Math 100

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3.3 Biconditional Statements

To prove you have to show both implications.

$$P \iff Q \tag{1}$$
$$(P \implies Q) \wedge (Q \implies P) \tag{2}$$

Example, show m is even if and only if m^2 is even.

First m^2 is even if m is even.

$$m = 2n \tag{3}$$
$$m^2 = (2n)^2 \tag{4}$$
$$(2n)^2 = 4n^2 = 2(2n^2) \tag{5}$$

Next, show that if m is odd, m^2 cannot be even.

$$m = 2n + 1 \tag{6}$$
$$m^2 = (2n+1)^2 \tag{7}$$
$$= 4n^2 + 4n + 1 \tag{8}$$
$$= 2(2n^2 + 2n) + 1 \tag{9}$$

□

Proof By Cases

Exhaustively prove all cases.