AER210 VECTOR CALCULUS and FLUID MECHANICS

Midterm Test # 2

Duration: 1 hour, 50 minutes

1 December 2022

Closed Book, no aid sheets, but non-programmable calculators are allowed

Instructor: Prof. Alis Ekmekci

Family Name: Solutions

Given Name:

Student #:

TA Name/Tutorial #:

FOR MARKER USE ONLY		
Question	Marks	Earned
1	15	
2	11	
3	8	
4	20	
5	8	
6	10	
7	18	
8	10	
TOTAL	100	

$\tau = \mu \frac{du}{dy}$ $\quad - \nabla p + \rho \vec{g} = \rho \vec{a}$ $\quad \frac{p}{\rho} + \frac{V^2}{2} + gz = constant$ (Bernoulli equation)

The gravitational acceleration: g = 10 m/s^2

$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} + \dot{B}_{out} - \dot{B}_{in}$ (Reynolds Transport Theorem for a mass-dependant property B)

 Name: ________________

1) a) [12 points] Indicate true (T) or false (F):

T In Newtonian fluids, the shear stress varies linearly with the deformation rate.

T In a room, for air at rest, the pressure variation with an elevation change is negligibly small.

F In flow regions close to solid surfaces (i.e., in the boundary layer regions), the viscous effects are negligible.

T No shear stresses exist in a hydrostatic fluid.

F In a steady fluid, flow properties (such as velocity, pressure, and density) are independent of time as well as location.

T For an object immersed in a hydrostatic fluid, the buoyant force acting on the object is independent of the density of the object.

F In an unsteady flow, dye/smoke injection into a fluid flow reveals streamlines.

T Force equals mass times acceleration is Newton's 2nd law written in the Lagrangian form.

F A tiny neutrally buoyant electronic pressure probe is released into the inlet of a water pump and transmits 2000 pressure readings per second as it passes through the pump. This is an Eulerian measurement.

F Bernoulli equation is valid in unsteady, compressible, frictionless flows along a streamline.

T Mass in a fluid system is always constant, even in an unsteady flow.

T Reynolds Transport Theorem can be applied to both steady and unsteady flows.

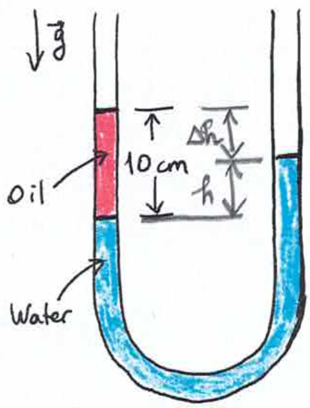
b) [3 points] Please connect the flowline names on the left to their definitions on the right with a line.

Streamline •	• a line that connects the particles that have passed through the same point at a previous time
Pathline •	• a line tangent to the velocity vectors
Streakline •	• a line traced out by a particle as it moves from one point to another in the flow

Name: ____________________

2) a) [4 points] The U-tube manometer shown below has two fluids, water and oil, and both ends of the manometer are exposed to atmospheric pressure. If ρ_{oil} is the density of oil and ρ_{water} is the density of water, $\rho_{oil} = 0.8\,\rho_{water}$. Find the height difference between the free water surface and the free oil surface.

$\Delta h = ?$



$0 + \rho_{oil} \cdot \cancel{g} \cdot (0.1) - \rho_{water} \cdot \cancel{g} \cdot h = 0$

$h = (0.1)\,\dfrac{\rho_{oil}}{\rho_{water}} = 0.08\,m = 8\,cm$

$\Delta h = 10 - 8 = 2\,cm$

b) [3 marks] Bernoulli equation is given below. Indicate the meaning of each term on the left-hand side:

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = constant$$

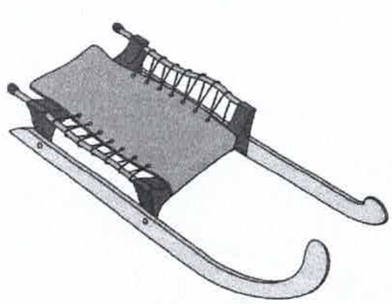
$\frac{p}{\rho}$: energy due to pressure per unit mass

$\frac{V^2}{2}$: kinetic energy per unit mass

gz: potential energy per unit mass

c) [4 points] The sled shown in the figure below slides along on a thin horizontal layer of water, which is sandwiched between the ice on the ground and the sled runners. The total horizontal force the water puts on the runners equals F = 5 N when the sled's speed is 5 m/s. The total area of the runners in contact with the water is 0.01 m^2, and the viscosity of the water is $\mu = 10^{-3}$ Pa.s. Assuming a linear velocity distribution in the water layer, determine the thickness of the water layer under the runners.

$V = 5\,m/s$



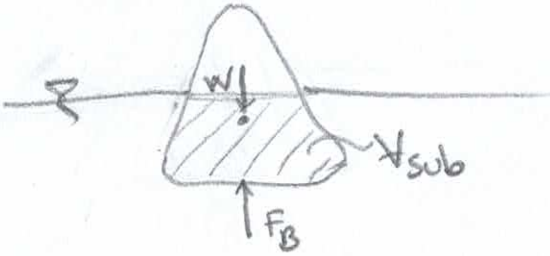
$\tau = \dfrac{F}{A} = \mu \dfrac{du}{dy} \Rightarrow \dfrac{F}{A} = \mu \dfrac{\Delta u}{\Delta y}$

$\Delta y = \mu\, \Delta u \cdot \dfrac{A}{F} \Rightarrow \Delta y = 10^{-3}\, \cancel{5} \cdot \dfrac{0.01}{\cancel{5}} = 10$

$\Delta u = V - 0 = V = 5\,m/s$

 Name: ________________

3) a) [4 points] An iceberg (with density $\rho_{iceberg} = 0.90$ kg/m^3) floats in the ocean (with density $\rho_{ocean} = 1.025$ kg/m^3). What percent of the volume of the iceberg is under water?



$$\overbrace{\rho_{ocean}\cdot \cancel{g}\cdot V_{sub}}^{F_B} = \overbrace{\rho_{iceberg}\,\cancel{g}\cdot V_{iceberg}}^{W}$$

$$\frac{V_{sub}}{V_{iceberg}} = \frac{\rho_{iceberg}}{\rho_{ocean}} = \frac{0.90}{1.025} = 0.878$$

% 87.8 is under water.

b) A golf ball manufacturer wants to study the effect of the dimple size on the distance a golf ball travels. A model ball five times larger than the size of a regular golf ball is installed in a wind tunnel (at the same pressure and temperature conditions). If the independent dimensionless parameter for this problem is the Reynolds number, $Re = \rho V D/\mu$, where ρ is the fluid density, V is the flow speed, D is the diameter of the ball and μ is the fluid viscosity:

b$_1$) [2 points] What should the speed of the wind tunnel be to simulate a golf ball speed of 60 m/s?

b$_2$) [2 points] As a second test, the large-scale golf model generated for the wind tunnel is to be tested in a water flow tunnel. What should be the flow speed in this water tunnel?

$\rho_{air} = 1.23$ kg/m^3, $\rho_{water} = 1000$ kg/m^3, $\mu_{air} = 1.8\times10^{-5}$ N.s/m^2, $\mu_{water} = 1.1\times10^{-3}$ N.s/m^2.

b$_1$) In wind tunnel; In air

$\rho_m = \rho_p$

$\mu_m = \rho_p$

$$(Re)_{model} = (Re)_{prototype} \Rightarrow \frac{\cancel{\rho_m} V_m D_m}{\cancel{\mu_m}} = \frac{\cancel{\rho_p} V_p D_p}{\cancel{\mu_p}}$$

$$V_m D_m = V_p D_p$$

$$V_m = V_p \left(\underbrace{\frac{D_p}{D_m}}_{\frac{1}{5}}\right) = \frac{60}{5} = 12 \text{ m/s.}$$

b$_2$) Method I

$$\frac{\rho_{m,air} V_{m,air}\cdot \cancel{D_{m,air}}}{\mu_{m,air}} = \frac{\rho_{m,water} V_{m,water} \cancel{D_{m,water}}}{\mu_{m,water}} \quad (D_{m,air} = D_{m,water})$$

$$V_{m,water} = \frac{\rho_{m,air}\cdot V_{m,air}}{\mu_{m,air}} \frac{\mu_{m,water}}{\rho_{m,water}} = \frac{1{,}23}{1.8\times10^{-5}}\cdot 12\cdot\frac{1.1\times10^{-3}}{1000} \Rightarrow V_{m,water} = 0{,}9 \text{ m}$$

Method 2

b₂) One could also solve this by ensuring similarity between the prototype (original) ball and the model in water tunnel.

$$\frac{\rho_{m,water} \cdot V_{m,water} \cdot D_{m,water}}{\mu_{m,water}} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$V_{m,water} = \frac{\rho_p}{\rho_{m,water}} \cdot V_p \cdot \frac{D_p}{D_{m,water}} \cdot \frac{\mu_{m,water}}{\mu_p}$$

$$= \frac{1.23}{1000} \cdot 60 \cdot \frac{1}{5} \cdot \frac{1.1 \times 10^{-3}}{1.8 \times 10^{-5}}$$

$$V_{m,water} = 0.9 \, m/s$$

Continuation to page: 4

 Name: ____________________

4) For the rectangular gate placed between the fixed top wall and the bottom floor, as shown in the figure below, $\alpha = 45°$, $y_1 = 1$ m, $y_2 = 3$ m, gate width $w = 1$ m. Determine the **closing moment** under the action of the hydrostatic forces when the gate is at $\alpha = 45°$ using:

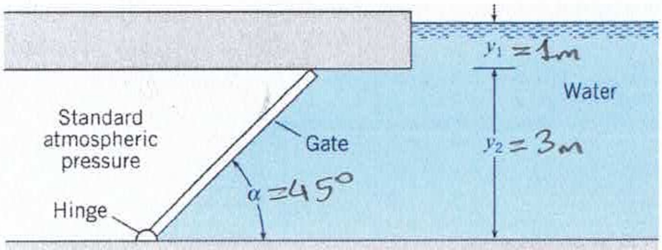
a) [8 points] the pressure-prism method

b) [8 points] the integration method

c) [2 points] Determine the **opening moment** if the gate itself weighs 90 kN.

d) [2 points] Will the gate fall or stay in position under the action of the hydrostatic and gravity forces?

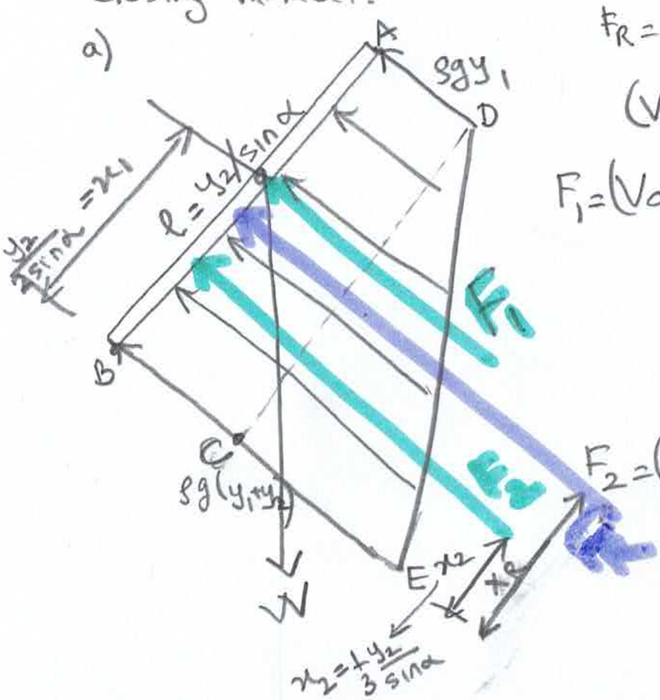
The density of the water is $\rho = 1000$ kg/m³, and the gravitational acceleration is $g = 10$ m/s².



$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

Closing moment:

a)



$F_R = F_1 + F_2$

F_1: $(\text{Volume})_{ABCD}$; F_2: $(\text{Volume})_{CDE}$

$\ell = y_2/\sin\alpha$

$F_1 = (\text{Volume})_{ABCD} = \rho g y_1 \ell w = \rho g y_1 \frac{y_2}{\sin\alpha} w$

$= (1000)(10)(1)\frac{3}{\frac{1}{\sqrt{2}}}\cdot(1)$

$\boxed{F_1 = 42{,}426.41 \text{ N}}$

$\ell = y_2/\sin\alpha$

$F_2 = (\text{Volume})_{CDE} = \rho g y_2 \ell \cdot \frac{w}{2} = \rho g y_2 \cdot \frac{y_2}{\sin\alpha}\frac{w}{2}$

$= (1000)(10)\cdot 3\cdot \frac{3}{\frac{1}{\sqrt{2}}}\cdot\frac{(1)}{2}$

$\boxed{F_2 = 63639.61 \text{ N}}$

$F_R = F_1 + F_2 = \boxed{106{,}066 \text{ N}}$

$F_R \cdot x_R = F_1 \cdot x_1 + F_2 \cdot x_2$

$F_R \cdot x_R = F_1 \cdot \frac{y_2}{2\sin\alpha} + F_2 \cdot \frac{y_2}{3\sin\alpha} \Rightarrow \boxed{x_R = 1.697 m}$

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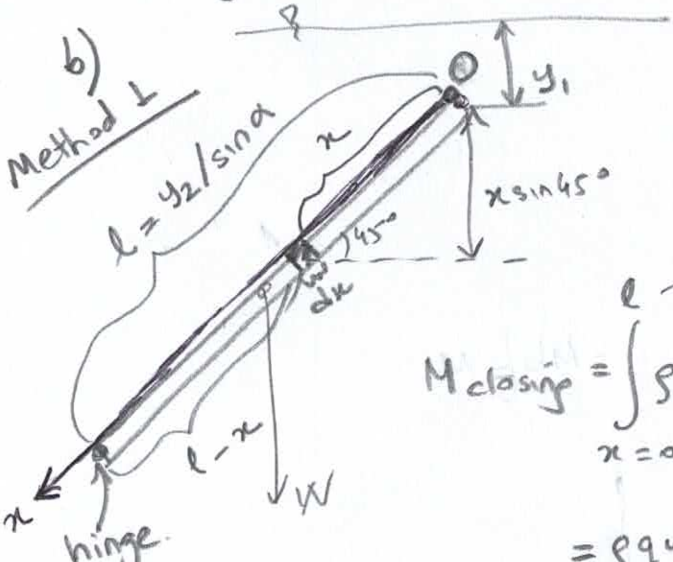
$$M_{closing} = F_R \cdot x_R = (106{,}066).(1.69) = 180{,}000 \text{ N.m}$$

b)

Method 1



here $l = y_2/\sin\alpha$

$$M_{closing} = \int_{x=0}^{l} \rho g\,(y_1 + x\sin 45).w.(l - x)\,dx$$

$$= \rho g w \int_0^l (y_1 l - x y_1 + l x \sin 45 - x^2 \sin 45)\,dx$$

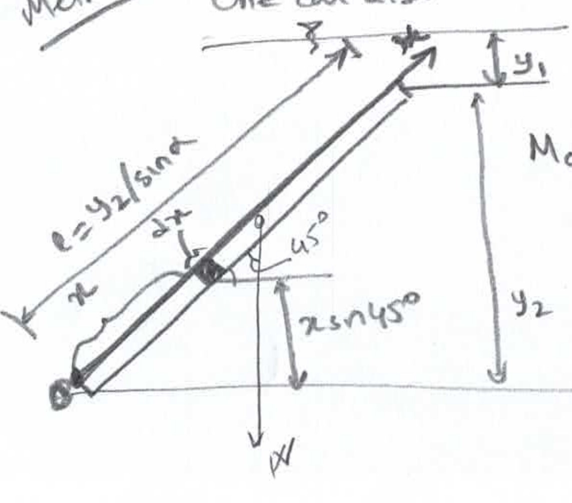
$$= \rho g w \left[y_1 l x - \frac{x^2}{2} y_1 + l\frac{x^2}{2}\sin 45 - \frac{x^3}{3}\sin 45 \right]\Bigg|_{x=0}^{l}$$

$$= \rho g w \left[y_1 l^2 - \frac{l^2}{2} y_1 + \frac{l^3}{2}\sin 45 - \frac{l^3}{3}\sin 45 \right]$$

$$= (1000)(10)(1).\left[1.\left(\frac{3\sqrt{2}}{1}\right)^2 - \frac{(3\sqrt{2})^2}{2}.1 + \frac{(3\sqrt{2})^3}{2}.\frac{1}{\sqrt{2}} - \frac{(3\sqrt{2})^3}{3}\frac{1}{\sqrt{2}} \right]$$

$$= 180{,}000 \text{ N.m}$$

Method 2: One can also select to put the origin of the x axis on the hinge



$l = y_2/\sin 45°$

$$M_{closing} = \int_0^l x.\rho g\,(y_1 + y_2 - x\sin 45°)\,dx$$

$$= \rho g \int_0^{y_2/\sin 45°} \left(x(y_1+y_2) - x^2\sin 45\right)dx$$

$$= \rho g \left[\frac{x^2}{2}(y_1+y_2) - \frac{x^3}{3}\sin 45 \right]\Bigg|_0^{y_2/\sin 45}$$

$$M_{closing} = \rho g \left[\frac{y_2^2}{(\sin 45)^2} \cdot \frac{1}{2} (y_1 + y_2) - \frac{y_2^3}{(\sin 45)^3} \cdot \frac{1}{3} \cdot \sin 45 \right]$$

$$= (1000)(10) \left[\frac{3^2}{\frac{1}{\cancel{2}}} \cdot \frac{1}{\cancel{2}} (4) - \frac{3^3}{\frac{1}{2} \cdot \cancel{\left(\frac{1}{\sqrt{2}}\right)}} \cdot \frac{1}{3} \cdot \cancel{\left(\frac{1}{\sqrt{2}}\right)} \right]$$

$$= (1000)(10) \left[36 - 18 \right]$$

$$= 180,000 \text{ N.m}$$

c) Opening moment : comes from weight $W = 90$ kN $= 90,000$ N.

$$M_{opening} = W \cancel{\sin 45°} \cdot \left(\frac{y_2}{\cancel{\sin 45}} \right) \frac{1}{2} = W . \frac{y_2}{2} = 90,000 . \frac{3}{2} = 135,000 \text{ N.m}$$

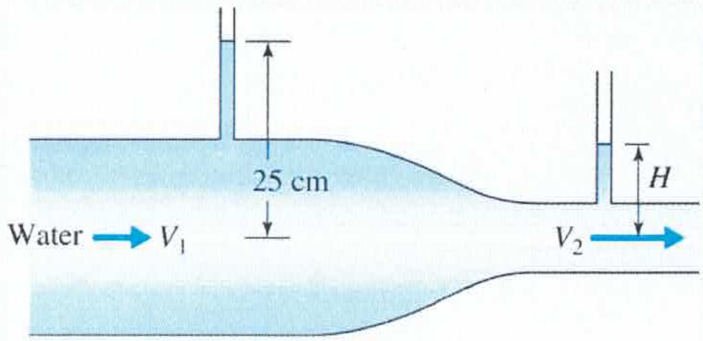
d) $M_{closing} > M_{opening}$ $\Rightarrow$ The gate will stay closed in its position.

Continuation to page 6

$V_2 = 1.125$ m/s

Name: ______________________

5) [8 points] In the water contraction shown in the picture below, water flows steadily with a velocity $V_1 = 0.5$ m/s and ~~$V_1 = 0.125$ m/s~~. Two piezometer tubes are attached to the pipe at sections 1 and 2. Neglecting any frictional losses during contraction, determine the height H.



Bernoulli :

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g\cancel{z_2}$$

$$\boxed{P_1 = \rho g \cdot (0.25)}$$

$$\boxed{P_2 = \rho g H}$$

plugging these into the Bernalli eqn. we get the following:

$$\frac{\cancel{\rho} g (0.25)}{\cancel{\rho}} + \frac{V_1^2}{2} = \frac{\cancel{\rho} g H}{\cancel{\rho}} + \frac{V_2^2}{2}$$

$$H = 0.25 + \frac{(V_1^2 - V_2^2)}{2g}$$

$$H = 0.25 + \frac{(0.5^2 - 1.125^2)}{20}$$

$$H = 0.199 \text{ m}$$

6) [10 points] A solid particle falls through a viscous liquid. The falling velocity, V, is believed to be a function of the fluid density, ρ_f, the particle density, ρ_p, the fluid viscosity, μ, the particle diameter, D, and the acceleration due to gravity, g. Apply dimensional analysis **choosing the repeating variables as ρ_f, D, g** to determine the dimensionless (π) groups for this problem and re-write the relationship between the dimensional variables in dimensionless form.

$$V = f(\rho_f, \rho_p, \mu, D, g)$$

$$\left.\begin{aligned}
[V] &= \frac{L}{T} \\
[\rho_f] &= \frac{M}{L^3} \\
[\rho_p] &= \frac{M}{L^3} \\
[\mu] &= \frac{M}{LT} \\
[D] &= L \\
[g] &= \frac{L}{T^2}
\end{aligned}\right\}$$

\# of variables = 6

\# of reference dimensions = 3 (M, L, T)

From Buckingham Pi Theorem

(# of Π terms) = (# of variables) − (min # of reference dim)

\# of Π terms = 6 − 3 = 3 $\Leftarrow$ 3 Π terms to be determined

Choose repeating variables as: ρ_f, D, g

$$\Pi_1 = V (\rho_f)^a (D)^b (g)^c$$

$$M^0 L^0 T^0 = \frac{L}{T} \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2a}}$$

$$M^0 L^0 T^0 = L^{1-3a+b+c} M^a T^{-1-2c}$$

$$1 - 3a + b + c = 0 \Rightarrow b = -1 + 3a - c = -1 + 0 + \frac{1}{2} \Rightarrow b = -\frac{1}{2}$$

$$a = 0$$

$$-1 - 2c = 0 \Rightarrow c = -\frac{1}{2}$$

$$\Pi_1 = \frac{V}{\sqrt{gD}}$$

$$\Pi_2 = \rho_p (\rho_f)^a D^b g^c$$

$$M^0 L^0 T^0 = \frac{M}{L^3} \cdot \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$$

$$= M^{1+a} L^{-3-3a+b+c} T^{-2c}$$

EXTRA PAGE

$1+a=0 \rightarrow \boxed{a=-1}$

$-3-3a+b+c=0 \Rightarrow -\cancel{3}+\cancel{3}+b+0=0 \Rightarrow \boxed{b=0}$

$-2c=0 \Rightarrow \boxed{c=0}$

$$\boxed{\Pi_2 = \frac{\rho_p}{\rho_f}}$$

$\Pi_3 = \mu(\rho_f)^a(D)^b(g)^c$

$M^0L^0T^0 = \frac{M}{LT} \cdot \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$

$= M^{1+a} L^{-1-3a+b+c} T^{-1-2c}$

$1+a=0 \Rightarrow \boxed{a=-1}$

$-1-3a+b+c=0 \Rightarrow -1+3+b-\frac{1}{2}=0 \Rightarrow \boxed{b=-\frac{3}{2}}$

$-1-2c=0 \Rightarrow \boxed{c=-\frac{1}{2}}$

$$\boxed{\Pi_3 = \frac{\mu}{\rho_f D^{3/2} g^{1/2}}}$$

New relationship $\Rightarrow$ $\Pi_1 = f_2(\Pi_2, \Pi_3)$

or

$$\frac{V}{\sqrt{gD}} = f_2\left(\frac{\rho_p}{\rho_f}, \frac{\mu}{\rho_f D^{3/2} g^{1/2}}\right)$$

Continuation to
page: 8

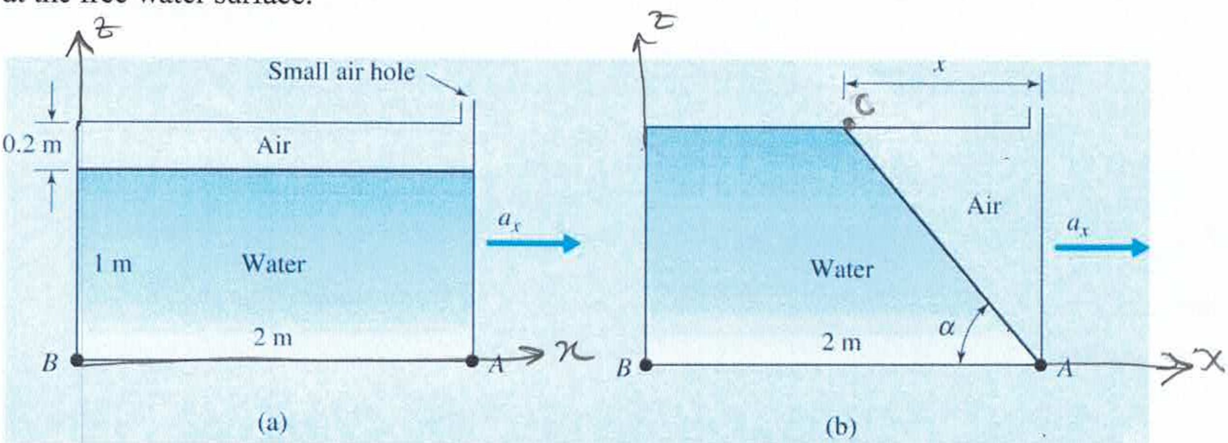
Name: ____________________

7) [18 points] The tank shown in figure (a) below is accelerated to the right with a constant acceleration a_x. As shown in the figure, this tank has a small air hole at its top right corner. The tank has a height of 1.2 m, a length of 2 m (which is the distance between points A and B in the figure), and a width of 1 m (which is the dimension into the page). Before the start of the motion, the height of the still water in the tank is 1 m, and the height of the air is 0.2 m, as also depicted in figure (a).

a. (8 marks) Starting from the equation $-\nabla p + \rho\vec{g} = \rho\vec{a}$ and showing how you arrive at the result, calculate the acceleration a_x needed to cause the free surface to touch the point A, as shown in figure (b) below.
b. (4 points) Find the pressure at point B for the situation depicted in figure (b) below.
c. (6 points) Determine the total force acting on the bottom of the tank again for the situation depicted in figure (b) below.

The density of water: $\rho = 1000\,\frac{kg}{m^3}$, the gravitational acceleration: $g = 10\,\frac{m}{s^2}$.

Hints: #1) As no water spills out, equating the air volume before and during the motion would give you the distance x marked in figure (b). #2) Note that pressure equals 0 Pa (gage pressure) at the free water surface.



- I selected to place the origin of the coordinate system at B. (personal choice)

$$-\vec{\nabla} p + \rho\vec{g} = \rho\vec{a}$$

$$-\left(\frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k}\right) - \rho g\vec{k} = \rho a_x\vec{i}$$

$\vec{i}$: $\frac{\partial p}{\partial x} = -\rho a_x$

$\vec{j}$: $\frac{\partial p}{\partial y} = 0$

$\vec{k}$: $\frac{\partial p}{\partial z} = -\rho g$

$p = p(x, z) \rightarrow dp = \underbrace{\frac{\partial p}{\partial x}}_{-\rho a_x} dx + \underbrace{\frac{\partial p}{\partial z}}_{-\rho g} dz$

$dp = -\rho a_x dx - \rho g dz$

integrate

$$\boxed{p = -\rho a_x x - \rho g z + C}$$

 Name: ____________

EXTRA PAGE

- As no water spills out, equating the air volume before and during the motion would give the distance x.

$$(V_{air})_{stationary} = (V_{air})_{in\ motion}$$

$$(0.2)(2).(1) = \frac{(1.2)(x).(1)}{2} \Rightarrow \boxed{x = \frac{2}{3}m \cong 0.667m}$$

∴ The coordinates of point A: $x=2$, $z=0$

∴ The coordinates of point C: $x = 2 - x = 2 - \frac{2}{3} = 1.334$, $z = 1.2$

At point A $\Rightarrow p(x=2, z=0)=0 \Rightarrow 0 = -\rho a_x.(2) - \cancel{\rho g.(0)} + C \Rightarrow \boxed{C = 2\rho a_x}$

At point C $\Rightarrow p(x = \underbrace{2-\frac{2}{3}}_{1.334}, z=1.2) = 0 \Rightarrow 0 = -\rho a_x\left(2-\frac{2}{3}\right) - \rho g(1.2) + C$

$$0 = -\cancel{\rho}a_x\left(2-\frac{2}{3}\right) - \cancel{\rho}g(1.2) + 2\cancel{\rho}a_x$$

$$0 = a_x\left(\cancel{-2} + \frac{2}{3} + \cancel{2}\right) - 1.2g$$

$$a_x = \frac{1.2.g.3}{2} = \frac{1.2.10.3}{2}$$

$$\boxed{a_x = 18\ m/s^2}$$

b) pressure at point B = ?

$p = -\rho a_x x - \rho g z + 2\rho a_x$ & the coordinates of B: $x=0$, $z=0$

$$P_B = p(x=0, z=0) = -\cancel{\rho a_x.(0)} - \cancel{\rho g(0)} + 2\rho a_x$$

$$P_B = 2\rho a_x = 2.\ 1000.\ 18 \Rightarrow P_B = 36{,}000\ \frac{N}{m^3}$$

EXTRA PAGE

c) Total force acting on the bottom of the tank $= F_{BA} = ?$

$$p(x,z) = -\rho a_x x - \rho g z + 2\rho a_x$$

Along BA $\Rightarrow z=0 \Rightarrow P_{BA} = p(x, z=0) = -\rho a_x x - \rho g \overset{z=0}{(0)} + 2\rho a_x$

$$P_{BA} = \rho a_x (2-x)$$

$$F_{BA} = \int_{x=0}^{2} P_{BA} \cdot w \, dx = \int_{x=0}^{2} \rho a_x (2-x) w \, dx$$

$$= \rho a_x w \int_0^2 (2-x) \, dx$$

$$= \rho a_x w \left[2x - \frac{x^2}{2}\right]\Bigg|_{x=0}^{x=2}$$

$$= \rho a_x w \Big[\underbrace{4-2}_{2}\Big]$$

$$= \underset{1000}{\rho}\ \underset{18}{a_x}\ \underset{1}{w} \cdot (2)$$

$$\boxed{F_{BA} = 36{,}000 \text{ N}}$$

continuation to page 10

8) a) [5 points] Using the Reynolds Transport theorem, derive the conservation of mass equation for a control volume (in other words, derive the Eulerian form of the continuity equation).

Hint: *Start with the conservation of mass equation for a fluid system. Then, use the Reynolds Transport Theorem to convert the conservation of mass equation from a form applicable to a system to a form applicable to a control volume. Remember that the Reynolds Transport Theorem for a mass-dependant fluid parameter* **B = mb** *can be written as:*

$$\frac{dB_{sys}}{dt} = \frac{dB_{CV}}{dt} + \dot{B}_{out} - \dot{B}_{in}$$

$B = m$

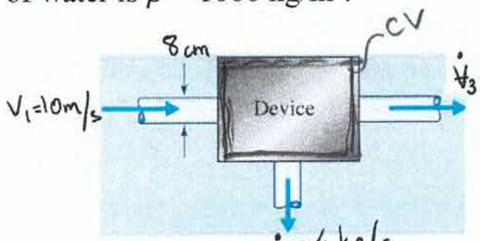
$$\cancel{\frac{dm_{sys}}{dt}} = \frac{dm_{cv}}{dt} + \dot{m}_{out} - \dot{m}_{in}$$

$\frac{dm_{sys}}{dt} = 0$ (conservation of mass for a fluid system)

$$\boxed{\frac{dm_{cv}}{dt} + \dot{m}_{out} - \dot{m}_{in} = 0}$$

– There may be a second version of this solution which can be found in the next page!

b) [5 points] Water flows in and out of a device as shown in the figure below. Calculate the rate of change of the mass of water (dm/dt) in the device. Note that the pipes carrying the water in and out of the device have circular cross-section. As shown in the figure, the following are given: the velocity is V_1 = 10 m/s and the pipe diameter is d_1 = 8 cm for section 1, the mass flow rate is $\dot{m}_2$ = 4 kg/s for section 2, and the volume flow rate is $\dot{V}_3$ = 0.008 m³/s for section 3. The density of water is ρ = 1000 kg/m³.



$\frac{dm_{cv}}{dt} + \dot{m}_{out} - \dot{m}_{in} = 0$ (continuity eqn.)

$\dot{m}_1 = \rho . V_1 A = 1000 . 10 . \pi (0.04)^2$

$\boxed{\dot{m}_1 = 50.265 \text{ kg/m}^3}$

$\boxed{\dot{m}_2 = 4 \text{ kg/s}}$ (given)

$\dot{m}_3 = \rho \dot{V}_3 = (1000)(0.008) = \boxed{8 \text{ kg/s} = \dot{m}_3}$

From continuity:

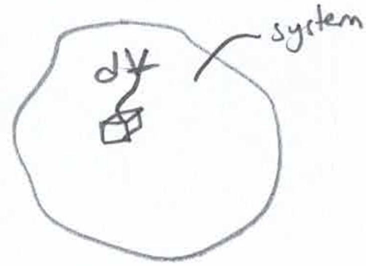
$$\frac{dm_{cv}}{dt} + \underbrace{\dot{m}_2 + \dot{m}_3}_{\dot{m}_{out}} - \underbrace{\dot{m}_1}_{\dot{m}_{in}} = 0 \Rightarrow \frac{dm_{cv}}{dt} + 4 + 8 - 50.26 = 0$$

$$\boxed{\frac{dm_{cv}}{dt} = 38.26 \frac{kg}{s}}$$

8) a) Second Method:

The following is also acceptable as an answer:

$$\frac{d}{dt}(m_{sys}) = 0$$ ⇐ conservation of mass for a fluid system



$$B = m b$$

m 1

$$\cancel{\frac{dm_{sys}}{dt}} = \frac{dm_{cv}}{dt} + \iint_S \rho \vec{V} \cdot dA$$

0

$$0 = \frac{d}{dt} \iiint_{CV} \rho \, d\forall + \iint_{CS} \rho \vec{\forall} \cdot d\vec{A}$$

(or $\forall$)

$$\boxed{\frac{d}{dt} \iiint_{CV} \rho \, d\forall + \iint_{CS} \rho \vec{\forall} \cdot d\vec{A} = 0}$$

Continuation to page 11