# **CIV102F Assignment # 2 – September 20-22, 2022**

Due September 27-29, (before assigned tutorial time), 2022

## **General Instructions**

- There are five questions on this assignment. All questions must be attempted; however, only one question will be graded.
- Submissions which are incomplete and do not contain a serious attempt to solve each question will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires both the correct process and answer.
- All final answers must be reported using slide-rule precision (ie, four significant figures if the first digit is a "1", three otherwise), and engineering notation for very large or very small quantities.
- Submissions must be prepared neatly and be formatted using the requirements discussed in the course syllabus. Marks will be deducted for poor presentation of work.

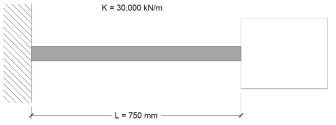
# **Assignment-Specific Instructions**

- When material properties are not provided in the question, use the table of material properties in Appendix A of the course notes if needed.
- Question 6 is optional.

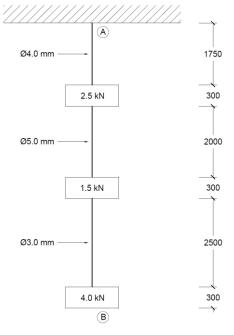
#### Linear Elastic Material (Stress-Strain) Behaviour

1. In class, we learned that using Hooke's law and the definitions of stress and strain, we can derive the spring constant "K" for a system by adjusting the choice of material and geometry. For the rectangular prism member shown below, calculate the required cross-sectional area and volume of material required to obtain the specified value of K for each of the three materials suggested below. Identify which material should be used to obtain the (i) cheapest. (ii) lightest, (iii) strongest and (iv) toughest equivalent springs.

- a. Low Alloy Steel
- b. Aluminum Alloy
- c. Carbon Fibre



**2.** The undeformed lengths of the system of cables and blocks are shown to the right. What is the total length of AB including the deformation due to the load? Assume that the blocks are rigid and that the cables are weightless. The Young's Modulus of the cables is 200,000 MPa. Calculate the total energy stored in the three cables assuming that they remain linear elastic. Note that all measurements provided are in mm and  $\varnothing$  denotes a diameter measurement (i.e.  $\varnothing 4.0 \text{ mm} = 4.0 \text{ mm}$  diameter).



**3.** The longest bridge span in the world is the main span of the Akashi Kaikyo bridge in Japan, completed in 1998. The central span of this suspension bridge is 1991 metres long (a half hour walk), and the deck is 36 m wide. If the total load which the deck must carry is 27.5 kPa (kN/m²), design the main cables of this bridge. The drape of the main cables between the towers and the centre of the bridge is 201.2 metres. The two main cables which support the deck are to be made of a collection of small wires 5.23 mm in diameter with a rupture stress of 1770 MPa. Calculate the number of wires needed in each cable if the *required factor of safety against rupture* is 2.20.



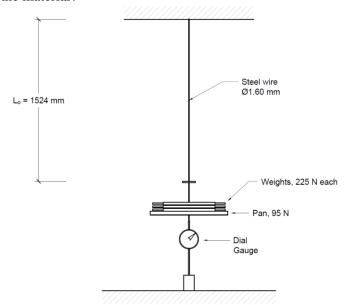
### Nonlinear Elastic-Plastic Material (Stress-Strain) Behaviour

**4.** Suppose that the steel wire shown below has a stress-strain curve described by the following set of equations:

$$\sigma_{s} = \begin{cases} E_{s}\varepsilon_{s} & 0.000 \leq \varepsilon_{s} \leq 0.002 \\ \sigma_{yield} & 0.002 \leq \varepsilon_{s} \leq 0.040 \\ 4000(\varepsilon_{s} - 0.040) + \sigma_{yield} & 0.040 \leq \varepsilon_{s} \leq 0.065 \\ \sigma_{ult} & 0.065 \leq \varepsilon_{s} \leq 0.100 \\ -1500(\varepsilon_{s} - 0.100) + \sigma_{ult} & 0.100 \leq \varepsilon_{s} \leq 0.200 \end{cases}$$

Where  $E_s = 200,000$  MPa,  $\sigma_{yield} = 400$  MPa and  $\sigma_{ult} = 500$  MPa. For the following question, assume that a wire with the geometry described in the figure below is being tested to determine its stress-strain behaviour.

- a) Plot the relationship between the wire tension P and the elongation  $\Delta$  if the wire was tested by gradually stretching the wire in small increments of  $\Delta$  and measuring the tension in the wire. The graph does not need to be to scale along the  $\Delta$  axis but should highlight all key points. This testing methodology is called *displacement control*.
- b) On the same graph, plot the relationship between the wire tension P and the elongation  $\Delta$  if the wire was tested by placing 225 N weights onto the 95 N loading pan and then measuring the change in length (i.e. in the same manner as the demonstration in tutorial 2). This testing methodology is called *force control*.
- c) Explain the differences in results. Which experimental method would be preferred if one was trying to measure the "full" stress-strain curve of the material?



**5.** A 2 m long wire made of low alloy steel is attached to a ceiling and has a rigid catch-plate securely attached to the other end. The diameter of the wire is 16 mm for the top 1 m, and 6 mm for the bottom 1 m. Ignore the self-weight of the wire and the catch-plate and neglect strain hardening for this question. The maximum strain the steel can sustain before rupture,  $\varepsilon_{\text{rupture}}$ , can be taken as 0.252 mm/mm.

h

A 3 kg weight is dropped onto the catch-plate from a height h:

- a) What is the maximum value of h which does not cause any permanent deformations in the wire?
- b) The weight is now dropped from a height of 1.9 m and then removed from the catch-plate. What is the maximum deformation of the wire due to the weight? What is the deformation of the wire after the weight is removed?
- c) If the weight is dropped repeatedly from a height of 1.9 m, the wire will eventually break. How many drops does it take?
- d) Suppose we repeated the process with a wire which was identical to the one shown but had a uniform diameter of 6 mm over the entire 2 m length. Can this new wire withstand more drops than the nonuniform wire? Why? A qualitative answer is acceptable.

### **Advanced Analytical Methods**

**6 (Optional).** When performing structural analysis, we typically calculate the stresses in the structure assuming that the initial geometry of the structure does not change due to the applied loads. This is usually a reasonable assumption because these deformations are typically small. However, sometimes these deflections are large and need to be considered as they modify how the load is carried – these are called **second-order effects**.

Complete the following questions if the steel wire ABC has a diameter of 3 mm and an ultimate strength of 1500 MPa. The geometry shown in the figure is the initial geometry of the wire system. Points A and C are fixed in place and do not move.

- a) Calculate the forces and stresses in the wires AB and BC. How much longer does each wire become due to the applied load? How far does the 2000 N load move downwards? What is the factor of safety of this structure?
- b) Because the wires have gotten longer, the geometry of the system has changed, which has caused the tension in the wires to change as well. Re-calculate the forces and stresses in wires AB and BC using the deformed lengths of AB and BC you calculated in part a to define the system's geometry. Has the factor of safety increased or decreased?
- c) Using the stresses calculated in part b, calculate the length of wires AB and BC caused by carrying the 2000 N load. Are these values consistent with the lengths used to compute the stresses? Based on your answer, estimate how much the 2000 N load has moved downwards.
- d) Comment on the differences between an analysis using the initial geometry only and an analysis which includes second-order effects. Can you think of an example where neglecting second-order effects may be dangerous?

