The Edward S. Rogers Sr. Department
of Electrical & Computer Engineering
UNIVERSITY OF TORONTO

ECE259: Electromagnetism

Final exam - Monday April 22, 2019

Instructors: Tome Kosteski and Micah Stickel

Last name: ....

First name: ....

Student number: ....

Instructions

- Duration: 2 hour 30 minutes (14:00 to 16:30)
- Exam Paper Type: A. Closed book. Only the aid sheet provided is permitted.
- Calculator Type: 2. All non-programmable electronic calculators are allowed.
- **Only answers that are fully justified will be given full credit!**

Marks:

Q1: /18	Q2: /20	Q3: /20	Q4: /21	Q5: /21	TOTAL: /100

Question 1

For the following 6 questions, select the correct answer or answers (1 mark) and provide a brief justification (2 marks). [18 marks total, all equally weighted]

1.1 Consider two wires that have the same resistance. If wire A is made of a material with a conductivity of 4.1×10^7 S/m and wire B is made of a material with a conductivity of 5.8×10^7 S/m then it is possible that

(a) Wire A has a smaller cross-sectional area than wire B if each wire is of equal length.

(b) Wire A has a smaller cross-sectional area than wire B if wire A is longer than wire B.

(c) Both wires have the same cross-sectional area if wire A is longer than wire B.

(d) Both wires have the same cross-sectional area if wire B is longer than wire A.

(e) None of the above are possible.

1.2 Two positive point charges separated by a distance, d, in free space exert a force of 5.2×10^{-9} N on each other. When a homogenous material fills the space between and around the two point charges, the force exerted on each charge is equal to 2.6×10^{-9} N. The relative dielectric permittivity (or dieletric constant) of the material is

(a) Equal to 1.0

(b) Equal to 2

(c) Greater than 2

(d) Equal to 0.5

(e) Equal to $2\varepsilon_0$

(f) None of the above

1.3 Consider a positive point charge $Q = 1$ mC moving in free space with a velocity $\mathbf{u} = 4\mathbf{a}_\theta$ m/s at a radius of $R = 2$ m in the presence of a magnetic field given by $\mathbf{B} = 0.1\mathbf{a}_\phi$ T. Which of the following additions to this situation, if any, will enable Q to remain at the same distance from the origin as it moves from $\theta = 0°$ to $\theta = 180°$? For this problem you can assume that the Lorentz force is the dominant force, and so you can ignore all other forces.

(a) Surround the moving point charge with a charged metallic shell with total charge $Q_2 = -1$mC and radius $R_2 = 3$ m

(b) Place a positive point charge $Q_2 = 6.4\pi\varepsilon_0$ C at the origin

(c) Place a negative point charge $Q_2 = -1$ mC at the origin

(d) Place a charged metallic shell with charge density $\rho_S = 1.6\varepsilon_0$ C/m^2 and radius $R = 1$ m centered about the origin.

(e) Nothing is needed. The point charge will remain at this radius $R = 2$ m as it moves along this arc.

(f) None of the above

1.4 An ac voltage generator is connected to a capacitor. If the dielectric in the capacitor has an electrical conductivity of 10^{-1} S/m and is otherwise ideal while the wire connecting the generator to the capacitor has an electrical conductivity of 10^7 S/m and is otherwise ideal then;

(a) In the wire the conduction current is equal to the displacement current

(b) In the dielectric the conduction current is zero and the displacement current is non-zero

(c) In the dielectric the displacement current is equal to the conduction current in the wire

(d) In the wire the displacement current is zero and the conduction current is non-zero

(e) None of the above

1.5 Consider the following set of statements related to the process of magnetization. Identify those that are true by selecting one of the answers (a) to (f) below.

(i) A soft magnetic material is one that usually is associated with large hysteresis loss.

(ii) A diamagnetic material usually has a magnetic susceptibility that is small but positive, i.e., $\chi_m \gtrsim 0$

(iii) A hard magnetic material is typically characterized by a large value of H_c (coercive field intensity).

(iv) A ferromagnetic material usually has large relative permeabilities, i.e., $\mu_r \gg 1$,

(v) A soft magnetic material is typically characterized by a large value of B_r (residual flux density)

(a) Statements (i), (ii) and (iv) are true

(b) Statements (ii), (iii), and (v) are true

(c) Statements (iii), (iv), and (v) are true

(d) Statements (iii) and (iv) are true

(e) Statements (i), (iv), and (v) are true

(f) None of the above statements are true

1.6 If an electric scalar potential field is given by $V(x, y, z) = 3x^2y - yz$ then which statement is false:

(a) $x^2y = 1$ is an equipotential line on the xy-plane

(b) At a point in space given by $(1, 0, -1)$ both V and $\mathbf{E}$ are zero.

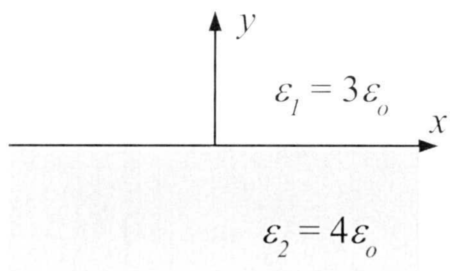
(c) The point in space given by $(2, -1, 4)$ lies on an equipotential surface with $V = -8$ Volts.

(d) At a point in space given by $(2, -1, 4)$ the electric field intensity is $\mathbf{E} = 12\mathbf{a}_x - 8\mathbf{a}_y + \mathbf{a}_z$ V/m.

(e) None of the above

Question 2

2.1 Two homogenous dielectric media share an uncharged boundary on the xz-plane as shown below. If $\mathbf{E}_1 = -5\mathbf{a}_x - 3\mathbf{a}_y + 4\mathbf{a}_z$ V/m calculate the following;



(a) The polarization vector in material 1, $\mathbf{P}_1$. [2 marks]

(b) The electric field intensity vector in material 2, $\mathbf{E}_2$. [2 marks]

(c) The angle $\mathbf{E}_2$ makes with the y-axis. [2 marks]

(d) The energy density in each region. [2 marks]

Question 2 (continued)

2.2 Consider a standard air-filled 100 pF parallel plate capacitor. Each plate is square with an area of 565 cm^2 and the distance between them is d. The potential difference between the top plate and the bottom plate is +300 V (i.e. the top plate is at a higher potential than the bottom plate). Assume that the space between the plates is vacuum and ignore fringing. Calculate the following:

(a) The charge density on each plate. [2 marks]

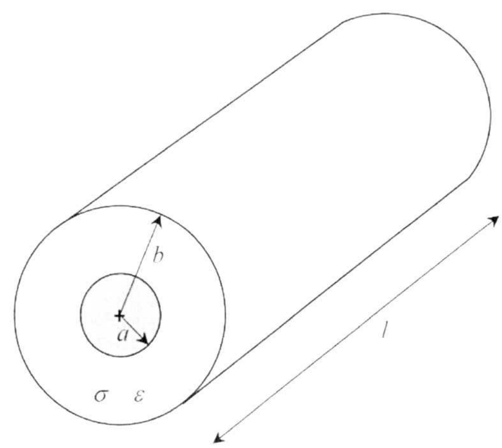
(b) The electrostatic force acting on each plate. [2 marks]

(c) Calculate the change in electrostatic energy if a homogenous dielectric material with a relative permittivity, ε_r, of 2.55 is carefully inserted between the plates and indicate whether the change is positive or negative. You can assume that the 300 V source has been removed from the capacitor before the dielectric piece is added. [3 marks]

Question 2 (continued)

2.3 A coaxial cable of length l carries a current of I_0 and is illustrated below. The radius of the inner perfect conductor is a while the radius of the outer perfect conductor is b. The material between the conductors has an electrical conductivity of σ and an electrical permittivity of ε. Ignoring fringing, show that the power dissipated by the cable is given by: [5 marks]

$$P = \frac{I_0^2 \ln(b/a)}{2\pi\sigma l}$$



Question 3

3.1 If $\mathbf{H} = 20r^2\mathbf{a}_\phi$ A/m then calculate the following;

(a) Calculate the current density, $\mathbf{J}$. [3 marks]

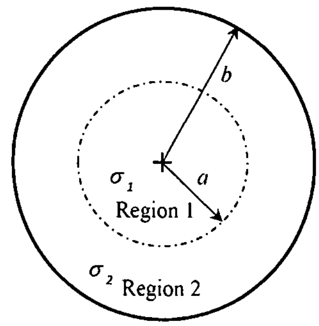
(b) Calculate the total current passing through the surface $0 \leq r \leq 1$, $0 \leq \phi \leq 2\pi$, $z = 0$. [3 marks]

Question 3 (continued)

3.2 The $z = 0$ plane carries a surface current $\mathbf{J}_S = 10\mathbf{a}_x$ A/m. There is one conducting wire that is situated at $y = 0$, $z = 6$ conducting a current, I_0, along the $\mathbf{a}_x$ direction. Calculate the value of I_0 such that $\mathbf{H} = 0$ A/m at position $(0, 0, 3)$. [6 marks]

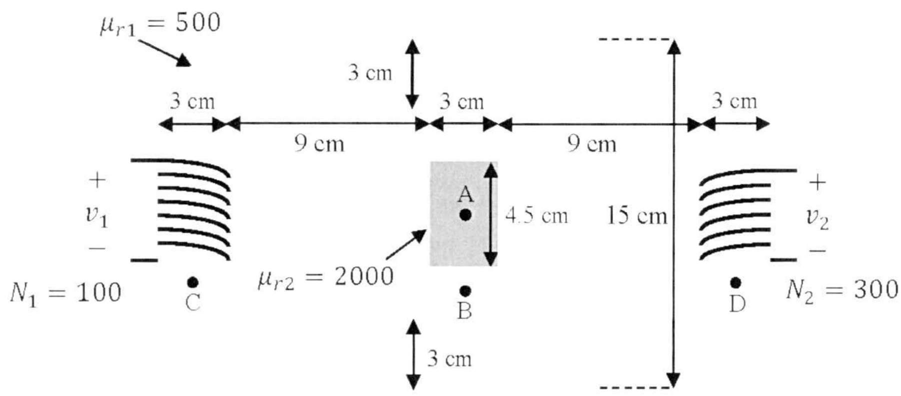
Question 3 (continued)

3.3 The cross-section of a very long electrically conducting wire is illustrated below. The conducting wire is comprised of two electrically conducting materials each with a different electrical conductivity. The material in region 1 has an electrical conductivity, $\sigma_1 = 1.0 \times 10^7$ S/m and the electrical conductivity in region 2 is $\sigma_2 = 4.0 \times 10^7$ S/m. Radius $a = 2$ mm and radius $b = 3$ mm. The total current that the conducting wire conducts is 0.1 A (out of the page in the positive $\mathbf{a}_z$ direction). If the wire lies centered about the z-axis, calculate $\mathbf{H}$ everywhere. [8 marks]



Question 4

Consider the magnetic circuit shown below. For this problem you can ignore the effects of fringing fields, and you can assume the core has a square cross-section (i.e., it extends 3 cm into the page). Both coils are tightly wound around the core.



4.1 For the magnetic circuit above, answer the following True/False questions. Briefly justify each of your answers with appropriate descriptions and/or calculations. [6 marks]

(a) (True / False) The self-inductance of coil 1 is larger than the self-inductance of coil 2 (i.e., $L_{11} > L_{22}$).

(b) (True / False) The magnetization vector, $\mathbf{M}$, is larger at point A than it is at point B.

(c) (True / False) If both coils are excited with sources that have the same voltage, V_0, then the magnetic flux at point C is larger than that at point D.

Question 4 (continued)

4.2 For the magnetic circuit shown on the previous page, determine:

(a) The mutual inductance of the system. [8 marks]

Question 4.2 (continued)

(b) The stored energy in the system if the coils are excited with input currents of $I_1 = 2$ A and $I_2 = 2$ A. You can assume the input currents flow into the coils from the positive terminal of the defined voltages (i.e., left to right for coil 1 and right to left for coil 2) [7 marks]

Question 5

A square conducting loop with side length w moves with a constant velocity of $\mathbf{u} = u_0\mathbf{a}_x$ in a region where the magnetic field is given by:

$$\mathbf{B} = \begin{cases} 0 & \text{if } y > 0 \\ B_0 e^{-x}\mathbf{a}_z & \text{if } y \leq 0 \end{cases}$$

At $t = 0$ s, the center of the loop passes through the origin, $(x, y) = (0, 0)$, so the position of the left side length of the loop can be described as $x(t) = u_0 t - \frac{w}{2}$.

(a) Draw a picture of this situation at $t = 0$ s and clearly label all relevant aspects of this problem. [4 marks]

(b) Determine the induced electromotive force on the loop. [10 marks]

Question 5(b) (continued)

(c) Calculate the induced current in the loop at $t = 2$ s if it is known that the loop consists of copper wires ($\sigma = 5.8 \times 10^7$ S/m), that have a circular cross section with radius $r = 0.5$ cm and that $w = 10$ cm, $B_0 = 7.47$ mT, and $u_0 = 1.65$ m/s. Indicate the direction this current flows at this time on your drawing above, and briefly justify your reasoning for this direction. For this problem you can ignore the effect of the loop's self-inductance. [7 marks]

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