CHE 260: THERMODYNAMICS AND HEAT TRANSFER

QUIZ #2

18th NOVEMBER 2013

NAME:

STUDENT ID NUMBER:

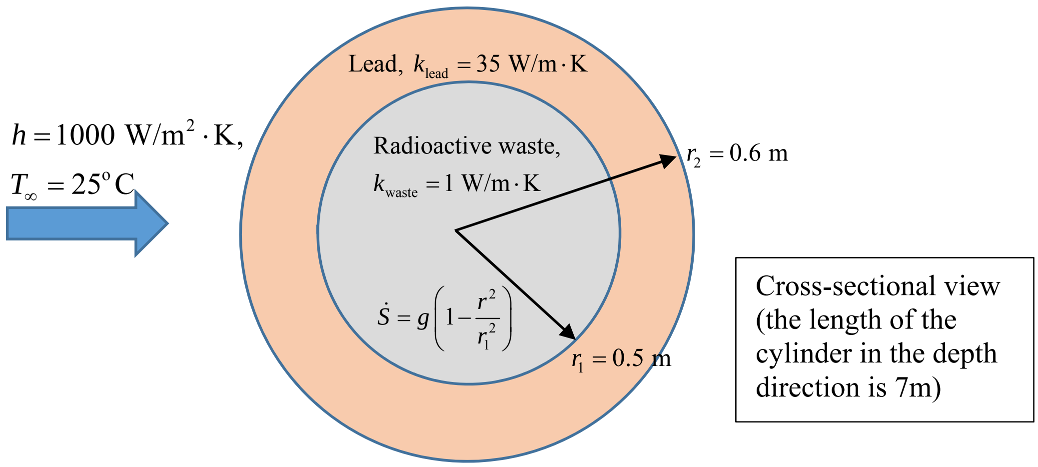
Q1	Q2	Q3	Q4A	Q4B	Q4C	Q4D	Total
25	25	25	5	5	5	10	100

INSTRUCTIONS

1. This examination is open textbook, open notes, but closed internet, closed all communication devices.
2. Please write legibly. If your handwriting is unreadable, your answers will not be assessed.
3. Answers written in pencil will NOT be re-marked. This is University policy.
4. For all problems, you must present the solution process in a step by step fashion for partial marks.
5. **Answers written on the blank, left sides of the answer book will not be assessed. Use the left sides for rough work only.**

Q.1. [25 points]

Solid radioactive waste (k_{waste} = 1 W/m·K) is packed inside a hollow, cylindrical lead container (k_{lead} = 35 W/m·K) with inner and outer diameters of 1.0 and 1.2 m, respectively. The length of the cylinder is 7 m. There is negligible thermal contact resistance at the boundary between the two phases. Thermal energy is generated within the waste material at a rate of $\dot{S} = g\left(1 - \frac{r^2}{r_1^2}\right)$ where g = 5 x 10^5 W/m^3, r is the radial co-ordinate measured from the center of the cylinder, and r_1 = 0.5 m is the inner radius of the container. The outer surface is exposed to water at 25°C, with a heat transfer coefficient of 1000 W/m^2·K.



Answer the following questions:

(a) Starting with the main heat conduction equation [see end of question booklet for this], for steady state conditions, write down the governing equations and boundary conditions for the temperature distributions in the radioactive solid phase, and the lead shell. Assume negligible contact resistance between the radioactive solid and the lead shell.

Solution:

For steady state conditions, assuming that temperature profiles that depend only on the radial co-ordinate [i.e. do not depend on the axial (z) or azimuthal (θ) co-ordinates], the governing equations for the waste and lead sections of the cylindrical container are

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{waste}}}{dr}\right) = -\frac{\dot{S}}{k_{\text{waste}}} = -\frac{g}{k_{\text{waste}}}\left(1 - \frac{r^2}{r_1^2}\right), \qquad (1)$$

and

$$\frac{k_{\text{lead}}}{r}\frac{d}{dr}\left(r\frac{dT_{\text{lead}}}{dr}\right) = 0\,, \qquad (2)$$

The boundary conditions are as follows:

Convection at the outer surface:

$$-k_B \left.\frac{dT_B}{dr}\right|_{r=r_2} = h\left(T_B\big|_{r=r_2} - T_\infty\right), \tag{3}$$

Temperature and flux continuity:

$$T_{\text{waste}}\big|_{r=r_1} = T_{\text{waste}}\big|_{r=r_1}, \tag{4}$$

$$-k_{\text{waste}} \left.\frac{dT_A}{dr}\right|_{r=r_1} = -k_{\text{waste}} \left.\frac{dT_B}{dr}\right|_{r=r_1}, \tag{5}$$

A fourth boundary condition is required. This is that the temperature at the center of the cylindrical core is finite.

$$T_{\text{waste}}\big|_{r=0} = \text{finite} \tag{6}$$

<u>2 points each for each governing equation, 1.5 points for each boundary condition. (2 * 2 + 1 * 4 = 8 points)</u>

(b) Determine the rate of heat transfer at steady state from the outer surface of the container at steady state.

<u>Solution</u>:

$\dot{Q}$ can also be obtained by realizing that the total heat lost from the surface is also the total heat generated within the volume of the inner cylindrical core.

$$\dot{Q} = \int_0^{r_1} \dot{S}\left(2\pi r dr\right)L = \int_0^{r_1} g\left(1-\frac{r^2}{r_1^2}\right)\left(2\pi r dr\right)L$$

$$= 2\pi Lg\int_0^{r_1}\left(r-\frac{r^3}{r_1^2}\right)dr = 2\pi Lg\left.\left(\frac{r^2}{2}-\frac{r^4}{4r_1^2}\right)\right|_0^{r_1}$$

$$= 2\pi Lg\left(\frac{r_1^2}{2}-\frac{r_1^2}{4}\right)$$

$$= \frac{1}{2}\pi r_1^2 Lg$$

With the given values, $\dot{Q} = 137.44\ \text{kW}$.
<u>4 points for derivation, and 1 point for the correct value</u>.

(c) Derive the steady-state temperature distributions in the radioactive waste, and in the lead shell. Evaluate the temperature at the center of the cylinder.

Solution:

Multiplying Eq. (1) by r and integrating w.r.t r, we get

$$r\frac{dT_{\text{waste}}}{dr} = -\frac{g}{k_{\text{waste}}}\left(\frac{r^2}{2} - \frac{r^4}{4r_1^2}\right) + c_1$$

Dividing by r, we get

$$\frac{dT_{\text{waste}}}{dr} = -\frac{g}{4k_{\text{waste}}}\left(\frac{r}{2} - \frac{r^3}{4r_1^2}\right) + \frac{c_1}{r}$$

Integrating the above equation w.r.t r, we get

$$T_{\text{waste}} = -\frac{g}{k_{\text{waste}}}\left(\frac{r^2}{4} - \frac{r^4}{16r_1^2}\right) + c_1 \ln r + c_2 \tag{7}$$

We also integrate Eq. (2) twice to get

$$T_{\text{lead}} = c_3 \ln r + c_4. \tag{8}$$

The boundary condition (6) when applied to Eq. (7) requires $c_1 = 0$ in order to eliminate the singular log term.

$$T_{\text{waste}} = -\frac{g}{4k_{\text{waste}}}\left(r^2 - \frac{r^4}{4r_1^2}\right) + c_2 \tag{9}$$

Applying the boundary condition (5),

$$-k_{\text{waste}}\left(-\frac{g}{4k_{\text{waste}}}\left(2r_1 - \frac{r_1^3}{r_1^2}\right)\right) = -k_{\text{lead}}\frac{c_3}{r_1},$$

which yields

$$c_3 = -\frac{g\, r_1^2}{4k_{\text{lead}}}. \tag{10}$$

The temperature T_{lead} becomes

$$T_{\text{lead}} = -\frac{g\ r_1^2}{4k_{\text{lead}}}\ln r + c_4. \qquad (11)$$

Applying the boundary condition (3),

$$-k_{\text{lead}}\left(-\frac{g\ r_1^2}{4k_{\text{lead}}}\frac{1}{r_2}\right) = h\left(-\frac{g\ r_1^2}{4k_{\text{lead}}}\ln r_2 + c_4 - T_\infty\right). \qquad (12)$$

This gives c_4 as

$$c_4 = \frac{g\ r_1^2}{4hr_2} + \frac{g\ r_1^2}{4k_{\text{lead}}}\ln r_2 + T_\infty. \qquad (13)$$

The temperature profile in the lead shell, therefore,

$$T_{\text{lead}} = -\frac{g\ r_1^2}{4k_{\text{lead}}}\ln r + \frac{g\ r_1^2}{4hr_2} + \frac{g\ r_1^2}{4k_{\text{lead}}}\ln r_2 + T_\infty = \frac{g\ r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r}\right) + \frac{g\ r_1^2}{4hr_2} + T_\infty. \qquad (14)$$

Applying the last boundary condition (4), we get

$$-\frac{g}{4k_{\text{waste}}}\left(\frac{3r_1^2}{4}\right) + c_2 = \frac{g\ r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r}\right) + \frac{g\ r_1^2}{4hr_2} + T_\infty.$$

and this yields c_2 as

$$c_2 = \frac{3r_1^2 g}{16k_{\text{waste}}} + \frac{g\ r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r}\right) + \frac{g\ r_1^2}{4hr_2} + T_\infty.$$

The temperature distribution in the radioactive waste is, therefore,

$$T_{\text{waste}} = \frac{g}{16k_{\text{waste}}}\left(3r_1^2 - 4r^2 + \frac{r^4}{r_1^2}\right) + \frac{g\ r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r}\right) + \frac{g\ r_1^2}{4hr_2} + T_\infty. \qquad (15)$$

The temperature at the center is

$$T_{\text{waste}}\Big|_{r=0} = \frac{3r_1^2 g}{16k_{\text{waste}}} + \frac{g\ r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r_1}\right) + \frac{g\ r_1^2}{4hr_2} + T_\infty \qquad (17)$$

Plugging in the values, we get the temperature at the center as 23438$^{\text{o}}$C ,which is extremely high, would probably melt/vaporize the waste. The waste needs to be stored in tubes that are an order of magnitude smaller, so that the maximum temperature may be controlled.

[4 points each for the correct temperature distributions in the lead and radioactive shell, 2 points for the center temperature].

(d) Evaluate the temperature at the inside surface of the lead shell. If the melting point of lead is 323°C, is the choice of lead as the shell material a good design choice?

Solution:

The temperature at the interface between the waste and lead shell

$$T_{\text{waste}}\Big|_{r=r_1} = \frac{g\,r_1^2}{4k_{\text{lead}}}\ln\left(\frac{r_2}{r_1}\right) + \frac{g\,r_1^2}{4hr_2} + T_\infty \qquad (16)$$

Plugging in the values we get this temperature as 238.87°C. As far as the local temperature is concerned, this is adequate, as it will not melt the lead shell.

[2 points for the lead/waste interface temperature].

Q.2. [25 points]

A spherical cryosurgical probe may be embedded in a diseased tissue for the purpose of freezing and destroying the tissue. Consider a probe of 3 mm diameter whose surface is maintained at -30°C when embedded in a tissue at 37°C. At steady state, a spherical layer of frozen tissue forms around the probe at a temperature of 0°C existing at the phase front between the frozen and normal tissues. If the thermal conductivity of frozen tissue is approximately 1.5 W/m-K, and heat transfer at the phase front may be characterized by an effective convection coefficient of 50 W/m^2·K, what is the thickness of the layer of frozen tissue (assuming negligible perfusion)? [**Hint:** Draw a picture of the situation first. This problem is not longer than a few lines if you think about it right. Use a resistance approach to set it up. You should get a quadratic equation for the thickness of the frozen tissue; pick the physical root.]

Solution :

KNOWN: Diameter and surface temperature of a spherical cryoprobe. Temperature of surrounding tissue and effective convection coefficient at interface between frozen and normal tissue.

FIND: Thickness of frozen tissue layer.

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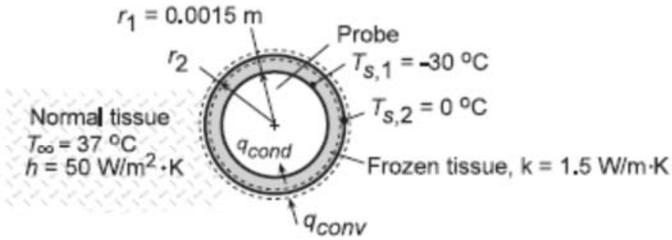


ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible contact resistance between probe and frozen tissue, (3) Constant properties, (4) Negligible perfusion effects.

ANALYSIS: Performing an energy balance for a control surface about the phase front, it follows that

$$q_{conv} - q_{cond} = 0$$

Hence,

$$h\left(4\pi r_2^2\right)\left(T_\infty - T_{s,2}\right) = \frac{T_{s,2} - T_{s,1}}{\left[(1/r_1) - (1/r_2)\right]/4\pi k}$$

$$r_2^2\left[(1/r_1) - (1/r_2)\right] = \frac{k}{h}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)}$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = \frac{k}{hr_1}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)} = \frac{1.5\,\mathrm{W/m\cdot K}}{\left(50\,\mathrm{W/m^2\cdot K}\right)(0.0015\,\mathrm{m})}\left(\frac{30}{37}\right)$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = 16.2$$

$$\left(r_2/r_1\right) = 4.56$$

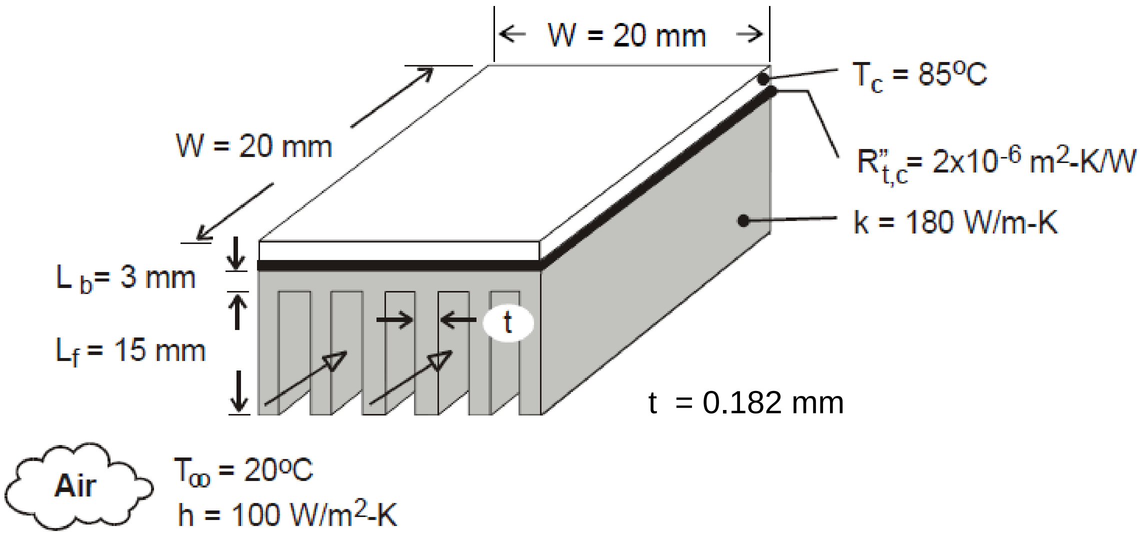
It follows that r_2 = 6.84 mm and the thickness of the frozen tissue is

$$\delta = r_2 - r_1 = 5.34\,\mathrm{mm} \qquad <$$

<u>15 points for the correct expressions for conductive and convective heat transfer, and 5 points for quadratic equation, 5 points for the correct thickness.</u>

Q.3. [25 points]

A square isothermal silicon chip of width $W = 20$ mm is soldered to a square aluminum heat sink ($k = 180$W/m·K) of equivalent width. The heat sink has a base thickness of $L_b = 3$ mm and an array of 11 rectangular fins, each of length $L_f = 15$ mm and a fin thickness of $t = 0.182$ mm. Air flow at $T_\infty = 20$°C is maintained through channels formed between the fins, which results in a convection coefficient of $h = 100$ W/m^2·K. The solder joint has a thermal contact resistance of $R''_{t,c} = 2\times10^{-6}$ m^2·K/W.



If the maximum allowable chip temperature is 85°C, what is the corresponding value of the chip power $\dot{Q}$? Use the resistance approach.

Solution:

KNOWN: Dimensions and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c} / W^2 = 2\times10^{-6}\text{m}^2\cdot\text{K}/\text{W}/(0.02\text{m})^2 = 0.005\ \text{K}/\text{W}$ and $R_{t,b} = L_b / k\left(W^2\right)$

$= 0.003\text{m}/180\ \text{W}/\text{m}\cdot\text{K}\left(0.02\text{m}\right)^2 = 0.042\ \text{K}/\text{W}$. From Eqs. (3.108), (3.107), and (3.104)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02\text{m} \times 0.015\text{m} = 6 \times 10^{-4}\ \text{m}^2$ and $A_b = W^2 - N(tW) = (0.02\text{m})^2 - 11(0.182 \times 10^{-3}\ \text{m} \times 0.02\text{m}) = 3.6 \times 10^{-4}\ \text{m}^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\ \text{W/m}^2\text{·K}/180\ \text{W/m·K} \times 0.182 \times 10^{-3}\text{m})^{1/2}\ (0.015\text{m}) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.92) yields

$$\eta_f = \frac{\tanh mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\ \text{m}^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{(85-20)°\text{C}}{(0.005 + 0.042 + 2.00)\text{K}/\text{W}} = 31.8\ \text{W} \qquad <$$

<u>**Correct resistance circuit – 5 points, calculation of correct fin resistance 10 points, calculation of the two other resistances 3 points each, 4 points for the correct heat transfer rate.**</u>

Q.4A. [5 points]

A hot liquid produced in a microfluidic reactor is transported using a plastic tube to a storage chamber. Currently, a tube with the inner diameter (ID) of 1.5 cm and a wall thickness of 0.1 cm is being used for this transport. (The wall thickness is the difference between the outer and inner radius of the tube). However, this tube is found to be inadequate in restricting the heat loss, and the tube wall thickness needs to be increased to improve the insulation. A tube vendor provides tubes of the same wall material with the desired 1.5 cm ID, but with three other wall thicknesses: 0.2 cm, 0.3 cm and 0.4 cm. If radiation effects are negligible, as a heat transfer engineer, which tube would you pick? You must justify your choice to get credit. Take the thermal conductivity of the plastic to be 0.1 W/m·K, and the heat transfer coefficient due to natural convection in air to be 10 W/m²·K.

<u>Solution</u>:

If r_1 and r_1+T are the inner and outer radii of the tube of thickness T, then the combined thermal resistance of the tube wall and convective heat transfer is

$$R=\frac{\ln\left(\left(r_1+T\right)/r_1\right)}{k2\pi L}+\frac{1}{h2\pi r_2 L}=\frac{1}{2\pi Lhr_1}\left[\left(\frac{hr_1}{k}\right)\ln\left(1+T/r_1\right)+\frac{1}{\left(1+T/r_1\right)}\right].$$

where $k = 0.1$ W/m·K is the thermal conductivity of the tube wall, and $h = 10$ W/m^2·K is the convective heat transfer coefficient. Here r_1=0.75 cm. Therefore

$$R=\frac{1}{2\pi Lhr_1}\left[0.75\ln\left(1+T/0.75\right)+\frac{1}{\left(1+T/0.75\right)}\right], \text{ where } T \text{ is in cm}$$

Let us consider the ratio of the resistances of the new tubes to the original tube

$$\frac{R_T}{R_{0.1\text{ cm}}}=\frac{0.75\ln\left(1+T/0.75\right)+\frac{1}{\left(1+T/0.75\right)}}{0.75\ln\left(1+0.1/0.75\right)+\frac{1}{\left(1+0.1/0.75\right)}}=\frac{\left[0.75\ln\left(1+T/0.75\right)+\frac{1}{\left(1+T/0.75\right)}\right]}{0.9762}$$

Thus, $\frac{R_{0.2\text{ cm}}}{R_{0.1\text{ cm}}}=0.990$, $\frac{R_{0.3\text{ cm}}}{R_{0.1\text{ cm}}}=0.990$ and $\frac{R_{0.4\text{ cm}}}{R_{0.1\text{ cm}}}=0.996$.

There is hardly any change in the thermal resistances with the available tube thicknesses. If anything, these tubes will slightly increase the heat transfer rate. Therefore, all three tubes provided by the vendor will be inadequate for reducing the rate of heat transfer. We need to find a tube with thickness that is much larger, or wrap the tube with thick insulation of a better insulating material.

2 points for realizing it is a critical insulation radius problem, 3 points for calculating the resistances and showing that none of the tubes will work.

Q.4B. [5 points]

In an experiment to measure the thermal conductivity of a polymeric liquid, a concentric cylindrical cell with heaters contacting the inner and outer cylindrical surfaces is used. Due to the temperature gradients, circulation currents are setup in the cell, with a velocity of 2 mm/min. The scientists performing this experiment claim that these convection currents are so weak that their influence on the heat transfer process is negligible; hence they are truly measuring conductive heat transfer in the cylindrical gap. Would you trust the thermal conductivities reported from such an experiment? Explain why/why not. The gap between the cylinders is 2 cm. The density of the liquid is 1000 kg/m^3, its specific heat capacity is 2500 J/kg·K, and the true thermal conductivity of the liquid is 0.3 W/m·K.

Solution:

To estimate the effect of convection on the thermal conductivity measurement, we can calculate the Peclet number

$$\mathrm{Pe} = \frac{UD}{\alpha}$$

where U is the velocity of fluid motion, D is the gap width, and α is the thermal diffusivity.

Given U = 2 mm/ min = 3.333×10^{-5} m/s, D = 0.02 m. The thermal diffusivity can be obtained as

$$\alpha = \frac{k}{\rho C_p} = \frac{0.3}{1000\times2500} = 1.2\times10^{-7}\ \mathrm{m^2/s}$$

The Peclet number is

$$\mathrm{Pe} = \frac{3.333\times10^{-5}\times0.02}{1.2\times10^{-7}} = 5.55$$

Since the Peclet number is O(1), the contribution of convection to heat transfer cannot be neglected. Hence, the thermal conductivity measurements from this research group cannot be trusted.

<u>3 points for realizing that the Peclet number needs to be calculated, 2 points for the correct Pe value and conclusion.</u>

Q.4C. [5 points]

The diurnal cycle (the cycle of day and night) heats and cools the earth's surface. Obtain an approximation for the depth of penetration of the thermal front into the earth's crust due to this cycle. Take the thermal diffusivity of earth's crust to be 1 mm^2/s.

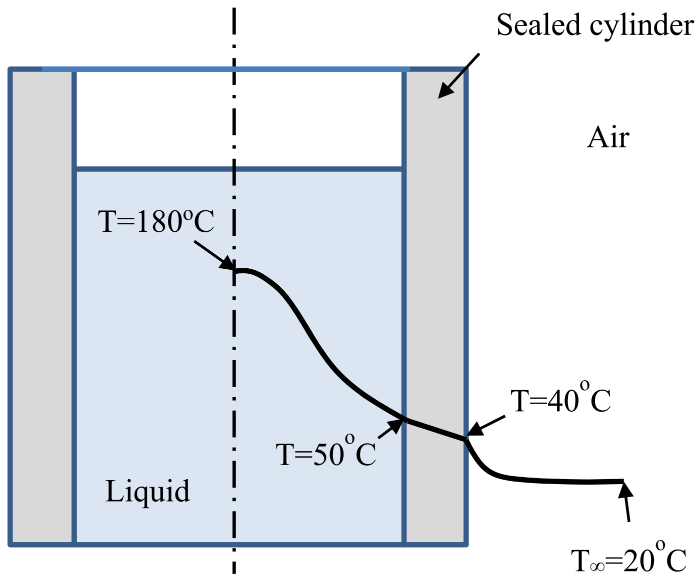
<u>Solution</u>:

To estimate the depth, we use the diffusion length scale $\sqrt{\alpha t}$, where a is the thermal diffusivity of the earth's crust and t is the time scale of the dynamic process. Here the appropriate time scale to use is the time period of diurnal cycle, which is 24 hours or 86400 seconds. Thus, the characteristic depth of penetration of the thermal front into the earth's crust is expected to be $\sqrt{10^{-6}\times86400} = 0.2939$ m or about 30 cm.

The characteristic depth may also be obtained from a 12 hour period, $\sqrt{10^{-6}\times43200} = 0.2078$ or about 21 cm. The depth is therefore, on the order of few tens of centimeters.

<u>3 points for realizing that the thermal diffusion length scale needs to be calculated, 2 points for the correct depth value.</u>

Q.4D. [10 points]



(The temperature profile is shown only on the right half of the picture. The cylinder is axisymmetric)

You are working in a chemical manufacturing facility. In one manufacturing step in this facility, a hot liquid is cooled down in a sealed, cylindrical vessel from a temperature of T_0 = 200°C to room temperature. According to the existing practice in the facility, the liquid in the cylinder is left stagnant (unmixed), and cooling occurs due to natural convection in the air external to the cylinder. The liquid itself is highly viscous, and natural convection within the liquid can be neglected as a heat transfer mechanism.

Your boss is interested in speeding up the rate of cooling. A senior engineer in your group suggests during a group discussion that if you switched from natural to forced convection on the outside by adding some blowing fans, you can improve the rate of heat transfer. Before suggesting this idea to your boss, you measure the temperature profile in the system some time after the hot liquid is loaded, and it appears as shown in the figure (assume that this profile is the same across the depth of the cylinder). Neglect radiation in this problem, and answer the following questions:

(a) Going by your knowledge of the relationship between temperature profiles and thermal resistances, would you accept the senior engineer's suggestion? Why/Why not?

<u>Solution</u>:

I would not accept the suggestion. The temperature profile suggests that the dominant resistance to heat transfer is in the liquid phase, since that is where we have the largest temperature drop. It would not make sense to decrease the convective heat transfer resistance on the outside, which is relatively small; this would influence the heat transfer rate only weakly.

2 points for connecting temperature differences to resistances, 3 points for the realizing that the largest resistance is in the liquid.

(b) What would be your suggestion to improve the rate of heat transfer? Why?

Solution:

I would install a mixing device within the tank to introduce forced convection, which would reduce the heat transfer resistance on the liquid side. The mixing has to be kept gentle to avoid viscous dissipating heating. So the addition of the mixer may also need to be accompanied by the attachment of fins on the liquid side of the cylinder.

5 points for a correct suggestion.