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Family Name, Given Name (please print)   Student Number   Tutorial Group and Leader's Name

PHY293 –Vibrations and Waves

FALL 2015 Midterm Test

Duration: 1 hour.

Aids allowed: Only calculators from a list of approved calculators as issued by the Faculty Registrar are allowed.

Before starting, please print your name, tutorial group, and student number at the top of this page and at the top of the exam booklet. Please also put down relevant information (including your name and student number) in the multiple-choice answer sheet.

There are four questions in this mid-term, with two being standard question-and-answer (Q&A) questions and two being multiple-choice (MC) questions.

For MC questions, only the final answers will be marked by a machine.

For Q&A questions, partial credit will be given for partially correct answers. So, please show any intermediate calculations that you do and write down, in a clear fashion, any relevant assumptions you are making along the way.

Some possibly useful equations:

	Amplitude	Velocity	Power
Peak Frequency	$\omega = \omega_0\sqrt{1-1/2Q^2}$	$\omega = \omega_0$	$\omega = \omega_0$
Peak Value	$A_m = \frac{a_0 Q}{\sqrt{1-\frac{1}{4Q^2}}}$	$V_m = a_0\omega_0 Q$	$P_m = \frac{1}{2}ma_0^2\omega_0^3 Q$
General	$A(\omega) = \frac{a_0\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+(\gamma\omega)^2}}$	$V(\omega) = \frac{a_0\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2/\omega^2+\gamma^2}}$	$< P(\omega) >= P_{\mathrm{m}}\frac{\gamma^2}{(\omega_0^2-\omega^2)^2/\omega^2+\gamma^2}$
	$\tan\delta = \frac{\omega\gamma}{(\omega_0^2-\omega^2)}$		$< P >= P_{\mathrm{m}}\frac{\gamma^2/4}{(\omega_0-\omega)^2+\gamma^2/4} \quad Q \gg 1$

Do not separate the stapled sheets of the question paper. Hand in the question sheets together with your exam booklet at the end of the test.

Good luck!

1. (10 marks) Consider, as in lectures, two simple pendula, each of mass m and of length l, coupled by a spring with spring constant k. Which of the following statements is/are true?
   I) The first normal mode has a frequency that is independent of k.
   II) The second normal mode has a frequency that is independent of k.
   III) With the first normal mode, the two masses move together.
   IV) With fixed values of m, k and g, the larger the value of l, the longer the period of the oscillation for the first normal mode.

   A. Only statements I) and II).
   B. Only statements I), II) and III).
   C. Only statements I) , III) and IV).
   D. Only statements II), III) and IV).
   E. All statements I), II), III) and IV).

Please write down your answer to Question 1 in the **multiple-choice answer sheet** provided.

2. (10 Marks) Consider a simple harmonic oscillator with both forcing and damping. Which of the following statements is/are true?
   I) The power dissipation is maximal when the driving frequency ω is the same as the natural frequency, $\omega_{0.}$
   II) The full width half heights (FWHH) of the power resonance curve is γ.
   III) In light damping, the oscillator oscillates at the natural frequency, $\omega_{0..}$

   A. Only statement I) is always true.
   B. Only statement II) is always true.
   C. Only statement III) is always true.
   D. Only statements I) and II) are always true.
   E. Statements I), II) and III) are always true.

Please write down your answer to Question 2 in the **multiple-choice answer sheet** provided.

3. (40 marks)

   A mass of 5kg is suspended on a spring and set oscillating. Owing to damping, the equation of motion is given by $m\ddot{x} + b\dot{x} + k\,x = 0$ where m=5 kg, b is a damping coefficient, and k is the spring constant. It is observed that the amplitude reduces to 80% of its initial value after 2 oscillations. It takes 0.5 seconds to do them. Calculate the following.

   (a) (8 marks) The angular frequency of the damped oscillation.
   (b) (8 marks) The natural frequency.
   (c) (8 marks) The value of b.
   (d) (8 marks) The value of the spring constant, k.
   (e) (8 marks) The value of the Q-factor.

4. (40 marks)

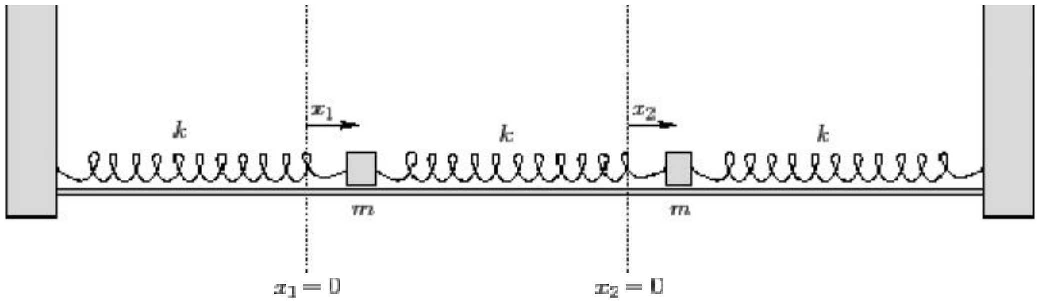

Figure 1: Two degree of freedom mass-spring system.

Consider a mechanical system consisting of two identical masses $m$that are free to slide over a frictionless horizontal surface. Suppose that the masses are attached to one another, and to two immovable walls, by means of three identical light horizontal springs of spring constant k, as shown above in Figure 1. The instantaneous state of the system is conveniently specified by the displacements of the left and right masses, $x_1(t)$ and $x_2(t)$, respectively.

(a) (8 marks) Find the normal frequencies of oscillations.
(b) (8 marks) Find the normal modes of oscillations.
(c) (8 marks) Write down the general solutions for all time of $x_1(t)$ and $x_2(t)$.
(d) (16 marks) Suppose at an initial time, $t = 0$, we have $x_1(0) = a$, $\dot{x}_1(0) = 0$, $x_2(0) = 0$ and , $\dot{x}_2(0) = 0$. Solve for $x_1(t)$ and $x_2(t)$.

The End