UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE AND ENGINEERING
FINAL EXAMINATION, DECEMBER 2021

CIV102F – Structures and Materials – An Introduction to Engineering Design

Examiners --- A. Kuan and E.C. Bentz

Permissible Aids: Non-programmable calculator, printed or handwritten notes

IMPORTANT INSTRUCTIONS

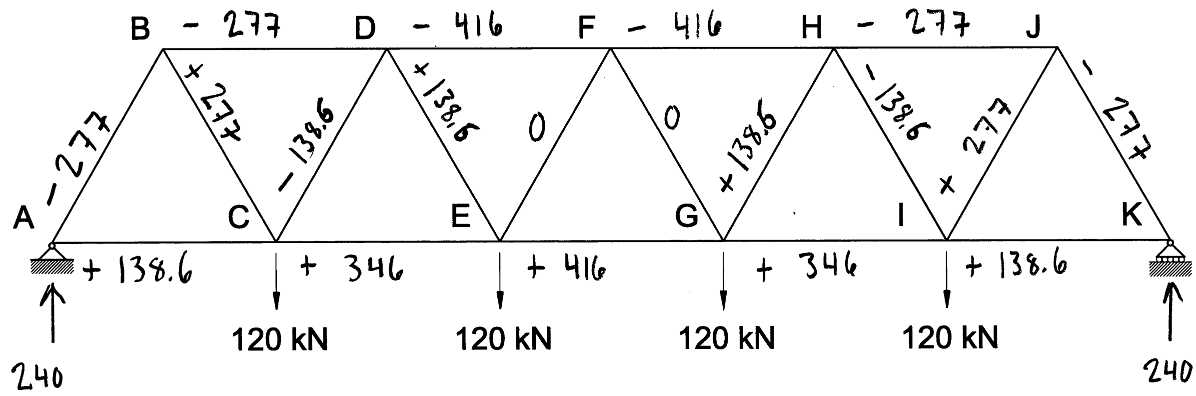
1. You have a total of 2.5 hours to complete and submit the final exam.
2. There are four questions on the exam. Attempt all questions; any questions left blank will receive a grade of zero. Part marks will be awarded for incomplete answers.
3. Report all final answers using slide-rule precision (i.e., four significant figures if the first digit is a "1", three otherwise).
4. If you need more space, you may write on the back side of the page. Indicate this on the front side of the page so none of your work is missed.
5. Write neatly and draw a box around your final answers.

Full Name: Solution

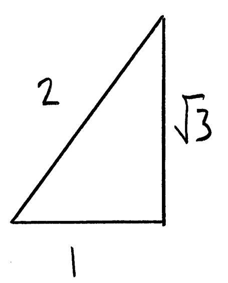
Student Number: ______________________

Question No.	Score	Possible Points
1		34
2		31
3		25
4		20
Total		110

1. The truss shown below supports a 30 m pedestrian bridge and is made from steel hollow structural sections with a yield stress of 350 MPa. When the bridge is crowded with people, it supports four 120 kN loads as shown below. **<u>Each member is 6 m long.</u>**



1(a). Calculate the axial force in each member of the truss due to the 120 kN loads. Indicate the calculated member forces on the provided drawing. Use the convention +ve for tension and -ve for compression. Use the provided space below for intermediate/sample calculations if needed. **(10 marks)**



1(b). Each of the horizontal chord members are HSS 203×203×6.4, and each of the diagonal web members are HSS 152×152×6.4. Is the truss **safe** under the 120 kN loads? Yes or no? At what value of the loads will the structure be on the boundary between safe and unsafe? **(6 marks)**

Note: When solving this problem, you do not need to check if the slenderness ratio, L/r, exceeds the allowable limit of 200.

$\sigma_y = 350$ MPa

$\ell = 6$ m

Chords

HSS 203 × 203 × 6.4 $\quad A = 4900 \text{ mm}^2$

$I = 31.3 \times 10^6 \text{ mm}^4$

$P_{yield} = \sigma_y \cdot A = 350 \times 4900 = \underline{\underline{1715 \text{ kN}}}$

$P_{buckle} = \dfrac{\pi^2 E I}{L^2} = \dfrac{\pi^2 \times 200{,}000 \times 31.3 \times 10^6}{6000^2} = \underline{\underline{1716 \text{ kN}}}$

Safe loads w/ FOS

yield $= \dfrac{1715}{2} = \underline{\underline{858 \text{ kN}}}$

buckle $= \dfrac{1716}{3} = \underline{\underline{572 \text{ kN}}}$ ← governs

Highest chord force = 416 kN < 572 kN; chords are safe.

Webs

HSS 152 × 152 × 6.4 $\quad A = 3610 \text{ mm}^2$

$I = 12.6 \times 10^6 \text{ mm}^4$

$P_{yield} = 350 \times 3610 = \underline{\underline{1264 \text{ kN}}}$

$P_{buckle} = \dfrac{\pi^2 \times 200{,}000 \times 12.6 \times 10^6}{6000^2} = \underline{\underline{691 \text{ kN}}}$

Safe loads w/ FOS

yield $= \dfrac{1264}{2} = \underline{\underline{632 \text{ kN}}}$

buckle $= \dfrac{691}{3} = \underline{\underline{230 \text{ kN}}}$ ← governs

Highest web force = 277 kN > 230 kN. Webs unsafe.

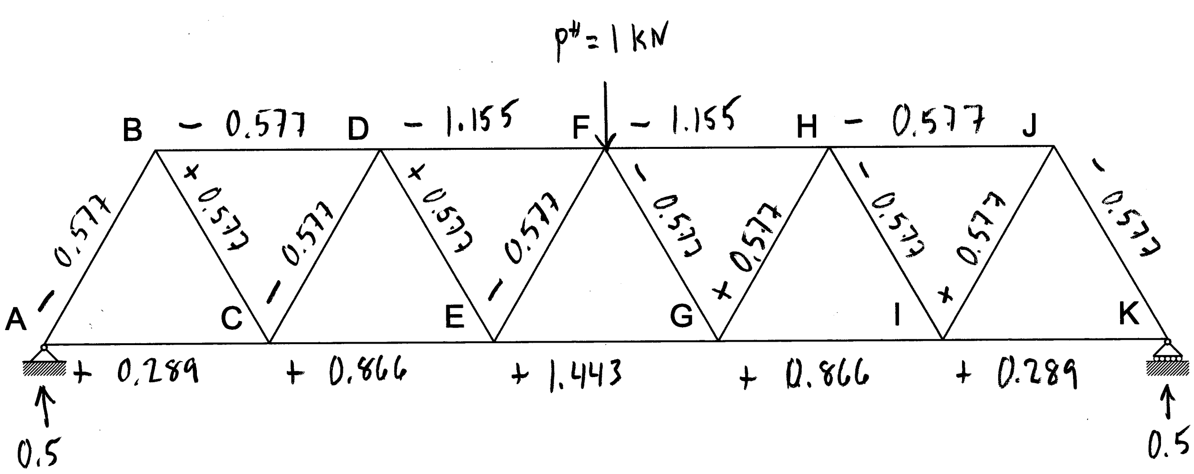
$\boxed{\text{Value of loads to be on boundary: } 120 \times \dfrac{230}{277} = 99.6 \text{ kN each}}$

1(c). Using the Method of Virtual Work, calculate the vertical deflection of **Joint F** due to the application of the 120 kN loads. Use the provided drawing of the truss to determine the virtual forces, and fill in the following table to perform your calculations. Recall that E = 200,000 MPa for steel. Note that the table lists the members for only one half of the truss. **(10 marks)**

Note: Two blank columns have been provided for intermediate calculations if needed.

Member	Member Force (kN)	Area (mm^2)	k kN/mm		Length (mm)	Δl (mm)	Virtual Force (kN)	Work (J)
AB	-277		120.3			- 2.30	-0.577	1.327
BC	+277					+ 2.30	+ 0.577	1.327
CD	-138.6	3610				- 1.152	- 0.577	0.665
DE	+138.6					+ 1.152	+ 0.577	0.665
EF	0				6000	0	- 0.577	0
BD	-277		163.3			- 1.696	- 0.577	0.979
DF	- 416	4900				- 2.55	- 1.155	2.945
AC	+138.6					+ 0.849	+ 0.289	0.245
CE	+346					+ 2.12	+ 0.866	1.836
EG	+ 416	4900			6000	+2.55	+1.443	3.680

9.989 J



$$k = \frac{AE}{L_o}$$

$$2 \times 9.989 + 3.680 = 1 \times \Delta_F$$

$$\Delta_F = 23.7 \text{ mm}$$

1(d). Consider the situation where a smaller crowd of people is standing on the bridge. When standing still, the resulting point loads on the structure will be 90 kN each instead of the 120 kN loads considered in parts (a) to (c).

Using your answer from part (c), calculate the midspan displacement under this smaller load and the resulting natural frequency of the loaded bridge. What will the maximum midspan displacement be if the crowd starts moving, producing loads of 90 ± 30 kN at a frequency of 3.5 Hz? Will any of the members yield or buckle? When performing your calculations, use a damping ratio of β = 0.02. **(8 marks)**

$$\Delta_{90} = \Delta_{120} \times \frac{90}{120} = 23.7 \times \frac{90}{120} = \underline{\underline{17.78 \text{ mm}}}$$

$$f_n = \frac{17.76}{\sqrt{17.78}} = 4.21 \text{ hz}$$

$$\frac{f}{f_n} = \frac{3.5}{4.21} = 0.831$$

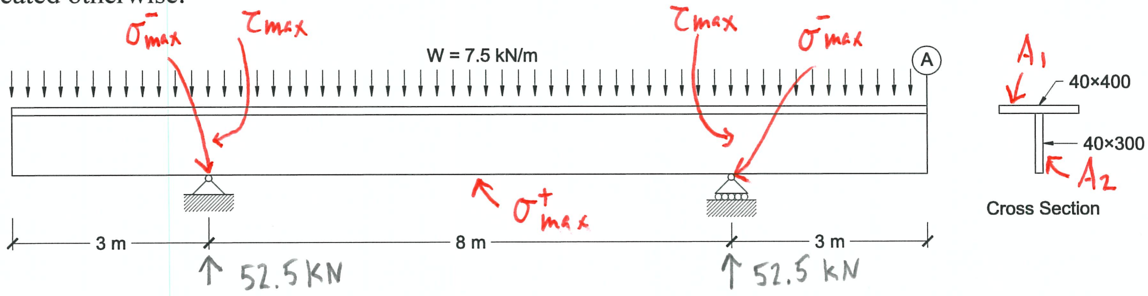
$$DAF = \frac{1}{\sqrt{(1-0.831^2)^2 + (2 \times 0.02 \times 0.831)^2}} = \underline{\underline{3.21}}$$

Effective point load: $P_{dynamic} = 90 + 3.21 \times 30 = \underline{\underline{186.3 \text{ kN}}}$

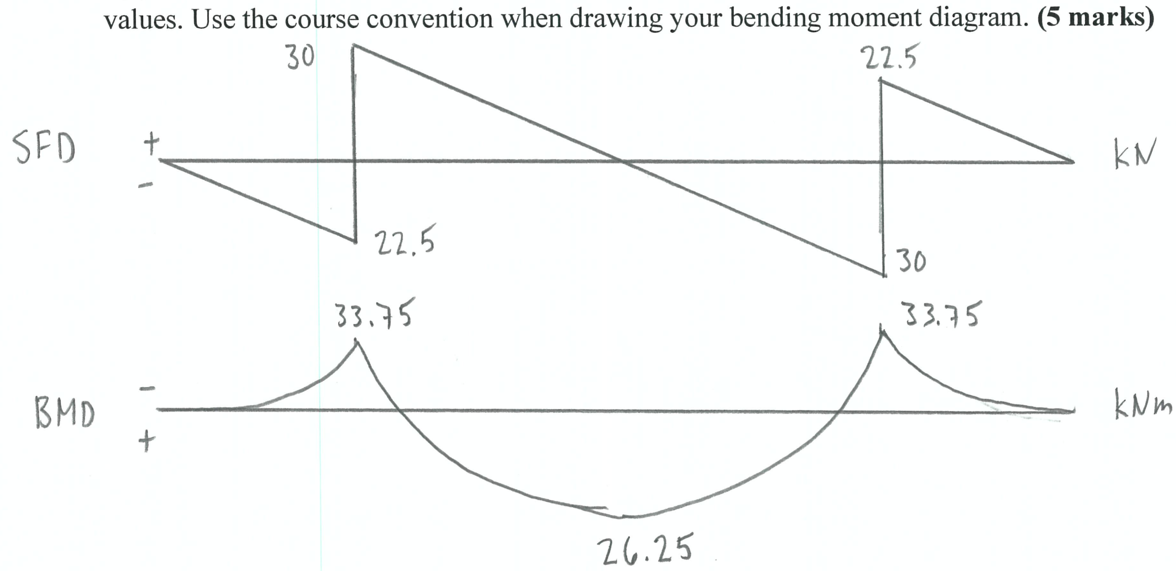
$$\Delta_{max} = \Delta_{120} \times \frac{186.3}{120} = 23.7 \times \frac{186.3}{120} = \underline{\underline{36.8 \text{ mm}}}$$

Forces in members increase by a factor of $\frac{186.3}{120} = 1.553$. No members fail because each have factors of safety which are at least 2.0.

2. The timber T-beam shown below has been fabricated by gluing together two pieces of wood. The beam is 14 m long, is supported by two supports 8 m apart, and is subjected to a uniformly distributed load, which includes self weight, of 7.5 kN/m. The wood has an ultimate tensile strength of 70 MPa, an ultimate compressive strength of 50 MPa, and a Young's modulus of 11,000 MPa. All dimensions are in mm unless indicated otherwise.



2(a). Draw the shear force and bending moment diagrams for the beam. Calculate and show important values. Use the course convention when drawing your bending moment diagram. **(5 marks)**



2(b). Determine the location of the centroidal axis and the value of I for the cross-section. **(5 marks)**
Note: there is more space at the top of page 7 for you to complete this calculation.

$A_1 = 40 \times 400 = 16,000\ mm^2$

$A_2 = 40 \times 300 = 12,000\ mm^2$

$\left.\begin{array}{l} y_1 = 320\ mm \\ y_2 = 150\ mm \end{array}\right\}$ from bottom

$I_{o1} = \frac{400 \times 40^3}{12} = 2.13 \times 10^6\ mm^4$

$I_{o2} = \frac{40 \times 300^3}{12} = 90 \times 10^6\ mm^4$

$d_1 = 320 - 247 = 73\ mm$

$d_2 = 150 - 247 = 97\ mm$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{16,000 \times 320 + 12,000 \times 150}{16,000 + 12,000}$

$\boxed{\bar{y} = 247\ mm \text{ from bottom}}$

$I = I_{o1} + A_1 d_1^2 + I_{o2} + A_2 d_2^2 = 2.13 \times 10^6 + 16,000 \times 73^2 + 90 \times 10^6 + 12,000 \times 97^2$

$\boxed{I = 290 \times 10^6\ mm^4}$

2(c). Determine the maximum flexural compressive stress, the maximum flexural tensile stress, and the maximum shear stress which occur in the beam. Indicate on the provided drawing where these maximum stresses occur. **(8 marks)**

Flexural stresses

$y_{top} = h - \bar{y} = 340 - 247 = 93\ mm$

$y_{bot} = \bar{y} = 247\ mm$

$\sigma = \frac{My}{I}$

at $M = +26.25\ kNm$:

$\sigma_{top} = \frac{26.25 \times 93}{290 \times 10^6} = -8.42\ MPa$

$\sigma_{bot} = \frac{26.25 \times 247}{290 \times 10^6} = +22.4\ MPa$

at $M = -33.75\ kNm$

$\sigma_{top} = \frac{33.75 \times 93}{290 \times 10^6} = +10.82\ kN$

$\sigma_{bot} = \frac{33.75 \times 247}{290 \times 10^6} = -28.7\ MPa$

$\therefore$ Max tensile stress is $+22.4\ MPa$, max comp. stress $= -28.7\ MPa$. See figure.

Shear stress

$V = 30\ kN$

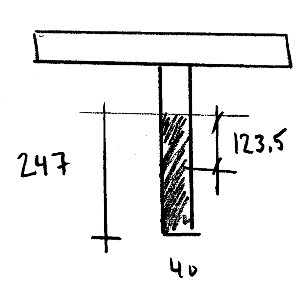
$b = 40\ mm$

$I = 290 \times 10^6\ mm^4$

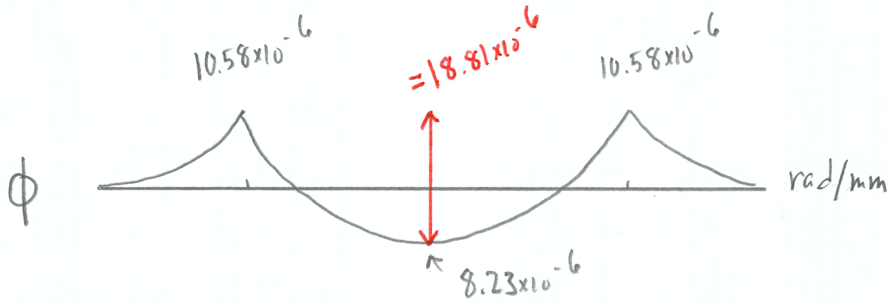
$$\tau = \frac{VQ}{Ib} = \frac{30 \times 1.220 \times 10^6}{290 \times 10^6 \times 40} = 3.16\ MPa$$

$Q = 247 \times 40 \times 123.5$

$Q = 1.220 \times 10^6\ mm^3$

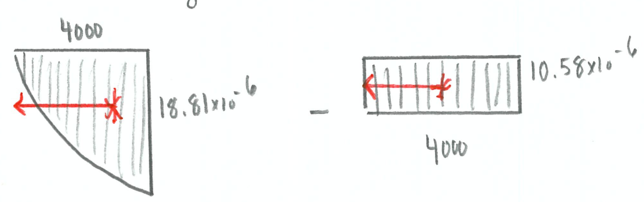


2(d). Draw the curvature diagram and calculate the vertical displacement of the beam at the midspan. Also determine the slope of the beam at point A, located at the far right side of the structure. **(7 marks)**
Note: report only the magnitudes of the slope and displacement.

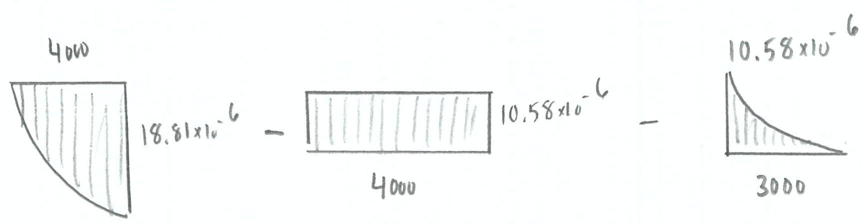


$\Delta_{mid} = \delta_{support-midspan}$
$\theta_{mid} = 0$
symmetry

$$\Delta_{mid} = \frac{2}{3}\times 4000 \times 18.81\times10^{-6} \times \frac{5}{8}\times 4000 - 4000\times 10.58\times10^{-6} \times \frac{1}{2}\times 4000 \Rightarrow \boxed{\Delta_{mid} = 40.8\text{ mm}}$$



$$\theta_A = \frac{2}{3}\times 4000 \times 18.81\times10^{-6} - 4000\times 10.58\times10^{-6} - \frac{1}{3}\times 3000 \times 10.58\times10^{-6} \Rightarrow \boxed{\theta_A = 2.74\times10^{-3}\text{ rad} \; (0.1570^\circ)}$$



2(e). Calculate the factor of safety against a tension failure and the factor of safety against a compression failure (do not check the FOS against a shear failure). Using these two values, determine the overall factor of safety. Will the overall factor of safety increase or decrease if the beam is flipped upside down? Explain. **(6 marks)**

$$FOS_{tension} = \frac{70}{22.4} = 3.13$$

$$FOS_{comp.} = \frac{50}{28.7} = 1.742$$

$\boxed{\text{Overall FOS} = 1.742}$

If beam is flipped, $y_{top} = 247$ mm & $y_{bot} = 93$ mm. The new stresses are:

$M = +26.25$: $\sigma_{top} = -22.4$ MPa, $\sigma_{bot} = +8.42$ MPa

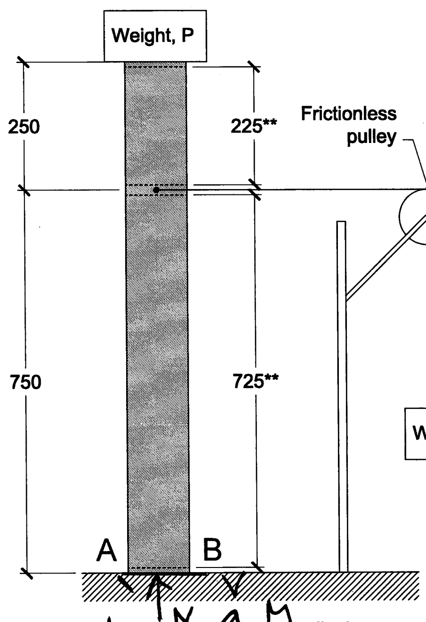
$M = -33.75$: $\sigma_{top} = +28.7$ MPa, $\sigma_{bot} = -10.82$ MPa.

$$FOS_{tens} = \frac{70}{28.7} = 2.44$$

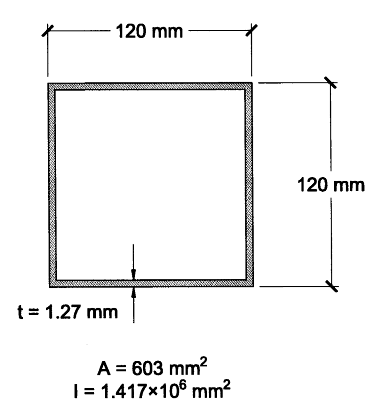
$$FOS_{comp} = \frac{50}{22.4} = 2.23$$

$\boxed{\text{new FOS} = 2.23\text{, got stronger}}$

3. Although the design-build project in CIV102 has traditionally involved building a bridge, another possibility is to build a tower. Consider the 1 m tall tower structure shown below. The tower is loaded using two weights which each have a value of P. One weight is applied to the top of the tower, subjecting it to an axial compression. The second weight applies a horizontal force to the tower 750 mm above the base using a pulley system. The tower has four diaphragms whose spacings are shown in the figure, and the base of the tower is rigidly connected to a table below. All dimensions are in mm unless indicated otherwise.



Elevation View



Cross Section View

The matboard has the following properties:

Tensile Strength	$\sigma^{+}_{ult} = 30$ MPa	Young's Modulus	E = 4000 MPa
Compressive Strength	$\sigma^{-}_{ult} = 6$ MPa	Poisson's Ratio	$\mu = 0.2$
Shear Strength	$\tau_{ult} = 4$ MPa	Thickness	t = 1.27 mm

3(a). Calculate the reaction forces if P = 100 N. At the base of the structure, calculate the tensile stress at point A and the compressive stress at point B caused by the combined loading. Determine where the axial stress equals zero at the base, and state its location relative to the left side of the tower (i.e., from point A). **(7 marks)**

Reaction Forces

$$\sigma = \frac{P}{A} + \frac{My}{I}$$

$\Sigma F_x = 0 \quad V = 100$ N (left)

$\Sigma F_y = 0 \quad N = -100$ N (−, so in compression)

$\Sigma M = 0 \quad M = 100 \times 750 = 75{,}000$ Nmm

Stresses

$y = 60$ mm

at A: $\sigma_A = \frac{-100}{603} + \frac{75{,}000 \times 60}{1.417 \times 10^6} = \underline{\underline{+3.01 \text{ MPa}}}$

at B: $\sigma_B = \frac{-100}{603} - \frac{75{,}000 \times 60}{1.417 \times 10^6} = \underline{\underline{-3.34 \text{ MPa}}}$

$\sigma = 0$ location

$0 = \frac{-100}{603} + \frac{75{,}000 \times y}{1.417 \times 10^6} \Rightarrow y = 3.13$ mm left of middle

$\rightarrow$ $\boxed{\sigma = 0 \quad 56.9 \text{ mm to the right of point A}}$

3(b). Calculate the values of P causing a compression failure (P_1) and a tension failure (P_2). **(4 marks)**

Crushing

at $P = 100$, $\sigma_B = -3.34 MPa$

$\sigma_B = -6$ MPa if $\boxed{P_1 = \frac{100 \times 6}{3.34} = 179.6 N}$

tension

at $P = 100 N$, $\sigma_A = +3.01$ MPa

$\sigma_A = +30$ MPa if $\boxed{P_2 = \frac{100 \times 30}{3.01} = 997 N}$

3(c). Calculate the values of P causing plate buckling failures due to flexural compression. Consider both possible plate buckling cases (P_3 and P_4). **(4 marks)**
Note: Use the clear distance between edges to find "b" when using the plate buckling equations.

Case 1 buckling:

$t = 1.27$ mm

$b = 120 - 2t = 117.5$ mm

$$\sigma_{crit} = \frac{4 \times \pi^2 \times 4000}{12(1-0.2^2)} \left(\frac{1.27}{117.5}\right)^2 = 1.601 \text{ MPa}$$

$\sigma_B = -1.601$ MPa if $\boxed{P_3 = \frac{100 \times 1.601}{3.34} = 47.9 N}$

Case 3 buckling:

$t = 1.27$ mm

$b = (120 - 56.9) - 1.27 = 61.8$mm

$$\sigma_{crit} = \frac{6 \times \pi^2 \times 4000}{12(1-0.2^2)} \left(\frac{1.27}{61.8}\right)^2 = 8.68 \text{ MPa}$$

to find σ, $y = 60 - 1.27 = 58.73$ mm

$$-8.68 = \frac{-P_4}{603} - \frac{750 P_4 \times 58.73}{1.417 \times 10^6}$$

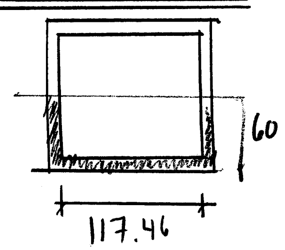
$\boxed{P_4 = 265 N}$

3(d). Calculate Q and the values of P causing a matboard shear failure (P_5) and a shear buckling failure (P_6). **(5 marks)**

Calculating Q

$Q = 2 \times (1.27 \times 60 \times 60/2) + 1.27 \times 117.46 \times (60 - \frac{1}{2} \cdot 1.27)$

$Q = 13{,}428 \text{ mm}^3$



Shear failure

$4 = \dfrac{P_5 \times 13428}{1.417 \times 10^6 \times 2.54} \Rightarrow \boxed{P_5 = 1072 \text{ N}}$

Shear buckling

$a = 725 \text{ mm}$

$h = 117.46 \text{ mm}$

$\tau_{crit} = \dfrac{5 \times \pi^2 \times 4000}{12(1 - 0.2^2)} \left[\left(\dfrac{1.27}{117.46}\right)^2 + \left(\dfrac{1.27}{725}\right)^2 \right] = \underline{\underline{2.06 \text{ MPa}}}$

$2.06 = \dfrac{P_6 \times 13428}{1.417 \times 10^6 \times 2.54} \Rightarrow \boxed{P_6 = 552 \text{ N}}$

3(e). What value of P will cause the tower to fail? What will be the cause of failure? **(2 marks)**

Fails at $P_3 = 47.9$ N. Case 1 plate buckling. $(k = 4)$

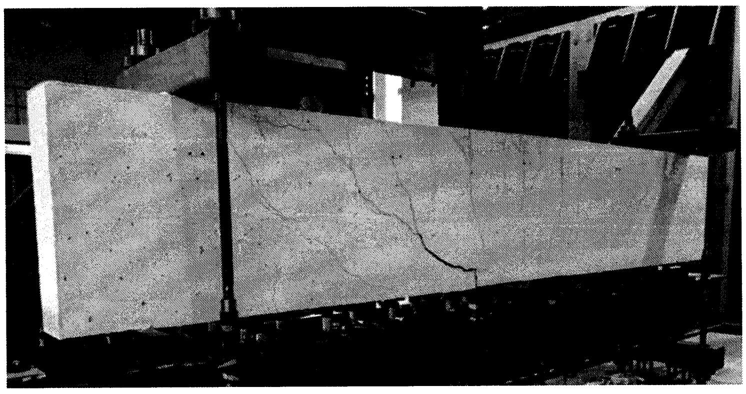
3(f). The shear buckling equation presented in class was derived for a beam that is carrying bending moments and shears forces only. Will this equation underestimate or overestimate the critical shear stress when applied to this tower which also carries an axial compression? Explain. **(3 marks)**

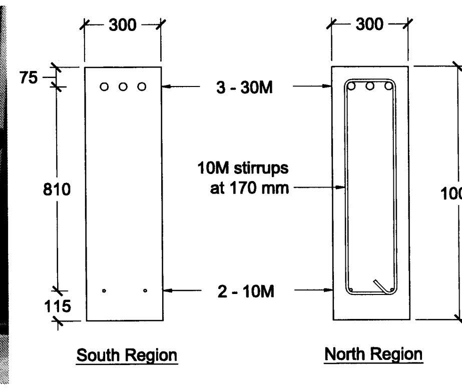
The shear buckling equation will overestimate τ_{crit} in this case.

The extra compression from the axial load will cause buckling to take place at a lower shear stress.

4. The reinforced concrete beam shown in the photo below was tested by Ms. Nicole Butkovic on Nov. 26, 2021, as part of her MASc research project. The specimen was uniformly loaded from below using a series of hydraulic jacks – a schematic explaining the loading apparatus can be found below. Because the loads were applied from below, the self-weight of the beam should be neglected when performing your calculations. The south region of the beam did not contain any shear reinforcement, while the north region of the beam contained two-leg 10M stirrups spaced apart at 170 mm.

All dimensions are in mm unless indicated otherwise.





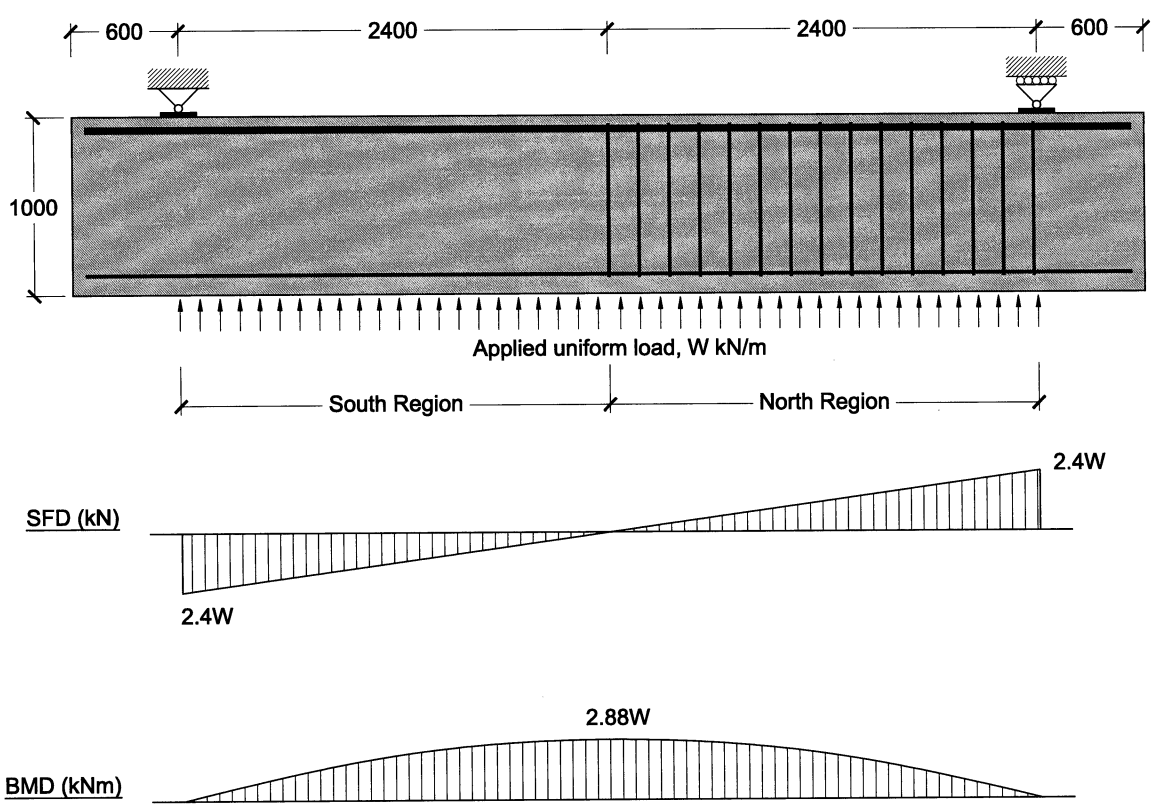


Photo of tested beam (top left), cross section details (top right), elevation view (middle), SFD and BMD showing key values (bottom).

The compressive strength of the concrete at the time of testing was f_c' = 41 MPa, and the yield strengths of the 10M and 30M steel bars were f_y = 400 MPa. The Young's modulus of the concrete can be taken as E_c = 28,000 MPa.

4(a). Calculate the highest value of the bending moment and corresponding value of W which can be **safely** resisted by the beam so that the tensile stress in the longitudinal reinforcement does not exceed $0.6 \times f_y = 240$ MPa and the compressive stress in the concrete does not exceed $0.5 \times f_c' = 20.5$ MPa. Neglect the longitudinal steel in compression when performing your calculations. **(8 marks)**

$A_s = 3 \times 700 = 2100 \text{ mm}^2$

$b = 300 \text{ mm}$

$d = 115 + 810 = 925 \text{ mm}$

$n = \dfrac{200{,}000}{28{,}000} = 7.14$

$\rho = \dfrac{2100}{300 \times 925} = 0.00757$

$n\rho = 0.0541$

$k = \sqrt{(0.0541)^2 + 2 \times 0.0541} - 0.0541$

$k = 0.279$

$j = 1 - \frac{1}{3} \times 0.279 = 0.907$

$jd = \underline{\underline{839 \text{ mm}}}$

Steel: $240 = \dfrac{M}{2100 \times 839} \Rightarrow M = \underline{\underline{423 \text{ kNm}}}$

Concrete: $20.5 = \dfrac{0.279}{1 - 0.279} \times \dfrac{M}{2100 \times 839 \times 7.14} \Rightarrow \underline{\underline{666 \text{ kNm}}}$

Steel is critical; $M_{max} = 423 \text{ kNm}$

$W = \dfrac{423}{2.88} = 146.9 \text{ kN/m}$

4(b). Calculate the value of W which would cause the longitudinal reinforcement to yield and hence cause a flexural failure. **(2 marks)**

$M_{yield} = 2100 \times 400 \times 839 = 705 \text{ kNm}$

$W = \dfrac{705}{2.88} = 245 \text{ kN/m}$

4(c). Calculate the highest shear force and corresponding value of W which will cause a shear failure in the **south** region of the beam which does not contain shear reinforcement. **(3 marks)**

No shear reinforcement; $V_s = 0$

$$V_{ult} = \frac{230\sqrt{40.5}}{1000 + 0.9 \times 925} \times 300 \times 839 = \underline{\underline{202 \text{ kN}}}$$

$$\boxed{W = \frac{202}{2.4} = 84.3 \text{ kN/m}}$$

4(d). Calculate the highest shear force and corresponding value of W which will cause a shear failure in the **north** region of the beam which contains shear reinforcement. **(4 marks)**

check min reinf: $A_v = 2 \times 100 = 200 \text{ mm}^2$

$$\frac{A_v f_y}{b_w s} = \frac{200 \times 400}{300 \times 170} = 1.569 \text{ MPa} > 0.06\sqrt{41} = 0.384 \text{ MPa}$$ Satisfies check ✓

check V_{max}: $V_{max} = 0.25 \times 41 \times 300 \times 839 = \underline{\underline{2580 \text{ kN}}}$

$$V_{ult} = V_c + V_s = 0.18\sqrt{41} \times 300 \times 839 + \frac{200 \times 400 \times 839}{170} \cot 35° = \underline{\underline{854 \text{ kN}}},$$ V_{max} does not govern

$$\boxed{W = \frac{854}{2.4} = 356 \text{ kN/m}}$$

4(e). Using your results from questions 4(b) to 4(d), predict the value of W which causes failure of the beam. Does the beam fail in flexure or shear? State approximately where the failure takes place. **(3 marks)**

Fails in shear near south support at $W = 84.3$ kN/m.