Name:____________________

Student #:_________________

Q1:____Q2:____Q3:____Q4:____Q5:____Q6:____

Total:_________

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

October 31, 2013

Instructor – W.R. Cluett

Closed book.

Allowable calculators: Casio FX-991MS or Sharp EL-520X (suffixes may differ)

All questions of equal value.

All work to be marked <u>must</u> appear on front of page. Use back of page for rough work only.

Given information:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}; \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix}; \ proj_{\vec{d}}\vec{u} = \frac{\vec{u}\cdot\vec{d}}{\|\vec{d}\|^2}\vec{d}$$

Q1: Find the matrix for the transformation that projects each point in R^3 (3-D) perpendicularly onto the plane $7x - y + 3z = 0$.

Q2: Consider a system of equations $AX = B$ and assume that it has at least two solutions for *X*, say X_1 and X_2.

a) Show that for all real values of the scalar *t*, $tX_1 + (1-t)X_2$ is also a solution.

b) Justify the assertion (statement) that if a system of linear equations has two solutions then it has infinitely many solutions.

c) Establish that this set of solutions contains a line. Give the equation of this line in vector form.

Q3: The vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent if and only if the only linear relationship among the vectors:

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$

is the trivial one, i.e. scalars $c_1 = c_2 = c_3 = 0$. Determine whether this set of 3 vectors is linearly dependent or independent:

$$\begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ -23 \end{bmatrix}$$

Q4: Let matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$. Is there some vector $\vec{v}$ such that

$T(\vec{v}) = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$ for the linear transformation $T(\vec{v}) = A\vec{v}$? If so, find $\vec{v}$.

Q5: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where the elements of matrix A are arbitrary real numbers.

a) Establish that the eigenvalues of the matrix A are given by:

$$\frac{(a+d)}{2} \pm \sqrt{\left[\frac{(a-d)}{2}\right]^2 + bc}$$

b) Establish the condition on the matrix A in order that it has two distinct (different) real eigenvalues.

c) Explain what happens to the eigenvalues if $(a-d)^2 = -4bc$. What are the eigenvalues in this case? What are the eigenvectors in this case? You may assume that $a \neq d$.

Q6:

a) Two lines are given:

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 2 \end{bmatrix} + t\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 10 \\ 2 \end{bmatrix} + s\begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$ where t and s are scalars. Do they have a point in common? If so, find this point.

b) In R^2 (2-D), two random lines will likely have a point in common. Is the same to be expected in R^3 (3-D)? Explain your answer.

c) Answer the related question about a random line and a random plane in R^3 (3-D).