PHYS 293 – Quiz 3 Version A Name: ________________ Student Number: ________________ 18 Nov 2021

[Instructions: Solve the two questions below. Show all the steps to your solution; you do not have to derive any equations included on the Formula Sheet. Number of points awarded for each question is included in the brackets. Partial marks will be awarded.

You are allowed: a non-communicating calculator, a one-page, two-sided Formula Sheet (can be annotated)].

1. A rocketship is launched from Earth intending to travel to Betelgeuse, a red-giant star 650 light-years away. Assume that the rocketship and the Earth are inertial reference frames.

   a. What would be the minimum speed of the rocket if the trip were to take 10 years from the perspective of the astronauts on the rocketship? How long would the trip take from observers on Earth (ignore any periods of acceleration or deceleration of the rocketship). [3 points]

   b. The rocketship is at maximum speed as it passes Jupiter, which has a diameter of 140,000 km. How long does it take to pass Jupiter from the perspective of the astronauts? [2 points]

   c. The rocket communicates with the Earth by pulsing a laser. If the Earth receiver can only detect light with a wavelength less than 350 nm, given that the rocket is flying toward a bright red star, what is the minimum frequency of the laser ON the rocketship. [3 points]

d. Sketch the Minkowski diagram for the worldline of the rocketship with Earth at the origin. Assume that the coordinate system has the rocketship travelling in the $\hat{x}$ direction. [2 points]

2. A photomultiplier tube uses cesium as the photocathode that releases photoelectrons when light strikes the plate with a given frequency range.

   a. If cesium has a work function of W = 2.14 eV, what range of frequencies is the photomultiplier sensitive to? [3 points]

   b. Plot the energy of the photoelectrons as a function of the frequency of the light. Indicate on the $\hat{x}$ axis the frequency. [2 points]

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1. A rocketship is launched from Earth intending to travel to Betelgeuse, a red-giant star 650 light-years away. Assume that the rocketship and the Earth are inertial reference frames.

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t_A = time in astronaut's frame = 10y

$\frac{L_E}{c}$ = "distance" in Earth's frame = 650y

Due to length contraction, $L_A = L_E/\gamma$

Speed $= v = \frac{L_A}{t_A}$ or $\beta = \frac{v}{c} = \frac{1}{t_A}\left(\frac{L_A}{c}\right)$

but $L_A = L_E/\gamma$, so

$$\Rightarrow \beta = \frac{1}{t_A}\left(\frac{L_E}{\gamma c}\right) \Rightarrow \gamma\beta = \frac{L_E}{t_A} = 65$$

$$\frac{\beta}{\sqrt{1-\beta^2}} = 65 \Rightarrow \beta = 0.999882$$

$$\therefore \boxed{v = 0.999882\ c}$$

Due to time dilation,

$$t_E = \gamma t_A$$

$$= \frac{1}{\sqrt{1-\beta^2}}(10y)$$

$$\boxed{t_E = 65.0077 \text{ years}}$$

b. The rocketship is at maximum speed as it passes Jupiter, which has a diameter of 140,000 km. How long does it take to pass Jupiter from the perspective of the astronauts? [2 points]

$$d_E = 140{,}000 \text{ km} = 1.4 \times 10^8 \text{ m}$$

$$d_A = \frac{d_E}{\gamma}$$

$$\therefore t_A = \frac{d_A}{v} = \frac{d_E}{\gamma v} = \frac{d_E}{\gamma \beta c}$$

We have that $\gamma\beta = 65$ from (a), so

$$t_A = \frac{1.4 \times 10^8 \text{ m}}{(65)(3 \times 10^8 \text{ m})}$$

$$\boxed{t_A = 0.0071 \text{ s}}$$

c. The rocket communicates with the Earth by pulsing a laser. If the Earth receiver can only detect light with a wavelength less than 350 nm, given that the rocket is flying toward a bright red star, what is the minimum frequency of the laser ON the rocketship. [3 points]

$$\frac{f_{sender}}{f_{receiver}} = \sqrt{\frac{1+\beta}{1-\beta}}$$

(sender and receiver are moving away from each other)

Using $f = \frac{c}{\lambda}$, we can write

$$\frac{\lambda_r}{\lambda_s} = \sqrt{\frac{1+\beta}{1-\beta}}$$

Setting $\lambda_r = 350 \text{ nm}$, we obtain

$$\lambda_s = \sqrt{\frac{1-\beta}{1+\beta}}\ \lambda_r$$

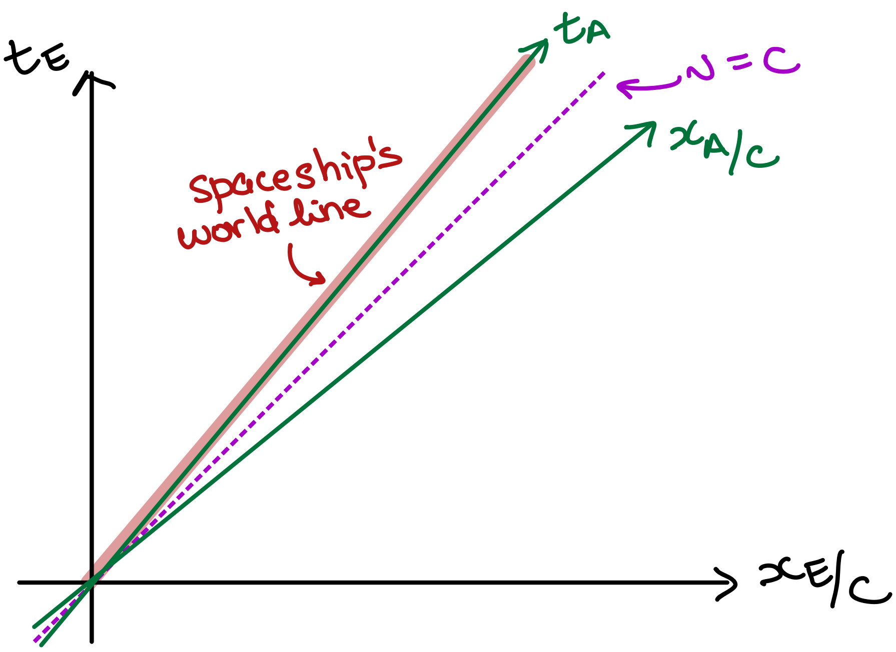
$$= 2.61 \text{ nm}$$

$\therefore$ The max. wavelength of the laser on the spaceship is $\lambda_{s,max} = 2.61$ nm,

which translates to a MINIMUM frequency of $f_{s,min} = \dfrac{c}{\lambda_{s,max}} = 1.39 \times 10^{17}$ Hz

$$\boxed{f_{s,min} = 1.39 \times 10^{17} \text{ Hz}}$$

d. Sketch the Minkowski diagram for the worldline of the rocketship with Earth at the origin. Assume that the coordinate system has the rocketship travelling in the $\hat{x}$ direction. [2 points]



2. A photomultiplier tube uses cesium as the photocathode that releases photoelectrons when light strikes the plate with a given frequency range.

   a. If cesium has a work function of W = 2.14 eV, what range of frequencies is the photomultiplier sensitive to? [3 points]

$W = 2.14 \text{ eV}$

W gives us the minimum frequency for which the photoelectric effect is seen

$$\Rightarrow hf_{min} = W$$

$$\Rightarrow f_{min} = \frac{W}{h}$$

$$= \frac{2.14 \text{ eV}}{6.582 \times 10^{-16} \text{ eV} \cdot \text{s}}$$

$$= 3.25 \times 10^{15} \text{ Hz}$$

$$\therefore \text{Range is } \boxed{f \geq 3.25 \times 10^{15} \text{ Hz}}$$

   b. Plot the energy of the photoelectrons as a function of the frequency of the light. Indicate on the $\hat{x}$ axis the frequency. [2 points]