Scientific Memos

G.A.

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1 Charge Gap

$$C(N, N_e) \equiv E(N, N_e + 1) + E(N, N_e - 1) - 2E(N, N_e), \tag{1}$$

where N_e is the number of electrons and N the number of atoms. Because the energy has to be extensive, we have $E(N, N_e) = Ne(N_e/N)$, where e is an intensive function. We take the limit of $N \to \infty$ with $N_e/N = n$ constant. Therefore, *as long as e" exists* $\boxed{C(N, N_e) \to e''(n)/N.}$

2 Spin Gap

$$S(N, N_\uparrow, N_\downarrow) \equiv (E(N, N_\uparrow + 1, N_\downarrow - 1) + E(N, N_\uparrow - 1, N_\downarrow + 1))/2 - E(N, N_\uparrow, N_\downarrow) \tag{2}$$

where $N_\uparrow$ ($N_\downarrow$) is the number of electrons up (down) and N the number of atoms. Because the energy has to be extensive, we have $E(N, N_\uparrow, N_\downarrow) = Ne(n_\uparrow, n_\downarrow)$, where e is an intensive function. We take the limit of $N \to \infty$ with $N_\uparrow/N = n_\uparrow$ constant, and similar for $N_\downarrow$. Therefore, *as long as e" exists* $\boxed{S(N, N_e) \propto 1/N.}$

3 The Binding Energy

By definition, the binding energy

$$B(N, N_e) \equiv E(N, N_e + 2) + E(N, N_e) - 2E(N, N_e - 1), \tag{3}$$

where N_e is the number of electrons and N the number of atoms. Therefore, *as long as e" exists*

$$E(N, N_e + 2) = Ne(N_e/N + 2/N) = N\left[e(n_0) + \frac{2}{N}e'(n_0) + \left(\frac{2}{N}\right)^2 e''(n_0)/2 + \cdots\right], \tag{4}$$

where $n_0 \equiv N_e/N$. Finally $\boxed{B(N, N_e) = e''(n_0)/N.}$