UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

Final Exam, December 15, 2023

DURATION: 2.5 hours

Second Year – Engineering Science

CHE260H1 – Thermodynamics and Heat Transfer

Calculator Type: 1 (Any, non-communicating)

Exam Type: A (Closed Book)

Examiner: A. W. H. Chan

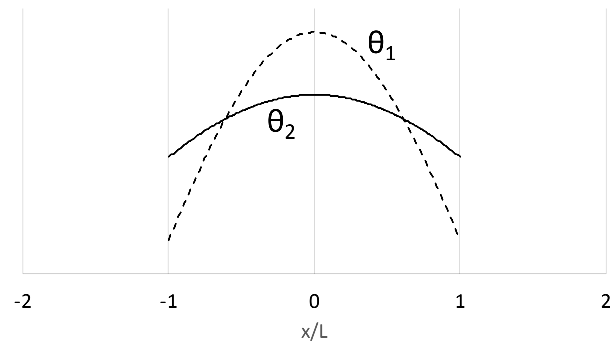
Instructions:

- For multiple choice questions, no work is needed or graded. No partial marks.
- For long form questions, work must be shown. Marks will be deducted if reasoning is unclear, even if the final answer is correct.
- Write legibly. Illegible handwriting will not be accepted.
- Number of significant figures should correspond to information provided in the question.
- If iterative calculations are needed, make a physically justifiable assumption and go through the calculations for 1 iteration. Then demonstrate how you would carry out the subsequent iterations to refine your answer without further calculations.

Question 1. Multiple choice questions (12 marks)

Circle the correct answer only. Circling multiple answers will automatically be graded as zero. (2 marks and no partial marks for each part)

A.



The above graph depicts two temperature profiles of 1D transient heat conduction of a wall. Which temperature profile (θ) is associated with a **higher** Bi? ($\theta = \frac{T-T_\infty}{T_i-T_\infty}$)

I. $\boldsymbol{\theta_1}$

II. θ_2

III. They are associated with the same Bi.

IV. There is not enough information.

Higher Bi means greater spatial differences in temperature. $\boldsymbol{\theta_1}$ shows greater temperature differences within the wall.

B. A hot object is placed in a cold water bath. The temperature at the center of the object will be close to the temperature at the surface when the material has a ______________________.

I. low thermal conductivity

II. high density

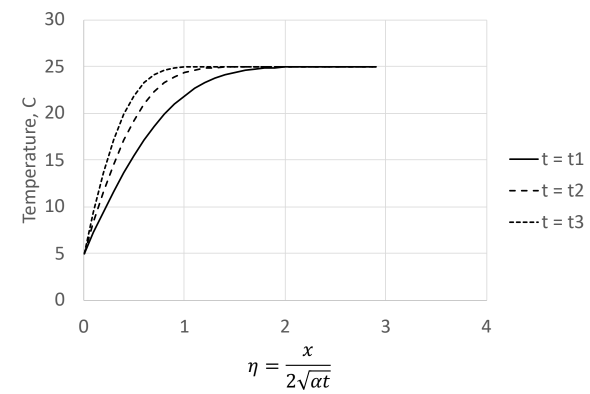
III. low heat capacity

IV. large surface area to volume ratio

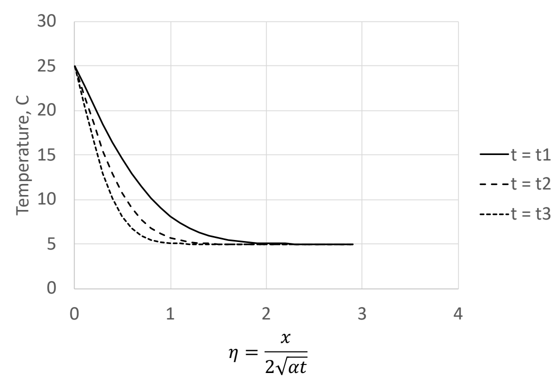
Large S/A is associated with lower Bi, and smaller temperature differences within the material

C. A semi-infinite solid is initially at 25C. At t = 0, the temperature at the surface (x=0) is immediately cooled and maintained at 5C. Which one of the following graphs would represent the plot of temperature (T) vs. $\eta = \frac{x}{2\sqrt{\alpha t}}$? Note that t1<t2<t3.

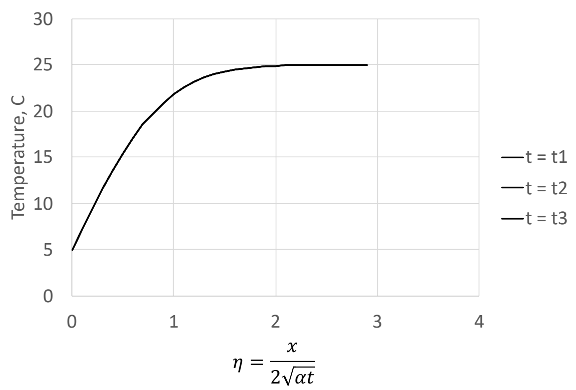
I.



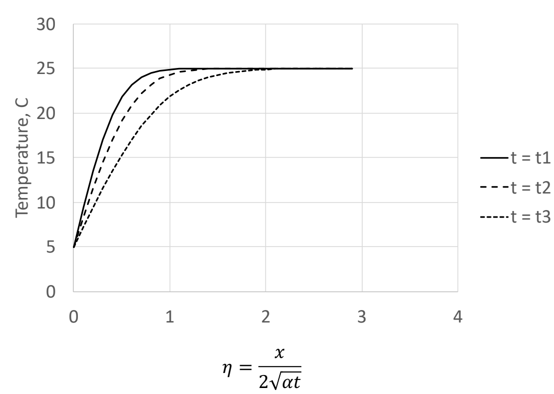
II.



III. (all the lines are on top of each other)



IV.



The temperature profile as a function of $\eta = \frac{x}{2\sqrt{\alpha t}}$ is simply the error function with no dependence on time. The time dependence is embodied within η.

D. For **laminar** flow over a flat plate, how does the local heat transfer coefficient (h_x) depend on distance from the leading edge of the plate (x)? Why?

I. h_x increases with x because the thermal boundary layer is growing.

II. h_x decreases with x because the thermal boundary layer is growing.

III. h_x increases with x because the thermal boundary layer is shrinking.

IV. h_x decreases with x because the thermal boundary layer is shrinking.

V. h_x increases with x because of increasing turbulence.

IV. h_x stays constant over the plate when flow is laminar.

The thermal boundary layer is growing with x, and the temperature gradient is smaller, resulting in a smaller local heat transfer coefficient.

E. For cold liquid metal (Pr < 0.1) flowing over a hot flat plate in laminar flow, the thickness of the thermal boundary layer is ____________ that of the momentum boundary layer.

I. greater than

II. less than

III. the same as

IV. not related to

Pr describes the thickness of the momentum boundary layer to that of the thermal boundary. Low Pr means the thermal boundary layer is thicker than the momentum boundary (or thermal diffusivity > momentum diffusivity)

F. Which of the following will improve effectiveness of heat fins?

I. increase the ratio of the cross-sectional area to the perimeter

II. use a longer fin

III. use a less conductive material

IV. change the air flow around the heat fin from turbulent to laminar

$\epsilon = \sqrt{\frac{kp}{hA_c}}$**: increases by decreasing h, increasing k, or increasing p/Ac. Changing the air flow from turbulent to laminar will decrease h, and hence increase effectiveness.**

Question 2. (8 marks)

It is holiday season! You will be making baked pork chop in the oven. Meat has thermal conductivity of 0.44 W/m-K and a thermal diffusivity of 1.32×10^{-7}m/s. The oven is a convection oven with a convective heat transfer coefficient of 125 W/m^2-K. You will set the oven temperature for 204C (about 400F). You can assume there is no radiation, and heat transfer is 1 dimensional.

You put a 5.6-cm thick pork chop (initially at room temperature of 25C) in the oven and set a timer. You want to make sure the pork chop is fully cooked (so your guest do not get sick), and meat is considered raw if it is below 62C.

A. How long will it take to cook the pork chop in the oven, in minutes? (3 marks)

Calculate Bi (0.5 mark):

$$Bi = \frac{h(L/2)}{k} = \frac{(125)(0.028)}{0.44} = 7.95$$

Use Table 11-2 to find A_1, λ_1 (0.5 mark):

$$A_1 = 1.2570, \lambda_1 = 1.3978$$

Find centerline temperature (0.5 mark for recognizing that centerline temperature is the lowest and it needs to exceed 62C):

$$\theta_0 = \frac{62 - 204}{25 - 204} = 0.7933$$

Use one-term approximation (0.5 mark):

$$0.7933 = 1.2570 e^{(-1.3978)^2 Fo}$$

$$Fo = 0.2356$$

Confirm Fo>0.2 for one-term approximation (0.5 mark)

Find time (0.5 mark):

$$t = \frac{Fo\left(\frac{L}{2}\right)^2}{\alpha} = 1400s = 23min$$

B. The pork chop will start to char at a temperature of 150C. Your guests do not like burnt pork. Will the pork chop char when it is fully cooked? (2 marks)

$\frac{\theta_s}{\theta_0} = \cos(\lambda_1) = \cos(1.3978) = 0.172$ (1 mark)

$$\frac{T_s - 204}{62 - 204} = 0.172$$

$T_s = 180$ (1 mark)

→ surface temperature exceeds 150C.

C. Assume in part B you found that your pork chop would indeed char. What can you change practically to avoid charring while still making sure that the pork chop is fully cooked? Explain qualitatively using the relevant equations. (3 marks)

1 mark for examining Bi; as it is an indication of temperature gradient within the conducting material.

1 mark for recognizing that one needs to reduce Bi to avoid large temperature difference between the centerline (lowest temperature) and the surface (highest temperature)

1 mark for proposing any practical measure to reduce Bi, which may include:

- Increase k
- Decrease h (e.g. reduce convection)
- Decrease thickness of the pork
- Lower T_∞

Question 3. (6 marks)

Two semi-infinite solids A and B, initially at uniform temperature ($T_{A,i}$ and $T_{B,i}$), are brought into contact. They instantly achieve equal temperature at the contact surface (assuming that contact resistance is negligible). Show that the temperature at the contact surface is:

$$T_s = \frac{\sqrt{k_A \rho_A c_{pA}} T_{A,i} + \sqrt{k_B \rho_B c_{pB}} T_{B,i}}{\sqrt{k_A \rho_A c_{pA}} + \sqrt{k_B \rho_B c_{pB}}}$$

For A: k_A is thermal conductivity, ρ_A is density c_{pA} is heat capacity.

For B: k_B is thermal conductivity, ρ_B is density c_{pB} is heat capacity.

At the interface the heat flux out of A is the same as the heat flux into B (2 marks), i.e.

$$q''_{AS} = q''_{BS}$$

Apply correct surface heat flux formula for infinite solids (2 mark):

$$-\frac{k_A(T_S - T_{A,i})}{\sqrt{\pi \alpha_A t}} = -\frac{k_B(T_{B,i} - T_A)}{\sqrt{\pi \alpha_B t}}$$

Rearranging (1 mark for partly correct, 2 marks for fully correct):

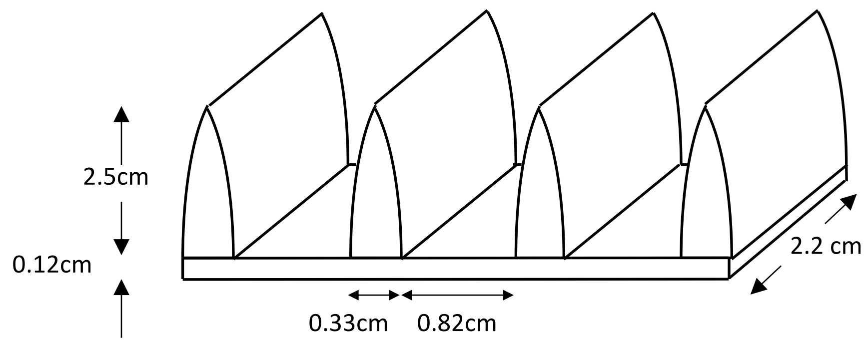
$$-\sqrt{k_A \rho_A c_{pA}}(T_S - T_{A,i}) = \sqrt{k_B \rho_B c_{pB}}(T_S - T_{B,i})$$

$$\sqrt{k_A \rho_A c_{pA}} T_{A,i} + \sqrt{k_B \rho_B c_{pB}} T_{B,i} = \sqrt{k_A \rho_A c_{pA}} T_S + \sqrt{k_B \rho_B c_{pB}} T_S$$

$$T_s = \frac{\sqrt{k_A \rho_A c_{pA}} T_{A,i} + \sqrt{k_B \rho_B c_{pB}} T_{B,i}}{\sqrt{k_A \rho_A c_{pA}} + \sqrt{k_B \rho_B c_{pB}}}$$

Question 4. (6 marks)

You need to cool a CPU chip to prevent it from overheating (chip will fail if the temperature exceeds 55C). You found this heat sink at an electronics shop:



The 4 fins are parabolic, and are made of aluminum (thermal conductivity of 237 W/m-K). The dimensions of the fins and geometry are shown in the diagram (not drawn to scale).

The base of the heat sink is a 1.2mm thick layer of epoxy (thermal conductivity of 1.20 W/m-K). The chip is 2.2 cm x 3.78 cm (same as the base of heat sink) and dissipates 3.5 W. The air temperature is 25C. The convective heat transfer coefficient is 55 W/m^2-K.

A. Will the chip fail if the heat sink was not installed? (1 mark)

$$\dot{Q}_{nofin} = hA_C(T_b - T_\infty)$$

$$3.5W = (55)(0.022)(0.0378)(T_b - 25)$$

$$T_b = 101C$$

Yes the chip will fail. (1 mark, no part marks)

B. What is the surface temperature of the chip if the heat sink is installed on the chip? (5 marks)

Use table to find A_{fins} and η:

$$m = \sqrt{2h/kt} = 11.859$$

$$C_1 = \sqrt{1 + \left(\frac{t}{L}\right)^2} = 1.00867$$

$A_{1\,fin} = 0.001104$ m2 per fin, or $A_{fins} = 0.004413$ m2 (1 mark)

$\eta = 0.9248$ (1 mark)

Calculate overall resistance (1 mark, 0.5 for each term):

$$R_{total} = \frac{L_{epoxy}}{k_{epoxy}A_{base}} + \frac{1}{h\left(A_{nofins} + \eta A_{fins}\right)}$$

Steps (1 marks for all the correct steps, subtract 0.5 for any minor mistakes):

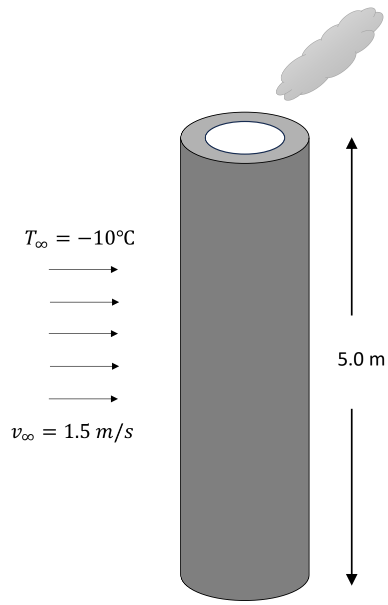
- $A_{no\,fins} = 3(0.0082)(0.022) = 5.41 \times 10^{-4} m^2$
- $\frac{L_{epoxy}}{k_{epoxy}A_{base}} = \frac{0.0012}{(1.2)(0.022)(0.0378)} = 1.203K/W$
- $\frac{1}{h(A_{nofins}+\eta A_{fins})} = \frac{1}{(55)[5.41\times10^{-4}+(0.9248)(0.004413)]} = \frac{1}{(55)(0.00046)} = 3.94K/W$

1 mark for correct final answer (subtract 0.5 for incorrect number of significant figures)

$\dot{Q} = \frac{T_b - T_\infty}{R_{total}} = \frac{T_b - 25C}{1.2+3.9} = 3.5W \rightarrow T_b = 43C$

Question 5. (8 marks)

On a cold day, you use your fireplace to enjoy a warm home. The smoke from the fireplace exits your home through a 5.0 m long chimney. The chimney, made out of clay bricks (k = 1.0 W/m-K), has an inner **diameter** of 0.74 m and outer **diameter** of 1.54 m. On this cold day, the air temperature is -10 C, and the wind speed across the chimney is 1.5 m/s.



A. Calculate the outer surface temperature of the chimney by assuming that the inner surface of the chimney is constant at 65C. (Reminder: read the note in the exam instructions about iterative calculations.) (6 marks)

Assume a reasonable film temperature (anything between -10 and 65C) and evaluate thermal properties (**1 mark**):

Assume $T_f = 25C$: (see Table below for different assumed Tf)

$k = 0.02551 W/mK$, $\alpha = 2.14 \times 10^{-5} m^2/s$, $\nu = 1.56 \times 10^{-5} m^2/s$, Pr = 0.7296

Calculate Re using outer diameter (**1 mark**)

$Re_D = \frac{v_\infty D}{\nu} = \frac{(1.5)(1.54)}{1.56 \times 10^{-5}} =$ 148000 (or tabulated values below)

Calculate Nu and h using equation provided (**1 mark**):

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + (0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \quad \textbf{(19-28)}$$

Nu = 285

$h = Nu\frac{k}{D} = 4.7\ W/m^2K$ (or tabulated values below)

Calculate individual and overall resistances (**1 mark**):

$R_{conv} = \frac{1}{h(\pi D_{outer}L)} = \frac{1}{(4.7)(\pi)(1.54)(5)} = 0.0087K/W$ (or tabulated values)

$$R_{cond} = \frac{\ln\left(\frac{r_{outer}}{r_{inner}}\right)}{2\pi k_{concrete}L} = 0.0233K/W$$

$R_{total} = 0.032K/W$

Calculate heat transfer rate and outer surface temperature (**1 mark**):

$$\dot{Q} = \frac{T_{inner} - T_{\infty}}{R_{total}} = 2336W$$

$$T_{outer} = T_{\infty} + \dot{Q}R_{conv} = 10C$$

Verify film temperature (1 mark):

$$T_f = \frac{10 + (-10)}{2} = 0C$$

Reevaluate thermal properties at 0C and repeat calculation.

Assumed film temperature	k, W/mK	alpha, m2/s	nu, m2/s	Pr	Re	A	B	Nu	h, W/m2K	R_conv, K/	R_cond, K/W	R_total, K/W	Q, W	T_outer, C	T_film, C
-10	0.02288	1.696E-05	1.252E-05	0.73820755	184505	2.119E+02	1.767E+00	334.44	4.97	0.00832	0.0233	0.0316	2370	9.7	-0.1
0	0.02364	1.818E-05	1.338E-05	0.7359736	172646	2.047E+02	1.736E+00	318.57	4.89	0.00845	0.0233	0.0318	2360	9.9	0.0
5	0.02401	1.880E-05	1.382E-05	0.73510638	167149	2.013E+02	1.721E+00	311.18	4.85	0.00852	0.0233	0.0318	2355	10.1	0.0
10	0.02439	1.944E-05	1.426E-05	0.73353909	161992	1.980E+02	1.707E+00	304.10	4.82	0.00858	0.0233	0.0319	2350	10.2	0.1
15	0.02476	2.009E-05	1.470E-05	0.73170732	157143	1.949E+02	1.694E+00	297.35	4.78	0.00865	0.0233	0.0320	2346	10.3	0.1
20	0.02514	2.074E-05	1.516E-05	0.73095468	152375	1.918E+02	1.681E+00	290.86	4.75	0.00871	0.0233	0.0320	2341	10.4	0.2
25	0.02551	2.141E-05	1.562E-05	0.72956562	147887	1.888E+02	1.668E+00	284.62	4.71	0.00877	0.0233	0.0321	2337	10.5	0.2
30	0.02588	2.208E-05	1.608E-05	0.72826087	143657	1.860E+02	1.656E+00	278.71	4.68	0.00883	0.0233	0.0322	2332	10.6	0.3
35	0.02625	2.277E-05	1.655E-05	0.72683355	139577	1.832E+02	1.644E+00	272.96	4.65	0.00888	0.0233	0.0322	2328	10.7	0.3
40	0.02662	2.346E-05	1.702E-05	0.7254902	135723	1.805E+02	1.633E+00	267.51	4.62	0.00894	0.0233	0.0323	2324	10.8	0.4
45	0.02699	2.416E-05	1.750E-05	0.72433775	132000	1.779E+02	1.622E+00	262.24	4.60	0.00899	0.0233	0.0323	2320	10.9	0.4
50	0.02735	2.487E-05	1.798E-05	0.72295939	128476	1.753E+02	1.612E+00	257.19	4.57	0.00905	0.0233	0.0324	2316	11.0	0.5
60	0.02808	2.632E-05	1.896E-05	0.72036474	121835	1.705E+02	1.592E+00	247.63	4.52	0.00916	0.0233	0.0325	2309	11.1	0.6
70	0.02881	2.780E-05	1.995E-05	0.7176259	115789	1.660E+02	1.573E+00	238.80	4.47	0.00925	0.0233	0.0326	2302	11.3	0.7

B. How much would be heat transfer rate change if the wind speed was zero? (2 marks)

Nusselt number is the ratio of convection to conduction in the fluid. If wind speed is zero, there is no advection, and heat transfer would be through conduction only. (1 mark)

Nu would be approximately 1 → R_{conv} would increase by a factor of about 300 to approximately 2.68K/W, and $R_{total} \approx R_{conv}$, which is about 100 times higher than in part A. As a result, the heat transfer rate would decrease by about a factor of 100. (1 mark)

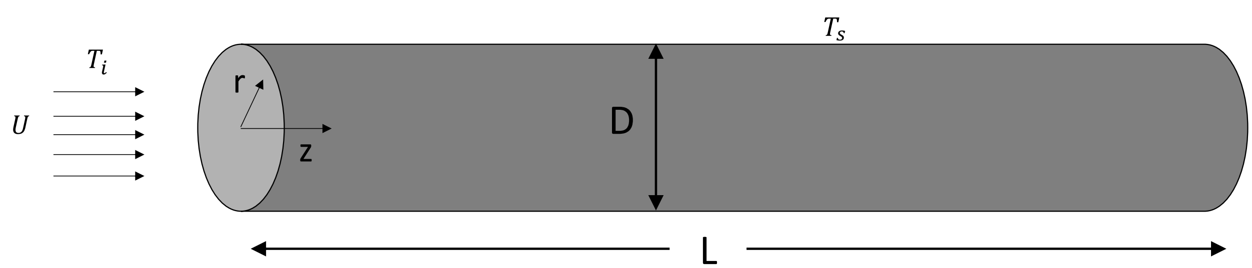
Question 6. (8 marks)

The heat balance in cylindrical coordinates is as follows:

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right]$$

In cylindrical coordinates, r represents the radial coordinate (distance from the center-line) and z represents the axial coordinate (distance along the center-line). (Minor note: we have removed some terms here by assuming that heat transfer is the same in all directions from the centerline). v_r is the fluid velocity in the r direction, and v_z is the fluid velocity in the z-direction.

Here we will examine heat transfer in a pipe. In this problem, the fluid is flowing through a pipe, with **inner** diameter D, and length L. The average linear velocity in the z-direction is U. The inlet temperature of the fluid is T_i and the pipe wall temperature is constant at T_s.



A. We will make two additional assumptions:
    - Steady state
    - $v_r \ll v_z$

    Show how these two assumptions simplify the heat balance equation. (1 mark)

Equation simplifies to:

$$\rho c_p \left(v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right]$$

(1 mark, no part marks)

B. For this heat transfer problem (flow through circular pipe), determine appropriate scales for variables in the heat balance equation. Create dimensionless variables. (2 marks)

$v_z \sim U$ $\rightarrow v_z^* = \frac{v_z}{U}$

$(T - T_i) \sim (T_s - T_i)$ $\rightarrow \theta^* = \frac{(T-T_i)}{(T_s-T_i)}$

$r \sim D \text{ or } R = \frac{D}{2}$ $\rightarrow r^* = \frac{r}{D}$

$z \sim L$ $\rightarrow z^* = \frac{z}{L}$

(0.5 marks each)

C. Render the heat equation dimensionless and show the important dimensionless numbers for this problem. (3 marks)

1.5 marks (0.5 for starting to plug in dimensionless variables, 1 mark if mostly there, full 1.5 if the form of the equation yields Re and Pr)

$$\frac{\rho c_p U}{L}\left(v_z^* \frac{\partial \theta^*}{\partial z^*}\right) = k\left[\frac{1}{D^2}\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial \theta^*}{\partial r^*}\right) + \frac{1}{L^2}\frac{\partial^2 \theta^*}{\partial z^{*2}}\right]$$

$$\frac{\rho c_p U D}{k}\left(v_z^* \frac{\partial \theta^*}{\partial z^*}\right) = \left(\frac{L}{D}\right)\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial \theta^*}{\partial r^*}\right) + \left(\frac{D}{L}\right)\frac{\partial^2 \theta^*}{\partial z^{*2}}$$

$$\left(\frac{\mu c_p}{k}\right)\left(\frac{\rho U D}{\mu}\right)\left(v_z^* \frac{\partial \theta^*}{\partial z^*}\right) = \left(\frac{L}{D}\right)\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial \theta^*}{\partial r^*}\right) + \left(\frac{D}{L}\right)\frac{\partial^2 \theta^*}{\partial z^{*2}}$$

Important dimensionless numbers (0.5 marks each):

1. $Pr = \frac{\mu c_p}{k}$
2. $Re_D = \frac{\rho U D}{\mu}$
3. $\frac{L}{D}$

D. Explain the physical meaning of each dimensionless number. (2 marks)

$Pr = \frac{\mu c_p}{k}$ ratio of momentum diffusivity to thermal diffusivity (0.5 mark)

$Re_D = \frac{\rho U D}{\mu}$ ratio of inertial stress to viscous stress (0.5 mark)

$\frac{L}{D}$: aspect ratio (ratio of axial length to radial length) (1 mark)

Extra pages:

**if you write here, it will be very helpful to make a note under the original question and refer to these pages

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