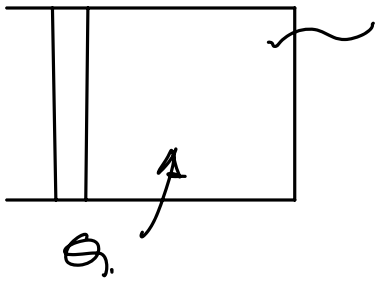
CHE 260

QUIZ 1 2023 - SOLUTION

1)



$P_1 = 100\ kPa$ $\quad P_2 = 700\ kPa$

$T_1 = 10^\circ C$ $\quad T_2 = 290^\circ C$

$V_1 = 0.2\ m^3$

$$m = \frac{P_1 V_1}{R T_1} = \frac{100\ kPa \times 0.2\ m^3}{2.07703 \frac{kJ}{kg\,K} \times (273+10)\,K} = 0.034\ kg$$

$$V_2 = \frac{m R T_2}{P_2} = \frac{0.034\ kg \times 2.07703 \frac{kJ}{kg\,K} \times 563\,K}{700\ kPa}$$

$$V_2 = 0.057\ m^3$$

$$P_1 V_1^n = P_2 V_2^n \Rightarrow \left(\frac{V_2}{V_1}\right)^n = \frac{P_1}{P_2}$$

$$\Rightarrow n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln(V_2/V_1)} = \frac{\ln\left(\frac{100\ kPa}{700\ kPa}\right)}{\ln\left(\frac{0.057\ m^3}{0.2\ m^3}\right)} = 1.550$$

$$w_{12} = -\int_{V_1}^{V_2} P\,dV = \frac{P_2 V_2 - P_1 V_1}{n-1}$$

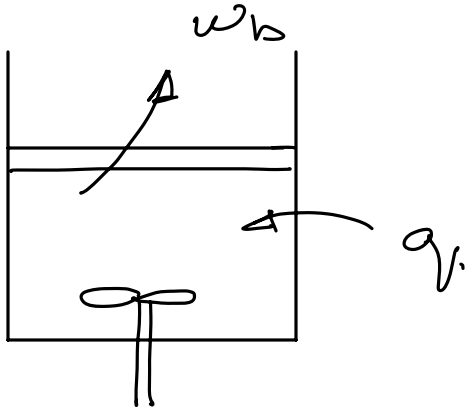
$$w_{12} = \frac{700\ kPa \times 0.057\ m^3 - 100\ kPa \times 0.2\ m^3}{1.550 - 1} = 36.18\ kJ$$

$$\Delta U = m C_v (T_2 - T_1)$$

$$= 0.034\ kg \times 3.1156 \frac{kJ}{kg\,K} (290-10)^\circ C = 29.66\ kJ$$

$$Q = \Delta U - W = 29.66\ kJ - 36.18\ kJ = -6.52\ kJ$$

2)



Air
$P_1 = 400\,kPa$
$T_1 = 17^\circ C$

$w_{shaft} = 75 \frac{kJ}{kg}$

$$q + w_{shaft} + w_b = \Delta u = \underbrace{c(T_2 - T_1)}_{=0,\ isothermal}$$

$$\Rightarrow \quad q = -w_{shaft} - w_b$$

For an isothermal process $PV = C$

$$w_b = P_1 V_1 \ln \frac{V_1}{V_2}$$

$$w_b = RT_1 \ln \frac{V_1}{V_2}$$

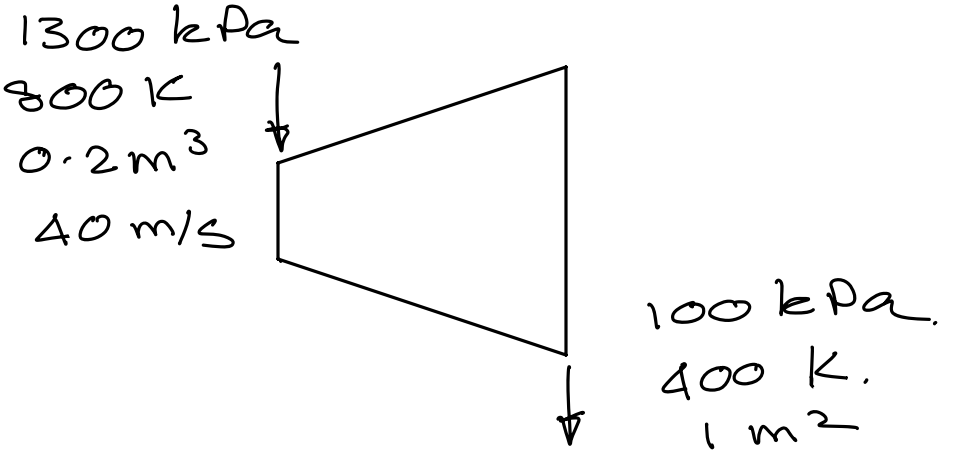
$$w_b = 0.287 \frac{kJ}{kg\,K} \times 290\,K \times \ln\left(\frac{1}{3}\right)$$

$$w_b = -91.44\ kJ/kg$$

$$q = -75\ kJ/kg + 91.44\ kJ/kg$$

$$q = 16.44\ kJ/kg$$

(3)



$$v_1 = \frac{RT_1}{P_1} = \frac{0.287\ kJ/kgK \times 800\ K}{1300\ kPa} = 0.1766\ \frac{m^3}{kg}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{0.2\ m^2 \times 40\ m/s}{0.1766\ m^3/kg} = 45.3\ kg/s$$

$$v_2 = \frac{RT_2}{P_2} = \frac{0.287\ kJ/kgK \times 400\ K}{100\ kPa} = 1.148\ \frac{m^3}{kg}$$

$$V_2 = \frac{\dot{m} v_2}{A_2} = \frac{45.3\ kg/s \times 1.148\ m^3/kg}{1\ m^2} = 52.0\ m/s$$

$$\dot{Q} + \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

($\dot{Q} = 0$, $z_2 - z_1 = 0$)

$$\dot{W} = \dot{m}\left[C_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}\right]$$

$$T_{avg} = \frac{800\ K + 400\ K}{2} = 600K \Rightarrow C_p = 1.051\ \frac{kJ}{kgK}$$

$$\dot{W} = 45.3\ \frac{kg}{s}\left[1.051\ \frac{kJ}{kgK}(800 - 400)K + \frac{(52\ m/s)^2 - (40\ m/s)^2}{2 \times 10^3\ \frac{J}{kJ}}\right]$$

$$\dot{W} = 19069\ kW = 19.1\ MW.$$