Q3:
Consider matrix A:

$$A = \begin{bmatrix} +4 & -1 & -1 & -1 \\ -1 & +4 & -1 & -1 \\ -1 & -1 & +4 & -1 \\ -1 & -1 & -1 & +4 \end{bmatrix}$$

The inverse of matrix A is known to be of the form:

$$A^{-1} = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}$$

a) Write two algebraic equations in terms of the unknowns a and b that must hold true for $AA^{-1} = I$.

b) Solve for the unknowns a and b in part (a).

c) By determining its rank, show that matrix B is **NOT** invertible, where:

$$B = \begin{bmatrix} +3 & -1 & -1 & -1 \\ -1 & +3 & -1 & -1 \\ -1 & -1 & +3 & -1 \\ -1 & -1 & -1 & +3 \end{bmatrix}$$

Q3: a) GIVEN A AND A^{-1}, THEN WE KNOW,

$$AA^{-1} = I$$

TAKE ENTRY (1,1) OF AA^{-1},

$$4a - b - b - b = 1$$
$$\text{OR} \quad 4a - 3b = 1 \quad (1)$$

TAKE ENTRY (1,2) OF AA^{-1},

$$4b - a - b - b = 0$$
$$\text{OR} \quad 2b - a = 0 \quad (2)$$

b) SOLVING FOR a AND b,

FROM (2), $a = 2b$

FROM (1), $4(2b) - 3b = 1$

$$8b - 3b = 1$$
$$5b = 1$$
$$b = \frac{1}{5}$$

$\therefore \quad a = \frac{2}{5}, \quad b = \frac{1}{5}$

c) $\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} R1 * \frac{1}{3}$

$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{8}{3} & -\frac{4}{3} & -\frac{4}{3} \\ 0 & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} \\ 0 & -\frac{4}{3} & -\frac{4}{3} & \frac{8}{3} \end{bmatrix} \begin{matrix} \\ R2+R1 \\ R3+R1 \\ R4+R1 \end{matrix}$

$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -2 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix} \begin{matrix} \\ R2 * \frac{3}{8} \\ R3 * (-\frac{3}{7}) \\ R4 * (\frac{-3}{7}) \end{matrix}$

$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -\frac{3}{2} & \frac{3}{2} \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} \end{bmatrix} \begin{matrix} \\ \\ R3-R2 \\ R4-R2 \end{matrix}$

$$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{matrix} \\ \\ R3*(-\frac{2}{3}) \\ R4*(\frac{2}{3}) \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} R4-R3$$

RANK = 3, $\therefore$ NOT FULL RANK $(3<4)$
$\therefore$ NOT INVERTIBLE.