BHao_Assign3

Problem Set 1

1) The rank of a matrix is equal to the number of pivot columns

- First, convert $\mathbf{X}$ to $\mathbf{U}$

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

- Swaps rows 2 and 3

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ -1 & 0 & 1 & 3 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

- Add row 1 to row 3

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 4 & 7 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

- Subtract 2 times row 2 from row 3

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & 5 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

- Subtract 5 times row 1 from row 4

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & 5 \\ 0 & -6 & -17 & -23 \end{bmatrix}$$

- Add 6 times row 2 to row 4

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & 5 \\ 0 & 0 & -29 & -17 \end{bmatrix}$$

- Add 29/8 times row 3 to row 4

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 8 & 5 \\ 0 & 0 & 0 & 9/8 \end{bmatrix}$$

U contains 4 pivot columns; as such **X** and **U** have a rank equal to 4

2) Maximum rank = n; minimum rank = 1

3) Columns 2 and 3 are multiples of column 1 and thus cannot be pivot columns; as such, there is only 1 pivot column, and the rank of **B** is equal to 1. The elimination process below will demonstrate the same result.

- Convert **B** to **U**

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

- Subtract 3 times row 1 from row 2; subtract 2 times row 1 from row 3

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Problem Set 2

To find the eigenvalues, solve det(λI - A) = 0.

$$\begin{bmatrix} \lambda - 1 & -2 & -3 & \lambda - 1 & -2 \\ 0 & \lambda - 4 & -5 & 0 & \lambda - 4 \\ 0 & 0 & \lambda - 6 & 0 & 0 \end{bmatrix}$$

Add left-to-right diagonals and subtract right-to-left diagonals to get the characteristic polynomial

$$(\lambda - 1)(\lambda - 4)(\lambda - 6) or \lambda = 1, 4, 6$$

For $\lambda_1 = 6$, solve the following for **v**

$$\begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 16 \\ 25 \\ 10 \end{bmatrix}$$

For $\lambda_1 = 4$, solve the following for **v**

$$\begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

For $\lambda_1 = 1$, solve the following for **v**

$$\begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

In the last case, since the coefficient for v_1 is zero, v_1 can be any real number.