PHY294, Winter 2023, QUIZ IV Monday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ___________________; **Student #**: ___________________; **Tutorial group**: __________

I. Consider two ideal monatomic gases. They occupy volumes V_1 and V_2, which are taken equal $V_1 = V_2$. The atoms in one of the gases are roughly four times heavier than the ones in the other, i.e. $m_2 = 4m_1$. In addition, gas 1 consists of N_1 atoms and gas 2 of N_2 atoms, with $N_2 = 8N_1$, i.e. the number density of gas 2 is eight times higher. The gases are kept in separate vessels but initially at the same temperature. The temperature of each gas can be individually varied by heating or cooling.

We have learned that at sufficiently high T, both gases are classical and obey the ideal gas law and the equipartition theorem. Suppose the gases get cooled; as we also learned, at sufficiently low T, classical equipartition and the ideal gas law will fail. Estimate the ratio of temperatures T_1 (for gas 1) and T_2 (for gas 2) where the two gases are expected to cease to obey the classical ideal gas law, i.e. estimate the ratio T_1/T_2. In other words, which gas has to be cooled more, and by how much, in order to make it non-classical?

$$\frac{\hbar}{\sqrt{mkT}} \sim \left(\frac{V}{N}\right)^{1/3} \rightarrow \frac{\hbar^2}{mkT} \sim \left(\frac{V}{N}\right)^{2/3} \rightarrow T_* \sim \frac{1}{m}\left(\frac{N}{V}\right)^{2/3}$$

$$T_{*1}/T_{*2} \sim \frac{m_2}{m_1}\left(\frac{N_1}{N_2}\right)^{2/3} \sim 4 \cdot \left(\frac{1}{8}\right)^{2/3} \sim \frac{4}{2^2} \sim 1$$

$\therefore$ @ same T_* (become non-classical)

(shorter $\lambda_{th.}$ of gas 2
compensated by smaller avge distance)

10 points

II. For an Einstein solid of N oscillators of frequency ω, there are two regimes of temperature, the high- and low-T regimes. What determines the temperature range of these regimes? In which regime is classical equipartition obeyed? Would it be appropriate to call one regime "disordered" and the other one "ordered"? Which one is which? (Qualitative answers only please!)

$kT \ll \hbar\omega$ – quantum – ordered ($S \to 0$ $T \to 0$)

$kT \gg \hbar\omega$ – classical – disordered ($S \to \infty$ $T \to \infty$)

equipartition

8 points

III. What is the magnetization of an electronic paramagnet of N spins of magnetic moment μ each, placed in a magnetic field B, in the low-temperature regime, as $T \to 0$? Is this an ordered or a disordered phase? (No derivations, please!)

$M\big|_{T \to 0} = \mu N$, the max value

ordered! $S \to 0$

7 points

Total number of points: 10+8+7=25.

The thermal de Broglie wavelength of a particle of mass m at temperature T is $\lambda_{th.d.B} \sim \frac{\hbar}{\sqrt{mkT}}$. The number density N/V of a gas of N molecules in volume V determines the average distance between particles. The relation between the latter and the thermal de Broglie wavelength determines whether the gas is classical or not. Classical equipartition relates the average energy per degree of freedom to the temperature. The Boltzmann constant is $k \sim 10^{-23} J/K$. The free energy of a system in thermal contact with a reservoir at temperature T (a (T, N, V)-system) is $F = E - TS$, where E is the energy and S the entropy. A simple harmonic oscillator of frequency ω has energy levels $E_n = \hbar\omega n$, $n = 0, 1, 2...$ (neglecting the "zero point" energy). A spin $s = \pm 1$ of magnetic moment μ placed in magnetic field B has energy $E = -\mu B s$. The magnetization of a magnet is its total magnetic moment.

PHY294, Winter 2023, QUIZ IV Tuesday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ___________________; **Student #**: ___________________; **Tutorial group**: ___________

I. Consider two ideal monatomic gases. They occupy volumes V_1 and V_2, one of which is eight times larger $V_1 = 8V_2$. The atoms in gas 1 are roughly four times heavier than the ones in the other, i.e. $m_1 = 4m_2$. In addition, gas 1 consists of N_1 atoms and gas 2 of N_2 atoms, with equal numbers $N_1 = N_2$. Thus, the number density of gas 2 is eight times higher. The gases are kept in separate vessels but initially at the same temperature. The temperature of each gas can be individually varied by heating or cooling.

We have learned that at sufficiently high T, both gases are classical and obey the ideal gas law and the equipartition theorem. Suppose the gases get cooled; as we also learned, at sufficiently low T, classical equipartition and the ideal gas law will fail. Estimate the ratio of temperatures T_1 (for gas 1) and T_2 (for gas 2) where the two gases are expected to cease to obey the classical ideal gas law, i.e. estimate the ratio T_1/T_2. In other words, which gas has to be cooled more, and by how much, in order to make it non-classical?

$$\frac{\hbar}{\sqrt{mkT}} \sim \left(\frac{V}{N}\right)^{1/3} \longrightarrow \frac{\hbar^2}{mkT} \sim \left(\frac{V}{N}\right)^{2/3} \longrightarrow T_* \sim \frac{1}{m}\left(\frac{N}{V}\right)^{2/3}$$

$$\frac{T_{*1}}{T_{*2}} \sim \frac{m_2}{m_1}\left(\frac{V_2}{V_1}\right)^{2/3} = \frac{1}{4}\cdot\left(\frac{1}{8}\right)^{2/3} = \frac{1}{4}\cdot\frac{1}{2^2} = \frac{1}{64}$$

$$\frac{m_2}{m_1} = \frac{1}{4} \quad \frac{V_2}{V_1} = \frac{1}{8}$$

gas 1 needs to be cooled more

shorter wavelength λ_{th}
+
larger avge distance

10 points

II. For an Einstein solid of N oscillators of frequency ω, there are two regimes of temperature, the high- and low- T regimes. What is the energy of the solid in the $T \to 0$ limit? What about its heat capacity, considered in the same limit? Is this an ordered or disordered regime? (Qualitative answers suffice, no derivations or long expressions needed.)

$T \to 0$ $E \to 0$ & $C \to 0$ $\hbar\omega \gg kT$

"ordered" $S \to 0$

8 points

III. An electronic paramagnet of N spins ± 1 of magnetic moment μ each is placed in a magnetic field B. There are two temperature regimes, where the behaviour of the paramagnet is different. What determines the low- and the high- T regimes? Describe qualitatively the behaviour of the magnetization in the two regimes. Which regime is ordered and which one is not? (no derivations!)

$kT \gg \mu B$

$\downarrow$

$M \to 0$ as $T \to \infty$

$\frac{M}{\mu N} = \frac{\mu B}{kT}$ (Curie)

$\uparrow$

not needed.

(disordered)

$kT \ll \mu B$

$\downarrow$

$M \to \max$ $(= \mu N)$ as $T \to 0$

(ordered)

7 points

Total number of points: 10+8+7=25.

The thermal de Broglie wavelength of a particle of mass m at temperature T is $\lambda_{th.d.B} \sim \frac{\hbar}{\sqrt{mkT}}$. The number density N/V of a gas of N molecules in volume V determines the average distance between particles. The relation between the latter and the thermal de Broglie wavelength determines whether the gas is classical or not. Classical equipartition relates the average energy per degree of freedom to the temperature. The Boltzmann constant is $k \sim 10^{-23} J/K$. The free energy of a system in thermal contact with a reservoir at temperature T (a (T, N, V)-system) is $F = E - TS$, where E is the energy and S the entropy. A simple harmonic oscillator of frequency ω has energy levels $E_n = \hbar\omega n$, $n = 0, 1, 2...$ (neglecting the "zero point" energy). A spin $s = \pm 1$ of magnetic moment μ placed in magnetic field B has energy $E = -\mu B s$. The magnetization of a magnet is its total magnetic moment.

PHY294, Winter 2023, QUIZ IV Wednesday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ___________

I. Consider two ideal monatomic gases. They occupy volumes V_1 and V_2, initially taken to be equal $V_1 = V_2$. The atoms in gas 2 are roughly four times heavier than the ones in the other, i.e. $m_2 = 4m_1$. In addition, gas 1 consists of N_1 atoms and gas 2 of N_2 atoms, with equal numbers $N_1 = N_2$. Thus, the initial number densities of the gases are equal. The gases are kept in separate vessels but at the same temperature, which is kept fixed throughout.

We have learned that at sufficiently low density, both gases are classical and obey the ideal gas law and the equipartition theorem. We assume that at the initial values of the volumes and temperature (given above), the gases obey the classical gas laws. Suppose now the gases get compressed. As we also learned, upon increasing the density, classical equipartition and the ideal gas law will fail. Estimate the ratio of volumes V_1' (for gas 1) and V_2' (for gas 2) where the two gases are expected to cease to obey the classical ideal gas law, i.e. estimate the ratio V_1'/V_2'. In other words, which gas does one need to compress more, and by how much, to make it non-classical?

$$\frac{\hbar}{\sqrt{mkT}} \sim \left(\frac{V}{N}\right)^{1/3} \longrightarrow \frac{\hbar^2}{mkT} \sim \left(\frac{V}{N}\right)^{2/3} \quad \therefore \quad V_* \sim \frac{N}{m^{3/2}}$$

$$\frac{V_1'}{V_2'} = \left(\frac{m_2}{m_1}\right)^{3/2} = 4^{3/2} = 8$$

#2 needs to be compressed more (shorter λ_{th})

10 points

II. What is the energy of an Einstein solid of N oscillators of frequency ω in the high-T regime? Is classical equipartition obeyed? Roughly at what temperatures is it valid? (Quick answers suffice, no derivations needed.)

$\underline{\hbar\omega \ll kT}$, $E = NkT$

<u>by equipartition.</u>

8 points

III. Consider an electronic paramagnet of N spins ± 1 of magnetic moment μ placed in magnetic field B. How does the magnetization behave in the high temperature limit? Is this an ordered or disordered phase? (no derivations)

$M \to 0$ as $kT \gg \mu B$

disordered

(Curie, not needed

$\left(\frac{M}{\mu N} = \frac{\mu B}{kT}, \; kT \to \infty \right)$

7 points

Total number of points: 10+8+7=25.

The thermal de Broglie wavelength of a particle of mass m at temperature T is $\lambda_{th.d.B} \sim \frac{\hbar}{\sqrt{mkT}}$. The number density N/V of a gas of N molecules in volume V determines the average distance between particles. The relation between the latter and the thermal de Broglie wavelength determines whether the gas is classical or not. Classical equipartition relates the average energy per degree of freedom to the temperature. The Boltzmann constant is $k \sim 10^{-23} J/K$. The free energy of a system in thermal contact with a reservoir at temperature T (a (T, N, V)-system) is $F = E - TS$, where E is the energy and S the entropy. A simple harmonic oscillator of frequency ω has energy levels $E_n = \hbar\omega n$, $n = 0, 1, 2...$ (neglecting the "zero point" energy). A spin $s = \pm 1$ of magnetic moment μ placed in magnetic field B has energy $E = -\mu B s$. The magnetization of a magnet is its total magnetic moment.

PHY294, Winter 2023, QUIZ IV Thursday.

Answer all questions on the exam paper. Duration: 20 minutes.

Name: ____________________; **Student #**: ____________________; **Tutorial group**: ___________

I. Consider two ideal monatomic gases. They occupy volumes V_1 and V_2, initially taken to be equal $V_1 = V_2$. The atoms in gas 1 are roughly four times heavier than the ones in the other, i.e. $m_1 = 4m_2$. In addition, gas 1 consists of N_1 atoms and gas 2 of N_2 atoms, with equal numbers $N_1 = N_2$. Thus, the initial number densities of the gases are equal. The gases are kept in separate vessels but at the same temperature, which is kept fixed throughout.

We have learned that at sufficiently low density, both gases are classical and obey the ideal gas law and the equipartition theorem. We assume that at the initial values of the volumes and temperature (given above), the gases obey the classical gas laws. Suppose now the gases get compressed. As we also learned, upon increasing the density, classical equipartition and the ideal gas law will fail. Estimate the ratio of volumes V_1' (for gas 1) and V_2' (for gas 2) where the two gases are expected to cease to obey the classical ideal gas law, i.e. estimate the ratio V_1'/V_2'. In other words, which gas does one need to compress more, and by how much, to make it non-classical?

$$\frac{\hbar}{\sqrt{mkT}} \sim \left(\frac{V}{N}\right)^{1/3} \rightarrow \frac{\hbar^2}{mkT} \sim \left(\frac{V}{N}\right)^{2/3} \qquad V_* \sim \frac{N}{m^{3/2}}$$

$$\frac{V_1'}{V_2'} = \left(\frac{m_2}{m_1}\right)^{3/2} = 4^{3/2} = 8$$

#2 needs to be compressed more (shorter λ_{th})

10 points

II. Consider a paramagnet of N spins ± 1 of magnetic moment μ placed in magnetic field B. What is the free energy of the magnet in the zero-temperature limit? Would you call this an ordered or a disordered phase? (Qualitative answers suffice, no derivations needed.)

$F = E - TS$, $T \to 0$ $F \to E$

$E = -\mu B N$ all spins up

ordered! $(\uparrow\uparrow \ldots \uparrow)$ $S \to 0$

8 points

III. An Einstein solid has N oscillators of frequency ω. What is the energy of the solid in the high-T limit? What about its heat capacity? Is this an ordered or a disordered phase? (No derivations needed.)

$kT \gg \hbar\omega$

$E \simeq NkT$

$C = Nk$

disordered

7 points

Total number of points: 10+8+7=25.

The thermal de Broglie wavelength of a particle of mass m at temperature T is $\lambda_{th.d.B} \sim \frac{\hbar}{\sqrt{mkT}}$. The number density N/V of a gas of N molecules in volume V determines the average distance between particles. The relation between the latter and the thermal de Broglie wavelength determines whether the gas is classical or not. Classical equipartition relates the average energy per degree of freedom to the temperature. The Boltzmann constant is $k \sim 10^{-23} J/K$. The free energy of a system in thermal contact with a reservoir at temperature T (a (T, N, V)-system) is $F = E - TS$, where E is the energy and S the entropy. A simple harmonic oscillator of frequency ω has energy levels $E_n = \hbar\omega n$, $n = 0, 1, 2...$ (neglecting the "zero point" energy). A spin $s = \pm 1$ of magnetic moment μ placed in magnetic field B has energy $E = -\mu B s$. The magnetization of a magnet is its total magnetic moment.