PHY180 - MECHANICS

Examiner - J. H. Thywissen

18 December 2017

Duration: 2.5 hours

Instructions: No notes or outside material may be used. A scientific calculator is permitted, so long as it has no connectivity, and has no graphing functions.

Show work when appropriate. The exam is out of 100 points, not all questions are weighted equally. **Give numerical answers to 2 significant figures.**

$$g = 9.80 \text{ m s}^{-2}$$

$$x = (-B \pm \sqrt{B^2 - 4AC})/(2A)$$

$$\frac{d}{dt}\sin\omega_0 t = \omega_0 \cos\theta$$

$$\frac{d}{dt}\cos\omega_0 t = -\omega_0 \sin\theta$$

$$v_f^2 = v_i^2 + 2a_x(x_f - x_i)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha_x(\theta_f - \theta_i)$$

$$\vec{a} = r\alpha\hat{t} - r\omega^2\hat{r}$$

$$\vec{a}_C = 2\vec{\omega} \times \vec{v}_{\text{rot}}$$

$$\frac{dv}{dt} + \frac{b}{m}(v - v_T) = 0$$

$$\int_0^b e^{-t/\tau}dt = \tau(1 - e^{-b/\tau})$$

$$v(t) = v_0\, e^{-t/\tau} + v_T(1 - e^{-t/\tau})$$

$$\tau = m/b$$

$$R = -0.5D\rho A v^2$$

$$K = \frac{1}{2}mv^2 \qquad K_R = \frac{1}{2}I\omega^2$$

$$p_x = mv_x \qquad L_z = I\omega_z$$

$$M_{\text{tot}}\vec{r}_{\text{CM}} = \sum m_i \vec{r}_i$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} yc - zb \\ -xc + za \\ xb - ya \end{bmatrix}$$

$$\ddot{x} + \omega_0^2 x = 0$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\omega_0 = \sqrt{k/m} \quad \text{or} \quad T_0 = 2\pi/\omega_0$$

$$x = A\cos[\omega_0 t + \phi]$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

$$x = Ae^{-\gamma t/2}\cos[\omega_d t + \phi]$$

$$\omega_d^2 = \omega_0^2 - \gamma^2/4 \qquad \gamma = b/m$$

$$I = I_{\text{CM}} + MD^2$$

$$I_{\text{CM}} = \frac{2}{5}MR^2 \quad \text{sphere}$$

$$I_{\text{CM}} = \frac{1}{2}MR^2 \quad \text{solid cylinder or disk}$$

$$I_{\text{CM}} = \frac{1}{12}ML^2 \quad \text{thin rod}$$

$$I_{\text{CM}} = \frac{2}{3}MR^2 \quad \text{thin spherical shell}$$

$$I_{\text{CM}} = \frac{1}{12}M(a^2 + b^2) \quad \text{rectangular plate}$$

$$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$$

$$R_E = 6.37 \times 10^6\,\text{m} \qquad M_E = 5.97 \times 10^{24}\,\text{kg}$$

$$M_{\text{Sun}} = 1.99 \times 10^{30}\,\text{kg}$$

$$R_{\text{E,Orb}} = 1.50 \times 10^{11}\,\text{m}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3 \quad \text{or} \quad \omega^2 = GM/a^3$$

$$L^2 = GMm^2b^2/a$$

$$E = -GMm/2a$$

$$v_{\text{esc}}^2 = 2GM/R$$

1. [15 pts.] A particle is moving in an one-dimensional line with an acceleration that changes as a function of time t

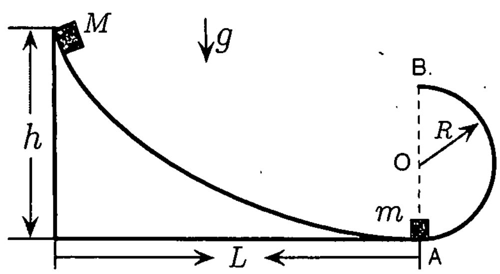
$$a_x(t) = 5.0 - t \quad (\text{m/s}^2)$$

If the particle starts ($t = 0$) at position $x = 10.0$ m and with an initial velocity $v_x = -8.0\,\text{m/s}$. Then

(a) [5 pts.] At what instants in time t will the particle be stopped?

(b) [5 pts.] How far has the particle moved by $t = 3.0\,\text{s}$?

(c) [5 pts.] At what time is the particle moving fastest in the $+\hat{\imath}$ direction?



2. [16 pts.] A frictionless track is connected smoothly to a circular, frictionless track of radius R at point A. The highest point of the circular track is B. Initially, an object m is at rest at point A and another object of mass M is at hold still at a height h at the track. The direction of the gravity points downwards. At $t = 0$, the object M is released.

(a) [3 pts.] What is the velocity of M just before it hits the object m?

(b) [5 pts.] Suppose that when object M hits object m, they immediately stick together. What is the final speed of the combined object, immediately after the collision? (We'll call this v_A for the next part.)

(c) [8 pts.] After the collision, the combined object with mass $M + m$ starts to move along the circular track. What is the minimal speed v_A at which the combined object must start the loop, to arrive at B without detaching from the track?

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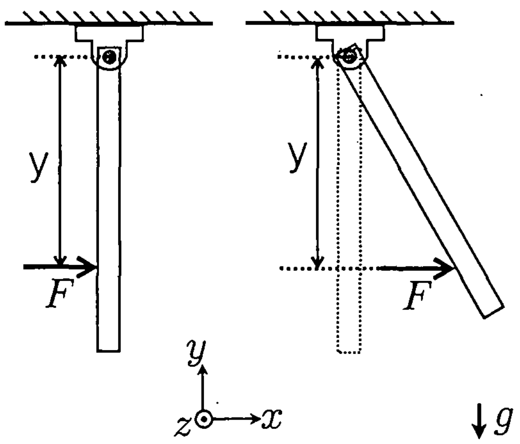
3. [15 pts.] A long, narrow rod of mass $M = 0.80\,\mathrm{kg}$ and length $L = 4.0\,\mathrm{m}$ hangs from a fixed pivot, and is initially vertical and at rest. An external force $F = 2.0\,\mathrm{N}$ pushes along $\hat{\jmath}$ at constant height, as shown in the FIGURE BELOW, with $y = 3.0\,\mathrm{m}$. After the rod has moved through $\Delta\theta = \pi/6$ (or 30° degrees),

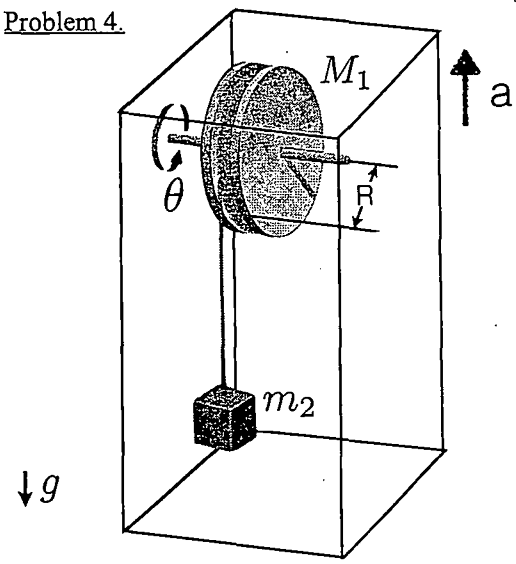
   (a) [6 pts.] How much work is done by F? (Give a numerical answer.)

   (b) [9 pts.] What is the final kinetic energy of the rod? (Give a numerical answer.)

Problem 3.



Problem 4.



4. [16 pts.] An elevator accelerates upwards at a. Inside, a cube hangs from a cylindrical spool of radius R, which is mounted to the sides of the elevator (SEE FIGURE ABOVE). The spool has moment of inertia I_{CM} (about its axis) and mass M_1; the cube has mass m_2 and side-length s. What is the **angular acceleration** of the spool about its axis? Give a simplified answer in terms of a, g, s, R, M_1, m_2, and I_{CM}. You can assume the rope has no mass.

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Total Pages = 4.

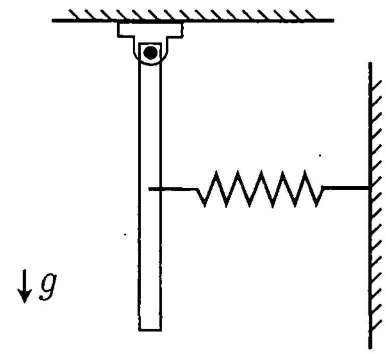
5. [21 pts.] Consider a binary system of two stars, with mass M and $2M$. Assume that each star has a circular orbit with the same period, and that the distance between the stars is D. Give an expression (in terms of M, D, and G) for

   (a) [8 pts.] The angular frequency ω of orbit.

   (b) [7 pts.] The kinetic energy

   (c) [6 pts.] The potential energy

Problem 6



6. [17 pts.] A long, narrow rod of length L and mass M hangs vertically from a frictionless pivot. It is connected, half-way up its length, to a side wall by a spring of strength k. If pushed slightly to one side, what is its **period of oscillation**? *Hint:* Use a small-angle approximation.

End of exam. Enjoy the holidays!

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