PHY294 MIDTERM TEST, 10 Feb 2020 — 9:10am-10:30am (80 min)

Useful formulae

$$\begin{aligned} 1\text{eV} &= 1.6\times 10^{-19}\text{J} \quad ; \quad \hbar = 1.05\times 10^{-34} kg\ m^2/s \quad ; \quad 2\sin a\sin b = \cos(a-b)-\cos(a+b) \\ e^{ix} &= \cos x + i\sin x \quad ; \quad \cos 60^\circ = 1/2 \quad ; \quad \sin 60^\circ = \sqrt{3}/2 \quad ; \quad \cos(90^\circ-\theta) = \sin\theta \end{aligned}$$

Oscillator angular frequency with spring constant K:

$$\omega = \sqrt{\frac{K}{m}} \tag{1}$$

Time-dependent Schrodinger equation in 1D

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t)$$

Time-independent Schrodinger equation in 1D

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\phi(x) + V(x)\phi(x) = E\phi(x)$$

Probability current density

$$J(x,t) = \frac{\hbar}{2mi}\left(\psi^*(x,t)\frac{\partial\psi(x,t)}{\partial x} - \psi(x,t)\frac{\partial\psi^*(x,t)}{\partial x}\right)$$

Harmonic oscillator

$$\begin{aligned} E_n &= (n+\frac{1}{2})\hbar\omega \qquad (n=0,1,2\ldots) \\ \phi_0(x) &= (\frac{m\omega}{\pi\hbar})^{1/4}\exp\left[-\frac{m\omega x^2}{2\hbar}\right] \quad ; \quad \phi_1(x) = (\frac{m\omega}{\pi\hbar})^{1/4}\sqrt{\frac{m\omega}{2\hbar}}\,2x\exp\left[-\frac{m\omega x^2}{2\hbar}\right] \\ \phi_2(x) &= (\frac{m\omega}{\pi\hbar})^{1/4}\frac{1}{\sqrt{2}}\,(1-2\frac{m\omega x^2}{\hbar})\exp\left[-\frac{m\omega x^2}{2\hbar}\right] \end{aligned}$$

Particle in a 1D box $0 < x < L$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}; \qquad \phi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \qquad (n=1,2\ldots)$$

Fourier transforms

$$\tilde{\phi}(k) = \int_{-\infty}^{+\infty}\frac{dx}{\sqrt{2\pi}}\,e^{-ikx}\,\phi(x) \quad ; \quad \phi(x) = \int_{-\infty}^{+\infty}\frac{dk}{\sqrt{2\pi}}\,e^{+ikx}\,\tilde{\phi}(k)$$

Integrals

$$\begin{aligned} \int_{-\infty}^{\infty} dx\ e^{-\alpha x^2} &= \sqrt{\frac{\pi}{\alpha}} \quad ; \quad \int_{-\infty}^{\infty} dx\ x^2 e^{-\alpha x^2} = \frac{\sqrt{\pi}}{2\alpha^{3/2}} \\ \int_{-\infty}^{\infty} dx\ x^4 e^{-\alpha x^2} &= \frac{3\sqrt{\pi}}{4\alpha^{5/2}} \quad ; \quad \int_{-\infty}^{\infty}\frac{dx}{\sqrt{2\pi}}\,e^{-\alpha x^2}\,e^{-ikx} = \sqrt{\frac{1}{2\alpha}}\,e^{-\frac{k^2}{4\alpha}} \\ \int_{-\infty}^{\infty}\frac{dx}{\sqrt{2\pi}}\,x e^{-\alpha x^2}\,e^{-ikx} &= -\sqrt{\frac{1}{2\alpha}}\,\frac{ik}{2\alpha}\,e^{-\frac{k^2}{4\alpha}} \end{aligned}$$

I. Polarization (5 points)
You are given three linear polarizers placed one after the other. The first two are lined up vertical, while the final polarizer is rotated by 60° with respect to the vertical. You now rotate the angle θ of the middle polarizer all the way around by 360° until you are back where you started. If you are told that N_0 photons emerge on average per second **after** the first polarizer, sketch the average number of photons to emerge at the final output (after all three polarizers) as a function of the angle θ, taking care to label important angles and photon numbers - e.g.., show angles where the photon number is maximum and minimum, and indicate the average photon number at these angles, as well as at $\theta = 0$.

II. Probability current (5 points)
Consider a quantum particle confined to a box of size L with infinitely high walls. The particle starts off in the normalized initial state

$$\psi(x,0) = \sqrt{\frac{1}{L}}\left[\sin(\frac{2\pi x}{L}) + \sin(\frac{3\pi x}{L})\right]$$

Calculate the probability current density as a function of space and time. What is the period with which this current will oscillate at any given point in space? What is the probability to find this particle to have energy E_2 as a function of time, where E_n labels the energies with $n = 1, 2, \ldots$.

III. Particle in a box (5 points)
Consider a particle in the ground state of a box of size L with infinitely high walls at $x = 0, L$. If the right wall of the box is suddenly moved to $x = 2L$ at time $t = 0$, so the wavefunction has no time to change, the particle will now find itself in a superposition of stationary states of the **new** box. What is the probability at that instant $t = 0^+$ to find the particle in the **ground state of the new box?**

IV. Harmonic oscillator momentum (5 points)
Consider a particle in the first excited state of a harmonic oscillator, with the normalized wavefunction

$$\phi_1(x) = (\frac{m\omega}{\pi\hbar})^{1/4}\sqrt{\frac{m\omega}{2\hbar}}\, 2x \exp\left[-\frac{m\omega x^2}{2\hbar}\right]$$

Determine the most likely outcome(s) when you measure the momentum of this particle.