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DAA - CS575

Theory Assignment 1

Q1]

Function	Function	O	Ω	Θ
A	B	$A = O(B)$	$A = \Omega(B)$	$A = \Theta(B)$
n^4	$n^3 \log n$	no	yes	no
$n\sqrt{n}$	n^2	yes	no	no
$(n+1)!$	$n!$	no	yes	no
$\log n$	n^k where $k>0$	yes	no	no
$\sum_{i=1}^{n} (i+1)$	$\frac{8}{3} n^2 + n$	yes	yes	yes
$\sum_{i=0}^{n-1} 3^i$	$\sum_{i=0}^{n} 9^{i-1}$	no	yes	no

Q2] Prove the following using the original definitions of O, Ω, Θ, o and ω.

a] $3n^3 + 50n^2 + 4n - 9 \in O(n^3)$

According to Big "oh" (O) :-

if $g(n) \in O(f(n))$

then relation is

$0 \leq g(n) \leq c\,f(n)$

for all $n \geq N$

where C & N are positive constants.

After Substituting the values in the relation we get,

$0 \leq 3n^3 + 50n^2 + 4n - 9 \leq cn^3$ for all $n \geq N$

We know that when we have multiple variables in an equation, we will add all the coefficient of variables present in the eq^n and keep the highest power of the variable.

$0 \leq 3n^3 + 50n^2 + 4n - 9 \leq 3n^3 + 50n^3 + 4n^3$ for $n \geq 1$

$0 \leq 3n^3 + 50n^2 + 4n - 9 \leq 57n^3$ for $n \geq 1$

choose $C = 57$ and for all $n \geq 1$

both the values of C and N are positive

$\therefore$ We can say $3n^3 + 50n^2 + 4n - 9 \in O(n^3)$.

b) $1000n^3 \in \Omega(n^2)$

Sol^n According to big omega (Ω):-

if $g(n) \in \Omega(f(n))$

then relation is

$g(n) \geq c f(n) \geq 0$ for all $n \geq N$

where c & N are positive constants.

After substituting the values in the relation, we get,

$1000n^3 \geq cn^2 \geq 0$ for all $n \geq N$

$1000n^3 \geq cn^2$

dividing both side by n^2.

$1000n \geq c$

Over here for any positive value of c the eq^n would satisfy.

lets choose $c = 2000$

$1000n \geq 2000$

$n \geq 2$

choose $N = 2$ and $c = 2000$

c) $10n^3 + 7n^2 \in \omega(n^2)$

Sol^n According to small omega (ω)

$g(n) \in \omega(f(n))$ for every positive c

then the relation will be,

$g(n) \geq cf(n) \geq 0$ for $n \geq 1$

After substituting the values in the relation we get,

$10n^3 + 7n^2 \geq cn^2 \geq 0$ for $n \geq 1$

$10n^3 + 7n^2 \geq cn^2$ for $n \geq 1$

$n^2(10n + 7) \geq cn^2$

Eliminating n^2 from both the sides.

$10n + 7 \geq c$ for $n \geq 1$

choose $c = 27$

$10n + 7 \geq 27$

$10n \geq 20$

$n \geq 2$

choose $N = 2$.

d) $78n^3 \in o(n^4)$

Sol^n According to small oh ('o')

$g(n) \in o(f(n))$ for every positive c

then the relation will be,

$0 < g(n) < c\,f(n)$ for $n \geq 1$

After substituting the values in the relation we, get,

$0 < 78n^3 < cn^4$ for $n \geq 1$

$78n^3 < cn^4$ for $n \geq 1$

dividing both sides by n^3

$78 < cn$ for $n \geq 1$

choose $c = 39$, we get:-

$78 < 39n$ for $n \geq 1$

$n > 2$

~~bothe~~ both the values of c and N are positive.

e) $n^2 + 3n - 10 \in \Theta(n^2)$

Sol According to the theta (Θ)

$g(n) \in \Theta(f(n))$

then the relation will be,

$0 \leq c(f(n)) \leq g(n) \leq d(f(n))$ for $n \geq N$

where c, d, and N are positive constants.

After substituting the values we get,

$0 \leq c(n^2) \leq n^2 + 3n - 10 \leq d(n^2)$

Solving for the first part

$0 \le cn^2 \le n^2+3n-10$ for all $n \ge N$

$cn^2 \le n^2+3n-10$ for $n \ge N$

put $\boxed{c = 1}$

$n^2 \le n^2+3n-10$ for $n \ge N$

$n^2 - n^2 \le 3n-10$ for $n \ge N$

$0 \le 3n-10$ for $n \ge N$

$3n \ge 10$ for $n \ge N$

$n \ge \frac{10}{3}$

choose $N = 3$ and $c = 1$

Now solving for the second part.

$n^2+3n-10 \le dn^2$ for $n \ge N$

if we choose $\boxed{d = 4}$ we get.

$n^2+3n-10 \le 4n^2$ for $n \ge N$

We can write this $4n^2$ as n^2+3n^2

$n^2+3n-10 \le n^2+3n^2$ for $n \ge N$

Nowe as studied earlier for the coefficient of the variable with highest power.

We can state that the above eq^n holds true for $n \ge N$. where N is a positive integer.

So $n^2+3n-10 \le n^2+3n^2$ for $n \ge 1$

choosing $N = 1$

The values of the coefficient are $c = 1$ and $d = 4$.

Q3] Prove the following using limits.

a) $n^{1/n} \in \Theta(1)$

Sol $\lim_{n \to \infty} \frac{(n^{1/n})}{(1)}$

As we know $x = e^{\ln x}$, we can use the same over here.

$$n^{1/n} = e^{\ln(n)^{1/n}}$$
$$= e^{\frac{1}{n}\ln(n)}$$
$$= e^{\frac{\ln(n)}{n}}$$

for $n \to \infty$ $\frac{\ln(n)}{n}$ will become 0.

then $e^{\frac{\ln(n)}{n}}$ will become e^0 for $n \to \infty$

$$\lim_{n \to \infty} \frac{e^{\frac{\ln(n)}{n}}}{1} = \frac{e^0}{1} = \frac{1}{1} = 1$$

Hence we have proved that $n^{1/n}$ belongs to $\Theta(1)$

$$n^{(1/n)} \in \Theta(1)$$

b) $4^n \in \omega(n^k)$

Soln $\lim_{n \to \infty} \frac{4^n}{n^k}$

Using the L'Hospital's Rule, we get. (Taking derivative)

$$\lim_{n \to \infty} \frac{(4^n)'}{(n^k)'}$$

After taking derivative we get.

$$= \lim_{n \to \infty} \frac{4^n \log 4}{k\, n^{k-1}}$$

this is a $\frac{\infty}{\infty}$ term,

So apply L'Hospital Rule once again.

$$= \lim_{n \to \infty} \frac{4^n (\log 4)^2}{k(k-1)\, n^{(k-2)}}$$

again we are getting $\frac{\infty}{\infty}$ term.

taking L'Hospital Rule once again.

$$= \lim_{n \to \infty} \frac{4^n (\log 4)^3}{k(k-1)(k-2)\, n^{(k-3)}}$$

this is again $\frac{\infty}{\infty}$.

We will ~~solve~~ ~~and we~~ apply L'Hospital rule till either the numerator or denominator is not ∞.

After taking derivatives multiple time, we get.

$$= \lim_{n \to \infty} \frac{4^n (\log 4)^k}{k!}$$

Now the term in denominator is indepent of n.

$\therefore$ Now denominator will not tend to ∞.

$$= \lim_{n\to\infty} 4^n \frac{(\log 4)^k}{k!} \rightarrow \text{Constant term.}$$

$$= \infty \times c$$

$\hookrightarrow$ Constant term.

$$= \infty.$$

$\therefore$ by definition. we know that.

If $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$ then $f(n) = \omega(g(n))$

Hence proved,

$$4^n \in \omega(n^k)$$

c) $\log^3 n \in o(n^{0.5})$

Sol^n

$$\lim_{n\to\infty} \frac{\log^3 n}{n^{0.5}}$$

$$n^{0.5} = n^{1/2}$$

Since $\frac{\log^3 n}{n^{1/2}}$ tends to $\frac{\infty}{\infty}$ when $n \to \infty$.

We will apply L'Hospital Rule.

$$= \lim_{n\to\infty} \frac{3\log^2 n}{\frac{1}{2} n^{-1/2}} \times \frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{6 \log^2 n \times n^{1/2}}{n} = \frac{6 \log^2 n}{n^{1/2}} \quad \left[\frac{n^{1/2}}{n} = \frac{1}{n^{1/2}} \right]$$

$$\lim_{n \to \infty} \frac{6 \log^2 n}{n^{1/2}}$$

Since this term is becoming $\frac{\infty}{\infty}$ for $n \to \infty$

Apply L'Hospital Rule once again.

$$= \lim_{n \to \infty} \frac{6 \times 2 \log n}{n \times \frac{1}{2} n^{-1/2}}$$

$$= \lim_{n \to \infty} \frac{12 \times 2 \log n}{n \times n^{-1/2}}$$

$$\lim_{n \to \infty} \frac{24 \log n}{n^{1/2}}$$

again the term is becoming $\frac{\infty}{\infty}$ for $n \to \infty$.

So Apply L'Hospital Rule once again.

$$= \lim_{n \to \infty} \frac{24}{n \times \frac{1}{2} n^{-1/2}}$$

$$= \lim_{n \to \infty} \frac{48}{n \times n^{-1/2}} = \frac{48}{n^{1/2}}$$

$$= \lim_{n \to \infty} \frac{48}{n^{1/2}}$$

when $n \to \infty$ $\frac{1}{n^{1/2}} = \frac{1}{\infty} = 0.$

$$= \lim_{n \to \infty} \frac{48}{n^{1/2}}$$

$$= 0.$$

By definition we know that.

If $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ then $f(n) \in o(g(n))$

Hence proved,

$\log^3 n \in o(n^{0.5})$

Q4] Order the functions below by increasing growth rates.

$n^n, n, n \ln(n), n^{1/2}, 2^{\log n}, \ln n, 10, n^{1/n}, \sqrt{2}^{\log n}, n!,$
$\lg(n^{10}), 2^n$

The correct order would be.

$10, n^{1/n}, \ln(n), \lg n^{10}, n^{1/2}, \sqrt{2}^{\lg n}, n, 2^{\lg n}, n \ln(n), 2^n,$
$n!, n^n.$

Q5] Let $f(n)$ and $g(n)$ be asymptotically positive functions. For each of the following conjectures, either prove it is true or provide a counter example to show it is not true.

~~Sol~~

a] $(f(n) + g(n)) \in \Theta(\max(f(n), g(n)))$.

Sol^n To prove the conjectures $(f(n)+g(n)) \in \Theta(\max(f(n), g(n)))$ we need to prove 2 things.

① $f(n) + g(n) \in O(\max(f(n), g(n)))$

② $f(n) + g(n) \in \Omega(\max(f(n), g(n)))$

Lets prove the first part.

① $f(n) + g(n) \in O(\max(f(n), g(n)))$

$$f(n) + g(n) \leq c_1 (\max(f(n), g(n)))$$

max$(f(n), g(n))$ can be either $f(n)$ or $g(n)$
lets consider $f(n)$ is max.

$$f(n) + g(n) \leq c_1 f(n)$$

We know that if $f(n)$ is greater than $g(n)$
so $2 \times f(n) \geq f(n) + g(n)$
This inequality will hold true for any value of c_1 greater than or equal to 2.
Lets consider $c_1 = 3$.

$$f(n) + g(n) \leq 3 f(n).$$

~~This~~ If both $f(n)$ & $g(n)$ asymptotically positive then the above inequality holds true.
~~Hence~~
Same we can prove for $g(n)$ being the max of max $(f(n), g(n))$

Hence $f(n) + g(n) \in O(\max(f(n), g(n)))$.

Now lets prove for 2^{nd} part.

② $f(n) + g(n) \in \Omega(\max(f(n), g(n)))$

max$(f(n), g(n))$ can be either $f(n)$ or $g(n)$.
lets consider $f(n)$ is max.

$$c_2 f(n) \leq f(n) + g(n)$$

If $f(n)$ and $g(n)$ both are asymptotically positive.
lets consider $c_2 = 1$ then,

$$f(n) \leq f(n) + g(n)$$

As both the functions are asymptotically positive then the above inequality holds true.

Same we can prove for $g(n)$ as max from $\max(f(n), g(n))$

Hence, $f(n) + g(n) \in \Omega(\max(f(n), g(n)))$.

Since we have proved both the conditions. the conjecture $(f(n) + g(n)) \in \Theta(\max(f(n), g(n)))$ holds true.

b] $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$.

Sol^n

To prove the above conjecture, lets take a look at the (O) notation.

By the the definition of Big oh (O), we can say that

$0 \le f(n) \le c \cdot g(n)$ for all $n \ge N$

where C & N are positive constants.

To prove the above inequality and as $f(n)$ and $g(n)$ are asymptotically positive functions. We can consider any value of $f(n)$ and $g(n)$

Lets consider $f(n) = n^2$ and $g(n) = n^3$.

Substituting these values in the above inequality we get.

$0 \le n^2 \le cn^3$ for all $n \ge N$

this above inequality will satisfy for $n \ge N$

~~So keeping N~~

lets consider $\boxed{C = 3}$

$0 \le n^2 \le 3n^3$ for $n \ge N$

this ~~eq~~ inequality will remain true for $n \ge 1$. choose $\boxed{N = 1}$

Hence we have proved that

$f(n) \in O(g(n))$ for $C = 3$ and $N = 1$

Now to verify, lets prove $2^{f(n)} \in O(2^{g(n)})$
lets consider the earlier values of f(n) & g(n).

$$2^{f(n)} = 2^{n^2}$$
$$2^{g(n)} = 2^{n^3}$$

Substituting these values in the Big oh notation.

$$0 \leq 2^{n^2} \leq c2^{n^3} \qquad \text{for } n \geq N$$

lets keep the same value of c,
Now the eq^n becomes.

$$0 \leq 2^{n^2} \leq 3 \times 2^{n^3} \qquad \text{for } n \geq N$$

The function 2^{n^3} will grow much faster and will always be greater than 2^{n^2} for the values of $n \geq 2$.

$$0 \leq 2^{n^2} \leq 3 \times 2^{n^3} \qquad \text{for } n \geq 2$$

choose $\boxed{N = 2}$

The above inequality gets satisfied for the values of c = 3 and N = 2. Hence we have proved that the conjecture is true

$f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$

c) $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$

Sol) Lets try to prove this conjecture.

$f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$.

According to the definition of big oh (O)

$$0 \leq f(n) \leq c \cdot g(n) \qquad \text{for } n \geq N$$

As f(n) and g(n) are asymptotically positive lets choose $\boxed{f(n) = n^2}$ and $\boxed{g(n) = n^3}$

Substituting the values we get.

$0 \leq n^2 \leq cn^3$ for $n \geq N$

lets choose $c = 3$.

$0 \leq n^2 \leq 3n^3$ for $n \geq N$

choose $N = 1$.

The above inequality holds true for $c = 3$ and $N = 1$.

Now lets try $g(n) \in \Omega(f(n))$

According to the definition of Omega (Ω):-

$0 \leq c.f(n) \leq g(n)$ for $n \geq N$

for the same values of $f(n)$ & $g(n)$ we get

$0 \leq cn^2 \leq n^3$ for $n \geq N$

lets take $\boxed{c = 1}$

$0 \leq n^2 \leq n^3$ for $n \geq N$.

The above ~~eq~~ inequality will hold true for $n \geq 1$

choose $\boxed{N = 1}$

Hence, we have proved that the conjecture $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$ holds true.

d) $f(n) \in O(g(n))$ implies $lg(f(n)) \in O(lg(g(n)))$, where $lg(g(n)) \geq 1$ and $f(n) \geq 1$ for sufficiently large n,

Sol. $f(n) \in O(g(n))$

According to the definition of big oh (O)

$0 \leq f(n) \leq c\,g(n)$ for $n \geq N$

lets consider $f(n) = 10^{n^2}$
$g(n) = 10^{n^2+2}$

Substituting these values.

$0 \leq 10^{n^2} \leq c\,10^{n^2+2}$ for $n \geq N$

lets choose $\boxed{c = 2}$

$0 \leq 10^{n^2} \leq 2 \times 10^{n^2+2}$ for $n \geq 1$

The above inequality holds true for $n \geq 1$

Now lets take a try to prove
$lg(f(n)) \in O(lg(g(n)))$.

$0 \leq \log(f(n)) \leq c \log(g(n))$ for $n \geq N$

Now lets put the same value of $f(n)$ & $g(n)$.

$0 \leq \log(10^{n^2}) \leq c \log(10^{n^2+2})$ for $n \geq N$

$0 \leq n^2 \log 10 \leq c \times (n^2+2) \log 10$. for $n \geq N$

$[\log_{10} 10 = 1]$

$0 \leq n^2 \leq c \times (n^2+2)$ for $n \geq N$

choosing the same value of $\boxed{c = 2}$

$0 \leq n^2 \leq 2(n^2+2)$ for $n \geq N$

$0 \leq n^2 \leq 2n^2+4$. for $n \geq N$

The above inequality holds true for $n \geq 1$

So choose $\boxed{N = 1}$ & $\boxed{c = 2}$

Hence, we have proved that the conjecture $f(n) \in O(g(n))$ implies $\log(f(n)) \in O(\log(g(n)))$ holds true.

Q6] Prove that for all integers $n > 0$

$$\left(\sum_{i=1}^{n} i\right)^2 = \sum_{i=1}^{n} (i)^3$$

by mathematical induction. Divide your proof into the three required parts: Induction base, Induction Hypothesis and Induction steps

Sol: Lets divide the proof into 3 parts.

Part 1:- Induction Base:-

for base case $n = 1$

$$L.H.S = \left(\sum_{i=1}^{n} i\right)^2 = \left(\sum_{i=1}^{1} i\right)^2 = (1)^2 = 1$$

$$R.H.S = \sum_{i=1}^{n} i^3 = \sum_{i=1}^{1} i^3 = 1^3 = 1$$

So for our Induction base $LHS = RHS$.

Part 2:- Induction hypothesis:-

Assume our equation $LHS = RHS$ is true for some positive integer $n = K$.
Here k is our assumed positive integer.
So our eq^n becomes.

$$\left(\sum_{i=1}^{K} i\right)^2 = \sum_{i=1}^{K} (i)^3$$

This is our Induction hypothesis.

Part 3:- Induction Step:-

Now we have assumed that LHS = RHS for $n=k$
Lets try to prove ~~fo~~ LHS = RHS for $n = k+1$

Substituting the value in eq^n we get.

$$\left(\sum_{i=1}^{k+1} i\right)^2 = \sum_{i=1}^{k+1} (i)^3$$

Lets solve for LHS first.

$$L.H.S = \left(\sum_{i=1}^{k+1} i\right)^2$$

We know that

$$= \left(\underbrace{1+2+3+\ldots\ldots k}+(k+1)\right)^2$$

we can replace this by $\sum_{i=1}^{k} i$

$$= \left(\left(\sum_{i=1}^{k} i\right) + (k+1)\right)^2$$

$$\left[(a+b)^2 = a^2 + 2ab + b^2\right]$$

$$= \left(\sum_{i=1}^{k} i\right)^2 + 2\left(\sum_{i=1}^{k} i\right)(k+1) + (k+1)^2$$

From ~~id~~ induction hypothesis we know that

$$\left(\sum_{i=1}^{k} i\right)^2 = \sum_{i=1}^{k} i^3$$

replace this value in above eq^n.

$$= \sum_{i=1}^{k} i^3 + 2\left(\sum_{i=1}^{k} i\right)(k+1) + (k+1)^2$$

$$\left[\sum_{i=1}^{k} i = \frac{k(k+1)}{2}\right.$$

using formule]

$$= \sum_{i=1}^{k} i^3 + \cancel{2} \times \left(\frac{k(k+1)}{\cancel{2}}\right)(k+1) + (k+1)^2.$$

$$= \sum_{i=1}^{k} i^3 + k(k+1)^2 + (k+1)^2$$

taking $(k+1)^2$ as ~~com~~ common

$$= \sum_{i=1}^{k} i^3 + (k+1)^2(k+1)$$

$$\text{LHS} = \sum_{i=1}^{k} i^3 + (k+1)^3$$

Now Lets solve for RHS

$$\text{RHS} = \sum_{i=1}^{k+1} (i)^3$$

~~We can split this~~ summation as $\sum_{i=r}^{k+1}$

We can rewrite this as $\sum_{i=1}^{k} (i)^3 + (k+1)^3$

$$\text{RHS} = \sum_{i=1}^{k} (i)^3 + (k+1)^3$$

$$= \text{LHS}$$

Hence Proved, LHS = RHS

Q7] Consider the following algorithm

```
for (i=2; i<=n; i++) {
    for (j=0; j<=n) {
        cout << i << j;
        j = j + ⌊n/4⌋;
    }
}
```

a) What is the output when $n=4$?
b) What is the time complexity $T(n)$. You may assume that n is divisible by 4.

Sol^n Lets first solve for a)

a) Output when $n=4$ will be :-

20 21 22 23 24   30 31 32 33 34   40 41 42 43 44

the value of i will go from 2 to 4
the value of j will go from 0 to 4.
thus we gets the output as above.

b) Solving for time complexity $T(n)$.
If we consider n is divisible by 4, then,
the values of n will be 4, 8, 12, 16 ......

The outer loop runs from
$i=2$ to $i \le n$

for $n=4$	$i=2$ to $i \le 4$	3 iterations
for $n=8$	$i=2$ to $i \le 8$	7 iterations.
for $n=12$	$i=2$ to $i \le 12$	11 iterations.

So the outer loop is running for $(n-1)$ times.

Now ~~the~~ for the inner loop.
$j=0$ to $j \le n$.
the values of n will dee same.

for $n=4$	$j=0$	to $j \le 4$	5 iterations
for $n=8$	$j=0$	to $j \le 8$	9 iterations
for $n=12$	$j=0$	to $j \le 12$	13 iterations

The ~~outer~~ inner loop is running for $(n+1)$ times

For each ~~of~~ outer loop, inner loop runs for $(n+1)$ times.

So the $T(n) = (n-1)(n+1) = (n^2-1)$

$\boxed{\therefore\ T(n) = O(n^2)}$.

Q8] What is the time complexity $T(n)$ of the nested loops below? For simplicity, you may assume that n is a power of 2. That is, $n=2^k$ for some positive integer k. Give some justification for your answer.

Sol^n.

```
for (i=1; i<=n; i++){
    j=n;
    while (j>=1) {

        <body of the while loop> //Needs θ(1)
        j = ⌊j/2⌋
    }
```

The outer for loop runs from.
$i=1$ to $i \le n$.
So it runs for total of 'n' times.

T(n) of outer loop will be n.

For inner loop, we have a while ~~condition~~. loop. We have 2 basic operation one is $\lfloor \frac{n}{2} \rfloor$ and other one is $\theta(1)$. So,

$$T(n) = \begin{cases} \theta(1) & \text{for } n=1 \\ T(\lfloor \frac{n}{2} \rfloor) + \theta(1) & \text{for } n \geq 1 \end{cases}$$

We have got $T(n) = T(\lfloor \frac{n}{2} \rfloor) + \theta(1)$

To get the value of $T(\lfloor \frac{n}{2} \rfloor)$ lets substitute n as $\frac{n}{2}$.

$$T\left(\lfloor \frac{n}{2} \rfloor\right) = T\left(\left\lfloor \frac{n/2}{2} \right\rfloor\right) + \theta(1)$$

$$T\left(\lfloor \frac{n}{2} \rfloor\right) = T\left(\lfloor \frac{n}{4} \rfloor\right) + \theta(1)$$

We can put this value in the eq^n of T(n).
The eq^n of T(n) becomes,

$$T(n) = \left(T\left(\lfloor \frac{n}{4} \rfloor\right) + \theta(1)\right) + \theta(1).$$

Similarly we can solve for $T\left(\lfloor \frac{n}{4} \rfloor\right)$

$$T(n) = \left(T\left(\left\lfloor \frac{n}{8} \right\rfloor\right) + \Theta(1)\right) + \Theta(1) + \Theta(1)$$

$(8 = 2^3)$ and we have $3\ \Theta(1)$

Simplifying the Eq^n we get.

$$T(n) = T\left(\left\lfloor \frac{n}{2^k} \right\rfloor\right) + K\Theta(1).$$

let $n = 2^k$

$$T(n) = T\left(\left\lfloor \frac{2^k}{2^k} \right\rfloor\right) + k\Theta(1)$$

$$= T(1) + K\Theta(1)$$

(from base cond^n $T(1) = \Theta(1)$)

$$= \Theta(1) + k\Theta(1)$$

$$T(n) = (k+1)\Theta(1)$$

$$= \Theta(k)$$

Solving k.

$\because n = 2^k$

taking log.

$$\log n = \log 2^k$$

$$\log n = k \log 2$$

$$\boxed{\log n = k}$$

$$\boxed{T(n) = \Theta(\log n)}$$

This is for inner loop.

So For each iteration of outer loop the inner loop would run for $\Theta(\log n)$

So total time Complexity: $\boxed{T(n) = n(\log n)}$