Lattice Gauge Theory with Finite Groups

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1 Motivation

[Please see https://g1257.github.io/NSF-MSGI for a full description] This project is a continuation from last year's project, and its objective is the formulation of Hamiltonians on the lattice for quantum electrodynamics (QED) and quantum chromodynamics (QCD), but where their Lie groups have been replaced by *finite groups*. We are interested in finite groups because we want to use lattice gauge theory to simulate experiments in optical lattices: in these experiments the number of the gauge-field degrees of freedom must be finite. Lattice gauge theories [1] discretize space, and have been under study since the 1970s [2, 3, 4, 5]. No previous knowledge of lattice gauge theory or physics is needed. By working on this project, the student shall benefit from learning the foundations of physics. One of Clay Mathematics Institute's million dollar problems is—to state it without rigor—the formulation of a non-trivial Yang-Mills theory in realistic dimensions. (The work of Balaban is arguably the closest that researchers have come to succeeding in this formulation. In less than realistic dimensions, Balaban has used lattice gauge theory to achieve proofs of existence in the continuum, by taking the limit of the lattice spacing to zero, in the thermodynamic limit.)

In this project we are interested in lattice gauge theories because ultra-cold atomic optical lattice systems are now able to simulate them [6]. Quantum simulators may be able to solve problems where the time-to-solution depends exponentially on input size; the simulations have only now become possible thanks to the advent of quantum technologies, advances in low-temperature physics, and atomic control techniques in optical lattices. Cold-atom quantum simulators implement matter fields in the presence of artificially designed gauge fields, by suitably identifying the gauge degrees of freedom with the internal states of the atom—for instance, with the atomic spin. Experiments with fermions in the presence of such artificial fields have already been proposed and performed. In addition, an experiment reproducing 1D QED with few *qubits* has been reported. Determining the effectiveness of the lattice gauge theory in reproducing the main features and phenomenology of the target theory requires numerical study. The numerical simulation is not part of this project but I mention it for completeness. The density matrix renormalization group algorithm (DMRG) should be the preferred numerical method in this context, because it reduces the computational complexity without compromising accuracy, as it converges systematically to the exact result. Theory thus opens the door toward solving what have traditionally been difficult problems to analyze, including the investigation of phase transitions, non-perturbative phenomena, and dynamics.

2 Objectives

This project proposes to formulate Hamiltonians for lattice gauge QED and QCD with finite groups. No previous knowledge of lattice gauge theory is needed; approximately two weeks will be spent becoming familiar with the underlying theory and theoretical tools. The first objective for the student is to continue work on our manuscript from 2018; see https://g1257.github.io/NSF-MSGI. By completing the examples in the manuscript, the student will develop confidence in the understanding of the theory. For example, one of the remaining unwritten parts consists in formulating the Gauss law for the SU(2) case, as well as a summary of the SU(3) case. The second objective is to think of possible ways to approximate the SU(2) case by finite groups if that was somehow possible. But having in mind the theorem of Toyama, one would have to instead truncate SU(2), and such truncation would have to be written down. The third objective is to study the lattice spacing a tending to zero limit, and the number of lattice sites N tending to infinity, minding the order of these limits. We would like to find out if there is a infinite Hilbert space that is (in some sense) the limit of a to zero, and of N to infinity. These limits can be studied both in the Lagrangian picture, using analysis tools, or in the Hamiltonian picture, using algebraic tools.

References

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