Quiz 3 Solution (2016)

1(a) Strealine is a curve that is locally tangent to the velocity vector.

Pathline is the trajectory of a fluid parcel over a period of time.

Streakline is a curve that connects the locations of fluid parcels at a given time that passed a fixed location at an earlier time.

1(b) A Newtonian fluid is a fluid for which the shear stress is <u>linearly</u> related to the rate of strain.

1(c) A viscous flow is a flow for which the shear stress is not negligible.

1(d) For large Knudson number flows, the mean-free-path is larger than the length scale of the problem.

1(e) Reynolds number is the ratio of the inertial force to viscous force.

1 (f) No. $\sigma_{xx} = 2\mu \frac{\partial u}{\partial x}$, where $\frac{\partial u}{\partial x} \neq 0$.

$\frac{\partial u}{\partial x}$ is non-zero due to conservation of mass.

2. $\pi = U^a D^b \nu^c = \{[L][T]^{-1}\}^a \{[L]\}^b \{[L]^2 T^{-1}\}^c$

$= L^0 T^0$

$$\left.\begin{aligned} a+b+2c &= 0 \\ -a-c &= 0 \end{aligned}\right\} \quad \begin{aligned} a &= -c \\ b &= -c \end{aligned}$$

$\pi = U^a D^a \nu^{-a} = \left(\frac{UD}{\nu}\right)^a$

3. $u = x+y+2$ $\qquad \frac{dx}{x+y+2} = \frac{dy}{-(x+y)}$

$v = -(x+y)$

$x\,dy + y\,dy + 2\,dy = -x\,dx - y\,dx$

$\underbrace{x\,dy + y\,dx}_{d(xy)} + x\,dx + y\,dy + 2\,dy = 0 \longrightarrow$

$xy + x^2/2 + y^2/2 + 2y = c$

4. $\vec{a} = (g_x x^2, g_y y^2, g_z z^2)$

$$-\vec{\nabla} p + \rho \vec{g} = \rho \vec{a}$$

$$\vec{\nabla} p = \rho(\vec{g} - \vec{a})$$

$$\vec{\nabla} p = \rho\left(g_x(1-x^2), g_y(1-y^2), g_z(1-z^2)\right)$$

① $\dfrac{\partial p}{\partial x} = \rho g_x (1-x^2)$

② $\dfrac{\partial p}{\partial y} = \rho g_y (1-y^2)$

③ $\dfrac{\partial p}{\partial z} = \rho g_z (1-z^2)$

① $p = \rho g_x (x - x^3/3) + f_1(y,z) \rightarrow$

$\dfrac{\partial p}{\partial y} = \dfrac{\partial f_1}{\partial y} = \rho g_y (1-y^2) \rightarrow \quad f_1(y,z) = \rho g_y (y - y^3/3) + f_2(z)$

$\rightarrow p = \rho g_x (x - x^3/3) + \rho g_y (y - y^3/3) + f_2(z)$

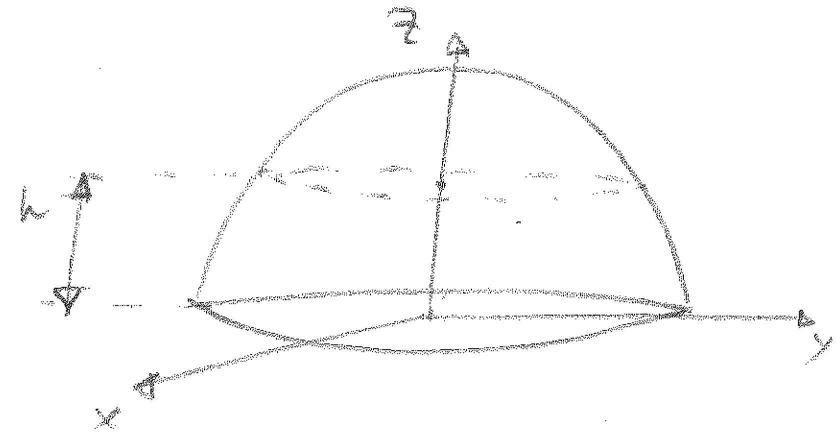
$\dfrac{\partial p}{\partial z} = \dfrac{d f_2}{dz} = \rho g_z (1-z^2) \rightarrow \quad f_2(z) = \rho g_z (z - z^3/3) + C$

$\rightarrow p(x,y,z) = \rho\left[g_x(x - x^3/3) + g_y(y - y^3/3) + g_z(z - z^3/3)\right] + C$

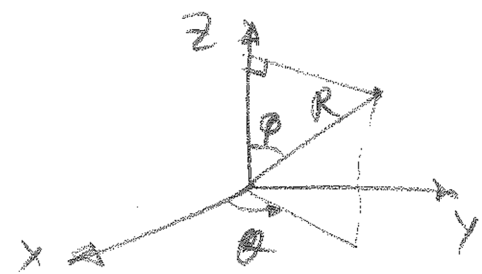
$p(0,0,0) = p_0 \rightarrow C = p_0 \rightarrow$

$p(x,y,z) = \rho\left[g_x(x - x^3/3) + g_y(y - y^3/3) + g_z(z - z^3/3)\right] + p_0$ 3/

5.



$$dA = R\,d\varphi \cdot R\sin\varphi\,d\theta$$



$$dF_z = \rho g\,(h - R\cos\varphi)\,R^2 \sin\varphi\,d\varphi\,d\theta\,\cos\varphi$$

$$F_z = 2\pi\rho g R^2 \int_{\varphi_0}^{\pi/2} (h - R\cos\varphi)\sin\varphi\cos\varphi\,d\varphi \,,$$

where $\cos\varphi_0 = \frac{h}{R}$

$$F_z = 2\pi\rho g R^2 \left\{ \frac{h}{2}\left[-\frac{1}{2}\cos 2\varphi\right]_{\varphi_0}^{\pi/2} - R\int_{\varphi_0}^{\pi/2} \sin\varphi\cos^2\varphi\,d\varphi \right\}$$

$$F_z = 2\pi\rho g R^2 \left\{ -\frac{h}{4}(-1 - \cos 2\varphi_0) - R\int_{\frac{h}{R}}^{0} -du\,u^2 \right\} \qquad u = \cos\varphi$$

$$F_z = 2\pi\rho g R^2 \left\{ \frac{h}{4}(1 + \cos 2\varphi_0) - \frac{R}{3}\left(\frac{h}{R}\right)^3 \right\}$$

$$F_z = 2\pi\rho g R^2 \left\{ \frac{h}{2}\left(\frac{h}{R}\right)^2 - \frac{R}{3}\left(\frac{h}{R}\right)^3 \right\}$$

$$F_z = 2\pi\rho g R^3 \cdot \frac{1}{6}\left(\frac{h}{R}\right)^3 = \frac{\pi}{3}\rho g R^3\left(\frac{h}{R}\right)^3$$

$$= \frac{\pi}{3}\rho g h^3$$