AER210 VECTOR CALCULUS and FLUID MECHANICS

Quiz 3

Duration: 70 minutes

10 November 2014

Closed Book, no aid sheets

Non-programmable calculators allowed

Instructor: Prof. Alis Ekmekci

Family Name: Ekmekci

Given Name: Alis

Student #: ______________________

TA Name/Tutorial #: ______________________

FOR MARKER USE ONLY		
Question	Marks	Earned
1	9	
2	10	
3	4	
4	10	
5	10	
6	10	
TOTAL	53	/50

Hints: $E_V = -\frac{dP}{dV\!\!\!/ / V\!\!\!/}$ $\quad \tau = \mu \frac{du}{dy}$ $\quad -\nabla p + \rho \vec{g} = \rho \vec{a}$

(Gravitational acceleration: $g = 10$ m/s^2)

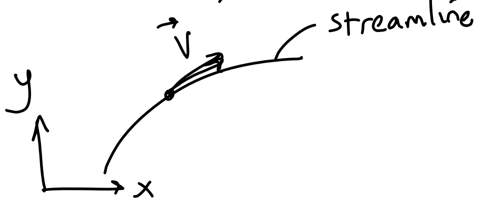
1) The velocity field in a flow is given as $\vec{V} = ax\vec{i} + bx^2\vec{j}$, where the constants a and b are: a = 1 s^{-1} and b = 1 $m^{-1}s^{-1}$.

(a) [**1 mark**] Is this a steady or unsteady flow? $\Rightarrow$ Steady

(b) [**1 mark**] Is this a one-, two- or three-dimensional flow? $\Rightarrow$ One-dimensional as $\vec{V} = \vec{V}(x)$

(c) [**3 marks**] Find an equation for the flow streamlines.

$u = ax$, $v = bx^2$



$$\underbrace{\frac{dy}{dx}}_{\text{slope of streamline}} = \underbrace{\frac{v}{u}}_{\text{slope of the velocity vectors.}} \Rightarrow \frac{dy}{dx} = \frac{bx^2}{ax}$$

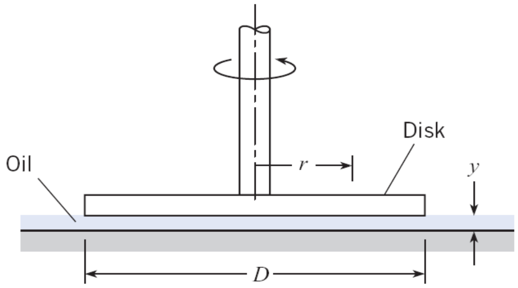
$$\frac{dy}{dx} = \frac{x^2}{x}$$

$$\int dy = \int x\,dx$$

$$\boxed{y = \frac{x^2}{2} + C_1}$$

(Note to TAs: If C is set to 0, that is also valid answer.)

(d) [**3 marks**] The device shown below consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil. If the rate of rotation is ω = 2 rad/s, what is the speed of oil in contact with the disk at r = 3 cm? If the oil viscosity is 0.01 N.s/m^2 and the spacing y is 2 mm, find the shear stress acting on the disk at r = 3 cm? (HINT: Assume linear velocity distribution between the disk and the boundary.)



- The speed of oil in contact with the disk at r=3cm is: [1mark]

$$(V)_{@r=3cm} = r\omega = (0.03) \times 2 = \boxed{0.06\ m/s}$$

$$\tau = \mu \frac{dV}{dy} = \mu \frac{V}{y} = \frac{(0.01)(0.06)}{0.002} = 0.30\ N/m^2 \quad [2marks]$$

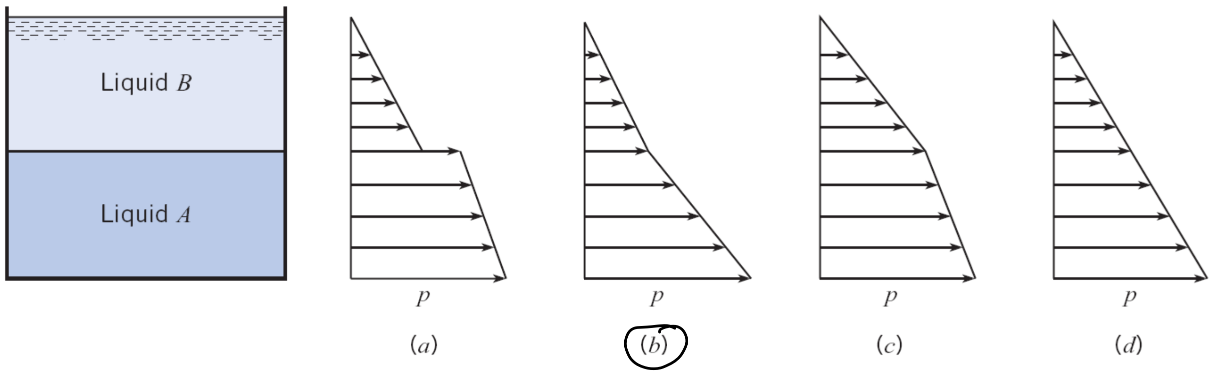
(e) [**1 marks**] What pressure increase must be applied to water to reduce its volume by 1%. Modulus of elasticity for water is $E_v = 2.2 \times 10^9$ Pa.

$$E_v = -\frac{dp}{d\forall/\forall} \Rightarrow dp = -E_v \frac{d\forall}{V} = -(2.2\times10^9)\left(-\frac{1}{100}\right) = 2.2\times10^7\ Pa$$

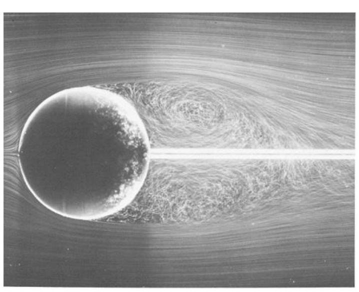
$\boxed{dp = 2.2\times10^7\ Pa = 22\ MPa}$ must be applied.

2) (a) Circle the true statement in the following:

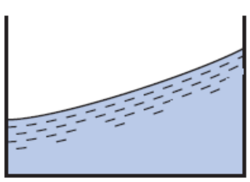
- [**1 mark**] Compressibility of a fluid is represented by a property called:
  - **a.** Kinematic viscosity
  - **b.** Density
  - **c.** Bulk modulus of elasticity
  - **d.** Change in pressure
- [**1 mark**] With an increase in temperature, the viscosity of gases
  - **a.** doesn't change
  - **b.** increases
  - **c.** decreases
- [**1 mark**] The reservoir in the figure contains two immiscible liquids of density ρ_A and ρ_B, respectively, one above the other. $\rho_A > \rho_B$. Which graph depicts the correct distribution of gage pressure along a vertical line through the liquids?



- [**1 mark**] Consider the smoke visualization of the flow over a sphere as shown in the picture below. We are seeing in this picture:

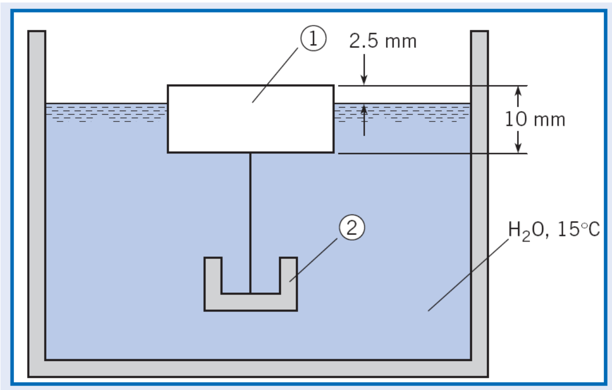


  - **a.** streamlines
  - **b.** streaklines
  - **c.** pathlines
  - **d.** constant pressure lines
- [**1 mark**] Given: The liquid orientation in a tank as shown. The conditions could be caused by:

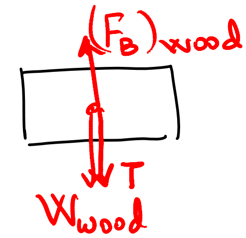


  - **a.** constant acceleration of the tank to the right
  - **b.** the tank being placed on a vehicle that travels at a constant speed about a circular track (center of the circle to the left of the vehicle)
  - **c.** the tank being placed on a vehicle that travels at a constant speed about a circular track (center of the circle to the right of the vehicle)
  - **d.** none of the above apply

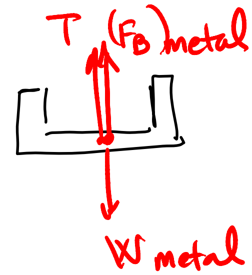
2) (b) [**5 marks**] The figure below shows a metal part (object 2) hanging by a thin cord from a floating wood block (object 1). The wood block's density is $\rho_{wood} = 294\ kg/m^3$ and the dimensions of the entire wood block is 50 × 50 × 10 mm. 7.5 mm height of the wood block is submerged as shown in the figure. The metal part (object 2) has a volume of 6600 mm^3. Find the tension force T in the cord and the mass of the metal part (object 2). The density of water is $\rho_{water} = 1000\ kg/m^3$ and the gravitational acceleration is $g = 10\ m/s^2$.



wood:



metal:



$$\left.\begin{aligned}(F_B)_{wood} &= W_{wood} + T \quad (\text{Eqn.}1)\\ W_{metal} &= (F_B)_{metal} + T \quad (\text{Eqn.}2)\end{aligned}\right\}$$

$$W_{wood} = \rho_{wood}\, g\, V_{wood} = 294 \times 10 \times (50 \times 50 \times 10) \times 10^{-9} = 0.0735\,N$$

$$(F_B)_{wood} = \rho_{water}\, g\, V_{\substack{submerged\\wood}} = 1000 \times 10 \times (50 \times 50 \times 7.5) \times 10^{-9} = 0.1875\,N$$

From Eqn.1 $\Rightarrow$ $T = (F_B)_{wood} - W_{wood} = 0.1875 - 0.0735$

$$\boxed{T = 0.114\,N}$$

$$(F_B)_{metal} = \rho_{water}\, g\, V_{metal} = 1000 \times 10 \times 6600 \times 10^{-9} = 0.066\,N$$

From Eqn.2 $\Rightarrow$ $W_{metal} = (F_B)_{metal} + T = 0.066 + 0.114$

$$W_{metal} = 0.18\,N = m_{metal}\, g$$

$$m_{metal} = \frac{0.18}{10} \boxed{= 0.018\,kg = 18\,gram}$$

3) [**4 marks**] An airfoil with a chord length (which is the length from its nose to tail) of $L_{chord} = 1$ m will be used in a flight speed of 50 m/s in standard atmospheric conditions. The performance of this airfoil will be evaluated by using a 1/10th scaled model of it.

(a) [**2 marks**] Find the air speed required in the experiment in order to maintain the same Reynolds number which is defined as $Re = (\rho V L_{chord})/\mu$.

(b) [**2 marks**] What velocity would be required if the experiment were performed in water?

Take the density and viscosity of air as $\rho_{air} = 1.2$ kg/m^3 and $\mu_{air} = 1.8\times10^{-5}$ Ns/m^2 respectively, and the corresponding properties of water as $\rho_{water} = 10^3$ kg/m^3 and $\mu_{water} = 10^{-3}$ Ns/m^2.

For the prototype : $(L_{chord})_p = 1m$, $V_p = 50 m/s$

For the model : $(L_{chord})_m = 0.1 m$

$$\Rightarrow Re = \frac{\cancel{\rho_{air}} V_p (L_{chord})_p}{\cancel{\mu_{air}}} = \frac{\cancel{\rho_{air}} V_m (L_{chord})_m}{\cancel{\mu_{air}}}$$ for testing in air

$$V_m = V_p \frac{(L_{chord})_p}{(L_{chord})_m} = 50 \times \frac{1}{0.1} = 500 \ m/s$$ for testing the model in air

$$\Rightarrow Re = \frac{\rho_{air} V_p (L_{chord})_p}{\mu_{air}} = \frac{\rho_{water} V_m (L_{chord})_m}{\mu_{water}}$$

$$V_m = \frac{(1.2)(50)(1)(10^{-3})}{(1.8 \times 10^{-5})(0.1)(10^3)} = 33.33 \ m/s$$ for testing the model in a water facility.

4) [**10 marks**] The terminal velocity V_T of small bubbles rising in unconfined liquids depends on the balance between buoyancy and drag forces. For the case of air bubbles rising in water, we might expect V_T to depend on the radius of the bubble R_b, the density of air ρ_A, the density of water ρ_w, the viscosity of water μ_w, the gravitational acceleration g, and the surface tension σ for the gas-liquid interface:

$$V_T = f(R_b, \rho_A, \rho_w, \mu_w, g, \sigma)$$

Using ρ_w, R_b and g as the repeating variables, determine the dimensionless (π) groups for this problem and re-write the original dimensional relationship in dimensionless terms.

<u>Hint:</u> *Note that surface tension is measured in force per unit length.*

$$V_T = f(R_b, \rho_A, \rho_w, \mu_w, g, \sigma)$$

$$[V_T] = \frac{L}{T}, \; [R_b] = L, \; [\rho_A] = \frac{M}{L^3}, \; [\rho_w] = \frac{M}{L^3}, \; [\mu_w] = \frac{M}{LT}, \; [g] = \frac{L}{T^2}, \; [\sigma] = \frac{M}{T^2}$$

# of variables = 7

# of reference dimensions = 3 (M, L, T)

Buckingham π theorem:

(# of π terms) = (# of variables) − (# of reference dimensions)

= 7 − 3

(# of π terms) = 4 ⇒ (4 π terms to be found)

Repeating variables: ρ_w, R_b, g

$$\pi_1 = V_T \rho_w^a R_b^b g^c$$

$$M^0 L^0 T^0 = \frac{L}{T} \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$$

$$M^0 L^0 T^0 = M^a L^{1-3a+b+c} T^{-1-2c}$$

$$\boxed{a = 0}$$

$$-1 - 2c = 0 \Rightarrow \boxed{c = -\frac{1}{2}}$$

$$1 - 3\underset{0}{\cancel{a}} + b + c = 0 \Rightarrow \boxed{b = -\frac{1}{2}}$$

$$\boxed{\pi_1 = \frac{V_T}{\sqrt{R_b g}}}$$

$$\pi_2 = \rho_A \rho_w^a R_b^b g^c$$

$$M^0 L^0 T^0 = \frac{M}{L^3} \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$$

$$M^0 L^0 T^0 = M^{1+a} L^{-3-3a+b+c} T^{-2c}$$

$$\boxed{c = 0}$$

$$1 + a = 0 \Rightarrow \boxed{a = -1}$$

$$-3 - 3a + b + \cancel{c} = 0 \Rightarrow \boxed{b = 0}$$

$$\boxed{\pi_2 = \frac{\rho_A}{\rho_w}}$$

EXTRA PAGE

$$\Pi_3 = \mu_w \rho_w^a R_b^b g^c$$

$$M^0L^0T^0 = \frac{M}{LT} \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$$

$$M^0L^0T^0 = M^{1+a} L^{-1-3a+b+c} T^{-1-2c}$$

$$1+a=0 \Rightarrow \boxed{a=-1}$$

$$-1-2c=0 \Rightarrow \boxed{c=-\frac{1}{2}}$$

$$-1-3a+b+c=0 \Rightarrow \underbrace{-1+3}_{2}+b-\frac{1}{2}=0$$

$$\boxed{b=-3/2}$$

$$\boxed{\Pi_3 = \frac{\mu_w}{\rho_w\sqrt{R_b^3 g}}}$$

$$\Pi_4 = \sigma \rho_w^a R_b^b g^c$$

$$M^0L^0T^0 = \frac{M}{T^2} \frac{M^a}{L^{3a}} L^b \frac{L^c}{T^{2c}}$$

$$M^0L^0T^0 = M^{1+a} L^{-3a+b+c} T^{-2-2c}$$

$$1+a=0 \Rightarrow \boxed{a=-1}$$

$$-2-2c=0 \Rightarrow \boxed{c=-1}$$

$$-3a+b+c=0 \Rightarrow 3+b-1=0$$

$$\boxed{b=-2}$$

$$\boxed{\Pi_4 = \frac{\sigma}{\rho_w R_b^2 g}}$$

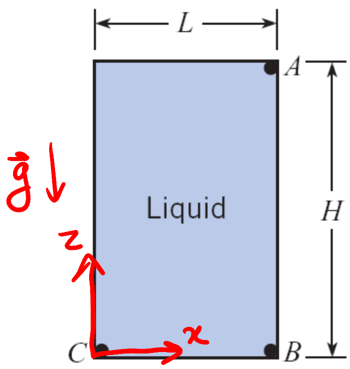
New relationship:

$$\Pi_1 = f_2(\Pi_2, \Pi_3, \Pi_4)$$

$$\boxed{\frac{V_T}{\sqrt{R_b g}} = f_2\left(\frac{\rho_A}{\rho_w}, \frac{\mu_w}{\rho_w\sqrt{R_b^3 g}}, \frac{\sigma}{\rho_w R_b^2 g}\right)}$$

Name: ____________________

(5) [**10 marks**] The closed tank shown, which is full of liquid, is accelerated downward at (2/3)g and to the right at 1g, where g is the gravitational acceleration. Here, L = 2.5 m, H = 3 m, and the liquid has a density of ρ = 1300 kg/m^3. Determine (p_C - p_A) and (p_B - p_A).



$a_x = 1g$

$a_z = -2/3\, g$

$$-\vec{\nabla} p - \rho g \vec{k} = \rho \vec{a}$$

$$-\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k}\right) - \rho g \vec{k} = \rho\left(a_x\vec{i} + a_z\vec{k}\right)$$

$$\begin{cases} \dfrac{\partial p}{\partial x} = -\rho a_x \\ \dfrac{\partial p}{\partial y} = 0 \\ \dfrac{\partial p}{\partial z} = -\rho(a_z + g) \end{cases}$$

Method 1: $p = p(x,z) \Rightarrow dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$

$$dp = -\rho a_x dx - \rho(a_z + g)\,dy$$

$$p = -\rho a_x x - \rho(a_z + g)y + C_1$$

$$p = -\rho g x - \rho\left(\frac{1}{3}g\right)y + C_1$$

@ point A $\Rightarrow x = L,\ y = H \Rightarrow P_A = -\rho g L - \frac{\rho g}{3}H + C$

@ point B $\Rightarrow x = L,\ y = 0 \Rightarrow P_B = -\rho g L + C_1$

@ point C $\Rightarrow x = 0,\ y = 0 \Rightarrow P_c = C_1$

$$P_C - P_A = C - \left(-\rho g L - \frac{\rho g}{3}H + C\right) = \rho g L + \frac{\rho g}{3}H = \rho g\left(L + \frac{H}{3}\right)$$

$$= 1300 \times 10 \times \left(2.5 + \frac{3}{3}\right) = \boxed{45500\ \text{Pa}}$$

$$P_B - P_A = \left(-\rho g L + C_1\right) - \left(-\rho g L - \frac{\rho g}{3}H + C_1\right)$$

$$= \frac{\rho g}{3}H = \frac{1300 \times 10 \times 3}{3} = \boxed{13000\ \text{Pa}}$$

Method 2:

$$\frac{\partial p}{\partial x} = -\rho a_x$$

$$\frac{\partial p}{\partial y} = 0$$

$$\frac{\partial p}{\partial z} = -\rho(a_z + g)$$

To find $p_B - p_A$:

$$\frac{\partial p}{\partial z} = -\rho(a_z + g)$$

$$\frac{p_B - p_A}{z_B - z_A} = -\rho\left(-\frac{2}{3}g + g\right)$$

$$p_B - p_A = -\rho\left(\frac{1}{3}g\right)(-H)$$

$$p_B - p_A = \rho \frac{gH}{3}$$

$$= \frac{1300 \times 10 \times \cancel{3}}{\cancel{3}} = \boxed{13000\text{ N}}$$

To find $p_C - p_A$, first let's find $p_C - p_B$

$$\frac{\partial p}{\partial x} = -\rho a_x \Rightarrow \frac{p_C - p_B}{x_C - x_B} = -\rho g \Rightarrow p_C - p_B = \rho g L$$

$x_C - x_B$ ← $-L$

$$= 1300 \times 10 \times 2.5$$

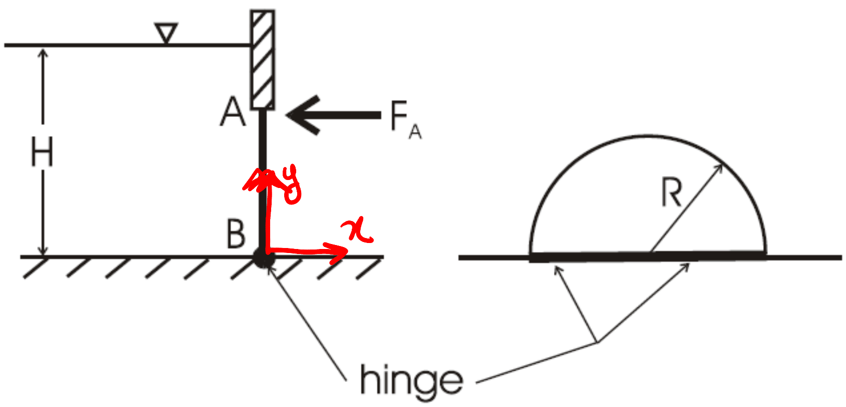
$$p_C - p_B = 32500\text{ N}$$

$$p_C - p_A = (p_C - p_B) + (p_B - p_A) = 32500 + 13000 = \boxed{45500\text{ N}}$$

Note to TAs: Both solution methods are valid.

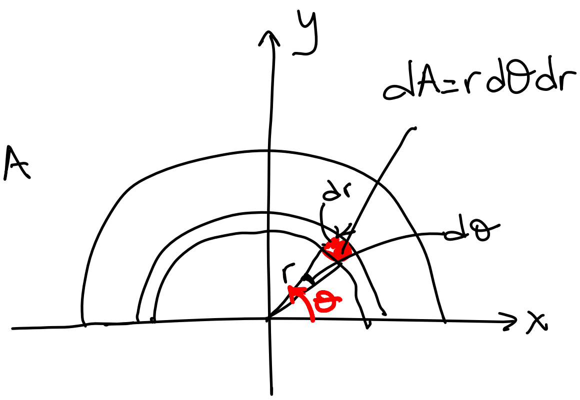
(6) [**10 marks**] A semicircular plane gate AB of radius R is hinged along B and held closed by horizontal force F_A applied at point A. The liquid to the left of the gate rises to a height H above the gate hinge. Integrate the pressure force on the gate to determine the required force F_A.

Hint: $\int \cos^2\theta \, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$; $\int \sin^2\theta \, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$; $\sin 2\theta = 2\sin\theta\cos\theta$



Closing moment: $M_{closing} = F_A \cdot R$

Opening moment: $M_{opening} = \iint_A y\, p \, dA$



$y = r\sin\theta$

$dA = r\,d\theta\,dr$

$$M_{opening} = \iint_A y\, p\, dA$$

$p = \rho g(H-y)$

$$= \int_{\theta=0}^{\pi}\int_{r=0}^{R} (r\sin\theta)\rho g(H - r\sin\theta)\, r\, d\theta\, dr$$

$$= \int_{\theta=0}^{\pi}\int_{r=0}^{R} H\rho g r^2 \sin\theta\, d\theta\, dr - \int_{\theta=0}^{\pi}\int_{r=0}^{R} \rho g \sin^2\theta\, r^3\, d\theta\, dr$$

$$= \int_{r=0}^{R} H\rho g r^2 dr \int_{\theta=0}^{\pi} \sin\theta\, d\theta - \int_{r=0}^{R}$$

$$= \int_{r=0}^{R} H\rho g r^2 dr \underbrace{\int_{\theta=0}^{\pi} \sin\theta \, d\theta}_{\left(-\cos\theta\right)\Big|_0^{\pi}} - \int_{r=0}^{R} \rho g r^3 dr \underbrace{\int_{\theta=0}^{\pi} \sin^2\theta \, d\theta}_{\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi}}$$

$$= H\rho g \frac{r^3}{3}\Big|_{r=0}^{R} \underbrace{\left(\cos 0 - \cos\pi\right)}_{(2)} - \rho g \frac{r^4}{4}\Big|_{r=0}^{R} \left(\frac{\pi}{2} - \underbrace{\cancel{\frac{1}{4}\sin 2\pi}}_{0} - \underbrace{\cancel{\frac{1}{4}\sin 0}}_{0}\right)$$

$$= \frac{2}{3}\rho g H R^3 - \frac{\pi}{8}\rho g R^4$$

$$\boxed{M_{opening} = \rho g R^3 \left(\frac{2}{3}H - \frac{\pi R}{8}\right)}$$

$$M_{opening} = M_{closing}$$

$$\rho g R^3 \left(\frac{2}{3}H - \frac{\pi R}{8}\right) = F_A R \Rightarrow \boxed{F_A = \rho g R^2 \left(\frac{2}{3}H - \frac{\pi R}{8}\right)}$$