Math 181
Day 15 Notes

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Archimedes

Archimedes The Sand Reckoner

Unclear whether Archimedes would have seen this work as anything important.

What do we know about Archimedes?

- Lived in Syracuse (Greek Island)
- His father was known to be an astronomer

  This is very thin evidence wise. We don't have any census or names for this time.

- Contemporaneous with a Roman invasion of Syracuse

  The dramatic tale is that Archimedes came up with many inventions to protect the city. And eventually the city fell and Archimedes was killed by a Roman soldier.

  Highly unlikely to be anything close to accurate. The invasion was real and he likely was involved, and probably died around the same time period. It seems very unlikely that he had many of the inventions he was credited with.

- Mathematical decoration on his grave
- Archimedian property

  Can always find a rational number between any real number and zero.

  $$\forall r \in \mathbb{R}$$

  $$\exists q \in \mathbb{Q}$$

  $$0 < q < r$$

- Integral Calculus?

  He discovered formulas for the volume of a sphere and circle using similar reasoning to integral calculus.

- Approximation of π

  He comes up with a range which correctly bounds π to three digits.

- Bouyancy

  He likely did not invent the concept of Bouyancy but is often attributed with a story where he invents the fact that objects displace water equal to their volume when submerged.

  This story also coins the term Eureka as the exclaimation Archimedes shouted when he came to a realization.

- Unpopular until after his Death?

  It is claimed his work was so ahead of its time that he wasn't given proper praise.

  It is also reasonable to think that the number of people who could read and understand Archimedes was small.

  Certainly by the first sources we have he was lauded as the pinnacle of mathematics.

The Sand Reckoner

- Universe means the sphere centered on the Earth and extending to the sun.
- Letter to King Gelon. Multiple times translated.

  King who ruled part of Sicily. We have good historical records of this person. Not as celebrated but does have good evidence and dates.

  Relationship is maybe a little condescending. Slightly informal, may be a translation issue. Also it is notably theatrical, literary would be the preferred term of Classicists. Assumes the King is knowledgable about the current mathematics and the current working mathematicians/astronomers.

  We should ask who Archimedes grew up around. Is he just comfortable with Kings and nobility? Or is he a common person who is showing his background.

  He is framing things as if he is the best and most knowledgable. The sentence about Zeuxippus could be translated as he worked with Zeuxippus or as if he was demonstrating he was better than Zeuxippus.
- He is stating his goal to calculate the amount of sand which could fit in the universe, and that there are numbers significantly larger than that.
- He mentions that Aristarchus of Samos proposed a theory of Heliocentrism in which the Earth orbits the sun and the universe is many times larger than what was previously thought.

  Some outside knowledge is that Aristarchus had a good method to calculate the size of the Sun. He was off by several orders of magnitude due to poor measurment tools, but was able to realize that the sun is several dozen times larger than the Earth. And therefore he proposed a Heliocentric model.

How many Grains of Sand can fit in the Universe?

We need at least a few assumptions.

1. Size of a grain of sand
2. Size of the Universe

Archimedes basically makes up the size of a grain of sand by taking a small unit of volume. To calculate the size of the Universe he is going to use astronomical data and geometry to estimate the size of the universe.

The second problem he is tackling is:

How to write down the answer?

Usual greek numerals only reach 999.

Why is Archimedes Answering These Questions?

What genre of text is this?

Not his usual fare. Archimedes was known for one upping mathematicians in well known difficult problems. These questions are not particularly famous.

Before the internet most math was in the form of textbooks and academic papers. Textbooks were for learning and giving background.

Euclid's Elements seem to fall into either textbook writing or Encyclopedia.

Archimedes letters were research letters which assumed large amounts of background.

- Seems like Archimedes is pushing back against incorrect views.
- Seems like he is playing with the math, for himself?
- Interested in extreme sizes.

  Calculus things, infintesimals, vs the universe.

  He mentions he gave more detail about his number system in a letter we do not have.

- Math communication?

  Philosophical and practical reasons to see the usefulness of exceedingly large numbers.

  Mathematics for broader audiences in more literary terms.

  The goal is to express how cool and awesome mathematicians are to think about these things. It also is a crossover between philosophy and mathematics via formal logic.