Math 181
Day 23 Notes

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How to study Counting Words in the History of Mathematics?

Ancient Greek has a similar structure to English.

English uses opaque words for zero through 10, then for 13-19 the prefix teen is used. 11 and 12 is odd, but 20 and above uses a system of prefixes.

Santa Barbara Native language had two bases, 10 and 4.

Had subtractive principle, some numbers are spoken as less than some fixed number.

The closest thing to this in English is time.

Formalizing Counting Words

Existing framework known as Generative Grammer.

Hurford

1. Formal Grammer for counting
2. Linguistic rules for converting numbers into words
3. Recognizing well formed words

As an example, 400 as four hundred vs. twenty twenty.

In english the second is not well formed since it does not follow standard convention.

What is a formal Grammer?

A formal grammer consists of:

1. Terminal symbols
2. Non Terminal Symbols
3. A distinguished non-terminal symbol called the Start State
4. A set of rules of the form $NT \rightarrow$ some symbols in the grammer

Examle:

1. Terminal symbols: $\{a, b\}$
2. Non Terminal Symbols: $\{S\}$
3. Rules: $\{S \rightarrow a|S, S \rightarrow b\}$

To produce a valid word in the grammer, we begin at the start state, then select rules to apply until we run out of rules.

ex:

$$S \quad (S \rightarrow aS) \tag{1}$$
$$aS \quad (S \rightarrow aS) \tag{2}$$
$$aaS \quad (S \rightarrow b) \tag{3}$$
$$aab \tag{4}$$
$$\tag{5}$$

The Language of a formal grammer is all words in terminal symbols produced from S by repeatedly applying rules. (Somtimes known as production rules)

We can produce trees from the process of generating words in the following way.

The root begins with the start symbol, at each level all avalible rules produce branches on this tree. The path from the root to a leaf is the order of which rules are applied to generate a specific word.

We can also construct a tree with a start symbol at the root, and each level are the symbols produced by applying some rule. For example the word aab in the above language would have a tree S, then a, S, then a, S, then b.

Formal Grammer For Numbers in English

1. Terminal Symbols $\{I, X\}$ (one, ten)
2. Non Terminal Symbols $\{N, P, M\}$ Which stand for Number, Phrase, and Multiply
3. Production Rules:
   - $N \to I$
   - $N \to IN$
   - $N \to P$
   - $N \to PN$
   - $P \to NM$
   - $M \to X$
   - $M \to NM$
4. Notes: N is our start symbol

For example:

$$N \tag{6}$$
$$IN \tag{7}$$
$$\ IN \tag{8}$$
$$\quad IN \tag{9}$$
$$... \tag{10}$$

$$N \tag{11}$$
$$P \quad N \tag{12}$$
$$N \quad M \tag{13}$$
$$I \quad N \quad M \tag{14}$$
$$I \quad I \quad X \tag{15}$$
$$\tag{16}$$

Interpretation

Each node in a tree produced by the grammer has a value determined as follows.

- $I = 1$
- $X = 10$
- $N = x + y$
- $N = x$
- $M = x^y$
- $M = x$
- $P = x \cdot y$

The Plan

1. Convert Trees to Words
2. Rules for Recognizing Words
3. Rules for Generating Words

We can start categorizing trees with names. A tree with a N root, and I and three as children, that is four.

We can continue like this.

One Hundred is a tree with a phrase, a left child of one, and a right child of hundred.

A hundred is a tree with a multiply, with a two on one side and a ten on the other.

How could we analyze the same trees for Ancient Greek?

The tree for Ancient Greek is very structurally similar to English. Some languages are not similar.

Example Mixtec, parts of Southern Mexico Used pre contact in Mexico

- Terminal Symbols
  - 1
  - 10
  - 15
  - 20
- Non Terminal Symbols
  - N (number)
  - P (phrase)
  - M (multiply)
- Production Rules
  - $N \to 1$
  - $N \to 1 \quad N$
  - $N \to P$
  - $N \to P \quad N$
  - $P \to N \quad M$
  - $M \to 10$
  - $M \to 15$
  - $M \to 20$
  - $M \to N \quad M$

It is a fallacy to think cultures without highly developed mathematics have simple or primitive linguistic number systems.

Our ability to speak numbers linguistically is fundamental to our linguistic ability and doesn't substantially evolve over time.