

## MAT 194 MOCK MIDTERM II

*Prepared by David Simons and Marc Goudge (NΨ 1T6) for Peer Assisted Study Sessions.  
We imagine this will be somewhat representative of your midterm, but make no guarantees!*

1. Find the following antiderivatives:

- a)  $3x^4$       b)  $\cos(5x)$       c)  $3/x$       d)  $3x \exp(x^2/3)$       e)  $x(2x+5)^8$

2. a) Find  $dy/dx$  for  $y(x) = \int_{2x-1}^{3x^2} \sin(t^4) dt$

b) Given that  $f$  is continuous and  $C$  is a constant, find  $f(x)$  and  $C$  satisfying:

$$\int_0^x f(t) dt = \cos(x) + C + \int_0^x \frac{f(t)}{1+t} dt \quad \text{for all } x$$

3. Find the area bounded by the following three curves:

- the  $x$ -axis
- the line  $x=e$
- the curve implicitly defined by  $y = \begin{cases} 0, & x = 0 \\ \sqrt{\ln(\frac{x}{y})}, & x > 0 \end{cases}$

4. Find the volume formed by rotating the area bounded by the following three curves **around the line  $y = -2$** :

- the  $x$ -axis
- the line  $x = 1$
- the curve  $y = x^2$

a) Use the washer method

b) Use the shell method

5. a) A 1600-lbm elevator is suspended by a 200-ft cable that weighs 10 lbm/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft? (Note: *lbm* means “pound-mass”, equivalent to approximately 0.454 kg, whereas *lbf* means “pound-force”, equivalent to approximately 4.448 N)

b) Find the average value of  $f(t) = t \sin(t^2)$  on the interval  $[0,10]$ .

6. a) Show that  $f(x) = \int_1^x \sqrt{1+t^3} dt$  is a bijection and find  $(f^{-1})'(0)$

b) If  $f$  is a twice differentiable bijection with inverse function  $g$ , show that  $g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$

7. Use Riemann sums to show that  $\int_0^1 x^2 dx = \frac{1}{3}$ . Hint:  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$

8. Sketch all points  $(x,y) \in \mathbb{R}$  such that  $|x+y| \leq e^x$