# PGSS: Math Finance Notes

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## 1 Asset Classes

The class will focus on assets which are indistinguishable from each other in terms of uniqueness (i.e. one share of a stock is identical to another).  indicates a topic we will focus on and? indicates one that may be applicable.

- Real Estate
- ! Cash/Currencies
- ! Stocks → Equities
- Artwork → Collectibles
- Vehicles → Durable goods
- ! Certificates of Deposit (CDs) & Bonds → Fixed Income
- ? Cryptocurrencies
- NFTs
- ? Precious metals → Commodities

## 2 Trading

- The lowest selling price on a share is the **Asking Price**.
- The highest buying price on a share is the **Bid Price**.
- The **Bid-Ask spread** is the difference between bid price and ask price.
- The bid-ask spread tends to be smaller in actively traded, larger stocks.

### Derivative Securities

Securities that derive value from the values of other assets.

### Forward Contract

An agreement on the parameters of an exchange at a specific date before that date.

### Call/Stock Options

Gives a person the right (not an obligation) to sell an assets for a particular price on a particular day.

### Put Options

Gives the right (but not the obligation) to sell an asset.

# 3 Mathematical Models of Financial Markets

## Assumptions

1. Single price to buy or sell (no bid-ask spread)
2. Can buy or sell any amount without moving the price
3. No trading fees
4. No taxes

# 4 Interest

- Borrow some amount (**Principal**)
- Pay back at some later datePay an extra amount (**Interest**)
- Borrow $P$, repay $P + I$ at time $T > 0$ (**maturity**) at $t = 0$

## Why do banks charge interest on loans?

1. To compensate for default risk
2. To compensate for inflation risk
3. 'Time value of money'
4. Compensate for 'opportunity cost'

## Why do banks pay interest on deposits?

Investors demand it

## Why do banks accept deposits?

To borrow at a low interest rate and loan at a higher rate using your money

## 4.1 Interest Payments

### 4.1.1 Simple Interest

- Proportional to the size of the principal
- Proportional to the loaning period length

Given a loan with principal $P$, maturity $T$, and interest rate $r$ where maturity is in years: principal is in dollars (or some other currency), and the interest rate is a rate in units $\frac{1}{\text{year}}$ The formula for the total interest paid would be as follows:

$$I = r[\frac{1}{\text{year}}] \cdot P[\$] \cdot T[\text{years}]$$
$$I = rPT[\$]$$

The total payment would be

$$P + I = P + rPT$$
$$P + I = P(1 + rT)$$

*Note* These principles assume simple interest

Interest is earned uniformly throughout the loan. i.e. $A_0$ borrowed at $t = 0$ grows to $A_t = (1 + rT)A_0$

### 4.1.2 Compound Interest

Interest is *compounded* to the interest during the loan.

**Example**
\$100 borrowed for 1 year at interest rate $r$ compounded quarterly (3 months)

$$A_0 = \$100$$
$$t = \frac{1}{4}$$
$$A_{\frac{1}{4}} = 100 + (\frac{100}{4})r$$
$$A_{\frac{1}{4}} = 100(1 + \frac{r}{4})$$

The above becomes the new principal

$$t = \frac{1}{2}$$
$$A_{\frac{1}{2}} = [100(1 + \frac{r}{4})] + \frac{r}{4}[100(1 + \frac{r}{4})]$$
$$A_{\frac{1}{2}} = (1 + \frac{r}{4})[100(1 + \frac{r}{4})]$$
$$A_{\frac{1}{2}} = 100(1 + \frac{r}{4})^2$$

The above becomes the new principal again

$$t = \frac{3}{4}$$
$$A_{\frac{3}{4}} = (1 + \frac{r}{4})[100(1 + \frac{r}{4})^2]$$
$$A_{\frac{3}{4}} = 100(1 + \frac{r}{4})^3$$

The above becomes the new principal again

$$A_1 = 100(1 + \frac{r}{4})^4$$

More generally

$$A_T = A_0(1 + \frac{r}{m})^{mT}$$

Where $m$ is the number of compounding periods per year

Compound interest allows for exponential growth of interest.

The *quoted* interest rate depends on the compounding convention even if payments are the same.

#### 4.1.3 Negative Rates

- Interests rates can be negative
- But $(1 + \frac{r}{12})^{12T}$ must be positive
- Thus mathematically $r > -12$

## 4.2 The time value of money

If you were to receive \$1000 one year from now, what is the value of that future payment today?

*Note* A deposit of $\frac{1000}{(1+\frac{r}{12})^{12}}$ it will grow to $(\frac{1000}{(1+\frac{r}{12})^{12}})(1 + \frac{r}{12})^{12}$ or \$1000 at $t = 1$

We say $\frac{1000}{(1+\frac{r}{12})^{12}}$ is the **present value** of 1000 to be paid one year from now. Thus you can take a loan for that amount and pay off the loan with interest using the credit from the \$1000 you are to receive in a year (it all cancels).

For the present value of several payments, add up the values of each individual payment

## 4.3 Fixed Income Securities

#### 4.3.1 Zero Coupon Bonds

**Zero Coupon Bonds** make a single payment at a single time. They pay a face value $f$ at the maturity $T > 0$.

If we have a bank with interest rate $r$ the discounted present value of the ZCB is $P_0 = \frac{F}{(1+\frac{r}{12})^12T}$.

If we deposit $A_0 = \frac{F}{(1+\frac{r}{12})^12T}$ at $t = 0$ until $T_1$ then $A_T = F$.

### 4.3.2 Annuities

An **annuity** is a series of same-sized payments made at regular intervals. It will make payments of $\$A$, $m$ times per year, for $T$ years

*Example* An annuity makes payments of \$200 for 2 years. To rewrite it, you could make it a sum of ZCBs, each being a month apart from each other in 24 fixed payments (emulating the annuity).

This makes the present value of an annuity be

$$P_0^A = \sum_{i=1}^{24} \frac{200}{(1+\frac{r}{12})^i}$$

*Example* An annuity makes payments of \$500 quarterly for 1 year. The net present value is $P_0^{A_2} = \frac{500}{(1+\frac{r}{12})^3} + \frac{500}{(1+\frac{r}{12})^6} + \frac{500}{(1+\frac{r}{12})^9} + \frac{500}{(1+\frac{r}{12})^{12}}$ or $\sum_{i=1}^{4} \frac{500}{(1+\frac{r}{12})^{3i}}$

## 4.4 Untitled

### 4.4.1 Coupon Bonds