

Physics HW 2

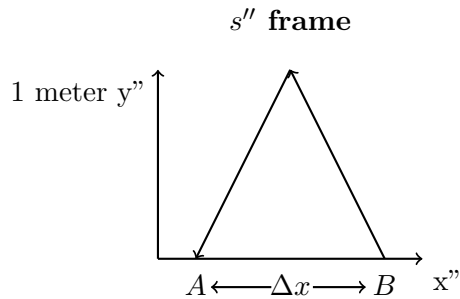
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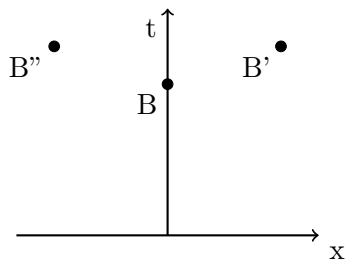
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1 The Invariant



$$\begin{aligned}
 & \Delta t^2 - (-\Delta x)^2 \\
 &= (2\sqrt{1 + \frac{\Delta x^2}{4}})^2 - (-\Delta x)^2 \\
 &= (4 - \Delta x)^2 - (\Delta x)^2 \\
 &= 4
 \end{aligned}$$



2 Michelson-Morley experiment

In the stationary frame speed, time, and length are constant

$$L_x = L_y$$

In the frame with motion, though

$$\begin{aligned}
 t_y &= 2\sqrt{l^2 + \frac{\Delta x^2}{4}} \\
 t_y &= 2\sqrt{l^2 + \frac{(\beta t_y)^2}{4}} \\
 t_y^2 &= 4(l^2 + \frac{(\beta t_y)^2}{4}) \\
 t_y^2 &= 4l^2 + \beta^2 t_y^2 \\
 t_y^2 - \beta^2 t_y^2 &= 4l^2 \\
 t_y^2(1 - \beta^2) &= 4l^2 \\
 t_y &= \frac{2l}{\sqrt{1 - \beta^2}}
 \end{aligned}$$

$$t_x = \frac{l'}{1 + \beta} + \frac{l'}{1 - \beta}$$

$$t_x = \frac{l'[(1 - \beta) + (1 + \beta)]}{1 - \beta^2}$$

$$t_x = \frac{2l'}{1 - \beta^2}$$

$$l = \gamma l'$$

$$l' = \frac{l}{\gamma}$$

$$t_x = \frac{2l}{\gamma(1 - \beta^2)}$$

$$t_x = \frac{2l\sqrt{1 - \beta^2}}{1 - \beta^2}$$

$$t_x = \frac{2l}{\sqrt{1 - \beta^2}}$$

$$t_x = t_y$$

3 Moving Balls

B because the length contraction will occur on the axis of movement.

4 Provisions in space

$$\beta = \frac{1}{2}$$

$$\gamma = \frac{1}{\sqrt{1 - 0.5^2}}$$

$$\gamma = \frac{1}{\sqrt{0.75}}$$

$$t = 20 \text{ years}$$

$$20 = \gamma t$$

$$t = 20\sqrt{0.75}$$

$$t = 10\sqrt{3} \text{ years}$$

$$t = 6322 \text{ days}$$

6322 meals.

5 Velocity Addition

$$\beta_x = \frac{\beta'_x + \beta}{1 + \beta\beta'}$$

$$\beta'_x = \frac{\beta''_x + \beta'}{1 + \beta'\beta''_x}$$

$$\beta'_x = \frac{0.9 + 0.9}{1 + 0.9(0.9)}$$

$$\beta'_x = 0.994475$$

$$\beta_x = \frac{(0.994475) + 0.9}{1 + 0.9(0.994475)}$$

$$\beta_x = 0.999708c$$

6 Break down of Galilean 8 Euclidean “slope” transformations

$$\begin{aligned}
 t' &= \gamma t \\
 0.99t &= t\sqrt{1 - \beta^2} \\
 0.99 &= \sqrt{1 - \beta^2} \\
 0.99^2 &= 1 - \beta^2 \\
 \beta &= \sqrt{1 - 0.99^2} \\
 \beta &= 0.141
 \end{aligned}$$

$$0.141c$$

7 Temporal order

All time-like events are in theory physically possible. In this case all time like events can be mapped to each other by Lorentz transformations and preserve the order where an event A that causes B will occur before B. Similarly light-like scenarios, with sufficient distance, will be able to preserve order of events. The issue comes when you reach space-like intervals where you exceed the speed of light. This is obviously impossible to physically happen, but also breaks down causation because if A is to occur, it will have caused B before it happens.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} G & -BG \\ BG & G \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} Gx' - BGy' \\ BGx' + Gy' \end{pmatrix}$$

$$x = Gx' - BGy'$$

$$x = Gx' - Gy' \tan \theta$$

$$G \equiv \frac{1}{\sqrt{1 + B^2}}$$

$$G = \frac{1}{\sqrt{1 + \tan^2 \theta}}$$

$$G = \cos \theta$$

$$x = x' \cos \theta - y' \cos \theta \tan \theta$$

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

$$\begin{aligned}
 &\sqrt{x^2 + y^2} \\
 &= \sqrt{(x' \cos \theta - y' \sin \theta)^2 + (x' \sin \theta + y' \cos \theta)^2} \\
 &= \sqrt{x'^2 \cos^2 \theta + y'^2 \sin^2 \theta + x'^2 \sin^2 \theta + y'^2 \cos^2 \theta} \\
 &= \sqrt{x'^2 (\cos^2 \theta + \sin^2 \theta) + y'^2 (\sin^2 \theta + \cos^2 \theta)} \\
 &= \sqrt{x'^2 + y'^2}
 \end{aligned}$$