

# Fourier Transforms

G V V Sharma\*

## CONTENTS

1	<b>Sinusoidal Response</b>	1
2	<b>Fourier Transform</b>	1
3	<b>Filtering</b>	2

**Abstract**—This manual provides a quick introduction to the Fourier transform.

## 1 SINUSOIDAL RESPONSE

- Fig. 1 shows an  $RC$  circuit with input  $x(t)$  and output  $y(t)$ . Show that

$$RC \frac{dy}{dt} + y(t) = x(t) \quad (1.1)$$

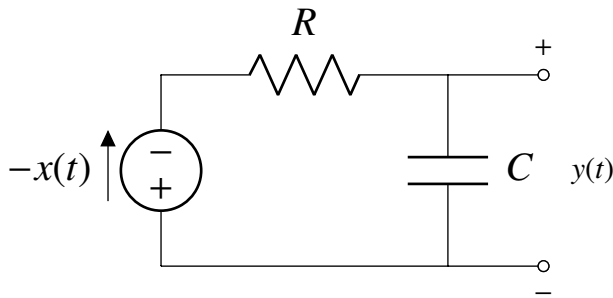


Fig. 1: RC Circuit

- Let  $x(t) = \cos 2\pi f_0 t$ . Show that

$$y(t) = \frac{1}{\sqrt{1 + (2\pi f_0 RC)^2}} \cos \left[ 2\pi f_0 \left( t - \tan^{-1} RC \right) \right] \quad (1.2)$$

\*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in. All content in this manual is released under GNU GPL. Free and open source.

by solving the differential equation in (1.1) using the integrating factor.

(1.3)

## 2 FOURIER TRANSFORM

The Fourier transform of a signal  $g(t)$  is defined as

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt \quad (2.1)$$

- Define the Dirac delta function as

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad (2.2)$$

$$\delta(t) = 0, \quad t \neq 0.$$

and show that

$$\delta(t) \stackrel{\mathcal{F}}{\rightleftharpoons} 1 \quad (2.3)$$

- Show that

$$\delta(t - t_0) \stackrel{\mathcal{F}}{\rightleftharpoons} e^{-j2\pi f t_0} \quad (2.4)$$

- Show that

$$g(t) \stackrel{\mathcal{F}}{\rightleftharpoons} G(f) \quad (2.5)$$

$$\Rightarrow G(t) \stackrel{\mathcal{F}}{\rightleftharpoons} g(-f)$$

assuming

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df. \quad (2.6)$$

(2.6) is known as the inverse Fourier transform.

- Show that

$$\cos 2\pi f_0 t \stackrel{\mathcal{F}}{\rightleftharpoons} \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)] \quad (2.7)$$

- Show that

$$\frac{dy}{dt} \stackrel{\mathcal{F}}{\rightleftharpoons} j2\pi f Y(f) \quad (2.8)$$

- Define

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \quad (2.9)$$

Show that

$$e^{-at}u(t) \xrightarrow{\mathcal{F}} \frac{1}{a + j2\pi f} \quad (2.10)$$

7. If  $x(t) = \delta(t)$  in (1.1), find  $y(t)$  using Fourier transforms. This is known as the *impulse response* and is denoted by  $h(t)$ .
8. Solve (1.1) for  $x(t) = \cos 2\pi f_0 t$  using Fourier transforms.
9. Verify that

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t - \tau) d\tau \quad (2.11)$$

This is known as the *convolution integral*.

10. Show that

$$x(t) * h(t) \xrightarrow{\mathcal{F}} X(f)H(f) \quad (2.12)$$

11. In (1.1), show that

$$H(f) = \frac{1}{1 + j2\pi fRC} \quad (2.13)$$

12. Show that

$$y(t) = |H(f_0)| \cos [2\pi f_0 t + \angle H(f_0)] \quad (2.14)$$

### 3 FILTERING

1. Plot  $|H(f)|$  for different values of  $RC$ . The following code plots Fig. 1.

```
import numpy as np
import matplotlib.pyplot as plt
import subprocess
import shlex

#Filter Characteristics

R = 5*1e2; #1K ohm resistance
C = 10e-6; #10 uF capacitance

#Plotting the filter amplitude response
f_0 = 50.0
f = np.linspace(-3*f_0, 3*f_0, 1e2)
s = 1j*2*np.pi*f

den = np.polyval([1,1],s*C*R);
H = 1/den;

plt.plot(f,abs(H))
plt.grid()# minor
plt.xlabel('$f$ (Hz)')
plt.ylabel('$|H(f)|$')
```

```
#Save figure
#plt.savefig('../figs/lpf.eps')
#plt.savefig('lpf.pdf')
#subprocess.run(shlex.split("termux-open lpf.
pdf"))
plt.show()
```

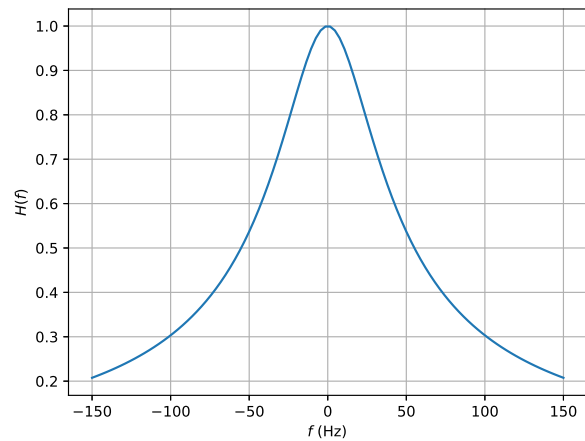


Fig. 1: Low Pass Filter

2. Sketch the output when  $x(t) = \cos 100\pi t + \cos 300\pi t$ ,  $R = 500\Omega$  and  $C = 10\mu\text{F}$ . Comment.