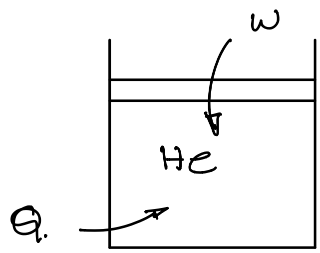
CHE 260 MID-TERM SOLUTION

2018

1)



$P_1 = 150\ kPa,\quad T_1 = 20°C$

$V_1 = 0.5\ m^3$

$P_2 = 400\ kPa \quad T_2 = 140°C$

$$m = \frac{P_1 V_1}{R T_1} = \frac{150\ kPa \times 0.5\ m^3}{2.0769\ \frac{kJ}{kg K} \times 293 K} = 0.1232\ kg$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow V_2 = \frac{T_2 P_1}{T_1 P_2} V_1$$

$$\Rightarrow V_2 = \frac{413 K}{293 K} \times \frac{150\ kPa}{400\ kPa} \times 0.5\ m^3$$

$$V_2 = 0.2643\ m^3$$

$$P_1 V_1^n = P_2 V_2^n$$

$$\Rightarrow \quad n \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{P_2}{P_1}\right)$$

$$\Rightarrow \quad n = \frac{\ln(400\ kPa / 150\ kPa)}{\ln(0.5\ m^3 / 0.2643\ m^3)} = 1.538$$

$$W_{12} = -\int_{V_1}^{V_2} P dV = -\frac{P_2 V_2 - P_1 V_1}{1-n} = \frac{mR(T_2 - T_1)}{n-1}$$

$$W_{12} = \frac{0.1232\,\text{kg} \times 2.0769\,\frac{\text{kJ}}{\text{kg K}}\,(413-293)\,\text{K}}{1.538 - 1}$$

$$W_{12} = 57.1\ \text{kJ}$$

Energy balance.

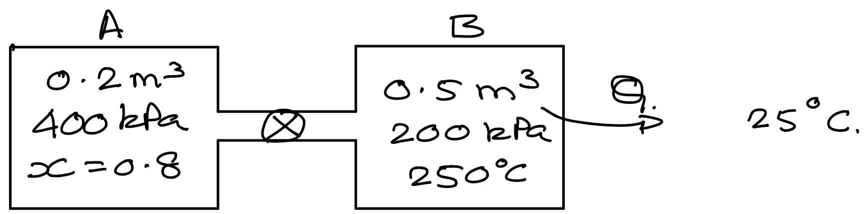
$$Q_{12} + W_{12} = U_2 - U_1$$

$$Q_{12} = m C_v (T_2 - T_1) - W_{12}$$

$$\Rightarrow Q_{12} = 0.1232\,\text{kg} \times 3.1156\,\frac{\text{kJ}}{\text{kg K}}\,(413-293)\,\text{K} - 57.2\,\text{kJ}$$

$$Q_{12} = -11.1\ \text{kJ}$$

2)



Tank A

$$\left.\begin{aligned} P_1 &= 400 \text{ kPa} \\ x_1 &= 0.8 \end{aligned}\right\} \begin{aligned} v_f &= 0.001084,\ v_g = 0.46242 \ \frac{m^3}{kg} \\ u_f &= 604.22,\ u_g = 2553.1 \ \frac{kJ}{kg} \end{aligned}$$

$$\begin{aligned} v_{1,A} &= v_f + x_1 (v_g - v_f) \\ &= 0.001084 + 0.8(0.46242 - 0.001084) \\ &= 0.37015 \ m^3/kg \end{aligned}$$

$$\begin{aligned} u_{1,A} &= u_f + x_1 (u_g - u_f) \\ &= 604.22 + 0.8(2553.1 - 604.22) \\ &= 2163.3 \ kJ/kg \end{aligned}$$

Tank B

$$\left.\begin{aligned} P_1 &= 200 \text{ kPa} \\ T_1 &= 250^\circ C \end{aligned}\right\} \begin{aligned} v_{1,B} &= 1.19890 \ m^3/kg \\ u_{1,B} &= 2731.4 \ kJ/kg \end{aligned}$$

$$m_A = \frac{V_A}{v_{1,A}} = \frac{0.2 \ m^3}{0.37015 \ m^3/kg} = 0.5403 \ kg$$

$$m_B = \frac{V_B}{v_{1,B}} = \frac{0.5 \ m^3}{1.19890 \ m^3/kg} = 0.4170 \ kg$$

$$m_2 = m_A + m_B = 0.5403 + 0.4170$$

$$m_2 = 0.9573 \ kg$$

$$V_2 = V_A + V_B = 0.2\,m^3 + 0.5\,m^3 = 0.7\,m^3$$

$$v_2 = \frac{0.7\ m^3}{0.9573\ kg} = 0.7312\ m^3/kg$$

At $T_2 = 25^\circ C$

$v_f = 0.001003\ m^3/kg \qquad u_f = 104.83\ kJ/kg$

$v_g = 43.340\ m^3/kg \qquad u_g = 2409.1\ kJ/kg$

$v_f < v_2 < v_g \Rightarrow$ saturated vapour.

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.7312 - 0.001003}{43.340 - 0.001003} = 0.01685$$

$$u_2 = u_f + x_2 (u_g - u_f)$$
$$= 104.83 + 0.01685 \times (2409.1 - 104.83)$$
$$= 143.66\ kJ/kg$$

Energy Balance

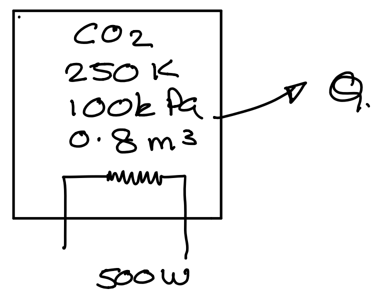
$$\cancel{W_{12}}^{\,0} + Q_{12} = U_2 - U_1$$

$$Q_{12} = m_2 u_2 - (m_A u_{A,1} + m_B u_{B,1})$$

$$Q_{12} = 0.9573 \times 143.66 - (0.5403 \times 2163.3 + 0.4170 \times 2731.4)$$

$$Q_{12} = -2170.3\ kJ$$

3)



$T_b = 300$ K.

$$m = \frac{P_1 V}{R T_1} = \frac{100\,\text{kPa} \times 0.8\,\text{m}^3}{0.1889\,\frac{\text{kJ}}{\text{kg K}} \times 250\,\text{K}} = 1.694\,\text{kg}$$

a) $$T_2 = \frac{P_2 V}{m R} = \frac{175\,\text{kPa} \times 0.8\,\text{m}^3}{1.694\,\text{kg} \times 0.1889\,\frac{\text{kJ}}{\text{kg K}}} = 437.5\,\text{K}.$$

b) Energy Balance

$$W_{elec} + Q = \Delta U = m c_v (T_2 - T_1)$$

$$Q = m c_v (T_2 - T_1) - \dot{W}_{elec} \Delta t$$

$$= 1.694\,\text{kg} \times 0.706\,\frac{\text{kJ}}{\text{kg K}} (437.5 - 250)\,\text{K} - 0.5\,\text{kW} \times 40\,\text{min} \times 60\,\frac{\text{s}}{\text{min}}$$

$$Q = -975.8\,\text{kJ}$$

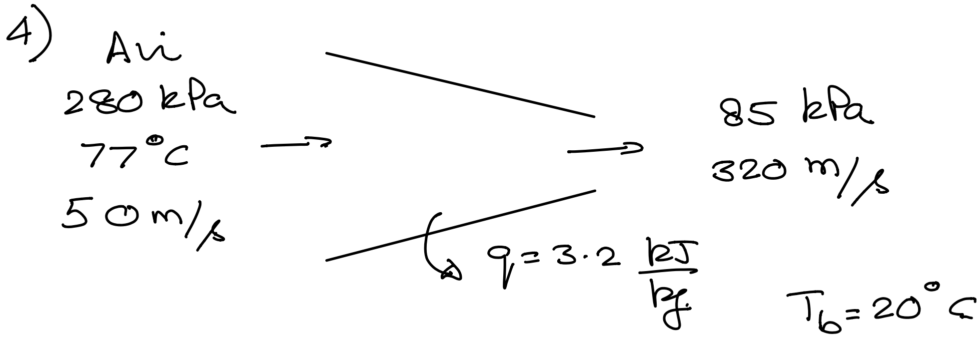
c) $$S_{gen} = \Delta S_{CO_2} + \Delta S_{surr}$$

$$S_{gen} = m\left(c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}\right) + \frac{Q}{T_b}$$

$$= 1.694\left(0.895 \ln \frac{437.5}{250} - 0.1889 \ln \frac{175}{100}\right) + \frac{975.8}{300}$$

$$S_{gen} = 3.92\,\text{kJ/K}.$$

4)



$$\cancel{w}^{\,0} + q = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

$$q = c_p (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$-3.2 \frac{kJ}{kg} = 1.005 \frac{kJ}{kg\,K} (T_2 - 77°C) + \frac{320^2 - 50^2}{2 \times 1000}$$

$$T_2 = 24.1°C = 297.1\ K$$

$$\Delta s_{gen} = \Delta s_{air} + \Delta s_{surr.}$$

$$\Delta s_{air} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

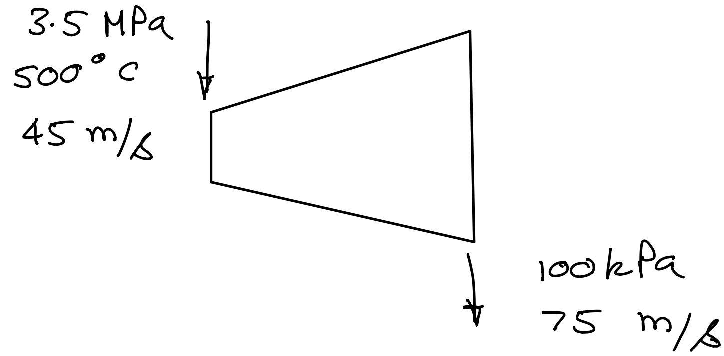
$$= 1.005 \ln \frac{297.1\ K}{350\ K} - 0.287 \ln \frac{85\ kPa}{280\ kPa}$$

$$= 0.1775\ kJ/kg\,K.$$

$$\Delta s_{surr} = \frac{q}{T_b} = \frac{3.2\ kJ/kg}{293\ K} = 0.0109 \frac{kJ}{kg\,K}$$

$$\Delta s_{gen} = 0.1775 + 0.0109 = 0.1884\ kJ/kg\,K$$

5)



$P_1 = 3.5$ MPa, $T_1 = 500°C$ $\Big\}$ $h_1 = 3451.7$ kJ/kg, $s_1 = 7.1593$ kJ/kgK

$P_2 = 100$ kPa $\quad s_f = 1.3028$ kJ/kgK

$s_g = 7.3589$ kJ/kgK

$h_f = 417.51$ kJ/kg

$h_{fg} = 2257.5$ kJ/kg.

For isentropic flow $s_2 = s_1 = 7.1593 \frac{kJ}{kg\,K}$

$$x_{2s} = \frac{s_2 - s_f}{s_g - s_f} = \frac{7.1593 - 1.3028}{7.3589 - 1.3028} = 0.9670$$

$$h_{2s} = h_f + x_2 h_{fg}$$

$$h_{2s} = 417.51 + 0.9670 \times 2257.5 = 2600.5 \frac{kJ}{kg}$$

Ideal work output

$$\dot{W}_s = \frac{\dot{W}_a}{\eta_t} = \frac{5000\,kW}{0.77} = 6493.5\,kW$$

Energy balance for ideal process.

$$\cancel{\dot{Q}}^{\,0} + \dot{W}_s = \dot{m}\left[(h_{2s} - h_1) + \frac{V_2^2 - V_1^2}{2}\right]$$

$$-6493.5 = \dot{m}\left[(2600.5 - 3451.7) + \frac{75^2 - 45^2}{2\times1000}\right]$$

$$\dot{m} = 7.645\,kg/s$$