Multiple Choice: Choose the best answer for each question. Circle the corresponding letter on the bubble sheet provided.

4. **[3.33 marks]** The following wave-function candidates are all zero for $|x| > 1$ and only non-zero for the interval $-1 < x < 1$. Without knowing anything about the system and its potential, which of the following candidate functions is a possible solution to the time-independent Schrödinger Equation?

   (a) None of the functions are possible solutions.

   (b) $\psi(x) = 1 - |x|$

   (c) $\psi(x) = 3x^2 - 1$

   (d) $\psi(x) = \sin(\pi x)$ correct

   (e) $\psi(x) = \frac{15}{4}(x^2 - x^4)$

5. **[3.33 marks]** Which one of the following statements about stationary states is correct?

   (a) None of the other answers is correct.

   (b) $\psi(x)$ will oscillate more quickly (small wavelength) in regions where the kinetic energy is highest. correct

   (c) Stationary states always have the highest probabilities at points where the kinetic energy is lowest.

   (d) Stationary states are independent of time, that is $\frac{\partial}{\partial t}\Psi(x, t) = 0$.

   (e) The sum of two stationary states is also a solution to the Schrödinger equation and is itself a stationary state.

6. **[3.33 marks]** Each of the following arguments attempts to invoke the uncertainty principle to explain some aspect of physics. Find the flawed argument.

   (a) An electron in a Hydrogen atom can absorb a photon in order to change its orbit. Since the process of changing orbits must take some amount of time, there is some uncertainty in that time. This means the absorption lines of the Hydrogen spectra must have non-zero widths due to the uncertainty principle.

   (b) In order to reach a temperature of absolute zero, particles must have zero kinetic energy and hence zero momentum. This is impossible since zero momentum necessarily implies zero uncertainty in momentum, violating the uncertainty principle.

   (c) Electron microscopes can see smaller objects than light-based microscopes. This is because for the same energy, electrons have more momentum than photons. The uncertainty principle thus requires that electrons have a smaller size than photons. correct

   (d) Two electrons are described by wave functions with the same average initial position and momentum. The wave packet with a smaller initial physical size will spread out more quickly in time due to the uncertainty principle.

   (e) A plane wave is squeezed through a small slit and as a result it spreads out upon leaving the slit. According to the uncertainty principle, the smaller the slit, the more it spreads out.

Written questions: Answer the following questions in the exam booklet provided. Explain your answers.

3. Helium has 2 protons (compared with Hydrogen which has only one proton) in its nucleus. Singly-ionized Helium has one electron. Find a model for the singly-ionized Helium atom as follows.

   (a) [**5 marks**] Explain the DeBroglie hypothesis as it applies to the electron while it orbits the Helium nucleus. Specifically, write the equation which relates momentum with radius and explain why this equation is plausible.

   The DeBroglie hypothesis is that electrons have a wavelength given by $\lambda = h/p$. (2 marks)

   For an electron orbiting a nucleus in a circular orbit its circumference should be an integer $n$ times the wavelength.

   $$nh/p = 2\pi r$$

   or

   $$pr = n\hbar$$

   (3 marks)

   (b) [**10 marks**] Using classical mechanics, electrostatics, and your answer to (a), show that the allowed energies for the electron should have the form $E_n = -54.4\,\mathrm{eV}/n^2$. You can assume that the helium nucleus does not move. For a circular orbit we have

   $$mv^2/r = kQq/r^2$$

   (2 marks)

   In our case, $q = e$ and $Q = 2e$. (2 marks)

   Inserting $pr = n\hbar$ into that equation gives (with some rearranging)

   $$r = \frac{n^2\hbar^2}{2mke^2}$$

   (2 marks)

   We know the total energy is

   $$E = KE + U = \frac{1}{2}mv^2 - kQq/r$$

   which we can combine with our equation for the circular motion $mv^2/r = kQq/r^2$ from above to get

   $$E = -kQq/2r = -ke^2/r$$

   (2 marks)

   Plug in $r = \frac{n^2\hbar^2}{2mke^2}$ into $E = -ke^2/r$ and you get

   $$E_n = -\frac{2mk^2e^4}{n^2\hbar^2}$$

   which, if you plug in the numbers, gives $E_n = -54.4\,\mathrm{eV}/n^2$ as required. (2 marks)

(c) [**5 marks**] What is the largest wavelength of light which an electron in the ground state of singly-ionized helium can absorb? Explain why your value is the largest wavelength.

The largest wavelength corresponds to the smallest energy difference based on $E = hc/\lambda$. (2 marks)

The least energy the electron in the ground state ($n = 1$) can absorb is enough to get it to $n = 2$. That energy is

$$-54.4\,\mathrm{eV}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 40.8\,\mathrm{eV}$$

(2 marks)

The wavelength is thus

$$\lambda = hc/E = 30.4\,\mathrm{nm}$$

(1 marks)

4. A demonstration of the photoelectric effect is set up where light with a single wavelength shines upon some metal. As a result, electrons are ejected from the metal. Answer the following questions.

(a) [**5 marks**] If the wavelength of light is 310 nm, electrons are ejected with speeds as high as $0.0024\,c$ but no higher. What is the work function of the metal?

The light has energy $E = hc/\lambda = 4.00\,\mathrm{eV}$. (2 marks)

The electrons have a maximum kinetic energy of $KE_{\max} = \frac{1}{2}mv^2 = 1.47\,\mathrm{eV}$. (2 marks)

The work function is just the difference between these two values: $2.53\,\mathrm{eV}$. (1 marks)

(b) [**5 marks**] The absolute cut-off in speeds in part (a) is a strong piece of evidence in favour of light being quantized (in the form of photons) rather than being waves. Explain why this observation supports photons over waves.

If electrons have a maximum kinetic energy for a given wavelength of light, this strongly implies that the energy of the light does not accumulate. If light was a wave, we would expect the energy to accumulate and sometimes produce very high energy electrons. This supports a particle vision of light since if light was a particle then you would expect it would have a certain amount of energy, and the electrons could never acquire more energy that what the photon had before they collide or the photon gets absorbed. (5 marks)

(c) [**5 marks**] If the wavelength is doubled to 620 nm while the metal remains the same, what are the speeds of the electrons produced, if any?

If we double the wavelength, we have the energy. The photons now have 2 eV. (2 marks)

Since this energy is less than the work function (2.53 eV), there cannot be any electrons produced! (3 marks)

Anyone who makes mistakes for either energy (especially the work function) and who subsequently concludes from their bad data that the process is possible should lose marks for part (b) but get full marks for this question provided they arrive at a sensible maximum speed given their values.

(d) [**5 marks**] The wavelength is set back to 310 nm. What changes, if any, would be observed if the intensity of the light doubled while keeping the initial wavelength of 310 nm constant? Be clear about what changes and what remains the same.

Changing the intensity only has one effect on the electrons: the number of electrons per second which are produced. (3 marks)

In this case, if we double the intensity, we should produce twice as many electrons per second as compared with the original (pre-doubled intensity) situation. (2 marks)