Physics HW 1

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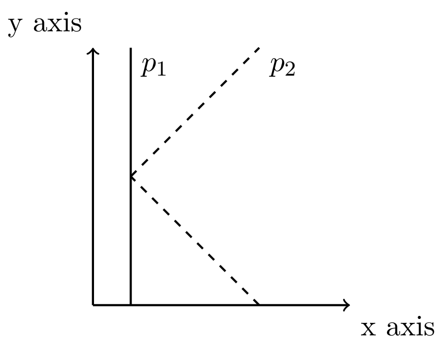
1 1D Collisions

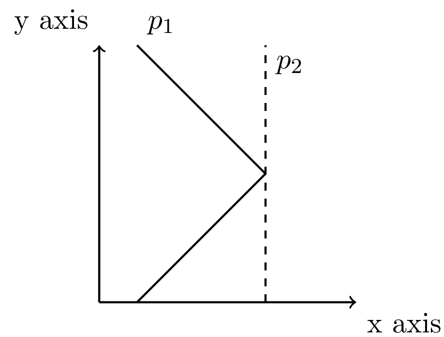
(a)

$$
\begin{aligned}
P &= mv \\
mv_i &= mv_f \\
v_i &= v_f
\end{aligned}
$$

(b) Frame p_1



(c) Frame p_2



(d)

Frame p_1

$$
\begin{aligned}
P_{1s} &= m(0) = 0 \\
P_{2s} &= m(-v) = -mv \\
E_{1s} &= \frac{1}{2}m(0)^2 = 0 \\
E_{2s} &= \frac{1}{2}m(-v)^2 = \frac{1}{2}mv^2
\end{aligned}
$$

Frame p_2

$$
\begin{aligned}
P_{1s} &= m(v) = mv \\
P_{2s} &= m(0) = 0 \\
E_{1s} &= \frac{1}{2}m(v)^2 = \frac{1}{2}mv^2 \\
E_{2s} &= \frac{1}{2}m(0)^2 = 0
\end{aligned}
$$

(e)

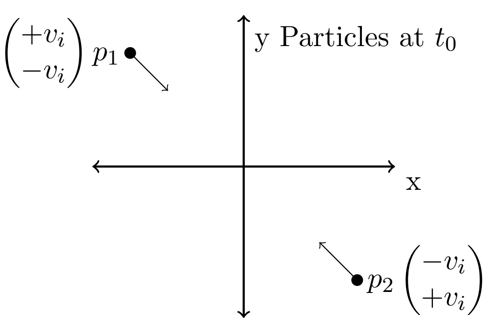
Frame p_1

$$P_{1f} = m(0) = 0$$
$$P_{2f} = m(v) = mv$$
$$E_{1f} = \frac{1}{2}m(0)^2 = 0$$
$$E_{2f} = \frac{1}{2}m(v)^2 = \frac{1}{2}mv^2$$

Frame p_2

$$P_{1f} = m(-v) = -mv$$
$$P_{2f} = m(0) = 0$$
$$E_{1f} = \frac{1}{2}m(-v)^2 = \frac{1}{2}mv^2$$
$$E_{2f} = \frac{1}{2}m(0)^2 = 0$$

2 2D Collisions

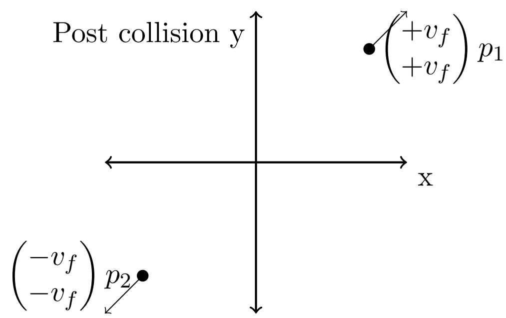


(a)

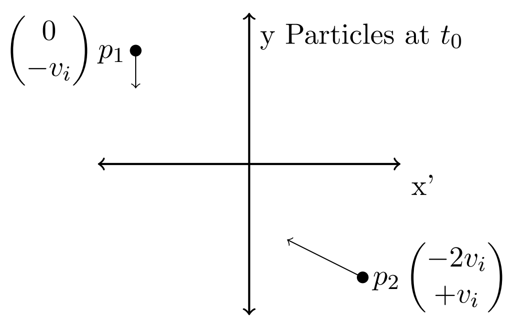
(b)

$$E = mv^2$$
$$\sum E_i = 2[\frac{1}{2}m(\|\vec{v_i}\|)]$$
$$\sum E_f = 2[\frac{1}{2}m(\|\vec{v_f}\|)]$$
$$\sum E_i = \sum E_f$$
$$\|\vec{v_f}\| = \|\vec{v_i}\|$$

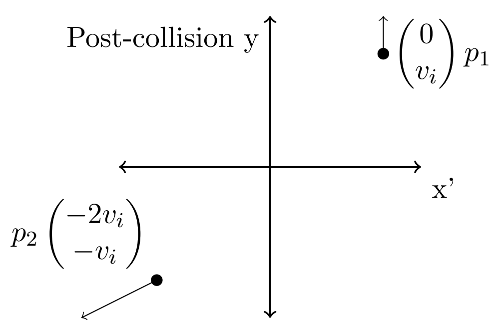
$$\overrightarrow{P_{1i}} + \overrightarrow{P_{2i}} = 0$$
$$\therefore \overrightarrow{P_{1f}} + \overrightarrow{P_{2f}} = 0$$
$$-\begin{pmatrix} +v_f \\ +v_f \end{pmatrix} = \overrightarrow{P_{2f}}$$
$$\overrightarrow{P_{2f}} = \begin{pmatrix} -v_f \\ -v_f \end{pmatrix}$$



(c)



(d)



(e)

$$v'_{1ix} = v_{1ix} - v_{frame}$$
$$v'_{1ix} = v_{ix} - v_i$$
$$v'_{1ix} = 0$$
$$\overrightarrow{v_{1i}} = \begin{pmatrix} 0 \\ -v_i \end{pmatrix}$$

$$v'_{2ix} = -v_{2ix} - v_{frame}$$
$$v'_{2ix} = -v_{ix} - v_i$$
$$v'_{2ix} = -2v_i$$
$$\overrightarrow{v_{2i}} = \begin{pmatrix} -2v_i \\ v_i \end{pmatrix}$$

$$v'_{1fx} = v_{1fx} - v_{frame}$$
$$v'_{1fx} = v_{ix} - v_i$$
$$v'_{1fx} = 0$$
$$\overrightarrow{v_{1f}} = \begin{pmatrix} 0 \\ v_i \end{pmatrix}$$

$$v'_{2fx} = -v_{2ix} - v_{frame}$$
$$v'_{2fx} = -v_{ix} - v_i$$
$$v'_{2fx} = -2v_i$$
$$\overrightarrow{v_{2f}} = \begin{pmatrix} -2v_i \\ -v_i \end{pmatrix}$$

(f)

Frame 1

$$P_{1xi} = +mv_i$$
$$P_{1yi} = -mv_i$$
$$P_{2xi} = -mv_i$$
$$P_{2yi} = +mv_i$$
$$E_{1i} = \frac{1}{2}m||\overrightarrow{v_i}||^2$$
$$E_{1i} = \frac{1}{2}m(\sqrt{v_i^2 + (-v_i)^2})^2$$
$$E_{1i} = mv_i^2$$
$$E_{2i} = \frac{1}{2}m||\overrightarrow{v_i}||^2$$
$$E_{2i} = mv_i^2$$

$$P_{1xf} = +mv_f$$
$$P_{1yf} = +mv_f$$
$$P_{2xf} = -mv_f$$
$$P_{2yf} = -mv_f$$
$$E_{1f} = \frac{1}{2}m||\overrightarrow{v_f}||^2$$
$$E_{1f} = \frac{1}{2}m(\sqrt{v_f^2 + v_f^2})^2$$
$$E_{1f} = mv_f^2$$
$$E_{2f} = \frac{1}{2}m||\overrightarrow{v_f}||^2$$
$$E_{2f} = mv_f^2$$

Moving Frame

$$P'_{1xi} = m(0) = 0$$
$$P'_{1yi} = -mv_i$$
$$P'_{2xi} = -2mv_i$$
$$P'_{2yi} = +mv_i$$
$$E'_{1i} = \frac{1}{2}m||\overrightarrow{v'_{1i}}||^2$$
$$E'_{1i} = \frac{1}{2}m(\sqrt{0^2 + (-v_i)^2})^2$$
$$E'_{1i} = \frac{1}{2}mv_i^2$$
$$E'_{2i} = \frac{1}{2}m||\vec{v_i}||^2$$
$$E'_{1i} = \frac{1}{2}m(\sqrt{(-2v_i)^2 + (v_i)^2})^2$$
$$E'_{1i} = \frac{5}{2}mv_i^2$$

$$P'_{1xf} = m(0) = 0$$
$$P'_{1yf} = +mv_f$$
$$P'_{2xf} = -2mv_f$$
$$P'_{2yf} = -mv_f$$
$$E'_{1f} = \frac{1}{2}m||\vec{v_f}||^2$$
$$E'_{1f} = \frac{1}{2}m(\sqrt{0^2 + v_i^2})^2$$
$$E'_{1f} = \frac{1}{2}mv_i^2$$
$$E'_{2f} = \frac{1}{2}m||\vec{v_f}||^2$$
$$E'_{1f} = \frac{1}{2}m(\sqrt{(-2v_i)^2 + (-v_i)^2})^2$$
$$E'_{2f} = \frac{5}{2}mv_i^2$$

3 Matrix Multiplication

(a)

$$A \times C = \begin{pmatrix} a_{11}c_1 + a_{12}c_2 \\ a_{21}c_1 + a_{22}c_2 \end{pmatrix}$$

(b)

$$A \times B = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

(c)

$$n \times A = \begin{pmatrix} na_{11} & na_{11} \\ na_{21} & na_{21} \end{pmatrix}$$

4 Proton Collisions in Newtonian Mechanics

$$0 = mv_{f1}^2 \sin\theta + mv_{f2}^2 \sin -\theta$$
$$v_{f1} = v_{f2}$$

$$E_i = \frac{1}{2}mv_i^2$$
$$E_f = 2[\frac{1}{2}mv_f^2$$
$$E_f = mv_f^2$$
$$E_f = E_i$$
$$\frac{1}{2}mv_i^2 = mv_f^2$$
$$v_f = \frac{v_i}{\sqrt{2}}$$

$$
\begin{aligned}
P_{xi} &= P_{xf} \\
mv_i &= 2mv_f \cos\theta \\
v_i &= 2v_f \cos\theta \\
v_i &= 2m(\frac{v_i}{\sqrt{2}}) \cos\theta \\
\frac{\sqrt{2}v_i}{2v_i} &= \cos\theta \\
\theta &= \arccos\frac{\sqrt{2}}{2} \\
\theta &= \frac{\pi}{4} = 45^\circ
\end{aligned}
$$

5 (Challenge Problem) General Linear Coordinate Transformations

(a)

$$
\begin{aligned}
\begin{pmatrix} x \\ t \end{pmatrix} &= \begin{pmatrix} a & b \\ e & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
\text{Given } x &= vt \text{ when } x' = 0 \\
\begin{pmatrix} vt \\ t \end{pmatrix} &= \begin{pmatrix} a & b \\ e & f \end{pmatrix} \begin{pmatrix} 0 \\ t' \end{pmatrix} \\
\begin{pmatrix} vt \\ t \end{pmatrix} &= \begin{pmatrix} bt' \\ ft' \end{pmatrix} \\
t &= ft' \\
vt &= bt' \\
vft' &= bt' \\
b &= vf
\end{aligned}
$$

(b)

$$
\begin{aligned}
\begin{pmatrix} x \\ t \end{pmatrix} &= \begin{pmatrix} a & vf \\ e & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
\text{Given } x' &= vt' \text{ when } x = 0 \\
\begin{pmatrix} 0 \\ t \end{pmatrix} &= \begin{pmatrix} a & vf \\ e & f \end{pmatrix} \begin{pmatrix} -vt' \\ t' \end{pmatrix} \\
\begin{pmatrix} 0 \\ t \end{pmatrix} &= \begin{pmatrix} -avt' + vft' \\ -evt' + ft' \end{pmatrix} \\
avt' &= vft' \\
a &= f
\end{aligned}
$$

(c)

$$
\begin{aligned}
\begin{pmatrix} x \\ t \end{pmatrix} &= \begin{pmatrix} f & vf \\ e & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
T(v_1)T(v_2) &= T(v_f) \\
&= \begin{pmatrix} f_1 & v_1 f_1 \\ e_1 & f_1 \end{pmatrix} \begin{pmatrix} f_2 & v_2 f_2 \\ e_2 & f_2 \end{pmatrix} \\
&= \begin{pmatrix} f_1 f_2 + f_1 v_1 e_2 & f_1 v_2 f_2 + f_1 v_1 f_2 \\ e_1 f_2 + f_1 e_2 & e_1 v_2 f_2 + f_1 f_2 \end{pmatrix} \\
f_1 f_2 + f_1 v_1 e_2 &= e_1 v_2 f_2 + f_1 f_2 \\
\frac{f_1 v_1}{e_1} &= \frac{f_2 v_2}{e_2} \\
g &= \frac{vf}{e} \\
e &= \frac{vf}{g}
\end{aligned}
$$

(d)

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} f & vf \\ \frac{vf}{g} & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

$$\begin{aligned}
T(v)T(-v) &= T(0) \\
&= \begin{pmatrix} f & vf \\ \frac{vf}{g} & f \end{pmatrix} \begin{pmatrix} f & -vf \\ \frac{-vf}{g} & f \end{pmatrix} \\
&= \begin{pmatrix} f^2 - \frac{v^2 f^2}{g} & -vf^2 + vf^2 \\ \frac{vf^2}{g} - \frac{vf^2}{g} & f^2 - \frac{v^2 f^2}{g} \end{pmatrix} \\
&= \begin{pmatrix} f^2 - \frac{v^2 f^2}{g} & 0 \\ 0 & f^2 - \frac{v^2 f^2}{g} \end{pmatrix} \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \begin{pmatrix} f^2 - \frac{v^2 f^2}{g} & 0 \\ 0 & f^2 - \frac{v^2 f^2}{g} \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \begin{pmatrix} x'(f^2 - \frac{v^2 f^2}{g}) \\ t'(f^2 - \frac{v^2 f^2}{g}) \end{pmatrix} \\
x &= x'(f^2 - \frac{v^2 f^2}{g}) \\
1 &= f^2 - \frac{v^2 f^2}{g} \\
f &= \frac{1}{\sqrt{1 - \frac{v^2}{g}}}
\end{aligned}$$

Take $x' = v_* t'$

$$\begin{aligned}
\begin{pmatrix} x \\ t \end{pmatrix} &= \frac{1}{\sqrt{1 - \frac{v_*^2}{v_*^2}}} \begin{pmatrix} 1 & v_* \\ -\frac{v_*}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
t &= \frac{1}{\sqrt{1 - \frac{v_*^2}{v_*^2}}} (-\frac{x' v_*}{v_*^2} + t') \\
t' &= \frac{x'}{v_*} \\
x' &= v_* t'
\end{aligned}$$

(e)

$$\begin{aligned}
\begin{pmatrix} x \\ t \end{pmatrix} &= \frac{1}{\sqrt{1 - \frac{v^2}{g}}} \begin{pmatrix} 1 & v \\ \frac{v}{g} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix} \\
g &= v_*^2 \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \frac{1}{\sqrt{1 - \frac{v^2}{v_*^2}}} \begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}
\end{aligned}$$

(f)

$$
\begin{aligned}
{v_*}^2t^2 - x^2 &= v_*^2t'^2 - x'^2 \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \frac{1}{\sqrt{1-\frac{v^2}{v_*^2}}}\begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix} \\
\text{Let } \gamma &= \frac{1}{\sqrt{1-\frac{v^2}{v_*^2}}} \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \gamma \begin{pmatrix} 1 & v \\ \frac{v}{v_*^2} & 1 \end{pmatrix}\begin{pmatrix} x' \\ t' \end{pmatrix} \\
\begin{pmatrix} x \\ t \end{pmatrix} &= \gamma \begin{pmatrix} x' + vt' \\ x'\frac{v}{v_*^2} + t' \end{pmatrix} \\
x &= \gamma(x' + vt') \\
t &= \gamma(x'\frac{v}{v_*^2} + t') \\
v_*^2[\gamma(x'(\frac{v}{v_*^2}) + t')]^2 &- [\gamma(x' + vt')]^2 \\
&= \gamma^2[[v_*^2(x'(\frac{v}{v_*^2}) + t')^2] - (x' + vt')^2] \\
&= \gamma^2[(\frac{x'^2v^2}{v_*^2} + t'^2v_*^2) - x'^2 - v^2t'^2] \\
&= \frac{1}{1-\frac{v^2}{v_*^2}}[t'^2(v_*^2 - v^2) - x'^2(1 - \frac{v^2}{v_*^2})] \\
&= \frac{t'^2(v_*^2 - v^2)}{1-\frac{v^2}{v_*^2}} - x'^2 \\
&= v_*^2t'^2 - x'^2
\end{aligned}
$$

(g) $v_* = \infty$

6 How long does it take light to travel a foot?

$$
\begin{aligned}
1\text{ft} &= ct \\
t &= \frac{1\text{ft}}{c} \\
c &= 3 \cdot 10^8 \text{ from BYJU's} \\
t &= 0.3048\text{m} \cdot \frac{1\text{s}}{3 \cdot 10^8\text{m}} \\
t &= 1.016 \cdot 10^{-9}\text{s}
\end{aligned}
$$

7 Two Runners

Runner B

$$t_B = \frac{L}{v}$$

Runner A

$$
\begin{aligned}
t_A &= \frac{\frac{L}{2}}{2v} + \frac{\frac{L}{2}}{\frac{v}{2}} \\
t_A &= \frac{L}{4v} + \frac{L}{v}
\end{aligned}
$$

$$
\begin{aligned}
\frac{L}{4v} + \frac{L}{v} &> \frac{L}{v} \\
t_A &> t_B
\end{aligned}
$$

Runner B wins