CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ FOR HEAT TRANSFER

21st NOVEMBER 2016

NAME:

STUDENT ID NUMBER:

Q1	Q2	Q3A	Q3B	Q3 bonus	Total
20	20	5	5	5	50 (with bonus, 53)

INSTRUCTIONS

1. This examination is closed book. Only one Letter-sized aid sheet is permitted.
2. Only type 3 calculators are permissible.
3. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
4. Answers written in pencil will NOT be re-marked. This is University policy.
5. For all problems, you must present the solution process in a step by step fashion for partial marks.
6. **ANSWERS WRITTEN ON THE BLANK, LEFT SIDES OF THE ANSWER BOOK WILL NOT BE ASSESSED. USE THE LEFT SIDES FOR ROUGH WORK ONLY. THIS WILL BE IMPLEMENTED STRICTLY THIS YEAR.**

Q.1. [20 points] HEAT TRANSFER FROM A HOT SPHERE INTO A STAGNANT FLUID

Consider a sphere of radius *a* whose entire volume is maintained at a uniform temperature T_0 . The sphere is placed in a fluid medium that is at a constant temperature T_∞ far away from the sphere. The thermal conductivity of the fluid is k_f . The fluid is stagnant everywhere. In this problem, convection effects and radiation can be ignored, and conduction is the only mechanism for heat transfer from the sphere into the fluid. The sphere is made of a hydrophobic (water-hating) material, and the fluid is hydrophilic (water-loving). As a result, a layer of extremely fine air bubbles is adsorbed on the surface of the sphere, which leads to a *contact* resistance to heat transfer between the sphere and the fluid. The boundary condition applicable at the sphere surface is, therefore,

$$T_0 - T\big|_{r=a} = \dot{q}_r\big|_{r=a} R_c,$$

where R_c is the contact resistance in ^{o}C-m^2/W.

Answer the following questions:

(a) **[13 points]** Beginning from the energy conservation equation in the spherical co-ordinate system [see last page], determine the *steady-state* temperature distribution *in the fluid*. Specify the governing equations and boundary conditions clearly. What are the types of boundary conditions (Dirichlet, Neumann or Robin)? Note that the domain for the governing equation will be $a \le r < \infty$, so boundary conditions have to be applied at $r = a$ and for $r \to \infty$.

Solution:

The governing equation for the energy balance in the spherical co-ordinate system is

$$\rho C_p \frac{\partial T}{\partial t} = \frac{k_f}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{k_f}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{k_f}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(\frac{\partial T}{\partial \phi}\right) + \dot{S}$$

At steady state, $\dfrac{\partial T}{\partial t} = 0$.

There is no heat source or sink in the fluid, hence $\dot{S} = 0$.

The problem is spherically symmetric. Therefore, there are no temperature variations in θ and ϕ directions, implying that $\frac{\partial T}{\partial \theta} = 0$ and $\frac{\partial T}{\partial \varphi} = 0$. Also, this means that the temperature is only a function of the radial co-ordinate.

Thus, the governing equation reduces to the following form:

$\frac{k_f}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$ **[Arguments leading to governing equation: 5 points]**

The boundary conditions are $T_0 - T\big|_{r=a} = -k_f \left.\frac{dT}{dx}\right|_{r=a} R_c$, and $T\big|_{r\to\infty} = T_\infty$.

[Boundary conditions: 2 points]

Integrating once, we have

$$r^2\frac{dT}{dr} = c_1$$

Dividing by r^2 and integrating once more, we have

$T = -\frac{c_1}{r} + c_2$ **[General form of T equation: 2 points]**

Applying the boundary condition at $r \to \infty$

$$T_\infty = 0 + c_2 \quad \Rightarrow \quad c_2 = T_\infty.$$

Thus,

$$T = -\frac{c_1}{r} + T_\infty$$

Applying the boundary condition at $r = a$, we have

$$T_0 - \left(-\frac{c_1}{a} + T_\infty\right) = -k_f\frac{c_1}{a^2}R_c,$$

Thus, $c_1 = -\frac{a\left(T_0 - T_\infty\right)}{\left(1 + \frac{k_f R_c}{a}\right)}$.

The temperature distribution is, therefore,

$$T = \frac{a}{r}\frac{\left(T_0 - T_\infty\right)}{\left(1+\frac{k_f R_c}{a}\right)} + T_\infty$$ **[Applying BC and substituting constants: 4 points]**

(b) **[3 points]** Calculate the heat transfer coefficient, h, if it is defined as

$$h = \frac{\dot{q}_r\big|_{r=a}}{T_0 - T_\infty}.$$

Hence, determine the Nusselt number for this heat transfer process, defined as

$$\mathrm{Nu} = \frac{ha}{k_f}.$$

Solution:

$$\dot{q}_r\big|_{r=a} = -k_f \left.\frac{dT}{dr}\right|_{r=a} = \frac{k_f\left(T_0 - T_\infty\right)}{a\left(1+\frac{k_f R_c}{a}\right)}$$

$$h = \frac{\dot{q}_r\big|_{r=a}}{T_0 - T_\infty} = \frac{k_f\left(T_0 - T_\infty\right)}{a\left(1+\frac{k_f R_c}{a}\right)\left(T_0 - T_\infty\right)} = \frac{k_f}{a\left(1+\frac{k_f R_c}{a}\right)}.$$ **[2 points for HTC]**

$$\mathrm{Nu} = \frac{ha}{k_f} = \frac{1}{\left(1+\frac{k_f R_c}{a}\right)}.$$ **[Nu expression: 1 point]**

(c) **[1 point]** Determine the temperature of the fluid at the sphere surface.

Solution:

$$T\big|_{r=a} = \frac{\left(T_0 - T_\infty\right)}{\left(1+\frac{k_f R_c}{a}\right)} + T_\infty.$$

(d) **[3 points]** Determine the heat flux and temperature of the fluid at the sphere surface for (i) low contact resistances and (ii) high contact resistances. What is the dimensionless number that determines how big or small the contact resistance is?

Solution:

The heat flux decreases monotonically as the contact resistance increases.

When R_c = 0, $\dot{q}_r\big|_{r=a} = \dfrac{k_f\left(T_0 - T_\infty\right)}{a}$

When $\dfrac{k_f R_c}{a} >> 1$ or $R_c >> \dfrac{a}{k_f}$, $\dot{q}_r\big|_{r=a} = \dfrac{k_f\left(T_0 - T_\infty\right)}{a\dfrac{k_f R_c}{a}} = \dfrac{\left(T_0 - T_\infty\right)}{R_c}$.

For large contact resistances, the flux is inversely proportional to the contact resistance. **[1 point]**

Similarly,

When R_c = 0, $T\big|_{r=a} = \dfrac{\left(T_0 - T_\infty\right)}{1} + T_\infty = T_0$.

When $\dfrac{k_f R_c}{a} >> 1$ or $R_c >> \dfrac{a}{k_f}$, $T\big|_{r=a} = \dfrac{\left(T_0 - T_\infty\right)}{\dfrac{k_f R_c}{a}} + T_\infty$.

Once $\dfrac{\left(T_0 - T_\infty\right)}{\dfrac{k_f R_c}{a}} << T_\infty$, $T\big|_{r=a} \to T_\infty$

Thus for very low contact resistance, the temperature of the fluid at the sphere surface is T_0, while for very large resistances, it is equal to the ambient fluid temperature, T_∞. **[1 point]**

The dimensionless number the determines the importance of the contact resistance is $\dfrac{k_f R_c}{a}$. **[1 point]**

2. [20 points] HEAT LOSSES THROUGH THE ROOF

A home has two identical rooms A and B. The roof of room A is constructed out of 40 mm thick wooden planks $\left(k_w = 0.11\ \mathrm{W/m \cdot K}\right)$. The area of each roof is 8 m x 12 m. While building Roof B, the contractor ran out of 40 mm thick wooden planks. Therefore, he constructed the 40 mm thick roof B by using a pair of 20 mm wooden planks nailed to each other using 40 mm long cylindrical metal nails of thermal conductivity $k_m = 50\ \text{W/m}\cdot\text{K}$. The diameter of each nail is 4 mm, and a total of 20,000 nails are used over the entire roof area.

The inner surface of each roof is maintained at 20°C. On a clear winter night, the ambient air is reported to be at 10°C, while the night sky temperature for radiation heat transfer is -18°C. The emissivity of the surface is 0.9. If these external conditions are identical for the two roofs, **compare the rates of heat loss through the roofs of rooms A and B, and find the outer surface temperatures of the two roofs**. Consider both convective and radiative heat losses, and ignore the contact resistance between the two nailed planks in roof B.

<u>Be sure to draw the thermal circuit for this problem.</u> **(3 points for roof A circuit, 3 points for roof B circuit)**

Note: The Newton-Raphson iterative formula for finding the root *x** of a function *f* (*x*), such that *f* (*x**) = 0, is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

Solution:

Room A (total 8 marks with circuit)

$A_{\text{wood}} = 8\times 12\ \text{m}^2$.

$R_{ceil} = \dfrac{L_{\text{wood}}}{k_{\text{wood}}A_{\text{wood}}} = \dfrac{4\times 10^{-2}}{0.11\times 96} = 0.003788\ ^{\text{o}}\text{C/W}$. **(1 mark for resistance calc)**

An energy balance gives

$$\dot{Q} = \frac{(T_1 - T_s)}{R_{\text{ceil}}} = hA(T_s - T_\infty) + \varepsilon\sigma A\left[(T_s + 273)^4 - (T_{\text{surr}} + 273)^4\right]$$

$$\frac{(20 - T_s)}{0.003788} = 15\times 96\times(T_s - 10) + 0.9\times 5.67\times 10^{-8}\times 96\times\left[(T_s + 273)^4 - (-18 + 273)^4\right].$$

$4.899\times10^{-6}\times(T_s+273)^4+1.704\times10^3 T_s-4.039\times10^4=0.$ **(1.5 marks for correct equation and reaching this point)**

The solution to this equation is $T_s=6.23^{\mathrm{o}}\mathrm{C}$. **(1.5 marks for Newton-Raphson calculation + answer)**

$\dot{Q}=\dfrac{(T_1-T_s)}{R_{\mathrm{ceil}}}=\dfrac{(20-6.23)}{0.003788}=3.635\,\mathrm{kW}$. **(1 mark for correct Q)**

Room B (total 10 marks with circuit)

$$A_{nails}=20000\times\pi\frac{\left(4\times10^{-3}\right)^2}{4}=\ 0.2514\ \mathrm{m}^2.$$

$A_{wood}=8\times12-0.2514=\ 95.75\ \mathrm{m}^2$. **(1 mark for proper area calculations)**

$$R_{ceil}=\left(\frac{1}{R_{wood}}+\frac{1}{R_{nails}}\right)^{-1}=\left(\frac{1}{\dfrac{L_{wood}}{k_{wood}A_{wood}}}+\frac{1}{\dfrac{L_{nail}}{k_{nail}A_{nails}}}\right)^{-1}$$ **(2 marks for resistance calculation)**

$$=\left(\frac{0.11\times95.75}{40\times10^{-3}}+\frac{50\times0.2514}{40\times10^{-3}}\right)^{-1}=0.001731\ ^{\mathrm{o}}\mathrm{C/W}.$$

$$\frac{(20-T_s)}{0.001731}=15\times96\times(T_s-10)+0.9\times5.67\times10^{-8}\times96\times\left[(T_s+273)^4-(-18+273)^4\right]$$

$4.899\times10^{-6}\times(T_s+273)^4+2.018\times10^3 T_s-4.667\times10^4=0.$ **(1.5 marks for correct equation and reaching this point)**

Solving for T_s, we get $T_s=7.99^{\mathrm{o}}\mathrm{C}$. **(1.5 marks for Newton-Raphson calculation + answer)**

$\dot{Q}=\dfrac{(T_1-T_s)}{R_{\mathrm{ceil}}}=\dfrac{(20-7.99)}{0.001731}=6.938\,\mathrm{kW}$. **(1 mark for correct Q)**

With the addition of nails, while the roof surface temperature does not change significantly, the rate of heat loss is nearly doubled. This was a poor choice by the contractor. **(2 marks for correct statement)**

3A. [5 points] CRUSHED ICE OR ICE CUBES?

Choose one of (a) through (c) for the first question, and justify it with one of (1) through (5). <u>Full credit if and only if answers to both questions are correct.</u>

You have a glass of tea in a well-insulated cup that you would like to cool off before drinking. You also have 2 ice cubes to use in the cooling process and an equivalent mass of crushed ice. Assuming no energy is lost from the tea into the room, which form of ice (cubes or crushed) added to your tea will give a lower final drink temperature?

(a) The crushed ice

(b) The ice cubes

(c) Either will lower the drink temperature the same amount

Because...

(1) Energy transfer is proportional to the mass of ice used

(2) Crushed ice will melt faster and will transfer energy from the tea faster

(3) Ice cubes contain less energy per mass than crushed ice so tea will cool more

(4) Ice cubes have a higher heat capacity than crushed ice

(5) Crushed ice has more surface area so energy transfer rate will be higher

Solution:

(c) and (1).

3B. [5 points] CRITICAL INSULATION RADIUS FOR A NANOSPHERE?

In class, you have seen the concept of the critical insulation radius for spherical particles. There is a critical radius of the sphere, below which the addition of a spherical insulation shell leads to an increase in the rate of heat transfer. However, these arguments were made assuming that the heat transfer coefficient, h, is a constant, i.e. independent of the radius of the insulated sphere. For a spherical nanoparticle, if the radius *after* insulation is R, it can be shown that $h = \frac{k_f}{R}$, where k_f is the thermal conductivity of the fluid. If that is the case, is there a critical insulation radius for a spherical nanoparticle?

Solution:

$$\mathrm{R_{eff}} = \frac{1}{4\pi k_I}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{1}{h4\pi R_2^2} = \frac{1}{4\pi k_I}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{1}{\frac{k_f}{R_2}4\pi R_2^2}$$

$$= \frac{1}{4\pi k_I R_1}+\frac{1}{4\pi R_2}\left(\frac{1}{k_f}-\frac{1}{k_I}\right)$$

[Deriving the expression: 2 points]

The effective thermal resistance with the insulation, relative to the resistance in the absence of the insulation is

$$\frac{\mathrm{R_{eff}}}{\mathrm{R_{eff,\ no\ ins}}} = \frac{\frac{1}{4\pi k_I R_1}+\frac{1}{4\pi R_2}\left(\frac{1}{k_f}-\frac{1}{k_I}\right)}{\frac{1}{4\pi k_f R_1}} = \frac{k_f}{k_I}+\frac{R_1}{R_2}\left(1-\frac{k_f}{k_I}\right)$$

$$=1+\frac{k_f}{k_I}-1+\frac{R_1}{R_2}\left(1-\frac{k_f}{k_I}\right)$$

$$=1+\left(\frac{k_f}{k_I}-1\right)\left(1-\frac{R_1}{R_2}\right)$$

Since $\left(1-\frac{R_1}{R_2}\right)>0$, whether $\mathrm{R_{eff}}$ increases or decreases relative to $\mathrm{R_{eff,\ no\ ins}}$ depends only on $\left(\frac{k_f}{k_I}-1\right)$. **[Resistance comparison: 2 points]**

Thus, there is no critical radius of insulation in this case. If $k_f > k_I$, the resistance always increases with the addition of insulation. If $k_f < k_I$, the resistance always decreases with the addition of insulation. **[Explanation: 1 point]**

Alternative explanation: The expression for the heat transfer coefficient is the expression in the conduction limit, as demonstrated in problem 1 of this quiz. Basically, by adding a spherical shell of the insulation, you are removing a spherical shell layer of the fluid. So, whether the resistance to heat transfer increases or decreases depends on the thermal conductivity of the insulation relative to the fluid. If the insulation has the lower thermal conductivity, the resistance increases. If the fluid has the lower thermal conductivity, replacing it with the higher conductivity insulation lowers the total thermal resistance.

Bonus (5 points, no partial credit): Using the appropriate dimensionless number, show that, for a spherical nanoparticle placed in water, the Brownian motion of the nanoparticle will not influence its heat exchange with the fluid, i.e. the heat transfer is always in the conduction-dominated regime, never in the convection-dominated regime.

Solution:

The appropriate dimensionless number to consider here is the Peclet number $Pe = \frac{UR}{\alpha_f}$.

This needs a measure of the velocity *U* of the sphere. The motion of a spherical nanoparticle is driven by random kicks of thermal energy of the magnitude kT, where k is the Boltzmann constant and T is the temperature. The characteristic Brownian force corresponding to these energy kicks is $\frac{kT}{R}$. According to Stokes law, the characteristic velocity of the sphere for this magnitude of the force is $U = \frac{kT/R}{6\pi\mu R} = \frac{kT}{6\pi\mu R^2}$, where μ is the viscosity of the fluid. The Peclet number becomes, therefore,

$$Pe = \frac{UR}{\alpha_f} = \frac{\frac{kT}{6\pi\mu R^2} R}{\alpha_f} = \frac{kT}{6\pi\mu R\alpha_f}$$

At room temperature, using the properties of water, we have, for a 1 nanometer radius sphere

$$Pe = \frac{\frac{8.314}{6.023\times10^{23}}\times 298}{6\pi\times10^{-3}\times10^{-9}\times10^{-7}} = 2.2\times10^{-6} << 1$$

Larger particles will only reduce Pe even further. Since Pe<<1, the heat transfer process is always in the conduction dominated limit; the Brownian motion of the particle does not influence heat transfer.