ESC103F Engineering Mathematics and Computation: Tutorial #6

Question 1: Consider matrix A:

$$A = \begin{bmatrix} 0.4 & 1-c \\ 0.6 & c \end{bmatrix}$$

a) Find the eigenvalues and eigenvectors of matrix A (they may depend on c).

b) Show that A has just one line of eigenvectors when $c = 1.6$.

Question 2: A person wants to combine three foods (fish, bread, and vegetables) to make a meal with specified contributions of protein, carbohydrates and fat. Each gram of fish has 0.8 grams of protein and 0.2 grams of fat. Each gram of bread has 0.25 grams of protein, 0.7 grams of carbohydrates and 0.05 grams of fat. Each gram of vegetables has 0.3 grams of protein, 0.6 grams of carbohydrates, and 0.1 grams of fat.

How many grams each of fish (x_1), bread (x_2) and vegetables (x_3) should the person consume if the person desires to end up with 2.95 grams of protein, 3.3 grams of carbohydrates and 0.75 grams of fat? Solve this problem by first formulating it as a system of linear equations, writing it in the form $AX = B$, and then solving it using Gaussian elimination.

Question 3: Consider the system of linear equations:

$$\begin{aligned} x_1 + x_2 - x_3 &= 5 \\ 2x_1 + x_3 - x_4 &= 2 \\ 2x_2 - 3x_3 + x_4 &= 8 \end{aligned}$$

a) Give the general solution to the system.

b) Use your solution from part a) to determine the solution to the system when $x_3 = x_4 = 2$.

Question 4: For the system given by:

$x + y + z = 9$

$x + 5y + 10z = 44$

a) Use Gaussian elimination to find the reduced normal form of this system.

b) Using your answer to part a), write the vector solution to this system and identify any free variables.

c) Using your answer to part b), find all positive integers for x, y and z that satisfy this system.

Question 5: Consider the attached figure that shows the flow rates of oil in a network of pipes. By applying the conservation of mass principle at each of the nodes, we can develop the following mathematical model of the network:

$$
\begin{aligned}
x_1 + x_3 &= 200 \\
x_1 + 25 &= x_2 \\
x_2 + x_4 &= x_6 + 175 \\
x_3 + 150 &= x_4 + x_5 \\
x_5 + x_6 &= 200
\end{aligned}
$$

where the x_i's denote the unknown flow rates.

a) Write this system of equations in the standard form *AX=B*.

b) Solve this network model for the unknown flow rates by first taking the system of equations to its reduced normal form.

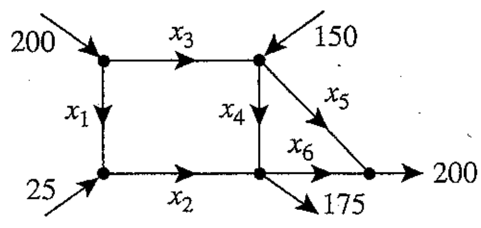
c) Using your solution to part b), find the unknown flow rates if $x_4 = 50$ and $x_6 = 0$.

Question 6: Consider the attached figure that shows an electric circuit. By applying Kirchhoff's Laws and Ohm's Law, a mathematical model of this circuit has been developed and is presented on the same figure.

a) Write this system of equations in the standard form *AX=B*.

b) Solve this circuit model for the unknown currents I_1 through I_5 by first taking the system of equations to its reduced normal form.

c) Why does a row of zeros appear in *M'* (the augmented matrix obtained after applying the Gaussian elimination algorithm)? Does this row change the solution?



200
x_3
150
x_1
x_4
x_5
x_6
25
x_2
175
200

Consider the electric circuit in the figure below.

Using Kirchoff's Laws and Ohm's law, we can model this circuit as a system of linear equations.

The equations that describe the current flowing in each branch of the circuit are as follows:

$$-I_1 + I_4 = 0$$
$$I_1 - I_2 - I_5 = 0$$
$$I_2 - I_3 = 0$$
$$I_3 - I_4 + I_5 = 0$$
$$I_2 + I_3 - I_5 = 10$$
$$I_1 + I_4 + I_5 = 10$$