Math 181

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Cont.

Lemma 1:

If $\frac{a}{b} = \frac{c}{d}$ then, $\frac{a+b}{b} = \frac{c+d}{d}$

Lemma 2:

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{a+c}{b+d}$

Something to note is both lemmas are usually considered in modern math to be simple algebraic manipulation, not worth noting.

By hypothesis, the ratio between two consecutive terms of a geometric series is always the same.

$$\frac{a_{n+1}}{a_n} = \frac{a_n}{a_{n-1}} = ...$$

By Lemma 1:

$$\frac{a_{n+1} - a_n}{a_n} = \frac{a_n - a_{n_1}}{a_{n-1}} = ...$$

By Lemma 2:

$$\frac{a_2 - a_1}{a_1} = \frac{(a_{n+1} - a_n) + (a_n - a_{n-1}) + ...(a_2 - a_1)}{a_n + a_{n-1} + ... + a_2 + a_1}$$

Then we simplify

$$\frac{a_2 - a_1}{a_1} = \frac{a_{n+1} - a_1}{a_n + a_{n-1} + ... + a_2 + a_1}$$

How did Euclid express this proof?

Theorem:

If as many numbers as we please be continued in proportion, and there be subtracted from the second and last numbers equal to the first, then, as the excess of the second is to the first, so will the excess of the last be to all those before it.

Lemma 1:

If, as whole is to whole, is a number subtracted to a number subtracted, the remainder will also be to the remainder as whole is to whole.

Lemma 2:

If there be as many numbers as we please in proportion, then as one of the antecedents is to one of the consequences, so are all antecedents to all the consequences.

If we have a bunch of ratios:

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = ...$$

Then:

$$\frac{a_1 + a_2 + a_3 + ...}{b_1 + b_2 + b_3 + ...} = \frac{a_1}{b_1}$$

Lemma 3:

If four numbers be proportionate, they will also be proportionate alternatively.

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

The core concept of the proof is the same, however he only does $n = 3$, and he is unable to leverage modern algebraic manipulation.

He must instead perform algebra geometrically.

The modern approach for understanding Greek mathematics is to combine anthropological approaches and mathematical approaches. The goal is to understand how they thought not just rewriting their results in terms of modern mathematics.

Archimedes

Letter about the sand Reckoner. Most of his work was found in the form of letters with other mathematicians. The Sand Reckoner was written to one of the Kings of Syracuse. The question is framed as how many grains of sand can you fit in the universe, and the book is made to make Achimedes' mathematics more digestible by the non mathematician King.

It touches astronomy and trigonometry, it also pushes the boundaries of Greek numerical notation. So Archimedes has to develop new notation to describe arbitarily large numbers.