

Fourier Transforms

G V V Sharma*

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Abstract—This manual provides a quick introduction to the Fourier transform.

1 SINUSOIDAL RESPONSE

- Fig. 1 shows an RC circuit with input $x(t)$ and output $y(t)$. Show that

$$RC \frac{dy}{dt} + y(t) = x(t) \quad (1.1)$$

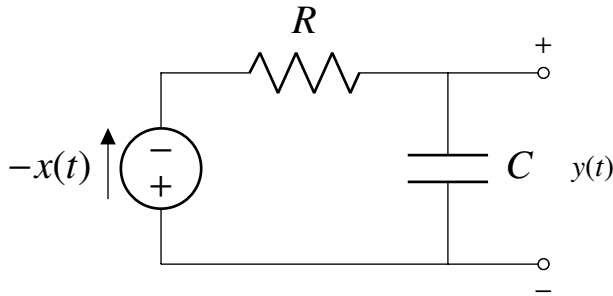


Fig. 1: RC Circuit

- Let $x(t) = \cos 2\pi f_0 t$. Show that

$$y(t) = \frac{1}{\sqrt{1 + (2\pi f_0 RC)^2}} \cos [2\pi f_0 (t - \tan^{-1} RC)] \quad (1.2)$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

by solving the differential equation in (1.1) using the integrating factor.

(1.3)

2 FOURIER TRANSFORM

The Fourier transform of a signal $g(t)$ is defined as

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi f t} dt \quad (2.1)$$

- Define the Dirac delta function as

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (2.2)$$

$$\delta(t) = 0, \quad t \neq 0.$$

and show that

$$\delta(t) \stackrel{\mathcal{F}}{\Leftrightarrow} 1 \quad (2.3)$$

- Show that

$$\delta(t - t_0) \stackrel{\mathcal{F}}{\Leftrightarrow} e^{-j2\pi f t_0} \quad (2.4)$$

- Assuming

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{2\pi f t} df, \quad (2.5)$$

show that

$$g(t) \stackrel{\mathcal{F}}{\Leftrightarrow} G(f) \quad (2.6)$$

$$\Rightarrow G(t) \stackrel{\mathcal{F}}{\Leftrightarrow} g(-f)$$

- Show that

$$\cos 2\pi f_0 t \stackrel{\mathcal{F}}{\Leftrightarrow} \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)] \quad (2.7)$$

- Show that

$$\frac{dy}{dt} \stackrel{\mathcal{F}}{\Leftrightarrow} j2\pi f Y(f) \quad (2.8)$$

- Define

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \quad (2.9)$$

Show that

$$e^{-at}u(t) \xrightarrow{\mathcal{F}} \frac{1}{a + j2\pi f} \quad (2.10)$$

7. If $x(t) = \delta(t)$ in (1.1), find $y(t)$ using Fourier transforms. This is known as the *impulse response* and is denoted by $h(t)$.
8. Solve (1.1) for $x(t) = \cos 2\pi f_0 t$ using Fourier transforms.
9. Verify that

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau) d\tau \quad (2.11)$$

This is known as the *convolution integral*.

10. Show that

$$x(t) * h(t) \xrightarrow{\mathcal{F}} X(f)H(f) \quad (2.12)$$

11. In (1.1), show that

$$H(f) = \frac{1}{1 + j2\pi fRC} \quad (2.13)$$

12. Show that

$$y(t) = |H(f_0)| \cos [2\pi f_0 t + \angle H(f_0)] \quad (2.14)$$

3 FILTERING

1. Plot $|H(f)|$ for different values of RC . The following code plots Fig. 1.

```
import numpy as np
import matplotlib.pyplot as plt
import subprocess
import shlex

#Filter Characteristics

R = 5*1e2; #1K ohm resistance
C = 10e-6; #10 uF capacitance

#Plotting the filter amplitude response
f_0 = 50.0
f = np.linspace(-3*f_0, 3*f_0, 1e2)
s = 1j*2*np.pi*f

den = np.polyval([1,1],s*C*R);
H = 1/den;

plt.plot(f,abs(H))
plt.grid()# minor
plt.xlabel('$f$ (Hz)')
plt.ylabel('$|H(f)|$')
```

```
#Save figure
#plt.savefig('../figs/lpf.eps')
#plt.savefig('lpf.pdf')
#subprocess.run(shlex.split("termux-open lpf.
pdf"))
plt.show()
```

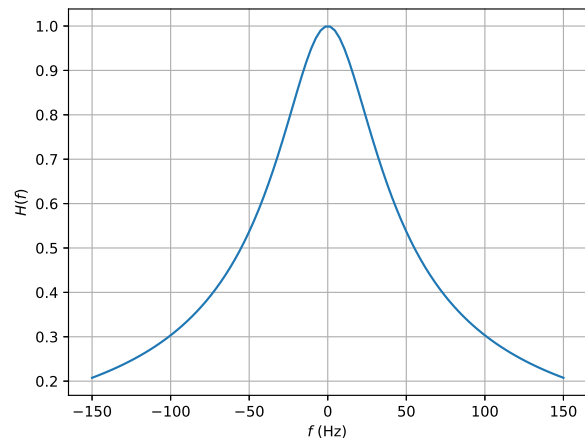


Fig. 1: Low Pass Filter

2. Sketch the output when $x(t) = \cos 100\pi t + \cos 300\pi t$, $R = 500\Omega$ and $C = 10\mu F$. Comment.