TUTORING 6 SOLUTIONS

a) $A = \begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} .4-\lambda & 1-c \\ .6 & c-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (.4-\lambda)\lambda - .6(1-c) = 0$$

$$\lambda^2 - (.4+c)\lambda + (c-.6) = 0$$

$$\lambda = \frac{(.4+c) \pm \sqrt{(.4+c)^2 - 4(c-.6)}}{2}$$

$$= \frac{(.4+c) \pm \sqrt{c^2 - 3.2c + 2.56}}{2}$$

$$= \frac{(.4+c) \pm (c-1.6)}{2}$$

$$\lambda = c - 0.6,\ 1 \quad \text{(EIGENVALUES)}$$

$\lambda_1 = c - 0.6$

$$\begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix} \vec{u} = (c-0.6)\vec{u} \qquad \vec{u} \neq \vec{0}$$

LET $\vec{u} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = (c-0.6)\begin{bmatrix} x \\ y \end{bmatrix}$$

$$.4x + (1-c)y = (c-0.6)x$$

$$.6x + cy = (c-0.6)y$$

$$.4x + y - cy = cx - 0.6x$$

$$x - cx + y - cy = 0$$

$$(1-c)x + (1-c)y = 0$$

$$\therefore x = -y$$

$$.6x + \cancel{cy} = \cancel{cy} - 0.6y$$

$$\therefore x = -y$$

$$\therefore \vec{u} = \begin{bmatrix} -y \\ y \end{bmatrix} = y\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

EIGENVECTORS ASSOCIATED WITH λ_1 ARE ALL VECTORS PARALLEL TO $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

$\lambda_2 = 1$

$$\begin{bmatrix} .4 & 1-c \\ .6 & c \end{bmatrix} \vec{u} = \vec{u} \qquad \vec{u} \neq \vec{0}$$

$$.4x + (1-c)y = x \Rightarrow .6x = (1-c)y$$

$$.6x + cy = y \Rightarrow .6x = (1-c)y$$

$$\therefore x = \frac{1-c}{.6} y$$

$$\therefore \vec{u} = \begin{bmatrix} \frac{1-c}{0.6} y \\ y \end{bmatrix} = y \begin{bmatrix} \frac{1-c}{0.6} \\ 1 \end{bmatrix}$$

EIGENVECTORS ASSOCIATED WITH λ_2 ARE ALL VECTORS PARALLEL TO $\begin{bmatrix} \frac{1-c}{0.6} \\ 1 \end{bmatrix}$.

b) WHEN $c = 1.6$ $\lambda = 1, 1$

EIGENVECTORS ASSOCIATED WITH BOTH EIGENVALUES ARE ALL VECTORS PARALLEL TO $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Q2:

X_1: FISH (g) X_2: BREAD (g) X_3: VEG. (g)

$$
\begin{aligned}
.8x_1 + .25x_2 + .3x_3 &= 2.95 \\
0x_1 + .7x_2 + .6x_3 &= 3.3 \\
.2x_1 + .05x_2 + .1x_3 &= .75
\end{aligned}
$$

$$M = \left[\begin{array}{ccc|c} .8 & .25 & .3 & 2.95 \\ 0 & .7 & .6 & 3.3 \\ .2 & .05 & .1 & .75 \end{array}\right]$$

$$\Downarrow$$

$$M' = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] \text{(RNF)}$$

$\therefore$ SOLUTION IS 2g FISH
3g BREAD
2g VEG.

Q3:

$$\begin{array}{ccc} A & X & = \; B \end{array}$$

a) $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & 0 & 1 & -1 \\ 0 & 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 8 \end{bmatrix}$

$$M = [A \mid B]$$

$$\Downarrow$$

$$M' = \left[\begin{array}{cccc|c} 1 & 0 & .5 & -.5 & 1 \\ 0 & 1 & -1.5 & .5 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \text{(RNF)}$$

LEADING VARIABLES: x_1, x_2

FREE VARIABLES: x_3, x_4

$$x_1 = 1 - .5x_3 + .5x_4$$

$$x_2 = 4 + 1.5x_3 - .5x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -.5 \\ 1.5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} .5 \\ -.5 \\ 0 \\ 1 \end{bmatrix}$$

b) $x_1 = 1 + 2(-.5) + 2(.5) = 1$; $x_2 = 4 + 2(1.5) + 2(-.5) = 6$

$x_3 = x_4 = 2$

Q4:

a) $M = [A|B]$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & 5 & 10 & 44 \end{array}\right]$$

NOT RNF.

USE GE TO FIND M' (RNF OF M).

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 1 & 5 & 10 & 44 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 4 & 9 & 35 \end{array}\right] R2-R1$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & 1 & 9/4 & 35/4 \end{array}\right] R2 \div 4$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5/4 & 1/4 \\ 0 & 1 & 9/4 & 35/4 \end{array}\right] R1-R2$$

$$M' = \left[\begin{array}{ccc|c} 1 & 0 & -5/4 & 1/4 \\ 0 & 1 & 9/4 & 35/4 \end{array}\right] \text{(RNF)}$$

b) M' EQUATIONS ARE:

$$X = \tfrac{1}{4} + \tfrac{5}{4} Z$$
$$Y = \tfrac{35}{4} - \tfrac{9}{4} Z$$

X + Y ARE LEADING
Z IS FREE

$$\therefore \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1/4 \\ 35/4 \\ 0 \end{bmatrix} + Z \begin{bmatrix} 5/4 \\ -9/4 \\ 1 \end{bmatrix}$$

Z IS A FREE VARIABLE.

c) LOOKING FOR POSITIVE INTEGERS (X, Y, Z) THAT SATISFY THIS SOLUTION.

$Z = 1 \Rightarrow X = 6/4 \quad Y = 26/4$

$Z = 2 \Rightarrow X = 11/4 \quad Y = 17/4$

$Z = 3 \Rightarrow X = 16/4 = 4 \quad Y = 8/4 = 2$

$Z = 4 \Rightarrow X = 21/4 \quad Y = -1/4$

BEYOND $Z = 3$, VALUES OF Y ARE NEGATIVE.

$\therefore$ $(4, 2, 3)$ ARE THE ONLY POSITIVE INTEGERS THAT SATISFY THIS SOLUTION FOR (X, Y, Z).

Q5:

a)

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 200 \\ -25 \\ 175 \\ -150 \\ 200 \end{bmatrix}$$

$$\underset{A}{\Uparrow} \qquad \underset{X}{\Uparrow} \qquad \underset{B}{\Uparrow}$$

b)

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{array}\right]$$

$\Downarrow$ RNF VIA GAUSSIAN ELIMINATION

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 0 & -1 & 150 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{(RNF)}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 150 \\ 175 \\ 50 \\ 0 \\ 200 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

x_4, x_6 FREE

c) $x_4 = 50 \qquad x_6 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 150 \\ 125 \\ 50 \\ 0 \\ 200 \\ 0 \end{bmatrix} + 50 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 100 \\ 125 \\ 100 \\ 50 \\ 200 \\ 0 \end{bmatrix}$$

Q6:

a)

$$M = \left[\begin{array}{ccccc|c} -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 10 \\ 1 & 0 & 0 & 1 & 1 & 10 \end{array}\right]$$

$\Uparrow$ A $\qquad$ $\Uparrow$ B

$$X = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} \qquad AX = B$$

b)

$$M' = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\therefore \quad X = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 0 \end{bmatrix}$$

NOTE: NO FREE VARIABLES; EVERY VARIABLE IS A LEADING VARIABLE.

c) A ROW OF ZEROS APPEARS IN M' BECAUSE OF SOME REDUNDANCY IN THE MODEL EQUATIONS. SINCE THE LAST EQUATION IN M' IS SATISFIED FOR ALL I's, THIS LAST ROW DOES NOT CHANGE THE SOLUTION.