UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Term Test II

First Year — Program 5

MAT185H1S — Linear Algebra

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Student Name:			
	Last Name	First Names	
Student Number:		Tutorial Section:	TUT

Instructions:

1. Attempt *all* questions.
2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question.
4. *No* aid is permitted.
5. The duration of this test is 90 minutes.
6. There are 9 pages and 5 questions in this test paper.

For Markers Only		
Question	**Value**	**Mark**
A		
1	10	
B		
2	10	
C		
3	10	
4	10	
5	10	
Total	50	

A. Definitions and Statements

Fill in the blanks.

1(a). The *span* of $\{\boldsymbol{v}_1 \ldots \boldsymbol{v}_n\} \subset \mathcal{V}$, a vector space, is defined as

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/2

1(b). The span of the empty set (span $\emptyset$), in a vector space, is

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/2

1(c). Give a mathematical test for the set of vectors $\{\boldsymbol{v}_1 \ldots \boldsymbol{v}_n\}$ to be *linearly dependent*.

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/2

1(d). State the *contrapositive* of the Fundamental Theorem of Linear Algebra.

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/2

1(e). A *basis* for the space of symmetric 2×2 matrices is

/2

B. True or False

Determine if the following statements are true *or* false *and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.*

2(a). Every vector space possesses at least two subspaces.

2(b). The set $\{1, cos2x, \sin^2 x, e^x\}$ is linearly independent in the function space $\mathcal{F}(\mathbb{R})$.

2(c). The set $\{1+x, 2-x^2, 3+5x^2, 7-2x\}$ is linearly independent in $\mathbb{P}_2(\mathbb{R})$.

2(d). If $\dim \mathcal{V} = n$ and M is a finite subset of $\mathcal{V}$ containing $k < n$ vectors, then M is linearly independent.

2(e). If $\mathcal{U} \cap \mathcal{W} = \mathcal{U}$ and $\dim \mathcal{U} = \dim \mathcal{W}$, then $\mathcal{U} = \mathcal{W}$.

C. Problems

3. Let M and N be two finite nonempty subsets of a vector space $\mathcal{V}$.

(a) Prove that $\operatorname{span}(M \cap N) \subseteq \operatorname{span} M \cap \operatorname{span} N$.

(b) Is $\operatorname{span}(M \cap N) = \operatorname{span} M \cap \operatorname{span} N$? Justify your answer.

3(a). Prove that $\operatorname{span}(M \cap N) \subseteq \operatorname{span} M \cap \operatorname{span} N$.

/5

3(b). Is span $(M \cap N) = \text{span}\, M \cap \text{span}\, N$? Justify your answer.

/5

4. Given that $\mathcal{S} = \{p \in \mathbb{P}_4 \,|\, p(1) = p(-1) = 0\}$ is a subspace of $\mathbb{P}_4$, find a basis for $\mathcal{S}$ and determine the dimension of $\mathcal{S}$.

. . . cont'd

4. ...cont'd

/10

5. Let $\mathcal{V}$ be a vector space and let $M \neq \emptyset$ be a subset of n vectors of $\mathcal{V}$. We say that M is a *maximal linearly independent set* if and only if

(i) M is linearly independent

(ii) If $\boldsymbol{x} \in \mathcal{V} \backslash M$ (*i.e.*, $\boldsymbol{x} \in \mathcal{V}$ but $\boldsymbol{x} \notin M$), then $M \cup \{\boldsymbol{x}\}$ is linearly dependent

Show that M is a basis for $\mathcal{V}$ if and only if M is a maximal linearly independent set.

. . . cont'd

5. . . . cont'd

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