ECE 30
Day 17 Notes

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Agenda

- Review Homework 6
- Ampere - Maxwell Law
- Faraday's Law

Homework 6 Overview

Circular motion happend when centripedal force is experienced and the speed is constant.

Given a particle moving in uniform circular motion,

$$|a_c| = \frac{v^2}{r}$$

This is a requirement and helpful for solving the questions.

Ampere

Three facts:

1. $dB \propto r, ds$
2. $|dB| \propto \frac{1}{r^2}$
3. $|dB| \propto I, |ds|$
4. $|dB| \propto sin(\theta)$

We can combine into:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \vec{r}}{r^2}$$

Therefore:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times \vec{r}}{r^2}$$

This results in:

$$\vec{B} = \frac{\mu_0 I}{2\pi a}$$

Where a is the orthogonal distance to the conductor.

This means the magnetic field is only a function of the distance from the conductor.

Ampere - Maxwell

Since the magnetic field is a function of distance, at each point along a conductor the magnetic field is radially symmetric.

$$|\vec{B}| = \frac{\mu_0 I}{2\pi a}$$

Note that, μ_0 is the permeability of free space, it is different than ϵ_0 which is for electric fields.

Consider ds as a small length along the magnetic field line.

That means $\vec{B}$ and $d\vec{s}$ are parallel.

Therefore:

$$\vec{B} \cdot d\vec{s} = Bds$$

Since B is constant for a given loop,

$$\oint \vec{B} \cdot d\vec{s} = \oint Bds = \frac{\mu_0 I}{2\pi a} \oint ds \tag{1}$$

$$= \mu_0 I \tag{2}$$

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

This means that the line integral above applies to all closed loops. The magnetic field in a closed loop is entirely determined by the current flowing through the loop.

Consider a Capacitor. The surface of the wire leading up to the Capacitor must follow Ampere's Law.

If we imagine a surface which passes between the plates of the capacitor, we find that:

$$\oint \vec{B} \cdot d\vec{s} = 0$$

Because I is zero between the plates.

To fix this Maxwell said there was a current flowing between the plates.

$$I_d \triangleq \epsilon_0 \frac{d\phi_E}{dt}$$

He called this "displacement current". ϕ_E is called Electric Flux.

The flux of a field is the field multiplied by the area.

$$\phi_E = \int\int \vec{E} \cdot d\vec{A}$$

Whre $d\vec{A}$ is an area vector. It is a small area with a direction and size.

$$d\vec{A} = d\vec{x} \times d\vec{y}$$

Where dx and dy are perpendicular.

Example:

For a flux through a circle we get:

$$\int\int E dA \tag{3}$$
$$= E\int\int dA \tag{4}$$
$$= \pi r^2 E \tag{5}$$

The Ampere Maxwell Law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + \epsilon_0 \frac{d\phi_E}{dt}) = \mu_0 (I_c + I_d)$$

We can see this means that if an electric field is changing it will cause a magnetic field. This is the fourth Maxwell Equation.

Faraday's Law

If a changing Electric field creates a magnetic field, could a changing magnetic field create an electric field?

If you hold a magnet still near a loop of wire the voltage will stay at zero.

If you move the magnet into the loop the voltage will become negative.

If you move the magnet out of the loop the voltage becomes positive.

The electric field will move to counteract the changing magnetic field.

A moving Magnet induces a voltage in the loop.

Be can derive Faraday's Law:

$$\oint E \cdot d\vec{s} = \frac{-d\phi_B}{dt}$$

The change in the magnetic flux over time is the inverse of a line integral of the Electric field which encompasses the surface.

Important:

1. A changing magnetic field causes a changing electric field.
2. The Direction of $\vec{E}$ induced by $\vec{B}$ will always act to reduce $\vec{B}$

The x means the magnetic field is traveling into the page.