CIV102F Assignment # 3 – September 29, 2021

Due Wednesday October 6, 2021 at 23:59 Toronto time

General Instructions

- There are four questions on this assignment. All questions must be attempted; however, only one question will be graded.
- Submissions which are incomplete and do not contain a serious attempt to solve each question will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires both the correct process and answer.
- All final answers must be reported using slide-rule precision (ie, four significant figures if the first digit is a "1", three otherwise), and engineering notation for very large or very small quantities.
- Submissions must be prepared neatly and be formatted using the requirements discussed in the course syllabus. Marks will be deducted for poor presentation of work.

Assignment-Specific Instructions

- For Question 1, recall that the material properties for bone can be found in the course notes appendix A.
- Part d in Question 3 can be solved qualitatively, although it may be helpful to perform supporting calculations to understand what is happening.
- Recall that gravitational potential energy, W, can be calculated as W = mgh.

Free Vibration

1. Bob Loblaw, a 65 kg bungee jumper, leaps off a bridge attached to a linear elastic bungee cord which has a modulus of elasticity E=8 MPa. It has a cross-sectional area of 400 mm^2, and an unstressed length of 100 m. Neglecting the weight of the bungee cord, evaluate the following:

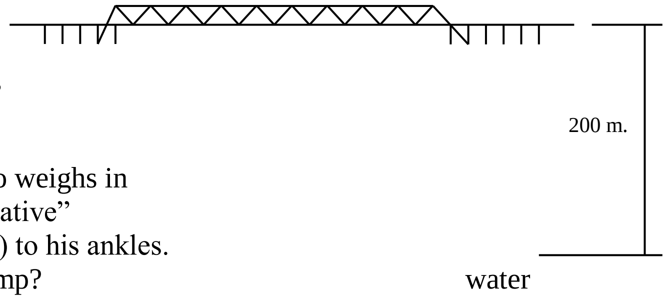
(a) How far into the gorge does Bob plunge?

(b) What is the maximum force in the bungee cord?

(c) Where does Bob come to rest? (i.e. Stop bobbing up and down)

(d) At what frequency does Bob bob up and down?

(e) After watching Bob's jump, his brother Rob, who weighs in at 100 kg, decides that he is going to be "conservative" and ties 4 bungee cords (each one just like Bob's) to his ankles. What problem will Rob experience during his jump?



Nonlinear Material Behaviour

2. Suppose that the steel wire shown below has a stress-strain curve described by the following set of equations:

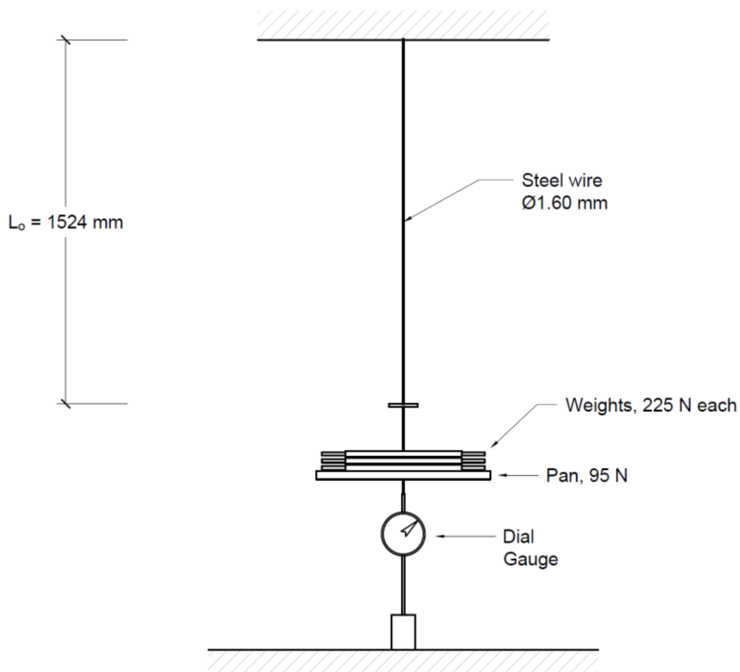
$$\sigma_s = \begin{cases} E_s \varepsilon_s & 0.000 \le \varepsilon_s \le 0.002 \\ \sigma_{yield} & 0.002 \le \varepsilon_s \le 0.040 \\ 4000(\varepsilon_s - 0.040) + \sigma_{yield} & 0.040 \le \varepsilon_s \le 0.065 \\ \sigma_{ult} & 0.065 \le \varepsilon_s \le 0.100 \\ -1500(\varepsilon_s - 0.100) + \sigma_{ult} & 0.100 \le \varepsilon_s \le 0.200 \end{cases}$$

Where E_s = 200,000 MPa, σ_{yield} = 400 MPa and σ_{ult} = 500 MPa. For the following question, assume that a wire with the geometry described in the figure below is being tested to determine its stress-strain behaviour.

a) Plot the relationship between the wire tension P and the elongation Δ if the wire was tested by gradually stretching the wire in small increments of Δ and measuring the tension in the wire. The graph does not need to be to scale along the Δ axis but should highlight all key points. This testing methodology is called *displacement control*.

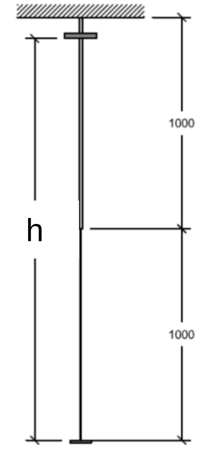
b) On the same graph, plot the relationship between the wire tension P and the elongation Δ if the wire was tested by placing 225 N weights onto the 95 N loading pan and then measuring the change in length (i.e. in the same manner as the demonstration in tutorial 2). This testing methodology is called *force control*.

c) Explain the differences in results. Which experimental method would be preferred if one was trying to measure the "true" stress-strain curve of the material?



Nonlinear Material Behaviour

3. A 2 m long wire made of low alloy steel is attached to a ceiling and has a rigid catch-plate securely attached to the other end. The diameter of the wire is 16 mm for the top 1 m, and 6 mm for the bottom 1 m. Ignore the self-weight of the wire and the catch-plate and neglect strain hardening for this question. The maximum strain the steel can sustain before rupture, $\varepsilon_{rupture}$, can be taken as 0.252 mm/mm.



A 3 kg weight is dropped onto the catch-plate from a height h:

a) What is the maximum value of h which does not cause any permanent deformations in the wire?
b) The weight is now dropped from a height of 1.9 m and then removed from the catch-plate. What is the maximum deformation of the wire due to the weight? What is the deformation of the wire after the weight is removed?
c) If the weight is dropped repeatedly from a height of 1.9 m, the wire will eventually break. How many drops does it take?
d) Suppose we repeated the process with a wire which was identical to the one shown but had a uniform diameter of 6 mm over the entire 2 m length. Can this new wire withstand more drops than the nonuniform wire? Why? A qualitative answer is acceptable.

Moment of Inertia

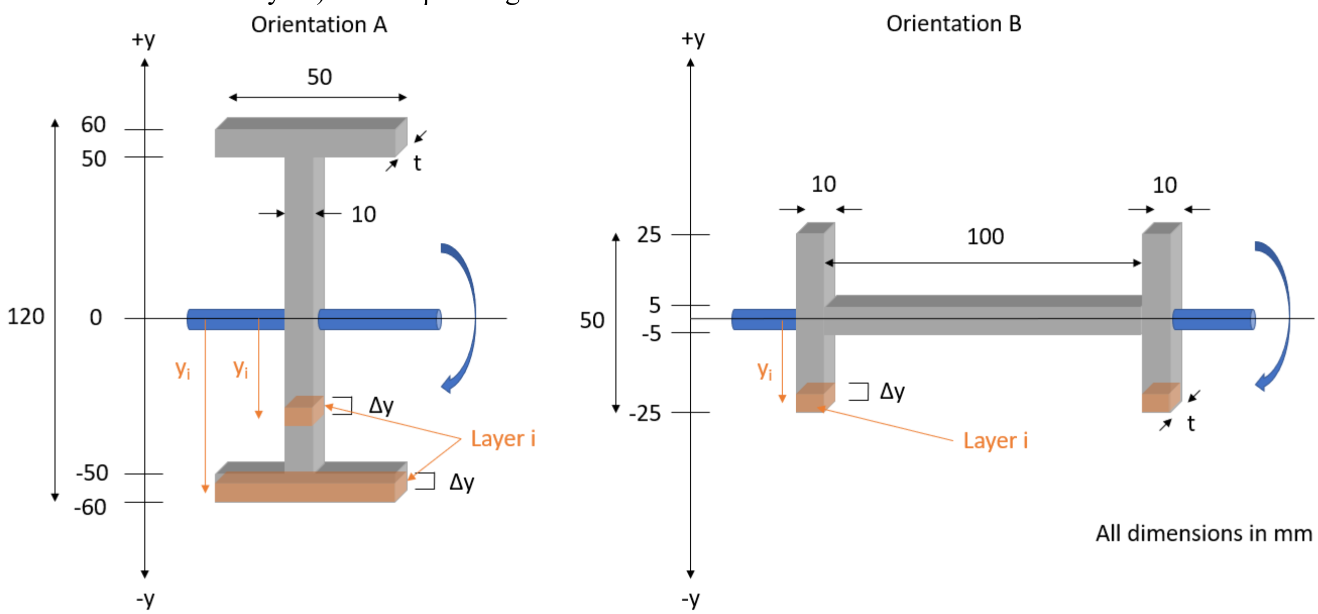
4. The "I" shaped object shown below is oriented in two different ways about different axis of rotation. In lecture, we solved for the mass moment of inertia, I_m, of a rectangular object by discretizing the object into layers and using the summation:

$$I_m = \sum_{i=1}^{n} I_{mi} = \sum_{i=1}^{n} {y_i}^2 \Delta m_i = \sum_{i=1}^{n} {y_i}^2 \rho t b_i \Delta y = \rho t \sum_{i=1}^{n} {y_i}^2 b_i \Delta y$$

where:

$I_{mi} = Incremental\ mass\ moment\ of\ inertia\ of\ layer\ i$
$y_i = Distance\ between\ the\ axis\ of\ rotation\ and\ the\ center\ of\ layer\ i$
$\Delta m_i = Incremental\ mass\ of\ layer\ i$
$\rho = Density\ of\ the\ material$
$t = Out\ of\ plane\ depth$
$b_i = Width\ of\ layer\ i$
$\Delta_y = Incremental\ thickness\ of\ layer\ i$

a) Using the same method, calculate the mass moment of inertia, I_m, of the "I" shaped object in orientation A (I_{mA}) and orientation B (I_{mB}). Set Δy = 5 mm (therefore, in orientation A, there will be 120/5 = 24 layers to sum over and in orientation B, there will be 50/5 = 10 layers) and set ρ = 1 kg/mm^3 and t = 1 mm. Excel can be used to calculate the summations.



b) If the object is set into rotation with the same angular acceleration, α, in both orientations, calculate how much more moment is required to rotate the object in orientation A vs orientation B as a ratio.

c) Calculate the *area* moment of inertia, I, of orientation A ($I = I_m / \rho t$). Provide your answer in mm^4.

d) Imagine if a large number of the "I" shaped objects were attached rigidly back-to-back to form a beam in both orientations as seen in the figure below. **Without doing any calculations**, which orientation do you think will be harder to bend (in order words, provide stronger resistance against bending)?