PHYS 293 – Quiz 3 Version B Name: ________________ Student Number: ________________ 18 Nov 2021

[Instructions: Solve the two questions below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.

You are allowed: a non-communicating calculator, a one-page, two-sided Formula Sheet (can be annotated)].

1. The rocket Blue Origin carried William Shatner to the upper atmosphere – a height of 105 km – on 13 October 2021. Assume that the Blue Origin and the Earth are inertial reference frames.

   a. Blue Origin rose to its apogee (point of highest ascent) at a speed of 3,600 km. What was the time to reach apogee as seen from the ground (keep at least 7 significant figures!)? Assume the speed was constant from lift-off to apogee. What was the difference in time to reach apogee as measured on Blue Origin? [3 points]

$v = 3600\ km/h = 1000\ m/s \quad , \quad d_e = 105\ km = 105 \times 10^3\ m$

<u>Earth frame</u>

$\Delta t_e = \frac{d_e}{v} = 105.0000000000\,s$

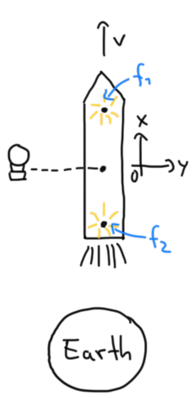
<u>Rocket frame</u>

Length contraction: $d_r = \frac{d_e}{\gamma} \Rightarrow \Delta t_r = \frac{d_r}{v} = \frac{1}{\gamma}\frac{d_e}{v} = \frac{1}{\gamma}\Delta t_e$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad , \quad \beta = \frac{v}{c} = \frac{1000\ m/s}{3\times10^8\ m/s} \Rightarrow \Delta t_r = 104.9999999994\,s$

$\Rightarrow |\Delta t_e - \Delta t_r| = 6\times10^{-10}\,s = 0.6\ ns$

   b. Two astronauts, one at each end of the rocketship and separated by 20 meters, flashed light signals exactly in unison when the midpoint of the rocketship passed an observation balloon that was stationary at an altitude of 15 km. Would the observers on the balloon have detected the signals as being simultaneous? If not, which one would have been received first on the balloon? [4 points]



<u>Flashes of light in rocket frame:</u>

Set $x=0$ at midpoint of rocket, light signals at $t=0$

$f_1: (ct_1, x_1) = (0, 10m)$

$f_2: (ct_2, x_2) = (0, -10m)$

<u>Balloon frame:</u> Use Lorentz transformation

$f_1: (ct_1', x_1') = (\gamma ct_1 - \gamma\beta x_1, \gamma x_1 - \gamma\beta ct_1) = (-\gamma\beta 10m, \gamma 10m)$

$f_2: (ct_2', x_2') = (\gamma ct_2 - \gamma\beta x_2, \gamma x_2 - \gamma\beta ct_2) = (+\gamma\beta 10m, -\gamma 10m)$

Not simultaneous in the balloon frame.
f_2 is detected first since here $\beta = \frac{v}{c} < 0$
(In the rest frame of the rocket the balloon moves in the $-\hat{x}$ direction.)

   c. The Blue Origin sent telemetry back to the Earth at a frequency of exactly 10 GHz. What frequency did the receiver on Earth have to be tuned to? [2 points]

<u>Relativistic Doppler shift</u>

Blue Origin and Earth are moving apart

$\Rightarrow \frac{f_s}{f_r} = \sqrt{\frac{1+\beta}{1-\beta}}$

$\Rightarrow$ Frequency on Earth: $f_r = \sqrt{\frac{1-\beta}{1+\beta}}\, f_s$ where $\beta = \frac{v}{c} = \frac{1000\ m/s}{3\times10^8\ m/s}$ and $f_s = 10\ GHz$

$= 9.99997\ GHz$

d. Blue Origin came to rest at the apogee for an instant. Was this observed at the same time on the rocketship and the Earth (taking into account that light had to travel back to the Earth)? Why or why not? [1 point]

Blue Origin coming to rest implies that at that instant its rest frame coincides with the Earth rest frame. However, light travels at finite speed ($c < \infty$) meaning that it takes time $t_L = \frac{d_a}{c} > 0$ ($d_a = 105\,\text{km}$) to travel from the apogee to Earth.

Therefore, on Earth Blue Origin coming to rest was observed later.

2. The photoelectric effect was explained by Einstein by assuming light could be considered as consisting of particles that transferred their energy to electrons in a metal surface, thereby creating photoelectrons.

   a. If this only happened for light with a frequency greater than $1.22 \times 10^{15}\ s^{-1}$, what is the minimum energy transferred to the electron to release it from the metal? [3 points]

$E_{pe} = h\nu - W$

Want to find $W \Rightarrow E_{pe} = 0 = h\nu_0 - W$ ($\nu_0 = 1.22 \times 10^{15}\,\text{Hz}$)

$\Rightarrow W = h\nu_0 = 8.08 \times 10^{-19}\,\text{J} = 5.046\,\text{eV}$

   b. Plot the energy of the photoelectrons as a function of the frequency of the light. Indicate on the $\hat{x}$ axis the frequency. [2 points]