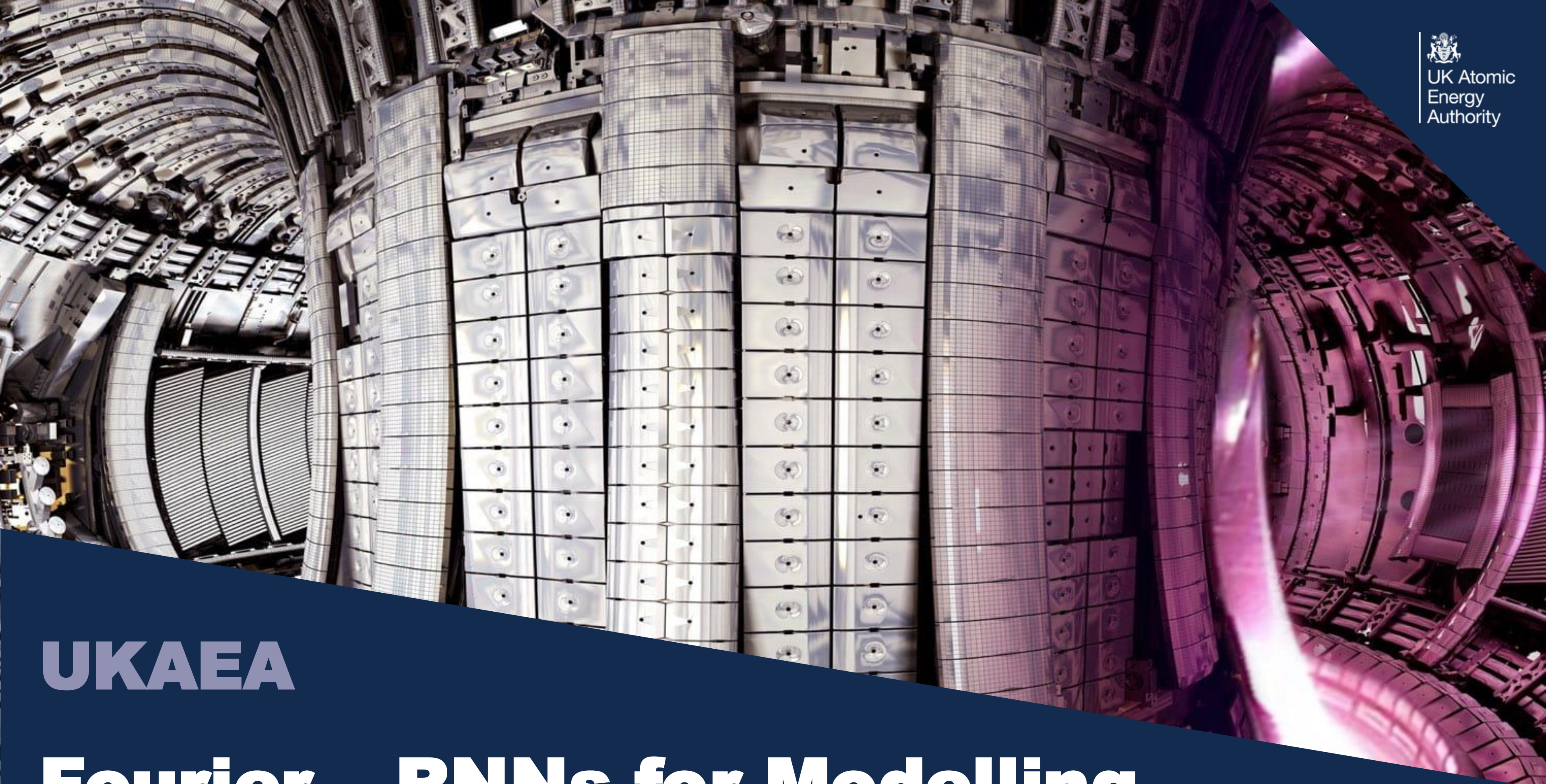

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Fourier – RNNs for Modelling Noisy Physics Data

Vignesh Gopakumar, Stanislas Pamela, Lorenzo Zanisi

IEEE ICMLA 2022

Outline

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- Neural Operator Learning
- Fourier Neural Operators
- Fourier-RNNs
- Experiments
- Scaling Up !!

Neural Operator Learning: Theory

Operator Learning : Learning the mapping between vector spaces of functions.

$$G_\theta : A \to U,\ \theta \,\epsilon\, \Theta$$

The Neural Network in the above configuration is seen as learning the approximation of Green's Function of the PDE.

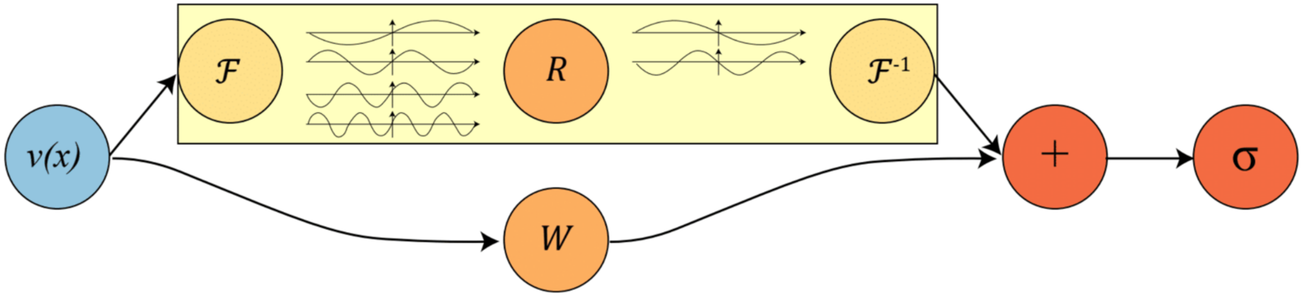
$$u_\lambda(x) = \int_D G_\lambda(x, y) f(y)\, \mathrm{d}y$$

$$\mathrm{NO}_\Theta(f) = \mathrm{NO}_\theta(f) = \int_D g_\theta(x, y) f(y)\, \mathrm{d}y.$$

Learning the dominant modes

Zongyi Li et al. Fourier neural operator for parametric partial differential equations, 2020
Emilia Magnani et al. Approximate Bayesian Neural Operators: Uncertainty Quantification for Parametric PDEs, 2022.

Fourier Layer



Schematic Layout of a Fourier Layer within FNO

$$y = \sigma\left(\mathcal{F}^{-1}(R\mathcal{F}(x)) + Wx\right)$$

Source : Zongyi et al. Fourier Neural Operator for Parametric Partial Differential Equations (ICLR 2021)

Autoregressive FNO

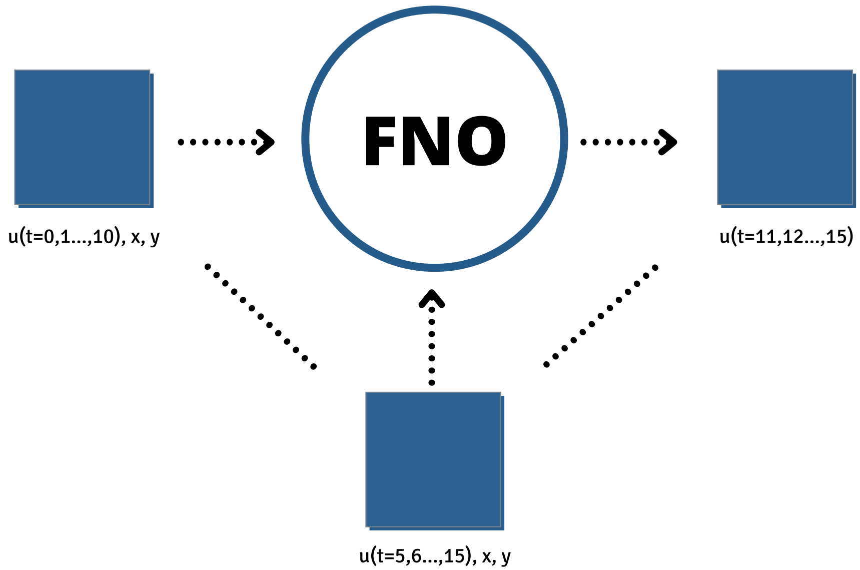
- Models Spatio-Temporal Evolution

- Inputs the first 10 time instances of the field variable to output the next 5 time instances.

- The 5 time instances output from the FNO is mixed with the last 5 time instances at the initial input to output the next 5 time time instances (11…15)

- Loop continued until the desired time length.

- FNO is trained to minimize the the reconstruction error (MSE) across the network output and the simulation.



Autoregressive FNO

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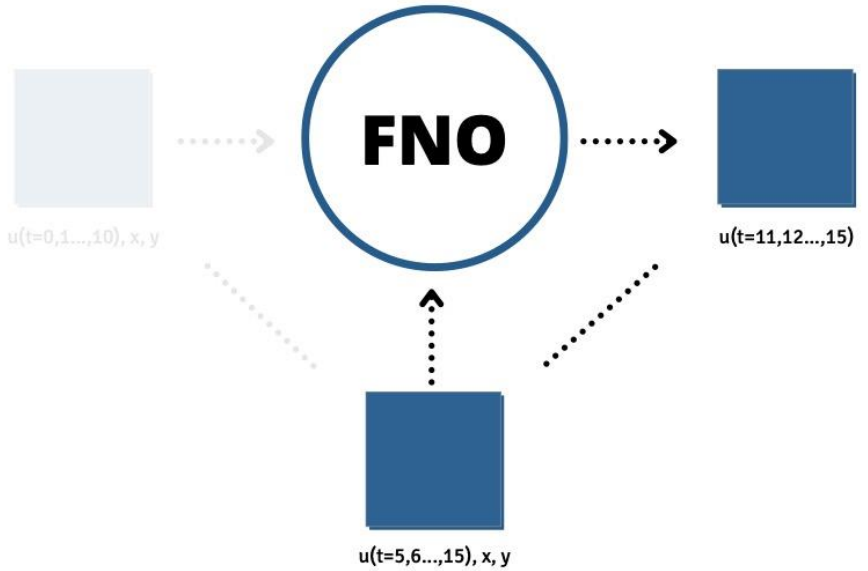
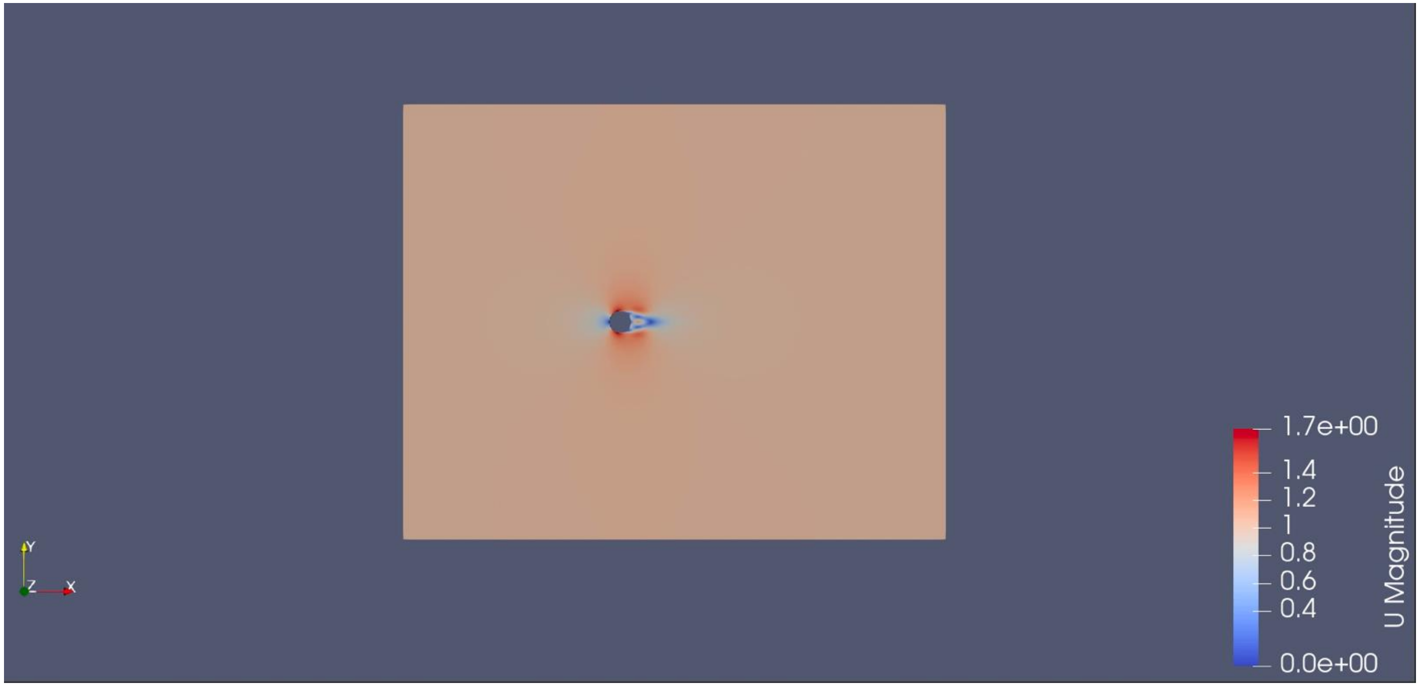


Vortex Shedding – Navier-Stokes flow around a cylinder

Initial Conditions : U_x= 1.0 m/s (left to right), U_y= 0 m/s, p = 0.0 Pa

Fixed cylinder size

Training dataset built across a range of Reynolds Numbers moving from the the Laminar regime to Turbulent regime.



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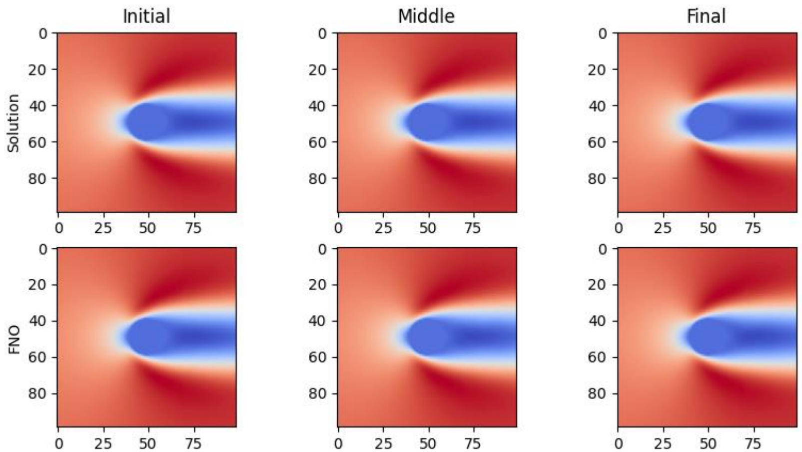
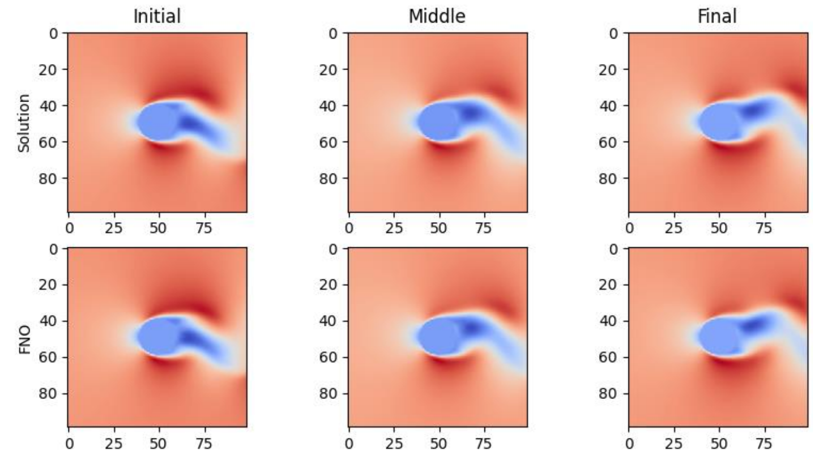
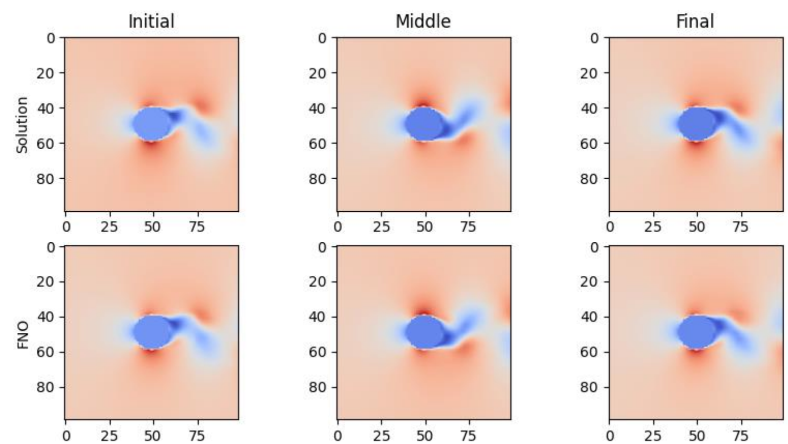
Re : 40 - 200



Re : 200 - 400





Re : 10000 - 100000

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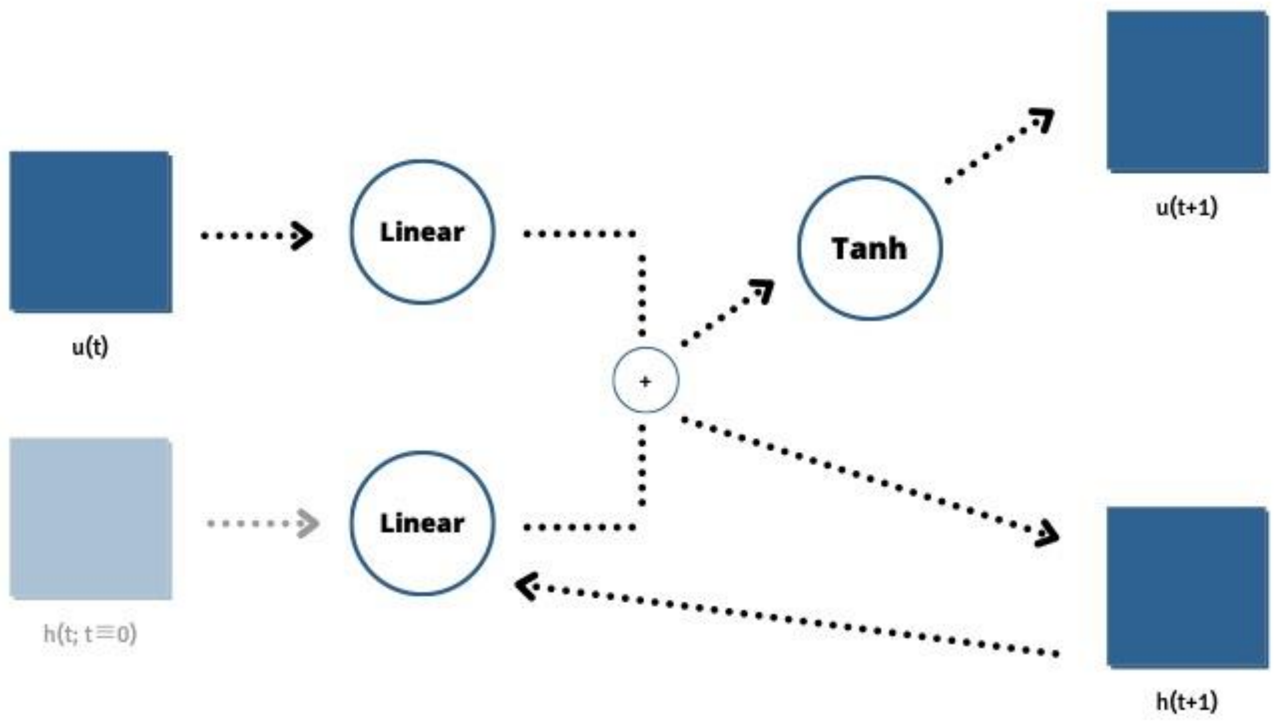
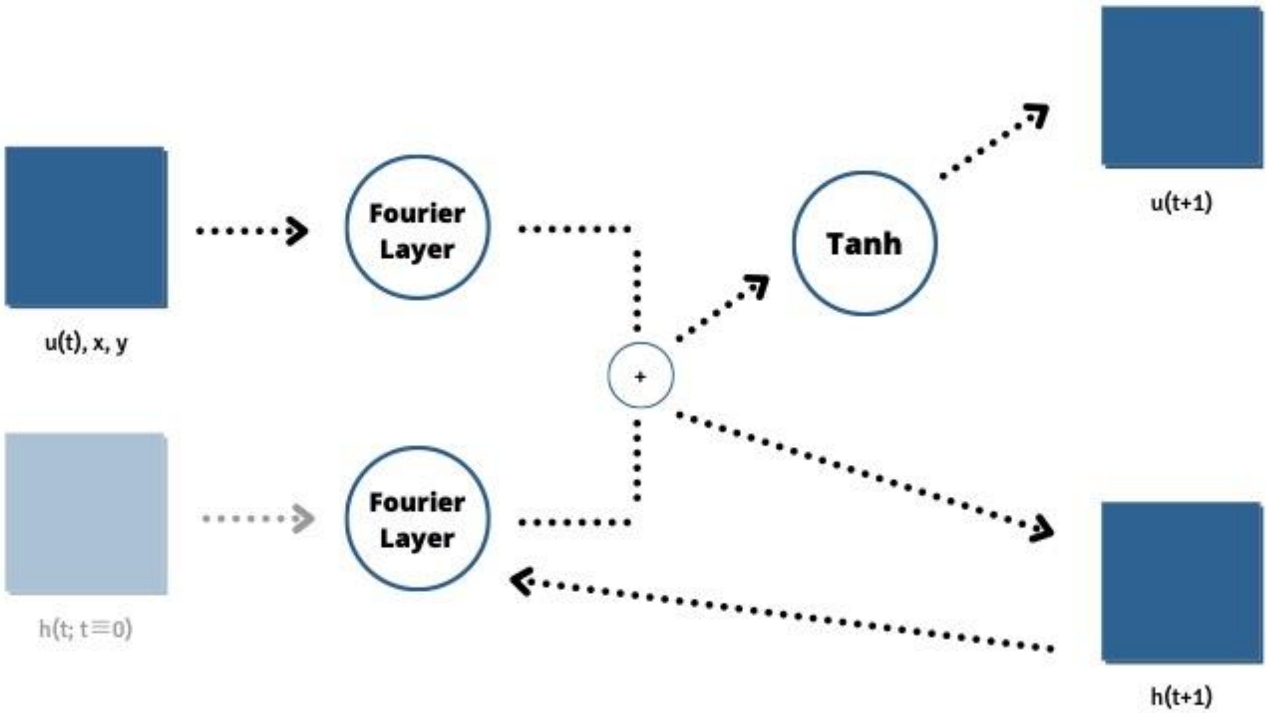
“But what happens when the data is Non-Markovian ?”

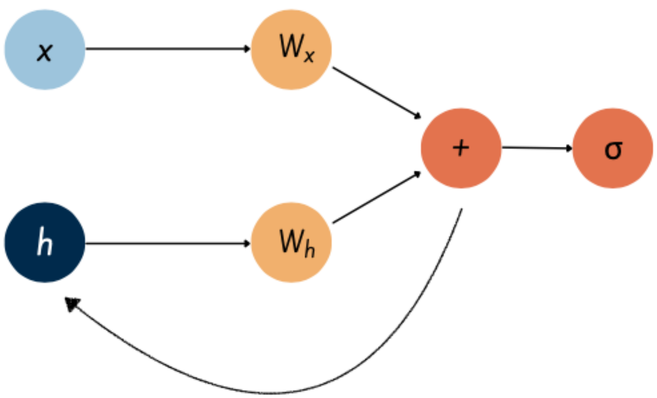
Recurrent Neural Networks (RNN)



Fourier - RNN

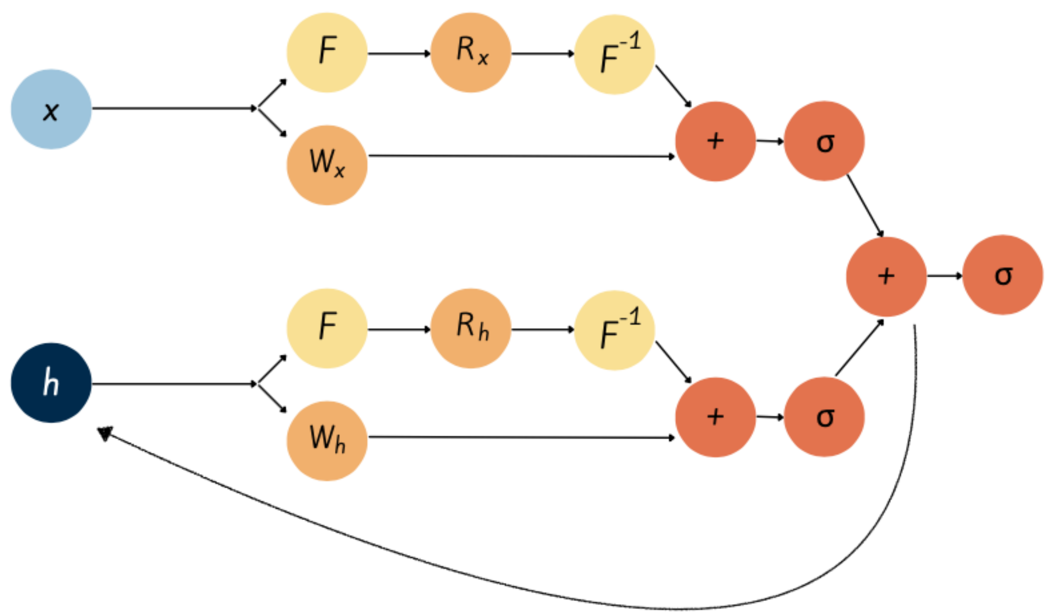


RNN Cell and F-RNN Cell



$$h_t = W_x x_t + W_h h_{t-1}$$
$$y_t = \sigma(h_t)$$

$$h_t = \mathcal{F}^{-1}\big(R_x \mathcal{F}(x_t)\big) + W_x x_t + \mathcal{F}^{-1}\big(R_h \mathcal{F}(h_{t-1})\big) + W_h h_{t-1}$$
$$y_t = \sigma(h_t)$$



Navier Stokes Flow with Noise

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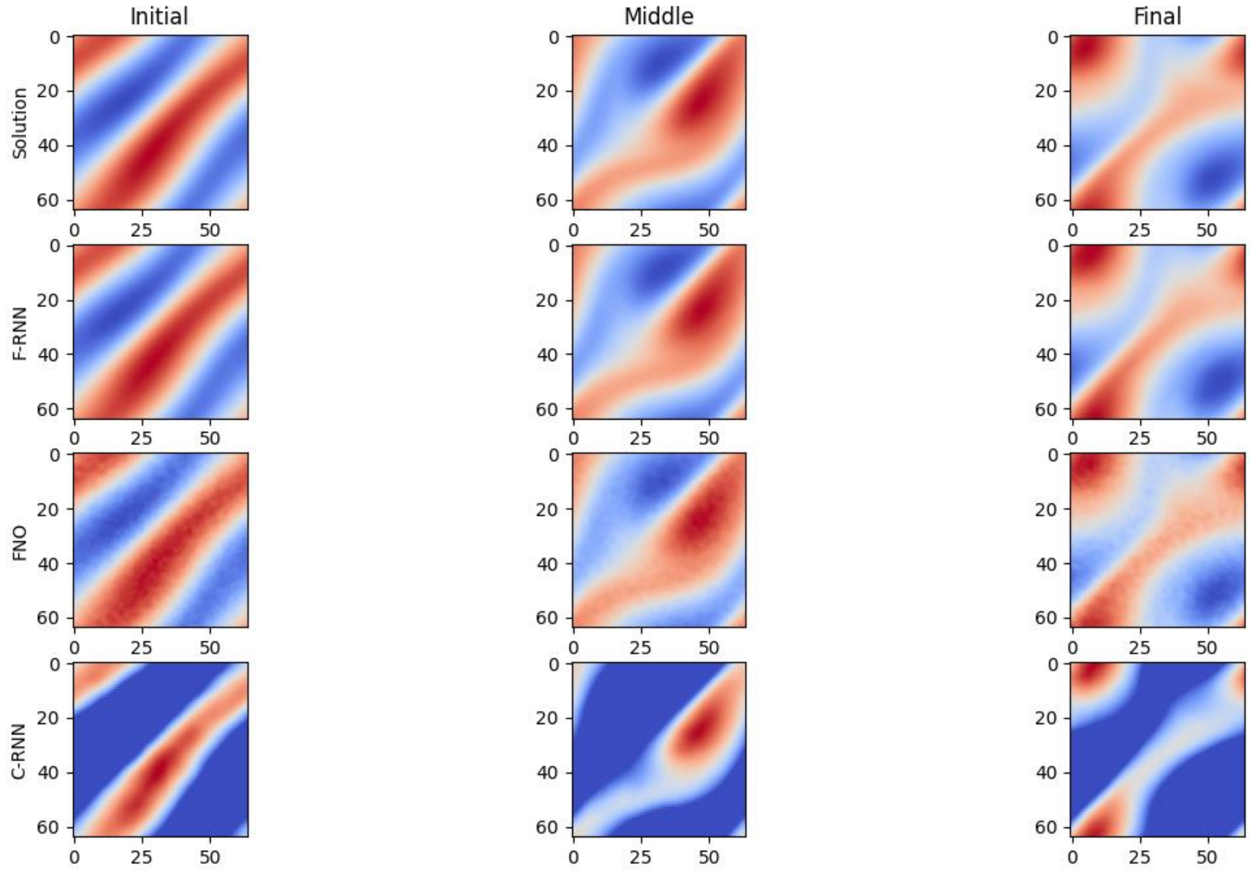


Noisy Data is given as:

$$\tilde{x} = x + \mathcal{N}(0, N)$$

Where, the noise factor is characterised by the variance of Normal Distribution centred at 0.

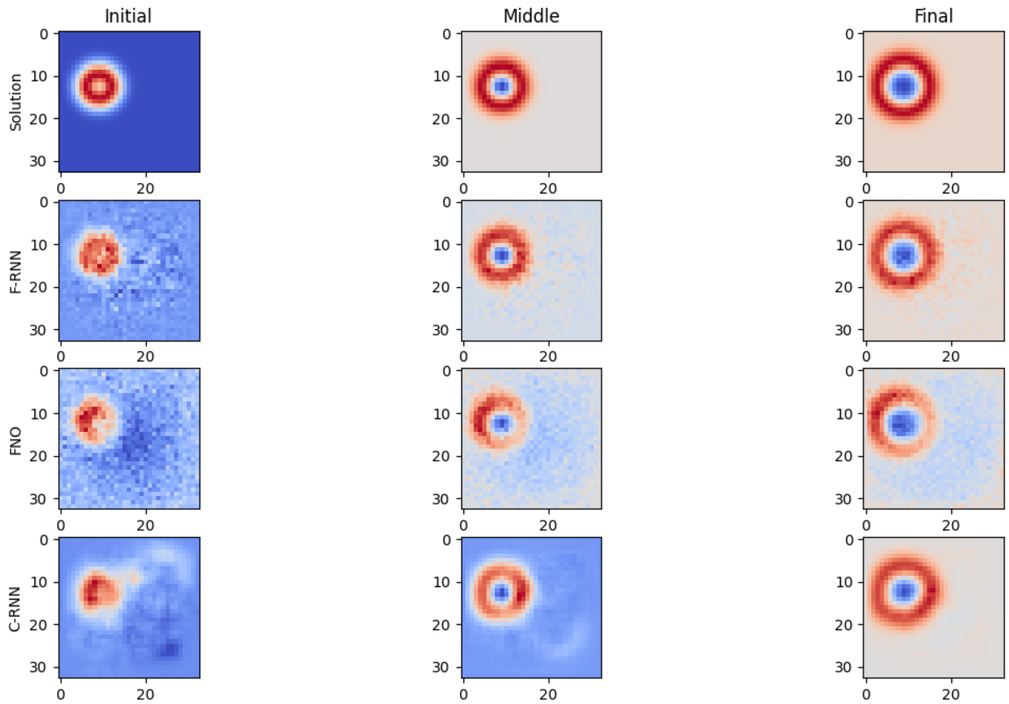
Navier Stokes Flow - Laminar

Model	N = 0.0	N = 0.05	N = 0.1	N = 0.25
C-RNN	0.4789	0.4785	0.4786	0.4791
FNO	**0.000365**	**0.0006603**	0.002565	0.01368
F-RNN	0.0008505	0.0009457	**0.001071**	**0.001499**

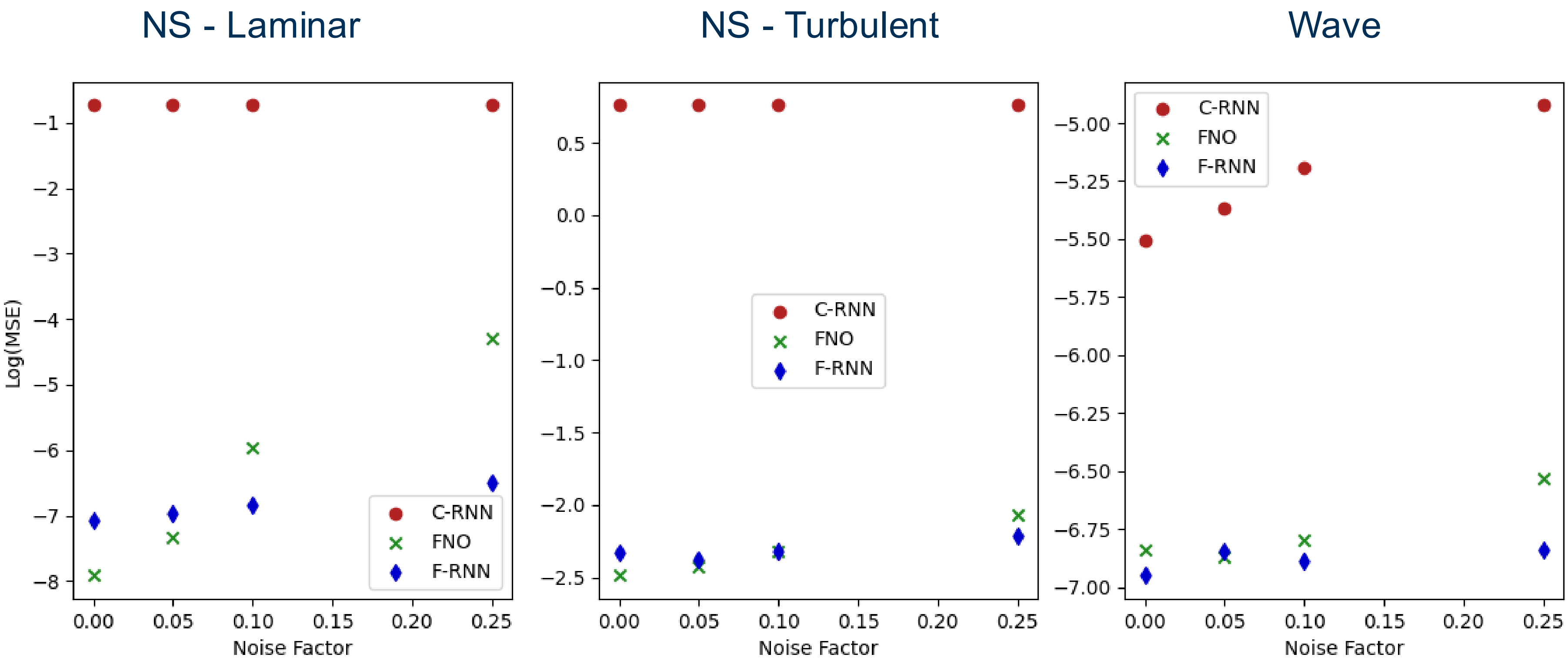
Navier Stokes Flow - Turbulent

Model	N = 0.0	N = 0.05	N = 0.1	N = 0.25
C-RNN	2.137	2.137	2.137	2.137
FNO	**0.08301**	**0.08792**	0.09808	0.1261
F-RNN	0.097	0.09234	**0.09793**	**0.1089**

Wave Dynamics with Noise



Model	N = 0.0	N = 0.05	N = 0.1	N = 0.25
C-RNN	0.004069	0.004656	0.005564	0.00727
FNO	0.001072	**0.001038**	0.001116	0.001461
F-RNN	**0.0009589**	0.001064	**0.001021**	**0.001073**



NS - Laminar
NS - Turbulent
Wave
Log(MSE)
Noise Factor
C-RNN
FNO
F-RNN
0.00
0.05
0.10
0.15
0.20
0.25
−1
−2
−3
−4
−5
−6
−7
−8
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−5.00
−5.25
−5.50
−5.75
−6.00
−6.25
−6.50
−6.75
−7.00