Math 181
Day 12 Notes

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Euclid and The Elements

- Most printed book after the Bible.
- Large influence on many mathematicians
- Influence on US Presidents

> President Garfield published a proof of pythagorean theorem. He was a teacher before and studied in depth.

> Abraham Lincoln also studied The Elements. He largly self studied, but one of the texts he studied was Euclid's elements.
>
> Story of how he went and memorized large portions of Euclid's elements before studying law. May be a tall tale but he definitely tried to train himself in mathematics.

- Foundational plane geometry
- Foundational Number theory

Very famous yet we don't have much of the context and few have actually read the text.

What do we know of Euclid and The Elements?

- We know very little
- Most of our primary fragments come from Egypt.
- First fragment is dated to 100AD, we could at most push back to 100BC. Full copies stop at around 300AD.

What about Euclid himself?

- The text begins with defining lines and points immediately.
- He continues defining terms and then gives out his famous five postulates.

> 1. To draw a straight line from any point to any point
> 2. To produce a finite straight line continuously in a straight line
> 3. To describe a circle with any center and distance
> 4. All right angles are equal
> 5. If two lines are parallel they do not meet.

- Some more axioms
  1. Equality is transitive
  2. Adding to both sides of an equality maintains the equality
  3. If you subtract from both sides of an equality the equality is maintained
  4. If two lines are the same, they have the same length
  5. The whole is greater than the part.
- He quickly starts proving various propositions culminating in the Pythagorean theorem in the first volume.
- It is not known why Euclid wrote his Elements. It seems unlikely to be a textbook. It seems likely that it was a compendium or collection of what the Greeks knew of geometry.
- Euclid came before Archimedes, around 300 BC. He also came before Plato (247 BC), after Alexander the Great, and Ptolomy.

> One text refers to him as Euclid of Alexandria. Which was an Egyptian port city.

Familiar Mathematics

The first 6 books of Euclid's Elements are about plane geometry.

Books 7-9 are about Number Theory.

One result from here is an algorithm for finding the greatest common factor between two integers.

Book 10 concerns irrational numbers. Culminates in a proof of the irrationality of $\sqrt{2}$

Books 11-13 concerns 3D geometry and the last book is about the platonic solids. However the last book may be a later addition.

Euclid's Number Theory

Euclid took a very geometric approach to all math. He used his postulates to build up number theory.

He also liked to use four terms for his examples of sequences. It was a literary convention at the time.

Euclid's Proposition 35

If we have a sequence of arbitrary length.

$$a_1, a_2, a_3, a_4, ...a_{n+1}$$

They are each in "continued proportion". This means they are a geometric series, or have a geometric relationship.

$$\frac{a_{i+1}}{a_i} = r$$

Next Euclid describes subtracting the first term from the last and second term.

$$a_{n+1} - a_1$$

$$a_2 - a_1$$

Then he says:

$$\frac{a_2 - a_1}{a_1} = \frac{a_{n+1} - a_1}{a_1 + a_2 + ...a_n}$$

Which more formally is:

$$\frac{a_2 - a_1}{a_1} = \frac{a_{n+1} - a_1}{\sum_{i=1}^{n} a_i}$$

We usually write geometric series as:

$$a_n = a_0 r^n$$

This is equivalent to the previous definition.

$$y = \frac{r^{n+1} - 1}{r - 1}$$

This is found by setting the series equal to y then multiplying by r and taking the difference between y and $y \cdot r$.

1. Is the formula correct?

$$\frac{a_2 - a_1}{a_1} = \frac{a_{n+1} - a_1}{\sum^n a_i} \tag{1}$$

$$\frac{a_1 * r - a_1}{a_1} = \frac{a_1 r^n - a_1}{a_1 \frac{r^n - 1}{r-1}} \tag{2}$$

$$\frac{a_1 * r - a_1}{a_1} = \frac{(a_1 r^n - a_1)(r-1)}{a_1(r^n - 1)} \tag{3}$$

$$r - 1 = \frac{a_1(r^n - 1)(r-1)}{a_1(r^n - 1)} \tag{4}$$

$$r - 1 = r - 1 \tag{5}$$

   This shows the formulas are the same, and it is correct. Both express the relationship between the ratio between two terms, and the sum of the sequence.

2. How does it compare to other formulas for geometric series?
3. How to make sense of the proof?

Euclid is not thinking like Nichomachus. Nichomachus thought geometrically but with a leaning towards abacus and calculative methods.