



**ESC195 - Calculus II**  
**Final Exam April 2023**

**Instructor: J. W. Davis**

First name (please write as legibly as possible within the boxes)

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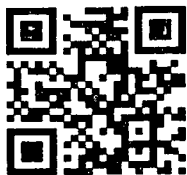
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**Closed book, no aid sheets, no calculators**  
**There are 12 questions; each question is worth 10 marks.**



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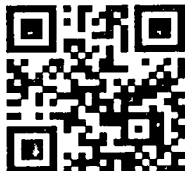


1. Evaluate the integrals:

a)  $\int \frac{\ln x}{x^2} dx$

b)  $\int \frac{x^3}{\sqrt{9+x^2}} dx$

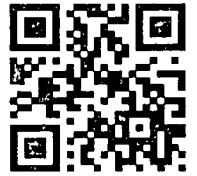
c)  $\int \frac{x}{(x+4)(2x-1)} dx$



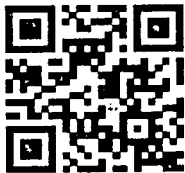
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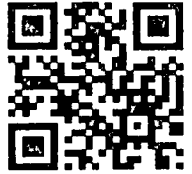
2. Sketch the curves and regions indicated and find an integral(s) representing the area of the region. Do not evaluate the integrals.
- (a) Inside one loop of the curve:  $r = 4 \cos 3\theta$ .
  - (b) Inside both  $r^2 = \sin 2\theta$  and  $r^2 = \cos 2\theta$ .
  - (c) Inside the cardioid  $r = 1 + \cos \theta$ , but outside the circle  $r = 3 \cos \theta$



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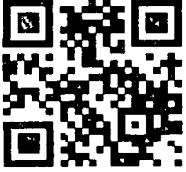


3. Evaluate each limit in two ways: (i) by l'Hospital's rule, (ii) by expanding the functions as Taylor series.

a)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

b)  $\lim_{x \rightarrow 0} \frac{2 \cos 2x - 2 + 4x^2}{2x^4}$

c)  $\lim_{x \rightarrow 0} \frac{e^{-2x} - 4e^{-x/2} + 3}{2x^2}$

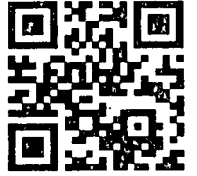


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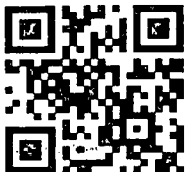


4. (a) Determine whether the series is convergent or divergent:

i)  $\sum_{k=1}^{\infty} k e^{-k^2}$

ii)  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

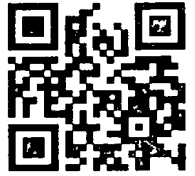
- (b) Suppose  $a_n > 0$  and  $\frac{a_{n+1}}{a_n} \geq \frac{n}{n+1}$  for all  $n$ . Show that  $\sum_{n=1}^{\infty} a_n$  diverges.



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5. Given the series:  $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

- (a) Find the radius and interval of convergence.
- (b) Show that the series is not convergent (conditionally or otherwise) for  $x < -2$ .



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6. Determine from first principles (that is by taking derivatives) the Taylor series for the function  $f(x) = \frac{1}{x^2}$  about  $x = 1$ . Express the series in  $\sum$  notation, and find the radius and interval of convergence.



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7. (a) Find an equation for the tangent plane to the surface  $xy^2 + 2z^2 = 12$  at the point  $(1, 2, 2)$ .

(b) The curve  $\vec{r} = \frac{1}{2}t^2\hat{i} + \frac{4}{t}\hat{j} + (\frac{1}{2}t - t^2)\hat{k}$  intersects the hyperbolic paraboloid  $x^2 - 4y^2 - 4z = 0$  at the point  $(2, 2, -3)$ . Find the angle of intersection.

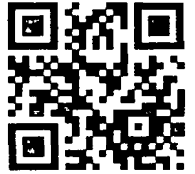


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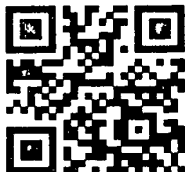
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8. Use the formal definition for the derivative of a multivariable function (the  $o(h)$  formulation) to find the gradient of:  $f(x, y, z) = xy + yz$ . Show that all remainder terms are  $o(h)$ .



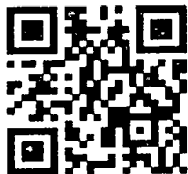
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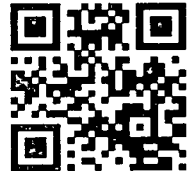
9. Find the maximum and minimum values of  $f(x, y) = \frac{x - y}{1 + x^2 + y^2}$  on the the upper half plane,  $y \geq 0$ . Provide a sketch of the region, and identify and show the locations of all critical points.



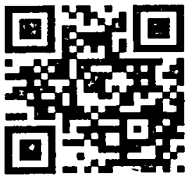
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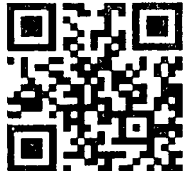
10. The plane  $x + y + 2z = 2$  intersects the paraboloid  $z = x^2 + y^2$  in an ellipse. Use Lagrange Multipliers to find the the points on this ellipse that are nearest and furthest from the origin.



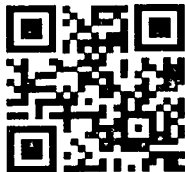
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11. Given  $\int_0^{2\pi} \frac{\cos \theta}{1 - y \cos \theta} d\theta = 2\pi \frac{1 - \sqrt{1 - y^2}}{y\sqrt{1 - y^2}}$ , where  $0 < y < 1$ , find  $\int_0^{2\pi} \ln(1 - y \cos \theta) d\theta$

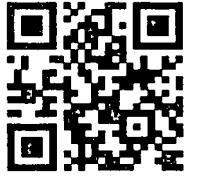


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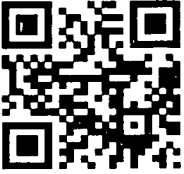
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12. Show that  $\int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} - \frac{1}{6^6} + \dots$ .



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