Homework Set #1

Due Date: Wednesday July 3rd

1) 1D Collisions

At time t_0 a particle (p_1) of mass m is traveling in the $+x$-direction with velocity v_i toward another particle (p_2) of mass m which is initially at rest. After the collision particle p_1 has stopped moving, and particle p_2 is moving with velocity v_f.

(a) Solve for v_f.

(a) Draw a "space-time" diagram of the collision from p_1s reference frame.

(a) Draw a "space-time" diagram of the collision from p_2s reference frame.

(b) What are the particles' initial momentum and energy in each frame ?

(b) What are the particles' final momentum and energy in each frame ?

2) 2D Collisions

At time t_0 a particle p_1 of mass m is moving with velocity $+v_i$ in the x-direction and -v_i in the y-direction (in vector notation $\vec{v_1^i} = (v_i, -v_i)$) towards another particle p_2 of mass m, which is moving with velocity $\vec{v_2^i} = (-v_i, +v_i)$. After the collision, p_1 is moving with $\vec{v_1^f} = (+v_f, +v_f)$.

(a) Draw a x-y picture of the particles and how they are moving at t_0.

(b) Solve for is v_f and $\vec{v_2^f}$.

(c) Draw a x-y picture of the particles and how they are moving after the collision.

(d) Draw this collision from the reference frame of someone moving with velocity v_i along the x-axis $v = (v_i, 0)$.

(a) Find the initial and final velocities of the particles in this other fram.

(b) What are the particles' initial momentum and energy in each frame ?

(b) What are the particles' final momentum and energy in each frame ?

3) Matrix Multiplication

Let, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ $C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(a) What is A× C ?

(b) What is A× B ?

(b) What is n$\times$ A ? where n is a number.

(c) Watch this: The adult way to think about matrices.

4) Proton Collisions in Newtonian Mechanics

A proton moving with speed v_i strikes a proton at rest in the lab frame. After the collision the protons are observed to scatter symmetrically about the x-axis. (ie: one proton makes an angle $+\theta$ with respect to the x-axis, the other makes an angle -θ.) What does Newtonian physics predict the angle between the protons should be in the lab frame ?

5) (Challenge Problem) General Linear Coordinate Transformations

Fill in the details for argument for the general coordinate transformation sketched in class. Write down the arbitrary linear transformation we assumed in class. The unknowns are arbitrary functions of v, but not (by assumption x,x',t, or t').

(a) Impose the constraint x=vt when x'=0. What is the most general form now?

(b) Impose the constraint x'=-vt' when x=0. What is the most general form now?

(c) Require that the combination of two transforms (v_1) and (v_2) yields another transform. (Hint: What relation do the diagonal elements have to have to form a valid transformation consistent with b)?) (Hint #2 This should yield a free parameter given by the separation of variables constant)

(d) You should now have one free parameter (the separation constant) and one unknown arbitrary function of v. Solve for the arbitrary function by requiring the two transforms (v) and ($-v$) to give the identity (ie: x'=x and t'=t)

(e) You should now have the most general linear coordinate transformation. Write the separation constant in terms of a velocity v_* as we did in class. Show that if something is moving with v_* in one frame it move with the speed in the other frame. (ie: $x' = v_* t' \implies x = v_* t$)

(f) Show that $v_*^2 t^2 - x^2$ is invariant under coordinate transformations.

(h) How does the most general coordinate transformation relate to the Galilean transformations in classical physics? eg: What choice does Newton make for v_* ? A pithy way to summarize the difference between Newtonian physics and Relativistic physics is simply a different choice for the free parameter in the general coordinate transformation.

6) How long does it take light to travel a foot ?

7) Two runners

Two runners (A and B) are racing along a straight track of length L. B travels at a constant speed v, A goes half of the way with speed $2v$and half of the way with speed v/2. Who wins?