

Lecture 6: 8 September 2023

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6.1 Linear Discriminant Analysis

In the following, r_t is an indicator variable to select elements belonging to a certain class.

1. Between class scatter is given by

$$(m_1 - m_2)^2 = (\mathbf{w}^\top \mathbf{m}_1 - \mathbf{w}^\top \mathbf{m}_2)^2 \quad (6.1)$$

$$= \mathbf{w}^\top (\mathbf{m}_1 - \mathbf{m}_2) (\mathbf{m}_1 - \mathbf{m}_2)^\top \mathbf{w} \quad (6.2)$$

$$= \mathbf{w}^\top \mathbf{S}_B \mathbf{w} \quad (6.3)$$

where we define

$$\mathbf{S}_B \triangleq (\mathbf{m}_1 - \mathbf{m}_2) (\mathbf{m}_1 - \mathbf{m}_2)^\top. \quad (6.4)$$

2. For any class, within class scatter is given by

$$s_i^2 = \sum_t (\mathbf{w}^\top \mathbf{x}_t - m_1)^2 r_t \quad (6.5)$$

$$= \sum_t \mathbf{w}^\top (\mathbf{x}_t - \mathbf{m}_1) (\mathbf{x}_t - \mathbf{m}_1)^\top \mathbf{w} r_t \quad (6.6)$$

$$= \mathbf{w}^\top \mathbf{S}_i \mathbf{w} \quad (6.7)$$

where we define

$$S_i \triangleq \sum_t (\mathbf{x}_t - \mathbf{m}_1) (\mathbf{x}_t - \mathbf{m}_1)^\top r_t. \quad (6.8)$$

3. For multiple classes, we define

$$\mathbf{S}_w \triangleq \sum_i \mathbf{S}_i. \quad (6.9)$$