

Lecture-13 Tp tr 6r

- Overfit Measure:  $E_{out}(H_{10}) - E_{out}(H_2)$

Learning Scenario Parameters:

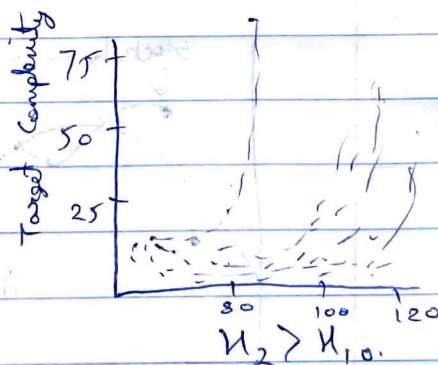
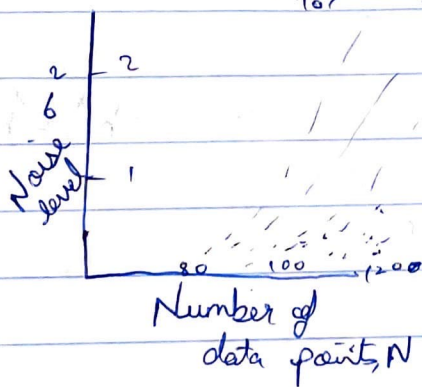
▢ Noise level  $\sigma^2$  (Variance)

▢ target complexity

▢

Favour  $H_{10}$  only if you have large dataset.

If the noise increases, prefer simpler model.  
 $H_{10} > H_2$



Noise level  $\sigma^2$  (Variance)  $\uparrow \uparrow$  overfitting

Target complexity  $\uparrow \uparrow$  overfitting

Complex no. of data points  $\uparrow \downarrow$  overfitting.

Noise: The Part of  $y$  we cannot model  
stochastic Noise.

- Error added to target function.

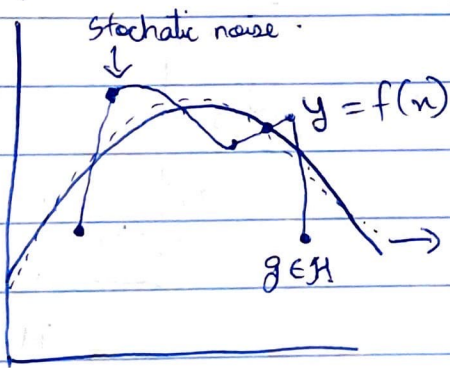
$$y = f(x) + \epsilon$$

$y$  → o/p of target func.  
 $x$  → input  
 $\epsilon$  → noise stochastic noise

Even on  
test point

$$\hat{y} = g(u).$$

★ We have no way to model stochastic noise.


$$\dots h^* \in \mathcal{H}$$

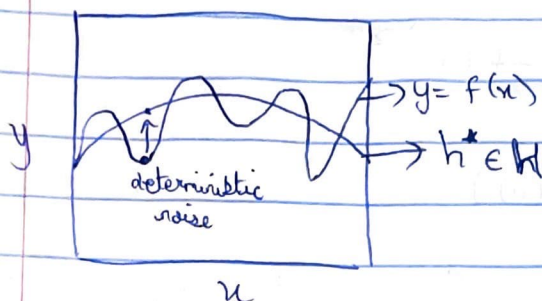
~~If~~ Instead of making the right curve ( $h^*$ ) we end up making  $g$  ~~not~~ because of stochastic noise

## ★ Stochastic & Deterministic Noise.

# Stochastic

## Deterministic Noise

the other part of  $y$  which cannot model.



We get this deterministic noise because we have already fixed the hypothesis set. & we don't know the target func<sup>n</sup>. Bec.

Choice of hypothesis set give rise to deterministic noise.

$$\begin{cases} y_n = f(x_n) \\ y_n = h^*(x_n) + \text{deterministic choice.} \end{cases}$$

target complexity  $\uparrow$  Data points  $\downarrow$  then simpler model.

## Stochastic noise

- random measurement errors
- It changes if we measure  $y_n$  again
- Independent of  $H$

## Deterministic noise

- $H$  cannot model  $f$ .
- Value remains const<sup>n</sup> even if we measure  $y_n$  again.
- Dependent of  $H$



$\mathbb{E}$  = Expected value.

Bias Variance Decomposition with stochastic noise.

$$\underset{\substack{\uparrow \\ \text{Expected value.}}}{\mathbb{E}} [F_{\text{out}}(x)] = \text{bias}(x) + \text{variance}(x)$$

$$\text{bias}(x) = (\bar{g}(x) - f(x))^2$$

$$\text{Var}(x) = \text{Var}(g(x))$$

$$F_{\text{out}}(x) = (g(x) - y)^2, \text{ where } y = f(x) + \epsilon$$

$$= (g(x) - f(x) - \epsilon)^2$$

$$= (g(x) - f(x))^2 - 2\epsilon(g(x) - f(x)) + \epsilon^2$$

$$\underset{\substack{\downarrow \\ \text{Expected Value.}}}{\mathbb{E}_x} [F_{\text{out}}] = \mathbb{E}_x [(g(x) - f(x))^2 - 2\epsilon(g(x) - f(x)) + \epsilon^2]$$

$$\bullet \mathbb{E}[\epsilon] = 0 \Rightarrow$$

$$\mathbb{E}_x [F_{\text{out}}] = \mathbb{E}_x [(g(x) - f(x))^2] + \mathbb{E}_x [\epsilon^2]$$

$$= \mathbb{E}_x [\text{bias}(x) + \text{Var}(x)] + \mathbb{E}_x [\epsilon^2]$$

$$F_{\text{out}} = \text{bias} + \epsilon^2 + \text{Var.}$$

$\downarrow$   
deterministic  
noise

$\downarrow$   
stochastic  
noise

$\downarrow$   
Indirect impact  
of noise

\* We have to find the best value of  $\lambda$  that will give us lower value pair of bias & variance.

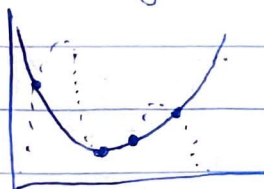
$$\nabla E_{\text{avg}}(w) = 0.$$

$$w_{\text{reg}} = (Z^T Z + \lambda I)^{-1} Z^T y.$$

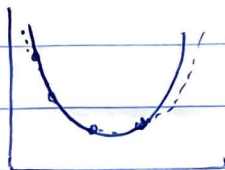
$$\text{Minimizing } E_{\text{in}}(w) + \frac{\lambda}{N} w^T w$$

$$\lambda = 0$$

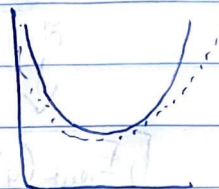
overfitting



$$\lambda = 0.0001$$

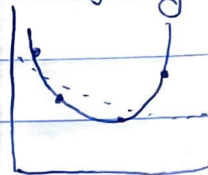


$$\lambda = 0.01$$



$$\lambda = 1.$$

Underfitting



More noise demands more Regularization.

