Name (printed legibly):________________

Student # (printed legibly):__________________

Q1:____Q2:____Q3:____Q4:____Q5:____Q6:____

Total:_________

UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

ESC103F – Engineering Mathematics and Computation

Term Test

November 2, 2015

Instructor – Professor W.R. Cluett

Closed book.

Allowable calculators:

Casio FX-991EX or FX-991ES PLUS or FX-991MS
or
Sharp EL-520X or EL-520W

All questions of equal value.

All work to be marked <u>must</u> appear on front of page. Use back of page for rough work only.

Given information:

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|};\ \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ -(x_1 z_2 - z_1 x_2) \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

$$proj_{\vec{d}}\,\vec{u} = \frac{\vec{u}\cdot\vec{d}}{\|\vec{d}\|^2}\vec{d};\ \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Q1: Consider the following system of linear equations:

$$2x_1 - x_2 = 1$$
$$-x_1 + 2x_2 - x_3 = 0$$
$$-x_2 + 2x_3 = 0$$

a) Write the augmented matrix associated with this system.

b) Solve this system by using the Gaussian elimination algorithm to take the augmented matrix to its reduced normal form.

c) Using your solution in part (b), express the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as a linear combination of three vectors $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.

Q2: Pick any three numbers (x, y, z) such that $x + y + z = 0$ (except for the case $x = y = z = 0$).

a) Find the angle between your vector $\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and your vector $\vec{v} = \begin{bmatrix} z \\ x \\ y \end{bmatrix}$.

b) Show that for any two non-zero vectors $\vec{u}$ and $\vec{v}$ as defined in part (a) where $x + y + z = 0$, $\vec{u} \cdot \vec{v} / (\|\vec{u}\| \|\vec{v}\|) = -0.5$.

Q3: Consider the vector $\vec{c} = \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix}$ and the points $a = (2,2,1)$ and $b = (1,0,0)$.

a) Project the vector $\vec{c}$ onto the plane that contains the origin, point *a* and point *b*.

b) Find a vector whose projection onto the plane in part (a) is the zero vector.

c) Find the 3x3 matrix that maps any vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ to the projection of this vector onto the plane in part (a).

Q4: Consider the matrix $\begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix}$ where c and d are unknowns.

a) Find values for c and d to give eigenvalues for the matrix equal to 4 and 7.

b) From part (a), find the eigenvectors associated with the eigenvalue equal to 4.

Q5: Consider these two parts as separate, unrelated questions.

a) If $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, draw all vectors $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ in the *x*-*y* plane with $\vec{u} \cdot \vec{v} = 5$.

b) Consider vectors $\vec{w}$ and $\vec{z}$ in R^3 where $\|\vec{w}\| = 5$ and $\|\vec{z}\| = 3$. What are the smallest and largest values of $\vec{w} \cdot \vec{z}$? What are the smallest and largest values of $\|\vec{w} - \vec{z}\|$? Show how you arrive at your answers.

Q6: The "cyclic" transformation T is defined by:

$$\begin{bmatrix} y \\ z \\ x \end{bmatrix} = T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right).$$

a) What is the matrix that summarizes this linear transformation T?

b) With $\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, what is the vector produced by $T(T(\vec{u}))$? What is the matrix that summarizes this composition of two linear transformations?

c) Examine the vector produced by applying T three times (i.e. $T(T(T(\vec{u})))$). What would be the vector produced by applying T 100 times to $\vec{u}$?