Reference Solutions for MA121 Final Review

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Abstract

Those references solutions are for QCC MA121 final review (Version Summer 2024). A html version can be found at https://fyeteaching.github.io/RefSolMA121/index.html. Please let me know if you see any mistakes. Thank you!

1. To convert an angle from radians to degree, multiply the angle by $\frac{180^\circ}{\pi}$. So

$$\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ.$$

2. Using the definition of cosine, the length of AC satsifies the equation $\cos 40^\circ = \frac{AC}{200 \text{ m}}$. So

$$AC = 200 \text{ m} \cdot \cos 40^\circ \approx 153.2 \text{ m}.$$

3. To get a cofunction with same value, replace the given angle by its complement. So

$$\csc\left(\frac{2\pi}{5}\right) = \sec\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \sec\sec\left(\frac{5\pi}{10} - \frac{4\pi}{10}\right) = \sec\left(\frac{\pi}{10}\right).$$

4. Let A be the end of the ramp on the street, B the end of the ramp at the entrance of the clinic, C is the point on the street right below B.

Figure 1: Right triangle with an angle 4 degrees and opposite side 2

The length of the ramp AB is the hypotenuse of the right triangle $\triangle ABC$. The length of the street AC is the adjacent side of the right triangle and the height of the building BC is the opposite side of the right triangle. The angle of elevation is the angle between the street and the ramp, that is, $\angle A$. With $\angle A = 4^\circ$, $BC = 2$ ft, the length of the ramp satisfies the equation $\tan 4^\circ = \frac{2 \text{ ft}}{AC}$. Solving for AC gives the length of the ramp

$$AC = \frac{2 \text{ ft}}{\tan 4^\circ} \approx 28.6 \text{ ft}.$$

5. The reference angle is the acute angle formed by the terminal side and the x-axis. Once the terminal side is determined, the reference angle can be found by measuring the angle between the terminal side and the x-axis. The angle of $242°$ has the terminal side in the third quadrant. The reference angle is $242° - 180° = 62°$.

6. To convert an angle from radian to degree, multiply the angle by $\frac{180°}{\pi}$. So

$$135° = \frac{135\pi}{180} = \frac{3\pi}{4}.$$

7. From the definition of sine and cosine of an arbitrary angle θ, $x = r\cos\theta$ and $y = r\sin\theta$, where (x, y) is a point on the terminal side and r is the distance from the origin. Because $\csc\theta = 1/\sin\theta < 0$ and $\sec\theta = 1/\cos\theta > 0$, then a point on the terminal side has $y < 0$ and $x > 0$. So the terminal side is in the fourth quadrant.

8. The find $\sin\theta$ from a given point on the terminal side, divide the y-coordinate by the distance from the origin. In this case, the distance between $(-3, 4)$ and the origin is $\sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2} = 5$. So

$$\sin\theta = \frac{y}{r} = \frac{-3}{5}.$$

9. Coterminal angles are different angles that have the same terminal side. To find a coterminal angle, add or subtract a multiple of $360°$ (or 2π) to the given angle. Since the given angle is negative and desired coterminal angle is positive and less than $360°$. We add multiples of $360°$ so that the angle is positive and less than $360°$. So the desired coterminal angle is

$$-685° + 2 \cdot 360° = 35°.$$

10. To determine the exact value of $\sin\theta$ with the given measure of θ, one can use the reference angle θ_{ref}. If the terminal side is in Quadrant I or II ($y > 0$), then $\sin\theta = \sin\theta_{\text{ref}}$. Otherwise, $\sin\theta = -\sin\theta_{\text{ref}}$. In this case, the terminal side of $\dfrac{4\pi}{3}$ is in Quadrant III, and the reference angle is $\dfrac{\pi}{3}$. Therefore,

$$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

Remark: Trig of special angles in $[0, \frac{\pi}{2}]$ can be found using the left hand trick. See for example https://www.geogebra.org/m/cGKXJnxZ.

11. For a function $y = A\cos(Bx)$, the amplitude is $|A|$ and the period is $\dfrac{2\pi}{|B|}$. In this case, the amplitude is $|3| = 3$ and the period is $\dfrac{2\pi}{|\frac{\pi}{6}|} = 12$. To sketch the graph within one period, first find the 5 key points: the maximum, the minimum, the points on the middle. When inside function is simple Bx, the first point can be taken as $(0, A\cos(0)) = (0, -3)$. Let T be the period. The second point can be taken as $\left(\dfrac{T}{4}, A\cos(\dfrac{\pi}{2})\right) = (3, 0)$, the third point can be taken as $\left(\dfrac{T}{2}, A\cos(\pi)\right) = (6, 3)$, the fourth point can be taken as $\left(\dfrac{3T}{4}, A\cos(\dfrac{3\pi}{2})\right) = (9, 0)$,

and the fifth point can be taken as $(T, A\cos(2\pi)) = (12, -3)$. Then connect the points smoothly to get the graph.

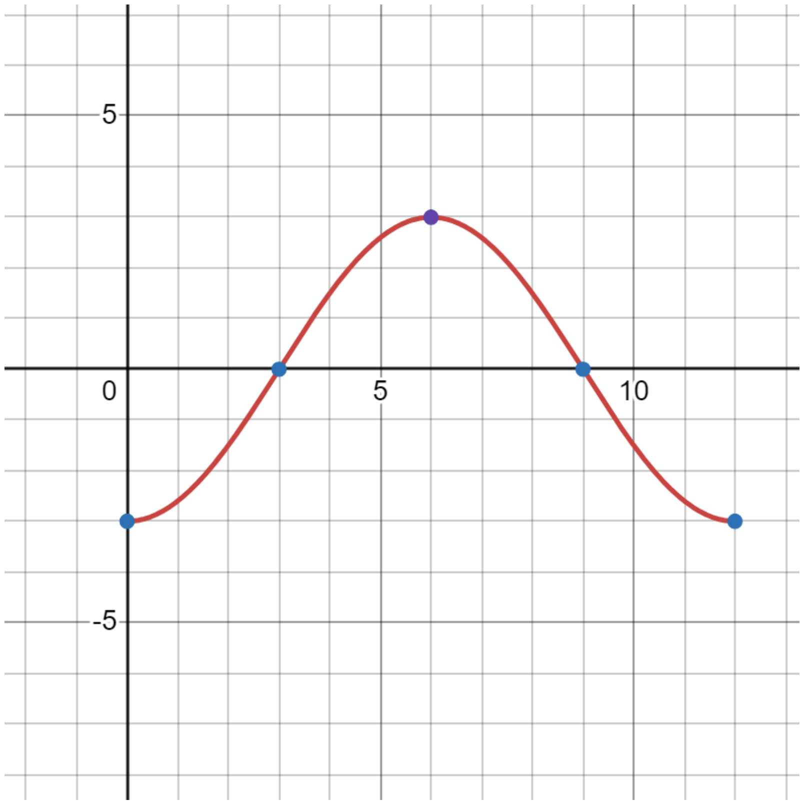

Figure 2: Graph of y=-3cos(pi/6*x)

12. The reference angle is the acute angle formed by the terminal side and the x-axis. Once the terminal side is determined, the reference angle can be found by measuring the angle between the terminal side and the x-axis. The angle of $\frac{8\pi}{9}$ has the terminal side in the second quadrant. The reference angle is
$$\pi - \frac{8\pi}{9} = \frac{\pi}{9}.$$

13. Given $\sin\beta$, the value of $\cos\theta$ can be find algebraically using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. Since $\sin\beta = \frac{8}{9}$ and *beta* is acute, $\cos\beta = \sqrt{1 - \sin^2\beta} = \sqrt{1 - \left(\frac{8}{9}\right)^2} = \frac{\sqrt{17}}{9}$.

   The value of $\cos\beta$ can be found using the right triangle. Let $\angle C$ be the right angle and A be the given angle. Take the hypotnuse $AB = 9$ and the opposite side $BC = 8$. Then by the geometric Pythogorean theorem, $AC = \sqrt{9^2 - 8^2} = \sqrt{17}$. So $\cos\beta = \frac{AC}{AB} = \frac{\sqrt{17}}{9}$.

14. Since the angle of 120° is in the second quadrant, the reference angle is $180^\circ - 120^\circ = 60^\circ$. The value of $\tan 120^\circ$ can be found using the reference angle. Since $\tan 60^\circ = \sqrt{3}$, then $\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}$.

15. Because $\tan\theta < 0$, the termial side of θ is in the second or the fourth quadrant. Because $\cos\theta < 0$, the terminal side of θ is in the second quadrant.

16. Let A be the observation point on the ground, B be the top of the building, and C be the bottom of the building. Note that $\angle C$ is the right angle, and $\angle A = 71^\circ$.

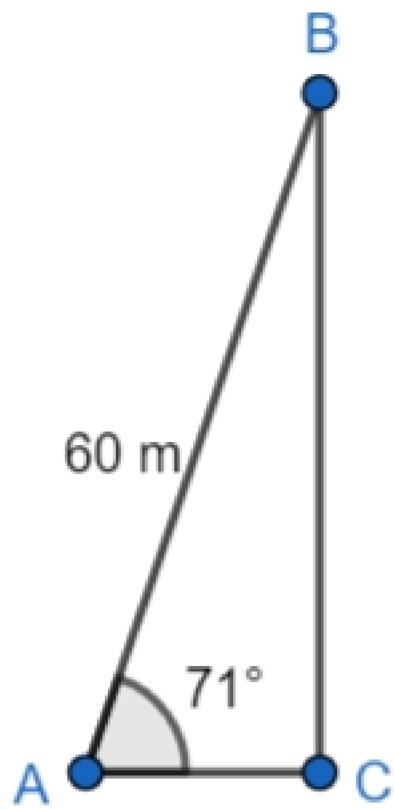

Figure 3: Right triangle with hypotenuse 60 and an angle 71 degrees

Because the distance between A and C is 60 meters. The hight of the building BC is the opposite side of the angle $\angle A$. By the definition of sine, $\sin 71^\circ = \dfrac{BC}{60 \text{ m}}$. Solving for BC gives the height of the building

$$BC = 60 \text{ m} \cdot \sin 71^\circ \approx 174.3 \text{ m}.$$

17. Since θ is acute and $\cos\theta = \dfrac{6}{7}$. Then

$$\sin\theta = \sqrt{1 - \left(\frac{6}{7}\right)^2} = \frac{\sqrt{13}}{7}.$$

18. Since A is in the third quadrant, then $\cos A$ is negative. Using the Pythagorean identity $\sin^2 A + \cos^2 A = 1$, we have $\cos A = -\sqrt{1 - \sin^2 A} = -\sqrt{1 - \left(-\dfrac{\sqrt{3}}{4}\right)^2} = -\dfrac{\sqrt{13}}{4}$. Using

the quotient identity $\tan A = \dfrac{\sin A}{\cos A}$, we have

$$\tan A = \frac{-\dfrac{\sqrt{3}}{4}}{-\dfrac{\sqrt{13}}{4}} = \frac{\sqrt{39}}{13}.$$

19. Similar to Question 11, the amplitude is $|A| = |2| = 2$, the period is $\dfrac{2\pi}{|B|} = \dfrac{2\pi}{|\frac{1}{3}|} = 6\pi$.

    The 5 key points are $(0,0)$, $(1.5\pi, 2)$, $(2\pi, 0)$, $(4.5\pi, -2)$, and $(6\pi, 0)$. Connect the points smoothly to get the graph.

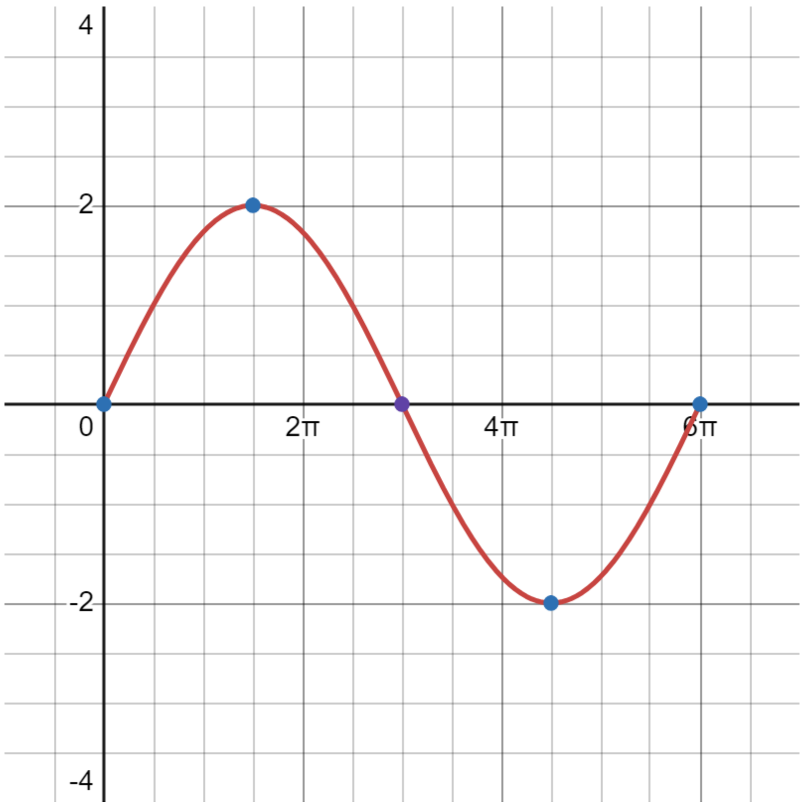

Figure 4: Graph of y=2sin(x/3)

20. Since $\tan\theta = -\frac{1}{2}$ and $\cos\theta > 0$, the angle θ is fourth quadrant. To find the values of the trigonometric functions without sign, we can apply the geometric method to θ_{ref} as see in Question 13. Consider the triangle with the opposite side 1 and the adjacent side 2. The hypotenuse is $\sqrt{1^2 + 2^2} = \sqrt{5}$. Then $\sin\theta_{\text{ref}} = \dfrac{1}{\sqrt{5}}$, $\cos\theta_{\text{ref}} = \dfrac{2}{\sqrt{5}}$. Therefore,

$$\sin\theta = -\frac{\sqrt{5}}{5},$$

$$\cos\theta = \frac{2\sqrt{5}}{5},$$

$$\tan\theta = -\frac{1}{2},$$
$$\csc\theta = -\sqrt{5},$$
$$\sec\theta = \frac{\sqrt{5}}{2},$$
$$\cot\theta = -2.$$

21. The cofunction with the same value as $\tan 78^\circ$ is $\cot(90^\circ - 78^\circ) = \cot 12^\circ$.

22. Since the given angle is greater than 2π, to find the cotermimal angle in $[0, 2\pi]$, we subtract multiples of 2π from the given angle. So the coterminal angle in $[0, 2\pi]$ is
$$\frac{17\pi}{5} - 2\pi = \frac{7\pi}{5}.$$

23. Let A be the bottom of the ladder, B the top of the ladder, and C the point on the ground right below B.

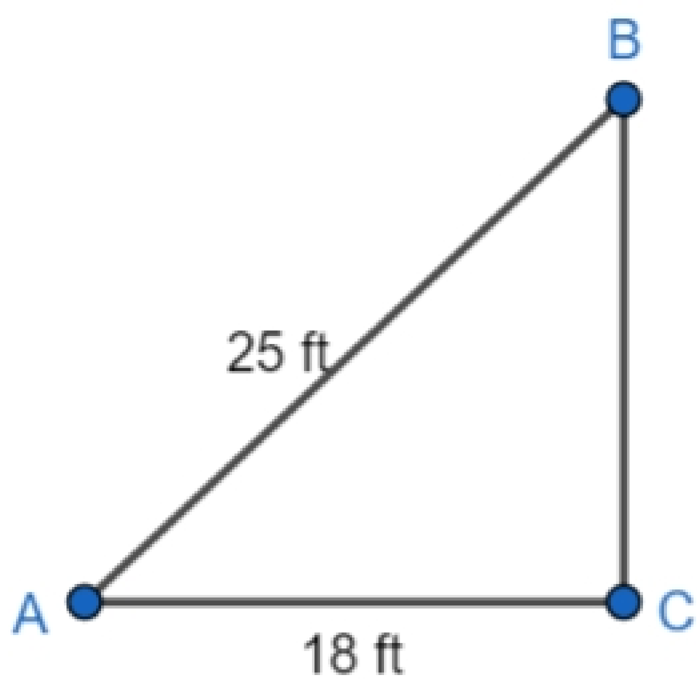

Figure 5: Right triangle with hypotenuse 25 and adjacent 18

Since the ladder is 25-foot long and the bottom of the ladder is 18 feet from the wall, we have $AC = 18$ and $AB = 25$. The angle formed by the ladder and the ground is $\angle A$, which statisfies the equation $\cos A = \dfrac{18}{25}$. Solving for A gives the angle
$$A = \cos^{-1}\left(\frac{18}{25}\right) \approx 44^\circ.$$

The height of the top of the ladder can be calculated by $BC = 25 \sin A \approx 25 \sin 44^\circ \approx 17.3$ feet.

Note that BC can also be found using the Pythagorean theorem:

$$BC = \sqrt{25^2 - 18^2} = \sqrt{625 - 324} = \sqrt{301} \approx 17.3 \text{ ft.}$$

24. Let A be the starting point of the road, B the ending point on the road, and C the point such that the triangle $\triangle ABC$ is a right triangle with C the right angle.

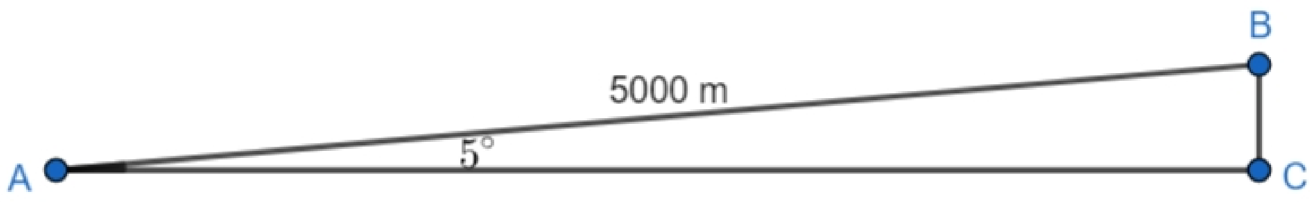

Figure 6: Right triangle with hypotenuse 5000 and an angle 5 degrees

Since the driving distance is 5000 meters and the angle of elevation is 5°, the increase in altitude is the opposite side BC of the right triangle. By the definition of sine, we have

$$\sin 5^\circ = \frac{BC}{5000 \text{ m}}.$$

Solving for BC gives the increase in altitude

$$BC = 5000 \text{ m} \cdot \sin 5^\circ \approx 436 \text{ m}.$$

25. To convert from radian to degree, multiply the angle by $\dfrac{180^\circ}{\pi}$. So

$$2 \cdot \frac{180^\circ}{\pi} \approx 114.59^\circ.$$

26. Since the point is on the unit circle, the distance from the origin is 1. The value of $\cos\theta$ is the x-coordinate of the point. So $\cos\theta = \dfrac{1}{2}$. The value of $\sin\theta$ is the y-coordinate of the point. Because the point is in the fourth quadrant, y-coordinate is negative. So by the Pythagorean identity,

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{1}{2}\right)^2} = -\frac{\sqrt{3}}{2}.$$

27. The distance r from the point on the termimal side to the origin is

$$r = \sqrt{x^2+y^2} = \sqrt{\left(-\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2} = 1.$$

Therofore, by the definition of trigonometric functions of an angle θ, we have

$$\sin\theta = \frac{y}{r} = \frac{24}{25},$$

$$\cos\theta = \frac{x}{r} = -\frac{7}{25},$$

$$\tan\theta = \frac{y}{x} = -\frac{24}{7},$$

$$\cot\theta = \frac{x}{y} = -\frac{7}{24}.$$

$$\sec\theta = \frac{r}{x} = -\frac{25}{7},$$

$$\csc\theta = \frac{r}{y} = \frac{25}{24},$$

28. Similar to Question, the distance r is $r = \sqrt{12^2 + (-5)^2} = 13$. Hence the trigonometric functions are

$$\sin A = \frac{y}{r} = -\frac{5}{13},$$

$$\cos A = \frac{x}{r} = \frac{12}{13},$$

$$\tan A = \frac{y}{x} = -\frac{5}{12},$$

$$\cot A = \frac{x}{y} = -\frac{12}{5}.$$

$$\sec A = \frac{r}{x} = \frac{13}{12},$$

$$\csc A = \frac{r}{y} = -\frac{13}{5},$$