Math 181

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Popular Math is partially about Historical Significance

- 1993 Andrew Wiles solved Fermat's Last theorem.
- Eureka reference to Archimedes.

> Archimedes lived on an island in the mediterranian. King comissioned a crown of gold, he thought that perhaps the crown was adultered with lead and bronze.
> Asked Archimedes to determine whether the crown was real or not. Was bathing and realized that water was displace according to his volume, got out of the bath and yelled Eureka while running naked through the streets.

Timeline:

1. 1993: Andrew Wiles proves Fermat's Last Theorem.

> For all $a, b, c \in \mathbb{Z}^+$
> $$a^n + b^n \neq c^n$$
> Given $n > 2$

2. 1994: Wiles finds error in his proof

> Fermat's theorem was an open problem for more than 350 years. Many of the best names in Mathematics announced false proofs.

3. 1996: Wiles fixes his proof

> At the time it was believed only 20 people could understand his arguement. Since then many have checked and become experts in this particular field.

Whos is Fermat?

- Pierre De Fermat was born 1601 in the south of France and lived there fore most of his life.

> Amateur mathematician, professional lawyer. Was in corrospondance with the leading mathematicians at the times.
> Most of his known work was from letters and corrospondance with other mathematicians.

> Wiles meanwhile was a professor at princeton, and did mathematicians professionally.

- What did he do?
  - In 1657 he wrote a challenge to the English mathematicians
    1. John Wallis
    2. William Brounecker

> $$x^3 + y^3 = z^3$$
> Has no positive integer solutions

> In his letter he said "One cube cannot be divided into two cubes"

- Why did he care?

  Inspired by Greek mathematics especially Arithmetica by Diophantis which dates to less than 300 AD.
  Several hundred years after Plato but still during a high point in Greek culture.

  Diophantis was focused mostly on solving polynomial equations.

  Fermat read a 1651 edition by Claude Gasper Bachet. Diophantis' work had been rediscovered around 1400.

  Most older Greek works were held in libraries managed by the Catholic Church, only in the 1400s were copies of older manuscripts made avalible to the wealthy public.

- How did we get Fermat's Last Theorem?

  Fermat's son collected and printed his father's work and notes into the margins of texts like Diophantis' and others.

  Fermat's Last Theorem was posited in the margins of his copy of Diophantis' book. He said he discovered a proof that he didn't have space to write down.

  The later consensus was that Fermat was incorrect about his proof, but it was important to the later history and discoveries of mathematics.

- Diophantis and Fermat built on math done on Pythagorean triples. $x^2 + y^2 = z^2$

  Some example solutions,
  - 3, 4, 5
  - 5, 12 ,13

  There are infinitely many solutions

  Supposedly analyzed by Pythagoras, a Greek Mathematician who predates Diophantis. (500BC)
  However solutions almost certainly were not originally found by Pythagoras.

"There is No More Evidence That Pythagoras did all this math as Hercules defeated the Hydra"

Our evidence of Pythagoras dates many years after he would have died. This question was analyzed by Greeks themselves, for example in Euclid's Elements (250BC).

Euclid is often called Greek, however he is often refferred to as Euclid of Alexandria which was a place in Egypt. Greek math is not purely mediterranean but borrows from many places around the world.

Moving Past Fermat

What Happened Between Fermat and Wiles?

- Lots of Partial Results that work for some values of n, but not all.
  - 1816: Sophie Germain

    Proved specific values of n, and published in the Paris Academy of Sciences.

  - Mid 1800s: German Professor Ernest Kummer

    Proved Fermat's theorem with the assumption that unique factorization held for extensions of the integers. Unfortunately this property was limited to integers.

    Also proved more cases of n.

- Wiles Proved the Stronger Modularity Conjecture of Yutaka Taniyama and Goro Shimura (from 1950s)

  Based on modular forms. Very technical and had more mathematical importance.

  Known before Wiles to imply Fermat's last theorem.

  Taniyama committed suicide after the conjecture, but Shimura was alive until Wiles time. Wiles tried to converse with him to figure out the name to go with, and he replied "The Shimura Theorem", so they went with the Modularity Conjecture instead.

  Wiles Used techniques developed by Victor Koyvagin and Matthias Flachin in the 1980s.

  Popular stories jump from Diophantis to Fermat to Wiles skipping over the rich history of collaboration and teamwork in the meantime.

What are some Lenses we can apply to Fermat's Last Theorem?

What Aspects of the Story could we study?

- View the historical development of relevant Mathematical ideas.

  A cartoon analysis would be to graph exponents proved vs time.
  We can see when Kummer or Germain proved cases of n.

- We could look at it through a Biographical lens.

  The Nova documentary focused on Wiles' personal story. He dedicated years of his life working in a study room in his attic trying to prove both Fermat's last theorem and the Modularity Conjecture.

  It put strain on his relationship and his career. After 7 years he announces his result, then announces its wrong, then 2 years later announces the correct proof. During those 7 years he didn't share his work.

  Pierre De Fermat has a less accessible life. We could still write a rough biography of his life but it would be more difficult.

- History of Texts

> Interesting story of how Diophantis' work was preserved until Fermat could read it. The work of creating a readable and accurate copy of his work was a scholarly achievement in itself.

- History of Institutions

> Often mathematicians will convince themselves that they've solved Fermat's last theorem and get shot down.

> Wiles was different not only because he was correct, but also because he had institutional support from Princeton. He made the announcement in front of Cambridge.
>
> Wiles also had different circumstances, he could work for 7 years without being fired in an american research university.

> Kummer was working in a German university.
> Fermat was working as a lawyer and did math as a hobby.
> Nobody knows how Euclid or Diophantis made their living.

Other lenses

Linguistically
: How math is written or notated.

History of Mathematic tools
: Where each tool comes from?

History of Geography
: How did geography affect culture?

Calculating Devices
: The ease of calculations and tools has increased.