Semidefinite Optimization and Relaxation

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Contents

Preface

This is the textbook for Harvard ENG-SCI 257: Semidefinite Optimization and Relaxation. Information about the offerings of the class is listed below.

2024 Spring

Time: Mon/Wed 2:15 - 3:30pm

Location: Science and Engineering Complex, 1.413

Instructor: Heng Yang

Teaching Fellow: Safwan Hossain

Syllabus

Acknowledgment

Notation

We will use the following standard notation throughout this book.

Basics

$\mathbb{R}$	real numbers
$\mathbb{R}_+$	nonnegative real
$\mathbb{R}_{++}$	positive real
$\mathbb{Z}$	integers
$\mathbb{N}$	nonnegative integers
$\mathbb{R}^n$	n-D column vector
$\mathbb{R}^n_+$	nonnegative orthant
$\mathbb{R}^n_{++}$	positive orthant
e_i	standard basic vector
$\Delta_n := \{x \in \mathbb{R}^n_+ \mid \sum x_i = 1\}$	standard simplex

Matrices

$\mathbb{R}^{m\times n}$	$m \times n$ real matrices
$\mathbb{S}^n$	$n \times n$ symmetric matrices
$\mathbb{S}^n_+$	$n \times n$ positive semidefinite matrices
$\mathbb{S}^n_{++}$	$n \times n$ positive definite matrices
$\langle A, B \rangle$ or $\bullet$	inner product in $\mathbb{R}^{m\times n}$
$\mathrm{tr}(A)$	trace of $A \in \mathbb{R}^{n\times n}$
$A^\top$	matrix transpose
$\det(A)$	matrix determinant
$\mathrm{rank}(A)$	rank of a matrix
$\mathrm{diag}(A)$	diagonal of a matrix A as a vector

$\mathrm{Diag}(a)$	turning a vector into a diagonal matrix
$\mathrm{BlkDiag}(A, B, \dots)$	block diagonal matrix with blocks $A, B, \dots$
$\succeq 0$ and $\preceq 0$	positive / negative semidefinite
$\succ 0$ and $\prec 0$	positive / negative definite
$\lambda_{\max}$ and $\lambda_{\min}$	maximum / minimum eigenvalue
$\sigma_{\max}$ and $\sigma_{\min}$	maximum / minimum singular value
$\mathrm{vec}(A)$	vectorization of $A \in \mathbb{R}^{m \times n}$
$\mathrm{svec}(A)$	symmetric vectorization of $A \in \mathbb{S}^n$
$\\|A\\|_{\mathrm{F}}$	Frobenius norm

Geometry

$\\|a\\|_p$	p-norm
$\\|a\\|$	2-norm
$B(o, r)$	ball with center o and radius r
$\mathrm{conv}(S)$	convex hull of set S
$\mathrm{cone}(S)$	conical hull of set S
$\mathrm{int}(S)$	interior of set S
∂S	boundary of set S
P°	polar dual of convex body
$\mathrm{SO}(d)$	special orthogonal group of dimension d
$\mathcal{S}^{d-1}$	unit sphere in $\mathbb{R}^d$

Optimization

KKT	Karush–Kuhn–Tucker
LP	linear program
QP	quadratic program
SOCP	second-order cone program
SDP	semidefinite program

Algebra

$\mathbb{R}[x]$	polynomial ring in x with real coefficients
deg	degree of a monomial / polynomial
$\mathbb{R}[x]_d$	polynomials in x of degree up to d
$[x]_d$	vector of monomials of degree up to d
$[\![x]\!]_d$	vector of monomials of degree d

Chapter 1

Mathematical Background

Chapter 2

Semidefinite Optimization