MATHEMATICS ELECTIVE: MATHEMATICAL FINANCE

Pennsylvania Governor's School for the Sciences 2024

Homework #3

Due: At class time on Tuesday, Jully 9.

Instructor:

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- 1. A coupon bond makes periodic "coupon payments" and a final "face value payment." A coupon bond is typically described in terms of
 - \bullet a face value F,
 - \bullet a maturity T,
 - a number of compounding periods m per year, and
 - a coupon rate q.

We'll assume that "today" is t = 0, that the first compounding period starts today, and that the maturity is an integer multiple of $\frac{1}{m}$ of a year.

This coupon bond makes coupon payments

$$C = q \times F \times \frac{1}{m}$$

at times

$$\{\frac{1}{m},\frac{2}{m},\frac{3}{m},\ldots,\frac{mT-1}{m},\frac{mT}{m}=T\}.$$

The bond also pays the *face value* at maturity. This payment of F is made at the same time as the final coupon payment, and is in addition to that payment. So the final payment at time $t = \frac{mT}{m} = T$ is C + F.

Consider a coupon bond with face value F = \$1000 and maturity T = 2 years, making m = 2 coupon payments per year at the coupon rate q = .05 = 5%.

- (a) What are amounts of the payments this bond makes, and at what times are these payments made?
- (b) Assume that the interest rate r for deposits and loans is r = .06 = 6, and that we are compounding monthly. Find the present value of each of these payments. What is the total present value of all the payments.
- (c) Describe how to construct a portfolio of zero coupon bonds that makes the same payments as this coupon bond.

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