Physics HW 2

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1 Lorentz Contraction

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$x' = \gamma x - \gamma\beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$t = \frac{x}{\beta}$$

$$t' = \gamma(t - \beta x)$$

$$t' = \gamma\left(\frac{x}{\beta} - \beta x\right)$$

$$t' = \gamma x\left(\frac{1}{\beta} - \beta\right)$$

$$t' = \gamma x\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma\beta t\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma\beta\left(\frac{x}{\beta}\right)\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma x\left(\frac{1}{\gamma^2\beta}\right)$$

$$t' = \frac{x}{\gamma\beta}$$

2 Time Dilation

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$x' = \gamma x - \gamma\beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$x = \beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$t' = \gamma t - \gamma\beta^2 t$$

$$t' = \gamma t(1-\beta^2)$$

$$t' = \gamma t\frac{1}{\gamma^2}$$

$$t' = \frac{t}{\gamma}$$

3 Mass Conversion Examples

1.

$$
\begin{aligned}
E &= mc^2 \\
\frac{100\text{J}}{\text{s}} \cdot 3.154 \cdot 10^7\text{s} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
3.154 \cdot 10^9\text{J} &= m(9 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
\frac{3.154 \cdot 10^9 \text{ kg} \cdot \text{m}^2}{\text{s}^2} &= m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
m &= \frac{3.154 \cdot 10^9}{9 \cdot 10^{16}} \text{ kg} \\
m &= \frac{3.154}{9 \cdot 10^7} \text{ kg} \\
m &= 3.504 \cdot 10^{-8} \text{ kg}
\end{aligned}
$$

2.

$$
\begin{aligned}
E &= mc^2 \\
1000\frac{\text{J}}{\text{s}} \cdot 310 \cdot 10^{12}\text{hr} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
1000\frac{\text{J}}{\text{s}} \cdot 1.116 \cdot 10^{18}\text{s} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
1.116 \cdot 10^{21} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} &= m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
m &= \frac{1.116 \cdot 10^{21}}{9 \cdot 10^{16}} \text{ kg} \\
m &= \frac{1.138 \cdot 10^5}{9} \text{ kg} \\
m &= 1.264 \cdot 10^4 \text{ kg}
\end{aligned}
$$

3.

$$
\begin{aligned}
2\text{hp} &= 1492\text{W} \\
1\text{lb} &= 4.53592 \cdot 10^{-1}\text{kg} \\
1492\text{W} \cdot t &= 4.53592 \cdot 10^{-1} \text{ kg } ((3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2) \\
1492\text{W} \cdot t &= 4.53592 \cdot 10^{-1} \text{ kg } (9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
1492\frac{\text{J}}{\text{s}} \cdot t &= 4.082328 \cdot 10^{16} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \\
1492\frac{\text{J}}{\text{s}} \cdot t &= 4.082328 \cdot 10^{16}\text{J} \\
t &= \frac{4.082328}{1492} \cdot 10^{16}\text{s} \\
t &= \frac{4.082328}{1492} \cdot 10^{16}\text{s} \\
t &= 2.7361448 \cdot 10^{13}\text{s} \\
t &\approx 867625 \text{ yrs}
\end{aligned}
$$

Humans lose weight via multiple pathways including, but not limited to, metabolism, heat, and sweat.

4.

$$
\begin{aligned}
\text{S.A. of the Earth} &= 5.1 \cdot 10^{14} \\
&\textit{Wikipedia} \\
\text{S.A. of the Earth under Light} &= 2.55 \cdot 10^{14} \\
\text{Power output of the Sun} &= 3.86 \cdot 10^{26}\text{W} \\
&\textit{Australian Space} \\
&\textit{Weather Forecasting} \\
&\textit{Center}
\end{aligned}
$$

$$E = mc^2$$
$$3.86 \cdot 10^{26}\text{W} \cdot 1\text{s} = m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2$$
$$3.86 \cdot 10^{26}\text{J} = m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2})$$
$$m = \frac{3.86 \cdot 10^{26}}{9 \cdot 10^{16}} \text{ kg}$$
$$m = 4.289 \cdot 10^9\text{kg lost per second}$$

$$W = (2.55 \cdot 10^{14})(1.4\text{kW})$$
$$W = 3.57 \cdot 10^{14}\text{kW}$$

$$E = mc^2$$
$$3.57 \cdot 10^{14}\text{kW} \cdot 3.154 \cdot 10^7\text{s} = m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2$$
$$11.25978 \cdot 10^{21}\text{J} = m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2})$$

$$m = \frac{11.25978 \cdot 10^{21}}{9 \cdot 10^{16}} \text{ kg}$$
$$m = 1.251 \cdot 10^5 \text{ kg in one year}$$

5.

$$E_i = 2\left[\frac{1}{2}mv^2\right]$$
$$E_i = (10^8)(44.704)^2$$
$$E_i = (10^8)(1998.448)$$
$$E_i = 1.998 \cdot 10^{11}\text{J}$$

$$E = mc^2$$
$$1.998 \cdot 10^{11}\text{J} = m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2})$$
$$m = \frac{1.998 \cdot 10^{11}}{9 \cdot 10^{16}} \text{ kg}$$
$$m = \frac{1.998}{9 \cdot 10^5} \text{ kg}$$
$$m = 0.222 \cdot 10^{-5} \text{ kg}$$

4 Relativistic Proton Collisions

$$E_i = \gamma m + m$$
$$E_f = 2\gamma_f m$$
$$P_i = \gamma m \beta$$
$$P_f = 2\gamma_f m \beta_f \cos\theta$$

$$\gamma m \beta = 2\gamma_f m \beta_f \cos\theta$$
$$\gamma \beta = 2\gamma_f \beta_f \cos\theta$$

$$\gamma m + m = 2\gamma_f m$$
$$\frac{\gamma + 1}{2} = \gamma_f$$

$$\sqrt{\gamma^2 - 1} = 2\sqrt{\gamma_f^2 - 1}\cos\theta$$
$$\sqrt{\gamma^2 - 1} = 2\sqrt{\left(\frac{\gamma+1}{2}\right)^2 - 1}\cos\theta$$
$$\cos\theta = \frac{\sqrt{\gamma^2 - 1}}{2\sqrt{\left(\frac{\gamma+1}{2}\right)^2 - 1}}$$
$$\cos\theta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2 + 2\gamma - 3}}$$
$$\cos\theta = \sqrt{\frac{(\gamma - 1)(\gamma + 1)}{(\gamma - 1)(\gamma + 3)}}$$
$$\cos\theta = \sqrt{\frac{\gamma + 1}{\gamma + 3}}$$
$$\theta = \arccos\sqrt{\frac{\gamma + 1}{\gamma + 3}}$$

$$\begin{aligned}
\cos^2 \frac{50\pi - 1}{200} &= \frac{\gamma + 1}{\gamma + 3} \\
(\gamma + 3)\left(\cos^2 \frac{50\pi - 1}{200}\right) &= \gamma + 1 \\
\gamma\left(\cos^2 \frac{50\pi - 1}{200}\right) - \gamma &= 1 - 3\left(\cos^2 \frac{50\pi - 1}{200}\right) \\
\gamma\left[\left(\cos^2 \frac{50\pi - 1}{200}\right) - 1\right] &= 1 - 3\left(\cos^2 \frac{50\pi - 1}{200}\right) \\
\gamma &= \frac{1 - 3\left(\cos^2 \frac{50\pi-1}{200}\right)}{\left(\cos^2 \frac{50\pi-1}{200}\right) - 1} \\
\gamma &= 1.0404 \\
\frac{1}{\sqrt{1 - \beta^2}} &= 1.0404 \\
\frac{1}{1 - \beta^2} &= 1.0824 \\
1 &= 1.0824 - 1.0824\beta^2 \\
0.0824 &= 1.0824\beta^2 \\
\beta &= \sqrt{\frac{0.0824}{1.0824}} \\
\beta &= 0.276 \\
\frac{v}{c} &= 0.276 \\
v &= 0.276c \\
v &= 8.28 \cdot 10^7 \frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}$$

$$\begin{aligned}
\theta &= \arccos \sqrt{\frac{\gamma + 1}{\gamma + 3}} \\
\theta &= \arccos \sqrt{\frac{(1.0404) + 1}{(1.0404) + 3}} \\
\theta &= \arccos \sqrt{0.505} \\
\theta &= \arccos 0.711 \\
\theta &= 0.78 \text{ rad}
\end{aligned}$$