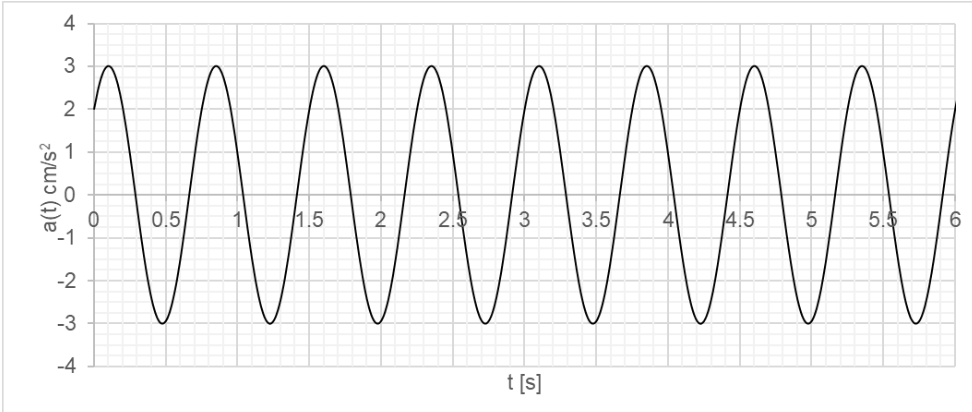
[Instructions: Solve the question below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.

You are allowed: a non-communicating calculator, a one-page formula sheet (can be annotated)].

Question

Mass $m = 0.215$ kg is attached to a spring of spring constant k. The mass is oscillating, according to $x(t) = A_0 \cos(\omega t + \phi_i)$, initially without any damping. The figure below shows the **acceleration** of the mass as a function of time.



a. What is the spring constant of the oscillation? [1 point]
   From the graph: $T = 0.75\ s$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{0.75s} = 8.38\ \mathrm{rad/s}$$

$$\omega_0^2 = \frac{k}{m} \rightarrow k = \omega_0^2 * m$$
$$= 15.1\ \mathrm{N/m}$$

NOTE: Units do not need to be substituted into the equations, final answer **requires** units for full marks.

b. What is the initial phase constant of the oscillation? [4 points]

$$x(t) = A\cos(\omega_0 t + \phi_i) \rightarrow a(t) = -A\omega^2\cos(\omega_0 t + \phi_i)$$
$$a(0) = -A\omega^2 cos\phi_i$$

NOTE: Realizing equation for acceleration needs to be used: 1 point

$$2.0 = -3.0\cos(\phi_i) \rightarrow \cos\phi_i = -\frac{2}{3}$$

NOTE: not realizing $A\omega^2$ is equal to 4.0 cm/s^2, missing the negative: deduction of 0.25 marks each.

$\phi_i = +2.30$ rad or $\phi_i = -2.30$ rad (answer 3.98 rad is also correct)

NOTE: Two possible solutions **HAVE TO** be considered. If only one is written without any justification, student is awarded 2/4 marks

As there are two solutions, the change of acceleration $\frac{da(t)}{dt}$ determines the correct one

$$\frac{da(t)}{dt}|_{t=0s} = -A\omega^3(-\sin\phi_i) = A\omega^3\sin\phi_i > 0 \therefore \sin\phi_i > 0 \rightarrow \ \phi_i = 2.30\ \mathrm{rad}$$

NOTE: Any check is fine (acceleration at t, sign of velocity). If the check is completed, but a wrong conclusion is drawn, student is awarded 3/4 marks.

Check with correct conclusion but wrong math/missing units: 3.5/4

Mass $m = 0.215$ kg is attached to a spring of spring constant k. The mass is oscillating, according to $x(t) = A_0 \cos(\omega t + \phi_i)$, initially without any damping. The figure below shows the **acceleration** of the mass as a function of time.

c. After the first 6.00 seconds the damping is introduced. Five full oscillations after that time the amplitude decreases by 6.25%.
Determine the damping coefficient. [2 points]

$$A(t) = A_0 e^{-\frac{\gamma}{2}t}$$

$$\frac{A}{A_0} = (1 - 0.0625) = e^{-\frac{\gamma}{2}5T} = e^{-\frac{\gamma}{2}*3.75}$$

$$\ln(0.9375) = -\frac{3.75}{2} * \gamma$$

$$\gamma = \frac{\ln(0.9375)}{-1.875} = 0.00344\frac{1}{\mathrm{s}}$$

NOTE: Marks are awarded for correct equation (0.5), correct time (0.5), correct math procedure (0.5) and providing the answer with units (0.5)

If all numbers are plugged in but the answer is incorrect (if it is positive), award full marks; it is clearly a calculation mistake and does not take away from the process.

d. Write the expression for the **position** of the mass as a function of time after the damping has been introduced (so starting at $t = 6.00\ \mathrm{s}$). Fill in all variables you can. [3 points]

As the oscillation is lightly damped (oscillation occurs while amplitude/energy decrease exponentially)

$$A(t) = A_0 e^{-\frac{\gamma t}{2}} \cos(\mathrm{\omega t} + \phi_i)$$

NOTE: 0.5 marks for realizing the type of motion AND including the equation for position NOT amplitude only.

$$A_0 = \frac{a_{max}}{\omega_o^2} = \frac{3.00}{8.38^2} = 0.0427\ \mathrm{cm}$$

NOTE: 0.5 marks; amplitude of 4.0 cm and 3.5 cm are incorrect (0 points); amplitude of 3.5 cm/ω_0^2 is worth 0.25 of a mark as the student shows awareness

$$\omega \approx \omega_0 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = 8.38\frac{\mathrm{rad}}{\mathrm{s}}$$

NOTE: 0.5 marks for checking While it is absolutely ok to use ω_0 as the frequency of the oscillation, there has to be some check that $\omega \approx \omega_0$ (even if it is of the "after 5 oscillations only 6.25% of energy is dissipated therefore Q is large enough to assume $\omega = \omega_0$").

$$\phi_i = 2.30 \text{ rad (recognize that at t} = 6.0 \text{ s the mass is back where it started)}$$

NOTE: 0.5 marks for realizing and selecting the value obtained in part b) (even if it was incorrect)

The final answer does not need to have all units. The answer is worth 1 mark, with 0.5 going into realizing γ from part c) needs to be used (halved!)

$$x(t) = (0.0427cm)e^{-\left(0.00860\frac{\text{rad}}{\text{s}}\right)t}\cos\left(\left(8.38\frac{\text{rad}}{\text{s}}\right)t + 2.30 \text{ rad}\right)$$