Small world and scale free networks

Python tutorial

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August 2th 2022

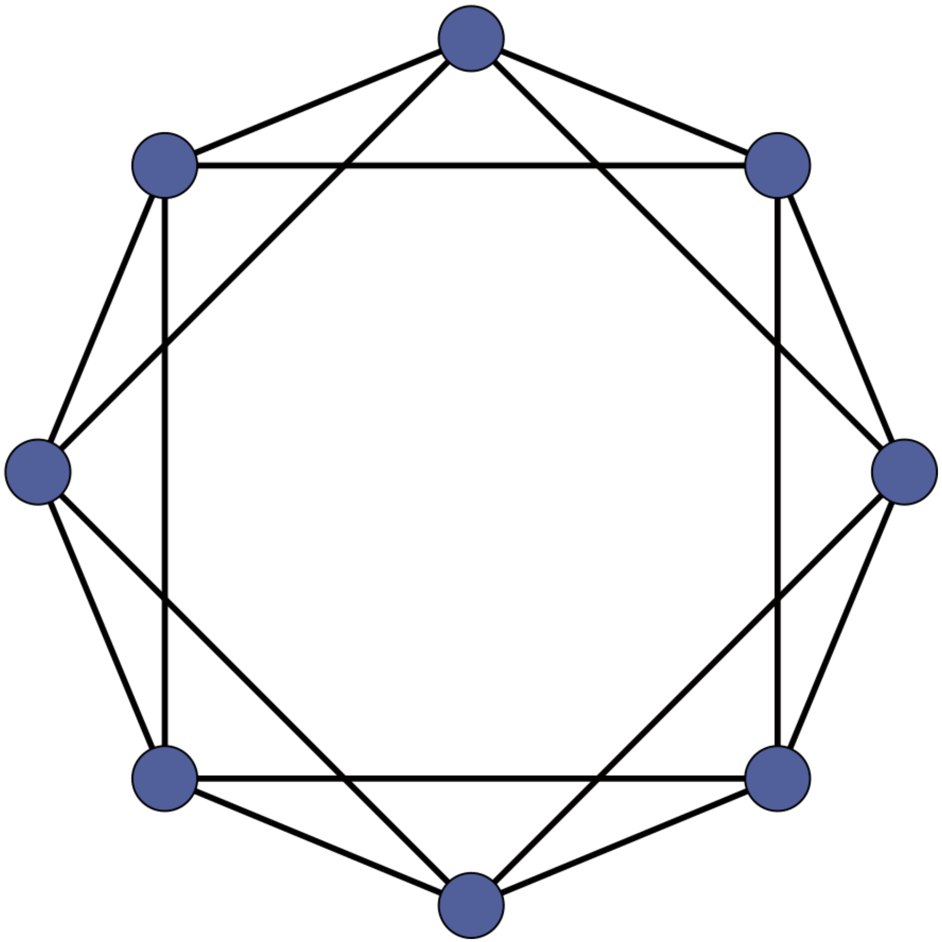
Some characteristics observed in real world networks

- Small world phenomenon

- Scale free property

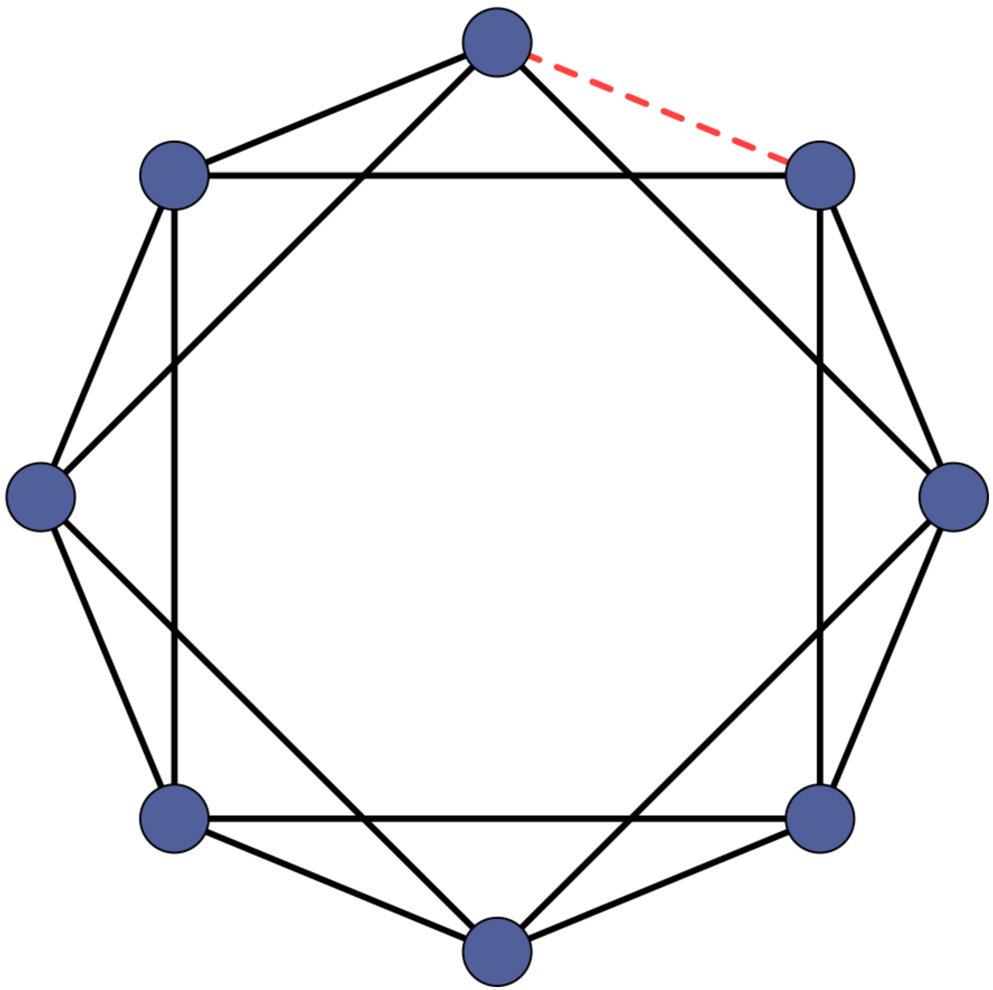
Watts-Strogatz model

Regular lattice



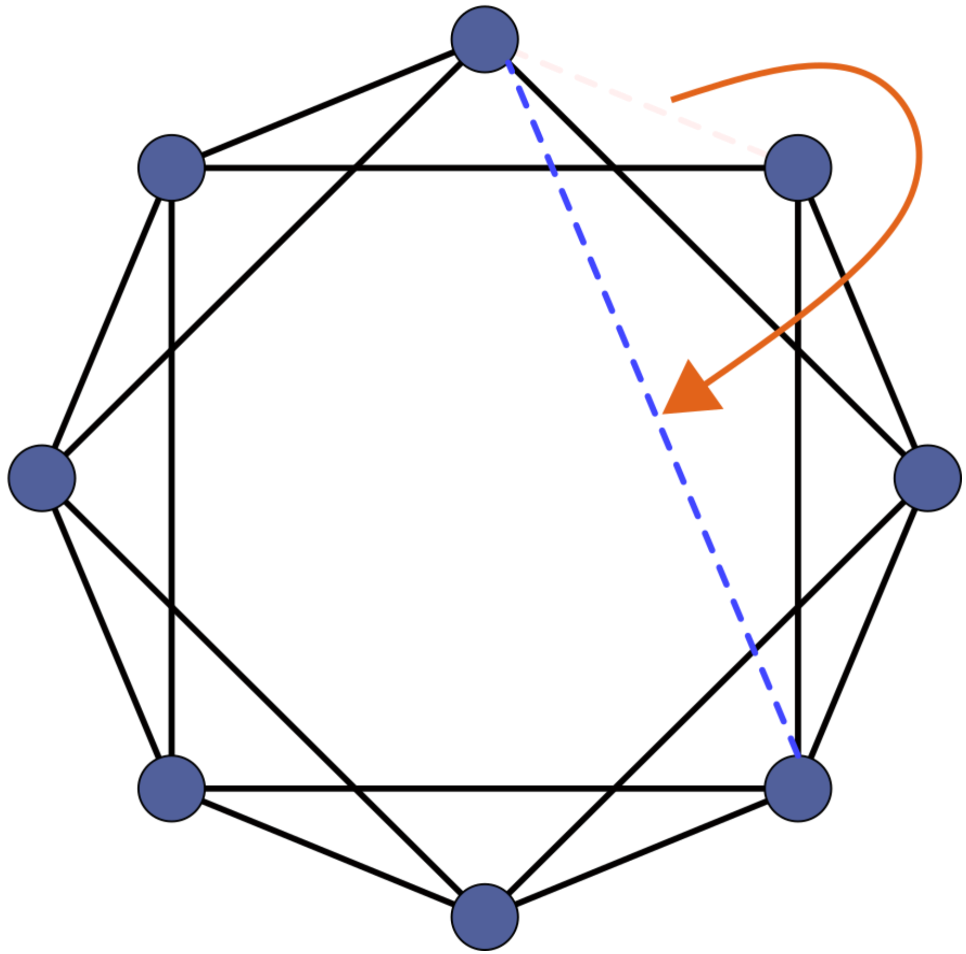
Watts-Strogatz model

The probability to reconnect an edge is p_r



Watts-Strogatz model

The probability to reconnect an edge is p_r



Scale free networks

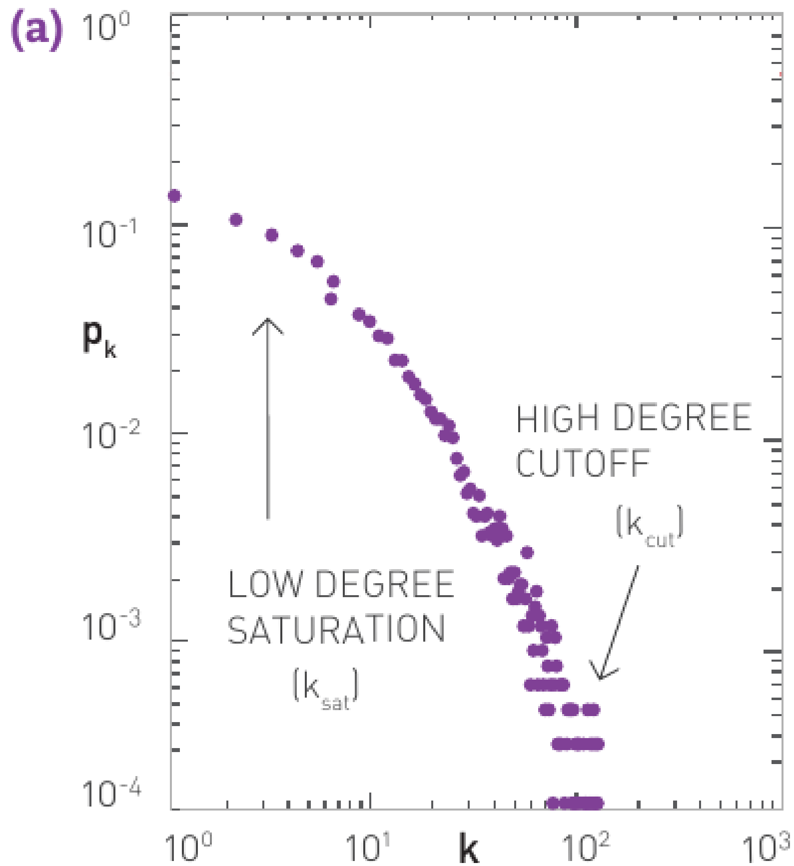
Power law degree distribution:

$$P(k) = Ck^{-\gamma}$$

How can we estimate the degree exponent?

Some considerations...

- In real networks, very small degrees or very high degrees are farther from the power law distribution.



Font: Barabási. *Network Science*

Parameter estimation

Para cada K_{min}:

$$\gamma = 1 + N\left[\sum_{i=1}^{N} \ln \frac{k_i}{K_{\min} - \frac{1}{2}}\right]^{-1}$$

Parameter estimation

Resulting pdf:

$$p_k = \frac{1}{\zeta(\gamma, K_{\min})} k^{-\gamma}$$

Parameter estimation

Resulting cdf:

$$P_k = 1 - \frac{\zeta(\gamma, k)}{\zeta(\gamma, K_{\min})}$$

Parameter estimation

Choose the K_{min} that minimizes the Kormogorov-Smirnov distance

$$D = max_{k \geq K_{\min}} | S(k) - P_k |$$

Goodness of fit

1. Compute D with the real data (D^{real})
2. Fit the power law distribution to the observed degrees. Use the resulting theoretical pdf to generate a synthetic sequence of N degrees
3. Compute $D^{synthetic}$ using the synthetic data
4. Repeat steps 2-3 M times.
5. Is D^{real} within the range of $D^{synthetic}$ values?

Goodness of fit