ECE 30

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Agenda

- HW Set 1 Posted
- Vectors and Scalars
- Reference Frames, vector Components, and Vector Sum
- Unit Vectors
- Vector Dot and Cross Products

Vectors and Scalars

What is a Vector?

In real life many quantities cannot be directly mapped to a real number. Even in one dimension, the sign indicates the direction.

Velocity

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}$$

Velocity is a representation of how displacement or position changes with time. It is a correlation with time.

In a context of calculus it is a kind of numerical integration. Velocity is the first derivative of position with respect to time.

$$v = \frac{d}{dt}x$$

Both velocity and displacement are vector quantities. Time is arguably vector but in classical physics is only a scalar.

Vectors are quantities with both direction and magnitude. They are denoted $\vec{v}$

Scalars are magnitudes only. They are written as s

Reference Frames

For vectors to make sense we need a defined start point. In application things like compasses, maps, gyroscopes, etc. can be used to define shared reference frames for navigation or calculations.

Reference frames are arbitrary, they are useful because they are relevant or shared.

Reference frames are a social construct lol

Reference frames are required to define direction. It is helpful to imagine our reference frames as composed of orthogonal components, of course this is only relevant when conceptualizing a single reference frame or in translating between reference frames.

Approaches to Vectors.

- Polar. The magnitude and direction are stored as two seperate values. ie: Angle and size. This is useful in navigation or rotationally symmetric contexts.
- Component. The vector is decomposed into multiples of some unit vector, which encodes the relative magnitudes along each unit vector. Unit vectors are the base of this system. Component vectors are useful for combining vectors or for using matricies.

Formulas:

1. Magnitude:

$$|\vec{v}| = \sqrt{\sum v_i^2}$$

   It is sometimes denoted:

$$v = \sqrt{\sum \vec{v}_i^2}$$

2. Direction:
   For 2D

$$tan(\theta) = \frac{v_2}{v_1}$$

   For 3D

$$(x, y, z) = (cos(\theta)sin(\phi), cos(\theta)cos(\phi), sin(\theta))$$

   For 2D direction is equivalent to mapping a circle to a line.
   For 3D direction is equivalent to mapping the surface of a sphere to a plane.

3. Summation:

$$\vec{a} = (a_1, a_2, ...a_i) \quad (1)$$
$$\vec{b} = (b_1, b_2, ...b_i) \quad (2)$$
$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, ...a_i + b_i) \quad (3)$$

   It can also be visualized by imagining laying each vector tip to tail, and the vector from the origin to the tip of the last vector is the sum.

   This property of summing nicely along components is one of the properties required for linearity and therefore linear algebra.

4. Polar Conversions in 2D

$$\vec{v} = (v_1, v_2) = (\theta, v) \quad (4)$$
$$v_1 = |\vec{v}|cos(\theta) \quad (5)$$
$$v_2 = |\vec{v}|sin(\theta) \quad (6)$$
$$v = \sqrt{v_1^2 + v_2^2} \quad (7)$$
$$\theta = atan2(v_2/v_1) \quad (8)$$

   Note: atan2 depends on the sign of the components.

Unit Vectors

We can conceptualize a vector as a sum of components. This can be represented algebraicly using symbols. One of the most famous vector quantities are complex numbers. $a + bi$

By convention three dimensional vectors in physics are written as:

$$a\vec{i} + b\vec{j} + c\vec{k}$$

This ties neatly into quaternions which are written

$$a\vec{i} + b\vec{j} + c\vec{k} + d$$

A unit vector corresponds to a single step along an orthogonal direction in our reference frame. Unit vectors define a reference frame and are arbitrary.

As an aside: we have already encountered two different unit vectors.
Polar coordinates use a degree which steps
around a circle as one unit, and a unit which moves away from the origin as a second vector.
Component coordinates use $\hat{x}$ and $\hat{y}$
It should be noted that Polar coordinates are nonlinear
and do not share many properties with components.

In the abstract, the way a linear operation works on the unit vectors define the way it works on the entire vector space.

It should be noted that unit vectors in physics are unitless.