Physics HW 2

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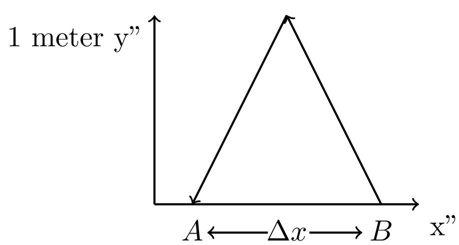
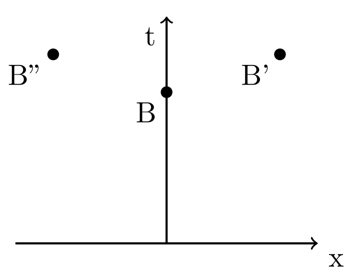
1 The Invariant

s'' frame



$$
\begin{aligned}
&\Delta t^2 - (-\Delta x)^2 \\
&= (2\sqrt{1 + \frac{\Delta x^2}{4}})^2 - (-\Delta x)^2 \\
&= (4 - \Delta x)^2 - (\Delta x)^2 \\
&= 4
\end{aligned}
$$



2 Michelson-Morley experiment

In the stationary frame speed, time, and length are constant

$$L_x = L_y$$

In the frame with motion, though

$$
\begin{aligned}
t_y &= 2\sqrt{l^2 + \frac{\Delta x^2}{4}} \\
t_y &= 2\sqrt{l^2 + \frac{(\beta t_y)^2}{4}} \\
t_y^2 &= 4(l^2 + \frac{(\beta t_y)^2}{4}) \\
t_y^2 &= 4l^2 + \beta^2 t_y^2 \\
t_y^2 - \beta^2 t_y^2 &= 4l^2 \\
t_y^2(1 - \beta^2) &= 4l^2 \\
t_y &= \frac{2l}{\sqrt{1 - \beta^2}}
\end{aligned}
$$

$$t_x = \frac{l'}{1+\beta} + \frac{l'}{1-\beta}$$
$$t_x = \frac{l'[(1-\beta)+(1+\beta)]}{1-\beta^2}$$
$$t_x = \frac{2l'}{1-\beta^2}$$

$$l = \gamma l'$$
$$l' = \frac{l}{\gamma}$$
$$t_x = \frac{2l}{\gamma(1-\beta^2)}$$
$$t_x = \frac{2l\sqrt{1-\beta^2}}{1-\beta^2}$$
$$t_x = \frac{2l}{\sqrt{1-\beta^2}}$$
$$t_x = t_y$$

3 Moving Balls

B because the length contraction will occur on the axis of movement.

4 Provisions in space

$$\beta = \frac{1}{2}$$
$$\gamma = \frac{1}{\sqrt{1-0.5^2}}$$
$$\gamma = \frac{1}{\sqrt{0.75}}$$
$$t = 20 \text{ years}$$
$$20 = \gamma t$$
$$t = 20\sqrt{0.75}$$
$$t = 10\sqrt{3} \text{ years}$$
$$t = 6322 \text{ days}$$

6322 meals.

5 Velocity Addition

$$\beta_x = \frac{\beta_x' + \beta}{1+\beta\beta'}$$
$$\beta_x' = \frac{\beta_x'' + \beta'}{1+\beta'\beta_x''}$$
$$\beta_x' = \frac{0.9+0.9}{1+0.9(0.9)}$$
$$\beta_x' = 0.994475$$
$$\beta_x = \frac{(0.994475)+0.9}{1+0.9(0.994475)}$$
$$\beta_x = 0.999708c$$

6 Break down of Galilean Transformations

$$
\begin{aligned}
t' &= \gamma t \\
0.99t &= t\sqrt{1-\beta^2} \\
0.99 &= \sqrt{1-\beta^2} \\
0.99^2 &= 1-\beta^2 \\
\beta &= \sqrt{1-0.99^2} \\
\beta &= 0.141
\end{aligned}
$$

$$0.141c$$

7 Temporal order

All time-like events are in theory physically possible. In this case all time like events can be mapped to each other by Lorenz transformations and preserve the order where an even A that causes B will occur before B. Similarly light-like scenarios, with sufficient distance, will be able to preserve order of events. The issue comes when you reach space-like intervals where you exceed the speed of light. This is obviously impossible to physically happen, but also breaks down causation because if A is to occur, it will have caused B before it happens.

8 Euclidean "slope" transformations

$$
\begin{aligned}
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} G & -BG \\ BG & G \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \\
\begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} Gx' - BGy' \\ BGx' + Gy' \end{pmatrix} \\
x &= Gx' - BGy' \\
x &= Gx' - Gy' \tan\theta \\
G &\equiv \frac{1}{\sqrt{1+B^2}} \\
G &= \frac{1}{\sqrt{1+\tan^2\theta}} \\
G &= \cos\theta \\
x &= x'\cos\theta - y'\cos\theta\tan\theta \\
x &= x'\cos\theta - y'\sin\theta \\
y &= x'\sin\theta + y'\cos\theta
\end{aligned}
$$

$$
\begin{aligned}
&\sqrt{x^2+y^2} \\
&= \sqrt{(x'\cos\theta - y'\sin\theta)^2 + (x'\sin\theta + y'\cos\theta)^2} \\
&= \sqrt{x'^2\cos^2\theta + y'^2\sin^2\theta + x'^2\sin^2\theta + y'^2\cos^2\theta} \\
&= \sqrt{x'^2(\cos^2\theta + \sin^2\theta) + y'^2(\sin^2\theta + \cos^2\theta)} \\
&= \sqrt{x'^2 + y'^2}
\end{aligned}
$$