The Barabási-Albert model
(preferential attachment)

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Some material and images are from (or adapted from):
A. Barabási, and M. Pósfai. Network science, Cambridge University Press, 2016

Questions:

- Why very different systems converge to a similar scale-free architecture?
- What are the mechanism responsible for the emergence of the scale-free property?

We move from describing a network's topology to modeling the evolution of a complex system

Two *hidden* assumptions of the Erdős-Rényi model

The random network model assumes a fixed number of nodes, N.

In real networks the number of nodes continually grows.

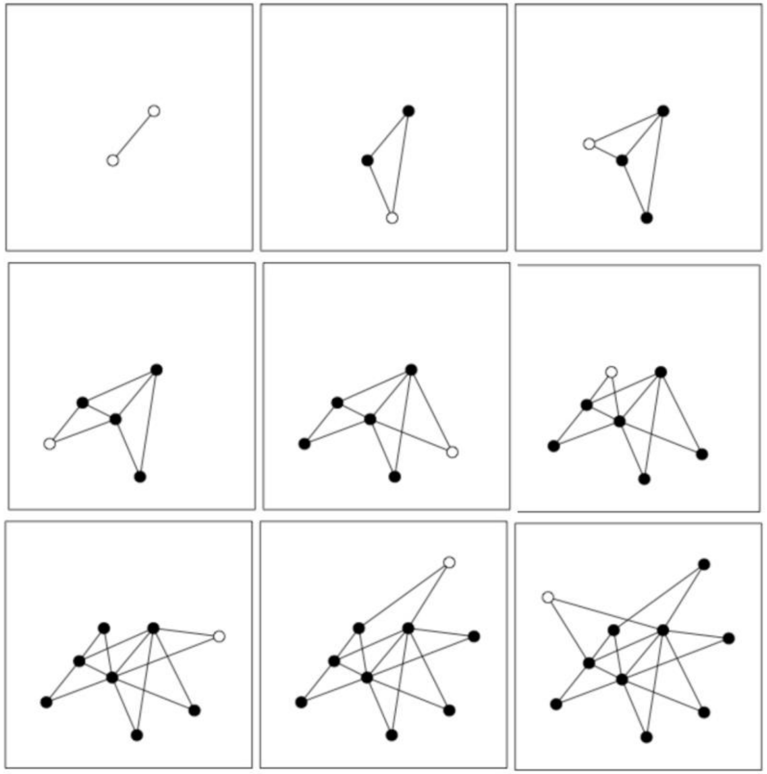
The random network model assumes that we randomly choose the interaction partners of a node.

In real networks new nodes prefer to link to the more connected nodes (preferential attachment)

The Barabási-Albert model

1. **Start** with m_0 nodes, the links between which are chosen arbitrarily (but each node has at least one link)
2. **Growth:** at each timestep we add a new node with m ($\leq m_0$) links connecting the new node to m nodes already in the network.
3. **Preferential attachment:** the probability $\Pi(k)$ that a link of the new node connects to node i depends on the degree k_i as

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$



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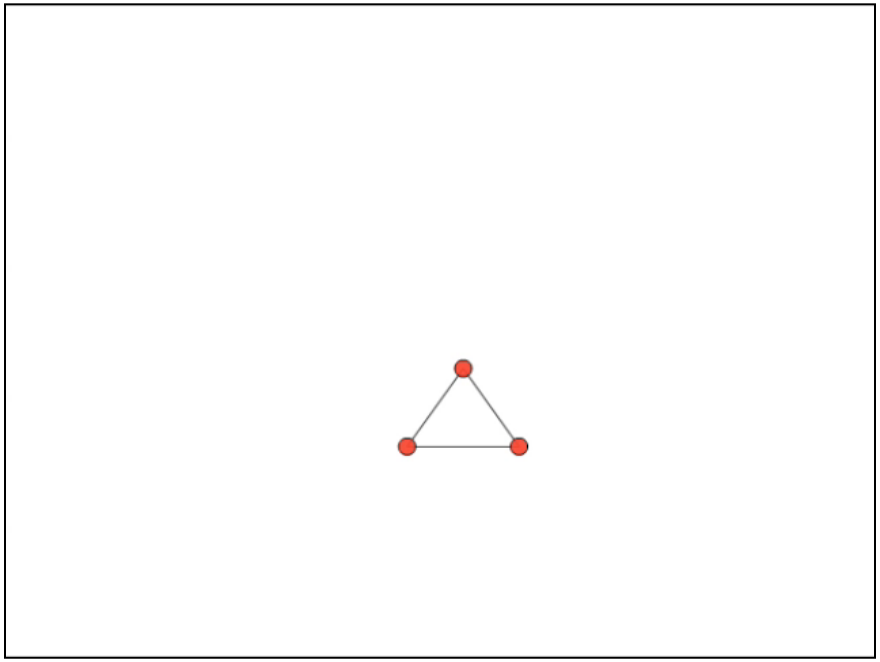
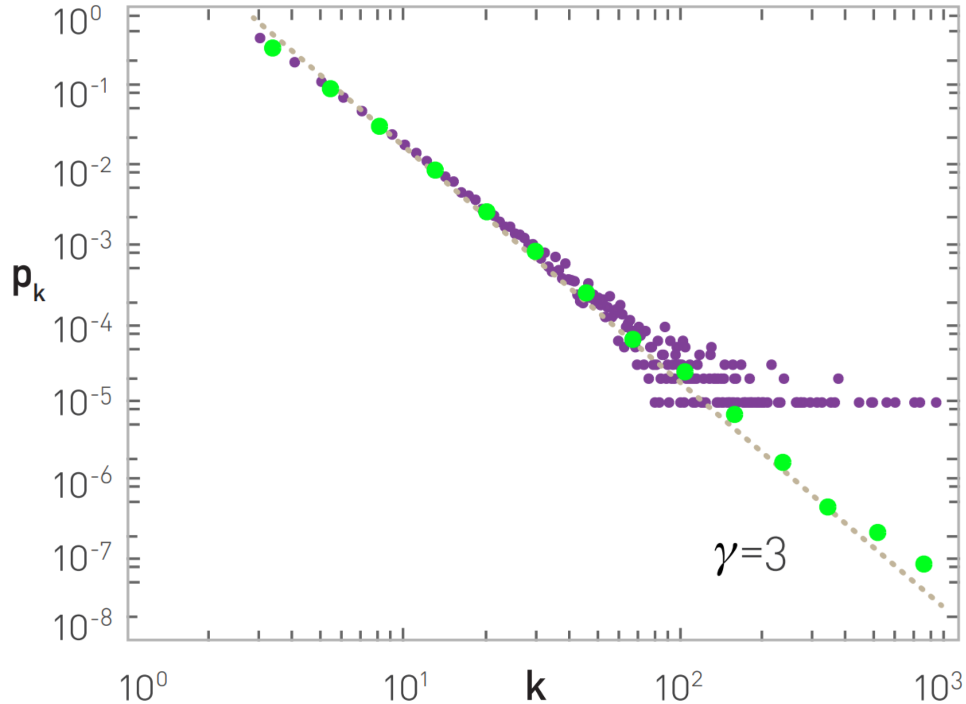




After t timesteps:

- $N = t + m_0$ nodes
- $m_0 + mt$ links
- power-law degree distribution with γ=3

The BA model indicates that two simple mechanisms, **growth** and **preferential attachment**, are responsible for the emergence of scale-free networks

Degree distribution

$$p_k = \frac{2m(m+1)}{k(k+1)(k+2)}$$

- For large k, p_k reduces to $p_k \sim k^{-3}$, or $\gamma = 3$
- The degree exponent γ is independent of m and m_0 parameters
- Degree distribution is independent of both t and N

On the coexistence of growth and preferential attachment

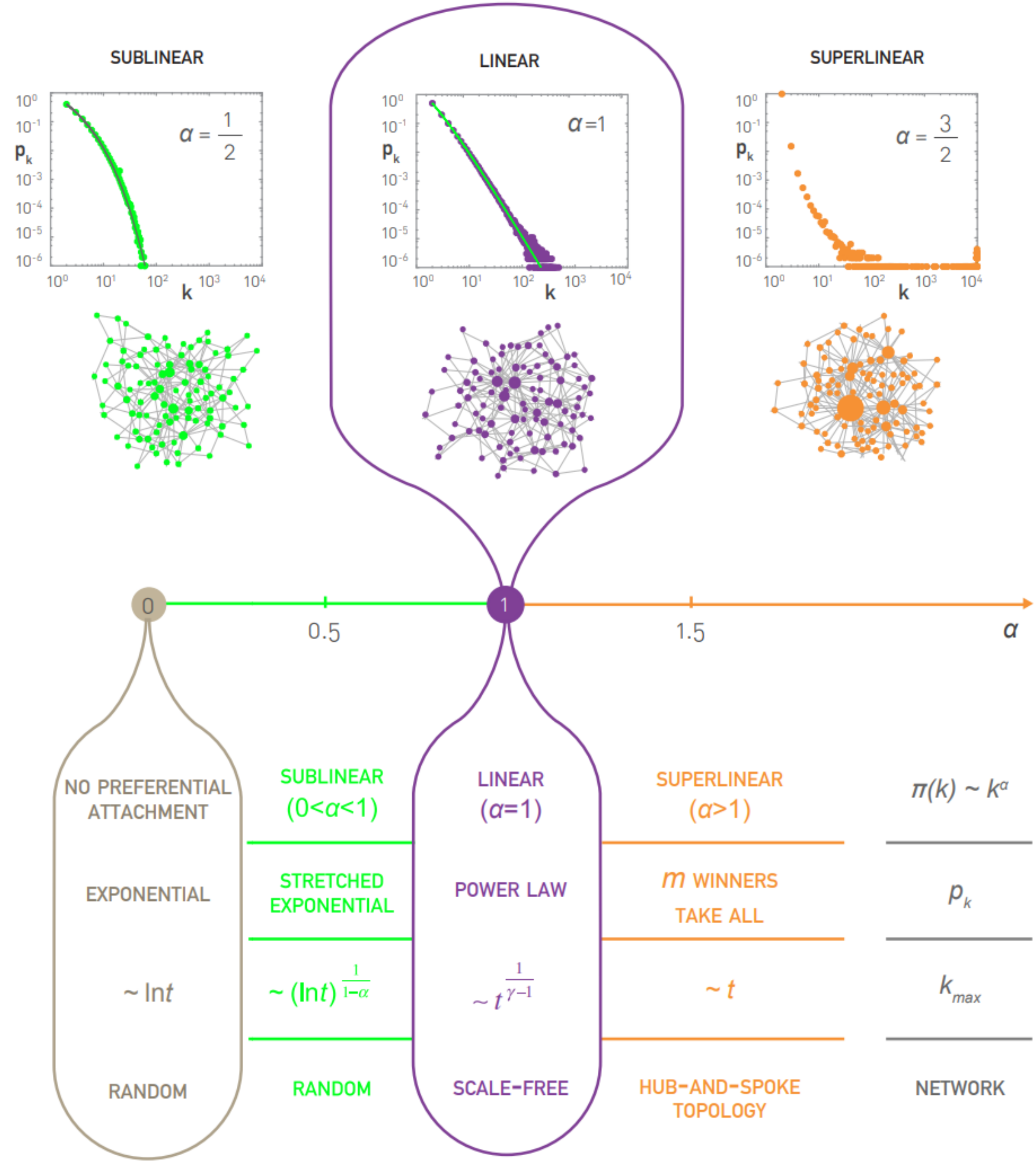
- The absence of preferential attachment eliminates the network's scale-free character and the hubs

- The absence of growth leads to convergence to a complete graph.

On the linearity of preferential attachment

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

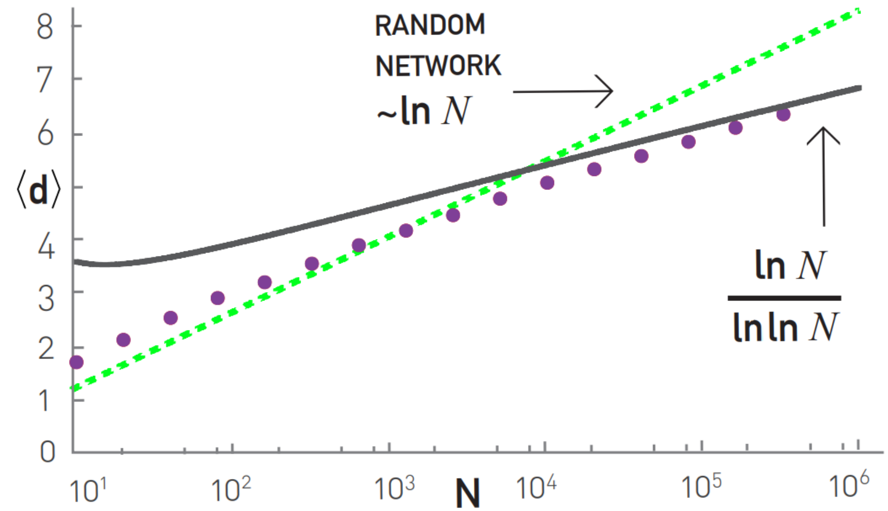
$$\Pi(k) \sim k^{\alpha}$$



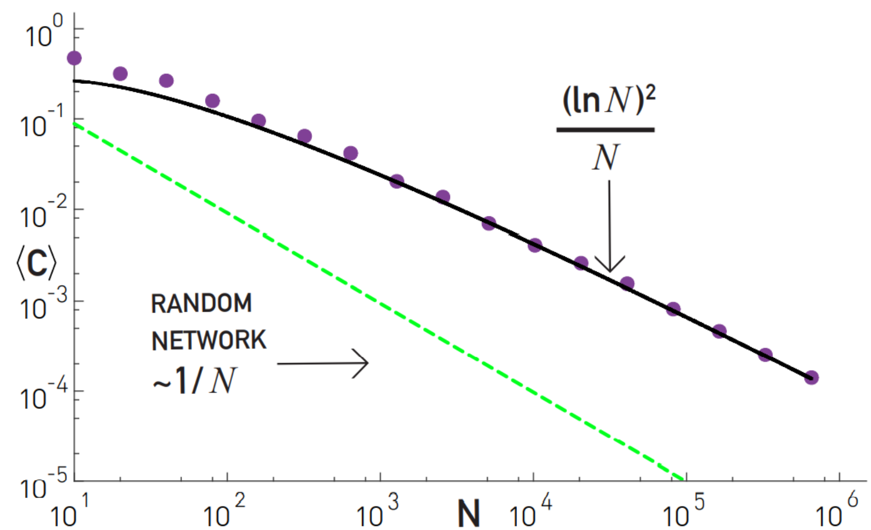
Diameter and clustering coefficient

$$\langle d \rangle \sim \frac{\ln N}{\ln \ln N}$$

the diameter
grows slower
than *ln N*



$$\langle C \rangle \sim \frac{(\ln N)^2}{N}$$



Important points

- Network structure and evolution are inseparable.
- *To understand the topology of a complex system, we need to describe how it came into being*
- growth and preferential attachment are <u>jointly</u> needed to generate scale-free networks
- The BA model is unable to describe many characteristics of real systems:
  - The model predicts γ=3 while the degree exponent of real networks varies between 2 and 5
  - the model generates only undirected networks.
  - many processes observed in networks, (e.g. linking to already existing nodes) are absent from the model.
- to understand the evolution of specific systems we need to incorporate the important details that contribute to the time evolution of these systems