NAME:______________________________

STUDENT NUMBER:________________________________

TUTORIAL GROUP: DAY OF THE WEEK:______________________TIME:_________

Time: 60 minutes

This is a closed-book exam worth a total of 60 points. Please answer all questions.

ONLY THE FOLLOWING ITEMS ARE ALLOWED ON YOUR DESK DURING THE EXAM:

1. **THIS EXAM BOOK –** It contains this cover page, three question pages and a formula sheet (which may be torn off). Make sure you start by putting your **NAME, ID NUMBER,** and **TUTORIAL GROUP** on the front (cover) page of the exam.
   The entire exam book (minus the formula sheet) **will be handed in** at the end of the exam and marked.
   a. **FORMULA SHEET**, annotated on the printed page. The formula sheet has to be printed from the file provided on Quercus.
2. **A CALCULATOR**, which can be anything that does NOT connect to wifi and does NOT communicate with other devices. ACCEPTABLE calculators include programmable and graphing calculators, scientific calculators, etc. UNACCEPTABLE calculators include: cell phones, tablets, laptops, etc.
3. **A PEN OR PENCIL.**
4. **YOUR STUDENT ID CARD**, used to check your identity.

If you are missing anything from the above items, raise your hand and ask an exam supervisor to supply what is missing. If you are missing an item that should have been brought by you (e.g., calculator, pen/pencil) there is a limited supply of extras and are on a first come, first served basis.

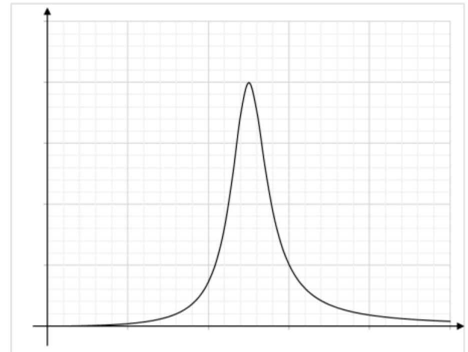
COMPLETE SOLUTION INCLUDES:

a) A sketch with a coordinate system, where appropriate. Where a diagram is provided, additional markings can be added to it.
b) Clear and logical workings, including quoting the equations that were used, proper substitutions of numbers (when necessary) with all steps clearly indicated.
c) The final answer with units and three significant figures.

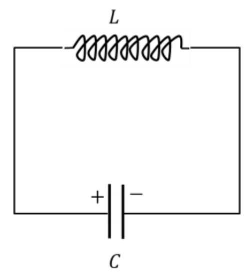
FOR OFFICE USE ONLY				
QUESTION	I	II	III	TOTAL
MARK				
MAXIMUM	20	20	20	60

Question I

1. Figure below shows a power resonance curve for a driven harmonic oscillator. Clearly label the graph, including the following values: $\bar{P}_{max}, \frac{\bar{P}_{max}}{2}, \bar{P}(\omega), \omega_{fwhh}, \omega_0, \gamma, \omega, \omega - \frac{\gamma}{2}, \omega + \frac{\gamma}{2}$. [4 marks]



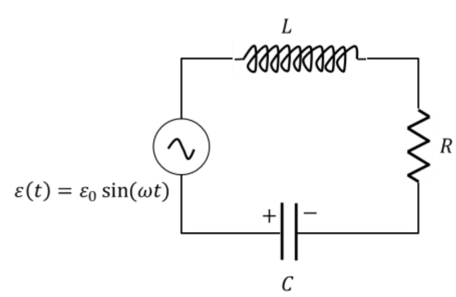
2. An LC circuit with a capacitor $C = 4.50 \times 10^{-5}$ F and a inductor $L = 0.225$ H is set up as shown in the picture with the maximum voltage across the capacitor set to 6.00 V.



   a. What is the resonance frequency of this circuit [2 points]

   b. At time $t = 0\ s$ the voltage across the capacitor is measured to be $V(t = 0\text{ s}) = -2.30\ V$ and the <u>absolute potential difference</u> between the plates of the capacitor is decreasing. If the potential across the capacitor is described according to the equation $V(t) = V_{C0}\cos(\omega t + \phi_o)$, what is the value of the phase constant ϕ_o? [6 points]

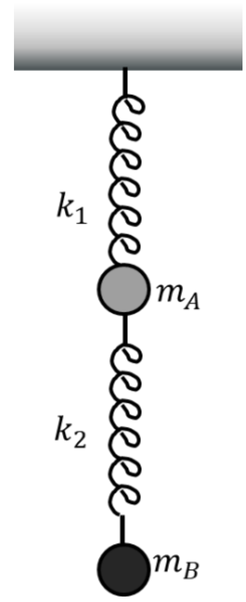
   c. The capacitor is discharged and the circuit is modified with a resistor $R = 16.0\ \Omega$ and a power supply providing alternating voltage $\varepsilon = \varepsilon_0 \sin(\omega t)$. At what driving frequency ω would the current of largest amplitude be observed in the circuit? [4 points]



   d. What is the value of the phase angle, δ, between the potential difference measured across the resistor and the potential difference across the source at that driving frequency? [4 points]

Question II

1. Two masses, $m_A = m$ and $m_B = 3m$ are connected vertically and attached to a rigid ceiling by two springs of spring constants $k_1 = 2k, k_2 = 3k,$ in such a way that k_1 connects mass m_A to a rigid support above the masses and spring k_2 connects mass m_B to mass m_A



   a. Assuming both masses are displaced downwards, draw arrows **clearly** indicating the direction of forces on each mass due to each spring attached to it. Label each force. *If it is impossible for you to draw the vectors in the picture, provide labels for the forces and their directions (e.g F_{12} – force on object 1 by agent 2, $+x$ direction).* [4 points]

   b. Write the equation of motion for each mass. Clearly label the equations so it is obvious, which mass it is for. [4 points]

   c. Assuming masses m_A and m_B move according to the equations $y_A(t) = A\cos(\omega t + \phi_A)$ and $y_B = B\cos(\omega t + \phi_B)$, determine the coefficient matrix $\mathbb{M}$ for the system. Express all elements of the matrix in terms of k and m. [6 points]

   d. Determine the normal frequencies of this oscillation. [6 points]

Question III

1. A string of length L and impedance Z is fixed at both ends. A driver, providing a **constant** power P for all frequencies is attached to the string, creating a set of standing waves of frequencies f_n and amplitudes A_n, one at the time. The speed of the wave in the string is v. Sketch a graph showing how the amplitude of the mode, A_n depends on the mode number. You can use logic or math to justify your dependence. [4 points]

2. An electrical power transmission line is made of a wire with linear mass density $\mu = 1.50\ \mathrm{kg/m}$, stretched under the tension force of $25.0\ \mathrm{kN}$. It is held up at the towers by insulators that hold the wire rigidly in the vertical direction (the position of the wire is constant at the tower).
Consider mechanical waves on one, $100\ \mathrm{m}$ long, section of wires between two towers, taking $x = 0\ \mathrm{m}$ at one of the towers. Ignore any possible sagging in the wire.
    a. What the the speed of the wave on the wire? [2 points]

    b. Wind is inducing a standing wave on the wire between two towers, with the wire moving vertically up and down with a maximum displacement of 12 cm. If the wind-induced wave has a frequency of $7.1\ \mathrm{Hz}$, what equation would describe the standing wave created on the string? Assume at time $t = 0$ s the string is completely straight, but in motion. [6 points]

    c. Which harmonic is it? [2 points]

    d. To reduce the potentially damaging standing waves, mechanical dampeners may be placed along the line. They are the most effective if they are placed at an antinode. Determine the set of locations that would indicate the most effective positions of the dampeners for reducing the $7.1\ \mathrm{Hz}$ standing waves on the transmission line. [6 points]

OSCILLATIONS

$\omega = 2\pi f = \frac{2\pi}{T}$	$\omega_0 = \sqrt{\frac{k}{m}}$	$\omega_0 = \sqrt{\frac{mgd}{I}}$	$\omega_0 = \frac{1}{\sqrt{LC}}$
$x(t) = A\cos(\omega t + \phi_i)$	$x(t) = A_0 \exp\left(-\frac{\gamma t}{2}\right)\cos(\omega t + \phi_i))$		$x(t) = A(\omega)\cos(\omega t - \delta)$
$x(t) = A\exp\left(-\frac{\gamma t}{2}\right) + Bt\exp\left(-\frac{\gamma t}{2}\right)$	$x(t) = A\exp\left(\left(-\frac{\gamma}{2} + \left(\left\lvert\omega_0^2 - \frac{\gamma^2}{4}\right\rvert\right)^{\frac{1}{2}}\right)t\right) + B\exp\left(\left(-\frac{\gamma}{2} - \left(\left\lvert\omega_0^2 - \frac{\gamma^2}{4}\right\rvert\right)^{\frac{1}{2}}\right)t\right)$		
$q_0(\omega) = \frac{\varepsilon_0}{\omega Z}$	$q(t) = q_0(\omega)\cos(\omega t - \delta)$	$Z = \sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}$	$i = \frac{dq}{dt}$
$V_R = i(t)R$	$V_C = \frac{q}{C}$	$V_L = L\frac{di}{dt}$	
$K = \frac{1}{2}mv^2$	$U = \frac{1}{2}kx^2$	$E(t) = E_0\exp(-\gamma t)$	$P = \frac{dE}{dt} = Fv$
$Q = \frac{\omega_0}{\gamma}$	$\omega^2 = \omega_0^2 - \frac{\gamma^2}{4}$		
$A(\omega) = \frac{a\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$		$\tan\delta = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$	
$\bar{P}(\omega) = \frac{b[v_0(\omega)]^2}{2}$	$\bar{P}_{max} = \frac{F_0^2}{2m\gamma}$	$\bar{P}(\omega) = \frac{F_0^2}{2m\gamma\left[\frac{4(\Delta\omega)^2}{\gamma^2} + 1\right]}$	

WAVES

$v = \lambda f$		$y(x,t) = f(x \pm vt)$		$y(x,t) = A\cos(kx \pm \omega t + \phi_i)$		
$k = \frac{2\pi}{\lambda}$		$y(x,t) = (A\sin(kx) + B\cos(kx))\cos(\omega t)$				
$v = \sqrt{\frac{F_T}{\mu}}$	$v = \sqrt{\frac{B}{\rho}}$	$v = \sqrt{\frac{Y}{\rho}}$	$v = \sqrt{\frac{\gamma RT}{M}}$	$\frac{\partial y^2}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$	$\omega = \frac{2\pi}{T}$	$f = \frac{1}{T}$
$\omega_n = \frac{n\pi v}{L}$		$\omega_n = \frac{n\pi v}{2L}$		$f_n = nf_1$	$E_n = \frac{1}{4}\mu\omega_n^2 A_n^2 L$	
$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$		$P_{ave} = \frac{1}{2}ZA^2\omega^2$	$I_{ave} = \frac{1}{2}Z_a A^2\omega^2$	$P = \mu vA^2\omega^2\sin^2(kx - \omega t + \phi_0)$		
$Z = \sqrt{\mu\tau}$	$Z_a = \sqrt{Y\rho}$	$Z_a = \sqrt{B\rho}$	$R = \frac{Z_1 - Z_2}{Z_1 + Z_2}$	$T = \frac{2Z_1}{Z_1 + Z_2}$		
$I(r) = I(r_0)\left[e^{-\alpha(r - r_0)}\right]\left(\frac{r_0}{r}\right)^{N-1}$		$v_g = \frac{d\omega}{dk}\Big\vert_{k=k_0}$	$v_g = v - \lambda\frac{dv}{d\lambda}$	$R^2 + \frac{Z_2}{Z_1}T^2 = 1$		
$c = (\mu_0\varepsilon_0)^{-\frac{1}{2}}$	$v = \frac{c}{n}$		$A_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi}{L}x\right)dx$			

MATHEMATICAL FORMULAE

$\cos\alpha + \cos\beta = 2\cos\left[\frac{\alpha+\beta}{2}\right]\cos\left[\frac{\alpha-\beta}{2}\right]$		$\cos\alpha - \cos\beta = -2\sin\left[\frac{\alpha+\beta}{2}\right]\sin\left[\frac{\alpha-\beta}{2}\right]$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$		$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$
$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$		$\tan^{-1}(x) = \{\theta, \theta + \pi\} + 2\pi n$ $\cos^{-1}(x) = \pm\theta + 2\pi n$
$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$		$\sin^{-1}(x) = \{\theta, \pi - \theta\} + 2\pi n$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$	$\tilde{A} = Ae^{j\theta} = A(\cos\theta + j\sin\theta)$
$\int\sin(ax)dx = -\frac{1}{a}\cos(ax)$		$\int x\sin(ax)\,dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$

CONSTANTS

$c = 3.00 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}}$	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T}\cdot\frac{\mathrm{m}}{\mathrm{A}}$	$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{C}^2}$	$g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}}$
$v_{sound\ at\ 20^\circ\mathrm{C}} = 343\,\frac{\mathrm{m}}{\mathrm{s}}$	$T_K = T_{^\circ\mathrm{C}} + 273.15^\circ\mathrm{C}$		