1. Photoelectric Effect

(a) The maximum kinetic energy of photoelectron is given by $KE = hf - \phi$,
where ϕ is the work fn.
Using $f = \frac{c}{\lambda}$, we get two equations with 2 unknowns;

$$2.3\ (eV) = \frac{hc}{200\times10^{-9}} - \phi \quad —①$$

$$0.9\ (eV) = \frac{hc}{253\times10^{-9}} - \phi \quad —②$$

① − ② gives

$$1.4 \times 1.6\times10^{-19} = hc\left(\frac{1}{200\times10^{-9}} - \frac{1}{253\times10^{-9}}\right)$$

Where we used $1\,eV = 1.6\times10^{-19}\,J$.

$\therefore\ h = 7.13\times10^{-34}\ J\cdot s$ (or $4.45\times10^{-15}\ eV\cdot s$)

(b) $$\phi = \frac{hc}{200\times10^{-19}} - 2.3\ (eV) =$$

$$= \frac{4.45\times10^{-15}\cdot 3\times10^{8}}{200\times10^{-19}} - 2.3 = 4.38\ (eV)$$

2.

(a) The energy difference is the energy of the emitted radiation.

$$\Delta E = \frac{hc}{\lambda} = \frac{4.14\times10^{-15}\,(\mathrm{eV\cdot s})\cdot 3\times10^{8}\,(\mathrm{m/s})}{5500\times10^{-10}\ \mathrm{m}}$$

$$= \underline{2.3\ (\mathrm{eV})}$$

(b) When the intensity falls with $I(t) = I_0 e^{-t/\tau}$, τ is called the lifetime of the excited state. Using the energy-time uncertainty,

$$\Delta E \approx \frac{\hbar}{\tau} = \frac{4.14\times10^{-15}\,(\mathrm{eV\cdot \cancel{s}})}{2\pi\cdot 2\times10^{-12}\,(\cancel{s})} = \underline{3.3\times10^{-4}\ (\mathrm{eV})}$$

3. (a) $$1 = \int_{-\infty}^{\infty} |\Psi(x,t)|^2\,dx$$

$$= \int_{-\infty}^{\infty} A^2 e^{-x^2/a^2}\,dx \qquad \text{(time-dependence cancels)}$$

$$= A^2 a\sqrt{\pi} \qquad \left(\text{used the formula } \int_{-\infty}^{\infty} e^{-\alpha^2 x^2}\,dx = \frac{\sqrt{\pi}}{\alpha}\right)$$

$$\therefore\ A = \frac{1}{\sqrt{a\sqrt{\pi}}}$$

(b) $\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx = \int_{-\infty}^{\infty} x A^2 e^{-x^2/a^2} dx = 0$

because the integrand is an odd fn. of x.

$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\Psi|^2 dx = \int_{-\infty}^{\infty} A^2 x^2 e^{-x^2/a^2} dx$

Change of variable $y \equiv \frac{x}{a}$.

$$\int_{-\infty}^{\infty} (ay)^2 e^{-y^2} (a\,dy) = a^3 \underbrace{\int_{-\infty}^{\infty} y^2 e^{-y^2} dy}_{\frac{\sqrt{\pi}}{2}}$$

$$\therefore \langle x^2 \rangle = A^2 \cdot a^3 \frac{\sqrt{\pi}}{2} = \frac{1}{\cancel{a}\cancel{\sqrt{\pi}}} \cdot \cancel{a^3}^{2} \cdot \frac{\cancel{\sqrt{\pi}}}{2}$$

$$= \frac{a^2}{2}$$

$$\therefore \sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{a^2}{2}$$

$$\therefore \sigma_x = \frac{a}{\sqrt{2}}$$

(c) $\langle p \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{p} \Psi \, dx = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar \frac{\partial \Psi}{\partial x} \right) dx$

since $\frac{\partial}{\partial x} e^{-x^2/2a^2} = -\frac{x}{a^2} e^{-x^2/2a^2}$ (odd fn).

The whole integrand is an odd fn. of x.

$\therefore \langle p \rangle = 0$.

4. (a) $1 = \int_{-\infty}^{\infty} |\Psi(x,0)|^2 dx$

$$= \int_{-\infty}^{\infty} |A\psi_1 + A\psi_2|^2 dx = A^2 \int_{-\infty}^{\infty} (\psi_1^*\psi_1 + \psi_1^*\psi_2 + \psi_2^*\psi_1 + \psi_2^*\psi_2) dx$$

The integration can be simplified greatly using the orthonormal condition, that is,

$$\int \psi_1^*\psi_1 dx = \int \psi_2^*\psi_2 dx = 1$$

$$\int \psi_1^*\psi_2 dx = \int \psi_2^*\psi = 0.$$

$\therefore$ $1 = A^2 \cdot (1 + 0 + 0 + 1) = 2A^2$

$\therefore$ $A = \frac{1}{\sqrt{2}}$

$\therefore$ $\Psi(x,0) = \frac{1}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}\psi_2(x)$

(b) Note that $\frac{2\pi^2\hbar^2}{ma^2} = 2^2\left(\frac{\pi^2\hbar^2}{2ma^2}\right)$

Therefore the question is asking about the probability of finding the particle in $n=2$ state. This is given by $|c_2|^2$

$\therefore$ $P = |c_2|^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

(c) This is obtained by using the general solution expression of time-dependent Schrödinger Eq.

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)\,e^{-i\frac{E_1}{\hbar}t} + \frac{1}{\sqrt{2}}\psi_2\,e^{-i\frac{E_2}{\hbar}t}$$

$$= \cancel{\frac{1}{\sqrt{2}}}\sqrt{\frac{\cancel{2}}{a}}\sin\left(\frac{\pi x}{a}\right)e^{-i\frac{\hbar\pi^2 t}{2ma^2}} + \cancel{\frac{1}{\sqrt{2}}}\sqrt{\frac{\cancel{2}}{a}}\sin\left(\frac{2\pi x}{a}\right)e^{-i\frac{4\hbar\pi^2 t}{2ma^2}}$$

$$= \frac{1}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right)e^{-i\frac{\hbar\pi^2 t}{2ma^2}} + \frac{1}{\sqrt{a}}\sin\left(\frac{2\pi x}{a}\right)e^{-i\frac{2\hbar\pi^2 t}{ma^2}}$$