UNIVERSITY OF TORONTO

FACULTY OF APPLIED SCIENCE AND ENGINEERING

Quiz 2, November 20, 2023

DURATION: 1.0 hours

Second Year – Engineering Science

CHE260H1 – Thermodynamics and Heat Transfer

Calculator Type: 1 (Any, non-communicating)

Exam Type: A (Closed Book)

Examiner: A. W. H. Chan

Instructions:

- For multiple choice questions, no work is needed or graded. No partial marks.
- For long form questions, work must be shown. Marks will be deducted if reasoning is unclear, even if the final answer is correct.
- Write legibly. Illegible handwriting will not be accepted.
- Number of significant figures should correspond to information provided in the question.

$\dot{Q}_{conduction} = -kA\frac{dT}{dx}$

$\dot{Q}_{convection} = hA(T_s - T_\infty)$

$\dot{Q}_{radiation} = \varepsilon\sigma A(T_s^4 - T_{surr}^4)$

$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4

$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$

$R_{convection} = \frac{1}{hA}$

$R_{wall} = \frac{L}{kA}$

$R_{radiation} = \frac{1}{h_{rad}A}$

$h_{rad} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr})$

$R_{cylinder} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi Lk}$

$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$

$R_c = \frac{\Delta T_{interface}}{\dot{Q}/A}$

$a = \sqrt{\frac{hP}{kA_c}}$

$\cosh(x) = \frac{e^x + e^{-x}}{2}$

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

For infinitely long fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-ax)$$

$$\dot{Q}_{fin,long} = \sqrt{hPkA_c}(T_b - T_\infty)$$

$$\eta_{fin,long} = \frac{1}{aL}$$

For insulated tip fin

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh\,[a(L - x)]}{\cosh\,(aL)}$$

$$\dot{Q}_{fin,ins} = \sqrt{hPkA_c}(T_b - T_\infty)\tanh(aL)$$

$$\eta_{fin,ins} = \frac{\tanh\,(aL)}{aL}$$

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\,fin}}$$

x	tanh(x)	x	tanh(x)
0	0	0.64	0.5649
0.04	0.04	0.68	0.5915
0.08	0.0798	0.72	0.6169
0.12	0.1194	0.76	0.6411
0.16	0.1586	0.8	0.664
0.2	0.1974	0.84	0.6858
0.24	0.2355	0.88	0.7064
0.28	0.2729	0.92	0.7259
0.32	0.3095	0.96	0.7443
0.36	0.3452	1	0.7616
0.4	0.3799	2	0.964
0.44	0.4136	3	0.9951
0.48	0.4462	4	0.9993
0.52	0.4777	5	0.9999
0.56	0.508	10	1
0.6	0.537		

Question 1. Multiple choice questions (8 marks)

Circle the correct answer only. Circling multiple answers will automatically be graded as zero. (2 marks and no partial marks for each part)

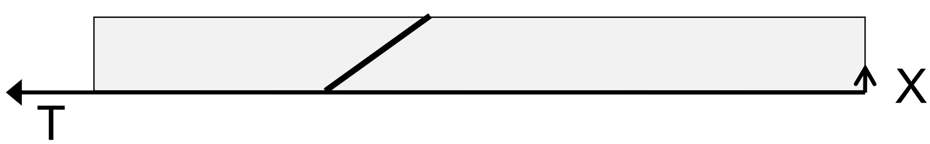
A. When will increasing insulation around a cylindrical pipe decrease heat transfer rate? (k: conductivity of pipe, h: heat transfer coefficient around insulation)

I. Always
II. When $r > k/h$
III. When $r < k/h$
IV. Never
V. There is not enough information.

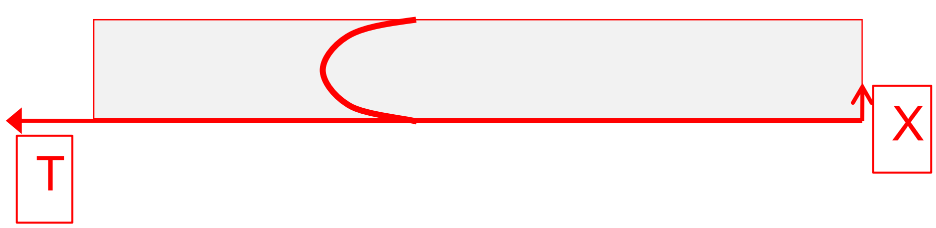
When $r > k/h$, the overall resistance increases with r and the heat transfer rate decreases.

B. Air cools the **top and bottom** of an electronic chip that uniformly generates heat. At steady-state, which plot represents the temperature profile within the chip?

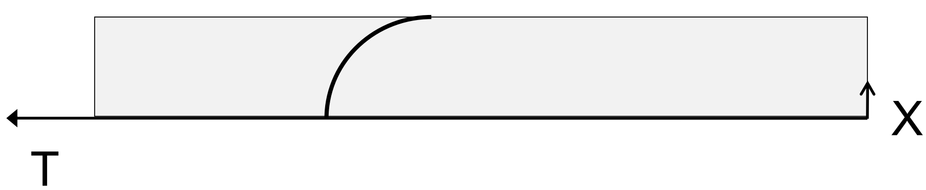
I.



II.



III.



II represents cooling from both top and bottom while heat is generated inside the chip.

C. In designing an effective fin to remove heat, one should:

I. Keep the ratio of the perimeter to the cross sectional area as high as possible

II. Use the longest fin

III. Use a less conductive material

IV. Increase contact resistance between different materials

Effectiveness of a heat fin scales with $\sqrt{\frac{kp}{hA_c}}$. Higher p/Ac gives higher effectiveness. While a longer fin can increase heat transfer rate, it is not effective once L approaches infinitely long.

D. If the thickness of a plane wall is doubled while maintaining the same temperatures on each side of the wall, the conductive heat transfer rate ($\dot{Q}$) ___________.

I. decreases

II. increases

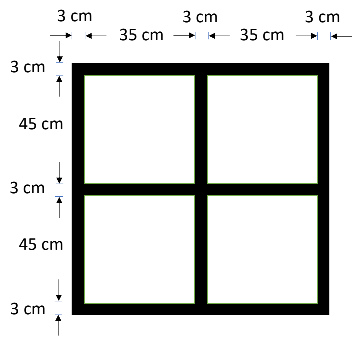
III. stays the same

IV. there is not enough information

Increasing L increases the resistance (L/kA) and decreases the heat transfer rate.

Question 2 (9 marks)

Consider a glass window with aluminum frame:



The thickness of the glass is 6 mm whereas the thickness of the aluminum frame is 3 cm. Glass has a thermal conductivity of 1.5 W/m·K and aluminum has a thermal conductivity of 240 W/m·K. Assume the outside surface temperature is −15°C and the inside surface temperature in the house is 23°C.

What is the overall thermal resistance through the window, in K/W? (6 marks) What is the total heat transfer rate, in W? (3 marks)

Successfully and clearly demonstrating…	Mark
$A_{glass} = 0.45 \times 0.35 \times 4 = 0.63m^2$	0.5
$A_{Al} = (0.35 \times 2 + 0.09) \times (0.45 \times 2 + 0.09) - A_{glass} = 0.152m^2$	0.5
$R_{glass} = \frac{L_{glass}}{k_{glass}A_{glass}} = (0.006K/W)$ (parentheses mean either correct formula or correct value)	1
$R_{Al} = \frac{L_{Al}}{k_{Al}A_{Al}}(= 0.0008K/W)$ (parentheses mean either correct formula or correct value)	1
$\frac{1}{R_{total}} = \frac{1}{R_{glass}} + \frac{1}{R_{Al}}$	2
Correct $R_{total} = 7.3 \times 10^{-4}K/W$ Subtract 0.5 for more than 3 significant figures	1
$\dot{Q} = \frac{T_{in} - T_{out}}{R_{total}}$	2
$\dot{Q} = 52{,}000W$ Subtract 0.5 for more than 3 significant figures, or different units	1

Question 3 (10 marks)

Hot water in your home with an average temperature 60°C is transported through a steel pipe (k = 50 W/m·K, outer diameter D_o = 3.0 cm, inner diameter D_i = 2.0 cm, and L = 15.0 m). The pipe is insulated with a 0.5-cm thick layer of gypsum plaster (k = 0.5 W/m·K). The insulated pipe is placed horizontally in your home where the average air temperature is 20°C. The hot water and the air heat transfer coefficients are estimated to be 800 and 200 W/m^2·K, respectively. Calculate (a) the **daily** rate of heat transfer from the hot water (in J per day) (7 marks), and (b) the temperature (in °C) on the outside surface of the gypsum plaster insulation. (3 marks)

Successfully and clearly demonstrating…	**Mark**
$R_{in} = \frac{1}{h_{in}A_{in}} (= \frac{1}{(800)(\pi)(0.02)(15)} = 0.00133\ K/W)$	1
$R_{steel} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{steel} L} (= 8.6 \times 10^{-5} K/W)$	1
$R_{insulation} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{insulation} L} (= 6.2 \times 10^{-3}\ K/W)$	1
$R_{out} = \frac{1}{h_{out}A_{out}} (= \frac{1}{(200)(2\pi)(0.02)(15)} = 2.65 \times 10^{-3} K/W)$	1
$R_{total} = R_{in} + R_{steel} + R_{insulation} + R_{out} (= 0.01027\ K/W)$	1
$\dot{Q} = \frac{T_{in} - T_{out}}{R_{total}} = 3894.84\ W$	1
$E = Qt = 3.4 \times 10^8 J$ Subtract 0.5 for more than 3 significant figures, or different units	1
$\dot{Q} = \frac{T_{s,out} - T_{out}}{R_{out}}$	2
$T_{s,out} = 30.3C$ or 30C Subtract 0.5 for more than 3 significant figures, or different units	1

Question 4 (13 marks)

A spoon is dipped into hot water (60°C), exposing 10 cm of its handle in air (25°C). The spoon is made of stainless steel (k = 15 W/m·K), and the handle is approximately a cylinder with a diameter of 0.3 cm. Assume the spoon handle is directly perpendicular to the water surface. The heat transfer coefficient is 10 W/m^2·K.

a. Sketch the temperature profile along the spoon handle (T as a function of y-position, y=0 at water surface). Explain the physical reason behind the trend in slope. (4 marks)
b. What is the heat transfer rate (in W) across the spoon handle? (5 marks)
c. What is the factor by which the spoon handle enhance heat transfer as a heat fin? How will this change if more of the spoon handle is extended into air? (4 marks)

Successfully and clearly demonstrating…	**Mark**
A decreasing temperature profile in the sketch	1
A decreasing slope in the sketch	1
Explanation: The rate of temperature decrease is decreasing because the temperature difference between the handle and the surrounding air is decreasing. At any given point on the handle, the heat conducted from the hot water is greater than the heat conducted away to further down the handle, because there is heat loss due to convection. Since the rate of heat conduction is proportional to dT/dx, dT/dx decreases with x.	2
$\dot{Q}_{fin,long} = \sqrt{hPkA_c}(T_b - T_\infty) = 0.111W$ or Add in tanh term: $\dot{Q}_{fin,ins} = \sqrt{hPkA_c}(T_b - T_\infty)\tanh(aL) = 0.11\text{W}$	3
Calculated corrected length, or checked that corrected length is close to fin length: $L = 0.1\,m \quad L_c = L + \frac{A_c}{p} = 0.10075m$	2
$\varepsilon = \dot{Q}_{fin}/\dot{Q}_{no\ fin}$	1
$\dot{Q}_{no\ fin} = hA_c(T_b - T_\infty)$ or $\varepsilon = \sqrt{kp/hA_c}$ or $\varepsilon = \sqrt{kp/hA_c}\tanh(aL)$	1
$\varepsilon = 45$ Subtract 0.5 for more than 3 significant figures, or different units	1
L is so long that additional length does not enhance heat transfer	1