ESC103F Engineering Mathematics and Computation: Tutorial #1

Question 1: Given points P(2,-1,4), Q(3,-1,2), A(0,2,1) and B(1,3,0), determine if $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are parallel.

Question 2: Let $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ be the vectors in standard position of two points P_1 and P_2. If the point M is $1/3^{\text{rd}}$ the way from P_1 to P_2, develop a general expression for the position vector $\overrightarrow{OM}$ in terms of $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$, and for the coordinates of the point M if P_1=(1,2,3) and P_2=(4,5,6).

Question 3: Let the points A, B, C, and D in the plane form a quadrilateral ABCD. Let E, F, G, and H be the midpoints of each side of the quadrilateral. Using a vector method approach, prove that the quadrilateral EFGH is a parallelogram.

Question 4: Let P be the point (2,3,-2) and Q the point (7,-4,1).

Find the midpoint of the line segment connecting P and Q.

Find the point on the line segment connecting P and Q that is $2/3^{\text{rd}}$ of the way from P to Q.

Question 5: Points A(-3, 2), B(1, -2) and C(7, 1) are given.

Find the coordinates of point D so that ABCD forms a parallelogram. Is this point unique?

Question 6: The linear combination of two vectors $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ span or fill a plane that goes through the origin. Each part of this question refers to this plane.

i) Describe in words the plane and sketch the plane.

ii) Consider linear combinations $c\vec{v} + d\vec{w}$. Write an expression for a single vector in terms of c and d that defines the plane.

iii) Using your result from part (ii), find a vector that is **not** in the plane.

Question 7: Find two different linear combinations of the three vectors $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$ that produce $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$? If you take **any** three vectors $\vec{u}, \vec{v}$ and $\vec{w}$, will there always be two different linear combinations that produce $\vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$?