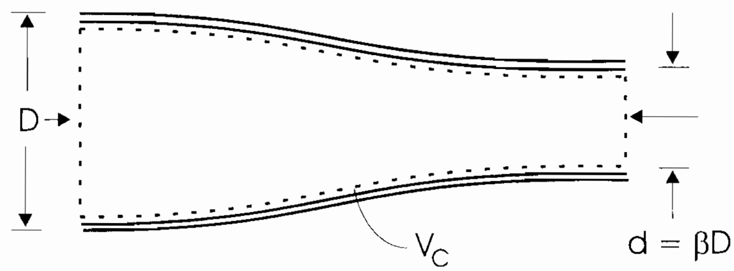
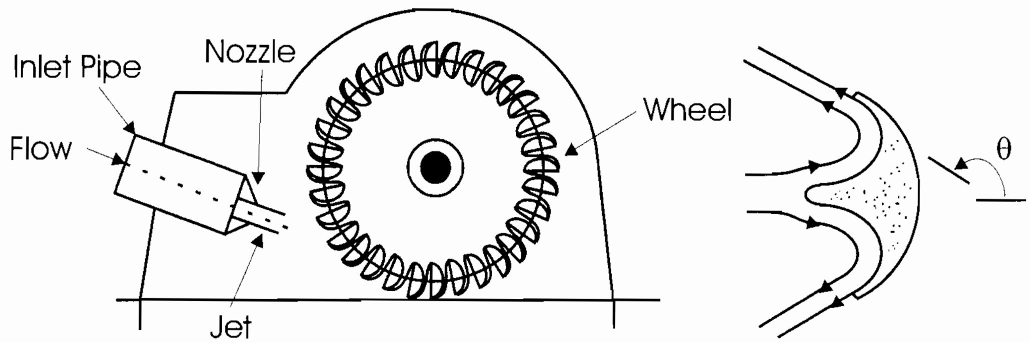
Exercises for Chapter 5

(5.1) A pipe of diameter D delivers incompressible liquid to the smoothly contoured nozzle of exit diameter $d < D$ as shown in the diagram. If the volume flux Q of a liquid of density ρ flows through the combination, by assuming that the flow is inviscid and may be treated as a filament, calculate the force F_N exerted on the nozzle. Note that, in this situation, the effect of the pressure difference across the ends of the filament must be correctly accounted for in the use of the momentum theorem, $\mathcal{T}$30.1 Hint: Use Bernoulli's theorem to estimate the pressure at the entrance to the nozzle, and treat the control volume depicted below as a filament. Then use the results of the analysis to estimate the force a fireman must exert to hold a hose by calculating F_N for $D =$ 6.0 cm and $d = 3.0$ cm, if water leaves the nozzle at a speed $q = 30$ m/s.

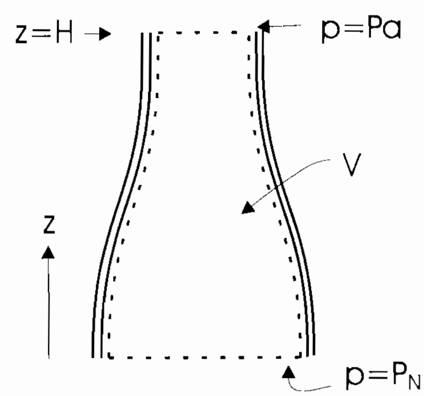


(5.2) A *Pelton Wheel* hydraulic turbine consists of a disc mounted on shaft free to rotate, with buckets attached to the disc at a mean radius R in such a way that they deflect a jet of water directed at them at speed V_J, with the surroundings being a chamber at atmospheric pressure. The overall concept of the turbine and a section through a typical bucket are depicted in the diagrams. The force exerted on the buckets by the delfecting stream causes the disk to rotate at angular speed Ω, thus moving the buckets at speed $V_B = R\Omega$, and performing work on the shaft. As depicted in the diagram, the performance of this machine can be determined by treating the jet as if it were a filament. The basic index of performance is the *hydraulic efficiency* η_H, which is the ratio of the rate P_W at which work is done on the buckets by the deflecting jets to the energy per unit mass of the undeflected jet, and m_j being the mass flux delivered by the nozzle, $E_K = m_J(\frac{1}{2}V_J{}^2)$. The water speed relative to the moving buckets as it impacts on the bucket is $V_R = V_J - V_B$; V_R is deflected by the angle θ depicted in the diagram, and is reduced in magnitude to a fraction k_f of its impact value. Calculate η_H as a function of the quantities given above. With all the other quantities fixed, show that, if the bucket speed $V_B = R\Omega = \frac{1}{2}V_J$, then η_H is a maximum. Find this maximum and P_W.

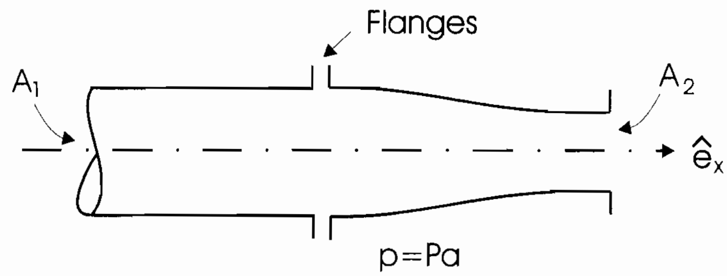


(5.3) Pelton wheel turbines are usually fed from reservoirs having very high heads H, in some cases as high as 1500 m. For a machine with $R = 1.0$ m, $\theta = \frac{3}{4}\pi$, and $k_f = 0.95$, and with a nozzle of exit diameter $d_J = 0.2$ m which is fed from a reservoir with $H =$ 800 m, calculate Ω and P_W by using Torricelli's law to estimate V_J. Assume that it operates at the point of maximum efficiency.

(5.4) A smooth contoured, gradually converging nozzle of exit area A_N has its axis directed vertically upwards as shown in the diagram. It is attached to a pipe of cross-sectional area $A_P = 2A_N$ which delivers water at a volume flow rate Q. The nozzle's height is H, and its interior volume is V. By assuming inviscid flow as in filament, find the force exerted on the nozzle by the water, and show that it is the sum of that acting in the absence of gravity plus that acting in the absence of flow.



(5.5) A pipe of cross-sectional area A_1 delivers water of density ρ at volume flux Q to a gradually tapering nozzle having its axis horzontal as shown in the diagram below. The cross-sectional area of the nozzle decreases from that of the pipe to A_2. By assuming that the flow in the nozzle is incompressible and frictionless, and that it can be treated as a filament, calculate the force in the bolts connecting the flanges on the pipe and the nozzle.

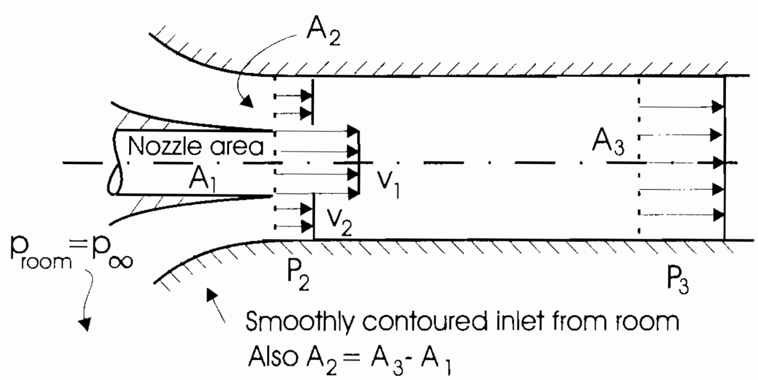


(5.6) Apply the abrupt enlargement analysis of example 5.6 to the nozzle geometry "D" in Example 1.4 to estimate the flow increase. Show that, if λ is the enlargement area ratio, equal to $(D_E/D_T)^2$ in Figure 1.6, the volume flux so obtained can be expressed as

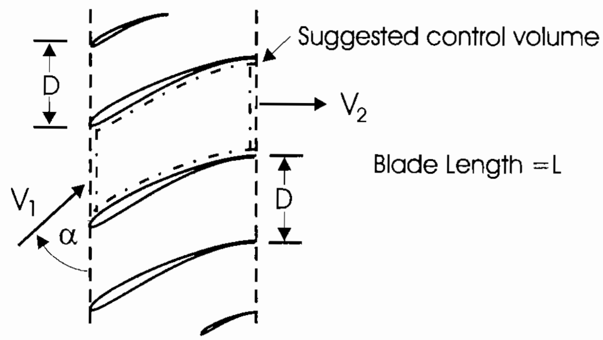
$$Q = R_\lambda A_T\sqrt{2gH} \quad \text{where} \quad R_\lambda = \frac{\lambda}{\sqrt{\lambda^2 - 2\lambda + 1}}$$

where $A_T = (\pi/4)D_T^2$ is the throat area, and H is the height above the nozzle axis of the liquid surface in the reservoir above the nozzle axis. Compare this prediction with the results in Figure 1.7. Show also that R_λ has a maximum at $\lambda = \sqrt{2}$, and that this maximum is 1.3066.

(5.7) A jet pump is a device for using a small mass flow of fluid at high velocity to move a larger mass flow against a given pressure rise $\Delta p = p_3 - p_\infty$, as shown in the diagram below. The jet emerging from the nozzle entrains the fluid to be pumped by viscous forces and turbulence. As in the abrupt enlargment problem the mixing is such that wall friction may be neglected, and that the velocity profile at the downstream station "3" can be assumed uniform. Assuming incompressible flow, by appropriate use of Bernoulli's and the momentum theorem find an expression for the entrained volume flow Q_e in terms of the jet flux Q_p and areas A_1 and A_3, and the imposed pressure rise Δp.



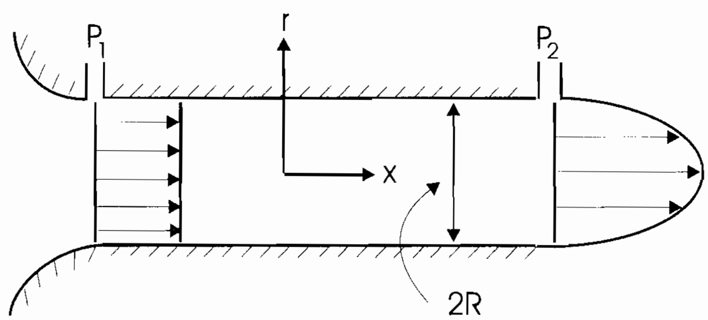
(5.8) A cascade is a set of aerofoils used to deflect a flow as shown in the diagram below. Assume that the flow through the blades is inviscid, incompressible and steady. It approaches the cascade at uniform speed V_1 and angle α to the entrance plane and leaves with uniform speed V_2 normal to the exit plane. Calculate in magnitude and direction the force exerted on a blade. In using the control volume suggested below, ignore the blade thickness.



(5.9) An incompressible fluid enters a horizontal pipe of diameter $D = 2R$ having its axis parallel to $\hat{\imath}_x$ with uniform velocity $V\hat{\imath}_x$ and pressure p_1. At a downstream section the pressure is p_2 and the velocity is parabolic; that is, with r being the radial distance from the tube axis

$$\vec{v} = u(r)\hat{\imath}_x = K\left[1 - \frac{r^2}{R^2}\right]\hat{\imath}_x$$

where K is a constant. This is depicted in the diagram below.



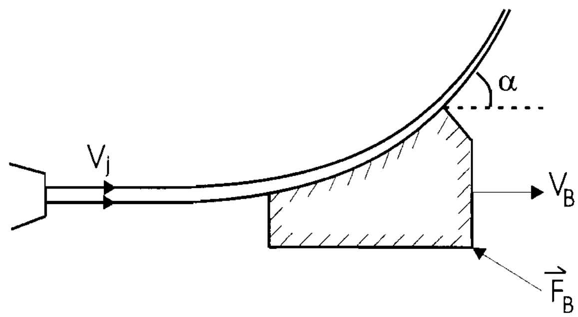
Show that the frictional force $\vec{F}$ exerted on the pipe walls between the stations where p_1 and p_2 are measured is given by

$$\vec{F} = \pi R^2\left[p_1 - p_2 - \frac{\rho V^2}{3}\right]\hat{\imath}_x$$

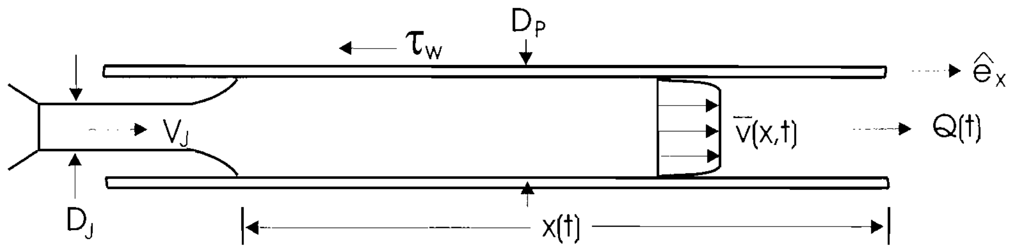
(5.10) This problem illustrates some aspects of fluid machinery. A thin jet of liquid emerges from a reservoir through a nozzle of diameter D_J at speed V_J. It strikes a block shaped in such a way that the jet is turned through an angle α as depicted in the diagram below. The block moves away from the nozzle at speed V_B parallel to the nozzle axis, the constant V_B being maintained by an opposing force $\vec{F}_B$. By assuming that the jet is a filament, and that friction on the block reduces the relative speed V_R on the block by an amount ηV_R during the turning process, calculate $\vec{F}_B$. Also define a hydraulic efficiency

$$\eta_H \doteq \frac{\text{power of} \vec{F}_B}{\text{kinetic energy flux out of nozzle}}$$

Calulate η_H and comment on the choice of α and V_B to maximise η_H.



(5.11) A nozzle directs a jet of liquid having diameter D_J and speed V_J into a horizontal open-ended pipe of diameter $D_P < D_J$ and length L and, at a distance $x(t)$ from the downstream end, abruptly fills it as shown in the diagram.



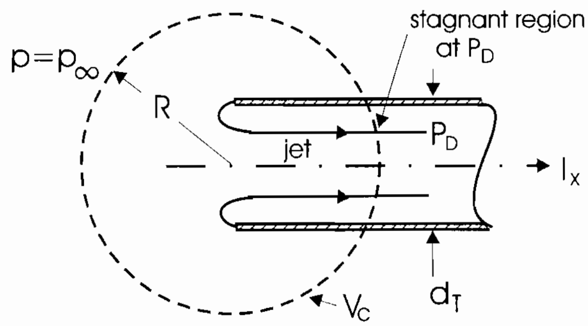
Using the global form of continuity and the momentum theorem show that, with $A_J = \pi D_J^2/4$ and $A_P = \pi D_P^2/4$, if $Q_J = A_J V_J$ is the volume flux out of the nozzle, the histories of the volume flux $Q(t)$ out of the downstream end and of $x(t)$ are governed by the following differential equations:

$$\frac{dx}{dt} = \frac{Q_1 - Q_2}{A_2 - A_1}$$

$$\frac{dQ}{dt} = \frac{1}{x}\left[\frac{Q_J^2}{A_J} - \frac{Q^2}{A_P} + \frac{(Q_J - Q)^2}{A_2 - A_1} - \left(\frac{2f}{\pi D_P^2}\right) x Q^2\right]$$

To obtain these equations assume that the flow in the filled portion of the pipe is turbulent so that the mean velocity $V_P = Q/A_P$ can be used for both calculating fluxes and estimating the effect of shear stress τ_w at the pipe wall through the friction factor of equation 3.5.12. Examine the behaviour of the equations in the limit $x \to 0$. Hint: 0using the ideas leading to equation 4.7.11 show first that, for steady flow in a pipe $\tau_w = (f/8)\rho V_M^2$.

(5.12) Example 1.4 introduces the concept of the vena-contrata associated with flow through an orifice. For the flat plate orifice of Figure 1.6, if the flow reduction from the predicted value of equation 1.1.17 is entirely due to the reduction of size of the jet, then the experimental results plotted in Figure 1.7 can be used to determine the extent of the contraction. Expressing the final cross-sectional area of the jet A_J as a fraction σ of A_T, $\sigma = 0.63$ approximately. For the general case, namely the flow of a viscous fluid through a nozzle of arbitrary geometry, prediction of σ is a major task. However, for one particular geometry, if one also assumes inviscid incompressible flow, this can be achieved. The geometry, depicted in the diagram below, is known as a *re-entrant* or *Borda* orifice; it is named after the French military engineer Jean Charles Borda [1733-1799], who first formulated the analysis that is the subject of this problem(Rouse and Ince, 1963, p. 125). Basically the Borda orifice is a sharp-edged tube extending into the interior of a reservoir. By an appropriate use of Bernoulli's and the momentum theorem, show that $\sigma = 0.5$. Hint: use a control volume in the form of a sphere or radius R centered on the orifice inlet plane and consider what happens in the limit $R \to \infty$.



(5.13) For the Borda orifice discussed in question 5.8, owing to fluid friction and turbulence, the flow downstream of the vena-contracta is much more complicated than first assumed; in particular it spreads out to fill the tube and, as in the abrupt enlargement problem of Example 5.6 [CHECK] there is an associated loss of mechanical energy. If Q is the volume flux through the orifice, and if A_1 is the cross-sectional area of the inside of the tube then, as usual, define a mean flow speed $U = Q/A_I$. Then the energy loss can be conveniently measure in units of $1/2\rho U^2$. If p_∞ is the reservoir pressure, and if p_d is the pressure measured on the tube wall a few diameters downstream of the entrance, according to Bernoulli's theorem, the ideal or frictionless value of p_d — namely p_{di} — is given by $p_{di} = p_\infty - 1/2\rho U^2$. But, owing to losses, p_d is less than this and can be expressed as

$$p_{di} - p_d = C_{EL}\frac{\rho U^2}{2}$$

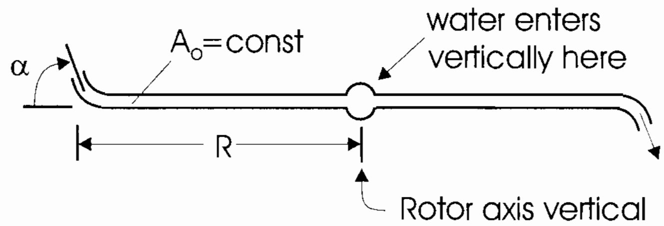
where C_{EL} is an energy loss coefficient. By adapting the analysis of the abrupt enlargment estimate the contraction coefficient σ by relating it to the measured C_{EL}.

(5.13) By assuming that the velocity profile is linear, use the von Kármán form of the momentum integral, equation5.?.?? to obtain an estimate of the skin friction generated on a flat plate immersed in and aligned with a uniform incompressible flow at speed U of a fluid having density ρ and viscosity μ. That is, assuming a finite boundary layer thickness $\delta(x)$, use

$$\frac{u(x,y)}{U} = \frac{y}{\delta(x)} \quad \text{for} \quad y < \delta$$

Determine the error incurred by this crude approximation.

(5.14) A lawn sprinkler discharges water in the horizontal plane so that it makes an angle α to the rotor as shown in the diagram below. The rotor is a single tube of constant cross-sectional area A_0 and discharges a total volume flux of Q through both arms, and the resisting torque imposed by the bearings and seals is T_0. Find the equation governing the angular velocity of the rotor and integrate it to find the equilibrium angular rotation speed.



(5.15) Liquid having density ρ emerges at speed V from a nozzle having exit area A which is pointing vertically downwards. It falls through a distance H into an open tank which is initially full of the liquid and which is supported on a weigh-scale as shown in the diagram below. The overflow water from the tank leaves horizontally through a spout as shown in the diagram. Before the jet flow is started, the weigh-scale records a weight W; what does the weigh-scale record when the jet is flowing?

(5.16) An incompressible liquid jet issues at speed U from a long narrow slot orifice of length L and width B and strikes a plate inclined at an angle α to the direction of the jet; as shown in the diagram below, on striking the plate the jet splits and leaves as two sheets of liquid. By assuming that the flow is inviscid, that gravitaional effects can be omitted, and that the sheets of liquid leave the plate competely parallel to it, calculate the force action on the plate and the division of the flow. Hint: use the momentum theorem in the $\hat{\imath}_s - \hat{\imath}_n$ coordinate system depicted in the diagram.

(5.17) On advantage of the von Kármán momentum integral is that it can be applied to time-averaged turbulent boundary layers. This fact and available data on turbulent flow in pipes was used by Prandtl in 1921 to obtain an estimate of the growth of a turbulent boundary layer on a flat plate immersed in a uniform stream as follows. In Section 3.5 we note that, for turbulent flow in a pipe of diameter D at mean speed $\bar{V}$, the pressure decrease Δp over a length L usually expressed in terms of the friction factor f introduced in the equation 3.5.12;

$$\Delta p = f(Re_D)\frac{L}{D}\left(\frac{1}{2}\rho\bar{V}^2\right) \quad \text{where} \quad Re_D = \frac{D\bar{V}\rho}{\mu}$$

in 1911 Blasius concluded that, for moderate Re_D the empirical formula

$$f = \frac{0.3164}{Re_D^{1/4}}$$

accurately described the data available to him. It was also known that the mean or time-average velocity profiles could be described as follows; with the pipe axis aligned along the x-direction, with r being the radial distance from the axis, with $R = D/2$, and with

$$\vec{\bar{v}}(\vec{r}) = \bar{u}(r)\hat{\imath}_x \quad , \quad \frac{\bar{u}(r)}{\bar{u}_{max}} = \left(\frac{[R-r]}{R}\right)^{1/7} \quad \text{where} \quad \bar{u}_{max} = \bar{u}(r=0)$$

where the bars over variables define time averages. Prandtl assumed that these two results could be applied to the turbulent boundary layer on the flat plate by taking the boundary layer thickness δ to be the counterpart of R and u_{max} to be the counterpart of the free stream speed U. Following Prandtl show that, using equation 5.?.??

$$\frac{\delta(x)}{x} = \frac{0.37}{Re_x^{1/5}} \quad \text{where} \quad Re_x = \frac{x\rho U}{\mu}$$

[ADD RESULTS FOR SKIN FRICTION, SEE SCHLCHTING PP 536-537]