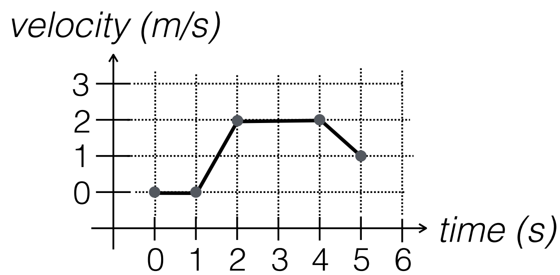
1. [15 pts.] How many dentists are there in Toronto? Give an order of magnitude, and explain your logic clearly.



2. [20 pts.] Consider the graph of velocity versus time shown above.

   (a) [5 pts.] What was the displacement between 0.0 s and 5.0 s ?

   (b) [5 pts.] What was the average velocity between 0.0 s and 5.0 s ?

   (c) [5 pts.] What was the average acceleration between 0.0 s and 5.0 s ?

   (d) [5 pts.] What was the instantaneous acceleration at 1.5 s ?

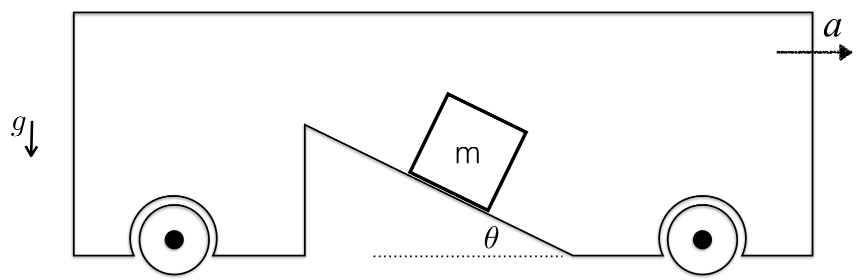
3. [20 pts.] The motion of a particle of mass 2.0 kg is described by

$$\vec{r} = (5.0\,t)\,\hat{\boldsymbol{\imath}} + (2.0\,t^4)\,\hat{\boldsymbol{\jmath}} - 9.5\,\hat{\boldsymbol{k}},$$

where t is time and $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$ are cartesian unit vectors. (Assume SI units for the numerical constants given here, so that $\vec{r}$ is in metres and t is in seconds.)

   (a) [10 pts.] What is the **average velocity** between $t = 0.0$ s and $t = 3.0$ s ?

   (b) [10 pts.] What is the **net force** applied to the particle at time $t = 2.0$ s ?



4. [30 pts.] A block of mass $m = 2.0\,\mathrm{kg}$ sits on an inclined ramp inside a railway car. The car is accelerating at a to the right, as indicated in the figure above. Between the block and the surface of the ramp, the coefficient of static friction is $\mu_s = 0.65$, and the coefficient of kinetic friction is $\mu_k = 0.40$. The angle of the ramp is $\theta = 30°$. Assume the block starts at rest with respect to the ramp.

   (a) [10 pts.] If $a = 0$, does the block start to slide down or stay at its initial height?

   (b) [20 pts.] For what values of a does the block start to slide up the ramp?

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5. [15 + 5 pts.] An object with mass $m = 0.20\,\mathrm{kg}$, initially at rest, is dropped at $t = 0$.

   (a) [5 pts.] In the absence of drag, what is $v(t)$? Specify which direction you mean for positive velocity.

   For parts (b), (c), and (d), include a drag force linearly proportional to velocity, with a drag coefficient $b = 0.10\,\mathrm{kg/s}$.

   (b) [5 pts.] What is the terminal speed of the object?

   (c) [5 pts.] At what time does the object reach $1 - 1/e^2$ or $\approx 86\%$ of terminal velocity?

   (d) [bonus 5 pts.] How far does the object fall in $4.0\,\mathrm{s}$? Compare to the answer in the absence of drag.

END OF EXAM.

no: 1

The population of Toronto is approximately $3{,}0 \cdot 10^6$ people.
Most people go once a year at the dental office on average (some go more often, some don't go at all). So, on an everyday basis, the flux of people must be

$$\approx \frac{\text{population}}{365 \text{ days}} \approx \frac{3{,}0 \cdot 10^6}{3{,}65 \cdot 10^2} \approx 8{,}2 \cdot 10^3 \sim 10^4 \text{ persons/day}$$

to meet the requirements of ~~offer~~ offer and demand. A dentist works about 8hrs a day and each appointment takes approximately 1h, which implies that each dentist can see 8 persons/day $\sim 10^1$ persons/day

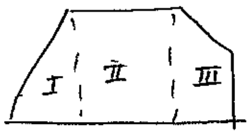
Thus, the number of dentists must be

$$\# \text{ dentist} \approx 8{,}2 \cdot 10^2 \frac{\text{persons}}{\text{day}} \left(8 \frac{\text{persons}}{\text{day} \cdot \text{dentist}} \right)^{-1} \approx 1{,}03 \cdot 10^3 \text{ dentists}$$

$$\Rightarrow \sim 10^3 \text{ dentists.}$$

The number of dentists in toronto is of the order of 10^3 dentists.

(2.) (a) $\Delta x = \int_i^f v\,dt$ = area under $v(t)$ curve.



area is
$\mathrm{I}: (2 \times 1)/2 = 1\ m$
$\mathrm{II}: (2 \times 2) = 4\ m$
$\mathrm{III}: \frac{1}{2} + 1 = 1.5\ m$
$\rightarrow 6.5\ m$

$\rightarrow \boxed{\Delta x = 6.5\ m}$

(b) Two ways of solving this:

- $V_{avg} = \frac{\Delta x}{\Delta t}$, and from (a) $\Delta x = 6.5\ m$,

  thus $V_{avg} = \frac{6.5\ m}{5\ s} = \boxed{1.3\ m/s}$

- OR, discuss this as $V_{avg} = \frac{1}{\Delta t}\int v\,dt$, but the math will be like (a) again.

(c) Two ways of solving this:

- $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{1\ m/s - 0\ m/s}{5\ s} = \boxed{0.20\ m/s^2}$

- OR, make a plot of $a(t)$ & take a weighted average: $a_{avg} = \frac{1s \times 2\ m/s^2 + 1s \times (-1\ m/s^2) + 3s \times (0\ m/s^2)}{5\ s} = 0.20\ m/s^2$

(d) Time $t = 1.5\ s$ is between $v = 0\ m/s$ @ $1\ s$, & $v = 2\ m/s$ @ $2\ s$. Assuming the acceleration is constant (as we did above), the slope is $\boxed{2.0\ m/s^2}$

③

$x(t) = 5,0\ [m/s] \cdot t\ [s]$

$y(t) = 2,0\ [m/s^4] \cdot t^4\ [s^4]$

$z(t) = -9,5\ [m]$

a) $v_{avg} = \frac{\Delta r}{\Delta t}$ $\Delta r = \vec{r}_f - \vec{r}_i$

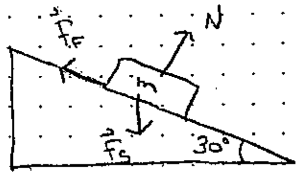
$\vec{r}(0) = \begin{pmatrix} 0 \\ 0 \\ -9,5 \end{pmatrix}$ $\vec{r}(3) = \begin{pmatrix} 15 \\ 162 \\ -9,5 \end{pmatrix}$

$\vec{v}_{avg} = \frac{\vec{r}(3) - r(0)}{3} = \boxed{(5.0\ \hat{i} + 54\ \hat{j})\ m/s}$

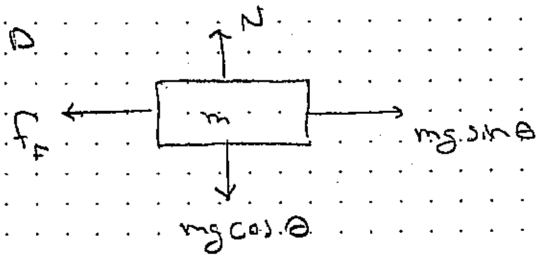
b) $\begin{matrix} x(t) \\ y(t) \\ z(t) \end{matrix} \xrightarrow{d/dt} \begin{matrix} v_x(t) = 5,0 \\ v_y(t) = 8,0\, t^3 \\ v_z(t) = 0 \end{matrix} \xrightarrow{d^2/dt^2} \begin{matrix} a_x(t) = 0 \\ a_y(t) = 24\, t^2 \\ a_z(t) = 0 \end{matrix} \Rightarrow \vec{a} = 24\, t^2\ \hat{j}$

$\vec{F}_{net} = m\vec{a} = 2,0 \cdot 24\, t^2\, \hat{j} = 48\, t^2\, \hat{j}$ $\vec{F}_{net}(t = 2.0s) = \boxed{192\ \hat{j}\ [N]}$

(4) a) $\theta = 30°$, $a = 0$



FBD



From $\sum F_\perp = 0$

$N = mg\cos\theta$

From $\sum F_\parallel = 0$

$mg\sin\theta = f_F$

The block will stay still if the maximum value of the static friction is bigger than the component of $\vec{F}_g$

$f_F \leq \mu_s N \Rightarrow f_F\big|_{max} = \mu_s N$

$= \mu_s mg\cos\theta =$

The block doesn't move if

$\frac{F_F|_{max}}{m} \geq g\sin\theta$

$\frac{f_F|_{max}}{m} = \mu_s g\cos\theta = (0{,}65)(9{,}8)\cos 30°$

$= 5{,}5 \ m/s^2$

$g\sin\theta = (9{,}8)\sin 30° = 4{,}9 \ m/s^2$

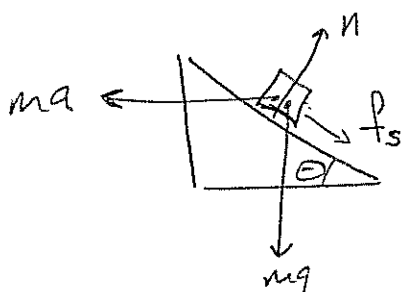
$\Rightarrow$ the block $\boxed{\text{stays still.}}$

(4b) Two ways of solving this problem:

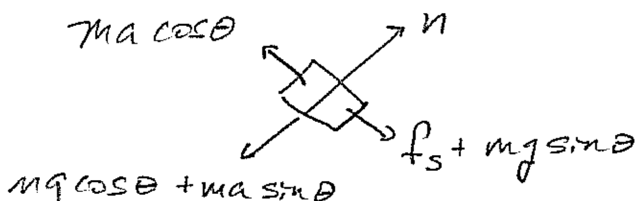
(I) work in a non-inertial frame, with an additional $-ma$ fictional force. Then $\sum \vec{F} = 0$ when block @ rest on ramp.

(II) Work in an inertial frame. Then $\sum \vec{F} = ma$ when block @ rest on ramp.

(4b) (in non-inertial frame)



resolve into axes parallel & perpendicular to the ramp:



Solve by (1) $\sum \vec{F} = 0$, then (2) And conditions that $f_s \leq \mu_s n$.

(1): $n = mg\cos\theta + ma\sin\theta$

$f_s + mg\sin\theta = ma\cos\theta$

Here, get same eqs for inertial frame, where $mg\cos\theta = \sum F_\perp = ma\sin\theta$ and $f_s + mg\sin\theta = \sum F_\parallel = ma\cos\theta$

(2) $\longrightarrow f_s = \cancel{m}(a\cos\theta - g\sin\theta) \overset{?}{\leq} \mu_s \cancel{m}(g\cos\theta + a\sin\theta)$

can plug in values for g & θ & μ_s here, or later:

$a\underbrace{(\cos\theta - \mu_s\sin\theta)}_{0.541} \overset{?}{\leq} \underbrace{g}_{9.8} \underbrace{(\mu_s\cos\theta + \sin\theta)}_{1.06}$

(accept $a \geq 19\,m/s^2$ as well.)

$\longrightarrow$ $a \leq 19\,m/s^2$ balance possible, so $\boxed{a > 19\,m/s^2}$ then block breaks static friction & slides up.

(3) (a) Under gravitational acceleration,

$v = v_0 + at$. Here, $a = g$ (downwards)

and $v_0 = 0$.

→ $\boxed{v = gt}$, with positive v pointing down (ie, aligned with gravity.)

or $\boxed{v = 9.8t}$

(b) The free body diagram is

, so $a = 0$ when $\Sigma F = 0$

or $mg = bv$

→ $\boxed{v_T = \frac{mg}{b} = 19.6\,\text{m/s} \text{ or } 20\,\text{m/s}}$

-1 pt if used 3 sig figs.

(c) The velocity is $v_T(1 - e^{-t/\tau})$, from a result derived in class, where $\tau = m/b = 2.0\,\text{s}$.

So, $v = v_T(1 - e^{-2})$ when $t = 2\tau$, or $\boxed{t = 4.0\,\text{s}}$

Can also find this answer from solving

$1 - e^{-t/\tau} = 0.86 \rightarrow -t/\tau = \ln(0.14)$

which is $t = 1.97\tau$ or $\boxed{t = 3.9\,\text{s}}$. Accept either answer with full credit.

5d
(bonus)

In the absence of drag

$$\Delta y = \frac{1}{2} g t^2 = \frac{1}{2}(9,8)(4)^2 = \boxed{78 \text{ m}}$$

(Free-Fall)

In the presence of drag

$$v(t) = v_T(1 - e^{-t/\tau}) \quad \text{where } \tau = \frac{m}{b} \quad v_T = g\tau \Rightarrow$$

$$x(t) = \int_0^t v(t)\,dt = v_T \int_0^t (1 - e^{-t/\tau})\,dt$$

$$= v_T(t + \tau e^{-t/\tau})\Big|_0^t$$

$$= v_T t + v_T \tau e^{-t/\tau} - (0) - \tau v_T$$

$$= v_T\left[t - \tau(1 - e^{-t/\tau})\right]$$

$$x(t=4, \tau=2) = (2,0)(9,8)\left[4,0 - 2,0(1 - e^{-2})\right]$$

$$= 19,6\left[4,0 - (2,0)(0,86)\right]$$

$$= 19,6\,(2,27)$$

$$= \boxed{44 \text{ m}}$$