UNIVERSITY OF TORONTO

Faculty of Applied Science and Engineering

Term Test I

First Year — Program 5

MAT185H1S — Linear Algebra

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1 February 2011

Student Name:

Last Name First Names

Student Number:

Instructions:

1. Attempt *all* questions.
2. The value of each question is indicated at the end of the space provided for its solution; a summary is given in the table opposite.
3. Write the final answers *only* in the boxed space provided for each question.
4. *No* aid is permitted.
5. The duration of this test is 90 minutes.
6. There are 9 pages and 5 questions in this test paper.

For Markers Only		
Question	**Value**	**Mark**
A		
1	10	
B		
2	10	
C		
3	10	
4	10	
5	10	
Total	50	

A. Definitions and Statements

Fill in the blanks.

1(a). The *closure axioms* of a vector space state that

/2

1(b). The mathematical notation to denote that U is a subset of V but not equal to V is

/2

1(c). The *"smallest" subspace* of a vector space $\mathcal{V}$ containing $\{\boldsymbol{v}_1 \cdots \boldsymbol{v}_n\}$ is

/2

1(d). The *commutativity theorem* for a vector space states that

/2

1(e). A function, $f : X \to Y$, is *injective* if

/2

B. True or False

Determine if the following statements are true *or* false *and indicate by "T" (for true) and "F" (for false) in the box beside the question. The value of each question is 2 marks.*

2(a). The set of points in $\mathbb{R}^2$ defined by the ellipse, $x^2/a^2 + y^2/b^2 = 1$, is closed under the standard scalar multiplication for $\mathbb{R}^2$. ☐

2(b). Any finite set of vectors in a vector space over $\mathbb{R}$ is never a subspace. ☐

2(c). Any vector space has an infinite number of subspaces. ☐

2(d). The solutions $T(x)$, the temperature distribution, to the steady-state heat equation,

$$\alpha \frac{d^2T}{dx^2} = 0$$

form a subspace of the vector space of functions. ☐

2(e). The solutions to the system of linear equations $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A} \in {}^m\mathbb{R}^n$ is a subspace of ${}^n\mathbb{R}$ if and only if $\mathbf{b} = \mathbf{0}$. ☐

C. Problems

3. Show that $\operatorname{span}\{x, x^2\} = \operatorname{span}\{x+x^2, x-x^2\}$ where x, x^2 are elements in the vector space $\mathbb{P}_n$.

...cont'd

3. . . . cont'd

/10

4. Let V be a set of elements endowed with operations of addition and scalar multiplication such that V satisfies all the axioms of a vector space *except* MIV, namely, $1\boldsymbol{v} = \boldsymbol{v},\ \forall \boldsymbol{v} \in V$.

From class, we also know that V possesses the following properties because none of these depend on MIV:

(*i*) If $\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{v} + \boldsymbol{w}$ then $\boldsymbol{u} = \boldsymbol{v},\ \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$. (This is the *right* cancellation theorem.)

(*ii*) $0\boldsymbol{v} = \boldsymbol{0},\ \forall \boldsymbol{v} \in V$.

(*iii*) $-(\lambda \boldsymbol{v}) = (-\lambda)\boldsymbol{v},\ \forall \lambda \in \Gamma, \forall \boldsymbol{v} \in V$.

Prove that $1\boldsymbol{v} = \boldsymbol{v},\ \forall \boldsymbol{v} \in V$, if and only if $-\boldsymbol{v} = (-1)\boldsymbol{v},\ \forall \boldsymbol{v} \in V$.

. . . cont'd

4. . . . cont'd

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5. Let $\mathcal{V}$ be a vector space and $\mathcal{S} \subseteq \mathcal{V}$. Prove that $\mathcal{S}$ is a subspace of $\mathcal{V}$ if and only if $\mathcal{S} \neq \emptyset$ and $\lambda_1 \boldsymbol{s}_1 + \lambda_2 \boldsymbol{s}_2 \in \mathcal{S}, \forall \lambda_1, \lambda_2 \in \Gamma$ and $\forall \boldsymbol{s}_1, \boldsymbol{s}_2 \in \mathcal{S}$.

. . . cont'd

5. . . . cont'd

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