Formalising large corner-free sets

Teaching my laptop to simplify asymptotics ~~automatically~~

Gareth Ma

University of Warwick

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Imagine...

You are Andrew Wiles.

You worked on a marvelous proof on a conjecture for 6 years, and finally publish it.

Two months later, a critical flaw was discovered, and your work is voided. ☹️

If only that computers can verify Mathematical proofs...

Formalise? What?

We can **formalise** proofs. Informally, it means to

Definition

Rewrite Mathematical proofs in a machine-understandable language.

The language I used is the `Lean 4` language + its Mathematics library `Mathlib 4`.

Formalise? What? 👀

Transitivity

Let P, Q, R be *logical* statements. If $P \implies Q$ and $Q \implies R$, then $P \implies R$.

Proof.

Suppose P holds. Then by $P \implies Q$, we know that Q holds. And since $Q \implies R$, we know that R holds. Hence, P implies R. $\square$

```
example {P Q R : Prop} (hPQ : P → Q) (hQR : Q → R) :
    P → R := by
  intro p
  have := hPQ p
  exact hQR this
```

Formalise what? (My project)

For my 3rd year project, I formalised a extremal combinatorics result in 2021 by Ben Green.

The resulting project is original work building on top of the `Lean 4` + `Mathlib 4` libraries.

To my knowledge, this is the **best** result of this type formalised in any theorem prover.

1. Type Theory

2. Conclusion

Flaws of set theory

In (naive) set theory, everything is a set. Numbers are encoded as nested sets, operations are set functions, etc.

There are many problems:

- $3 \subseteq 17$ is a valid question.
- Russell's Paradox: $A \in A$ holds.

Flaws of set theory

Slightly absurdly, the problem fundamentally stems from that **everything is a set**.

Idea

Flaws of set theory

Flaws of set theory

Flaws of set theory