MATHEMATICS ELECTIVE: MATHEMATICAL FINANCE

Pennsylvania Governor's School for the Sciences 2024

Homework #5

Due: At class time on Tuesday, July 16.

Instructor:
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1. (a) A zero coupon bond has face value $F = 1000$ and maturity $T = 1$ year. At time $t = 0$ the interest rate is $r(0, 1) = .03 = 3\%$. Find the arbitrage-free price $\mathcal{P}_0$ of the bond at time 0.

   One month later, the interest rate is the same, $r(\frac{1}{12}, 1) = .03 = 3\%$. Find the arbitrage-free price $\mathcal{P}_{\frac{1}{12}}$ of the bond a time $\frac{1}{12}$.

   Has the price increased or decreased?

   (b) Another zero coupon bond has face value $F = 5000$ and maturity $T = 5$ years. At time $t = 0$ the interest rate is $r(0, 5) = .04 = 4\%$. Find the arbitrage-free price of this bond at time 0.

   One month later, the interest rate has changed to $r(\frac{1}{12}, 5) = .041 = 4.1\%$. Find the arbitrage-free price of this bond a time $\frac{1}{12}$.

   Has the price increased or decreased?

2. Suppose that $r(0, \frac{1}{2}) = .02 = 2\%$ and that $r(0, 2) = .045 = 4.5\%$. Find the forward interest rate $f(\frac{1}{2}, 2)$ for deposits and loans between $t = \frac{1}{2}$ and $T = 2$.