Physics HW 3

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1 Lorentz Contraction

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$x' = \gamma x - \gamma\beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$t = \frac{x}{\beta}$$

$$t' = \gamma(t - \beta x)$$

$$t' = \gamma\left(\frac{x}{\beta} - \beta x\right)$$

$$t' = \gamma x\left(\frac{1}{\beta} - \beta\right)$$

$$t' = \gamma x\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma\beta t\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma\beta\left(\frac{x}{\beta}\right)\left(\frac{1-\beta^2}{\beta}\right)$$

$$t' = \gamma x\left(\frac{1}{\gamma^2\beta}\right)$$

$$t' = \frac{x}{\gamma\beta}$$

2 Time Dilation

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

$$x' = \gamma x - \gamma\beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$x = \beta t$$

$$t' = \gamma t - \gamma\beta x$$

$$t' = \gamma t - \gamma\beta^2 t$$

$$t' = \gamma t(1 - \beta^2)$$

$$t' = \gamma t\frac{1}{\gamma^2}$$

$$t' = \frac{t}{\gamma}$$

3 Mass Conversion Examples

1.

$$
\begin{aligned}
E &= mc^2 \\
\frac{100\text{J}}{\text{s}} \cdot 3.154 \cdot 10^7\text{s} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
3.154 \cdot 10^9\text{J} &= m(9 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
\frac{3.154 \cdot 10^9 \text{ kg} \cdot \text{m}^2}{\text{s}^2} &= m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
m &= \frac{3.154 \cdot 10^9}{9 \cdot 10^{16}} \text{ kg} \\
m &= \frac{3.154}{9 \cdot 10^7} \text{ kg} \\
m &= 3.504 \cdot 10^{-8} \text{ kg}
\end{aligned}
$$

2.

$$
\begin{aligned}
E &= mc^2 \\
1000\frac{\text{J}}{\text{s}} \cdot 310 \cdot 10^{12}\text{hr} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
1000\frac{\text{J}}{\text{s}} \cdot 1.116 \cdot 10^{18}\text{s} &= m(3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2 \\
1.116 \cdot 10^{21} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} &= m(9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
m &= \frac{1.116 \cdot 10^{21}}{9 \cdot 10^{16}} \text{ kg} \\
m &= \frac{1.138 \cdot 10^5}{9} \text{ kg} \\
m &= 1.264 \cdot 10^4 \text{ kg}
\end{aligned}
$$

3.

$$
\begin{aligned}
2\text{hp} &= 1492\text{W} \\
1\text{lb} &= 4.53592 \cdot 10^{-1}\text{kg} \\
1492\text{W} \cdot t &= 4.53592 \cdot 10^{-1} \text{ kg } ((3 \cdot 10^8 \frac{\text{m}}{\text{s}})^2) \\
1492\text{W} \cdot t &= 4.53592 \cdot 10^{-1} \text{ kg } (9 \cdot 10^{16} \frac{\text{m}^2}{\text{s}^2}) \\
1492\frac{\text{J}}{\text{s}} \cdot t &= 4.082328 \cdot 10^{16} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \\
1492\frac{\text{J}}{\text{s}} \cdot t &= 4.082328 \cdot 10^{16}\text{J} \\
t &= \frac{4.082328}{1492} \cdot 10^{16}\text{s} \\
t &= \frac{4.082328}{1492} \cdot 10^{16}\text{s} \\
t &= 2.7361448 \cdot 10^{13}\text{s} \\
t &\approx 867625 \text{ yrs}
\end{aligned}
$$

Humans lose weight via multiple pathways including, but not limited to, metabolism, heat, and sweat.

4.

$$
\begin{aligned}
\text{S.A. of the Earth} &= 5.1 \cdot 10^{14} \\
&\textit{Wikipedia} \\
\text{S.A. of the Earth under Light} &= 2.55 \cdot 10^{14} \\
\text{Power output of the Sun} &= 3.86 \cdot 10^{26}\text{W} \\
&\textit{Australian Space} \\
&\textit{Weather Forecasting} \\
&\textit{Center}
\end{aligned}
$$

$$
\begin{aligned}
E &= mc^2 \\
3.86 \cdot 10^{26}\mathrm{W} \cdot 1\mathrm{s} &= m(3 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}})^2 \\
3.86 \cdot 10^{26}\mathrm{J} &= m(9 \cdot 10^{16} \frac{\mathrm{m}^2}{\mathrm{s}^2}) \\
m &= \frac{3.86 \cdot 10^{26}}{9 \cdot 10^{16}}\ \mathrm{kg} \\
m &= 4.289 \cdot 10^9 \mathrm{kg}\ \text{lost per second}
\end{aligned}
$$

$$
\begin{aligned}
W &= (2.55 \cdot 10^{14})(1.4\mathrm{kW}) \\
W &= 3.57 \cdot 10^{14}\mathrm{kW}
\end{aligned}
$$

$$
\begin{aligned}
E &= mc^2 \\
3.57 \cdot 10^{14}\mathrm{kW} \cdot 3.154 \cdot 10^7\mathrm{s} &= m(3 \cdot 10^8 \frac{\mathrm{m}}{\mathrm{s}})^2 \\
11.25978 \cdot 10^{21}\mathrm{J} &= m(9 \cdot 10^{16} \frac{\mathrm{m}^2}{\mathrm{s}^2})
\end{aligned}
$$

$$
\begin{aligned}
m &= \frac{11.25978 \cdot 10^{21}}{9 \cdot 10^{16}}\ \mathrm{kg} \\
m &= 1.251 \cdot 10^5\ \mathrm{kg}\ \text{in one year}
\end{aligned}
$$

5.

$$
\begin{aligned}
E_i &= 2\left[\frac{1}{2}mv^2\right] \\
E_i &= (10^8)(44.704)^2 \\
E_i &= (10^8)(1998.448) \\
E_i &= 1.998 \cdot 10^{11}\mathrm{J}
\end{aligned}
$$

$$
\begin{aligned}
E &= mc^2 \\
1.998 \cdot 10^{11}\mathrm{J} &= m(9 \cdot 10^{16} \frac{\mathrm{m}^2}{\mathrm{s}^2}) \\
m &= \frac{1.998 \cdot 10^{11}}{9 \cdot 10^{16}}\ \mathrm{kg} \\
m &= \frac{1.998}{9 \cdot 10^5}\ \mathrm{kg} \\
m &= 0.222 \cdot 10^{-5}\ \mathrm{kg}
\end{aligned}
$$

4 Relativistic Proton Collisions

$$
\begin{aligned}
E_i &= \gamma m + m \\
E_f &= 2\gamma_f m \\
P_i &= \gamma m \beta \\
P_f &= 2\gamma_f m \beta_f \cos\theta
\end{aligned}
$$

$$
\begin{aligned}
\gamma m \beta &= 2\gamma_f m \beta_f \cos\theta \\
\gamma\beta &= 2\gamma_f \beta_f \cos\theta
\end{aligned}
$$

$$
\begin{aligned}
\gamma m + m &= 2\gamma_f m \\
\frac{\gamma+1}{2} &= \gamma_f
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{\gamma^2 - 1} &= 2\sqrt{\gamma_f^2 - 1}\cos\theta \\
\sqrt{\gamma^2 - 1} &= 2\sqrt{\left(\frac{\gamma+1}{2}\right)^2 - 1}\cos\theta \\
\cos\theta &= \frac{\sqrt{\gamma^2-1}}{2\sqrt{\left(\frac{\gamma+1}{2}\right)^2 - 1}} \\
\cos\theta &= \sqrt{\frac{\gamma^2-1}{\gamma^2+2\gamma-3}} \\
\cos\theta &= \sqrt{\frac{(\gamma-1)(\gamma+1)}{(\gamma-1)(\gamma+3)}} \\
\cos\theta &= \sqrt{\frac{\gamma+1}{\gamma+3}} \\
\theta &= \arccos\sqrt{\frac{\gamma+1}{\gamma+3}}
\end{aligned}
$$

$$\cos^2 \frac{50\pi - 1}{200} = \frac{\gamma + 1}{\gamma + 3}$$

$$(\gamma + 3)\left(\cos^2 \frac{50\pi - 1}{200}\right) = \gamma + 1$$

$$\gamma\left(\cos^2 \frac{50\pi - 1}{200}\right) - \gamma = 1 - 3\left(\cos^2 \frac{50\pi - 1}{200}\right)$$

$$\gamma\left[\left(\cos^2 \frac{50\pi - 1}{200}\right) - 1\right] = 1 - 3\left(\cos^2 \frac{50\pi - 1}{200}\right)$$

$$\gamma = \frac{1 - 3\left(\cos^2 \frac{50\pi-1}{200}\right)}{\left(\cos^2 \frac{50\pi-1}{200}\right) - 1}$$

$$\gamma = 1.0404$$

$$\frac{1}{\sqrt{1 - \beta^2}} = 1.0404$$

$$\frac{1}{1 - \beta^2} = 1.0824$$

$$1 = 1.0824 - 1.0824\beta^2$$

$$0.0824 = 1.0824\beta^2$$

$$\beta = \sqrt{\frac{0.0824}{1.0824}}$$

$$\beta = 0.276$$

$$\frac{v}{c} = 0.276$$

$$v = 0.276c$$

$$v = 8.28 \cdot 10^7 \frac{\mathrm{m}}{\mathrm{s}}$$

$$\theta = \arccos\sqrt{\frac{\gamma + 1}{\gamma + 3}}$$

$$\theta = \arccos\sqrt{\frac{(1.0404) + 1}{(1.0404) + 3}}$$

$$\theta = \arccos\sqrt{0.505}$$

$$\theta = \arccos 0.711$$

$$\theta = 0.78 \text{ rad}$$