ECE 30

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Dot and Cross Product (cont)

- Dot Product (An inner product in the space $\mathbb{R}^n$)

Measures how "aligned" two vectors are. Has two equivalent definitions.

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|cos(\theta)$$

Where θ is the angle between the vectors. This definition makes clear that the dot products is positive when the vectors have a small angle, zero when orthogonal, and negtive when opposing.

Another equivalent definition is as follows:

$$\vec{a} \cdot \vec{b} = \sum \vec{a}_i \vec{b}_i$$

For 2D:

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

Some Algebraic properties:

- $a \cdot b = b \cdot a$
- $a \cdot b \in \mathbb{R}$

And for Unit vectors

- $i \cdot i = 1$
- $i \cdot j = 0$
- $i \cdot -i = -1$

This also implies the dot product is linear. This is also obvious from the second definition.

The dot product is not reversible since it maps pairs of vectors to scalars.

- Cross Product (Analogous to an outer product)

Only works in 3 dimensions, in 2D usually can be substituted with per-product.

The cross product is defined as:

$$a \times b = |a||b|sin(\theta)w$$

Where w is a direction orthogonal to both a and b. w follows a right hand rule, which is an arbitrary choice.

In Geometric algebra, the outer product, in this context, actually has the same magnitude as the cross product but is instead defined as the plane which contains a and b. This is equivalent because orthogonal vectors can be used to define planes.

Some Properties:

- $a \times b = -b \times a$

Table of unit vectors

	i	j	k
i	0	k	-j
j	-k	0	i
k	j	-i	0

Agenda

- Displacement
- Instantaneous Acceleration
- Uniformly Accelerated Motion

Displacement

Our goal is to describe particle motion in 2D space. We need:

- Reference point
- Unit vectors

We can use a vector to describe a particle's position, with each component being the projection of the particle's position onto a real number line aligned with our unit vectors.

The **displacement** is defined as the difference between this vector at two points in time. The average velocity is defined as the ratio between the change in time and the displacement.

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_0 \tag{1}$$

$$\vec{v} = \frac{\Delta\vec{x}}{t_f - t_0} \tag{2}$$

This implies the following relation:

$$\vec{x}_f = \vec{x}_0 + \Delta t\vec{v}$$

For a continuous path this method will have error. We can imagine finer and finer samples, with smaller and smaller Δt values. We can express this via a limit.

$$v = \lim_{\Delta t \to 0} \frac{x_f - x_0}{t_f - t_0}$$

We can rewrite position as a function over time.

$$v = \lim_{\Delta t \to 0} \frac{p(t_f) - t(t_0)}{t_f - t_0}$$

We can see this is equivalent to a derivative which is defined as:

$$\frac{d}{dt}p(t) = \lim_{t \to 0} \frac{p(x+t) - p(x)}{t}$$

We can see these are identical since $t_f = t_0 + \Delta t$ and thus can be easily substituted to get us the derivative formula.

$\boxed{\text{Velocity is the derivative of position with respect to time.}}$

As a side note: You can imagine velocity as a conversion between time and displacement. Due to acceleration this conversion doesn't remain constant but you can always use it to calculate a change in position.