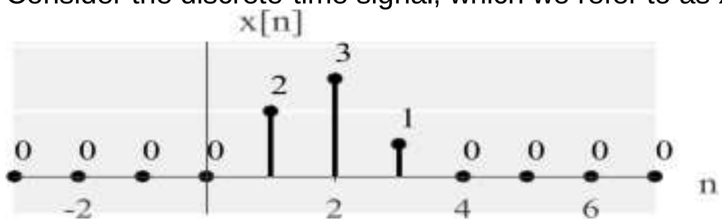
Concepts covered: Signal, systems, linear, superposition, time-invariant, impulse response, convolution

Assignment 1

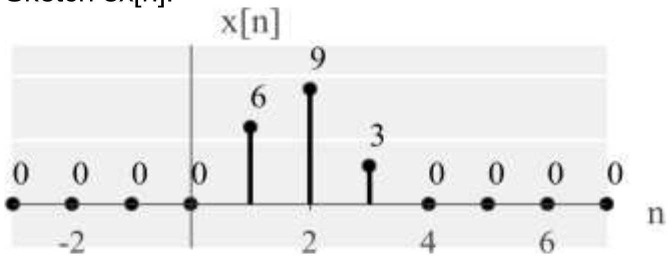
1. Go through the IEEE signal processing society web-page on the introduction of signal processing and write about any of the applications of signal processing you find interesting
2. Download an .mp3 file from web.
   i) Plot the corresponding DT signal say x[n].
   ii) Plot y[n]=x[2*n]. Save the DT signal as .mp3 file. Hear both the files and comment on the differences you perceive.

Assignment 2

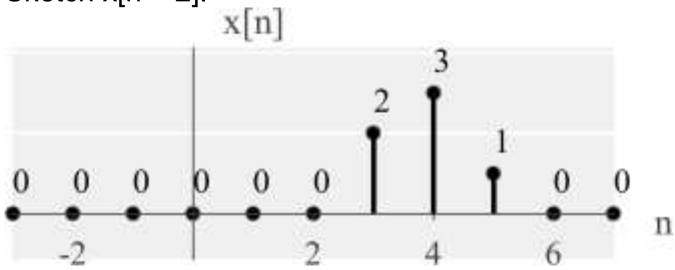
3. Consider the discrete-time signal, which we refer to as x[n]:



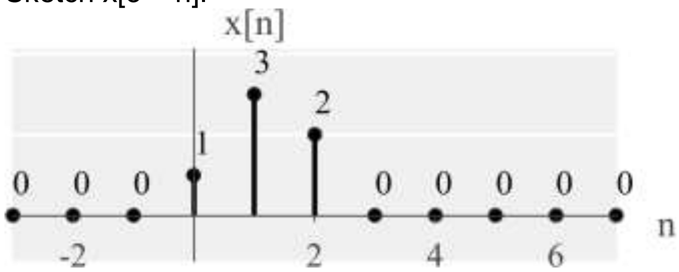
(a) Sketch 3x[n].



(b) Sketch x[n − 2].



(c) Sketch x[3 − n].



(d) Is x[n] an energy signal? Is x[n] an average power signal? Is x[n] a causal signal? Is x[n] a periodic signal?

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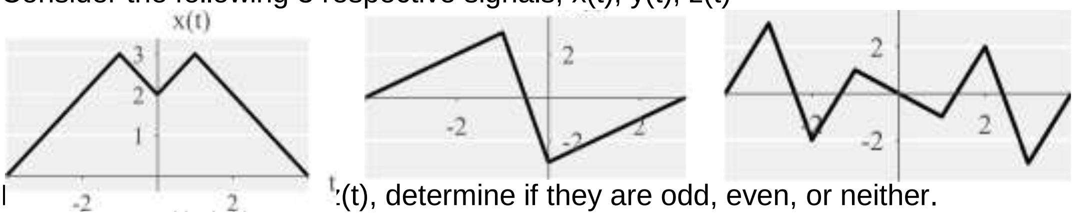
Solution: Energy signal: yes. Average power signal: no. Causal signal: yes. Periodic signal: no.

4. Consider the following 3 respective signals, x(t), y(t), z(t)



:(t), determine if they are odd, even, or neither.

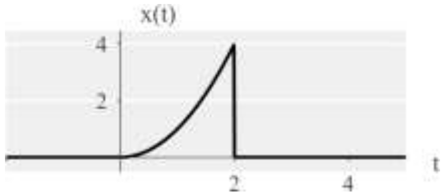
Solution:

- x(t) is even
- y(t) is neither (signal is symmetric, but not symmetric around zero)
- z(t) is odd

5. Consider the continuous-time signal

$$x(t) = \begin{cases} 0 & \text{if } t < 0 \\ t^2 & \text{if } 0 \le t \le 2 \\ 0 & \text{if } t > 2 \end{cases}$$

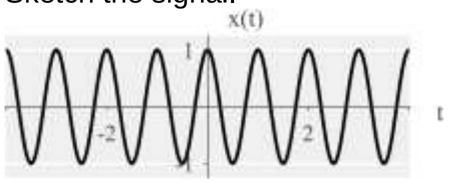
a. Sketch x(t). Is x(t) periodic?
Solution: Periodic: no.



b. Is x(t) causal?
Causal: yes.

c. Compute the energy, power of the signal.
Solution: The energy in x(t) is defined by $\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_0^2 t^4\, dt = \frac{32}{5}$. The power is $\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt = 0$.

6. Consider the continuous-time signal x(t) = cos(2t)
(a) Sketch the signal.



(b) Is the signal continuous or discrete?
Solution: Continuous
(c) Is the signal even, odd, or neither?
Solution: Even
(d) Is the signal causal, anti-causal, or acausal?
Solution: acausal
(e) Compute the energy of the signal?(f) Compute the power of the signal?
Solution:

$$E = \int_{-\infty}^{\infty} \cos^2(2t)\, dt = \infty.$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos^2(2t)\, dt = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \frac{1+\cos(4t)}{2}\, dt = \frac{1}{2}.$$

7. Consider the discrete-time signal
(a) (3 pts) Sketch x[n].

$$x[n] = \begin{cases} 0 & \text{if } n < -2 \\ -2n & \text{if } -2 \le n \le 2 \\ 0 & \text{if } n > 2 \end{cases}$$

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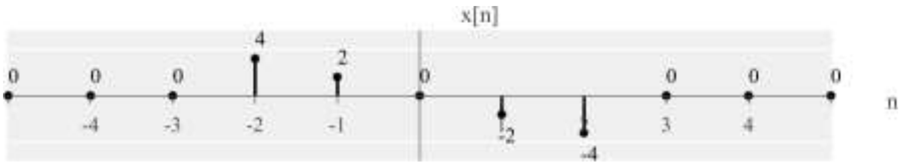
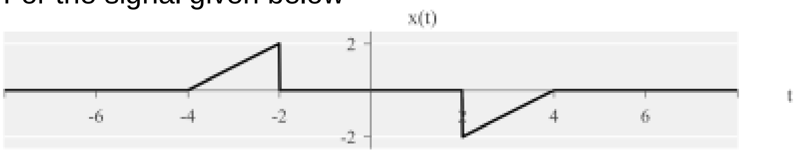


(b) (2 pts) Is the signal causal, anti-causal, or neither?
Solution: Neither

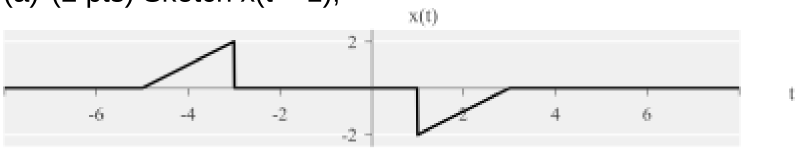
(c) (2 pts) Is the signal even, odd, or neither?
Solution: Odd

(d) (3 pts) If x[n] is an energy signal, compute its energy. If x[n] is a power signal, compute its power.
Solution: $\mathrm{E} = \sum_{-\infty}^{\infty} |\mathrm{x}[\mathrm{k}]|^2 = 40 < \infty$, So x[n] is an Energy signal.

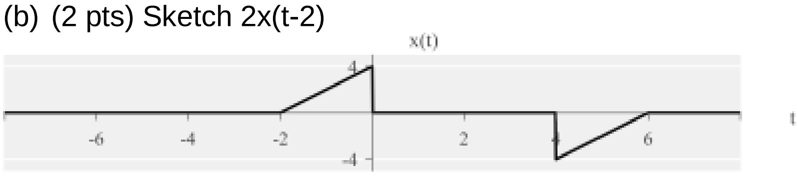
8. For the signal given below



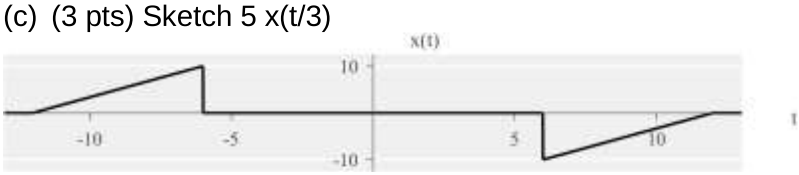
(a) (2 pts) Sketch x(t + 1),



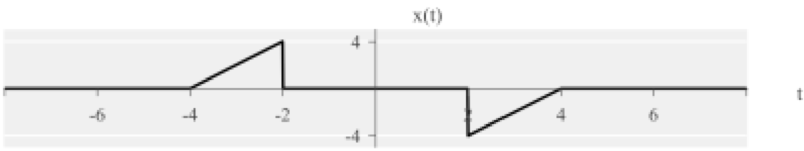
(b) (2 pts) Sketch 2x(t-2)



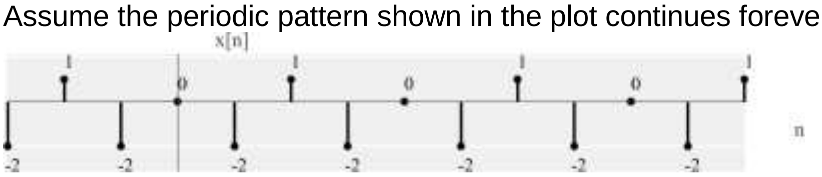
(c) (3 pts) Sketch 5 x(t/3)



(d) (3 pts) Sketch x(t) - x(-t)



9. Assume the periodic pattern shown in the plot continues forever



a) Determine the fundamental period of x[n].
Solution: The signal repeats every 4 samples. Therefore, $N_0 = 4$

b) Compute the energy, average power of x[n].
Solution: The signal is infinite and periodic. Therefore, $|x[n]|^2$ is always positive from $-\infty < n < \infty$. Hence $E = \infty$. The signal is infinite and periodic, so it has a finite power. The power is the average energy of the signal in one period. Energy in one

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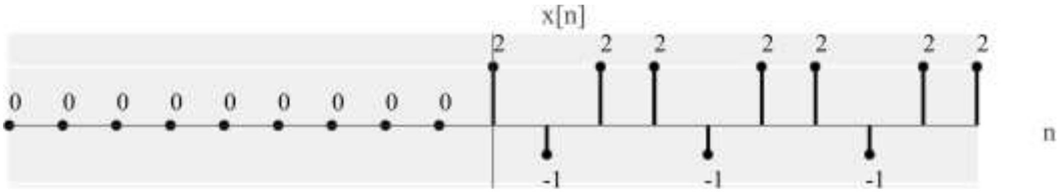
period is (−2)2 + (1)2 + (−2)2 = 9. The fundamental period is N_0 = 4. Therefore the power is 9/4

c) Is x[n] causal? Also, is x[n] even, odd, or neither?
 Solution: No. It is even

10. For the signal x[n], the periodic pattern shown in the plot continues for n ≥ 0 and the zeros continue for n < 0.



a) Express x[n] using step functions and/or impulse functions.

Solution: $x[n] = \sum_{k=0}^{\infty} 2\delta[n-3k] - 1\delta[n-3k-1] + 2\delta[n-3k-2]$

b) The signal y[n] = x[n] + x[?] is periodic. Determine ? so that this statement is true.

Solution: $? = -n - 1$. This can be verified visually.

c) Determine the fundamental period of y[n].

Solution:The fundamental period is N_0 = 3.

11. Let $x_1(t)$, $x_2(t)$, and $x_3(t)$ be periodic signals with fundamental periods of 1, 3, and 10, respectively. Also let $x_1(t)$, $x_2(t)$, and $x_3(t)$ have powers of 1, 2, and 3, respectively.
(a) Compute the fundamental period of z(t) = $x_1(t) + x_2(t) + x_3(t)$

Solution: The fundamental periods of each term are T_1 = 1, T_2 = 3, T_3 = 10. The least common multiple of the Fundamental period of z(t) = LCM(1, 3, 10) = 30.
(b) Based on our knowledge, can we compute the power of z(t)? If so, what is the power? If not,why?

Solution:You cannot compute the power of z(t). This is because
$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |z(t)|^2\, dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x_1(t) + x_2(t) + x_3(t)|^2\, dt = 0$
and we don't know the cross terms 2 $x_1(t)x_2(t)$, 2 $x_1(t)x_3(t)$, etc in the integration.

12. Determine if each signal is periodic. If it is, determine its fundamental period.
a. (2 pts) x(t) = 2 cos(6t + 3) + 1 0
Solution: cos(6t + 3) has a fundamental period T when
6T = 2π → T = π /3. Note that 3 shifts the signal and does not affect the time period. Similarly 10 is constant and therefore does not affect periodicity. So the fundamental period is T = π /3
b. (2 pts) x[n] = sin(3n) + sin(2n)
Solution: sin(3n) has a fundamental period N when

$$3N = 2\pi$$
$$N = (2\pi/3)$$

This is never an integer, so x[n] will not be periodic.
c. (3 pts) x(t) = cos(π/10t)+ 3 cos(π/20t)+ 4 sin(π/30t)
cos(π/10)t) has a fundamental period T_1 when

$$(\pi/10)T_1 = 2\pi \rightarrow T_1 = 20$$

cos((π /20)t) has a fundamental period T_2 when

$$(\pi/20)T_2 = 2\pi, \rightarrow T_2 = 40$$

sin((π /30)t) has a fundamental period T_3 when

$$(\pi/30)T_3 = 2\pi \rightarrow T_3 = 60$$

The least common multiple of 20, 40, and 60 is 120. Therefore fundamental period is T = 120.

d. (3 pts) $x[n] = (-1)^n$

Solution: $(-1)^{n+N_0}=(-1)^n$. The fundamental period is 2.

e. (2 pts) $x(t) = e^{-\pi jt/2} + e^{\pi jt}$

Solution: The period of the _rst term is

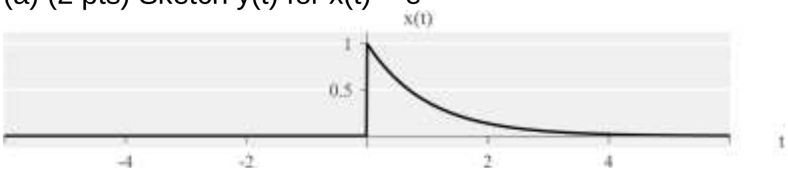
$$\pi T_1/2 = 2\pi$$
$$T_1 = 4$$

The period of the second term is

$$\pi T_2 = 2\pi$$
$$T_2 = 2$$

The fundamental period is 4.

13. Consider the following system with input x(t) and output y(t), y(t) = x(t)u(t)
 (a) (2 pts) Sketch y(t) for $x(t) = e^{-t}$



 (b) (3 pts) Is the system linear?

Solution: Yes. Proof: Given $y_1(t) = H\{x_1(t)\} = x_1(t)u(t)$, $y_2(t) = H\{x_2(t)\} = x_2(t)u(t)$

A system is linear if $H\{ax_1(t) + bx_2(t)\} = ay_1(t) + by_2(t)$.

If we evaluate this expression, we get

$$\begin{aligned} H\{ax_1(t) + bx_2(t)\} &= (ax_1(t) + bx_2(t))\, u(t) \\ &= ax_1(t)u(t) + bx_2(t)u(t) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

 (c) (3 pts) Is the system time-invariant?

Solution: No.

Given $y(t) = \mathcal{H}\{x(t)\} = x(t)u(t)$

A system is time-invariant if

$$y(t-\tau) = \mathcal{H}\{x(t-\tau)\}$$

The left-hand side is $y(t-\tau) = x(t-\tau)u(t-\tau)$.

The right-hand side is $\mathcal{H}\{x(t-\tau)\} = x(t-\tau)u(t)$.

Therefore, $y(t-\tau)\ 6\neq \mathcal{H}\{x(t-\tau)\}$

 (c) (2 pts) Is the system causal?

Solution: Yes. A system is causal if it only depends on current and/or prior inputs. If we plug t = 0 into the system, we get y(0) = x(0)u(0) . The output at the current time t = 0 only depends on the input x(t) at time t = 0 and no t > 0. Therefore, the system is causal.

14. Consider the continuous-time system defined by the following input-output relationship

$$y(t) = (x+a)^n = \sum_{m=-1}^{2} m x(t-m)$$

 (a) (3 pts) Sketch y(t) for the input x(t) = u(t)

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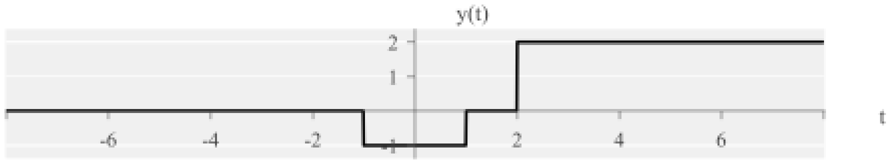
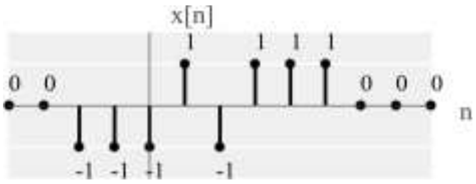
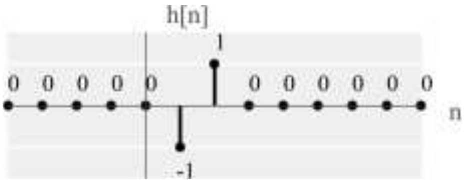


(b) (2 pts) Is the system causal?

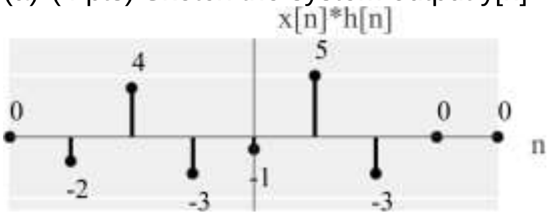
Solution: The solution is not causal because it depends on future values (i.e., when $m = -1$)

15. Consider the following discrete-time signal $x[n]$ and LTI system with impulse response $h[n]$. Assume the zeros continue forever for $n \to -\infty$ and $n \to \infty$.

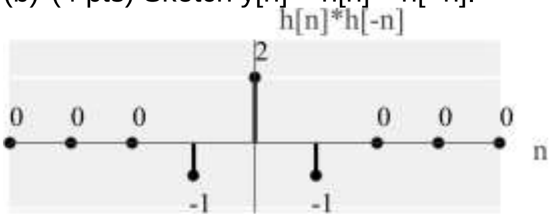




(a) (4 pts) Sketch the system output $y[n] = x[n] * h[n]$.



(b) (4 pts) Sketch $y[n] = h[n] * h[-n]$.



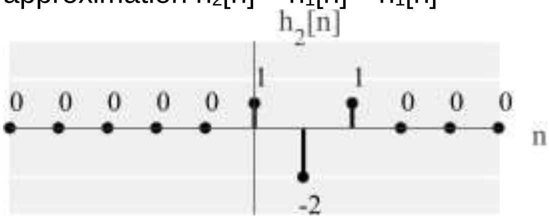
(c) (2 pts) Is the system with impulse response $h[n]$ causal?

Solution: The system is causal since $h[n] = 0$ for all $n < 0$.

16. Consider the following discrete-time LTI system with the following impulse response (this system performs a numerical approximation of the first derivative [times -1])

$$h_1[n] = -\delta[n] + \delta[n-1]$$

(a) (5 pts) Compute or sketch the impulse response for the second derivative approximation $h_2[n] = h_1[n] * h_1[n]$



(b) (5 pts) Compute/sketch the numerical approximation of the first derivative of a box $y[n] = h_1[n] * (u[n] - u[n-2])$