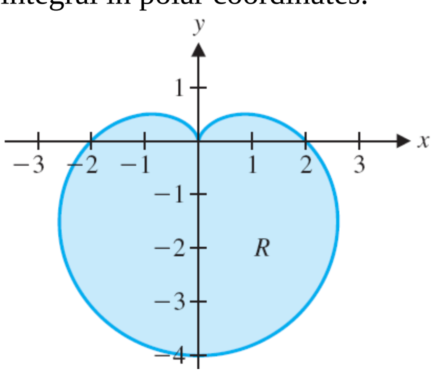
1) a) (4 marks) Evaluate the following double integral:

$$\int_0^1 \int_0^{2x} (x + 2y)dydx$$

b) (5 marks) Sketch the region over which the integration is defined and change the order of integration for the following double integral:

$$\int_0^1 \int_0^{2y} f(x,y)dxdy$$

c) (8 marks) Find the area inside the curve defined by $r = 2 - 2sin\theta$ by forming a double integral in polar coordinates.



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2) (8 marks) Use **<u>cylindrical coordinates</u>** to form the appropriate **<u>triple integral</u>** to find the volume of the solid given by the following surfaces: $z = 4 - x^2 - y^2$ and the xy-plane. Make sure to sketch the solid.

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3) (10 marks) Use **a double integral in polar coordinates** to find the volume of the solid that lies inside both the cylinder $x^2 + y^2 = 4$ and the ellipsoid $2x^2 + 2y^2 + z^2 = 18$. Provide a sketch of the volume.

4) (a) (6 marks) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$

where $\vec{F}(x, y, z) = x\vec{\imath} - z\vec{\jmath} + y\vec{k}$ and C is given by $\vec{r}(t) = 2t\vec{\imath} + 3t\vec{\jmath} - t^2\vec{k}, \quad -1 \leq t \leq 1$

(b) (7 marks) Evaluate the following line integral by Green's theorem where C is the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, \pi)$, $(0, \pi)$.

$$\oint_C e^x cosy\, dx + e^x siny\, dy$$

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5) (10 marks) If R is the region bounded by the lines

$$y = 2x - 1, \quad y = 2x + 5, \quad y = 1 - 3x, \quad y = -1 - 3x$$

using an appropriate coordinate transformation evaluate the following integral:

$$\iint_R (y + 3x)\,dA$$

6) The field $\vec{F} = (axy + z)\vec{\imath} + x^2\vec{\jmath} + (bx + 2z)\vec{k}$ is a conservative vector field.

a) (4 marks) Find a and b.

b) (6 marks) Find a potential function for $\vec{F}$.

c) (2 marks) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve from $(1, 1, 0)$ to $(0, 0, 3)$ that lies on the intersection of the surfaces $2x + y + z = 3$ and $9x^2 + 9y^2 + 2z^2 = 18$ in the octant $x \geq 0$, $y \geq 0$, $z \geq 0$.

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7) a) (10 marks) Evaluate the following line integral

$$\int_C (3x - y)ds$$

where curve C is the portion of the circle $x^2 + y^2 = 18$ traversed from $(3, -3)$ to $(3, 3)$.

b) (2 marks) Write the value of the integral if the curve C was traversed in the opposite direction (that is, clockwise from $(3, 3)$ to $(3, -3)$). Here, you are expected to write the value without re-evaluating the integral.

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8) (18 marks) Verify Stokes' theorem for the vector field $\vec{F}(x, y, z) = yz\vec{\imath} + 2xz\vec{\jmath} + y\vec{k}$ over the part of the plane $8x + y + 8z = 8$ in the first octant. Provide a sketch of the surface and the boundary curves.

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