Q1: I is wrong: it will pseudo-oscillate at pseudo-frequency $\omega_d = \omega_0\sqrt{1-\frac{1}{4Q^2}}$

II is correct.

III is wrong: the fastest return to equilibrium is for critical damping ($\gamma = 2\omega_0$)

IV is correct: $\omega_d < \omega_0 \Rightarrow T_d = \frac{2\pi}{\omega_d} > T_0 = \frac{2\pi}{\omega_0}$

The correct answer is C.

Q2: I is correct.

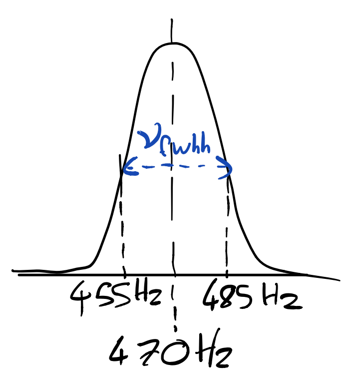
II. is wrong: an IC with 100% projection on a given mode will only trigger that mode. Modes are independent ⇒ That's it for the other modes.

III is wrong. We can't count degrees of freedom that easily, we need to count how many equations we need to solve the problem. Here, it is one per mass position, i.e. 2.

IV is wrong: we are missing the initial velocity!

The correct answer is D.

Q3: (a) First, in terms of frequency:



$$\nu_{fwhh} = 2 \times 15\ Hz = 30\,Hz$$

$$\omega_{fwhh} = 2\pi\nu_{fwhh} \approx 189\ rad.s^{-1}$$

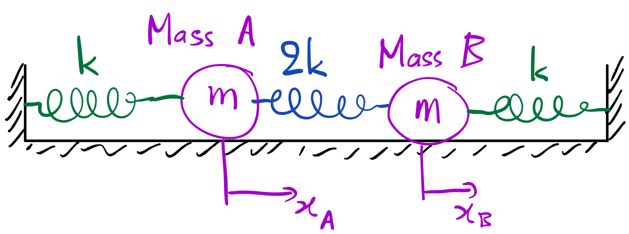
(b) When $\gamma \ll \omega_0$, or $Q \gg 1$, then $\omega_{fwhh} \approx \gamma$

$$\Rightarrow Q_0 = \frac{\omega_0}{\gamma} \approx \frac{\omega_0}{\omega_{fwhh}} \approx \frac{470\text{ Hz}}{30\text{ Hz}} \approx 15.7$$

Based on what we saw in class, $15.7 \gg 1$ and the calculation is valid.

(c) One of the interpretations of Q is $2\pi\{$number of osillations that happen within $1/\gamma\}$. $1/\gamma$ is also the e-folding decay scale of $E \Rightarrow$ The number of oscillations is about $\frac{15.7}{2\pi} \approx 2.5$

Q4.



(a) Mass A: $m\ddot{x}_A + kx_A + 2k(x_A - x_B) = 0$

Mass B: $m\ddot{x}_B + kx_B + 2k(x_B - x_A) = 0$

Or: $\ddot{x}_A + \frac{3k}{m}x_A - \frac{2k}{m}x_B = 0$

$\ddot{x}_B - \frac{2k}{m}x_A + \frac{3k}{m}x_B = 0$

$$\Rightarrow \ddot{\vec{X}} + P\vec{X} = 0 \quad \text{with} \quad P = \begin{bmatrix} 3\omega_s^2 & -2\omega_s^2 \\ -2\omega_s^2 & 3\omega_s^2 \end{bmatrix} \qquad \omega_s^2 = \frac{k}{m}$$

assuming $\vec{X} = \vec{A}\cos(\omega t + \phi) \Rightarrow (P - \omega^2 I_2)\vec{X} = 0$

(b) $\det(P-\omega^2 I_2) = (3\omega_s^2-\omega^2)^2 - 4\omega_s^2 = (3\omega_s^2 - 2\omega_s^2 - \omega^2)(3\omega_s^2 + 2\omega_s^2 - \omega^2)$

$a^2 - b^2 = (a-b)(a+b)$

$= (\omega_s^2 - \omega^2)(5\omega_s^2 - \omega^2)$

$\Rightarrow \omega_1 = \omega_s, \quad \omega_2 = \sqrt{5}\,\omega_s$

$m = 10\text{kg}, \quad k = 50 \text{ N.m}^{-1} \Rightarrow \omega_s^2 = 5 \text{ rad}^2.\text{s}^{-2}$

$\Rightarrow \omega_1 \approx \underbrace{2.23}_{=\sqrt{5}} \text{ rad.s}^{-1}; \quad \omega_2 = 5 \text{ rad.s}^{-1}.$

(c) Take 1st eqn: $\ddot{x}_A + 3\omega_s^2 x_A - 2\omega_s^2 x_B = 0$

$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \cos(\omega_s t) \Rightarrow -\omega_s^2 x_A + 3\omega_s^2 x_A - 2\omega_s^2 x_B = 0$

$\Rightarrow x_A = x_B$

"antisymmetric mode", central spring unstretched.

$\begin{bmatrix} x_A \\ x_B \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \cos(\sqrt{5}\,\omega_s t) \Rightarrow -5\omega_s^2 x_A + 3\omega_s^2 x_A - 2\omega_s^2 x_B = 0$

$\Rightarrow x_A = -x_B$

"symmetric mode", motion is symmetric around central location.