

# Reference Solutions for MA121 Final Review

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## Abstract

Those references solutions are for QCC MA121 final review (Version Summer 2024). A html version can be found at <https://fyeteaching.github.io/RefSolMA121/index.html>. Please let me know if you see any mistakes. Thank you!

1. To convert an angle from radians to degree, multiply the angle by  $\frac{180^\circ}{\pi}$ . So

$$\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ.$$

2. Using the definition of cosine, the length of  $AC$  satisfies the equation  $\cos 40^\circ = \frac{AC}{200 \text{ m}}$ . So

$$AC = 200 \text{ m} \cdot \cos 40^\circ \approx 153.2 \text{ m}.$$

3. To get a cofunction with same value, replace the given angle by its complement. So

$$\csc\left(\frac{2\pi}{5}\right) = \sec\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) = \sec\sec\left(\frac{5\pi}{10} - \frac{4\pi}{10}\right) = \sec\left(\frac{\pi}{10}\right).$$

4. Let  $A$  be the end of the ramp on the street,  $B$  the end of the ramp at the entrance of the clinic,  $C$  is the point on the street right below  $B$ .



Figure 1: Right triangle with an angle 4 degrees and opposite side 2

The length of the ramp  $AB$  is the hypotenuse of the right triangle  $\triangle ABC$ . The length of the street  $AC$  is the adjacent side of the right triangle and the height of the building  $BC$  is the opposite side of the right triangle. The angle of elevation is the angle between the street and the ramp, that is,  $\angle A$ . With  $\angle A = 4^\circ$ ,  $BC = 2 \text{ ft}$ , the length of the ramp satisfies the equation  $\tan 4^\circ = \frac{2 \text{ ft}}{AC}$ . Solving for  $AC$  gives the length of the ramp

$$AC = \frac{2 \text{ ft}}{\tan 4^\circ} \approx 28.6 \text{ ft}.$$

5. The reference angle is the acute angle formed by the terminal side and the  $x$ -axis. Once the terminal side is determined, the reference angle can be found by measuring the angle between the terminal side and the  $x$ -axis. The angle of  $242^\circ$  has the terminal side in the third quadrant. The reference angle is  $242^\circ - 180^\circ = 62^\circ$ .
6. To convert an angle from radian to degree, multiply the angle by  $\frac{180^\circ}{\pi}$ . So

$$135^\circ = \frac{135\pi}{180} = \frac{3\pi}{4}.$$

7. From the definition of sine and cosine of an arbitrary angle  $\theta$ ,  $x = r \cos \theta$  and  $y = r \sin \theta$ , where  $(x, y)$  is a point on the terminal side and  $r$  is the distance from the origin. Because  $\csc \theta = 1/\sin \theta < 0$  and  $\sec \theta = 1/\cos \theta > 0$ , then a point on the terminal side has  $y < 0$  and  $x > 0$ . So the terminal side is in the fourth quadrant.
8. To find  $\sin \theta$  from a given point on the terminal side, divide the  $y$ -coordinate by the distance from the origin. In this case, the distance between  $(-3, 4)$  and the origin is  $\sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2} = 5$ . So

$$\sin \theta = \frac{y}{r} = \frac{-3}{5}.$$

9. Coterminal angles are different angles that have the same terminal side. To find a coterminal angle, add or subtract a multiple of  $360^\circ$  (or  $2\pi$ ) to the given angle. Since the given angle is negative and desired coterminal angle is positive and less than  $360^\circ$ . We add multiples of  $360^\circ$  so that the angle is positive and less than  $360^\circ$ . So the desired coterminal angle is

$$-685^\circ + 2 \cdot 360^\circ = 35^\circ.$$

10. To determine the exact value of  $\sin \theta$  with the given measure of  $\theta$ , one can use the reference angle  $\theta_{\text{ref}}$ . If the terminal side is in Quadrant I or II ( $y > 0$ ), then  $\sin \theta = \sin \theta_{\text{ref}}$ . Otherwise,  $\sin \theta = -\sin \theta_{\text{ref}}$ . In this case, the terminal side of  $\frac{4\pi}{3}$  is in Quadrant III, and the reference angle is  $\frac{\pi}{3}$ . Therefore,

$$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}.$$

**Remark:** Trig of special angles in  $[0, \frac{\pi}{2}]$  can be found using the left hand trick. See for example <https://www.geogebra.org/m/cGKXJnxZ>.

11. For a function  $y = A \cos(Bx)$ , the amplitude is  $|A|$  and the period is  $\frac{2\pi}{|B|}$ . In this case, the amplitude is  $|3| = 3$  and the period is  $\frac{2\pi}{|\frac{\pi}{6}|} = 12$ . To sketch the graph within one period, first find the 5 key points: the maximum, the minimum, the points on the middle. When inside function is simple  $Bx$ , the first point can be taken as  $(0, A \cos(0)) = (0, -3)$ . Let  $T$  be the period. The second point can be taken as  $\left(\frac{T}{4}, A \cos\left(\frac{\pi}{2}\right)\right) = (3, 0)$ , the third point can be taken as  $\left(\frac{T}{2}, A \cos(\pi)\right) = (6, 3)$ , the fourth point can be taken as  $\left(\frac{3T}{4}, A \cos\left(\frac{3\pi}{2}\right)\right) = (9, 0)$ ,

and the fifth point can be taken as  $(T, A \cos(2\pi)) = (12, -3)$ . Then connect the points smoothly to get the graph.

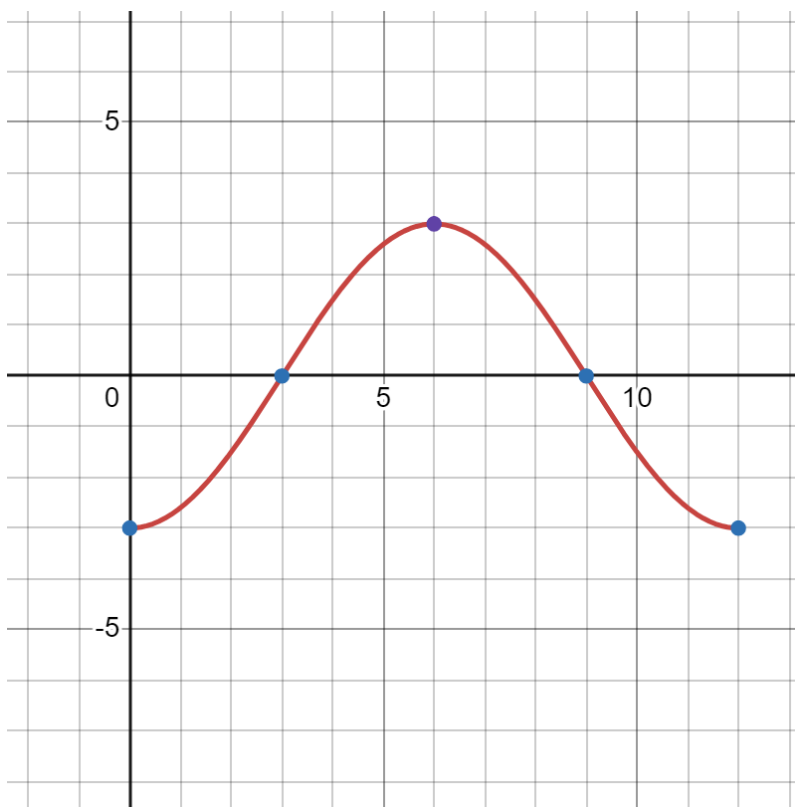


Figure 2: Graph of  $y = -3\cos(\pi/6 \cdot x)$

12. The reference angle is the acute angle formed by the terminal side and the  $x$ -axis. Once the terminal side is determined, the reference angle can be found by measuring the angle between the terminal side and the  $x$ -axis. The angle of  $\frac{8\pi}{9}$  has the terminal side in the second quadrant. The reference angle is

$$\pi - \frac{8\pi}{9} = \frac{\pi}{9}.$$

13. Given  $\sin \beta$ , the value of  $\cos \theta$  can be found algebraically using the Pythagorean identity  $\sin^2 \theta + \cos^2 \theta = 1$ . Since  $\sin \beta = \frac{8}{9}$  and  $\beta$  is acute,  $\cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \left(\frac{8}{9}\right)^2} = \frac{\sqrt{17}}{9}$ .

The value of  $\cos \beta$  can be found using the right triangle. Let  $\angle C$  be the right angle and  $A$  be the given angle. Take the hypotenuse  $AB = 9$  and the opposite side  $BC = 8$ . Then by the geometric Pythagorean theorem,  $AC = \sqrt{9^2 - 8^2} = \sqrt{17}$ . So  $\cos \beta = \frac{AC}{AB} = \frac{\sqrt{17}}{9}$ .

14. Since the angle of  $120^\circ$  is in the second quadrant, the reference angle is  $180^\circ - 120^\circ = 60^\circ$ . The value of  $\tan 120^\circ$  can be found using the reference angle. Since  $\tan 60^\circ = \sqrt{3}$ , then  $\tan 120^\circ = -\tan 60^\circ = -\sqrt{3}$ .

15. Because  $\tan \theta < 0$ , the terminal side of  $\theta$  is in the second or the fourth quadrant. Because  $\cos \theta < 0$ , the terminal side of  $\theta$  is in the second quadrant.
16. Let  $A$  be the observation point on the ground,  $B$  be the top of the building, and  $C$  be the bottom of the building. Note that  $\angle C$  is the right angle, and  $\angle A = 71^\circ$ .

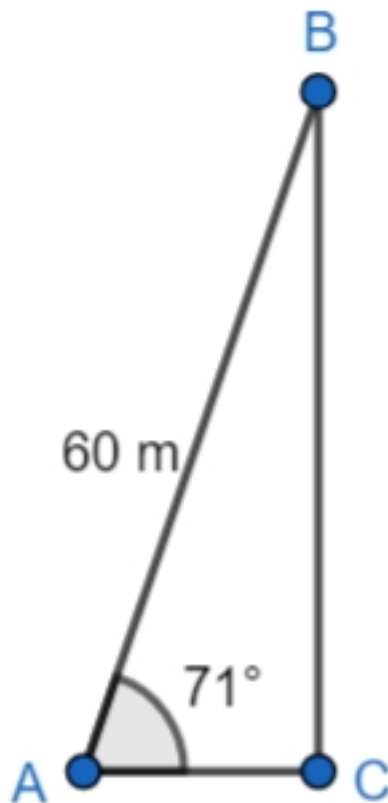


Figure 3: Right triangle with hypotenuse 60 and an angle 71 degrees

Because the distance between  $A$  and  $C$  is 60 meters. The height of the building  $BC$  is the opposite side of the angle  $\angle A$ . By the definition of sine,  $\sin 71^\circ = \frac{BC}{60 \text{ m}}$ . Solving for  $BC$  gives the height of the building

$$BC = 60 \text{ m} \cdot \sin 71^\circ \approx 174.3 \text{ m}.$$

17. Since  $\theta$  is acute and  $\cos \theta = \frac{6}{7}$ . Then

$$\sin \theta = \sqrt{1 - \left(\frac{6}{7}\right)^2} = \frac{\sqrt{13}}{7}.$$

18. Since  $A$  is in the third quadrant, then  $\cos A$  is negative. Using the Pythagorean identity

$$\sin^2 A + \cos^2 A = 1, \text{ we have } \cos A = -\sqrt{1 - \sin^2 A} = -\sqrt{1 - \left(-\frac{\sqrt{3}}{4}\right)^2} = -\frac{\sqrt{13}}{4}. \text{ Using}$$

the quotient identity  $\tan A = \frac{\sin A}{\cos A}$ , we have

$$\tan A = \frac{-\frac{\sqrt{3}}{4}}{-\frac{\sqrt{13}}{4}} = \frac{\sqrt{39}}{13}.$$

19. Similar to Question 11, the amplitude is  $|A| = |2| = 2$ , the period is  $\frac{2\pi}{|B|} = \frac{2\pi}{|\frac{1}{3}|} = 6\pi$ .

The 5 key points are  $(0, 0)$ ,  $(1.5\pi, 2)$ ,  $(2\pi, 0)$ ,  $(4.5\pi, -2)$ , and  $(6\pi, 0)$ . Connect the points smoothly to get the graph.

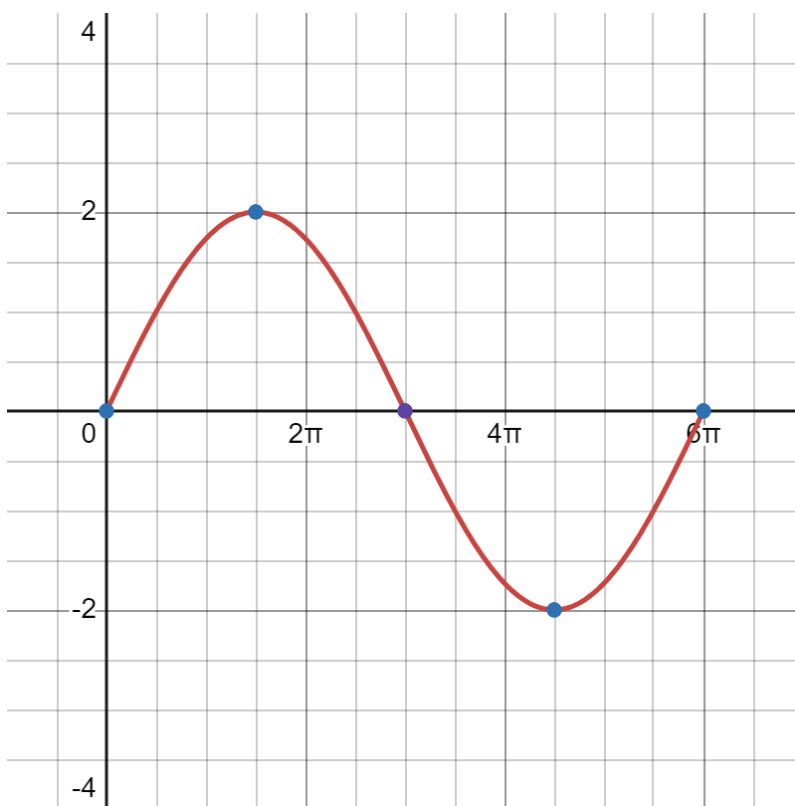


Figure 4: Graph of  $y=2\sin(x/3)$

20. Since  $\tan \theta = -\frac{1}{2}$  and  $\cos \theta > 0$ , the angle  $\theta$  is fourth quadrant. To find the values of the trigonometric functions without sign, we can apply the geometric method to  $\theta_{\text{ref}}$  as see in Question 13. Consider the triangle with the opposite side 1 and the adjacent side 2. The hypotenuse is  $\sqrt{1^2 + 2^2} = \sqrt{5}$ . Then  $\sin \theta_{\text{ref}} = \frac{1}{\sqrt{5}}$ ,  $\cos \theta_{\text{ref}} = \frac{2}{\sqrt{5}}$ . Therefore,

$$\sin \theta = -\frac{\sqrt{5}}{5},$$

$$\cos \theta = \frac{2\sqrt{5}}{5},$$

$$\tan \theta = -\frac{1}{2},$$

$$\csc \theta = -\sqrt{5},$$

$$\sec \theta = \frac{\sqrt{5}}{2},$$

$$\cot \theta = -2.$$

21. The cofunction with the same value as  $\tan 78^\circ$  is  $\cot(90^\circ - 78^\circ) = \cot 12^\circ$ .
22. Since the given angle is greater than  $2\pi$ , to find the coterminal angle in  $[0, 2\pi]$ , we subtract multiples of  $2\pi$  from the given angle. So the coterminal angle in  $[0, 2\pi]$  is

$$\frac{17\pi}{5} - 2\pi = \frac{7\pi}{5}.$$

23. Let  $A$  be the bottom of the ladder,  $B$  the top of the ladder, and  $C$  the point on the ground right below  $B$ .

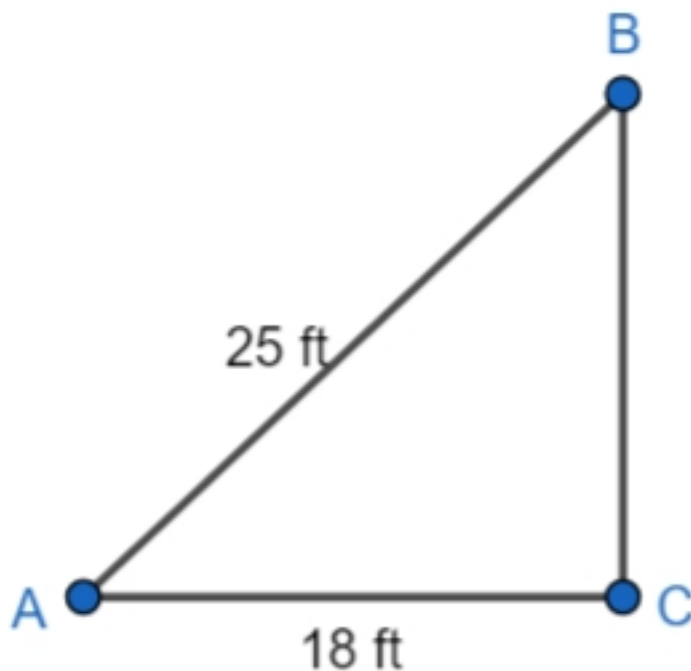


Figure 5: Right triangle with hypotenuse 25 and adjacent 18

Since the ladder is 25-foot long and the bottom of the ladder is 18 feet from the wall, we have  $AC = 18$  and  $AB = 25$ . The angle formed by the ladder and the ground is  $\angle A$ , which satisfies the equation  $\cos A = \frac{18}{25}$ . Solving for  $A$  gives the angle

$$A = \cos^{-1}\left(\frac{18}{25}\right) \approx 44^\circ.$$

The height of the top of the ladder can be calculated by  $BC = 25 \sin A \approx 25 \sin 44^\circ \approx 17.3$  feet.

Note that  $BC$  can also be found using the Pythagorean theorem:

$$BC = \sqrt{25^2 - 18^2} = \sqrt{625 - 324} = \sqrt{301} \approx 17.3 \text{ ft.}$$

24. Let  $A$  be the starting point of the road,  $B$  the ending point on the road, and  $C$  the point such that the triangle  $\triangle ABC$  is a right triangle with  $C$  the right angle.

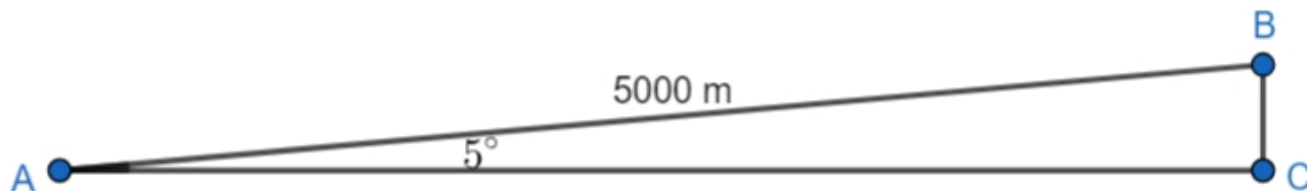


Figure 6: Right triangle with hypotenuse 5000 and an angle 5 degrees

Since the driving distance is 5000 meters and the angle of elevation is  $5^\circ$ , the increase in altitude is the opposite side  $BC$  of the right triangle. By the definition of sine, we have

$$\sin 5^\circ = \frac{BC}{5000 \text{ m}}.$$

Solving for  $BC$  gives the increase in altitude

$$BC = 5000 \text{ m} \cdot \sin 5^\circ \approx 436 \text{ m}.$$

25. To convert from radian to degree, multiply the angle by  $\frac{180^\circ}{\pi}$ . So

$$2 \cdot \frac{180^\circ}{\pi} \approx 114.59^\circ.$$

26. Since the point is on the unit circle, the distance from the origin is 1. The value of  $\cos \theta$  is the  $x$ -coordinate of the point. So  $\cos \theta = \frac{1}{2}$ . The value of  $\sin \theta$  is the  $y$ -coordinate of the point. Because the point is in the fourth quadrant,  $y$ -coordinate is negative. So by the Pythagorean identity,

$$\sin \theta = -\sqrt{1 - \cos^2 \theta} = -\sqrt{1 - \left(\frac{1}{2}\right)^2} = -\frac{\sqrt{3}}{2}.$$

27. The distance  $r$  from the point on the terminal side to the origin is

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2} = 1.$$

Therefore, by the definition of trigonometric functions of an angle  $\theta$ , we have

$$\sin \theta = \frac{y}{r} = \frac{24}{25},$$

$$\cos \theta = \frac{x}{r} = -\frac{7}{25},$$

$$\tan \theta = \frac{y}{x} = -\frac{24}{7},$$

$$\cot \theta = \frac{x}{y} = -\frac{7}{24}.$$

$$\sec \theta = \frac{r}{x} = -\frac{25}{7},$$

$$\csc \theta = \frac{r}{y} = \frac{25}{24},$$

28. Similar to Question, the distance  $r$  is  $r = \sqrt{12^2 + (-5)^2} = 13$ . Hence the trigonometric functions are

$$\sin A = \frac{y}{r} = -\frac{5}{13},$$

$$\cos A = \frac{x}{r} = \frac{12}{13},$$

$$\tan A = \frac{y}{x} = -\frac{5}{12},$$

$$\cot A = \frac{x}{y} = -\frac{12}{5}.$$

$$\sec A = \frac{r}{x} = \frac{13}{12},$$

$$\csc A = \frac{r}{y} = -\frac{13}{5},$$