25 October 2021

CHE 260F – Thermodynamics and Heat Transfer

Mid-Term Exam – 2021

You have 110 minutes to do the following five problems. You may use any type of non-communicating calculator. All questions are worth equal marks.

1) Air enters an adiabatic compressor at 100 kPa and 17°C at a rate of 2.4 m^3/s and it exits at 257°C. The compressor has an isentropic efficiency of 84%. Neglecting changes in kinetic and potential energies, determine (a) the exit pressure of air and (b) the power required to drive the compressor. Assume that for air R=0.287 kJ/kgK and c_p=1.004 kJ/kgK.

2) An isolated system contains a mass m of water. The water is initially divided into two equal parts, one at temperature T_1 and the other at temperature T_2. The two masses are mixed and allowed to come to equilibrium. If the specific heat of water is c, show that the entropy generated by mixing is:

$$S_{gen} = mc\ ln\left[\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right]$$

3) A piston-cylinder device with 2 kg of water at 1 MPa and 250°C is cooled with a constant load on the piston. This isobaric process ends when the water has reached a state of saturated liquid. Sketch the process on both *P-v* and *T-s* diagrams on which the vapour dome is marked. Find the work and heat transfer during this process.

4) Air enters a nozzle steadily at 280 kPa and 77°C with a velocity of 50 m/s and exits at 85 kPa and 320 m/s. The heat losses from the nozzle to the surrounding medium at 20°C are estimated to be 3.2 kJ/kg. Determine (*a*) the exit temperature and (*b*) the total entropy change for this process. Assume that for air R=0.287 kJ/kgK and c_p=1.004 kJ/kgK.

5) A steam turbine receives steam at a pressure of 1 MPa and a temperature of 300°C. The steam leaves the turbine at a pressure of 15 kPa. The work output of the turbine is measured to be 600 kJ/kg of steam flowing through the turbine. Determine (a) the isentropic efficiency of the turbine and (b) the quality of steam at the exit of the turbine.

Ideal gas equation

$$PV = NR_uT \qquad R_u = 8.314 \text{ kJ/kmol K}$$
$$PV = mRT \qquad R = R_u/M$$

Boundary Work

$$W_{12} = -\int_{V_1}^{V_2} P\,dV$$

For a constant pressure process

$$W_{12} = P_1(V_1 - V_2) = P_1V_1 - P_2V_2$$

For a polytropic process $PV^n = C$

$$W_{12} = P_1V_1 \ln\frac{V_1}{V_2} = P_2V_2 \ln\frac{V_1}{V_2} \quad \text{for } n=1$$

$$W_{12} = \frac{P_2V_2 - P_1V_1}{n-1} \qquad \text{for } n \neq 1$$

Flow work per unit mass of fluid

$$w_{\text{flow}} = Pv$$

Enthalpy $h = u + Pv$

Specific heats

$$c_v(T) \equiv \left(\frac{\partial u}{\partial T}\right)_v \text{ and } c_p(T) \equiv \left(\frac{\partial h}{\partial T}\right)_P$$

For an ideal gas

$$c_p = c_v + R$$
$$\Delta u = u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$
$$\Delta h = h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

Specific heat ratio $\gamma = \dfrac{c_p}{c_v} = \dfrac{\overline{c_p}}{\overline{c_v}}$

For a control volume

$$\dot{m} = \frac{AV}{v}$$

$$\dot{Q} + \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

Gibbs equation

$$ds = \frac{1}{T}du + \frac{P}{T}dv$$

For a liquid or solid

$$\Delta s = s_2 - s_1 = c_{avg}\int_{T_1}^{T_2}\frac{dT}{T} = c_{avg}\ln\frac{T_2}{T_1}$$

For an ideal gas

$$\Delta s = s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$\Delta s = s_2 - s_1 = c_v \ln\frac{P_2}{P_1} + c_p\ln\frac{v_2}{v_1}$$

$$\Delta s = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

Isentropic turbine efficiency

$$\eta_t = \frac{w_t}{w_{t,s}} = \frac{h_2 - h_1}{h_{2s} - h_1}.$$

Isentropic nozzle efficiency,

$$\eta_{nozzle} = \frac{\mathbf{V}_2^2}{\mathbf{V}_{2s}^2}.$$

Isentropic compressor or pump efficiency,

$$\eta_c = \frac{w_{c,s}}{w_c} = \frac{h_{2s} - h_1}{h_2 - h_1}.$$

For an isentropic process in an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{(\gamma-1)}; \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma};$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}; \quad Pv^{\gamma} = \text{constant}$$

For a saturated liquid-vapour mixture

$$x = \frac{\text{mass of vapour}}{\text{mass of mixture}} = \frac{m_g}{m}$$

$$u = \frac{m_g}{m}u_g + \frac{m_f}{m}u_f = xu_g + (1-x)u_f$$

$$h = \frac{m_g}{m}h_g + \frac{m_f}{m}h_f = xh_g + (1-x)h_f$$

$$s = \frac{m_g}{m}s_g + \frac{m_f}{m}s_f = xs_g + (1-x)s_f$$