AER210 VECTOR CALCULUS and FLUID MECHANICS

Quiz 4

Duration: 65 minutes

28 November 2013

Closed Book, no aid sheets

Non-programmable calculators allowed

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Family Name: Alis Ekmekci

Given Name: ____________________

Student #: ____________________

TA Name/Tutorial #: ____________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	13	
2	8	
3	6	
4	9	
5	6	
6	8	
TOTAL	50	/50

Name: ____________________

1) a) [2 marks] Explain the difference between the surface forces and body forces.

Surface forces are forces, such as pressure and frictional forces, which act on the surface of a fluid element.
Body forces are forces that are proportional to the mass of a fluid element, such as gravitational force.

b) [1 marks] Is the Lagrangian method of fluid flow analysis more similar to study of a system or control volume? Explain.

It is similar to the study of a system. You follow a particular system while analyzing its motion.

c) [2 marks] Please fill the following:

A streakline is a line that connects all fluid particles that have passed through the same point in space at a previous time.

A streamline is a line that is tangent to the local velocity vector at every point along the line at that instant.

d) [8 marks] Indicate whether the statement is True (T) or False (F).

T The Reynolds transport theorem can be applied to both scalar and vector quantities.

F There is always a pressure *decrease* across a sudden expansion in a pipe line.

F The Bernoulli equation can be used in boundary layers and in wake regions.

T In the drag coefficient plot for flow past a sphere, the sudden drop in drag at high Reynolds number is because the boundary layer suddenly becomes turbulent.

F Absolute pressure in a liquid of constant density doubles when the depth is doubled.

F Boundary layer is a region in which viscous forces may be neglected.

F The amount of mass entering a control volume have to be equal to the amount of mass leaving during an *unsteady-flow process*.

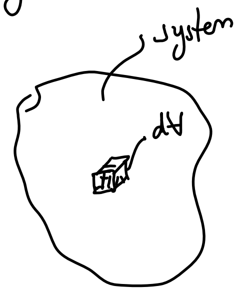
T The variation of pressure with elevation in steady incompressible flow with straight streamlines is the same as that in the stationary fluid.

2) [8 marks] Using the Reynolds Transport theorem derive the conservation of mass equation for a control volume (in other words, the integral form of the continuity equation).

Hint: The Reynolds Transport Theorem for a fluid parameter ***B = mb*** *can be written as:*

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} + \dot{B}_{out} - \dot{B}_{in} \text{ or } \frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} + \oiint_{CS} b\rho\vec{V}.\overrightarrow{dA}$$

$$\frac{d}{dt}(m_{sys}) = 0 \Leftarrow$$ (conservation of mass for a fluid system)



$$\underset{m}{B} = m\underset{1}{b}$$

$$\underbrace{\cancel{\frac{dm_{sys}}{dt}}}_{0} = \frac{dm_{cv}}{dt} + \oiint_{CS} \rho \vec{V}.d\vec{A}$$

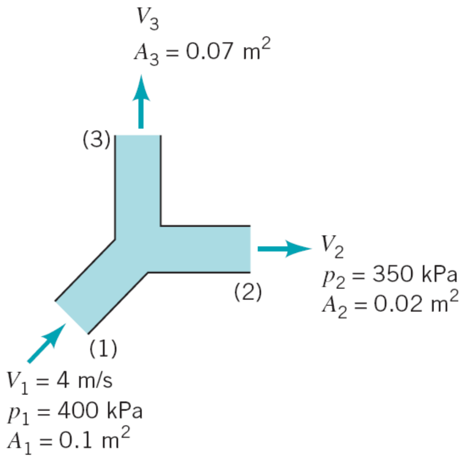
$$0 = \frac{d}{dt}\left(\iiint_{\forall} \rho\, d\forall\right) + \oiint_{A} \rho\vec{V}.d\vec{A}$$

(or CV) (or CS)

$$\boxed{\frac{d}{dt}\iiint_{\forall} \rho\, d\forall + \iint_{A} \rho\,\vec{V}.d\vec{A} = 0}$$

Continuity eqn in integral form

3) [6 marks]Water flows through a horizontal branching pipe as shown in the figure. Determine the pressure at section (3). Density of water is 1000 kg/m^3.



$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$ (continuity)

$$\boxed{V_1 A_1 = V_2 A_2 + V_3 A_3} \quad (1)$$

$$\boxed{\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{g z_2}} \quad (2)$$

$$\frac{400{,}\cancel{000}}{\cancel{1000}} + \frac{4^2}{2} = \frac{350{,}\cancel{000}}{\cancel{1{,}000}} + \frac{V_2^2}{2}$$

$V_2^2 = 116$

$$\boxed{V_2 = 10.77 \text{ m/s}}$$

From eqn (1); $V_3 = \frac{V_1 A_1 - V_2 A_2}{A_3} = \frac{(4)(0.1) - (10.77)(0.02)}{0.07} = 2.637$ m/s

$$\boxed{V_3 = 2.637 \text{ m/s}}$$

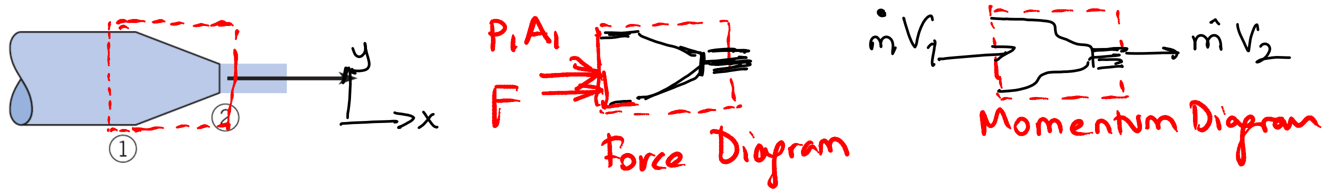
$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \frac{P_3}{\rho} + \frac{V_3^2}{2} + \cancel{g z_2} \Rightarrow \frac{400\,\cancel{000}}{\cancel{1000}} + \frac{4^2}{2} = \frac{P_3}{1000} + \frac{2.637^2}{2}$$

$$\boxed{P_3 = 406681.4 \text{ Pa} \approx 406.7 \text{ kPa}}$$

4) [9 marks] Water at 15°C flows through a nozzle that contracts from a diameter of 10 cm to 2 cm. The exit speed is $V_2 = 25$ m/s, and atmospheric pressure prevails at the exit of the jet. Neglect weight. Density of water is 1000 kg/m^3.

a) Calculate the pressure at section 1.

b) Calculate the force required to hold the nozzle stationary.



Continuity principle

$$\dot{m}_1 = \dot{m}_2 \Rightarrow A_1 V_1 = A_2 V_2 \quad \leftarrow V_2 = 25 \text{ m/s}$$

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{d_2^2}{d_1^2} V_2 = \frac{2^2}{10^2} \times 25 = \frac{4}{\cancel{100}_{4}} \times \cancel{25} = 1 \text{ m/s}$$

$$\boxed{V_1 = 1 \text{ m/s}}$$

$$\dot{m}_1 = \dot{m}_2 = \rho A_2 V_2 = 1000 \times \pi \frac{0.02^2}{4} \times 25 = 7.85 \text{ kg/m}^3$$

Bernoulli equation applied from 1 to 2

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{g z_1} = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + \cancel{g z_2}$$

$P_2 = P_{atm}$

$$P_1 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{1000}{2}\left(25^2 - 1^2\right) = 3.117 \times 10^5 \text{ Pa}$$

$$\boxed{P_1 \cong 312 \text{ kPa}}$$

Momentum principle (x-direction)

$$\Sigma F_x = \dot{m}\left(V_{2x} - V_{1x}\right)$$

$$F + p_1 A_1 = \dot{m}\left(V_2 - V_1\right)$$

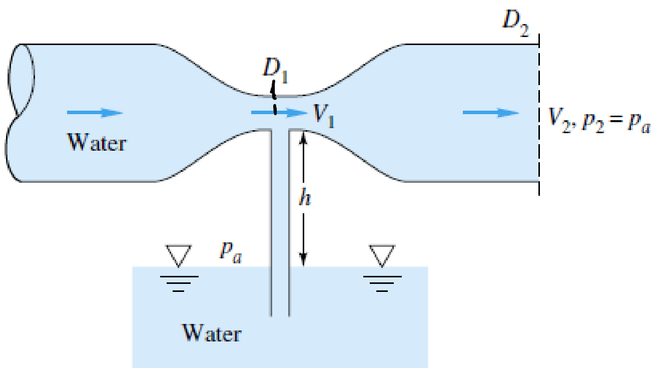
$$F = -p_1 A_1 + \dot{m}\left(V_2 - V_1\right)$$

$$= -\left(311.7\times10^3\right)\left(\frac{\pi \times 0.1^2}{4}\right) + (7.85)(25-1)$$

$$\boxed{F = -2259.7\ N}$$

Force on nozele = 2.26 kN to the left

5) [6 marks] A necked-down section in a pipe flow, called a venturi, develops a low throat pressure which can aspirate fluid upward from a reservoir, as shown in the figure below. Assuming no losses and one-dimensional flow in the venturi, derive an expression for the velocity V_1 which is just sufficient to bring reservoir fluid into the throat.



$$\frac{P_1}{\rho} + \cancel{gz_1} + \frac{V_1^2}{2} = \cancel{\frac{P_2}{\rho}} + \cancel{gz_2} + \frac{V_2^2}{2}$$

0

$$\boxed{\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2}}$$ ① Bernoulli

$$V_1 A_1 = V_2 A_2$$

$$\boxed{V_1 D_1^2 = V_2 D_2^2}$$ ② Continuity

$$0 - \rho g h = P_1 \Rightarrow \boxed{P_1 = -\rho g h}$$ ③

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{V_2^2}{2}$$

$$-gh + \frac{V_1^2}{2} = \left(V_1 \frac{D_1^2}{D_2^2}\right)^2 \frac{1}{2}$$

$$-gh + \frac{V_1^2}{2} = V_1^2 \frac{1}{2} \frac{D_1^4}{D_2^4}$$

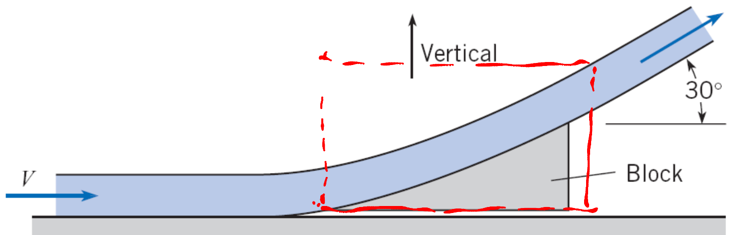
$$\frac{V_1^2}{2}\left(1 - \frac{D_1^4}{D_2^4}\right) = gh \Rightarrow \boxed{V_{1,min} = \sqrt{\frac{2gh}{\left(1 - \frac{D_1^4}{D_2^4}\right)}}}$$

6) [8 marks] Water strikes a block as shown and is deflected 30º. The mass flow rate of the water is 1 kg/s, and the inlet velocity is V = 10 m/s. The mass of the block is 1 kg. If the friction between the block and the surface exceeds 1.48N, the block will move.

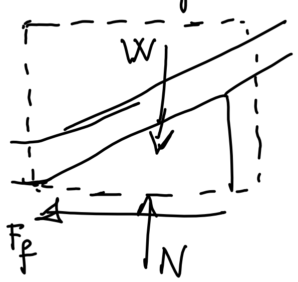
a) Determine normal and horizontal forces acting on the block;

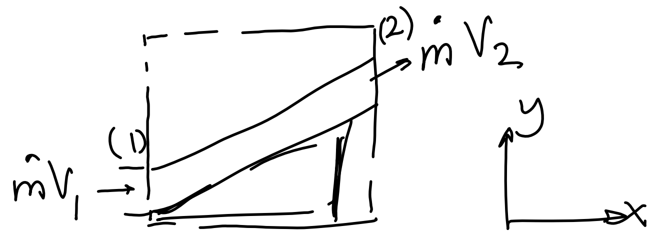
b) determine whether the block will move.

Neglect the weight of the water. Also, as the jet passes over the block neglect elevation changes. (Gravitational acceleration $g = 10\ m/s^2$ and the density of water is $\rho = 1000\ kg/m^3$).



Force Diagram





Bernoulli : $\cancel{\frac{P_1}{\rho}} + \frac{V_1^2}{2} + \cancel{gz_1} = \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + \cancel{gz_2} \Rightarrow V_1 = V_2 = V = 10\ m/s$

Momentum : $\sum \vec{F} = \dot{m}\vec{V}_2 - \dot{m}\vec{V}_1$

$$(-W + N)\vec{j} - F_f\vec{i} = \dot{m}\left(V\cos 30\,\vec{i} + V\sin 30\,\vec{j}\right) - \dot{m}V\vec{i}$$

x-direction:

$$-F_f = \dot{m}V\cos 30 - \dot{m}V$$
$$= \dot{m}V(1 - \cos 30)$$
$$= 1 \times 10 \times (1 - \cos 30) \Rightarrow \boxed{F_f = 1.34N}$$

EXTRA PAGE

y-direction:

$$-W + N = V \sin 30^\circ$$

$$N = W + V \sin 30$$

$$= mg + V \sin 30$$

$$= 1 \times 10 + 1 \times 10 \times \sin 30$$

$$\boxed{N = 15N}$$

$F_f = 1.34N < 1.48N \Rightarrow$ block will not slip