8pts

1. Time for light to travel E → S

2 is $T = \frac{\text{Distance}}{\text{velocity}}$ ← orbital radius

← $c = 3 \cdot 10^8$ m/s

3 Distance scale given by semimajor axis "a" of elliptical orbit: $a^3 \frac{4\pi^2}{GM_s} = T^2$

← 1 yr.

There is some variation as we approach or recede from the sun, but we are asked for an estimate.

1 $1\,yr = 365$ days $\times 24h \times 3600s$

using G & M_s from front page $\begin{cases} G = 6.7\times10^{-11}\ N\,m^2/kg \\ M_s = 2\times10^{30}\ kg \end{cases}$

Algebra: 2pts

$a = 1.5\times10^{11}$ m

& thus $T = \frac{1.5\times10^{11}\ m}{3\times10^{8}\ s} \sim 500\,s$

Order of magnitude: $\underline{10^3\,s.}$

1 pt off for 10^2 s.

20 pts

2. The CM follows ballistic motion, independent of the rotational degrees of freedom of the object.

$\vec{F} = m\vec{a}$: constant $\vec{a} = \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix}$

gives

4 (a) $\vec{r} = \begin{pmatrix} V_{xi} t \\ V_{yi} t - \frac{1}{2} g t^2 \\ 0 \end{pmatrix}$ or other equivalent forms, eg $\vec{r} = \vec{v}_i t + \frac{1}{2}\vec{a} t^2$ if $\vec{a}$ specified.

4 (b) $\vec{p} = m\vec{v} = \begin{pmatrix} MV_{xi} \\ MV_{yi} - gt \\ 0 \end{pmatrix}$ or other equivalent fors

For the second two parts of this problem, consider the sum of CM motion and rotation about the CM.

(6 pts) (c) $\vec{L} = \underbrace{\vec{r} \times \vec{p}}_{\text{changing as a function of time } t.} + \underbrace{I_{CM}\,\omega_i \hat{k}}_{\text{remains @ initial value.}}$

3 pts for knowing this

[continued]

now need to take the cross product of

$$\vec{r} \times \vec{p} = m\begin{pmatrix} v_{xi}t \\ v_{yi}t - \frac{1}{2}gt^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} v_{xi} \\ v_{yi} - gt \\ 0 \end{pmatrix}$$

$$= m\left(r_x v_y - r_y v_x\right)$$

$$= m\left(v_{xi}t \cdot (v_{yi} - gt) - (v_{yi}t - \tfrac{1}{2}gt^2)\, v_{xi}\right)$$

$$= -\frac{1}{2} m g v_x t^2$$

-3 pts for calc'ing cross product

& thus $\boxed{L_z = I_{cm}\omega_i - \frac{1}{2} m g v_{xi} t^2}$

(6 pts) (d) There are two approaches to finding the time-average torque: [full credit for either approach]

<u>Approach I:</u> $\Delta\vec{L} = \vec{\tau}_{avg}\,\Delta t$

so $\vec{\tau}_{avg} = \dfrac{\Delta\vec{L}}{\Delta t}$

we just found that $L_z(t) = L_z(t=0) - \frac{1}{2} m g v_{xi} t^2$,

so $\boxed{(\tau_{avg})_z = -\frac{1}{2} m g v_x t}$

[② d, cont.]

<u>Approach II:</u> Direct integration

$$\vec{\tau}_{avg} = \frac{1}{t}\int_0^t \vec{\tau}(t)\,dt$$

Here,

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{pmatrix} v_{xi}t \\ v_{yi}t - \frac{1}{2}gt^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -mg \\ 0 \end{pmatrix}$$

$$= -mgv_{xi}t\ \hat{k}$$

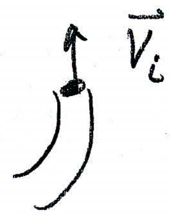
Thus $$(\tau_z)_{avg} = \frac{1}{t}\underbrace{\int_0^t (-mgv_{xi})\,t'\,dt'}_{-mgv_{xi}\frac{t^2}{2}}$$

$$\boxed{(\tau_z)_{avg} = -\frac{1}{2}mgv_{xi}t}$$

as before.

[8 pts]

(3.) Motion on a circle has velocity tangent to the circle, so when leaving the channel,



velocity is straight up.

Afterwards, there are no net forces applied, so the ball continues to move in a straight line (path B) by Newton's 1st law.

6/8 if answer given but no justification.

[24 pts]

(4) This problem is easy to solve if one considers the centre of mass motion first.

(a) For a constant force, there is constant acceleration. So

$$\Delta X_{cm} = \cancel{(V_{cm})_i}^{\to 0} + \frac{1}{2} a t^2$$

but $a_{cm} = \dfrac{F}{2M}$ ← M_{TOT}

& thus

$$\Delta X_{cm} = \frac{F}{4m} t^2$$

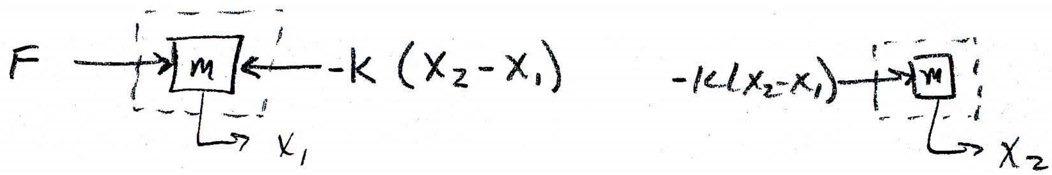
Since we know that $\Delta X_{cm} = \dfrac{x_1 + x_2}{2}$ after Δt

then $$\Delta t = \sqrt{\frac{4M\Delta X}{F}} = \sqrt{\frac{2M}{F}(x_1 + x_2)}$$

[check units: $\sqrt{\dfrac{kg \cdot m}{kg\, m/s^2}} = s$ ✓]

8 pts for this part. But if "Δt" used in later parts, either without solving or using the wrong answer for (a), then give full credit.

4. Alternate approach, starting with $F=ma$ & free body diagrams, B:



$$m\ddot{x}_1 = F - kx_1 + kx_2 \qquad m\ddot{x}_2 = -kx_2 + kx_1$$

Rewrite in terms of $\Delta x \equiv x_2 - x_1$

$\bar{x} = (x_1 + x_2)/2$

so that $2m\ddot{\bar{x}} = m\ddot{x}_1 + m\ddot{x}_2$

$= F - kx_1 + kx_2 - kx_2 + kx_1$

$= F$

so we see that $\ddot{\bar{x}} = \frac{F}{2m}$, ie that

$x_{cm} = \bar{x}$ has <u>constant</u> <u>acceleration</u>.

→ from here, back to previous page.

For other degree of freedom,

$$m\,\Delta\ddot{x} = m\ddot{x}_2 - m\ddot{x}_1 = -k(x_2-x_1) - F - k(x_2-x_1)$$

$$= -F - 2k\Delta x$$

One can solve this, but not relevant to any question asked. without F, $m\Delta\ddot{x} + 2k\Delta x = 0 \to$ <u>SHO</u> eq!

[8 pts]

(b) Next, knowing Δt, we can find the total impulse: $\vec{I} = \vec{F}_{av} \Delta t$ +3 pts for knowing this.

Here, all vectors in one direction (x), & F = const, so we are left with $I = F\Delta t$

$$= \sqrt{2FM(x_1 + x_2)}$$

Now given conservation of momentum, after F removed, p remains the same:

$\Delta \vec{P} = \vec{I}$ & $\Delta \vec{P} = \underbrace{M_{TOT}}_{2M} \vec{V}_{cm}$ (since $v_i = 0$)

+2 pts for knowing this

so

$$\boxed{V_{cm} = \sqrt{\frac{F}{2M}(x_1 + x_2)}}$$

2 pts for algebra, putting it together, etc.

[8 pts]

(c) Energy from cm motion is not equal to the work done, so whatever is left over must be ~~internal~~ (vibrational) energy:

$$\Delta E = W \quad \text{where} \quad \Delta E = \Delta K + \Delta E_{vib}$$

$$W = F x_1$$

$$\Delta K = K_f = \frac{1}{2} M_{TOT} V_{cm}^2 = \frac{1}{2}(2m)\left(\frac{F}{2m}(x_1 + x_2)\right)$$

$$\rightarrow \Delta E_{vib} = W - \Delta K$$

Key idea: 5/8 points.

$$= F x_1 - \frac{F}{2}(x_1 + x_2)$$

$$\boxed{\Delta E_{vib} = \frac{F}{2}(x_1 - x_2)}$$

Incorrect approaches to Q4

One cannot use $\Delta E_{vib} = \frac{1}{2} K (x_2 - x_1)^2$, because this actually gives only the increase in potential energy of the spring.

There is also a kinetic energy of the vibration which is $\frac{1}{2} m \dot{x}_2^2 + \frac{1}{2} m \dot{x}_1^2 - \frac{1}{2} M_{TOT} V_{cm}^2$

$$= \frac{1}{2} m \left(\dot{x}_2^2 + \dot{x}_1^2 - 2 \left(\frac{\dot{x}_1 + \dot{x}_2}{2} \right)^2 \right)$$

$$= \frac{1}{2} m \left(\dot{x}_1^2 + \dot{x}_2^2 - \frac{1}{2} \dot{x}_1^2 - \frac{1}{2} \dot{x}_2^2 - \dot{x}_1 \dot{x}_2 \right)$$

$$= \frac{1}{2} m \left(\frac{1}{2} \dot{x}_1^2 + \frac{1}{2} \dot{x}_2^2 - \dot{x}_1 \dot{x}_2 \right)$$

$$= \frac{1}{2} m \frac{(\Delta \dot{x})^2}{2} \quad \text{where} \quad \Delta x \equiv x_2 - x_1$$

(Anyway, we did not give k in the problem to avoid this direction)

Final velocities. If we were told k, ...

Since we can solve for both $\Delta\dot{x}$ & $\dot{x}_{cm}$ after the force, we can also find the final velocities for both masses.

$$\underbrace{\Delta E}_{Fx_1} = \underbrace{\Delta K_{cm}}_{\frac{F}{2}(x_1+x_2)} + \underbrace{\Delta U + \Delta K_{vib}}_{\frac{F}{2}(x_1-x_2)}$$

& also, $\Delta U = \frac{1}{2}k(x_2-x_1)^2$

so, $$\Delta K_{vib} = \underbrace{\frac{F}{2}(x_1-x_2) - \frac{1}{2}k(x_2-x_1)^2}_{-\frac{F\Delta x}{2} - \frac{1}{2}k(\Delta x)^2} = \frac{m}{4}(\Delta\dot{x})^2$$

$$\rightarrow \quad \Delta\dot{x} = \sqrt{\frac{4}{m}\left(-\frac{\Delta x}{2}\left(F+k\Delta x\right)\right)}$$

(Note $\Delta x < 0$ as defined & drawn...)

giving, a second equation for $\dot{x}_1$ & $\dot{x}_2$, over

$$\Delta K_{cm} = \frac{1}{2}m\dot{x}_{cm}^2 = \frac{F}{2}(x_1+x_2) \rightarrow \dot{x}_{cm} = \sqrt{\frac{F}{m}(x_1+x_2)}$$

combine to give

$$\dot{x}_1 = 2\frac{(x_1+x_2)}{2} + (x_1-x_2) = 2V_{cm} + \Delta\dot{x}$$

$$\& \quad \dot{x}_2 = 2V_{cm} - \Delta\dot{x}$$

[25 pts]

(5.) The problem asks questions about the platform, so the only thing we need to use about the car is that by Newton's third law,

[4 pts] (a) F (car on platform) $= - F$ (platform on car)

or $F_{cp} = -F_{pc}$

$$\longrightarrow \quad F_{cp} = -F_{pc}$$

Now if $a_{car} = F_{pc}/m_c$, then

$$F_{cp} = -m_c a_c = -m_c \left(0.1 \tfrac{m}{s^3}\right) t$$

where

$$\vec{F} = F_{cp}\hat{\imath} - m_c g \hat{\jmath}$$

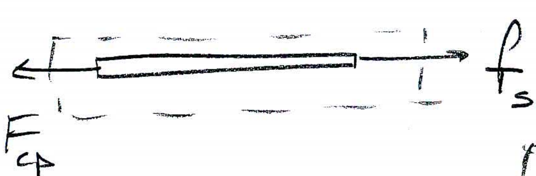
$$\boxed{F_{cp} = -\left(200 \tfrac{N}{s}\right) t}$$

[4 pts] (b) Static friction can hold up to

$f_{max} = \mu_s n$ key point: this is a maximum.

(In this case, $n = (m_c + m_p) g$)

At rest,

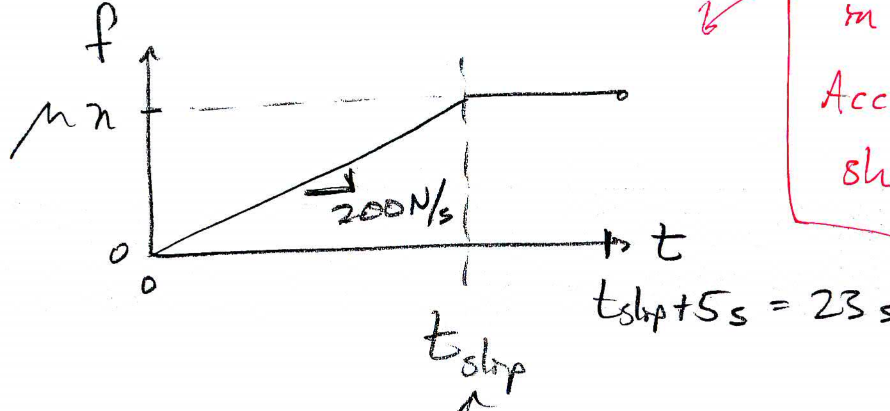


so $|f_s| = |F_{cp}| = 200t$

The max is reached at $t_{slip} = \dfrac{\mu(m_c+m_p)g}{F'}$ $\boxed{= 18s}$

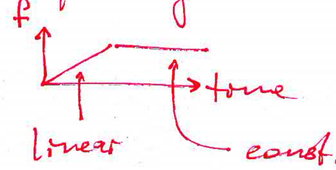
4 pts

(c) Plot of f:



answers will vary in level of detail.

Accept anything that shows



$\mu n = 0.15 \cdot 9.8 \cdot 2400\,kg = 3.5 \times 10^3\,N$

$t_{slip} = \frac{\mu n\, g}{m_c} = 18\,s$

Before t_{slip}

- Static friction
- $|f| = |\text{applied force}| = |m_c a_c| = 200\,\frac{N}{s}\,t$

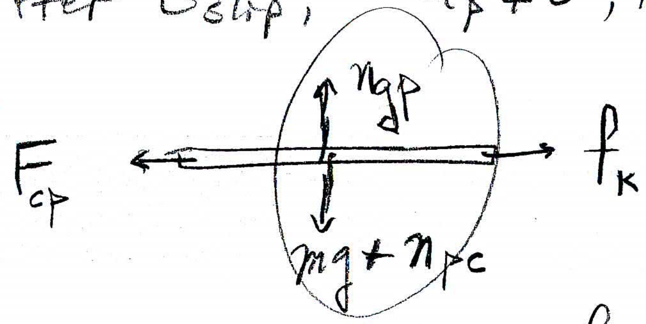
After t_{slip}

- Kinetic friction
- $|f| = \mu_k n = 3.5 \times 10^3\,N$ as given above

check slope: $18 \cdot 200 = 3600$ ✓ (close enough)

[6 pts] (d) upto & At t_{slip}, forces balanced, & $a_p = 0$.

After t_{slip}, $a_p \neq 0$; in x direction



forces still balance in y direction: $n_{gp} = m_p g + \underbrace{m_c g}_{n_{pc}}$

In x direction,

$$m_p a_p = f_K - F_{cp}$$

$$= f - m_c a_c(t) \quad , \quad \text{where } a_c = 0.1\,t$$

$$= -m_c 0.1 \left(t - \underbrace{f/_{0.1 m_c}}_{\text{this} = t_{slip}}\right)$$

$$= -\underbrace{m_c \cdot 0.1}_{200}\, t_+$$

$$\boxed{a_p = -0.5\, t_+}$$

$\left\{ \begin{array}{l} \text{magnitude: } 0.5\, t_+ \\ \text{direction: left or } -\hat{\imath} \end{array} \right.$

5(e) Energy lost to friction.

Nonconservative force is only kinetic friction.
It creates $\Delta E_{int} = f \cdot d$, where d = path length (relative motion of surfaces)

Here, $f = \mu_K n = 3528\,N \approx 3.5 \times 10^3\,N$

Path length given by a_p in last 5s:

$$a_p = -0.5\,t_+$$
$$V_p = -0.25\,t_+^2$$
$$x_p = -0.083\,t_+^3$$

integrate $0 \to t_+$

So @ $t_+ = 5s$, $d = |x_p| = 10.4m$

together, $\boxed{\Delta E_{int} = 3.7 \times 10^4\,J}$

Checks

A bit ridiculous about this problem is that the car goes a long way:

$$a_c = 0.1\,t$$

$$V_c = \frac{1}{2}\,0.1\,t^2$$

$$X_c = \frac{1}{6}\,0.1\,t^3$$

@ $t = 18s + 5s$, $X_c = 203\,m$

Also, what is the force on the car @ the end of its journey?

$$F = m_c\,0.1\,t_f$$

coefficient of friction must be what?

$$\mu_s m_c g \geq m_c\,0.1\,t_f$$

$$\mu_c \geq \frac{0.1}{9.8} \cdot (23s) = 0.23$$

fine, realistic enough.

[15pts]

6.

(a) $I = I_D + mr^2$

5 pts

$L = I\omega$ = conserved ← key idea: 3/5 points

Thus $K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{L_i^2}{I} = \frac{L_i^2}{2}\frac{1}{I_D+mr^2}$

(b) "Looks like" $U_{eff} = K_R = \frac{L^2}{2}\frac{1}{I_0+mr^2}$ in rotating frame.

(b) What is force? $-\frac{dU_{eff}}{dr} = \frac{-d}{dr}(L^2/2I_0)(1+mr^2/I_D)^{-1}$

5 pts

$$= -\frac{L^2}{2I_0}(-1)(1+mr^2/I_D)^{-2}\, 2mr/I_D$$

3 pts for knowing $(F_{int})_r = -\frac{dU_{eff}}{dr}$

$$= \boxed{\frac{L^2}{I_0^2}\frac{mr}{(1+mr^2/I_D)^2}}$$

2 pts for calculus

(c) but

5pts

$$\omega^2(r) = \frac{L^2}{(I_D+mr^2)^2}$$

3pts

so this is simply $\boxed{F = mr\omega^2(r)}$

Centrifugal force! Because $\vec{a}_c = -r\omega^2\hat{r}$ ← centripetal acceleration

and $\vec{F}_I = -m\vec{a}$

is inertial force when moving into non-inertial reference frame.

2 pts for argument.