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Abstract—This manual provides a simple introduction to digital signal processing.

1 SOFTWARE INSTALLATION

Run the following commands

```
sudo apt-get update
sudo apt-get install libffi-dev libsndfile1 python3
    -scipy python3-numpy python3-matplotlib
sudo pip install cffi pysoundfile
```

2 DIGITAL FILTER

2.1 Download the sound file from

```
wget https://raw.githubusercontent.com/
gadepall/
EE1310/master/filter/codes/Sound_Noise.wav
```

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- 2.2 You will find a spectrogram at <https://academo.org/demos/spectrum-analyzer>. Upload the sound file that you downloaded in Problem 2.1 in the spectrogram and play. Observe the spectrogram. What do you find?
Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.
- 2.3 Write the python code for removal of out of band noise and execute the code.

Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('Sound_Noise.wav'
    )

#sampling frequency of Input signal
sampl_freq=fs

#order of the filter
order=4

#cutoff frequency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and denominator
    polynomials respectively
b, a = signal.butter(order,Wn, 'low')

#filter the input signal with butterworth filter
output_signal = signal.filtfilt(b, a,
    input_signal)
```

```
#write the output signal into .wav file
sf.write('Sound_With_ReducedNoise.wav',
        output_signal, fs)
```

2.4 The output of the python script in Problem 2.3 is the audio file Sound_With_ReducedNoise.wav. Play the file in the spectrogram in Problem 2.2. What do you observe?

Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz.

3 DIFFERENCE EQUATION

3.1 Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \quad (3.1)$$

Sketch $x(n)$.

3.2 Let

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2),$$

$$y(n) = 0, n < 0 \quad (3.2)$$

Sketch $y(n)$.

Solution: The following code yields Fig. 3.2.

```
wget https://github.com/gadepall/EE1310/raw/
master/filter/codes/xnyn.py
```

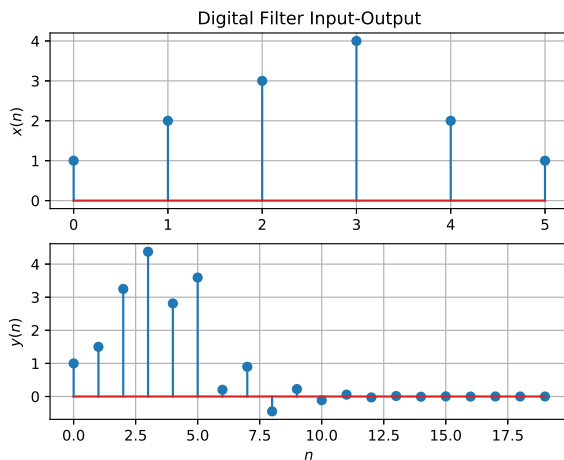


Fig. 3.2

4 Z-TRANSFORM

4.1 The Z-transform of $x(n]$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4.1)$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \quad (4.2)$$

and find

$$\mathcal{Z}\{x(n-k)\} \quad (4.3)$$

Solution: From (4.1),

$$\mathcal{Z}\{x(n-k)\} = \sum_{n=-\infty}^{\infty} x(n-k)z^{-n} \quad (4.4)$$

$$= \sum_{n=-\infty}^{\infty} x(n)z^{-n-1} = z^{-1} \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4.5)$$

resulting in (4.2). Similarly, it can be shown that

$$\mathcal{Z}\{x(n-k)\} = z^{-k}X(z) \quad (4.6)$$

4.2 Find

$$H(z) = \frac{Y(z)}{X(z)} \quad (4.7)$$

from (3.2) assuming that the Z-transform is a linear operation.

Solution: Applying (4.6) in (3.2),

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \quad (4.8)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (4.9)$$

4.3 Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.10)$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (4.11)$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (4.12)$$

Solution: It is easy to show that

$$\delta(n) \stackrel{\mathcal{Z}}{\rightleftharpoons} 1 \quad (4.13)$$

and from (4.11),

$$U(z) = \sum_{n=0}^{\infty} z^{-n} \quad (4.14)$$

$$= \frac{1}{1 - z^{-1}}, \quad |z| > 1 \quad (4.15)$$

using the formula for the sum of an infinite geometric progression.

4.4 Show that

$$a^n u(n) \stackrel{Z}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \quad (4.16)$$

4.5 Let

$$H(e^{j\omega}) = H(z = e^{j\omega}). \quad (4.17)$$

Plot $|H(e^{j\omega})|$. Comment. $H(e^{j\omega})$ is known as the *Discrete Time Fourier Transform* (DTFT) of $x(n)$.

Solution: The following code plots Fig. 4.5.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/dtft.
py
```

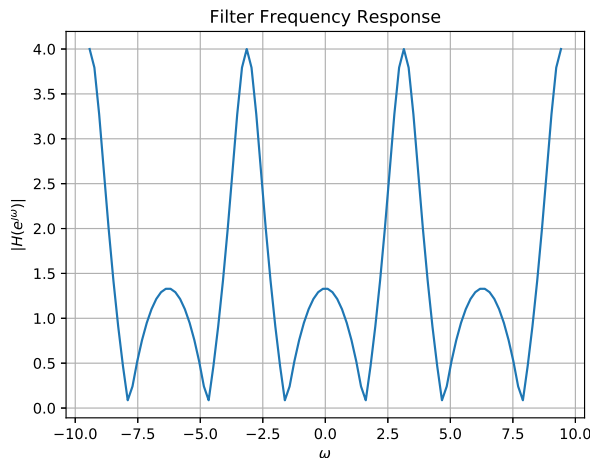


Fig. 4.5: $|H(e^{j\omega})|$

5 IMPULSE RESPONSE

5.1 Find an expression for $h(n)$ using $H(z)$, given that

$$h(n) \stackrel{Z}{\rightleftharpoons} H(z) \quad (5.1)$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse*

response of the system defined by (3.2).

Solution: From (4.9),

$$H(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (5.2)$$

$$\Rightarrow h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \quad (5.3)$$

using (4.16) and (4.6).

5.2 Sketch $h(n)$. Is it bounded? Convergent?

Solution: The following code plots Fig. 5.2.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/hn.py
```

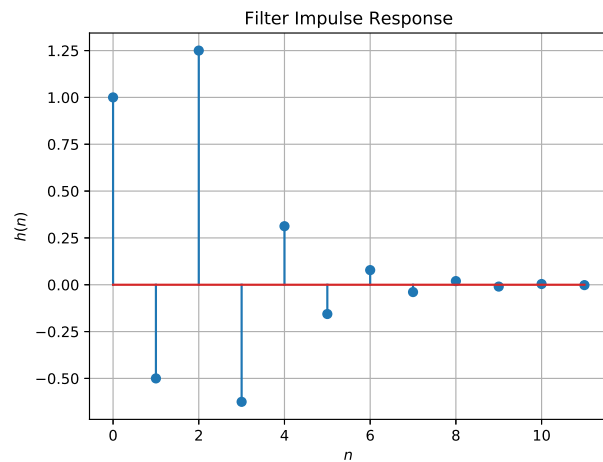


Fig. 5.2: $h(n)$ as the inverse of $H(z)$

5.3 The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \quad (5.4)$$

Is the system defined by (3.2) stable for the impulse response in (5.1)?

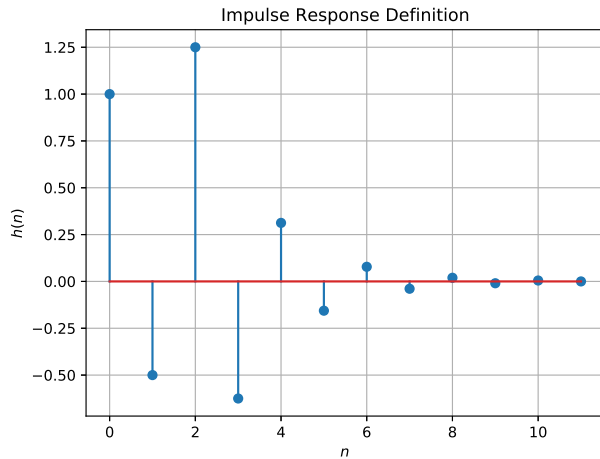
5.4 Compute and sketch $h(n)$ using

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2), \quad (5.5)$$

This is the definition of $h(n)$.

Solution: The following code plots Fig. 5.4. Note that this is the same as Fig. 5.2.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/hndef
.py
```

Fig. 5.4: $h(n)$ from the definition

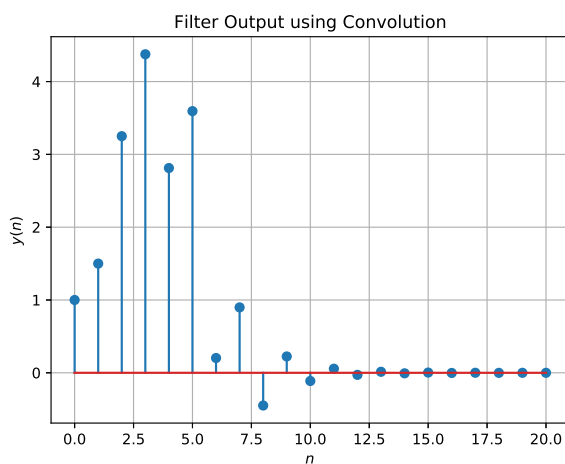
5.5 Compute

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad (5.6)$$

Comment. The operation in (5.6) is known as *convolution*.

Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/
ynconv.py
```

Fig. 5.5: $y(n)$ from the definition of convolution

5.6 Show that

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k) \quad (5.7)$$

6 DFT AND FFT

6.1 Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \quad (6.1)$$

and $H(k)$ using $h(n)$.

6.2 Compute

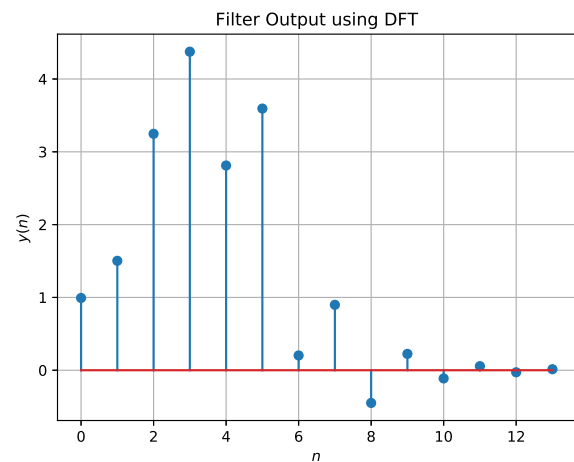
$$Y(k) = X(k)H(k) \quad (6.2)$$

6.3 Compute

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot e^{j2\pi kn/N}, \quad n = 0, 1, \dots, N-1 \quad (6.3)$$

Solution: The following code plots Fig. 5.5. Note that this is the same as $y(n)$ in Fig. 3.2.

```
wget https://raw.githubusercontent.com/
gadepall/EE1310/master/filter/codes/yndft.
py
```

Fig. 6.3: $y(n)$ from the DFT

6.4 Repeat the previous exercise by computing $X(k)$, $H(k)$ and $y(n)$ through FFT and IFFT.

6.5 Wherever possible, express all the above equations as matrix equations.

7 EXERCISES

Answer the following questions by looking at the python code in Problem 2.3.

7.1 The command

```
output_signal = signal.filtfilt(b, a,
                                input_signal)
```

in Problem 2.3 is executed through the following difference equation

$$\sum_{m=0}^M a(m) y(n-m) = \sum_{k=0}^N b(k) x(n-k) \quad (7.1)$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.

7.2 Repeat all the exercises in the previous sections for the above a and b .

7.3 What is the sampling frequency of the input signal?

Solution: Sampling frequency(fs)=44.1kHz.

7.4 What is type, order and cutoff-frequency of the above butterworth filter

Solution: The given butterworth filter is low pass with order=2 and cutoff-frequency=4kHz.

7.5 Modifying the code with different input parameters and to get the best possible output.