PGSS: Physics Notes

Etash Jhanji

1 Newtonian Physics

1.1 Position, Velocity, Acceleration, and Forces

Given that $x(t)$ is position as a function of time, $v(t)$ is velocity against time, and $a(t)$ is acceleration as a function of time.

$$v(t) = \frac{\Delta x(t)}{\Delta t} = \frac{dx}{dt}$$
$$a(t) = \frac{\Delta v(t)}{\Delta t} = \frac{dv}{dt}$$
$$a(t) = \frac{d^2x}{dt^2}$$

Any change in velocity, or the rate of change of position, is acceleration and must be due to a force (Newton's 1st)

$$F = ma$$
$$a = \frac{F}{m}$$

Mass (m) is inversely proportional to acceleration and force (F) is directly proportional

Thus more force means greater acceleration and more mass means less acceleration

$$F = G_n \frac{m_1 m_2}{(x_1 - x_2)^2}$$

Where G_n is the gravitation constant, m is the mass of each object, and x is the position of each object on a single axis, the above equation determines the force of attraction between two objects.

1.2 Momentum and Energy

Conserved quantities which make problems simpler to solve.

$$p = mv$$
$$E = \frac{1}{2}mv^2$$
$$\Delta E = 0$$
$$\sum_P E_i^P = \sum_P E_f^P$$

1.3 Example

Q Two particles of equal mass oppose each other exactly and travel at the same velocity that collide, producing two particles of equal masses that also oppose each other. One of the produced particles head $\frac{\pi}{4}$ radians elevated from the x-axis. What is the speed of the two produced particles?

A The original particles have opposite momenta which cancel each other out giving the system a momentum of 0. Thus the produced particles will also have to have the same component momenta to cancel each other out, defined as $p = mv_f$ where v_f is the final velocity of the produced particles. Using energy conservation we can also say energy is conserved from each original particle and that $v_f = v_i$.

2 Coordinates and Coordinate Changes

Frame of reference can be changed between particles to make algebraic calculations easier.

Each frame of reference has different numbers for calculations

Well-defined relations between frames allow conversion

Interrelated frames create an *equivalence class.*

2.1 Moving and and relating frames

- Given two frames s and s' where over time the frame s' moves in the positive x direction over time at a constant velocity

Note Visualize these two frames as a 2D set of axes with position (x) on the x-axis and time (t) on the y-axis. THey start off on top of each other at $t = 0$ before diverging.

- We know that $t = t'$ because time is a constant
- However we do not know the relation between x and x'
- We can determine $x \to x'(t') = x'(t) = x(t) - v_s t$
- Considering x and t as vectors.

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} x \\ t \end{pmatrix} \begin{pmatrix} 1 & -v_s \\ 0 & 1 \end{pmatrix}$$

- Now given frames x, x', and x'' where x' moves relative to x with a velocity of v_1 and x'' moves relative to x' with a velocity of v_2
- To determine the position of x'' we can use the following

$$\begin{aligned} \begin{pmatrix} x'' \\ t'' \end{pmatrix} &= \begin{pmatrix} 1 & -v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -v_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ t \end{pmatrix} \\ &= \begin{pmatrix} 1 & -v_1 - v_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ t \end{pmatrix} \\ &= \begin{pmatrix} 1 & -v_c \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ t \end{pmatrix} \\ &= \begin{pmatrix} x - v_c t \\ t \end{pmatrix} \end{aligned}$$

- You can also write the matrix as a *Galilean Transformation*

$$G(v_s) = \begin{pmatrix} 1 & v_s \\ 0 & 1 \end{pmatrix}$$

$$G(v_2) \cdot G(v_1) = G(v_1 + v_2)$$

- In rotates frames where one frame is turned θ you can use the following equations

$$\begin{aligned}
x' &= \cos\theta x + \sin\theta y \\
y' &= \sin\theta x + \cos\theta y \\
\begin{pmatrix} x' \\ y' \end{pmatrix} &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \\
R(\theta) &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \\
R(\theta_1) + R(\theta_2) &= R(\theta_1 + \theta_2)
\end{aligned}$$

2.2 How the physics changes in inertial frames

$$\begin{aligned}
x' &= v_s t \\
F(|x_1 - x_2|) &= F'(|x_1' - x_2'|) \\
x_1' &= x_1 - v_s t \\
x_2' &= x_2 - v_s t \\
x_1' - x_2' &= x_1 - x_2 \\
m &= m' \\
v_1' &= \frac{dx}{dt} = \frac{\Delta(x_1 - v_s t)}{\Delta t} \\
&= v_1 - v_s \\
a_1' &= \frac{\Delta v_1'}{\Delta t} = \frac{\Delta(v_1 - v_s)}{\Delta t} \\
&= \frac{\Delta v_1}{\Delta t} - \frac{\Delta v_s}{\Delta t} = a \\
a_1' &= a
\end{aligned}$$

Note If $F = ma$ (and momentum and energy) applied in one frame it applies in all others.

2.3 Why it doesn't work

- Electromagnetism
- changing electromagnetic fields can alter forces
- Speed of light: $c' = c - v_s$ breaking the constant velocity of light
- Determinism
- General coordinate transformations
- Experimentally untrue (Michaelson and Morely)

2.4 Linear Experiments

- $$\begin{aligned} x &= ax' + bt' \\ t &= ex' + ft' \end{aligned}$$
- When $x' = 0$, $x = v_s t$
- When $x = 0$, $x = -v_s t$
- OR
$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} x & b \\ e & f \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$
- $$\begin{aligned} T(v_2)T(v_1) &= T(v_c) \\ T(-v)T(v) &= T(0) \end{aligned}$$
- The entire scenario is parametrized by v_*
$$\begin{pmatrix} x \\ t \end{pmatrix} = \frac{1}{\sqrt{1 - (\frac{v_1}{v_*})^2}} \begin{pmatrix} 1 & v_s \\ \frac{v_s}{v_*^2} & 1 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$
In Galilean: $v_* = \infty$
$$x' = v_* t' \rightarrow x = v_* t$$

3 Matrices Crash Course

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a_{11}c_1 + a_{12}c_2 \\ a_{21}c_1 + a_{22}c_2 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

4 Principles of Relativity

- Given a flash of light at a point, the photons leave the point at a constant speed c
- If an observer is moving towards the light, it should appear that it is going faster than c whereas moving away should indicate movement slower than c, however this is not the case

4.1 Units Analogy

- Two surveying contractors are surveying CMU
- One set of surveyors views the campus from a frame on the x axis and the other from a rotated frame on the x' axis
- The differing orientations of the frames cause the surveyors to measure different distances and angles with different coordinates
- Hypothetically company 1 is composed on engineers and 2 is chemists
- Lets say the surveyors use meters for x distance and miles for the y distance.
- The physics student (our hero) determines a conversion factor k to convert between the two frames and measurements.
- $k = \frac{\text{m}}{\text{miles}}$
- Dist: $\sqrt{x^2 + (ky)^2} = \sqrt{x'^2 + (ky')^2}$

4.2 For Relativity

- Instead of measuring t in seconds, we will make $[t] = [ct]$ so that we have units of meters and all coordinates are in meters.

4.3 Example

$$(ct, x, y, x) \to (ct', x', y', z')$$
$$y = y'$$
$$z = z'$$

Only x movement is the simplest case.

4.4 Light clock

- Given a clock which uses the speed of light against a mirror to measure a tick from light omission at time A and reception at time B.
- Put this on a rocket. The entire frame is now moving and the light instead of being up and down now moves to the right simultaneously in an externally observational frame
- In the rocket frame however, the clock is the same and $\Delta x = 0$

-
$$t = t' = 0$$
$$x_A = 0$$
$$t_A = 0$$
In rocket
$$x'_A = 0$$
$$t'_B = 2m$$
$$\Delta x' = 0$$
$$\Delta t' = 2m$$
Lab frame
$$\Delta x > 0$$
$$\Delta t = 2\sqrt{1 + \frac{\Delta x}{x}^2}$$
$$\Delta t > \Delta t'$$

Moving clocks run slow.

Note

$$\Delta t^2 - \Delta x^2 = 4(1 + (\frac{\Delta x}{2})^2) - \Delta x^2$$
$$= 4$$

$$\Delta t'^2 - \Delta x'^2 \equiv (\text{Interval})^2$$
$$\Delta x^2 - \Delta y^2 \equiv (\text{length})^2$$

i.e. The sums form a hyperbola and circle with a radius of length, respectively.

Lorenz Geometry

When the interval $(\Delta t^2 - \Delta x^2)$ is

< 0 it is 'time-like'

$= 0$ it is 'light-like'

> 0 it is 'space-like'

This is a physical property of two events.

Note in Euclidean Geometry a straight path is shorter than a curved path whereas in Lorenz Geometry a curved path is shorter than a straight path.

4.5 Lorenz Transformations

$$\beta = \frac{v}{c}$$
$$\beta = [0, 1]$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\gamma = [1, \infty)$$
$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

4.6 Velocity Transformations

In a Galilean transformation $v_x = v'_x + v_s$ and $v_y = v'_y$.

In a Lorenz transformation, y velocity takes the form $\beta_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\gamma \Delta t'} = \frac{1}{\gamma}\beta'_y$

The X-component

$$
\begin{aligned}
\beta_x &= \frac{\Delta x}{\Delta t} \\
\beta_x &= \frac{\gamma \Delta x' + \beta\gamma\Delta t'}{\beta\gamma\Delta x' + \gamma\Delta t'} \\
\beta_x &= \frac{\Delta x' + \beta\Delta t'}{\beta\Delta x' + \Delta t'} \\
\beta_x &= \frac{\frac{\Delta x'}{\Delta t'} + \beta}{\beta\frac{\Delta x'}{\Delta t'} + 1} \\
\beta_x &= \frac{\beta_x' + \beta}{1 + \beta\beta_x}
\end{aligned}
$$