University of Toronto
Faculty of Applied Science and Engineering

Quiz 3 – November 9th, 2017
9:15 am – 10:15 am

SECOND YEAR – ENGINEERING SCIENCE

AER210F VECTOR CALCULUS and FLUID MECHANICS
Examiner: Philip McCarthy

Instructions:
(1) Closed book examination; Non-programmable calculator allowed, no other aids are permitted.
(2) Write your name, student number and tutorial group in the space provided below.
(3) Answer as many questions as you can. Parts of questions may be answered.
(4) Use the overleaf side of pages or the extra page at the end for additional work. Indicate clearly if you have continued a question to a second page
(5) Do not separate or remove any pages from this exam booklet.

Family Name: PHILIP MCCARTHY

Given Name: ______

Student Number: ______

TA Name/ Tutorial Session #: ______

FOR MARKER USE ONLY		
Question	Mark	Earned
1	13	
2	7	
3	~~10~~ 14	
4	10	
5	~~14~~ 10	
TOTAL	54	

Unless otherwise stated in the question, use:

Gravitational Acceleration – $g = 9.81\ m/s^2$
Density of Water – $\rho = 1000\ kg/m^3$

The exact mark breakdown for each question is only a guidance for the markers. Where questions have been answered in a different manner, the assignment of marks will be decided on a case-by-case basis.

1. (a) Indicate whether each statement is true or false: **[6 Marks]** 1

F Increasing the temperature of a gas will reduce the dynamic viscosity.

T A fluid with a low value of bulk modulus of elasticity will experience a large change in volume for a small change in pressure. 1

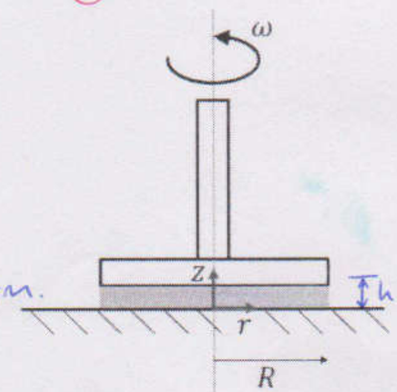
F The assumption that fluid is a continuum is valid for large Knudsen numbers, $Kn > 3$. 1

F Streamlines can cross one another if the fluid has sufficiently high velocity. 1

T A large Reynolds number indicates that the viscous forces are substantially smaller than the inertial forces. 1

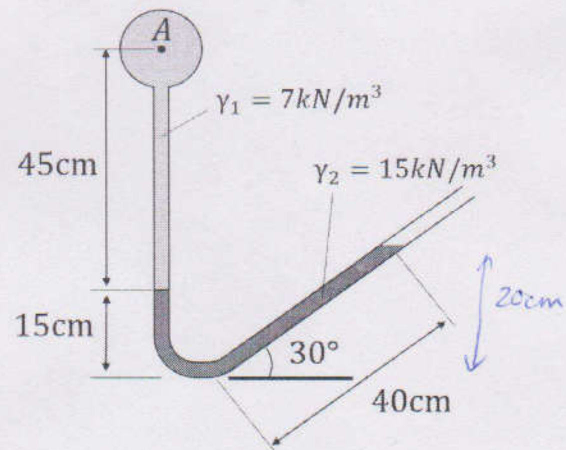
F The buoyancy force acts through the centre of mass of the solid it is acting upon. 1

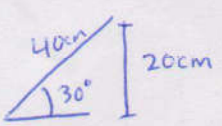
1. (b) A rotating disc viscosity meter can be used determine the viscosity of a fluid placed between the two plates, if the torque required to rotate the top plate is measured. Can this device be used for non-Newtonian fluids? Explain your answer. **[3 Marks]**



1 1
No, because the applied shear stress is not uniform. it varies from zero at the centre of the disc to $\mu \frac{\omega r}{h}$ at the edge. As non-Newtonian fluids do not have a constant viscosity for different shear stresses, 1 this device is not suitable.

1. (c) Find the gauge pressure at point A in the pipe. **[4 Marks]**





1

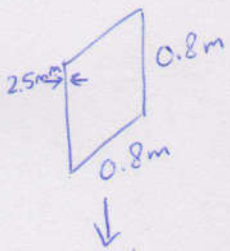
$\gamma_2(0.15) + \gamma_1(0.45) + P_A = \gamma_2(0.2)$ 1 1

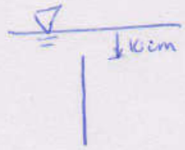
$P_A = \gamma_2(0.2 - 0.15) \mp \gamma_1(0.45)$

$P_A = 15000 \times 0.05 \pm 7000 \times 0.45$

$P_A = -2400\ Pa$ 1

2. (a) A metal plate 0.8 m × 0.8 m × 2.5 mm weighing 20 N is placed vertically in a large tank of oil, 10 cm below the surface. The oil has a specific gravity of 0.95 and dynamic viscosity of 1.5 N.s./m^2. Would you expect the plate to float, sink or remain at the same location? Explain your reasoning.
[3 Marks]





OIL SG = 0.95

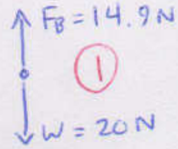
$\rho_{oil} = 950\ kg/m^3$

$w = 20N$

$V_{plate} = 0.8 \times 0.8 \times 0.0025$
$= 0.0016 m^3$

$F_B = \rho_{oil}\, g\, V_{plate} = 950 \times 9.81 \times 0.0016 = 14.9\,N$ (1)

$W = 20\,N$

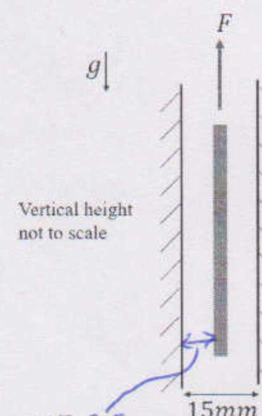


(1)

$W > F_B \Rightarrow$ It will sink (1)

OR $\rho_{plate} = \frac{W}{Vg} = 1274\ kg/m^3 \rightarrow \rho_{plate} > \rho_{oil} \Rightarrow$ It will sink

2. (b) The vertical walls of the tank are now moved closer together so that they are spaced 15 mm apart, with the plate located equidistant between the walls. The plate will be lifted out of the tank at a constant speed of 0.1 m/s. What force, *F*, is required to lift this plate? **[4 Marks]**



$y = \frac{15 - 2.5}{2} = 6.25mm$

The fluid in the tank is the same oil used in Question 2. (a), with specific gravity of 0.95 and dynamic viscosity of 1.5 N.s./m^2. Neglect end effects of the plate.

$F + F_B = W + F_\tau$ (1)

$F = W + F_\tau - F_B$

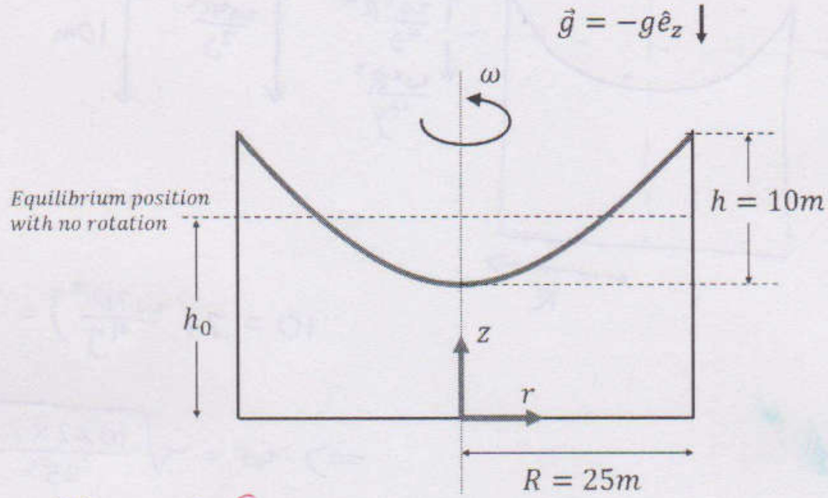
(1)

$\tau = \mu \frac{\Delta u}{\Delta y} = 1.5 \frac{0.1}{0.00625} = 24\ N/m^2 \rightarrow$ on each side of the plate.

$F_\tau = 2\tau A = 2 \times 24 \times 0.8^2 = 30.72\ N$ (1)

$F = 20 + 30.72 - 14.9 = \underline{35.82\,N}$ (1)

3. A Mercury miner and budding astronomer wants to build a liquid Mercury telescope in an old asteroid crater next to his house. The crater has a radius of $R = 25$ m and the maximum desired depth of the reflecting surface is $h = 10$ m. Find the rotation rate of the Mercury container in order to produce the desired reflecting surface. **[14 Marks]**



The pressure gradient in cylindrical coordinates is:

$$\vec{\nabla}p = \frac{\partial p}{\partial r}\hat{e}_r + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{e}_\theta + \frac{\partial p}{\partial z}\hat{e}_z$$

The general equation of motion for a fluid in which there are no shear stresses is:

$$-\vec{\nabla}p + \rho\vec{g} = \rho\vec{a}$$

$$\Rightarrow -\vec{\nabla}p - \rho g \hat{e}_z = \rho \vec{a}$$

$a_r = -\omega^2 r$ (1) $\quad -\frac{\partial P}{\partial r} = -\rho\omega^2 r \rightarrow \frac{\partial P}{\partial r} = \rho\omega^2 r$ (1)

$a_\theta = 0$ (1) $\quad -\frac{1}{r}\frac{\partial P}{\partial \theta} = 0$

$a_z = 0 \quad -\frac{\partial P}{\partial z} - \rho g = 0 \rightarrow \frac{\partial P}{\partial z} = -\rho g$ (1)

$P = p(r,z) \Rightarrow dp = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz = \rho\omega^2 r\,dr - \rho g\,dz$ (1)

Integrate $dp \Rightarrow P = \frac{\rho\omega^2 r^2}{2} - \rho g z + C_1$ (1) $\qquad$ @ $z = z_s$, $P = P_a$

$$z_s = \frac{\cancel{\rho}\omega^2 r^2}{2\cancel{\rho}g} + \frac{C_1 - P_a}{\rho g} = \frac{\omega^2 r^2}{2g} + \underbrace{\frac{C_1 - P_a}{\rho g}}_{C_2}$$

$$z_s = \frac{\omega^2 r^2}{2g} + C_2 \quad (1)$$

Equate volumes before and after:

$V_1 = \pi R^2 h_0 \qquad dV_2 = 2\pi r z_s\,dr$ (1)

$$V_2 = \int_0^R 2\pi r z_s\,dr = \int_0^R 2\pi r\left(\frac{\omega^2 r^2}{2g} + C_2\right)dr = \pi R^2\left(\frac{\omega^2}{4g}R^2 + C_2\right)$$

$V_1 = V_2$

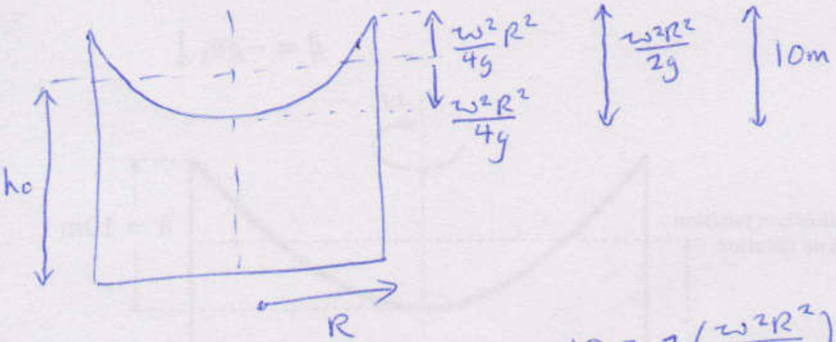
$$\cancel{\pi R^2} h_0 = \cancel{\pi R^2}\left(\frac{\omega^2}{4g}R^2 + C_2\right) \Rightarrow C_2 = h_0 - \frac{\omega^2}{4g}R^2 \quad (1)$$

$$z_s = h_0 + \frac{\omega^2}{4g}(2r^2 - R^2) \quad (1)$$

CONTINUE OVERLEAF

$@\ r=0 \quad z_s = h_0 + \frac{\omega^2}{4g}(0 - R^2) = h_0 - \frac{\omega^2 R^2}{4g}$ 1

$@\ r=R \quad z_s = h_0 + \frac{\omega^2}{4g}(2R^2 - R^2) = h_0 + \frac{\omega^2 R^2}{4g}$ 1



$$10 = 2\left(\frac{\omega^2 R^2}{4g}\right) = \frac{\omega^2 R^2}{2g}$$ 1

$$\Rightarrow \omega = \sqrt{\frac{10 \times 2 \times 9.81}{25^2}} = 0.56 \text{ rad/s}$$ 1

4. The size of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter, D, the jet velocity, U, the liquid density, ρ, the liquid dynamic viscosity, μ, and the liquid surface tension, σ.

Using D, U and ρ as the repeating variables, perform a dimensional analysis and re-write the original dimensional relationship in dimensionless form. **[10 Marks]**

Surface tension, σ, is measured in force per unit length.

$d = f(D, U, \rho, \mu, \sigma)$ 1

$D, U, \rho \rightarrow$ repeating variables

$$\left.\begin{aligned} d &= m \longrightarrow L \\ D &= m \longrightarrow L \\ U &= m/s \longrightarrow LT^{-1} \\ \rho &= kg/m^3 \longrightarrow ML^{-3} \\ \mu &= kg/ms \longrightarrow ML^{-1}T^{-1} \\ \sigma &= kg/s^2 \longrightarrow MT^{-2} \end{aligned}\right\} 1$$

$\Pi_1 = f(d\, D^a U^b \rho^c)$ $\quad L(L)^a(LT^{-1})^b(ML^{-3})^c$ 1

M	$0 = c$	$c = 0$
L	$0 = 1 + a + b - 3c$	$a = -1$
T	$0 = -b$	$b = 0$

$\Pi_1 = \frac{d}{D}$ 1

$\Pi_2 = f(\mu D^a U^b \rho^c)$ $\quad ML^{-1}T^{-1}(L)^a(LT^{-1})^b(ML^{-3})^c$ 1

M	$0 = 1 + c$	$c = -1$
L	$0 = -1 + a + b - 3c$	$a = 3c - b + 1 = -3 + 1 + 1 = -1$
T	$0 = -1 - b$	$b = -1$

$\Pi_2 = \frac{\mu}{DU\rho}$ 1

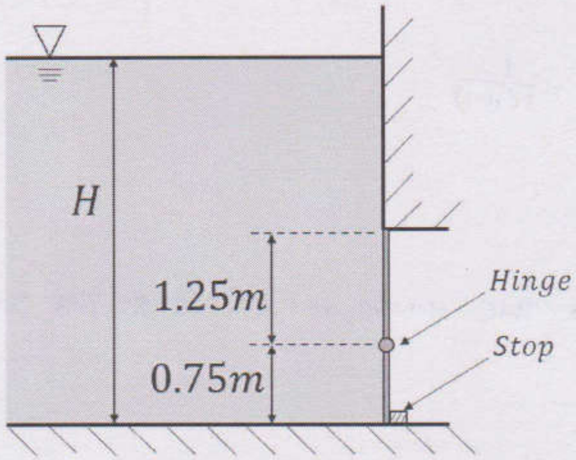
$\Pi_3 = f(\sigma D U \rho)$ $\quad MT^{-2}(L)^a(LT^{-1})^b(ML^{-3})^c$ 1

M	$0 = 1 + c$	$c = -1$
L	$0 = a + b - 3c$	$a = 3c - b = -3 + 2 = -1$
T	$0 = -2 - b$	$b = -2$

$\Pi_3 = \frac{\sigma}{DU^2\rho}$ 1

$$\frac{d}{D} = f\left(\underbrace{\frac{\mu}{DU\rho}}_{1}, \frac{\sigma}{DU^2\rho}\right) \quad 1$$

5. A rectangular gate with length $L = 2$ m and width $w = 2$ m is hinged 0.75 m from its lower end, shown in the figure. Due to the non-centred hinge, when the water is shallow, the hydrostatic pressure and the stopper at the bottom edge of the gate act to keep the gate closed. When the water level, H, raises above a certain value, the gate will open. Find the minimum water level that causes the gate to open. **[10 Marks]**



METHOD 1

CENTRE OF PRESSURE METHOD

CENTROID AT $(H-1)$

$$I_{xx} = \frac{bd^3}{12} = \frac{2\times 2^3}{12} = \frac{4}{3} \quad (1)$$

$$y_{cp} = \frac{I_{xx}}{y_{CG}A} + y_{CG} \quad (1)$$

$$y_{cp} = \frac{\frac{4}{3}}{4\times(H-1)} + (H-1) = \frac{1}{3(H-1)} + (H-1) \quad (1)$$

RESULTANT FORCE:

$$F_R = \rho g h_{CG} A = \rho g (H-1) A \quad (1)$$

FOR STATIC EQUILIBRIUM - JUST BEFORE THE GATE WILL OPEN - MOMENT ABOUT THE HINGE IS EQUAL TO ZERO.

TO TAKE MOMENT ABOUT THE HINGE

$$\text{So} \quad M = 0 \; (1) \qquad M = F_R \cdot \overset{(1)}{(}y_{cp} - (H-0.75)) \; (1)$$

$$M = 0 = F_R \cdot (y_{cp} - (H-0.75)) = \rho g A (H-1)(y_{cp} - (H-0.75))$$

$$\rho g A (H-1)\left(\frac{1}{3(H-1)} + (H-1) - (H-0.75)\right) = 0$$

$$\rho g A (H-1)\left(\frac{1}{3(H-1)} - 0.25\right) = 0$$

$$\frac{1}{3} - \frac{H-1}{4} = 0 \implies H = \frac{4}{3} + 1 = 2.3\dot{3}\text{ m} \quad (1) \leftarrow \text{NEUTRAL POINT}$$

$\therefore$ WHEN $H > 2.3\dot{3}$ m THE GATE WILL OPEN. (1) (1)

METHOD 2

WE KNOW THAT THE CENTRE OF PRESSURE IS BELOW THE CENTROID BUT INCREASE ELEVATION TOWARDS THE CENTROID AS THE WATER LEVEL RISES. IF WE FIND THE HEIGHT OF THE WATER THAT PLACES THE CENTRE OF PRESSURE AT THE HINGE, WE KNOW THAT ANY WATER LEVEL ABOVE THAT WILL CAUSE THE GATE TO OPEN.

$$y_{CP_c} = \frac{I_{xx}}{y_{CG}A} = \frac{\frac{bd^3}{12}}{(H-1)(2\times 2)} = \frac{4/3}{(H-1)4} = \frac{1}{3(H-1)}$$

RELATIVE TO THE CENTROID

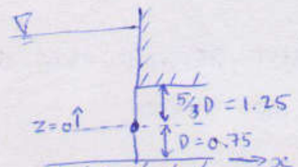
TO FIND CENTRE OF PRESSURE RELATIVE TO THE HINGE, OFFSET IT BY THE DISTANCE BETWEEN THE HINGE & CENTROID

$$y_{CP_H} = y_{CP_c} - \frac{1}{4} = \frac{1}{3(H-1)} - \frac{1}{4}$$

WE WANT THEM TO COINCIDE, SO $y_{CP_H} = 0 \Rightarrow 0 = \frac{1}{3(H-1)} - \frac{1}{4} \Rightarrow H = \frac{4}{3} + 1 = 2.3\dot{3}\,m$

THE GATE WILL OPEN WHEN THE CENTRE OF PRESSURE IS ABOVE HINGE,
HENCE WHEN $H > 2.3\dot{3}\,m$ 10

METHOD 3



$p = \rho g(H - D - z)$ 1

$d\vec{F}_p = pw\,dz\,\hat{i} \Rightarrow dF_p = \rho g w(H-D-z)dz$ 1

Moments about hinge M ↺

$dM = -z\,dF_p = -z\rho g w(H-D-z)dz = -\rho g w\left[(H-D)z - z^2\right]dz$ 1 1

Just before the gate opens, the moment is zero (Any extra water will cause a -ve moment and the gate will open).

$$M = -\rho g w \int_{-D}^{5/3 D} \left[(H-D)z - z^2\right] dz = 0 \Rightarrow -\rho g w\left[\frac{H-D}{2}\left(\frac{5}{3}\right)^2 D^2 - \frac{1}{3}\left(\frac{5}{3}\right)^3 D^3 - \frac{H-D}{2}D^2 - \frac{1}{3}D^3\right] = 0$$ 1 1 1

$\Rightarrow (H-D)\frac{8}{9}D^2 - \frac{152}{81}D^3 = 0 \Rightarrow H = \frac{19}{9}D + D$ where $D = 0.75\,m$

$$H = \frac{19}{9}(0.75) + 0.75 = 2.3\dot{3}\,m$$ 1

WHEN WATER LEVEL, $H > 2.3\dot{3}\,m$, THE GATE WILL OPEN.
1 1