CHE 260: THERMODYNAMICS AND HEAT TRANSFER

FINAL EXAMINATION FOR HEAT TRANSFER

13th DECEMBER 2018

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3	Q4	Q5	Total
15	15	15	15	15	75

INSTRUCTIONS

1. This examination is closed textbook, closed internet, closed all communication devices.
2. One aid sheet of size 8.5” x 11” aid sheet (both sides) is permitted.
3. Any non-communicating, non-programming, non-graphing calculator is permissible.
4. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
5. Answers written in pencil will NOT be re-marked. This is University policy.
6. For all problems, you must present the solution process in a step by step fashion for partial marks.
7. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY.**

Q.1. [20 points] WHODUNNIT?

Mrs. X returns home after work. Hungry and tired, she rushes to the freezer to lay her hands on her favorite snack – batter-coated, deep-fried potato balls. To her horror, she finds that the freezer door is open. There are two suspects: her stay-at-home naughty cat, and her absent-minded husband, Prof. X. Being a thermal engineer, Mrs. X uses a thermocouple and measures the temperatures at the surface and the center of the potato ball to be -5.7ºC and -10.1ºC, respectively. She also measures the temperature of the air in the open freezer to be 2.0ºC, which is much greater than the originally set temperature of -18.0ºC.

Assuming that

a) the *potato ball* was in thermal equilibrium with the environment of the freezer prior to the freezer door being opened;
b) the air in the freezer was at 2ºC immediately after the freezer was kept open;
c) each *potato ball* is a sphere of diameter 5 cm;
d) the warming of the *potato ball* was radially symmetric; and
e) radiative heat transfer effects may be ignored,

answer the following questions to help Mrs. X figure out who opened the freezer door:

(1) **[12 points]** How long ago was the freezer door opened? Take the thermal conductivity and thermal diffusivity of the *potato ball* to be 0.5 W/mºC and 10^{-7} m^2/s, respectively. What is the Biot number, and the convective heat transfer coefficient?

Notes/hints:

(a) Do not assume the *potato ball* to have properties of water anywhere in this problem. Use only the values of properties provided in this problem.

(b) You will encounter a transcendental equation in this problem. The Newton-Raphson method fails miserably for that equation, so don't try it. You will find the adjacent table useful:

y	sin(y)/y
0.000	1
0.314	0.984
0.628	0.935
0.942	0.858
1.257	0.757
1.571	0.637
1.885	0.505
2.199	0.368
2.513	0.234
2.827	0.109
3.142	0

(2) **[2 points]** Mrs. X checks the home alarm logs and finds that Prof. X returned from work 15 min before her. Who opened the freezer door, Prof. X or the cat? Or is there insufficient information to make a deduction?

(3) **[3 points]** How much heat was received by the *potato ball* in the time calculated in part (1)?

(4) **[3 points]** After what *additional* time will the *potato ball* begin to unfreeze, assuming that the unfreezing starts when the local temperature in the *potato ball* exceeds 0ºC?

Solution:

Part (1)

There is a significant difference between the center and surface temperatures, i.e. a non-negligible temperature gradient within the vada. Hence we cannot use the lumped capacitance model.

[2 points for making this decision. Zero credit for opting the lumped capacitance model.]

We assume the 1 term approximation to be valid and proceed.

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 t^*\right) \frac{\sin\left(\lambda_1 x^*\right)}{\lambda_1 x^*}$$

Surface, center, initial and ambient temperatures are all provided.

As the center is approached, i.e. for $x^* \to 0$,

$$\theta_0 = A_1 \exp\left(-\lambda_1^2 t^*\right), \qquad A_1 \exp\left(-\lambda_1^2 t^*\right) = \frac{(-10.1-2)}{(-18-2)} = 0.605.$$

At the surface, i.e. for $x^* = 1$,

$$\theta_S = A_1 \exp\left(-\lambda_1^2 t^*\right)\frac{\sin\lambda_1}{\lambda_1}, \qquad A_1 \exp\left(-\lambda_1^2 t^*\right)\frac{\sin(\lambda_1)}{\lambda_1} = \frac{(-5.7-2)}{(-18-2)} = 0.385.$$

<u>Equations for dimensionless center and surface temperature: 2 points</u>

<u>Calculation of dimensionless center and surface temperature: 1 point</u>

The ratio of the above two equations give

$$\frac{\sin\lambda_1}{\lambda_1} = 0.636.$$

From the table provided in the problem, $\lambda_1 \approx 1.571$.

<u>Determining λ1: 2 points</u>

From table 11-2 in the appendix, for this value of $A_1 \approx 1.273$, and Bi = 1.0.

<u>Getting A1 and Bi: 1 point</u>

We can now use the center temperature equation to deduce t^*.

$$t^* = \frac{1}{\lambda_1^2}\ln\left(\frac{A_1}{\theta_0}\right) = \frac{1}{1.571^2}\ln\left(\frac{1.273}{0.605}\right) = 0.301.$$

<u>Getting t*: 1 point</u>

Since t^*>0.2, the one term approximation is valid.

<u>Showing validity of one term approx.: 1 point</u>

The time that has elapsed is

$$t = \frac{r_0^2}{\alpha}t^* = \frac{\left(2.5\times10^{-2}\right)^2}{10^{-7}}0.301 = 1881 \text{ seconds or } 31.3 \text{ min.}$$

<u>Calculation of actual time: 1 point</u>

The heat transfer coefficient is

$$h = \text{Bi}\frac{k}{r_0} = 1\times\frac{0.5}{2.5\times10^{-2}} = 20 \;\; \text{W/m}^2\cdot{}^\circ\text{C}.$$

<u>Calculation of h: 1 point</u>

Part (2)

Prof. X arrived 15 min before Mrs. X. The freezer was opened 31 min before her arrival. Since we have ignored the time required for the freezer to establish to a steady state with the room, the time we have calculated in likely an underestimate. Hence Prof. X is not the culprit. It is the cat.

2 points for the reaching the correct conclusion

Part (3)

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_0 \frac{(\sin\lambda_1 - \lambda_1\cos\lambda_1)}{\lambda_1^3}$$

where

$$Q_{\max} = \rho C V (T_\infty - T_i) = \frac{k}{\alpha}\left(\frac{4}{3}\pi r_0^3\right)(T_\infty - T_i) = \frac{0.5}{10^{-7}}\frac{4}{3}\pi\left(2.5\times10^{-2}\right)^3\left[2-(-18)\right] = 6.545 \text{ kJ}.$$

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_0 \frac{(\sin\lambda_1 - \lambda_1\cos\lambda_1)}{\lambda_1^3} = 1 - 3\times0.605\frac{(\sin1.571 - 1.571\cos1.571)}{1.571^3} = 0.532.$$

$$Q = 6.545\times0.532 = 3.482 \text{ kJ}.$$

1 point for working out that $\rho C = \frac{k}{\alpha}$

1 point for $Q_{\max}$

1 point for Q

Part (4)

The *potato ball* will begin to unfreeze when the surface temperature reaches 0ºC; this is the location from where the unfreezing process will commence.

$$\theta_S = \frac{(0-2)}{(-18-2)} = A_1\exp\left(-\lambda_1^2 t^*\right)\frac{\sin\lambda_1}{\lambda_1}, \qquad A_1\exp\left(-\lambda_1^2 t^*\right)\frac{\sin(\lambda_1)}{\lambda_1} = 0.1.$$

$$t^* = \frac{1}{\lambda_1^2}\ln\left(\frac{A_1}{\theta_S}\frac{\sin\lambda_1}{\lambda_1}\right) = \frac{1}{1.571^2}\ln\left(\frac{1.273}{0.1}\frac{\sin1.571}{1.571}\right) = 0.848.$$

$$t = \frac{r_0^2}{\alpha} t^* = \frac{\left(2.5\times10^{-2}\right)^2}{10^{-7}} 0.848 = 5300 \text{ seconds or } 88.3 \text{ min.}$$

It would take about an hour and a half, or about an *additional* hour of time.

1 point for discussing when the unfreezing process begins
Calculation of surface dimensionless surface temperature: 0.5 point
Calculation of surface dimensionless time: 1 point
Calculation of actual time: 0.5 point

Q.2. [20 points] FIN PROBLEM

A 0.2 cm thick, 15 cm high, and 15 cm long circuit board houses 60 closely-spaced logic chips on one side, each dissipating 0.05 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 30 W/m·°C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to a medium at 30°C, with a heat transfer coefficient of 20 $W/m^2 \cdot {}^\circ C$.

(a) Determine the temperatures on the two sides of the circuit board. Use the resistance network approach and sketch the circuit.

(b) Now, a 0.2 cm thick, 15 cm high and 15 cm long aluminium plate (k = 237 W/m-K) with 900, 2-cm-long aluminium pin fins of rectangular profile each of diameter 0.25 cm is attached to the back side of the circuit board with a 0.02 cm thick epoxy adhesive (k = 1.8 W/m-K). Determine the new temperatures on the two sides of the circuit board. Again, use the resistance network approach and sketch the circuit.

Solution:

Cross-sectional area across which heat transfer takes place, A, is 15 cm x 15 cm = 0.0225 m^2.

The total heat being generated by the chips is 0.05 x 60 =3.0 W.

We will also assume that there is no heat loss from the face of the circuit board not exposed to air.

Part 1: Circuit board without fin block

The resistance network with only the circuit board is shown below

2 points for the circuit

$$R_{\text{board}} = \frac{\Delta x_{\text{board}}}{k_{\text{board}} A} = \frac{0.2\times 10^{-2}}{30\times 0.0225} = 2.963\times 10^{-3} \quad {}^{\circ}\text{C/W.}$$

1 point for Rboard

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{20\times 0.0225} = 2.222 \quad {}^{\circ}\text{C/W.}$$

1 point for Rconv

Total resistance is

$$R_{\text{eff}} = R_{\text{board}} + R_{\text{conv}} = 2.225 \quad {}^{\circ}\text{C/W.}$$

1 point for Reff

The total rate of heat transfer is

$$\dot{Q} = \frac{(T_1 - T_\infty)}{\mathrm{R}_{\text{eff}}} \Rightarrow T_1 = T_\infty + \dot{Q}\,\mathrm{R}_{\text{eff}} = 30 + 3.0\times 1.273 = 36.67 \ {}^{\circ}\text{C.}$$

1 point for T1

The temperature on the other side of the board is

$$\dot{Q} = \frac{(T_1 - T_2)}{\mathrm{R}_{\text{board}}} \Rightarrow T_2 = T_1 - \dot{Q}\,\mathrm{R}_{\text{board}} = 36.67 - 3.0\times 2.963\times 10^{-3} = 36.66 \quad {}^{\circ}\text{C.}$$

1 point for T2

Part 2: Circuit board with block

Note that the convective resistance is much greater than the resistance of the board. Hence it makes sense to add a fin block.

The resistance circuit in this case is

2 points for the circuit

$$R_{\text{glue}} = \frac{\Delta x_{\text{glue}}}{k_{\text{glue}} A} = \frac{0.02 \times 10^{-2}}{1.8 \times 0.0225} = 4.938 \times 10^{-3} \quad ^{\circ}\text{C/W}.$$

<u>1 point for Rglue</u>

$$R_{\text{Al-base}} = \frac{\Delta x_{\text{Al-base}}}{k_{\text{Al-base}} A} = \frac{0.2 \times 10^{-2}}{237 \times 0.0225} = 3.751 \times 10^{-4} \quad ^{\circ}\text{C/W}.$$

<u>1 point for RAl-base</u>

$$R_{\text{conv}} = \frac{1}{h\left[A_{\text{unfin}} + nA_{\text{fin}}\eta_{\text{fin}}\right]}.$$

To calculate the individual fin related parameters, we refer to the table provided with the question book under 'Pin fin of rectangular profile'.

$$m = \left(\frac{4h}{kD}\right)^{1/2} = \left(\frac{4 \times 45}{237 \times 0.25 \times 10^{-2}}\right)^{1/2} = 11.62 \text{ m}^{-1}.$$

<u>1 point for m</u>

$$L_c = L + \frac{D}{4} = 0.02 + \frac{0.0025}{4} = 2.063 \times 10^{-2} \text{ m}.$$

<u>1 point for L_c</u>

$$A_{\text{fin}} = \pi D L_c = \pi 0.0025 \times \ 2.0625 \times 10^{-2} = 1.620 \times 10^{-4} \text{ m}^2.$$

<u>1 point for A_{fin}</u>

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c} = \frac{\tanh\left(11.620 \times 2.0625 \times 10^{-2}\right)}{11.620 \times 2.0625 \times 10^{-2}} = 0.98128.$$

<u>1 point for eta</u>$_{fin}$

$$A_{\text{unfin}} = A - n\frac{\pi D^2}{4} = 0.0225 - 900 \times \frac{\pi \times 0.0025^2}{4} = 0.01808 \text{ m}^2.$$

<u>1 point for A_{unfin}</u>

$$R_{\text{conv}} = \frac{1}{h\left(A_{\text{unfin}} + nA_{\text{fin}}\eta_{\text{fin}}\right)} = \frac{1}{20\left(0.01808 + 900 \times 1.620 \times 10^{-4} \times 0.9590\right)} = 0.3167 \ ^{\circ}\text{C/W}.$$

<u>1 point for Rfin</u>

$$R_{\text{eff}} = R_{\text{board}} + R_{\text{glue}} + R_{\text{Albase}} + R_{\text{conv}}$$
$$= 2.963 \times 10^{-3} + 4.938 \times 10^{-3} + 3.751 \times 10^{-4} + 0.3167$$
$$= 0.3250 \ ^{\circ}\text{C/W}.$$

<u>1 point for Reff</u>

The temperature on the left face of the circuit board is

$T_1 = T_\infty + \dot{Q}\,\mathrm{R}_{\mathrm{eff}} = 30 + 3.0 \times 0.3250 = 30.97 \;\; {}^\circ\mathrm{C}.$

<u>1 point for T1</u>

The temperature on the right face of the board is

$T_2 = T_1 - \dot{Q}\,\mathrm{R}_{\mathrm{board}} = 30.97 - 3.2 \times 2.963 \times 10^{-3} = 30.96 \;\; {}^\circ\mathrm{C}.$

<u>1 point for T2</u>

Q.3. [20 points] A SLAB WITH A HEAT SOURCE AND CONVECTION AT ITS SURFACES

Consider a plane wall of thickness 2*L*. A constant heat sink $\dot{S}_0$ is present in the wall. The two faces of the plane wall are exposed to fluids at the same ambient temperature T_∞. The convective heat transfer coefficient on both faces is *h*. In this problem, you will investigate the temperature distribution in the wall <u>at steady state</u>.

Answer the following questions:

(1) **[5 points]** Starting from the energy balance equation for heat conduction, write down the governing equation and the boundary conditions for this problem.

Since this is a plane wall, we can use the Cartesian co-ordinates

$$\rho C \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{S}$$

Here the x co-ordinate is measured from the center plane of the wall normal to the area of the wall to exploit the symmetry in the problem. We assume one dimensional heat conduction, and ignore variations in y and z.

At steady state, the governing equation for a heat source of $\dot{S} = -\dot{S}_0$ is

$$k\frac{d^2 T}{dx^2} = \dot{S}_0$$

The two boundary conditions are symmetry at the center plane,

$$\left.\frac{dT}{dx}\right|_{x=0} = 0,$$

and the convection bc at the wall

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left(T\big|_{x=0} - T_\infty\right).$$

(2) **[5 points]** Scale the variables in this problem and render the governing equations and boundary conditions dimensionless. What are the temperature and length scales? You should get exactly one dimensionless parameter. What is the physical interpretation of this dimensionless parameter?

$$x^* = \frac{x}{L}, \quad \theta = \frac{T - T_\infty}{\Delta T_c}$$

$$\frac{k\Delta T_c}{L^2}\frac{d^2\theta}{dx^{*2}} = \dot{S}_0$$

$$\frac{d^2\theta}{dx^{*2}} = \frac{\dot{S}_0 L^2}{k\Delta T_c}$$

The heat sink and conduction terms balance each other to produce th temperature gradient. In the absence of the heat sink, ,the solution would be trivial $(\theta = 0)$.

Hence we set $\frac{\dot{S}_0 L^2}{k\Delta T_c} = 1$ or $\Delta T_c = \frac{\dot{S}_0 L^2}{k}$.

The governing equation becomes

$$\frac{d^2\theta}{dx^{*2}} = 1.$$

The boundary conditions become

$$\frac{d\theta}{dx^*}\bigg|_{x^*=0} = 0, \text{ and}$$

$$-\frac{k\Delta T_c}{L}\frac{d\theta}{dx^*}\bigg|_{x^*=1} = h\Delta T_c\, \theta\big|_{x^*=1}.$$

or $-\frac{d\theta}{dx^*}\Big|_{x^*=1} = \mathrm{Bi}\ \theta\big|_{x^*=1}$,

where $\mathrm{Bi} = \frac{hL}{k}$.

The Biot number or Bi is thenumber that measures the importance of convection relative to conduction

(3) **[5 points]** Without solving the governing equations, explain what temperature distributions should be expected in this problem in the limits of small and large values of the dimensionless parameter? Provide scaling estimates of the temperature difference between the center and the wall i.e. $T\big|_{x=L} - T\big|_{x=0}$ in these two limits.

If Bi>>1, the boundary condition $-\left.\frac{d\theta}{dx^*}\right|_{x^*=1} = \text{Bi}\ \theta\big|_{x^*=1}$ tells us that $\theta\big|_{x^*=1} << 1$, suggesting that $T\big|_{x=L} \approx T_\infty$. The temperature scale was shown to be $\Delta T_c = \frac{\dot{S}_0 L^2}{k}$. The center temperature, therefore, is $T_\infty + c\frac{\dot{S}_0 L^2}{k}$, where c is an order 1 negative constant.

In the opposite limit, Bi<<1, the boundary condition tells us that $-\left.\frac{d\theta}{dx^*}\right|_{x^*=1} << 1$, suggesting that temperature variations in the solid are weak. But then theta would not be scaled properly, because it would not span values between 0 and order 1. Also, this leaves us with no way of determining the scale of the absolute temperature, because we do not know the temperature at any point in the solid.

We need, therefore, to rescale the temperature

$$\theta = \frac{T - T_1}{\Delta T_c}$$

where T_1 is the temperature at x=L, and is unknown.

The scale for the temperature change in the solid is still $\Delta T_c = \frac{\dot{S}_0 L^2}{k}$. But the boundary condition has to be reworked.

$$-\frac{k\Delta T_c}{L}\frac{d\theta}{dx^*}\bigg|_{x^*=1} = h\Delta T_c\,\theta\big|_{x^*=1} + h\left(T_1 - T_\infty\right).$$

$$-\frac{d\theta}{dx^*}\bigg|_{x^*=1} = \mathrm{Bi}\,\theta\big|_{x^*=1} + \mathrm{Bi}\frac{\left(T_1 - T_\infty\right)}{\Delta T_c}.$$

For Bi<<1,

$$-\frac{d\theta}{dx^*}\bigg|_{x^*=1} \approx \mathrm{Bi}\frac{\left(T_1 - T_\infty\right)}{\Delta T_c}.$$

Since T1 is an unknown scale, the equation of the orders of the two terms in the above equation gives

$$\mathrm{Bi}\frac{\left(T_1 - T_\infty\right)}{\Delta T_c} \sim 1$$

Since Bi<<1, $\left(T_1 - T_\infty\right) >> \Delta T_c$

The temperature change within the solid is much smaller than the temperature change going from the wall to the ambient.

We can rearrange the above equation to show that

$$T_1 = T_\infty + c\frac{\Delta T_c}{\mathrm{Bi}} = T_\infty + c\frac{\dot{S}_0 L^2}{k\frac{hL}{k}} = T_\infty + c_1\frac{\dot{S}_0 L}{h}.$$

The center temperature is $T_1 + c_2\frac{\dot{S}_0 L^2}{k} = T_\infty + c_1\frac{\dot{S}_0 L}{h} + c_2\frac{\dot{S}_0 L^2}{k}$.

Again c1 and c2 are order 1 negative constants.

(4) **[5 points]** Solve the governing equation along with boundary conditions to get the temperature distribution. Confirm the predictions in made in part (3).

Integrating the governing equation with the original scaling, we get

$$\frac{d\theta}{dx^*} = x^* + a.$$

Symmetry bc at the center gives a =0.

$$\frac{d\theta}{dx^*} = x^*.$$

Integrating again,

$$\theta = \frac{x^{*2}}{2} + b.$$

Applying the convection bc at x*=1, we get

$$-1 = \mathrm{Bi}\left(\frac{1}{2} + b\right).$$

and hence

$$b = -\left(\frac{1}{\mathrm{Bi}} + \frac{1}{2}\right).$$

The temperature distribution is

$$\theta = -\frac{1 - x^{*2}}{2} - \frac{1}{\mathrm{Bi}},$$

or in dimensional terms

$$\frac{T - T_\infty}{\frac{\dot{S}_0 L^2}{k}} = -\frac{1 - \frac{x^2}{L^2}}{2} - \frac{k}{hL},$$

which can be rearranged to

$$T = T_\infty - \frac{\dot{S}_0 L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) - \frac{\dot{S}_0 L}{h}.$$

The temperature at the surface is

$$T = T_\infty - \frac{\dot{S}_0 L}{h},$$

and the temperature at the center is

$$T = T_\infty - \frac{\dot{S}_0 L^2}{2k} - \frac{\dot{S}_0 L}{h}.$$

Q. 4. [20 points] HEAT LOSSES FROM A STEAM PIPE

A composite steam pipe comprised of an inner stainless steel tube and a rigid urethane foam insulation is used to transport high temperature steam at 150 ºC from one building to another. The thermal resistances of the convection on the steam side and that of conduction through the steel section of the composite are negligible. The stainless steel pipe has an outer diameter of 0.5 m. The pipe is insulated with a rigid urethane foam (k = 0.026 W/mºC) of 10 cm thickness and is exposed to ambient air at -10ºC. The air moves in cross-flow fashion over the pipe with a velocity of 3 m/s. What is the heat loss per unit length, and what is the temperature at the outer surface of the insulation? Calculate the convective heat transfer coefficient using an appropriate correlation. Also, include radiative heat losses by accounting for a surrounding surface at a temperature of 250 K. The emissivity of the surface of the insulation is 0.95.

<u>Use the following properties of air: k = 0.02881 W/m-K, ρ = 1.028 kg/m^3, μ = 2.052 × 10^{-5} kg/m-s, α = 2.780 x 10^{-5} m^2/s. Ignore the film temperature considerations in the correlation.</u>

The Stefan Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4. **Note**: The Newton-Raphson iterative formula for finding the root x^* of a function $f\ (x)$, such that $f\ (x^*) = 0$, is $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$.

Solution:

Diameter of insulated object $D = 0.5 + 2\times10\times10^{-2} = 0.7$ m.

<u>Diameter of insulated object: 1 point</u>

$$\nu = \frac{2.052\times10^{-5}}{1.028} = 1.996\times10^{-5}\ \text{m}^2/\text{s}.$$

<u>Calculation of nu: 1 point</u>

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{3 \times 0.7}{1.996 \times 10^{-5}} = 1.052 \times 10^{5},$$

<u>Calculation of Re: 1 point</u>

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{1.996 \times 10^{-5}}{2.780 \times 10^{-5}} = 0.7180.$$

<u>Calculation of Pr: 1 point</u>

Re Pr > 0.2, so the following correlation can be used.

$$\mathrm{Nu}_{\mathrm{cyl}} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1 + \left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\mathrm{Re}}{282,000}\right)^{5/8}\right]^{4/5} = 223.8.$$

<u>Selecting correlation: 1 point</u>
<u>Justifying that correlation can be used (Re Pr>0.2): 1 point</u>
<u>Calculation of Nu: 1 point</u>

Thus, $h = \frac{k}{D}\mathrm{Nu}_{\mathrm{cyl}} = 9.211\ \mathrm{W/m^2\,{}^{o}C}$.

<u>Calculation of *h*: 1 point</u>

Rate of heat transfer

$$\frac{\dot{Q}}{L} = \frac{T_1 - T_S}{L\,\mathrm{R_{ins}}} = h\left(2\pi r_2\right)\left(T_S - T_\infty\right) + \varepsilon\sigma\left(2\pi r_2\right)\left(T_S^4 - T_{surr}^4\right)$$

<u>The overall energy balance equation: 5 points (Partial marks of 3 points to be provided if resistance circuit is provided but if equation is wrong)</u>

Here

$$R_{\mathrm{ins}} L = \frac{1}{2\pi k_{\mathrm{ins}}} \ln\left(\frac{r_2}{r_1}\right) = 2.060\ \frac{{}^{o}\mathrm{C\text{-}m}}{\mathrm{W}}.$$

<u>Calculation of insulation resistance : 1 point</u>

Assuming the temperature of the outer surface of the insulation to be x°C,

$$\frac{150 - x}{2.060} = 9.211\left(\pi \times 0.7\right)\left(x + 10\right) + 0.95 \times \sigma\left(\pi \times 0.7\right)\left(\left(x + 273\right)^4 - 250^4\right)$$

$$1.185 \times 10^{-7}\left(x + 273\right)^4 + 20.74x - 333.0 = 0$$

$$f(x) = 1.185\times10^{-7}(x+273)^4 + 20.74x - 59.78$$

$$f'(x) = 1.185\times10^{-7}\times(x+273)^3 + 20.74$$

Use Newton method

The temperature of the surface is x = -10.90ºC.

<u>Newton's method calculation: 4 points</u>

<u>Correct temperature: 1 point</u>

The rate of heat transfer per unit length is

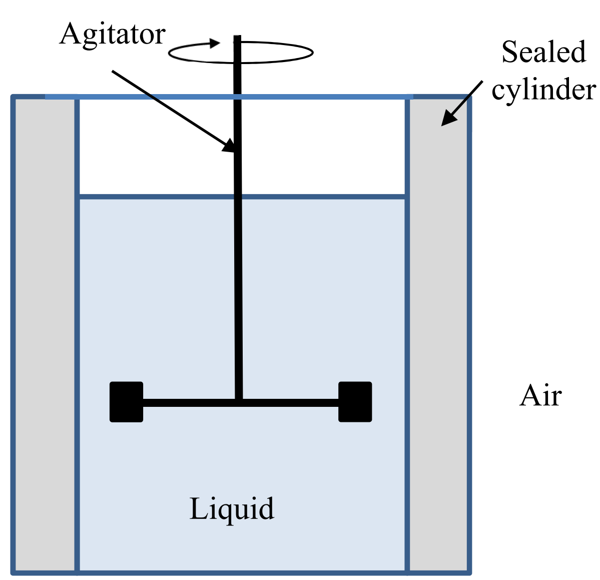
$$\frac{\dot{Q}}{L} = \frac{T_1 - T_S}{L\,\mathrm{R}_{\mathrm{ins}}} = \frac{150-(-10.91)}{2.060} = 78.11\text{ W/m.}$$

<u>Rate of heat transfer: 1 point</u>

Q.5. CONCEPT QUESTIONS

1. [7 points] FINS OR NO FINS?

You are working in a chemical manufacturing facility. In one manufacturing step in this facility, a hot, viscous and corrosive liquid is cooled down in a sealed, cylindrical vessel. To protect the cylinder against corrosion, it is made of a solid plastic, with an inner diameter of 1.4 m and a wall thickness 20 cm. The thermal conductivity of the plastic is 0.5 W/m·ºC. An agitator in the vessel gently mixes the viscous liquid, leading to a convective heat transfer coefficient of 40 W/m^2·ºC between the liquid and the inner surface of the cylinder. Heat is conducted through the walls of the cylinder, and then convected away by air on the outer surface of the cylinder, with an associated heat transfer coefficient of 30 W/m^2·ºC.



Your boss is interested in speeding up the rate of cooling, and asks you to come up with suggestions to achieve this. A senior engineer suggests during a team meeting that if you installed fins on the air side, where the heat transfer coefficient is lower, you could improve the heat loss rate. Neglecting radiation, answer the following questions:

(1) Would you accept the senior engineer's suggestion? Explain why/why not.

(2) Would the installation of fins on the liquid side improve the heat transfer rate? Explain why/why not.

(3) What would be your final proposal to your boss to improve the rate of heat transfer? Why?

Solution:

If we consider a unit length of the cylinder, the thermal resistance in (ºC-m/W) for the three resistors in series here are

$$R_{conv,i} = \frac{1}{h_i 2\pi r_i} = 0.00568 \text{ °C-m/W}$$

$$R_{wall} = \frac{1}{2\pi k} \ln\left(\frac{r_o}{r_i}\right) = 0.0800 \text{ °C-m/W}$$

$$R_{conv,o} = \frac{1}{h_o 2\pi r_o} = 0.00589 \text{ °C-m/W}$$

<u>Calculation of the three resistances: 3 points (1 point each)</u>

Answers to (1) and (2)
Installing fins reduces the convective heat transfer resistances. But in this problem, both the liquid and the air side convective resistances are weak compared to the tube wall resistance. There is no use of installing the fins on either side and further reducing these resistances.

<u>Realizing that fin installation is pointless in this problem on either side: 2 points</u>

Answer to (3)
My suggestion to the supervisor would be to improve the thermal conductivity of the wall material, or make the wall thinner, so as the reduce the dominant resistance in this network. The first could be achieved, for example, by having a bulk of the wall made out of a metal, and then providing a thin, corrosion resistant coating on the liquid side.

<u>Mentioning that the wall resistance needs to be cut down:1 point</u>
<u>Suggestions for improving heat transfer rate: 1 point</u>

2. [5 points] LATENT HEAT EFFECTS: CAN THEY BE IGNORED?

In class, there were several examples related to the concept of heat conduction, which involved a phase change (freezing of the beef, or deep frying of the ice-cream). In these examples, we ignored any latent heat effects in the calculations. Consider one such problem, e.g. the freezing of a cylindrical piece of beef, from 37ºC down to -10ºC. Assume the frozen and unfrozen pieces of beef to have properties identical to that of water. The latent heat of fusion of water is 33.44 kJ/kg. Was the neglect of the release of the latent heat of fusion during the freezing process appropriate in this problem?

Hint: Examine the total amount of energy exchanged per unit mass of beef.

Solution

The sensible heat change from 37oC to -10oC is $mC\Delta T$

Sensible heat content change: 1 point

The latent heat released upon freezing is $m\lambda$

Latent heat release: 1 point

The ratio of the two is $\dfrac{m\lambda}{mC\Delta T} = \dfrac{\lambda}{C\Delta T}$

Ratio: 1 point

If this ratio is small, the neglect of latent heat effects is justifiable . For the freezing beef problem,

$$\frac{\lambda}{C\Delta T} = \frac{33.44\ \dfrac{\text{kJ}}{\text{kg}}}{4.180\ \dfrac{\text{kJ}}{\text{kg}^\circ\text{C}} \times \left[37-(-10)\right]\ ^\circ\text{C}} = 0.17$$

Calculation of ratio: 1 point

The latent heat exchange is about 20% of the total heat exchange, so its effect is not completely negligible.

Comment on neglect of latent heat exchange: 1 point

Q.3C. [8 points] HEAT TRANSFER RATE FROM DRAG

A streamlined body is moving at a velocity of 3 m/s against air at an ambient temperature of 20°C. The body is maintained at a temperature of 50°C. The drag force acting on the object is 6.3 N. What is the rate of heat transfer between the body and fluid? For air, use a density of 1.1 kg/m^3, a specific heat capacity (at constant pressure) of 1007 J/kg°C, a momentum diffusivity of 1.7×10^{-5} m^2/s, and a Prandtl number of 0.72, and assume these values to be constant over the temperature range of concern.

Solution

Since the body is streamlined, the drag on it is dominated by skin drag as opposed to form drag. Also, since 0.6<Pr<60, we can use Colburn's analogy

$$\frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}^{1/3}} = \frac{C_f}{2}$$

$$\frac{\frac{hL}{k}}{\frac{UL}{\nu}\mathrm{Pr}^{1/3}} = \frac{1}{2}\frac{F_D}{A\frac{1}{2}\rho U^2}$$

Simplifying and rearranging the terms, we have

$$hA = \frac{k}{\nu}\mathrm{Pr}^{1/3}\frac{F_D}{\rho U} = \frac{\alpha\rho C}{\nu}\mathrm{Pr}^{1/3}\frac{F_D}{\rho U} = \frac{1}{\mathrm{Pr}^{2/3}}\frac{CF_D}{U} = \frac{1}{0.72^{2/3}}\frac{1007\times 6.3}{3} = 2632\frac{\mathrm{W}}{{}^{\circ}\mathrm{C}}.$$

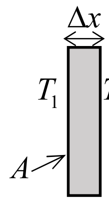
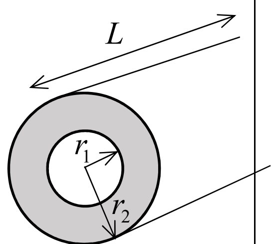
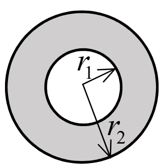
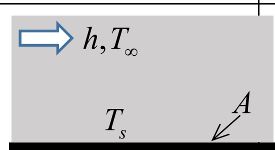
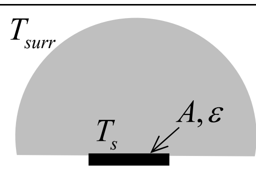
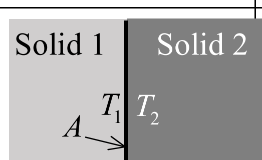
The rate of heat exchange is given by Newton's law of cooling

$$\dot{Q} = hA\left(T_S - T_\infty\right) = 2632\left(50-20\right) = 78960 \text{ J} = 78.96 \text{ kJ}.$$

Justification for using Colburn analogy: 3 points

Applying Colburn analogy formula (either in original or simplified form): 4 points

TABLE OF THERMAL RESISTANCES

Geometry / Situation	Schematic	Heat transferred (W)	Resistance (°C/W)
Slab (plane wall)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\Delta x}{kA}$
Straight cylindrical shell (Hollow cylinder)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi k L}$
Spherical shell (Hollow sphere)		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)}{4\pi k}$
Convective heat transfer		$\dot{Q} = \dfrac{T_s - T_\infty}{R_{\text{conv}}}$	$R_{\text{conv}} = \dfrac{1}{hA}$
Radiative heat transfer		$\dot{Q} = \dfrac{T_s - T_{surr}}{R_{\text{rad}}}$	$R_{\text{rad}} = \dfrac{1}{\varepsilon \sigma A \left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right)}$
Thermal contact resistance		$\dot{Q} = \dfrac{T_1 - T_2}{R}$	$R = \dfrac{R_c}{A}$ (R_c has units of °C-m²/W)

GOVERNING EQUATIONS FOR HEAT CONDUCTION IN VARIOUS CO-ORDINATE SYSTEMS

For a homogeneous medium with a constant specific heat and constant density

$$\rho C\frac{\partial T}{\partial t}=\nabla\cdot\left(k\nabla T\right)+\dot{S}$$

CARTESIAN CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_x=-k\frac{\partial T}{\partial x},\quad \dot{q}_y=-k\frac{\partial T}{\partial y},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant *k*</u>: $\rho C\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{S}$

CYLINDRICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_z=-k\frac{\partial T}{\partial z}$

<u>Constant *k*</u>: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\dot{S}$

SPHERICAL CO-ORDINATE SYSTEM

$$\rho C\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\dot{S}$$

Conductive flux components: $\dot{q}_r=-k\frac{\partial T}{\partial r},\quad \dot{q}_\theta=-k\frac{1}{r}\frac{\partial T}{\partial \theta},\quad \dot{q}_\phi=-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$

<u>Constant *k*</u>: $\rho C\frac{\partial T}{\partial t}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right]+\dot{S}$

TABLE 11–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x/L)$	$\lambda_n \tan \lambda_n = \mathrm{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r/r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \mathrm{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x/L)}{\lambda_n x/L}$	$1 - \lambda_n \cot \lambda_n = \mathrm{Bi}$

TABLE 11–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres ($\mathrm{Bi} = hL/k$ for a plane wall of thickness $2L$, and $\mathrm{Bi} = hr_o/k$ for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 11–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Total heat transferred:

$$\frac{Q}{Q_{\max}} = 1-\theta_0 \frac{\sin\lambda_1}{\lambda_1} \qquad \text{Plane wall}$$

$$= 1-2\theta_0 \frac{J_1(\lambda_1)}{\lambda_1} \qquad \text{Cylinder}$$

$$= 1-3\theta_0 \frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} \qquad \text{Sphere}$$

Here, $Q_{\max} = mC(T_\infty - T_i)$, where m and C and the mass and the specific heat capacity, respectively, of the solid. θ_0 is a dimensionless center temperature.

NUSSELT NUMBER CORRELATIONS

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\mathrm{Re}_{x,\,\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5\times10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $\quad C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}}, \qquad \mathrm{Re}_x < 5\times10^5$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}, \qquad \mathrm{Pr} > 0.6$$

Turbulent: $\quad C_{f,x} = \dfrac{0.059}{\mathrm{Re}_x^{1/5}}, \qquad 5\times10^5 \le \mathrm{Re}_x \le 10^7$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $\quad C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}}, \qquad \mathrm{Re}_L < 5\times10^5$

Turbulent: $\quad C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}, \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Combined: $\quad C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} - \dfrac{1742}{\mathrm{Re}_L}, \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

Rough surface, turbulent: $\quad C_f = \left(1.89 - 1.62\log\dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\quad \mathrm{Nu} = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}, \qquad \mathrm{Re}_L < 5\times10^5$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5\times10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\quad \mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\ \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\quad \mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\ \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$

Laminar: $\quad h = \dfrac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$

Turbulent: $\quad h = \dfrac{5[1-(\xi/x)^{9/10}]}{(1-\xi/L)}\,h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\quad \mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\quad \mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

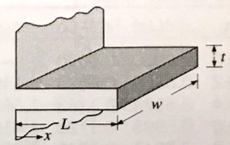
TABLE 10–3

Efficiency and surface areas of common fin configurations

Straight rectangular fins

$m = \sqrt{2h/kt}$
$L_c = L + t/2$
$A_{fin} = 2wL_c$

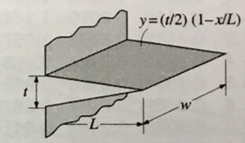
$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



Straight triangular fins

$m = \sqrt{2h/kt}$
$A_{fin} = 2w\sqrt{L^2 + (t/2)^2}$

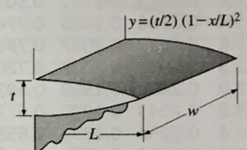
$$\eta_{fin} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$



Straight parabolic fins

$m = \sqrt{2h/kt}$
$A_{fin} = wL[C_1 + (L/t)\ln(t/L + C_1)]$
$C_1 = \sqrt{1 + (t/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL)^2 + 1}}$$

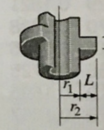


Circular fins of rectangular profile

$m = \sqrt{2h/kt}$
$r_{2c} = r_2 + t/2$
$A_{fin} = 2\pi(r_{2c}^2 - r_1^2)$

$$\eta_{fin} = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$$

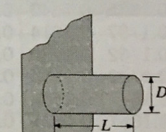
$$C_2 = \frac{2r_1/m}{r_{2c}^2 - r_1^2}$$



Pin fins of rectangular profile

$m = \sqrt{4h/kD}$
$L_c = L + D/4$
$A_{fin} = \pi DL_c$

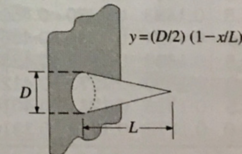
$$\eta_{fin} = \frac{\tanh mL_c}{mL_c}$$



Pin fins of triangular profile

$m = \sqrt{4h/kD}$
$A_{fin} = \frac{\pi D}{2}\sqrt{L^2 + (D/2)^2}$

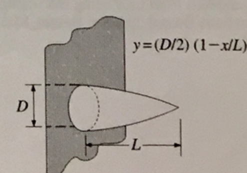
$$\eta_{fin} = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$$



Pin fins of parabolic profile

$m = \sqrt{4h/kD}$
$A_{fin} = \frac{\pi L^3}{8D}[C_3C_4 - \frac{L}{2D}ln(2DC_4/L + C_3)]$
$C_3 = 1 + 2(D/L)^2$
$C_4 = \sqrt{1 + (D/L)^2}$

$$\eta_{fin} = \frac{2}{1 + \sqrt{(2mL/3)^2 + 1}}$$



Pin fins of parabolic profile (blunt tip)

$m = \sqrt{4h/kD}$
$A_{fin} = \frac{\pi D^4}{96L^2}\left\{[16(L/D)^2 + 1]^{3/2} - 1\right\}$

$$\eta_{fin} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)}$$