MATHEMATICS ELECTIVE: MATHEMATICAL FINANCE

Pennsylvania Governor's School for the Sciences 2024

Homework #4

Due: At class time on Thursday, July 11.

Instructor:
David Handron, handron@andrew.cmu.edu

1. An annuity making four payments of $A = \$25$ per year for one year can be bought or sold today for $\mathcal{P}_0^A = \$97.67$.

   A zero coupon bond with face value \$1000 and maturity $T = 1$ year can be bought or sold today for $\mathcal{P}_0^Z = \$957.00$.

   (a) Determine the arbitrage-free price of an annuity making four payments of $A = \$125$ per year for one year

   (b) Determine the arbitrage-free price of a zero coupon bond with face value \$10,000 and maturity $T = 1$ year.

   (c) Determine the arbitrage-free price $\mathcal{P}_0^{CB}$ of a coupon bond with face value $F = \$10{,}000$ and maturity $T = 1$ year, making four coupon payments per year at the coupon rate $q = .05$.

2. An annuity makes payments of \$500 four times per year for one year. At time zero the interest rates at time $t = 0$ for deposits or loans of maturity $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$, are

$$\begin{aligned} r(\frac{1}{4}) &= .03 = 3\% \\ r(\frac{1}{2}) &= .04 = 4\% \\ r(\frac{3}{4}) &= .045 = 4.5\% \\ r(1) &= .0475 = 4.75\% \end{aligned}$$

   Find the arbitrage-free price of this annuity.