#### **UNIVERSITY OF TORONTO**

# FACULTY OF APPLIED SCIENCE AND ENGINEERING

FINAL EXAMINATION, December 19, 2022

DURATION: 2.5 hours

Second Year - Engineering Science

CHE260H1 – Thermodynamics and Heat Transfer

Calculator Type: 1

Exam Type: A

Examiner: J. Werber

Last Name:	 	 
First Name:	 	 
Casall.		

For all questions, please circle answers. While you can complete the exam as you like, please try (as best as you can) to use pages in order to ease grading. After finished, reassemble pages to be in order and turn in your exam. The exam is out of 100 pts.

**Problem 1: (15 points)** A refrigerated tractor trailer is traveling down a local road at 7 m/s. The trailer is 20-m long and 2.5 m wide. Wind on this day is essentially zero, and the bulk air temperature is 30 °C. The trailer ceiling comprises a thin aluminum wall and 10-cm thick insulating foam with a thermal conductivity of 0.06 W m<sup>-1</sup> K<sup>-1</sup>. The average exterior surface temperature of the trailer ceiling is 15 °C. What is the rate of heat transfer from the environment to the trailer through the trailer ceiling?

$$Nu = (0.037 Re^{0.8} - 871) Pr^{3}$$

$$= (0.037 \times (8.70 \times 10^{6})^{0.6} - 871) \times (0.71)^{3}$$

$$= 10980$$

$$Nu = hL = h = h = 10980 \times 0.0252 \text{min}$$

$$= 13.8 \text{min}$$

$$R_{con} = \frac{1}{hA} = \frac{1}{13.8 \text{min}} \times 50^{2} = 0.001446 \text{min}$$

$$\dot{Q} = (70^{\circ}C - 15^{\circ}C) = 10.4 \text{min}$$

$$(10376 \text{min})$$

**Problem 2. (20 points)** A 5-cm diameter sphere is taken out of an oven and initially has a uniform temperature of 130 °C. The sphere is placed in stagnant air with a bulk temperature of 20 °C and a convective mass transfer coefficient of 15 W m<sup>-2</sup> K<sup>-1</sup>.

- a) What is the lowest thermal conductivity of a material in this case where the lumped capacitance approach could be used?
- b) Assume the sphere is made of copper. How long will it take until sphere is cooled to 40 °C?
- c) Which of the following best describes the lumped capacitance approach? (Pick all that apply)
  - Resistance to conduction in the material outweighs resistance to convection from the material, leading to constant surface temperatures.
  - Resistance to convection outweighs resistance to conduction, leading to spatially uniform temperatures throughout the body.
  - uniform temperatures throughout the body.

    The T vs. t function is derived by comparing convection from a body and energy change in the body, not accounting for conduction at all.

Bi = 
$$\frac{1}{k}$$
  $C = \frac{4\pi r^2}{4\pi r^2} = \frac{2.5610^{-2}}{3}$ 

$$= 0.00833 \text{ m}$$

$$0.1 = 15 \frac{15 \times 0.00833}{k} \times 0.00833 \text{ m}$$

$$=) k = \frac{15 \times 0.00833}{0.1} = 1.2495 \frac{1}{\text{mK}}$$

milinum conductivity.

b) Coppr: 
$$h = 401 \frac{1}{nk}$$
 $p = 385 \frac{1}{hsk}$ 
 $C_p = 385 \frac{1}{hsk}$ 
 $= 78i = \frac{hL_c}{h} = \frac{15 \times 0.00837}{401 \frac{1}{nk}} = 0.00031$ 
 $b = \frac{h}{pL_cC_f} = \frac{15 \frac{1}{nk}}{(733 \frac{1}{nk}) \times 0.00833 m \times 385 \frac{1}{hsk}}$ 
 $= 5.236 \times 10^{-4} \text{ s}^{-1}$ 

$$= \frac{40-20}{130-20} = \frac{-5.236 \times 10^{-4} + }{= 0.181818}$$

$$-5.236 \times 10^{-4} + - l_{n} (0.181818) = -1.7047$$

**Problem 3. (20 Points)** A long circular duct is divided into a bottom half (1) and a top half (2). The diameter of the duct is 10 cm and the interior of the duct is under vacuum. The bottom half is 1100 K with an emissivity of 0.8. The top half is 400 K with an emissivity of 0.5. All interior surfaces are diffuse and gray.

- a) Find the view factors  $F_{12}$ ,  $F_{11}$ ,  $F_{21}$ , and  $F_{22}$
- b) Calculate the net rate of radiation heat transfer per meter of duct length from surface 1 to surface 2.

2 
$$400 \text{ K}$$
  $60.5$ 

$$(0 - A = 0.05 = 1 - 2)$$

$$A_1 = A_2 = A = TTD = 0.05 = 0.$$

$$\frac{1}{1-\epsilon_{1}} = \frac{-(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{1}}{A_{1}\epsilon_{1}} + \frac{1-\epsilon_{2}}{A_{1}\epsilon_{1}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{1}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{1}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{2}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{1}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{1}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{1}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{1}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{1}}}}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{2}}} = \frac{A\sigma(\tau_{1}^{4} - \tau_{2}^{4})}{\frac{1-\epsilon_{2}}{\epsilon_{1}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{2}}}}{\frac{1-\epsilon_{2}}{A_{1}\epsilon_{2}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{1-\epsilon_{2}}{\epsilon_{2}}}}{\frac{1-\epsilon_{2}}{A_{1}\epsilon_{2}} + \frac{1-\epsilon_{2}}{F_{11}} + \frac{$$

Problem 4. (25 points) Aquatic mammals are warm-blooded, with internal body temperatures of 37 °C. To stay warm in cold water, they are insulated by blubber or hair. For example, otters have a high-density layer of hair that traps air between their skin and the water.

Let's model an otter swimming at a steady-state velocity through cold water at 5 °C as a flat plate of 1-m length and 20-cm width, with a 0.5-cm thick layer of hair with thermal conductivity of 0.08 W m<sup>-1</sup> K<sup>-1</sup> and an under-side temperature of 37 °C. The water is still.

Determine the net heat loss when the otter is swimming at:

- a) 0.1 m/s
- b) 2 m/s

Water 5°C

New to such that 
$$A = \frac{37 - 5^{\circ}(C)}{0.3125 \times 10^{\circ}}$$
 $A = \frac{37 - 5^{\circ}(C)}{0.3125 \times 10^{\circ}}$ 
 $A = \frac{37 - 5^{\circ}(C$ 

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$$F_{ror} + \frac{5+10}{2} = 7.5^{\circ}C$$

$$F_{ror} + \frac{5}{10} = \frac{5+10}{2} = 7.5^{\circ}C$$

$$P_{ror} = \frac{1000 \frac{ks}{m^{2}}}{N^{2}}$$

$$N = \frac{1416 \times 10^{-6} \frac{Ns}{m^{2}}}{N^{2}}$$

$$V = \frac{1.4175 \times 10^{-6} \frac{m^{2}}{s}}{S}$$

a) 
$$V_{\infty} = 0.1 \frac{\pi}{5}$$

Re =  $\frac{pV_{\infty}L}{r} = \frac{V_{\infty}L}{2} = \frac{0.1 \frac{\pi}{5} \times 1^{m}}{1.4175 \times 10^{-6} \frac{\pi^{2}}{5}} = 7.055 \times 10^{+4}$ 
 $\Rightarrow Laminer$ 

$$N_{n} = 0.664 \times Re^{0.5} \times P_{r}^{1/3}$$

$$= 0.664 (7.055 \times 10^{4})^{0.5} (10.45)^{1/3} = 385.6$$

$$h = \frac{N_{n} \times h}{L} = \frac{385.6 \times 0.572 \times k}{1.5} \times 220.7 \times \frac{k}{n^{2}k}$$

=) 
$$R_{t_0+1} = 0.3125 + \frac{4.5}{220.7} = 0.3752 \frac{1}{12}$$
  
=)  $Q = \frac{37^{\circ}C-5^{\circ}C}{0.3352 \frac{1}{12}} = 95.5 \text{ W}$ 

b) 
$$V_{\infty} = 2\frac{\pi}{3}$$
 $Re = \frac{V_{\infty}^{2}L}{2} = \frac{2\frac{\pi}{3} \times 1^{n}}{1.4175 \times 10^{-6} \frac{\pi^{2}}{3}} = 1.411 \times 10^{-6} \Rightarrow 1 \text{ turbulen} L$ 
 $N_{n} = (0.037 \text{ Re}^{0.8} - 871) P_{r}^{V_{3}}$ 

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$$N_{n} = (0.037 \text{ Re}^{\circ} - 871) \text{ Fr}$$

$$= (0.037 \times (1.411 \times 10^{6})^{0.8} - 871) \text{ Re}(10.45)^{1/3}$$

$$= 4818.2$$

$$L = \frac{4818.2 \times 6.572 \frac{W}{NK}}{1-} = 2756 \frac{W}{M^{2}K}$$

$$= 7 R_{T0} + 1 = 0.3125 + \frac{5}{2751} = 0.3143 \frac{W}{N}$$

$$= 7 \hat{Q} = \frac{37'(-5')}{5.3140 \frac{K}{N}} = 101.8 W$$

Chech Ts assumption.

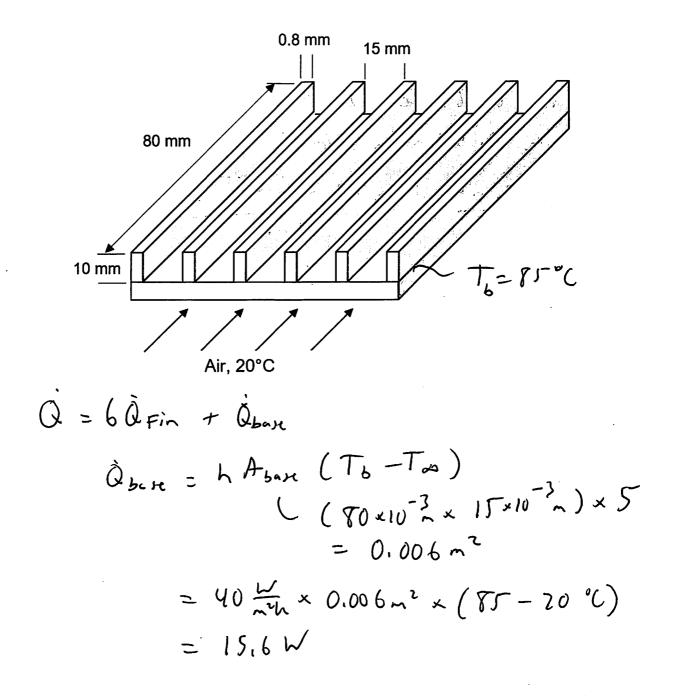
in a) 
$$h = 221 \stackrel{\checkmark}{=} u = 0.0226 \stackrel{\checkmark}{=}$$

in a)  $h = 221 \stackrel{\checkmark}{=} u = 0.0226 \stackrel{\checkmark}{=}$ 

$$\Delta T = 0 \times Rom = 2°C \Rightarrow Tsuf ~ 7°C$$

Close enough to 10°C

**Problem 5. (20 points)** An aluminum heat sink with six identical fins (as shown in the figure) is attached to an electronic device for cooling. Cooling air at 20 °C is supplied to give a convective heat transfer coefficient of 40 W m<sup>-2</sup> K<sup>-1</sup>. If the average base temperature is not to exceed 85 °C, calculate the overall heat dissipation from the sink, including accounting (as best you can) for heat transfer from the fin ends.



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# **Aid Sheet**

$$\dot{Q}_{conduction} = -kA \frac{dT}{dx}$$

$$\dot{Q}_{radiation} = \varepsilon \sigma A (T_s^4 - T_{surr}^4)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

$$R_{radiation} = \frac{1}{h_{rad}A}$$

$$R_{cylinder} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$$

For fluid temp change:  $Q = mC_p\Delta T$ 

For vaporization:  $Q = m\Delta \hat{H}_{\text{vap}}$ 

$$a = \sqrt{\frac{hP}{kA_c}}$$

# For infinitely long fin

$$\frac{T(x) - T_{\infty}}{T_b - T_{\infty}} = \exp(-ax)$$

$$\dot{Q}_{fin,long} = \sqrt{hPkA_c}(T_b - T_{\infty})$$

$$\eta_{fin,long} = \frac{1}{aL}$$

### For insulated tip fin

$$\frac{T(x) - T_{\infty}}{T_h - T_{\infty}} = \frac{\cosh \left[a(L - x)\right]}{\cosh \left(aL\right)}$$

$$\dot{Q}_{fin,ins} = \sqrt{hPkA_c}(T_b - T_{\infty}) \tanh(aL)$$

$$\eta_{fin,ins} = \frac{\tanh(aL)}{aL}$$

$$\dot{Q}_{convection} = hA(T_s - T_{\infty})$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$$

$$R_{convection} = \frac{1}{hA}$$
  $R_{wall} = \frac{L}{kA}$ 

$$R_{wall} = \frac{L}{kA}$$

$$h_{rad} = \varepsilon \sigma (T_s^2 + T_{surr}^2)(T_s + T_{surr})$$

$$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$
  $R_c = \frac{\Delta T_{interface}}{\dot{Q}/A}$ 

$$R_c = \frac{\Delta T_{interface}}{\dot{Q}/A}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \qquad \sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

x	tanh(x)	X	tanh(x)
0	0	0.64	0.5649
0.04	0.04	0.68	0.5915
0.08	0.0798	0.72	0.6169
0.12	0.1194	0.76	0.6411
0.16	0.1586	0.8	0.664
0.2	0.1974	0.84	0.6858
0.24	0.2355	0.88	0.7064
0.28	0.2729	0.92	0.7259
0.32	0.3095	0.96	0.7443
0.36	0.3452	1	0.7616
0.4	0.3799	2	0.964
0.44	0.4136	3	0.9951
0.48	0.4462	4	0.9993
0.52	0.4777	5	0.9999
0.56	0.508	10	1
0.6	0.537		

#### Forced convection:

$$Re = \frac{\rho V_{\infty} L}{\mu}$$
  $Nu = \frac{hL}{k}$   $Pr = \frac{\mu c_p}{k}$ 

$$Nu = \frac{hL}{k}$$

$$Pr = \frac{\mu c_p}{k}$$

For a flat plate

$$Nu = 0.664 Re^{0.5} Pr^{1/3}$$

$$Nu = 0.664 Re^{0.5} Pr^{1/3}$$
  $Re < 5 \times 10^5$ ,  $Pr > 0.6$ 

$$Nu = (0.037Re^{0.8} - 871)Pr^{1/3}$$
  $5 \times 10^5 < Re < 10^7$ ,  $0.6 < Pr < 60$ 

$$5 \times 10^5 < Re < 10^7$$

#### Radiation

Reciprocity: 
$$A_iF_{ij} = A_jF_{ji}$$

Summation: 
$$\sum_{j=1}^{N} F_{ij} = 1$$

For a two-surface enclosure: 
$$\dot{Q}_{12}=rac{\sigma(T_1^4-T_2^4)}{rac{1-arepsilon_1}{A_1arepsilon_1}+rac{1}{A_1F_{12}}+rac{1-arepsilon_2}{A_2arepsilon_2}}$$

#### **Transient Heat Transfer**

$$L_c = V/A$$

$$Bi = \frac{hL_c}{k_{solid}}$$

For lumped capacitance system (Bi < 0.1):

$$\frac{T(t)-T_{\infty}}{T_{i}-T_{\infty}}=\exp(-bt)$$

$$b = \frac{h}{\rho L_c c_p}$$

# Properties of metals

Metal	$\rho$ (kg/m <sup>3</sup> )	k (W/m°C)	c <sub>p</sub> (J/kg°C)		
Aluminum	2702	237	903		
Copper	8933	401	385		

TABLE 13 Water at saturation pressure

Temperature,		ıre,	Density, P (kg/m³)	Coefficient of Thermal Expansion, $\beta \times 10^4$ (1/K)	Specific Heat, C <sub>p</sub> (J/kg K)	Thermal Conductivity, k (W/m K)	Thermal Diffusivity, $\alpha \times 10^6$ (m <sup>2</sup> /s)	Absolute Viscosity, $\mu \times 10^6$ (N s/m $^6$ )	Kinematic Viscosity, $\nu \times 10^6$ (m <sup>2</sup> /s)	Prandtl Number, Pr	$\frac{g\beta}{\nu^2} \times 10^{-9}$ (1/K m <sup>3</sup> )
°F	K	•c	$\times 6.243 \times 10^{-2}$ = ( $\text{lb}_{\text{m}}/\text{ft}^3$ )	× 0.5556 = (1/R)		× 0.5777 = (Btu/h ft °F)	× 3.874 × 10 <sup>4</sup> = (ft <sup>2</sup> /h)	× 0.6720 = (lb <sub>m</sub> /ft s)	$\times 3.874 \times 10^4$ = (ft <sup>2</sup> /h)	- <del></del>	$\times$ 1.573 $\times$ 10 <sup>-2</sup> = (1/R ft <sup>3</sup> )
32	273	0	999.9	-0.7	4226	0.558	0.131	1794	1.789	13.7	_
41	278	5	1000	_	4206	0.568	0.135	1535	1.535	11.4	_
50	283	10	999.7	0.95	4195	0.577	0.137	1296	1.300	9.5	0.551
59	288	15	999.1	_	4187	0.585	0.141	1136	1.146	8.1	_
68	293	20	998.2	2.1	4182	0.597	0.143	993	1.006	7.0	2.035
77	298	25	997.1	_	4178	0.606	0.146	880.6	0.884	6.1	_
86	303	30	995.7	3.0	4176	0.615	0.149	792.4	0.805	5.4	4.540
95	308	35	994.1	_	4175	0.624	0.150	719.8	0.725	4.8	_
104	313	40	992.2	3.9	4175	0.633	0.151	658.0	0.658	4.3	8.833
113	318	45	990.2	_	4176	0.640	0.155	605.1	0.611	3.9	_
122	323	50	988.1	4.6	4178	0.647	0.157	555.1	0.556	3.55	14.59
167	348	75	974.9	_	4190	0.671	0.164	376.6	0.366	2.23	_
212	373	100	958.4	7.5	4211	0.682	0.169	277.5	0.294	1.75	85.09
248	393	120	943.5	8.5	4232	0.685	0.171	235.4	0.244	1.43	140.0
284	413	140	926.3	9.7	4257	0.684	0.172	201.0	0.212	1.23	211.7
320	433	160	907.6	10.8	4285	0.680	0.173	171.6	0.191	1.10	290.3
356	453	180	886.6	12.1	4396	0.673	0.172	152.0	0.173	1.01	396.5
392	473	200	862.8	13.5	4501	0.665	0.170	139.3	0.160	0.95	517.2
428	493	220	837.0	15.2	4605	0.652	0.167	124.5	0.149	0.90	671.4
464	513	240	809.0	17.2	4731	0.634	0.162	113.8	0.141	0.86	848.5
500	533	260	779.0	20.0	4982	0.613	0.156	104.9	0.135	0.86	1076
536	553	280	750.0	23.8	5234	0.588	0.147	98.07	0.131	0.89	1360
572	573	300	712.5	29.5	5694	0.564	0.132	92.18	0.128	0.98	1766

# Thermodynamic Properties of Gases

TABLE 28 Dry air at atmospheric pressure

Temperature,		e,	Density, P (kg/m³)	Coefficient of Thermal Expansion, $\beta \times 10^3$ (1/K)	Specific	Thermal Conductivity, k (W/m K)	Thermal Diffusivity, $\alpha \times 10^6$ (m <sup>2</sup> /s)	Absolute Viscosity, μ × 10 <sup>5</sup> (N s/m <sup>2</sup> )	Kinematic Viscosity, v × 10 <sup>6</sup> (m²/s)	Prandtl Number, Pr	$\frac{g\beta}{\nu^2} \times 10^{-8}$ (1/K m <sup>3</sup> )
۰F	K	°C	$\times 6.243 \times 10^{-2}$ = ( $\text{lb}_{\text{m}}/\text{ft}^3$ )	× 0.5556 = (1/R)	× 2.388 × 10 <sup>-4</sup> = (Btu/lb <sub>m</sub> °F)	× 0.5777 = (Btu/h ft °F)	$\times 3.874 \times 10^4$ = (ft <sup>2</sup> /h)	×0.6720 = (lb <sub>m</sub> /ft s)	$ \frac{\times 3.874 \times 10^4}{= (ft^2/h)} $		$\times 1.573 \times 10^{-2}$ = (1/R ft <sup>3</sup> )
32	273	0	1.252	3.65	1011	0.0237	19.2	17.456	13.9	0.71	1.85
68	293	20	1.164	3.41	1012	0.0251	22.0	18.240	15.7	0.71	1.36
104	313	40	1.092	3.19	1014	0.0265	24.8	19.123	17.6	0.71	1.01
140	333	60	1.025	3.00	1017	0.0279	27.6	19.907	19.4	0.71	0.782
176	353	80	0.968	2.83	1019	0.0293	30.6	20.790	21.5	0.71	0.600
212	373	100	0.916	2.68	1022	0.0307	33.6	21.673	23.6	0.71	0.472
392	473	200	0.723	2.11	1035	0.0370	49.7	25.693	35.5	0.71	0.164
572	573	300	0.596	1.75	1047	0.0429	68.9	29.322	49.2	0.71	0.0709
752	673	400	0.508	1.49	1059	0.0485	89.4	32.754	64.6	0.72	0.0350
932	773	500	0.442	1.29	1076	0.0540	113.2	35.794	81.0	0.72	0.0193
1832	1273	1000	0.268	0.79	1139	0.0762	240	48.445	181	0.74	0.00236

Source: K. Raznjevič, Handbook of Thermodynamic Tables and Charts, McGraw-Hill, New York, 1976.