UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAN – DECEMBER 2022

First Year Engineering Science

CIV102F –Structures & Materials – An Introduction to Engineering Design

Permissible Aids: Non programmable calculator, printed or handwritten notes and marked quiz/assigns

Examiner – E.C. Bentz

NAME: Solu[redacted]on

ID Number: πei[redacted]eiπei 01

Important Instructions

1. You have 2.5 hours to complete this exam. Use your time wisely.
2. There are four questions on the exam. Attempt all questions. Any questions left blank will receive a mark of zero. Part marks will be awarded for incomplete answers.
3. Report all final answers to slide-rule accuracy (four digits if first digit is 1, three otherwise)
4. If you need more space, use the back of the pages or extra pages at the end but indicate which page leads to which and which page each one came from.
5. Write neatly and draw a box around your final answer.

Question Number	Earned Grade	Maximum Grade
1		25
2		32
3		31
4		21
Total		109

1. The following crane structure is made of steel HSS members with a yield stress of 350 MPa. Answer the following questions about it. (25 marks total)

1(a) Solve for the reactions (3 marks)

$\Sigma M_A = 0 = -20 \cdot 4 - 100 \cdot 6 - 20 \cdot 2 + C_y \cdot 2$

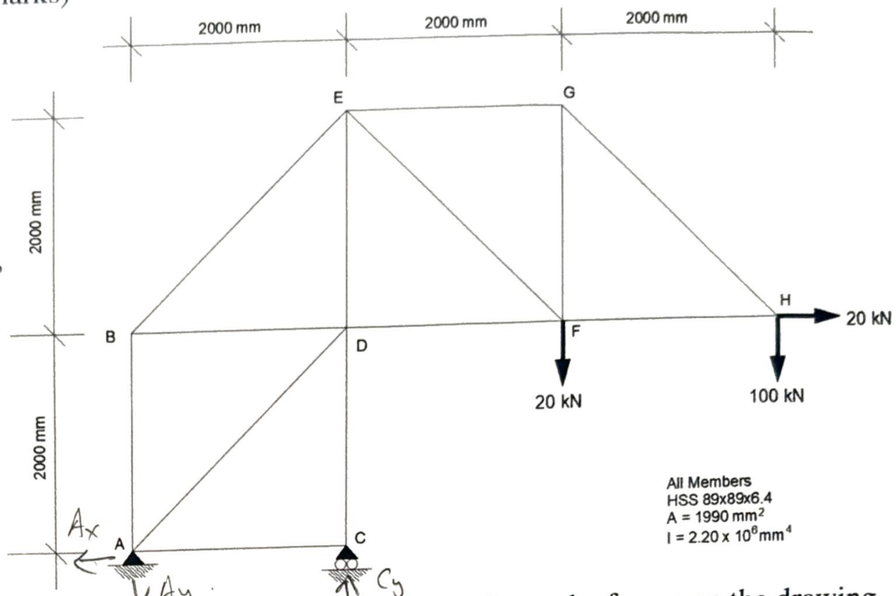
$\boxed{C_y = 360 \text{ kN}} \uparrow$

$\Sigma F_y = 0 = -A_y + C_y - 20 - 100$

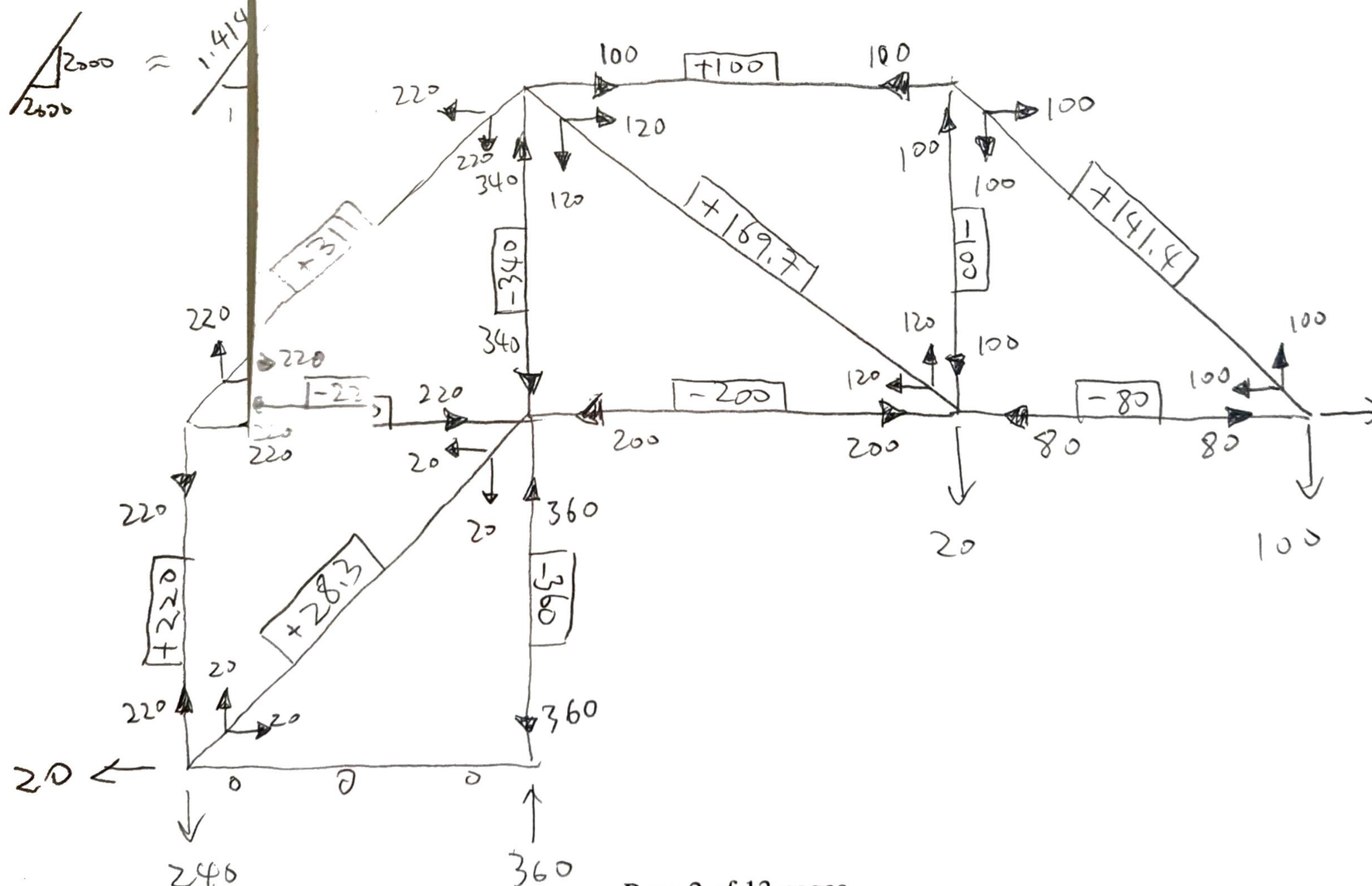
$\boxed{A_y = 240 \text{ kN}} \downarrow$

$\Sigma F_x = 0 = -A_x + 20$

$\boxed{A_x = 20 \text{ kN}} \leftarrow$



1(b) Solve for the internal axial forces using any method you wish. Indicate the forces on the drawing by writing numbers along the members: positive for tension, negative for compression, all in kN. (12 marks)



1(c) Given that all members in the truss are HSS 89x89x6.4 which have the geometric properties listed on the sketch, is the truss safe against the shown loading using the safety factors we use in this course? Determine a factor by which all the shown loads could be increased or decreased to make the structure right on the boundary of being safe and not being safe. (6 marks)

Tension

Max force = 311 kN

Safe capacity $= 1990\,mm^2 \times \frac{350\,MPa}{2} = 348\,kN$ ∴ safe

Compression

Max force = 360 kN

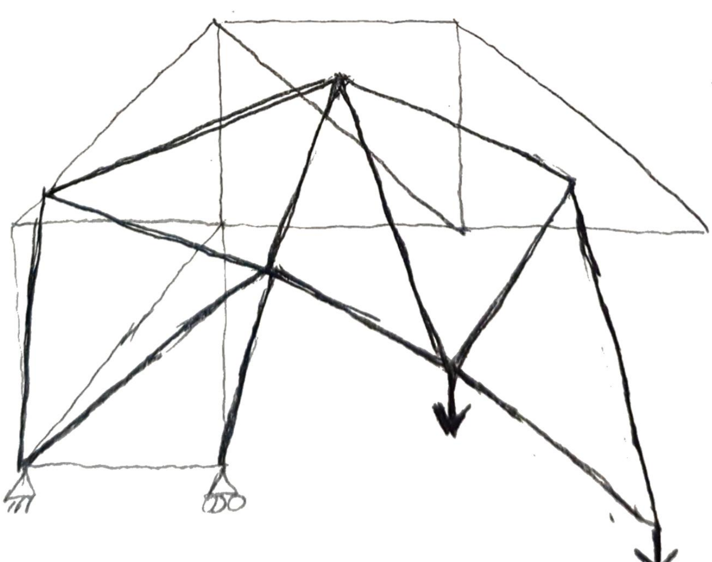
Safe capacity (yield) = 348 kN

Safe capacity (buckling) $= \frac{\pi^2 \cdot 200{,}000 \cdot 2.2\times10^6}{2000^2 \cdot 3} = 362\,kN$

$\Big\}$ min = 348 ∴ Not Safe

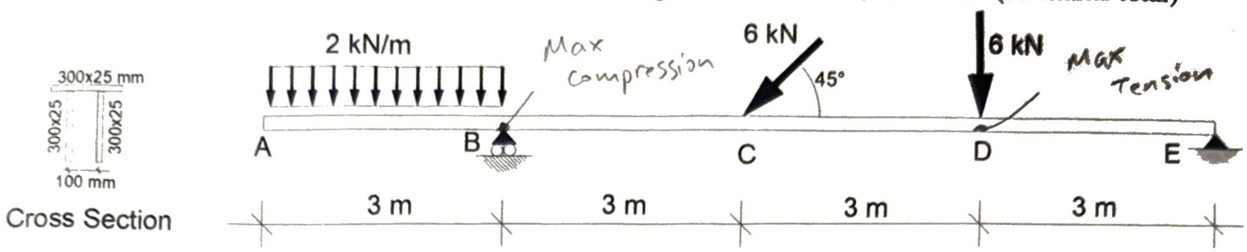
Scale Factor $\frac{348}{360} = \boxed{0.967}$

1(d) Rather than determining the displacement by virtual work, draw a sketch (not to scale and without doing any calculations) of how you think the truss would deform from the listed loads. Remember that truss members remain straight when loaded and that tension members get longer while compression members get shorter. (4 marks)

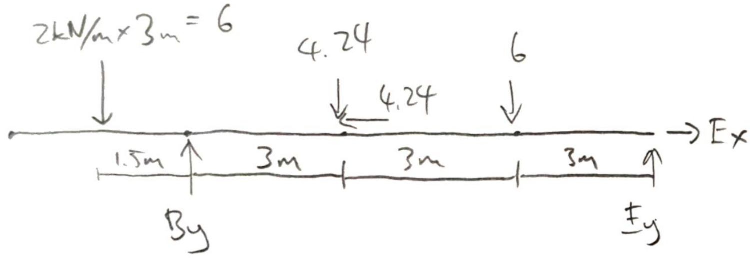


N.T.S.

2: The following 12 metre long timber beam was constructed by gluing three pieces of timber together and is subjected to the combination of UDL and two point loads shown. The three pieces glued together are all of rectangular cross section 300 x 25 mm as shown. The compressive capacity of the timber is 50 MPa while the tensile strength is 60 MPa and the Young's modulus is 12,000 MPa. (32 marks total)



2(a) Determine the reactions for this beam (3 marks)



$\sum F_x = 0 = E_x - 4.24$

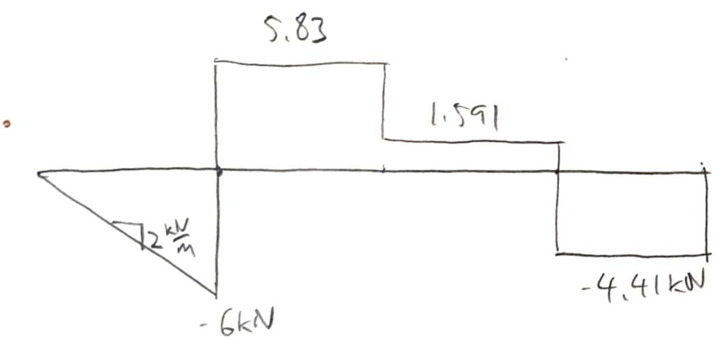
$\therefore \boxed{E_x = 4.24 kN \rightarrow}$

$\sum M_B = 0 = 6 \cdot 1.5 - 4.24 \cdot 3 - 6 \cdot 6 + E_y \cdot 9$

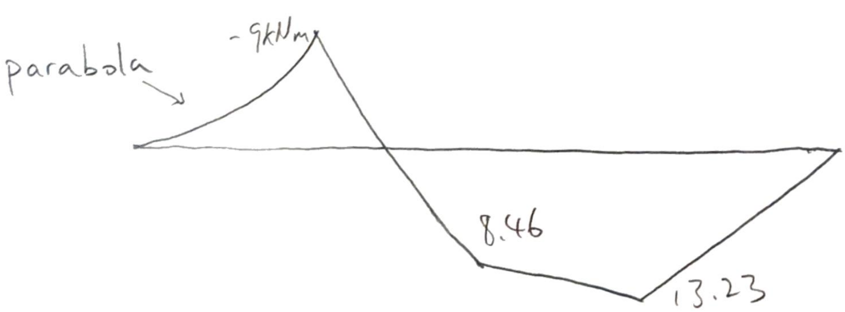
$\therefore \boxed{E_y = 4.41 kN \uparrow}$

$\sum F_y = 0 = B_y + E_y - 4.24 - 6$ ↑ $\boxed{B_y = 11.83}$

2(b) Determine the SFD, BMD, and axial force diagram for this beam. Show your sign convention and all important values on the drawings. (7 marks)



SFD ↑ ⊞ ↓

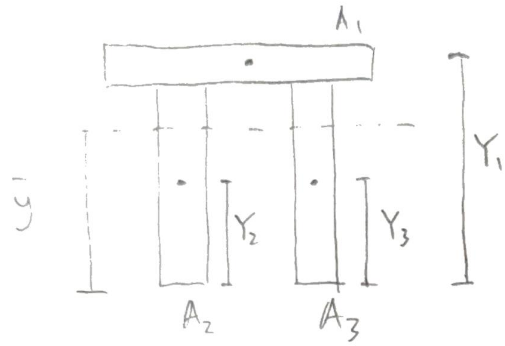


BMD

Axial Diagram

A B C D E

2(c) Determine the depth of the centroidal axis and the second moment of area of the cross section. (5 marks)



$$\bar{y} = \frac{\sum A_i Y_i}{\sum A_i}$$

$$= \frac{(300 \times 25) \cdot 312.5 + (300 \times 25 \times 2) \cdot 150}{300 \times 25 \times 3}$$

$$= \boxed{204\text{mm from bottom}}$$

$$I = I_1 + I_2 + I_3 + A_1 d_1^2 + A_2 d_2^2 + A_3 d_3^2$$

$d_1 = Y_1 - \bar{y} = 108.3$

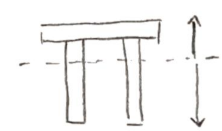
$d_2 = Y_2 - \bar{y} = 54.2$

$$= \frac{300 \cdot 25^3}{12} + 2 \cdot \frac{25 \times 300^3}{12} + 300 \times 25 (108.3^2)$$

$$+ 300 \times 25 \times 2 (54.2^2) = \boxed{245 \times 10^6 \text{mm}^4}$$

2(d) Determine the maximum values of tensile stress and compressive stress in the beam as well as the maximum shear stress. Indicate on the drawing at the start of this question where these maxima and minima are. (10 marks)

Check at max moment locations



$y_{top} = 120.8\text{mm}$

$y_{bot} = 204.2\text{mm}$

at B $M = -9\text{kNm}$

$$\sigma_{top} = \frac{9 \times 10^6 \times 120.8}{245 \times 10^6} = 4.43 \text{ MPa tens}$$

$$\sigma_{bot} = \frac{9 \times 10^6 \times 204}{245 \times 10^6} = \boxed{7.50 \text{ MPa comp}}$$

at C $M = 13.23$

$$\sigma_{top} = \frac{13.23 \times 10^6 \times 120.8}{245 \times 10^6} = 6.54 \text{ MPa comp}$$

$$\sigma_{bot} = \frac{13.23 \times 10^6 \times 204}{245 \times 10^6} = \boxed{11.05 \text{ MPa tens}}$$

Considering combined flex + axial $\left(\frac{My}{I} + \frac{P}{A}\right)$

$$\sigma_{tot} = -7.50 + \frac{4.24 \times 10^3}{22500} = \boxed{-7.31 \text{ MPa Comp.}}$$

$$\sigma_{tot} = 11.05 + \frac{4.24 \times 10^3}{22500} = \boxed{11.24 \text{ MPa tens}}$$

Check at max shear

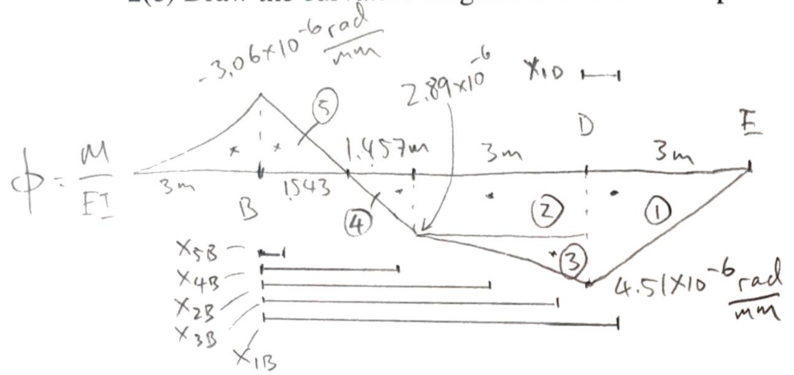
$V = 6\text{kN}$

$Q = 50 \times 204.2 \times \frac{204}{2}$

$= 1042 \times 10^3 \text{mm}^3$

$$\tau_{max} = \frac{6000 (1042)}{245 (50)} = \boxed{0.510 \text{ MPa}}$$

2(e) Draw the curvature diagram and find the displacement at point D in the beam. (5 marks)



$A_1 = \frac{1}{2}(3000)\,4.51\times10^{-6} = 6.765\times10^{-3}$

$A_2 = 8.67\times10^{-3}$ $\quad X_{1B} = 7100\,mm$

$A_3 = 2.43\times10^{-3}$ $\quad X_{2B} = 4500\,mm$

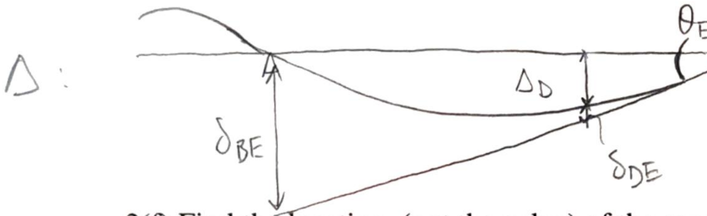
$A_4 = 2.11\times10^{-3}$ $\quad X_{3B} = 5000\,mm$

$A_5 = -2.36\times10^{-3}$ $\quad X_{4B} = 2514\,mm$

$X_{1D} = 1000\,mm$ $\quad X_{5B} = 514\,mm$

$\delta_{BE} = A_1 X_{1B} + A_2 X_{2B} + \dots A_5 X_{5B} = 102.6\,mm$

$\delta_{DE} = A_1 X_{1D} = 6.77\,mm$

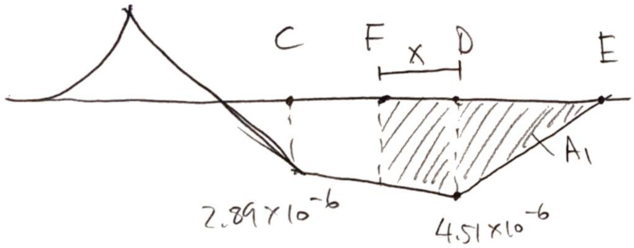


$\frac{\Delta_D + \delta_{DE}}{3m} = \frac{\delta_{BE}}{9m} \quad \therefore \Delta_D = \frac{\delta_{BE}}{3} - \delta_{DE} \; \boxed{= 27.4\,mm} \downarrow$

2(f) Find the location, (not the value) of the maximum displacement in this beam. Remember from calculus that the maximum displacement occurs where the slope is equal to zero. (2 marks)

$\theta_E = \frac{\delta_{BE}}{9000\,mm} = 0.01137\ rad$

Let the location of max disp. be F.



At F, $\theta_F = 0$

$\therefore \theta_E - \theta_F = (\text{Area btwn } F, E)$

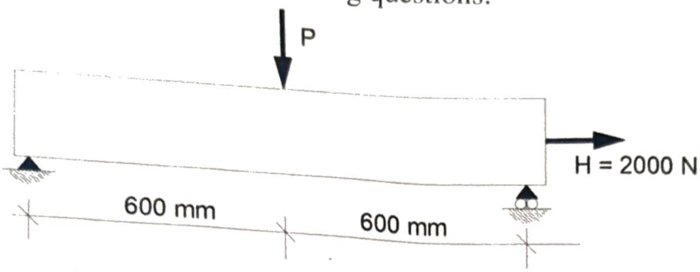
$\therefore 0.01137 - 0 = A_1 + A_{F-D}$

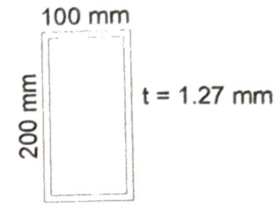
$0.004605 = (x)\cdot\left[4.51\times10^{-6} + \left(4.51\times10^{-6} - x\left(\frac{\overbrace{1.62\times10^{-6}}^{4.51-2.89}}{3000}\right)\right)\right]/2$

$x = 1.1m$

$\therefore$ The location of max disp is 1.1m left of D.

3) For next year's design-build project we are considering a change from what we did this year. The beam shown is simply supported with a vertical point load at midspan of a 1200 mm long span. We are thinking of allowing the designer to select how much constant axial load that they want applied to their bridge before we apply the vertical load. In this question there is a 2000 N force applied at the right side applying a constant axial load to the beam before the P load is applied and increased to failure. The cross section is a single-walled box with diaphragms every 100 mm along the length. The material properties are shown below. Answer the following questions:





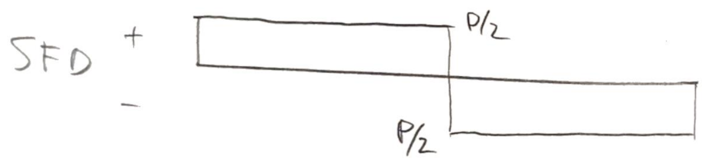
Cross Section

Axial compressive strength = 6 MPa
Axial tensile strength = 30 MPa
Young's Modulus = 4000 MPa

Thickness = 1.27 mm
Poisson's ratio = 0.2
Shear strength = 4 MPa

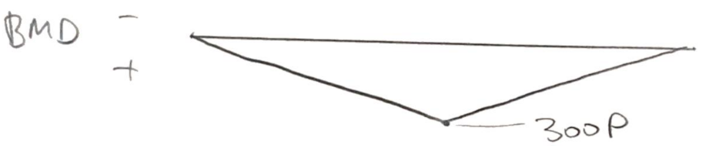
Area = 756 mm^2
I = 4.14x10^6 mm^4

3(a) Determine the SFD and BMD for this member as a function of P. What is the axial stress and is it in tension or compression? (3 marks)



$$\sigma_{axial} = \frac{P}{A} = \frac{2000\,N}{756\,mm^2}$$

$$= \boxed{2.64\ MPa\ (\text{tens})}$$



3(b) Find the stresses due to combined bending and axial load at the top and bottom of the cross section at the maximum moment location if P= 100 N. (5 marks)

$$M_{max} = 300P = 30\,000\ Nmm$$

$$\sigma_{top} = -\left(\frac{30000 \times 100}{4.14\times10^6}\right) + 2.64 = \boxed{1.915\ MPa\ \text{tens}}$$

Comp.

$$\sigma_{bot} = +\left(\frac{30000 \times 100}{4.14\times10^6}\right) + 2.64 = \boxed{3.36\ MPa\ \text{tens}}$$

tens

3(c) Determine the values of P that would cause the member to fail under combined loading so that the tensile stress limit is reached. Repeat this, if appropriate, for the maximum compressive strength limit as well. Recall that the horizontal force, H, is constant at 2000 N. (5 marks)

Tension Failure

$\sigma_{bot} = \sigma_{ult,\,tens}$

$$+\left(\frac{300P(100)}{4.14\times10^{6}}\right) + 2.64 = 30\,MPa$$

$$\boxed{P = 3776N}$$

Compression Failure

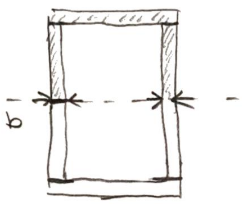
$\sigma_{top} = \sigma_{ult,\,comp}$

$$-\left(\frac{300P(100)}{4.14\times10^{6}}\right) + 2.64 = -6\,MPa$$

$$\boxed{P = 1192N}$$

3(d) Determine value of P to cause the member to fail by the shear stress reaching the maximum shear strength (5 marks)

Shear failure



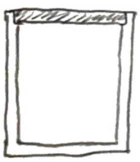
$$Q = 100\times1.27\times\left(100-\frac{1.27}{2}\right) + 2\left[1.27\times(100-1.27)\times\frac{(100-1.27)}{2}\right] = 25000\,mm^3$$

$\tau_{xy} = \tau_{ult}$

$$\frac{0.5P(25000)}{4.14\times10^{6}(2.54)} = 4$$

$$\boxed{P = 3365N}$$

3(e) Determine the applied load P to cause the member to fail by plate buckling in all potential modes of buckling, including shear (6 marks)



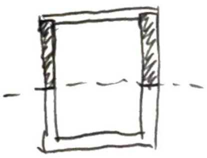
Case 1 buckling

$$\sigma_{crit} = \frac{4\pi^2E}{12(1-\mu^2)}\left(\frac{1.27}{100-1.27}\right)^2 = 2.09\,MPa$$

$\sigma_{bot} = \sigma_{crit}$

$$-\left(\frac{300P(100)}{4.14\times10^{6}}\right) + 2.64 = -2.09$$

$$\boxed{P = 652N}$$

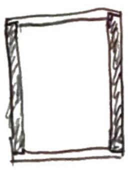


Case 3 buckling

$$\sigma_{crit} = \frac{6\pi^2E}{12(1-\mu^2)}\left(\frac{1.27}{100-1.27}\right)^2 = 3.14\,MPa$$

$$-\left(\frac{300P(100)}{4.14\times10^{6}}\right) + 2.64 = -3.14$$

$$\boxed{P = 798N}$$



Shear buckling

$$\tau_{crit} = \frac{5\pi^2E}{12(1-\mu^2)}\left[\left(\frac{1.27}{200-1.27}\right)^2 + \left(\frac{1.27}{100}\right)^2\right] = 3.50\,MPa$$

$\tau_{xy} = \tau_{crit}$

$$\frac{0.5P(25000)}{4.14\times10^{6}(2.54)} = 3.5$$

$$\boxed{P = 2940N}$$

3(f) What is the critical value of P that represents your estimate of the theoretical strength and what is the predicted failure mode. (2 marks)

min P = 652N, by flexural buckling of top plate.

3(g) Do you think the added axial load made it stronger or weaker? Make an estimate of how much of a difference it made? (5 marks)

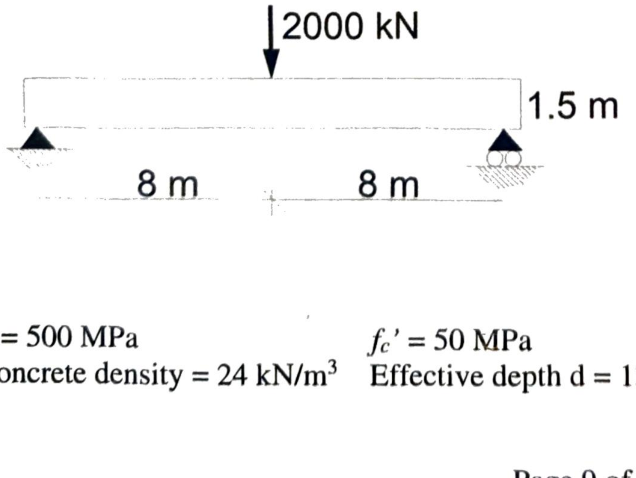
For critical equation, if H removed, 2.64 MPa → 0

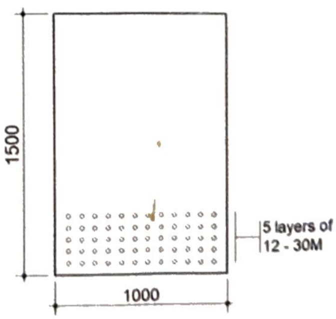
$$-\left(\frac{300P(100)}{4.14\times10^{6}}\right) + \cancel{2.64}^{\,0} = -2.09$$

$P = 288N$

$$\text{Factor } \frac{\text{with } H}{\text{w/o } H} = \frac{652N}{288N} = 2.26 \text{ times the difference}$$

4: Yesterday an announcement was made about the first controlled nuclear fusion experiment that produced more energy than was needed to make it occur. This supports the idea that humanity can indeed solve the global climate crisis in the coming years. To help make this occur more quickly, you have agreed on your summer job to help design a 16 metre long beam that is needed to resist 2000 kN at midspan as part of the proposed reactor core. You have recommended reinforced concrete and have concluded a 1000 mm wide and 1500 mm tall beam should work. Your initial calculations (which might be wrong) suggested 60-30M bars should be reasonable and this is your starting point Answer the following questions. Note the material properties below the sketch.





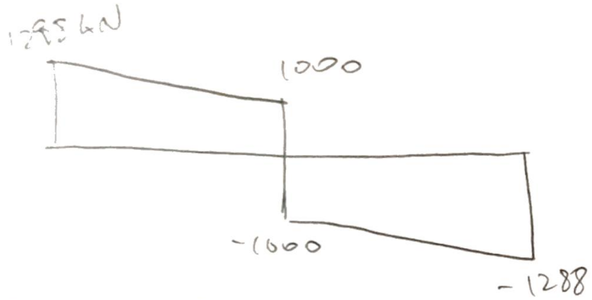
$f_y = 500$ MPa
$f_c' = 50$ MPa
Area of a 30M bar = 700 mm^2
Concrete density = 24 kN/m^3
Effective depth d = 1300 mm

4(a) Draw the SFD and BMD diagrams for this problem and identify the highest moment and shear force. Note that you cannot ignore self-weight for a beam as large as this. (5 marks)

$w = 24\ kN/m^3 \times 1m \times 1.5m = 36\ kN/m$

Reaction $A_y = 1000 + (36\ kN/m \times 16m)/2 = 1288\ kN$

SFD



BMD

4(b) Determine the applied stress in the steel reinforcement under the applied loads as well as the maximum stress in the concrete. Are these values safe using a safety factor of 2.0 for concrete and steel? (5 marks)

$d = 1300\ mm$, $A_s = 60 \times 700\ mm^2 = 42\ 000\ mm^2$

$b = 1000\ mm$, $E_c = 4500\sqrt{50} = 31800\ MPa$

$n = \frac{E_s}{E_c} = 6.29$, $\rho = \frac{A_s}{bd} = 0.0323$, $n\rho = 0.203$

$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho = 0.466$

$j = 1 - k/3 = 0.845$, $jd = 1099\ mm$

Steel $\quad f_s = \frac{M}{A_s jd} = \frac{9150 \times 10^6}{42000 \times 1099} = 198\ MPa < \frac{500}{2}$ ✓ ok

Concrete $\quad f_c = \frac{k}{1-k} \frac{M}{nA_s jd} = 27.5\ MPa \not< \frac{50}{2}$ ✗ not ok.

4(c) Perform the shear design for these beams as was described in class. Try to make two legs of 15M stirrups work with your design if you need stirrups. (a 15M bar has area of 200 mm^2) (10 marks)

1) Check $V_{max} = 0.25 f'_c b_w d_v$, $d_v = 0.9d = 1170$ mm

$= 7313$ kN $> V_{SFD} = 1288$ kN ✓ ok

2) No stirrups

$$V_c = \frac{230\sqrt{f'_c}\, b_w d_v}{(1000 + d_v)\, 2} = 438 \text{ kN} < 1288 \text{ kN}$$ ✗ Not ok

Need stirrups.

3) Add minimum stirrups ($A_v = 2 \times 200 = 400$ mm^2)

$\left(\frac{A_v f_y}{b_w S} = 0.06\sqrt{f'_c}\right)$ $\therefore S = 471$ mm

$V_c = \frac{1}{2} 0.18\sqrt{f'_c}\, b_w d_v = 745$ kN

$V_s = \frac{1}{2}\frac{A_v f_y d_v}{S} \cot 35° = 355$ kN

$V_{safe} = V_c + V_s = 1100$ kN < 1288 ✗ Not ok. Need more stir.

4) Select spacing $S = \frac{1}{2}\frac{A_v f_y d_v}{(V - V_c)} \cot 35° = 307$ mm $\boxed{\text{select } 300 \text{ mm}}$

4(d) Summarize your design with a sketch of the side of the beam showing the stirrups (1 mark)

stirrups spaced at 300 mm