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UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATIONS, DECEMBER 2015

CIV102H1F – Structures and Materials-
An Introduction to Engineering Design

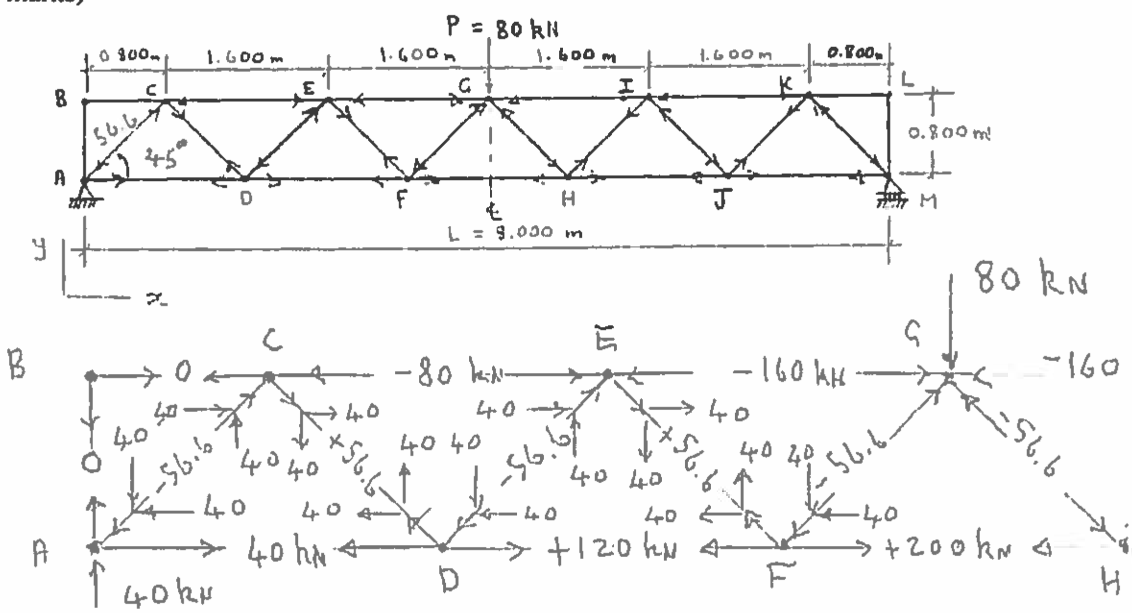
Examiner --- M.P.Collins

Permissible Aids: Notebook, calculator and set-square.

1	26
2	28
3	26
4	26
Total	106

1. The small steel truss shown below spans eight metres and supports a mid-span point load of 80 kN.

1(a). Calculate the axial force in the members of the truss due to the 80 kN load. Neatly write your calculated forces in the table on page 2. Use the convention +ve for tension and –ve for compression. Note that members AB, BC, KL and LM have zero axial force. (10 marks)



1(b). As shown in the table on page 2 the top chord and bottom chord members of the truss have a cross-sectional area of 11,200 mm² while the diagonal members all have a cross-sectional area of 1800 mm². Calculate the deformations of the members of the truss caused by the 80 kN load applied at joint G. List your results in the table on page 2. Use the convention +ve for extension and –ve for shortening. (6 marks)

1(c). Using the method of virtual work calculate the vertical deflection of joint G due to the application of the 80 kN load. Fill in the table below. Note that the table lists the members for only the left half of the truss. (10 marks)

Member	P (kN)	A (mm^2)	σ (MPa)	ε (mm/m)	L (m)	Δ (mm)	P* (kN)	Work(J)
CE	−80	11200	−7.14	−0.0357	1.60	−0.0571	−1	0.0571
EG	−160	11200	−14.29	−0.0714	1.60	−0.1142	−2	0.2284
AD	+40	11200	+3.57	+0.0179	1.60	+0.0286	+0.50	0.0143
DF	+120	11200	+10.71	+0.0536	1.60	+0.0857	+1.50	0.1286
FH	+200	11200	+17.86	+0.0893	0.80	+0.0714	+2.50	0.1785
AC	−56.6	1800	−31.4	−0.1572	1.131	−0.1778	−0.707	0.1258
DE	−56.6	1800	−31.4	−0.1572	"	−0.1778	−0.707	"
FG	−56.6	1800	−31.4	−0.1572	"	−0.1778	−0.707	"
CD	+56.6	1800	+31.4	+0.1572	1.131	+0.1778	+0.707	"
EF	+56.6	1800	+31.4	+0.1572	"	+0.1778	+0.707	"

$\Sigma = 1.236$ J

$1 \times \Delta_G = 2 \times 1.236$

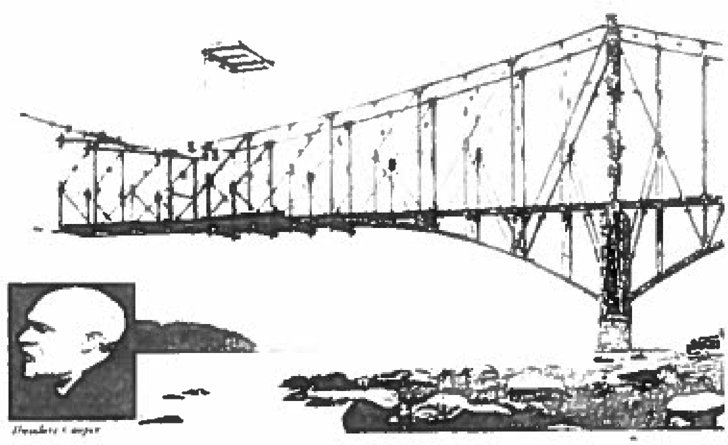
$\therefore \Delta_G = \underline{\underline{2.472}}$ mm

If FH $L = 1.60$ m

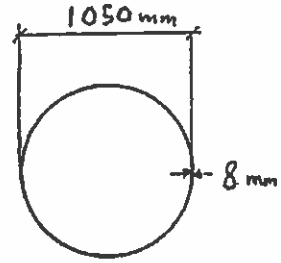
$\Sigma = 1.415 - 0.357/2$

≈ 1.236 J

2. On August 29th 1907 the Quebec Bridge over the St. Lawrence River collapsed under its own weight killing 76 construction workers. The bridge collapsed because the main compression members supporting the huge cantilever buckled. For the similar span Firth of Forth Bridge these critical compression members were hollow circular steel tubes made from bending and riveting steel plates. At Quebec, to reduce construction costs, these members were built up from four steel equal-leg angles held together by diagonal steel straps.



2. (a) A compression member is to be made as a hollow, circular steel tube. The exterior diameter of the tube is 1050 mm and the thickness of the steel wall is 8 mm. If the length of the compression member is 30 m what will be the predicted Euler buckling load? For a solid circular section, $I = \pi D^4/64$. What will be the compressive stress in the member when it buckles? (6 marks)



section of tube

$$I_o = \frac{\pi}{64}\left[d_o^4 - d_i^4\right] = \frac{\pi}{64}\left[1050^4 - 1034^4\right]$$

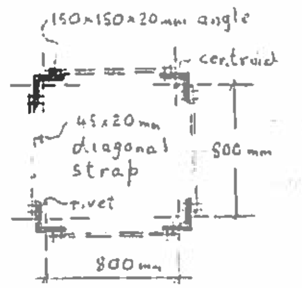
$$\approx \underline{\underline{3.555 \times 10^9 \text{ mm}^4}}$$

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 3.555 \times 10^9}{30\,000^2} = \underline{\underline{7796}} \text{ kN}$$

$$A = \frac{\pi}{4}\left[d_o^2 - d_i^2\right] = \frac{\pi}{4}\left[1050^2 - 1034^2\right] = \underline{26190 \text{ mm}^2}$$

$$\sigma_E = P_E/A = 7796 \times 10^3 / 26190 = \underline{\underline{298}} \text{ MPa}$$

2(b). As shown in the figure, a built-up member is to be made from four 150x150x20 mm steel angles riveted together by 45x20 mm rectangular steel straps. The four angles each have an area of 5600 mm² and an I value about their own centroid of 11.6×10^6 mm⁴. The distance between the centroids of the two upper angles and the two lower angles is 800 mm. Use the parallel axis theorem to calculate the I of the built up section. Count only the contributions of the four angles. Based on the calculated I what will be the predicted Euler buckling load if the length of the compression member is 30 m? What will be the compression stress in the angles when the member buckles? Comment on the efficiency of the built-up section. (10 marks)



section of built-up member

1.3% $\quad 896 \times 10^6$

$$I = 4\left[11.6 \times 10^6 + 5600 \times 400^2\right] \approx \underline{\underline{3.630 \times 10^9}} \text{ mm}^4$$

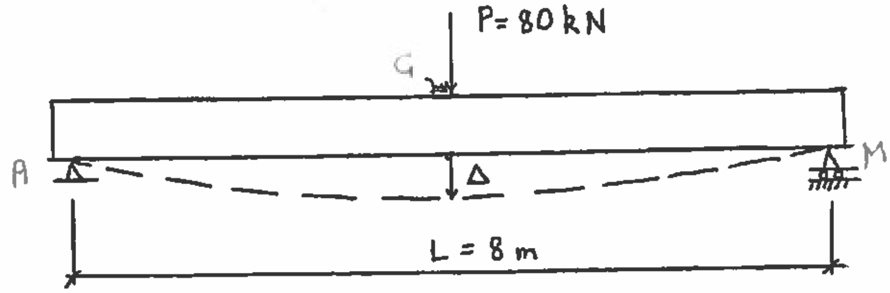
$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 3.630 \times 10^9}{30000^2} = \underline{\underline{7962}} \text{ kN}$$

$$A = 4 \times 5600 = \underline{\underline{22400 \text{ mm}^2}}$$

$$\sigma_E = P_E/A = 7962 \times 10^3 / 22400 = \underline{\underline{355}} \text{ MPa}$$

Based on these calculations it seems that the built-up square member is more efficient than the hollow circular tube as with less area it is predicted to carry more load. As it was also much easier to fabricate it seems like a good choice.

2 (c). The truss analysed in Question 1 has the same cross-section as the built-up member described in Question 2(b) above. Note that the area of the top and bottom chords are 2 x 5600 = 11200 mm^2 and that of the diagonals is 2 x 45 x 20 = 1800 mm^2. If we regard the truss as a beam subjected to a point load, P, at mid-span the deflection at mid-span can be found from the moment-area method as $PL^3/(48EI)$. Compare the value for deflection from this beam equation (use the I calculated in Question 2(b) above) with that calculated from the truss analysis in Question 1(c). Note that the beam equation neglects shear deformations. However, a considerable portion of the truss deformation is caused by change of length of the diagonal members, see table on page 2, which is equivalent to shear deformation. Calculate the I value that would be appropriate so that the $PL^3/(48EI)$ equation gives a deflection equal to that calculated in Question 1(c). With this reduced I value, which accounts for shear deformations in the built-up member, what is now the predicted buckling load of the 30-metre-long compression member? What will be the compressive stress in the member when buckling occurs. Comment on the dangers of estimating the buckling capacity of a truss like member by methods which neglect shear deformations. (12 marks)



If truss acts like a beam the mid-span deflection

$$\Delta_G = \frac{PL^3}{48EI} = \frac{80000 \times 8000^3}{48 \times 200000 \times 3.63 \times 10^9} = \underline{\underline{1.175}} \text{ mm}$$

But when calculated as a truss in Question 1 (c)

$$\Delta_G = \underline{\underline{2.472}} \text{ mm}$$

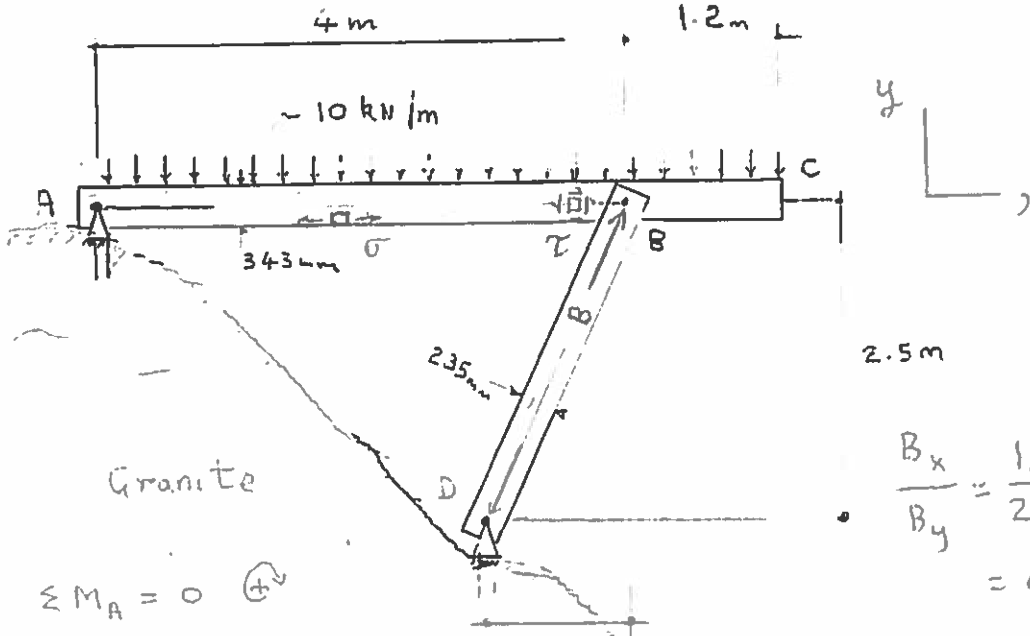
Thus the deformations of the web have increased the deflection by a factor of $2.472/1.175 = \underline{\underline{2.104}}$. This is equivalent to reducing the effective I to $3.63 \times 10^9 / 2.104 = \underline{\underline{1.725 \times 10^9}}$. Hence a more accurate estimate of the buckling load would be:

$$P_E = \frac{\pi^2 \times 200 \times 10^3 \times 1.725 \times 10^9}{30000^2} = \underline{\underline{3783}} \text{ kN}$$

$$\sigma_E = 3783 \times 10^3 / 22400 \quad \underline{\underline{169}} \text{ MPa}$$

Thus if the shear deformations of the square built-up member are ignored it's buckling load is overestimated by a factor of $\underline{2.10}$.

3. It is planned to build a cottage deck which will extend south 5.2 m over a steep, high granite face to provide a spectacular view over the lake below. The Cedar deck will be 8 m wide in the east-west direction and will be supported by five Hem-Fir frames like the one shown in the diagram below. Each frame will be supported by two steel pins, A and D, which in turn will be supported by steel plates anchored to the rock face. The frames consist of a horizontal beam, ABC, and an inclined strut, BD, pinned together at B. Each beam must safely resist a total uniformly distributed vertical load of 10 kN/m.



$$\frac{B_x}{B_y} = \frac{1.25}{2.50} = 0.50$$

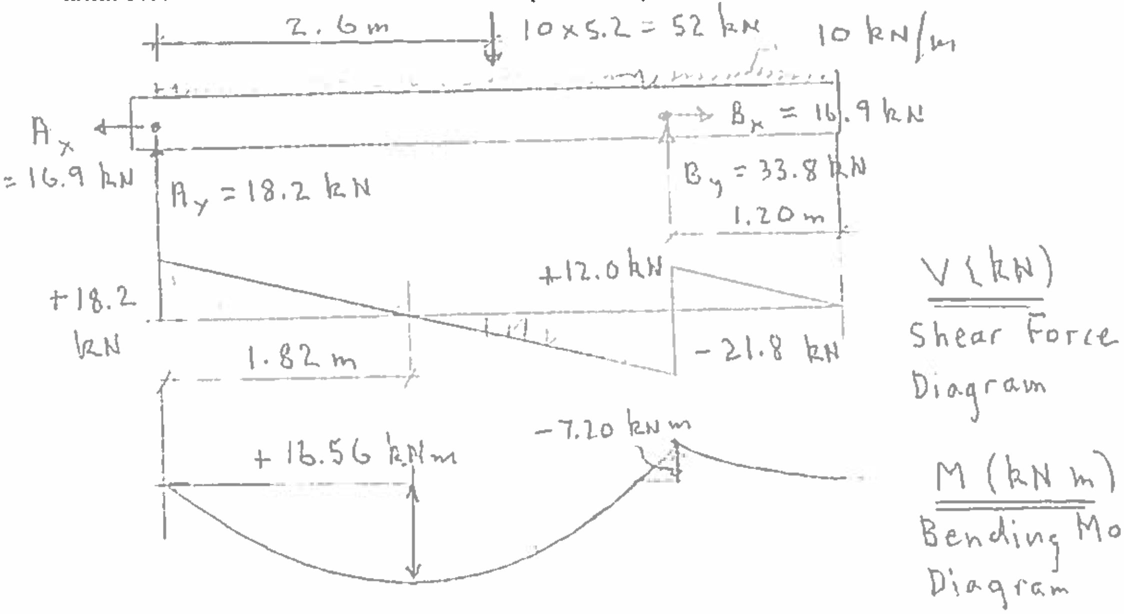
$\Sigma M_A = 0$

$52 \times 2.6 - B_y \times 4.0 = 0$

$B_y = \underline{33.8\ kN}$ $\quad B_x = \underline{16.9\ kN}$

$A_y = 52 - 33.8 = \underline{18.2\ kN}$

3(a). By considering a free body diagram of beam ABC and taking moments about A determine the vertical force component applied to the beam by the strut at B. Then draw the shear force diagram and the bending moment diagram for beam ABC. Also determine the axial forces in the beam and in the strut. (10 marks)



Axial force AB = $\underline{16.9}$ kN tension

Axial force DB $= \sqrt{33.8^2 + 16.9^2} \simeq \underline{37.8}$ kN compression

3(b). The initial choice of members is a 191x343 for beam ABC while for strut DB two 64x235 members have been chosen with one member on each side of the beam. Calculate the maximum longitudinal tensile stress in the beam due to the combined effects of axial load and bending moment. Also calculate the highest shear stress in the beam and the axial compressive stress in the strut. (6 marks)

191 x 343 $\quad A = 65500\ mm^2 \quad S_x = 3750 \times 10^3\ mm^3$

- $\sigma = +\frac{P}{A} + \frac{M}{S} = \frac{16900}{65500} + \frac{16.56 \times 10^6}{3750 \times 10^3} = 0.258 + 4.416 = \underline{4.67}$ MPa

- $\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{21.8 \times 10^3}{65500} = \underline{0.499\ MPa}$

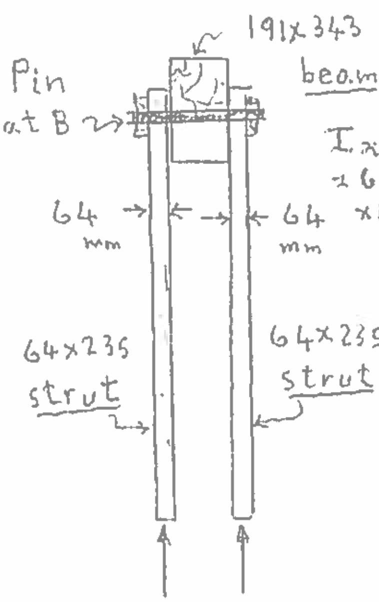
$2 - \underline{64 \times 235} - A = 2 \times 15000 = 30\,000\ mm^2$

$\sigma = -\frac{P}{A} = -\frac{37.8 \times 10^3}{30000} = -\underline{1.260}$ MPa

3(c). For Number 1 or Number 2 grade Hem-Fir it may be assumed that the safe stress for combined bending and axial load is 7.7 MPa, the safe shear stress is 0.53 MPa and the safe compressive stress for short struts is 7.0 MPa. In checking the buckling stress of slender struts an appropriate value of E is 7000 MPa. Does the initial choice of member sizes result in a safe design? If not suggest how the design may be improved. (10 marks)

- $4.67 < 7.7$ MPa $\therefore \sigma$ O.K.
- $0.499 < 0.53$ MPa $\therefore \tau$ O.K.
- $1.260 < 7.0$ MPa $\therefore$ compressive stress O.K. if member does not buckle.

Two 64x235 members with 191x343 beam



$I_x \approx 69.2 \times 10^6\ mm^4$

$L = \sqrt{1.25^2 + 2.5^2} = \underline{2.795}$ m

Struts will buckle by bending about their weak axis

$\approx 64 \times 235 \quad I_y = 5.13 \times 10^6\ mm^4$

$P_{safe} = \frac{\pi^2 EI}{3L^2} = \frac{\pi^2 \times 7000 \times 5.13 \times 10^6}{3 \times 2795^2}$

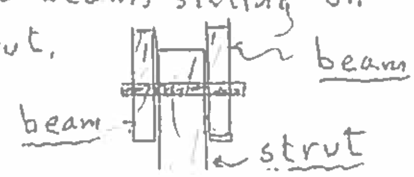
$= \underline{15.12}$ kN $< \underline{18.9}$ kN

Struts not safe!

$\frac{37.8\ kN}{2} \quad \frac{37.8\ kN}{2} = 18.9$ kN

$= 18.9$ kN

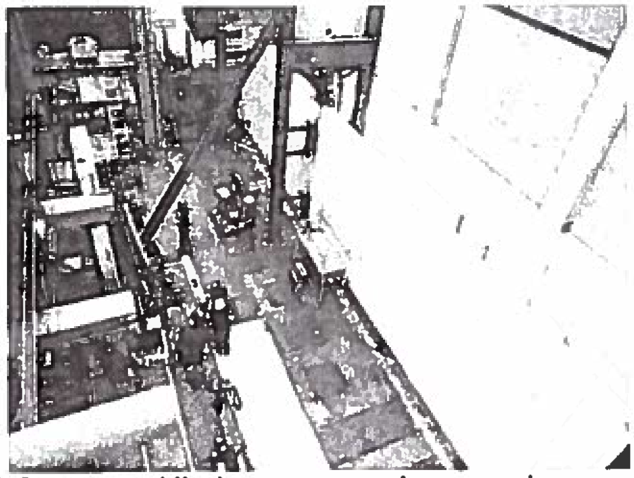
- Change design from one beam sitting on two struts to two beams sitting on one strut.



Watch https://www.youtube.com/watch?v=VQ2_tBF8mBs

Phil Quach - YouTube

4. In June 2015 the deepest reinforced concrete shear specimen ever tested was loaded to failure in the Structures Laboratory of the University of Toronto. The specimen represented a slice, 250 mm wide, cut from a four-metre-thick reinforced concrete slab. The specimen was simply supported over an east-west span of 19 m and was loaded by its own self weight of 24 kN/m and by a point load applied 7 m from the west end support. The longitudinal flexural tension reinforcement consisted of nine 30M bars with a yield strength of 573 MPa placed in three rows of three with their centroid 160 mm above the bottom surface of the concrete. Thus d = 3840 mm. The twelve-metre-long east shear span contained no shear reinforcement while the seven-metre-long west shear span had single 20M vertical bars at 1500 mm centres with a yield strength of 522 MPa giving a shear reinforcement stress index of $A_v f_y/(b_w s) = 300 \times 522/(250 \times 1500) = 0.418$ MPa. The concrete compressive strength, f_c', was 40 MPa while E_c = 29,900 MPa giving a modular ratio n = 200,000/29,900 = 6.69.



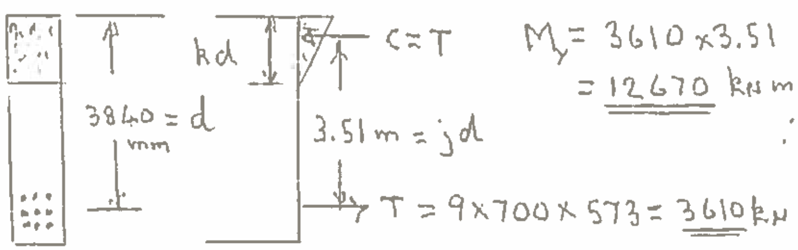
$w_{self\,weight} = 4.0 \times 0.25 \times 24 = \underline{24\ kN/m}$

4 (a). After calculating the flexural lever arm jd determine the moment required to cause the stress in the longitudinal reinforcing bars to reach their yield stress of 573 MPa. Allowing for the moment at seven metres from the west end caused by the self-weight of the large beam, what will be the magnitude of the point load, P, which will cause yielding of the flexural tension reinforcement? (8 marks)

$\rho = A_s/(bd) = 9 \times 700/(250 \times 3840) = 0.00656 \quad n\rho = 0.0439$

$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho = 0.256 \quad j = 1 - 0.256/3 = 0.915$

$jd = \underline{3512\ mm}$



$M_y = 3610 \times 3.51$
$= \underline{12670\ kN\,m}$

$M_{sw} = \underline{1008}\ kN\,m$

$\therefore\ M_p = 11662\ kN\,m$

$P = \underline{2640}\ kN$

4 (b). For sections with no shear reinforcement the Canadian Standards Association (CSA) simplified design method indicates that shear failure will occur when the shear stress in MPa reaches $230\sqrt{f_c'}/(1000 + 0.9d)$ where d is in mm and f_c' is in MPa. Based on this calculate the shear required to cause a shear failure in the east shear span of the tested beam. Allowing for the shear caused by the self-weight of the beam at the section distance d from the east support calculate the magnitude of the point load, P, predicted to cause a shear failure in the east shear span of the beam. (8 marks).

$\tau_{fail} = v_{fail} = 230\sqrt{40}/(1000 + 0.9 \times 3840) = 0.326\ MPa$

$V_{fail} = 0.326 \times 250 \times 3512 = \underline{286.6}\ kN$

Self-weight shear d from east support $= V_{sw}$

$= 24\left(\frac{19}{2} - 3.84\right) = \underline{135.8\ kN}$

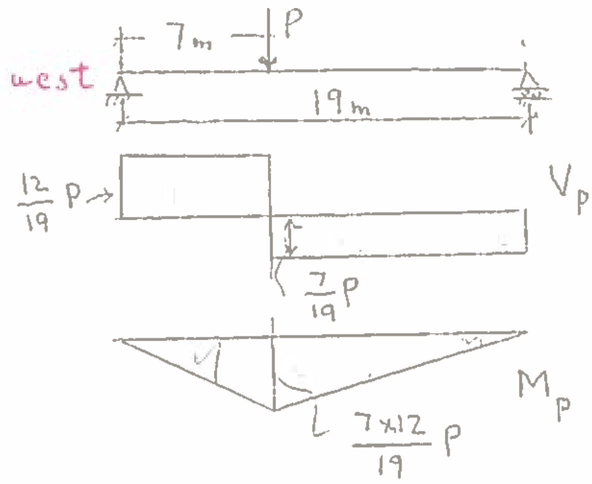
$V_{fail} = V_{sw} + V_P$

$286.7 = 135.8 + V_P$

$\therefore V_P = \underline{150.9 \text{ kN}}$

$= \frac{7}{19} P_{fail}$

$\therefore P_{fail} = \underline{410 \text{ kN}}$



4. (c). When the shear reinforcing stress index exceeds 0.06$\sqrt{f'_c}$ = 0.379 MPa the CSA simplified shear design method predicts that shear failure will occur when the shear stress reaches 0.180$\sqrt{f'_c}$ + cot 35° $A_v f_y/(b_w s)$. What is the CSA predicted shear strength for the west shear span? Allowing for the shear caused by the self-weight of the beam at the section distance d from the west support calculate the magnitude of the load, P, predicted to cause a shear failure in the west shear span of the beam. Which will happen first: failure in flexure under the point load; failure in shear in the east shear span; or failure in shear in the west shear span? (10 marks).

For west shear span $\frac{A_v f_y}{b_w s} = \frac{300 \times 522}{250 \times 1500} = 0.418 \text{ MPa} > 0.379 \text{ MPa}$

So $\tau_{fail} = v_{fail} = 0.180\sqrt{40} + \cot 35° \times 0.418$ MPa

$= 1.138 + 0.597 = 1.735$ MPa

$V_{fail} = 1.735 \times 250 \times 3512 = 1524$ kN

Self-weight shear V_{sw}, at section d from west support $= 135.8$ kN

$\therefore V_P = 1524 - 135.8 = 1388 \text{ kN} \approx \frac{12}{19} P_{fail}$

$\therefore P_{fail} = \underline{2197 \text{ kN}}$

Predicted sequence of failure:

1. Shear failure east shear span P=410 kN
2. Shear failure west shear span P=2197 kN
3. Flexural failure under point load P≈2640 kN

In experiment: East span shear failure when P=685 kN

After east end clamped: CSA detailed method P=620 kN

West end shear failure when P=2162 kN