PHY 180: Tutorial questions

1. Calculate the gravitational force between two masses of 1 kg each, separated by 1 m. How much smaller is this force compared to that between the earth and a 1 kg mass on the earth's surface?
   How much smaller is the gravitational potential energy between the two objects compared to that between the earth and a 1 kg mass on the earth's surface?
2. How long is one Jupiter-year, given that Jupiter's distance from the sun is 5 AU?
3. A planet of mass M has radius R. Assuming that a satellite is launched radially outwards, what is the minimum velocity so that the satellite never returns to the planet?

   This is called the *escape velocity*.
4. Estimate the numerical value of the escape velocity for earth, and for the asteroid Ryugu ($m = 4.5 \times 10^{11}$ kg, R = 1 km).
5. Consider an object with mass M, which is extremely dense so that its radius is negligible. At what distance r_s from it does the escape velocity equal c, the speed of light?

   The distance r_s is called the "Schwarzschild radius". Any object that is smaller than its Schwarzschild radius cannot emit any light from its surface, because even light would not have the required escape velocity – this is called a *black hole*.
6. At what distance from the earth should a satellite be placed in a circular orbit, so that it always remains vertically above a fixed point on earth, despite the earth's rotation?
7. The mass of Jupiter is 320 times the mass of the earth, and its orbital period is T = 12 earth years. For this problem, you can ignore the rotation of Jupiter and the earth around their own axes. You may assume that the orbits of Jupiter and the earth can be approximated as circles.
   (a) Given that the angular momentum of the earth due to its orbit around the sun has magnitude L_0 , what is the magnitude of the orbital angular momentum of Jupiter around the sun?
   (b) What is the ratio of the kinetic energies of Jupiter and the earth?
8. A robotic probe lands on a small asteroid of mass $M = 10^{12}$ kg, and radius R = 1 km. The entire asteroid (assume it is spherical) is rotating at 10 rpm about its north-south axis.
   (a) With what acceleration does an object fall on its surface, at the north pole?
   (b) With what acceleration does an object fall on its surface, at the equator?
9. Four masses M = 10 kg each are arranged at the corners of a square with side length a = 1 m. What is the gravitational force on a particle of mass m = 1 kg at
   (a) the centre of the square,
   (b) the midpoint of a side of the square.
10. Four masses M = 10 kg each are arranged at the corners of a square with side length a = 1 m.
    (a) What is the gravitational force on a particle of mass m = 1 kg at the centre of the square?
    (b) What is the gravitational potential on the particle at the centre of the square?
11. The planet Venus moves in an approximately circular orbit with radius 0.72 AU. Given this fact (and knowing what you know about the earth's orbit), calculate the speed (in m/s) with which Venus moves.
12. The planet Jupiter has a moon named Ganymede, which orbits Jupiter at a radius $R = 10^6$ km, with a period of 7 days. The mass of Ganymede is negligible compared to the mass of Jupiter. Given these facts (and knowing what you know about the earth's orbit and the mass of the sun), estimate the mass of Jupiter.
13. Two masses m_1 = 1 kg and m_2 = 2 kg are connected together by a spring, and the resonance frequency of this system is 2 Hz. If the mass m_2 is changed to 4 kg, calculate the new resonance frequency.

14. Diatomic molecules behave like harmonic oscillators, for small vibrations around the equilibrium bond length. A carbon monoxide molecule contains a carbon nucleus ($m_C = 2 \times 10^{-26}$ kg) and an oxygen nucleus ($m_O \approx 3 \times 10^{-26}$ kg), and has a vibrational resonance frequency of 30 THz (= 3×10^{13} Hz). Calculate the effective spring constant of this system. What is the total energy contained in the vibrating molecule if the amplitude of oscillations is 1 angstrom (10^{-10} m)?

15. A uniform bar of mass m = 1 kg and length ℓ = 1 m is suspended vertically from a pivot attached to one end. Assuming that its breadth and width are negligible compared to its length, calculate the frequency of small angular oscillations of the bar about its equilibrium configuration. Calculate the energy of this oscillator if the angular amplitude is 3.6°.

16. A hemispherical bowl of mass M and radius R rests on a flat table. Its centre of mass is located $R/3$ below the centre of the sphere. Calculate the frequency of small angular oscillations of the bowl about its equilibrium configuration.

17. The potential energy for a particle moving in one dimension turns out to be $V(x) = \frac{-1}{2}Ax^2 + \frac{1}{4}Bx^4$. Draw the energy diagram for this system. Calculate all the stable equilibrium points, and the frequency of small oscillations around these equilibrium points.

18. (Optional) Two bars with equal mass and length are attached to each other, and suspended in the form of a double pendulum (see the demo experiment). Construct the differential equations that describe the motion of this system.

19. A crane picks up a van weighing 1 metric ton, using a 10 m long steel cable. (You can neglect the weight of the cable relative to the van.) How much kinetic energy does the van have on average, if it swings from side to side with an amplitude of 1 m?

20. A 1 kg mass is attached to one end of a spring with constant k = 100 N/m. (Neglect the mass of the spring.) Another mass m is attached to the other end of the spring, so that the spring and two masses form an 1D harmonic oscillator.

    (a) What is the value of m if the natural resonance frequency of the oscillator is 0.1 Hz?

    (b) Draw a graph of the centre of mass momentum of the system of two masses, when the amplitude of oscillations is 10 cm.

21. A thin sheet of metal in the shape of a square has mass M and side length a. It is suspended vertically from a pivot located at one corner of the sheet. What is the frequency of small angular oscillations about the equilibrium configuration?

22. A particle of mass m moves in one dimension in the potential $V(x) = -Ax + Bx^2$, where A = 1 J/m and B = 4 J/m^2.

    (a) Draw an energy diagram for this system and indicate all the stable and unstable equilibrium points.

    (b) Calculate the frequency of small oscillations about all of the stable equilibrium points.

23. A particle of mass m moves in one dimension in the potential $V(x) = Ax^3 - Bx$, where A and B are equal to 1 in their respective SI units.

    (a) Draw an energy diagram for this system and indicate all the stable and unstable equilibrium points.

    (b) Calculate the frequency of small oscillations about all of the stable equilibrium points.

24. Two simple harmonic oscillators A and B with the same mass have natural resonance frequencies in the ratio $f_A : f_B$ = 1:3.

    (a) What is the ratio of their spring constants?

    (b) If they have the same peak kinetic energies when they oscillate, what is the ratio of their amplitudes?

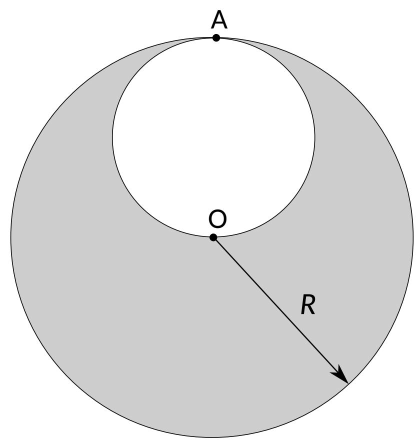
    (c) Draw a qualitative graph showing the potential energy curves for the two oscillators.

25. Calculate the moment of inertia of a square plate with mass M and side length a, about an axis that lies along one of the sides of the square.

26. A two-dimensional object with mass M has the shape shown in the shaded region of the figure.

    (a) Calculate its moment of inertia about an axis perpendicular to the page and passing through the point O.

    (b) Calculate its moment of inertia about an axis defined by the line joining the points O and A.



27. A solid ball with mass M = 1 kg and radius R = 10 cm is placed on a table and given a sharp impulse so that its centre of mass initially moves with velocity v = 1 m/s, with no rolling. The ball has a friction coefficient (both kinetic and static) $\mu = 0.2$ with the table.

    (a) How far does the ball travel before it starts rolling without slipping?

    (b) What is the total kinetic energy of the ball after it starts rolling without slipping?

    (c) What is the work done on the ball by friction after it is rolling without slipping?

28. A solid cylinder with mass M and radius R is suspended from a thin film of tape that is wrapped around it many times. The cylinder is held horizontally, with the tape exactly vertical, and released from rest. The tape unravels as the cylinder falls.

    (a) How long does the cylinder take to fall through a height ℓ?

    (b) How much rotational kinetic energy, and total kinetic energy, does it gain in falling through this height?

29. A spherical ball with mass M and radius R is placed at rest on a horizontal table. The coefficient of friction between the ball and table is μ (static and kinetic coefficients are equal). A horizontal force with magnitude F is applied to the ball at a height d above its centre of mass.

    (a) Draw a diagram showing all the forces and torques acting on the ball. Write down the equations of motion for the ball's centre of mass momentum and angular momentum.

    (b) If friction is negligible, what value of d ensures that the ball always rolls without slipping, regardless of the magnitude of the force?

30. (a) The uniform disk shown in the figure moves on the frictionless surface with a combination of rolling and slipping. Its mass is M and its radius is R. Its centre of mass moves to the right with speed v, and the disk rotates (about an axis passing through E and perpendicular to the page) clockwise with angular speed ω. Draw the velocity vector at each of the points A-E.

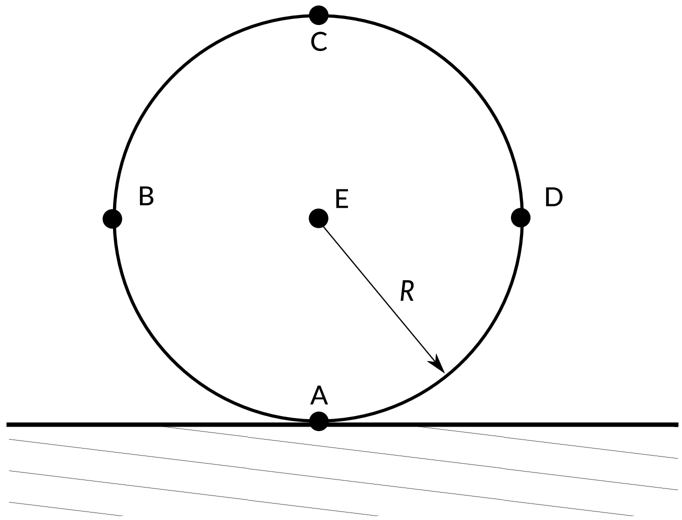
    (b) Calculate the total kinetic energy of the disk.

31. A thin sheet of metal in the shape of a square has mass M and side length a.

    (a) Calculate its moment of inertia for rotation about an axis perpendicular to the sheet, passing through one of the corners of the square.

    (b) Calculate its moment of inertia for rotation about an axis defined by the line joining two opposite corners of the square.

32. A star with mass equal to 10^{31} kg and radius equal to 10^9 m spins around its axis with angular speed $\omega = 10^{-6}$ rad/s. One day, the star explodes as a supernova and blows away a lot of gas, after which all that is left is a neutron star with 10% of the mass of the original star, and radius equal to 10^4 m. You may assume that the original star and the neutron star can be modeled as uniform spheres. Also assume that the angular momentum carried away by the gas from the explosion is negligible.



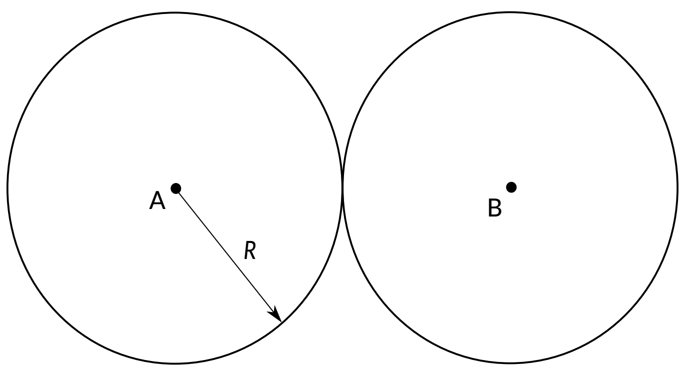
(a) Estimate the final angular speed of the neutron star.

(b) How much kinetic energy does the neutron star have?

33. Two uniform thin disks (each with mass M and radius R) are glued together into a composite object as shown in the figure.

    (a) Calculate its moment of inertia about an axis perpendicular to the figure, and passing through the point A.

    (b) Calculate its moment of inertia about an axis defined by the line joining the points A and B.



34. A carbon monoxide molecule can be modeled as two point masses (corresponding to the carbon nucleus, $m_C = 2 \times 10^{-26}$ kg, and the oxygen nucleus, $m_O \approx 3 \times 10^{-26}$ kg), separated by a distance $r = 10^{-10}$ m. Microwave spectroscopy shows that the molecule rotates 10^{11} times per second about an axis through its centre of mass, perpendicular to the line joining the atoms. Calculate the angular momentum of the molecule.

35. (a) Calculate the cross product of $\vec{A} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{B} = -1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

    (b) Construct a vector whose cross product with $\vec{C} = \hat{\imath}$ equals -1.

    (c) What is the magnitude of $\frac{(\vec{r} \times \vec{v}) \times \vec{v}}{r^2}$ if the vectors $\vec{v}$ and $\vec{r}$ are perpendicular? Which way does the vector point?

    (d) Calculate the area of a triangle with one vertex at the origin and the other two vertices at $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - 2\hat{\jmath}$.

36. A shaft attached to a constant-torque motor spins up to 60,000 rpm (revolutions per minute) in 20 s. If the moment of inertia of the shaft around the rotation axis is 10^{-5} kg m^2, calculate the torque generated by the motor. How many revolutions does the shaft make during these 20 s?

37. The motion of a particle is described by the vector $\vec{r}(t) = A\left(\cos \alpha t\ \hat{\imath} + \sin \alpha t\ \hat{\jmath}\right)$, with α = 2π rad/s.
    What is the shape of the trajectory when $A = B$?

Calculate the particle's angular velocity $\vec{\omega}$, and angular acceleration $\frac{d\vec{\omega}}{dt}$. (Recall that $\vec{\omega} = \frac{\vec{r}\times\vec{v}}{r^2}$.)
What is the particle's speed at $t = 0$ s? at $t = 0.5$ s?

38. A particle with mass m = 100 g moves in a straight line in the xy-plane with position $\vec{r} = (3t - 3)\ \hat{\imath}$ m + $2t\ \hat{\jmath}$ m (the time t is in seconds). Calculate the angular momentum of the particle as a function of time. What is the distance of the particle from the origin at its closest point of approach?

39. A 100 g mass is attached to a 1 m-long string (neglect the string's mass).
With what angular speed does the mass need to be whirled around so that it has 10 kg m^2/s of angular momentum?
What is its rotational kinetic energy?

40. A ring whose moment of inertia is 10 kg m^2 is spun up using a constant torque of 10 N m, from $t = 0$ until $t = 10$ s.
What is the angular speed at $t = 10$ s?
Draw (as accurately as possible) a graph of the angular momentum of the ring vs. time.

41. A moving particle bounces off an initially stationary disk, and makes the disk spin about its axis with an angular speed of 10 rpm. If the moment of inertia of the disk is 10^{-4} kg m^2, what is the angular momentum transferred to the disk by the collision? How much kinetic energy is contained in the spinning disk?

42. A particle of mass m = 10 g is attached to the origin by a massless string of length 10 cm, and executes circular motion in the xy-plane with angular momentum 10 kg m^2/s. What is the angular speed of the particle? At a certain time, the string is suddenly cut: with what linear speed does the particle move away?

43. A bicycle wheel can be modeled as thin ring, with mass M = 5 kg, and radius R = 50 cm. How much angular momentum is contained in one wheel of a bicycle moving with speed v = 10 m/s ? How much torque is applied by the brakes if this wheel comes to a stop in 2 s?

44. A simple pendulum consists of a mass m = 1 kg on the end of a string of length ℓ = 1 m. It is suspended in a gravitational field (assumed to point vertically down, with gravitational acceleration g = 10 m/s^2). The pendulum is displaced by an angle of 30° from the vertical direction. The origin is at the end of the string which is not attached to the mass. Calculate the magnitude of the torque acting on the pendulum. Draw a diagram showing the force vector and the torque vector.

45. A projectile of 100 g mass is fired at an angle of 30° to the horizontal, with an initial speed of 10 m/s.
    (a) Calculate the work done on the projectile by the gravitational force, between the launch point and the highest point of the trajectory.
    (b) Calculate the power expended on the projectile by the gravitational force at the highest point of its trajectory. (Recall that the power $P = \vec{F}\cdot\vec{v}$.)
    (c) Calculate the work done on the projectile by the gravitational force, between the launch point and the landing point.

46. A particle of mass M moves in one dimension, and experiences a force $F_x = -kx$. Calculate the work done in moving from $x = 0$ to $x = L$. What are the units of the quantity k ?

47. A particle of mass M moves on a plane, and experiences a force $\vec{F} = -kx\hat{\imath} - ky\hat{\jmath}$. Calculate the work done in moving from the point with coordinates $(0, 0)$ to the point $(4\,\mathrm{m}, 3\,\mathrm{m})$.

48. A particle of mass M moves from point A (the origin) to point B ($\vec{r}_B = 3\,\mathrm{m}\ \hat{\imath} + 3\,\mathrm{m}\ \hat{\jmath} + 2\,\mathrm{m}\ \hat{k}$), under the action of a force $\vec{F} = -3\hat{k}$ N. Calculate the change in the kinetic energy of the particle when it moves from A to B in a straight line.

49. A particle of mass M moves in a potential $V(x, y, z) = \frac{1}{2}k[(x-3)^2 + y^2 + z^2]$. Calculate the force vector at the origin.

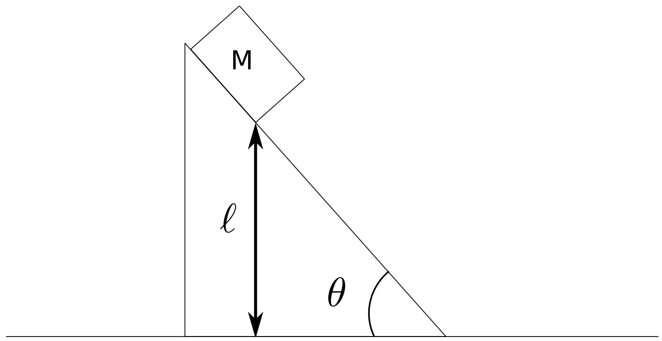
50. An object of mass M = 1 kg experiences a force $\vec{F} = -Ax\hat{\imath} - Ay\hat{\jmath} - Az\hat{k}$. The value of A is 10 N/m.
    (a) Calculate the work done on an object that moves along a circle of radius 20 cm in the xy-plane under the action of this force.
    (b) Calculate the work done on an object that moves from the point with coordinates $(3, 0, 0)$ to the origin along a straight line.

51. An object of mass M = 1 kg experiences a force $\vec{F} = -Bx\hat{i} - By\hat{j} + 2Bz\hat{k}$. The value of B is 10 N/m.

    (a) Calculate the work done on the object if it moves along a circle of radius 10 cm in the xy-plane under the action of this force.

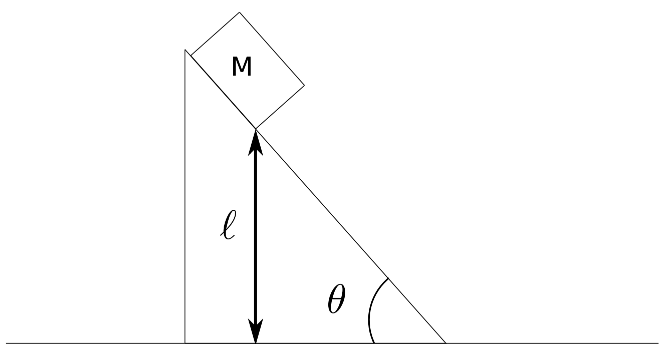
    (b) Calculate the work done on the object as it moves from the point with coordinates $(0, 0, 3)$ to the point $(0, 0, -3)$ in a straight line.

52. A block of mass M is positioned on a slope angled at $\theta = 45°$. What is the minimum value of the coefficient of static friction required to keep the block stationary?



53. A 20,000 kg TTC streetcar starts from rest and experiences a constant horizontal acceleration of 0.2 m/s^2 for 10 s. What is the work done by its motor?

54. A block of mass M is placed a slope angled at θ. It is initially pushed down the slope with speed u, from a height ℓ. The coefficient of sliding friction between the block and slope is μ_K. What fraction of its kinetic energy does the block lose by the time it gets down to the bottom?



55. A 20,000 kg TTC streetcar brakes from 10 m/s to a complete stop in 3 s. What is the work done in stopping the streetcar?

56. A projectile is launched with initial velocity $\vec{u} = 1\ \mathrm{m/s}\,\hat{\mathrm{i}} + 1\ \mathrm{m/s}\,\hat{\mathrm{j}}$.

    (a) What is the angle made by the initial velocity with the horizontal?

    (b) What is the maximum height reached by the projectile?

    (c) How far along the x-axis does the projectile travel from its launch point?

57. A ball of mass m is thrown down with initial velocity $\vec{u}$ in a gravitational field $\vec{g}$, and experiences an air resistance force $\vec{F} = -b\vec{v}$.

    (a) What are the units for the quantity b?

    (b) Write down the differential equation for the z-component of $\vec{v}$.

    (c) Sketch the graph of v_z versus time.

    (d) What is the maximum value of v_z reached by the ball?