Mathematics Elective: Mathematical Finance

Pennsylvania Governor's School for the Sciences 2024

Homework #6
Due: Thursday, July 18.

Instructor:
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1. Suppose that $r(0,1) = .035 = 3.5\%$ and that $r(0,5) = .05 = 5\%$. Explain how to replicate a forward loan of \$1000 between $t = 1$ and $T = 5$ using zero coupon bonds and find the forward interest rate $f(1,5)$ for this forward loan.

2. A security is available that will pay the holder

$$V_1 = \$10{,}000 \times \frac{r(\frac{11}{12}, 1)}{12}$$

at time 1.

   (a) Explain how to replicate this security. That is, explain what securities need to be bought or sold at time 0, and what adjustments or changes need to be made after that to ensure the portfolio's value at time 1 will be V_1.

   (b) Suppose that $r(0, \frac{11}{12}) = .039 = 3.9\%$ and that $r(0,1) = .04 = 4\%$. What is the arbitrage-free value V_0 of this security at time 0?