## PHY294, Winter 2017, Thermal Physics Term Test.

One $8.5 \times 11$ inches double-sided, hand-written aid sheet allowed. Duration: 75 minutes.

**I.** *Einstein solids with small energy per oscillator*

Consider an Einstein solid. Consider the case when $N \to \infty$, $q \to \infty$, but $q \ll N$; as usual, when we say $N \to \infty$, we really mean a very large number, like $10^{23}$. In words, let both the number of oscillators and energy be large (this is the so-called thermodynamic limit), but the energy per quantum is small. Remember that the energy of the solid is $E = \hbar\omega q$.

1. Find an expression for the multiplicity of the Einstein solid $\Omega(N, q)$, and *show* that $\Omega(N, q) \simeq \left(\frac{eN}{q}\right)^q$ in this limit.
2. Find the entropy and temperature in this limit. Is the temperature small or large, compared to $\hbar\omega/k$? How would you then call this $q \ll N$ limit?
3. Find the energy as a function of the temperature and determine the heat capacity $c_N = \frac{1}{N}\left(\frac{dE}{dT}\right)_N$ per oscillator. What is the limit of $c_N$ as $T \to 0$?

*30 points*

**II.** *Sharpness of multiplicity function*

Consider two Einstein solids. Let each of them consist of $N$ oscillators, each of frequency $\omega$, so that the quantum of energy is $\hbar\omega$. One of them has $q_1 = \frac{q}{2} + x$ quanta of energy and the other has $q_2 = \frac{q}{2} - x$ quanta. In words, the total number of quanta is $q$, and $x$ is the energy disbalance between the solids.

Consider the thermodynamic limit of large-$N$ and $q_i$, $i = 1, 2$, but $q_i \ll N$, as in the previous problem. The multiplicity of each solid is now *given* (making this problem independent) to be $\Omega(N, q_i) \simeq \left(\frac{eN}{q_i}\right)^{q_i}$.

Assuming now that the two solids are in thermal contact, find the thermodynamic equilibrium probability distribution for $x$, the energy disbalance between the solids. Show that, in the limit $x \to \infty$ but $x \ll q$, the probability distribution is Gaussian. What is the width of this Gaussian? Discuss the sharpness of the multiplicity function.

*30 points*

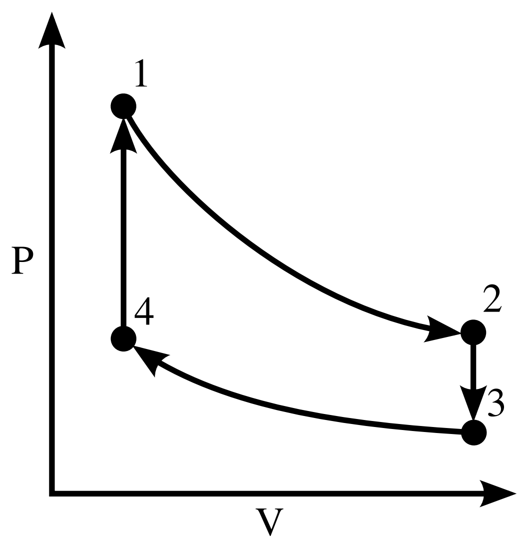
Turn over, please →

**III.** *A cyclic process: the "Stirling cycle"*

A monatomic ideal gas made of $N$ atoms undergoes the quasistatic cyclic process shown on the figure below in $p$-$V$ coordinates. Let the volumes of points 1 and 4 be $V_1$ and of points 2 and 3—$V_2$. The 4-1 and 2-3 parts of the cycle are, therefore, isochoric. The 1-2 and 3-4 parts are isothermal expansion and contraction, respectively, at temperatures $T$ and $T'$ ($T' < T$), respectively.

Compute the work done by the gas (this could be negative) during every part of the cycle. Compute also the heat absorbed or the heat given away during every part of the cycle. Find the ratio of the total work done by the gas during the entire cycle to the total heat absorbed.

This ratio could be called the 'efficiency' of the cycle, as it measures the 'benefit' (total work done by gas) over the 'cost' (heat absorbed); we shall learn that later in class, or you may have already learned it in other courses. What range of values can this ratio (the efficiency) take?




*30 points*

*Total number of points:* $30 + 30 + 30 = 90$.