① Approach: Assuming paper has about the density of wood, the ratio of volumes will tell us roughly how many pieces of paper we can get.

Tree: $20m \times \frac{\pi}{2}(0.5m)^2 \sim 5m^3$

Book: 500 pp in typical $8.5'' \times 11'' \times 1''$ textbook

$\longrightarrow 2\times10^{-3} \cdot (100\ in^3)\cdot(2.54cm/in)^3 \cdot 10^{-6} m^3/cm^3$

$2\cdot10^{-6}\ m^3$ volume/page

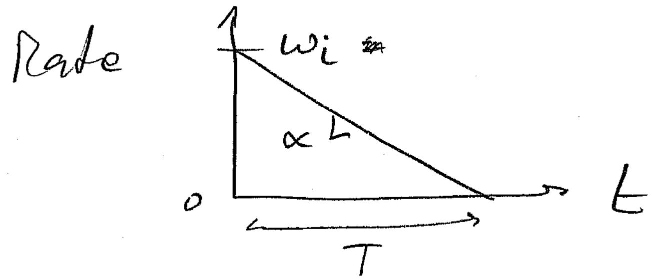
$\longrightarrow$ # pages $\approx \frac{\text{volume/tree}}{\text{volume/page}} \sim \frac{5m^3}{2\times10^{-6}m^3} \sim 3\times10^6$

So, between 10^6 & 10^7 pages.

Common mistakes:

- Guessing 1mm thickness (actually closer to 0.1mm)
-

(2.)



$\rightarrow \quad T = \omega_0/\alpha$

During this time, $\Delta\theta = \omega_0 T - \frac{1}{2}\alpha T^2$

substitute: $\quad \Delta\theta = \omega_0\left(\frac{\omega_0}{\alpha}\right) - \frac{1}{2}\alpha\left(\frac{\omega_0}{\alpha}\right)^2 = \frac{1}{2}\frac{\omega_0^2}{\alpha}$

& solve for α:

$$\alpha = \frac{\omega_0^2}{2(\Delta\theta)}$$

$\omega_0 = 3450 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{60\,\text{s}} = 361\ \text{s}^{-1}$

$\Delta\theta = 2\pi \cdot 52\,\text{rev} = 327$ radians

& thus $\underline{\alpha = 200.\ \text{s}^{-2}}$ (or $200.\ \text{rad/s}^2$) (or $2.00\times10^2\ \text{s}^{-2}$)

check units: $[\omega_0] = \text{s}^{-1}$, $[\Delta\theta] = 1$ ✓ ok.

Common mistakes

- $\Delta\theta = \frac{1}{2}\alpha t^2$ (assuming $\omega_0 = 0$?)
- Not converting revs → radians
- Not converting minutes → seconds

] full credit for: $33.2\ \text{rev/min}^2$, $7.19\times10^5\ \text{rad/min}^2$

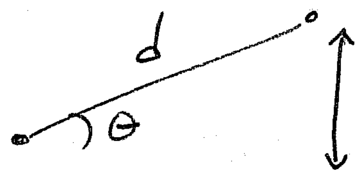
- Sign of α & ω: I used positive quantities above, but one could also use $\{\omega_0 < 0, \alpha > 0\}$ or $\{\omega_0 > 0, \alpha < 0\}$ (We have not discussed right-hand rule, so sign of rotational motion may be vaguely defined.)

3. (a) $\Delta K = K_f - K_i$

$= 0 - \frac{1}{2} m V_i^2$, $m = 4.20\,kg$, $V_i = 8.40\,m/s$.

$= \boxed{-148\,J}$

(b) $\Delta U = mg \underbrace{\Delta h}_{+d/2}$



$\Delta h = d \underbrace{\sin\theta}_{1/2}$

$= +mgd/2$, $m = 4.20\,kg$, $g = 9.80\,m/s^2$, $d = 6.00\,m$

$= \boxed{+123.\,J}$

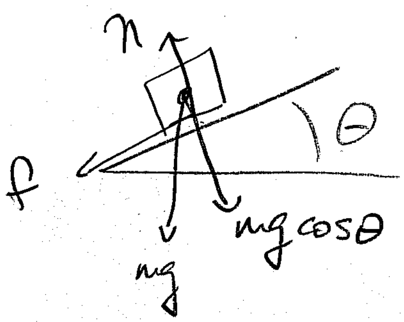
(c) By cons of energy, E_{int} must have gone up by $\Delta E_{int} = -\Delta K - \Delta U = \underline{25\,J}$ So this is the work done by friction:

$|W| = |f \cdot d| = \Delta E_{int}$

Knowing $d = 6.00\,m$, $f = \frac{\Delta E_{int}}{d} = \boxed{4.12\,N}$

(2 sig figs ok here)

(d) Coefficient of friction:



$\rightarrow n = mg\cos\theta$, & $f = \mu n$

so $\mu = \frac{f}{mg\cos\theta} = \boxed{0.115}$

(2 sig figs also ok here)

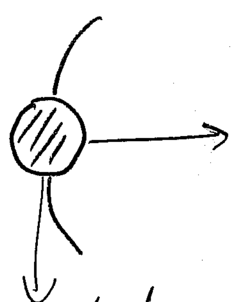
(4.) By conservation of E in the bead + wire + earth system, $E_i = E_A$

$$mgh = mgR + \frac{1}{2}mv^2$$

$$\longrightarrow \quad v^2 = 2(h-R)g \quad \text{or} \quad v = \sqrt{2(h-R)g}$$

[Note we must assume $h > R$, which is suggested by the drawing, & the fact that the bead arrives at point A at all, since it must have "done the loop" — ie, $h > 2R$.]

Now, what is acceleration?



centrifugal acceleration $= \dfrac{v^2}{R}$

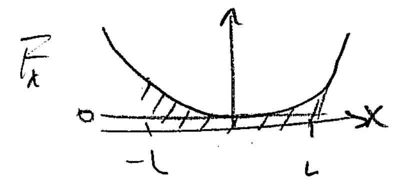
gravitational acceleration $= g$

$$\vec{a} = \left(2g\,\frac{h-R}{R}\right)\hat{\imath} - g\,\hat{\jmath}$$

[check units: $[g] = m/s^2$ ✓]

$$\text{or} \quad = \begin{bmatrix} 2g\left(\frac{h}{R}-1\right) \\ -g \end{bmatrix}$$

(3.)



(a) $W = \int_{-L}^{L} F\,dx = \int_{-L}^{L} Bx^2\,dx = \frac{1}{3}B\left[x^3\right]_{-L}^{L} = \boxed{\frac{2}{3}BL^3}$

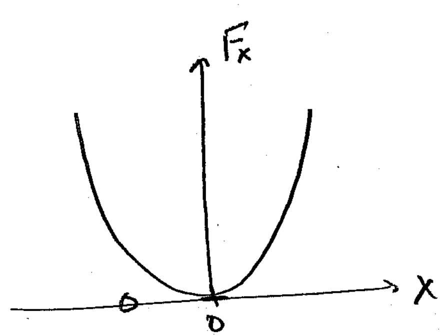
(b) If we define $U=0$ @ $x=0$, & work is from a conservative force, then

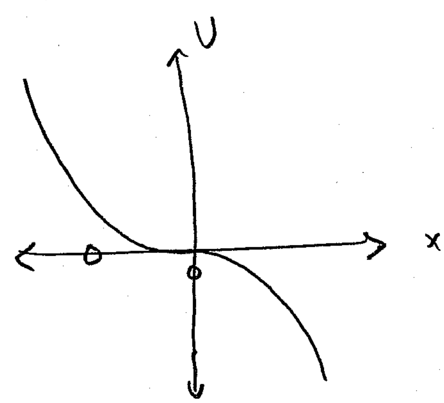
$$U = -\int_0^x F_x\,dx' = \boxed{-\frac{1}{3}Bx^3}$$

Check: $F_x = -\frac{dU}{dx} = \frac{1}{3}B\frac{d}{dx}(x^3)$

$$= Bx^2 \quad ✓$$

(c)





Key points:
- $U>0$ for $x<0$ } sign change
- $U<0$ for $x>0$ }
- $F>0$ for all x ← no sign change
- $F=0$ at $x=0$
- slope of U zero @ $x=0$

6. $\vec{r} = \langle 2t^3, 6t, -10 \rangle$

(a) $\vec{v} = \frac{d\vec{x}}{dt} = \langle 6t^2, 6, 0 \rangle$

$\frac{1}{2} m v^2 = \frac{1}{2} \overset{2.00}{m} (36t^4 + 36)$

@ $t = 0$, $\boxed{KE = 36.0\,J}$

(b) $\vec{a} = \frac{d\vec{v}}{dt} = \langle 12t, 0, 0 \rangle$

@ $t = 2.00\,s$, $\boxed{\vec{a} = \langle 24.0\,m/s^2, 0, 0 \rangle}$

(c) Two approaches: one is $P = \frac{dE}{dt}$, other $P = \vec{F} \cdot \vec{v}$.

$KE = 36t^4 + 36$

$\frac{d(KE)}{dt} = \boxed{144t^3 = P}$

or, $\vec{F} \cdot \vec{v} = m\vec{a} \cdot \vec{v} = 2kg \langle 12t, 0, 0 \rangle \cdot \langle 6t^2, 6, 0 \rangle$

$= \boxed{144t^3 = P}$