

Multi-dimensional NumPy arrays

ESS 116 | Fall 2024

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(Modified from Ethan Campbell and Katy Christensen's materials for UW's Ocean 215)

What we'll cover in this lesson

1. NumPy arrays – arithmetic, logical operations, indexing
2. NumPy functions and constants
3. Multi-dimensional NumPy arrays
4. More array functions

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Loading NumPy (“Numeric Python”)

Makes this package available to Python

`import numpy`

Package names are
usually all lowercase

This is a shortcut;
you can choose any name
but `np` is most common

`as np`

—
This part is
technically optional

Checking a package's version

```
1 import numpy as np
2
3 print(np.__version__)
```

☞ 1.18.5

That's a double
underscore: __



The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array([5, 6, 7, 8])
```

Similarities between lists and NumPy 1-D arrays

Both are **mutable** (they can be changed)

```
1 numbers = np.array([5,6,7,8])
2 numbers[1] = 13
3 print(numbers)
```

```
☞ [ 5 13  7  8]
```

Both are **iterable**

```
1 for num in numbers:
2     print(num)
```

```
☞ 5
   13
   7
   8
```

Both are compatible with **indexing** and **slicing**

```
1 print(numbers[-3:])
```

```
☞ [13  7  8]
```

Find length using **len()**

```
1 print(len(numbers))
```

```
☞ 4
```

Check membership using **in** and **not in**

```
1 print(13 in numbers)
2 print(14 in numbers)
```

```
☞ True
   False
```

Differences between lists and NumPy 1-D arrays

Lists

- Lists can contain a mix of object types (integers, strings, sub-lists, etc.)

- Lists are **computationally inefficient** (avoid using to store large data sets)

NumPy 1-D arrays

- Arrays can contain only a single object type (check using `.dtype`, change using `.astype()`)

```
1 numbers = np.array([5,6,7,8])
2 print(numbers.dtype)
3 print(numbers.astype(str))
```

```
☞ int64
   ['5' '6' '7' '8']
```

- Arrays are **fast for computation** and small in memory (great for big data)

Differences between lists and NumPy 1-D arrays

Lists

- Lists don't preserve scientific notation in floating-point numbers

```
1 print([3.5e9, 1.4e-3])
```

```
↳ [3500000000.0, 0.0014]
```

- Use Python's **in-place** `append()` or `extend()`, `insert()`, `del`, `reverse()`, `remove()`, `pop()`

```
1 numbers = [5, 6, 7, 8]
2 numbers.append([9, 10])
3 print(numbers)
```

```
↳ [5, 6, 7, 8, [9, 10]]
```

NumPy 1-D arrays

- Arrays preserve scientific notation

```
1 print(np.array([3.5e9, 1.4e-3]))
```

```
↳ [3.5e+09 1.4e-03]
```

- NumPy's `append()`, `insert()`, `delete()`, `flip()` functions are **not in-place**; note the different syntax; no functions to remove, pop

```
1 numbers = np.array([5, 6, 7, 8])
2 numbers = np.append(numbers, [9, 10])
3 print(numbers)
```

```
↳ [ 5  6  7  8  9 10]
```

Differences between lists and NumPy 1-D arrays

Lists

- Convert from list → array using:

```
1 my_list = [5,6,7,8]
2 my_array = np.array(my_list)
```

- Adding lists **concatenates** (joins) **them**:

```
1 a = [1,2,3,4]
2 b = [5,6,7,8]
3 print(a + b)
```

```
[> [1, 2, 3, 4, 5, 6, 7, 8]
```

NumPy 1-D arrays

- Convert from array → list using:

```
1 my_list1 = my_array.tolist()
2 my_list2 = list(my_array)
```

- Adding arrays actually **adds them**!*

```
1 a = np.array([1,2,3,4])
2 b = np.array([5,6,7,8])
3 print(a + b)
```

```
[> [ 6  8 10 12]
```

* Note that NumPy also has a `concatenate()` function.

Arithmetic operations with arrays

Arithmetic operators

| | |
|----|----------------|
| + | Addition |
| - | Subtraction |
| * | Multiplication |
| / | Division |
| ** | Exponential |
| % | Remainder |
| // | Floor |

Element-wise arithmetic between two or more arrays

```
1 a = np.array([1,2,3,4])
2 b = np.array([5,6,7,8])
3
4 print('a + b =', a + b)
5 print('a - b =', a - b)
6 print('a * b =', a * b)
```

```
a + b = [ 6  8 10 12]
a - b = [-4 -4 -4 -4]
a * b = [ 5 12 21 32]
```

Element-wise arithmetic with an array and a number

```
1 print('a + 10 =', a + 10)
2 print('10 * a =', 10 * a)
3 print('a / 10 =', a / 10)
4 print('a**2 =', a**2)
```

```
a + 10 = [11 12 13 14]
10 * a = [10 20 30 40]
a / 10 = [0.1 0.2 0.3 0.4]
a**2 = [ 1  4  9 16]
```

Element-wise operations require arrays to be the same dimensions

```
1 x = np.array([1,2,3])
2 y = np.array([11,12,13,14,15])
3
4 print(x + y)
```



```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-97-d5d99ad6233b> in <module>()
      2 y = np.array([11,12,13,14,15])
      3
----> 4 print(x + y)
```

ValueError: operands could not be broadcast together with shapes (3,) (5,)

Logical operations with arrays

Comparison operators

| | |
|--------------------|--------------------------|
| <code>==</code> | Equal |
| <code>!=</code> | Not equal |
| <code>></code> | Greater than |
| <code>>=</code> | Greater than or equal to |
| <code><</code> | Less than |
| <code><=</code> | Less than or equal to |

Element-wise comparisons between two arrays or an array and a number

```
1 u = np.array([1,2,3,4])
2 v = np.array([0,2,4,6])
3
4 print(u == v)           [False  True False False]
5 print(u < v)            [False False  True  True]
6 print(v != 0)           [False  True  True  True]
7 print(v <= 4)           [ True  True  True False]
```

Instead of comparing Boolean arrays with **and/or**, use **`np.logical_and()`** and **`np.logical_or()`**

```
1 bool1 = np.array([True,False,True])
2 bool2 = np.array([True,False,False])
3
4 print(np.logical_and(bool1,bool2))  [ True False False]
5 print(np.logical_or(bool1,bool2))   [ True False  True]
```

New indexing options with arrays

When you want to **access certain value(s)** in an array:

```
1 v = np.array([10, 11, 12, 13])
```

```
2
```

```
3 print(v[3])
```

```
4
```

```
5 print(v[[2, 3]])
```

```
6
```

```
7 print(v[v >= 12])
```

```
8
```

```
9 print(v[[False, False, True, True]])
```

Python prints:

13

Traditional list-style **single index**

[12 13]

Multiple indices retrieves multiple elements

[12 13]

Logical conditions also work...

[12 13]

... because they evaluate to **Boolean arrays**

When you want the **indices of certain values** in an array:

```
1 print(np.where(v >= 12))
```

```
2
```

```
3 print(np.where(v >= 12)[0])
```

(array([2, 3]),)

`np.where()` gives the indices at which a Boolean condition is satisfied...

[2 3]

... but you have to index into the result using [0]

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Most functions acting on NumPy arrays can be called two ways

```
x = np.array([10, 11, 12, 13])
```

np.sum(x) ← Evaluates to: 46

x.sum() ← Evaluates to: 46

NumPy functions can also be applied to lists

```
x = [10, 11, 12, 13]
```

np.sum(x) ← Evaluates to: 46

x.sum() ← Evaluates to: 46

Mathematical reductions (array → number)

```
x = np.array([10, 11, 12, 13])
```

| Function: | Purpose: | Evaluates to: |
|---------------------------|--------------------|---------------|
| <code>np.sum(x)</code> | Sum | 46 |
| <code>np.mean(x)</code> | Mean (average) | 11.5 |
| <code>np.median(x)</code> | Median | 11.5 |
| <code>np.max(x)</code> | Maximum value | 13 |
| <code>np.min(x)</code> | Minimum value | 10 |
| <code>np.std(x)</code> | Standard deviation | 1.11803... |

Mathematical constants (each return a `float`)

Constant value:

`np.pi`

`np.e`

`np.inf`

`np.nan`

Purpose:

π (pi)

e (Euler's number)

Positive infinity

“Not a Number”
(used as a placeholder for missing data)

Evaluates to:

`3.14159...`

`2.71828...`

`inf`

`nan`

Note:

```
1 print(5 * np.inf)
2 print(5 * np.nan)
```

`⇒ inf`
`nan`

Element-wise functions (number → number, or array → array)

Function:

`np.absolute([-2,-1])`

`np.round([5.23,5.29],1)`

`np.sqrt([4,9,16])`

`np.exp([0,1,2])`

`np.sin([0,np.pi/2])`

`np.cos([np.pi,2*np.pi])`

Purpose:

Absolute value

Round to a certain decimal place

Square root
(same as `**0.5`)

Exponential
(same as `np.e**`)

Sine (from radians)

Cosine

Evaluates to arrays:

`[2,1]`

`[5.2,5.3]`

`[2.,3.,4.]`

`[1.,2.718...,7.389...]`

`[0.,1.]`

`[-1.,1.]`

Functions to create new arrays

Function:

`np.zeros(4)`

`np.ones(4)`

`np.full(4, 2)`

`np.arange(4)`

`np.arange(0, 1, 0.25)`

`np.linspace(0, 1, 5)`

Purpose:

Array of given length
filled with zeros

Array of given length
filled with ones

Array of given length
filled with given value

Same as `range()`...

...except floats and fractional
increments are allowed

Returns the given number of
evenly spaced values from
start to end (both are inclusive)

Evaluates to arrays:

`[0., 0., 0., 0.]`

`[1., 1., 1., 1.]`

`[2, 2, 2, 2]`

`[0, 1, 2, 3]`

`[0., 0.25, 0.5, 0.75]`

`[0., 0.25, 0.5, 0.75, 1.]`

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The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array( [ 5 , 6 , 7 , 8 ] )
```

A one-dimensional (1-D) numpy array

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array([[1, 2, 3, 4], [5, 6, 7, 8]])
```

A two-dimensional (2-D) numpy array

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)

np.array([[1, 2, 3, 4], [5, 6, 7, 8]])

Inner list brackets

Outer list brackets

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])  
2 print(a)
```

```
[[1 2 3 4]  
 [5 6 7 8]]
```

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])  
2 print(a)
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| [| [| 1 | 2 | 3 | 4 |] | 0 |
| | [| 5 | 6 | 7 | 8 |] | 1 |
| |] | | | | | | |

First dimension (rows)

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])  
2 print(a)
```

| | | | | | | |
|---|---|---|---|---|---|---|
| [| [| 1 | 2 | 3 | 4 |] |
| | [| 5 | 6 | 7 | 8 |] |
| | | 0 | 1 | 2 | 3 | |

Second
dimension
(columns)

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])  
2 print(a)
```

| | | | | | |
|---|-----|---|---|-------|---|
| [| [1 | 2 | 3 | 4] | 0 |
| | [5 | 6 | 7 | 8]] | 1 |
| | 0 | 1 | 2 | 3 | |

First dimension (rows)

Second
dimension
(columns)

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])
2 print(a)
```

| | | | | | |
|---|-----|---|---|-----|---|
| [| [1 | 2 | 3 | 4] | 0 |
| | [5 | 6 | 7 | 8] | 1 |
| | | | | | |
| | 0 | 1 | 2 | 3 | |

A single number index gives the **items** of the outer list.

```
1 print(a[0])
2
```

```
[1 2 3 4]
```

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])
2 print(a)
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| [| [| 1 | 2 | 3 | 4 |] | | 0 |
| | [| 5 | 6 | 7 | 8 |] |] | 1 |
| | | 0 | 1 | 2 | 3 | | | |

A single number index gives the **items** of the outer list.

```
1 print(a[0])
2
```

[1 2 3 4]

Select specific items using the row and column index values

```
1 print(a[0,0])
2 print(a[1,0])
3 print(a[0,1])
4 print(a[0,3])
```

1
5
2
4

[row, column]

Slicing and indexing N-dimensional arrays

```
1 a = np.array([[1,2,3,4],[5,6,7,8]])
2 print(a)
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| [| [| 1 | 2 | 3 | 4 |] | | 0 |
| | [| 5 | 6 | 7 | 8 |] |] | 1 |
| | | 0 | 1 | 2 | 3 | | | |

Arrays can also be sliced with rows and columns

```
1 print(a[0,:2])
2 print(a[:,1])
```

```
[1 2]
[2 6]
```

A single number index gives the **items** of the outer list.

```
1 print(a[0])
2
```

```
[1 2 3 4]
```

Select specific items using the row and column index values

```
1 print(a[0,0])
2 print(a[1,0])
3 print(a[0,1])
4 print(a[0,3])
```

```
1
5
2
4
```

[row, column]

The NumPy array (`ndarray`)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array([[1, 2, 3, 4], [5, 6, 7, 8]])
```

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array([[[1, 2, 3, 4], [5, 6, 7, 8]],  
          [[9, 10, 11, 12], [13, 14, 15, 16]]])
```

A three-dimensional (3-D) numpy array

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array( [[ [1, 2, 3, 4], [5, 6, 7, 8]],  
            [[9, 10, 11, 12], [13, 14, 15, 16]] )
```

Outer brackets

Middle brackets

Inner brackets

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)



```
np.array([ [1, 2, 3, 4], [5, 6, 7, 8],  
          [9, 10, 11, 12], [13, 14, 15, 16] ])
```

Outer brackets

Middle brackets

Inner brackets

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)

```
np.array([ [ [1, 2, 3, 4], [5, 6, 7, 8] ],  
          [ [9, 10, 11, 12], [13, 14, 15, 16] ] ])
```

Outer brackets

Middle brackets

Inner brackets

The NumPy array (ndarray)

“N-dimensional array” (e.g. 1-D, 2-D, 3-D, 4-D, etc.)

```
1 b = np.array([[[1,2,3,4],[5,6,7,8]],  
2               [[9,10,11,12],[13,14,15,16]]])  
3 print(b)  
4
```

```
[[[ 1  2  3  4]  
  [ 5  6  7  8]]
```

```
[[ 9 10 11 12]  
 [13 14 15 16]]]
```

2 layers with 2 rows and 4 columns each

[layer, row, column]

```
1 print(b[0,1,3])
```

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- 4. More array functions**

Multi-dimensional NumPy arrays have more than just a length

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```


Multi-dimensional NumPy arrays have more than just a length

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

len()

Gives the number of
items in the outer list
dimension

Example

```
1 print(len(d1))
2 print(len(d2))
3 print(len(d3))
```

4
2
3

Multi-dimensional NumPy arrays have more than just a length

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

len()

Gives the number of
items in the outer list
dimension

size

Gives the total
number of items in
the array

Example

```
1 print(d1.size)
2 print(d2.size)
3 print(d3.size)
```

4
8
24

Notice there are no parentheses
at the end of this. This is because
size is not a function, but an
attribute of the array.

Multi-dimensional NumPy arrays have more than just a length

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

Example

```
1 print(d1.ndim)
2 print(d2.ndim)
3 print(d3.ndim)
```

1
2
3

len()

Gives the number of
items in the outer list
dimension

size

Gives the total
number of items in
the array

ndim

Gives the number of
dimensions in an
array

Multi-dimensional NumPy arrays have more than just a length

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

len()

Gives the number of items in the outer list dimension

size

Gives the total number of items in the array

ndim

Gives the number of dimensions in an array

shape

Gives the number of items in each dimension of an array

Example

```
1 print(d1.shape)
2 print(d2.shape)
3 print(d3.shape)
```

(4,)

(2, 4)

(3, 2, 4)

← Notice these are given as tuples

You can change the shape of a NumPy array

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```


You can change the shape of a NumPy array

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

reshape()

Changes the shape of
the array into a given
shape

Example

```
1 d1_to_d2 = d1.reshape((2,2))
2 print(d1_to_d2)
3 print()
4
5 d3_to_d2 = d3.reshape((2,12))
6 print(d3_to_d2)
```

```
[[1 2]
 [3 4]]
```

```
[[ 1  2  3  4  5  6  7  8  9 10 11 12]
 [13 14 15 16 17 18 19 20 21 22 23 24]]
```

You can change the shape of a NumPy array

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6               [[9,10,11,12],[13,14,15,16]],
7               [[17,18,19,20],[21,22,23,24]]])
```

reshape()

Changes the shape of
the array into a given
shape

flatten()

Creates a copy of an
array as a 1-D array

Example

```
1 print(d3)
2 print()
3 print(d3.flatten())
```

```
[[[ 1  2  3  4]
   [ 5  6  7  8]]
```

```
[[ 9 10 11 12]
 [13 14 15 16]]
```

```
[[17 18 19 20]
 [21 22 23 24]]
```

```
[ 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24]
```

You can change the shape of a NumPy array

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

reshape()

Changes the shape of
the array into a given
shape

flatten()

Creates a copy of an
array as a 1-D array

Example

```
1 print(d2)
2 print()
3 print(d2.transpose())
```

```
[[1 2 3 4]
 [5 6 7 8]]
```

```
[[1 5]
 [2 6]
 [3 7]
 [4 8]]
```

transpose()

Permutes (e.g.
rotates) the axes of an
array

Arithmetic operations with arrays

Arithmetic operators

| | |
|----|----------------|
| + | Addition |
| - | Subtraction |
| * | Multiplication |
| / | Division |
| ** | Exponential |
| % | Remainder |
| // | Floor |

Element-wise arithmetic between two or more arrays

```
1 a = np.array([1,2,3,4])
2 b = np.array([5,6,7,8])
3
4 print('a + b =', a + b)
5 print('a - b =', a - b)
6 print('a * b =', a * b)
```

```
a + b = [ 6  8 10 12]
a - b = [-4 -4 -4 -4]
a * b = [ 5 12 21 32]
```

Element-wise arithmetic with an array and a number

```
1 print('a + 10 =', a + 10)
2 print('10 * a =', 10 * a)
3 print('a / 10 =', a / 10)
4 print('a**2 =', a**2)
```

```
a + 10 = [11 12 13 14]
10 * a = [10 20 30 40]
a / 10 = [0.1 0.2 0.3 0.4]
a**2 = [ 1  4  9 16]
```

Element-wise operations require arrays to be the same dimensions

```
1 x = np.array([1,2,3])
2 y = np.array([11,12,13,14,15])
3
4 print(x + y)
```



```
-----
ValueError                                Traceback (most recent call last)
<ipython-input-97-d5d99ad6233b> in <module>()
      2 y = np.array([11,12,13,14,15])
      3
----> 4 print(x + y)
```

ValueError: operands could not be broadcast together with shapes (3,) (5,)

Element-wise operations require arrays to broadcast to the same dimensions

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

Example

```
1 print(d2)      [[1 2 3 4]
2 print()        [5 6 7 8]]
3 print(d1)      [1 2 3 4]
4 print()
5
6 print(d2+d1)    [[ 2  4  6  8]
                  [ 6  8 10 12]]
```

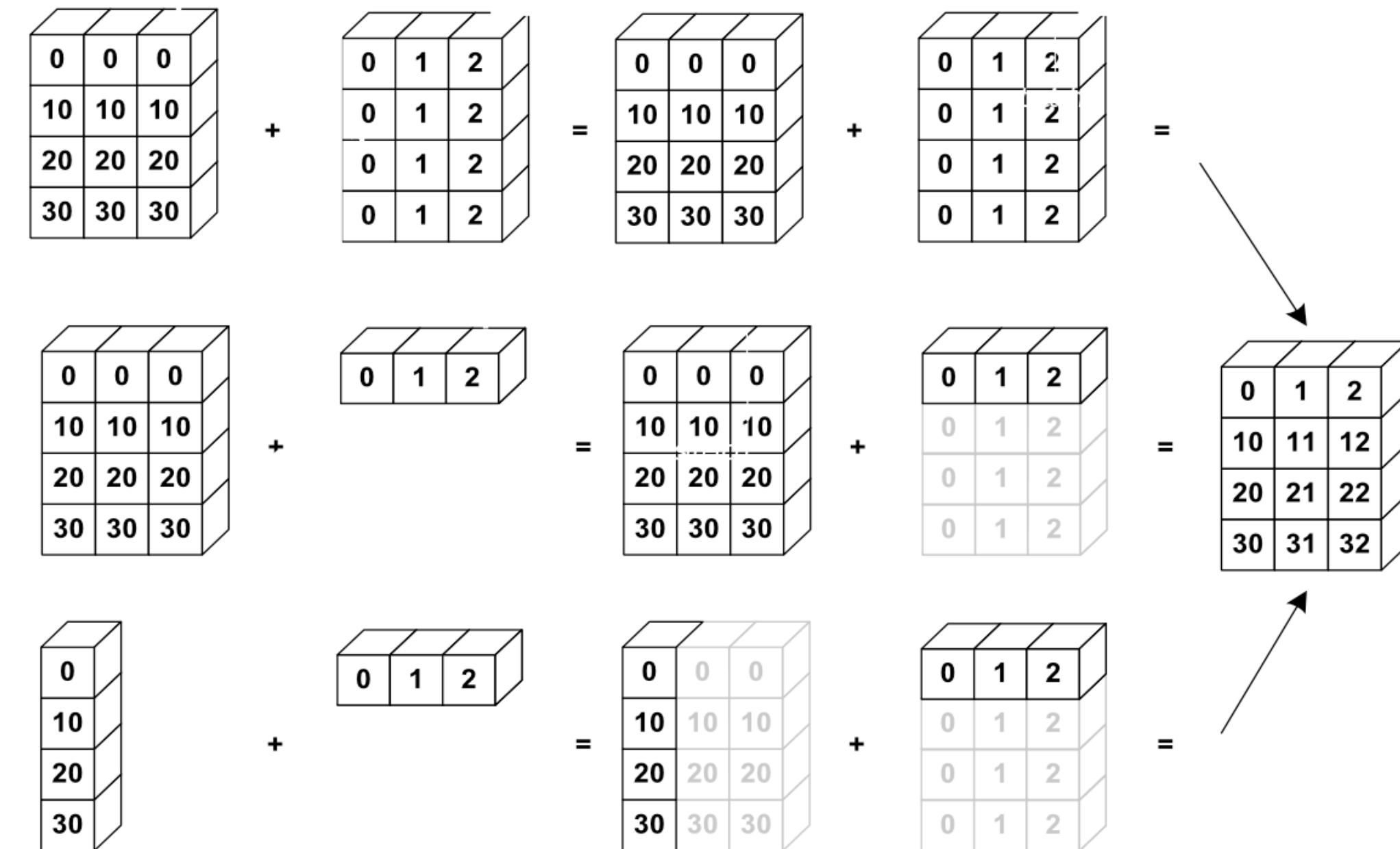


Image: http://scipy-lectures.org/_images/numpy_broadcasting.png

You can combine NumPy arrays

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

You can combine NumPy arrays

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

vstack()

Stacks arrays on top
of each other
vertically

Example

```
1 print(d1)
2 print()
3 print(d2)
4 print()
5
6 print(np.vstack((d1,d2)))
```

```
[1 2 3 4]
```

```
[[1 2 3 4]
 [5 6 7 8]]
```

```
[[1 2 3 4]
 [1 2 3 4]
 [5 6 7 8]]
```


You can combine NumPy arrays

```
1 d1 = np.array([1,2,3,4])
2
3 d2 = np.array([[1,2,3,4],[5,6,7,8]])
4
5 d3 = np.array([[[1,2,3,4],[5,6,7,8]],
6                [[9,10,11,12],[13,14,15,16]],
7                [[17,18,19,20],[21,22,23,24]]])
```

vstack()

Stacks arrays on top
of each other
vertically

hstack()

Stacks arrays
horizontally

Example

```
1 d1_vert = np.array([[1],[2],[3],[4]])
2 print(d1_vert)
3 print()
4 print(d2.transpose())
5 print()
6
7 print(np.hstack((d1_vert,d2.transpose())))
```

```
[[1]
 [2]
 [3]
 [4]]
```

```
[[1 5]
 [2 6]
 [3 7]
 [4 8]]
```

```
[[1 1 5]
 [2 2 6]
 [3 3 7]
 [4 4 8]]
```

Mathematical reductions (array → number)

```
x = np.array([10, 11, 12, 13])
```

| Function: | Purpose: | Evaluates to: |
|---------------------------|--------------------|---------------|
| <code>np.sum(x)</code> | Sum | 46 |
| <code>np.mean(x)</code> | Mean (average) | 11.5 |
| <code>np.median(x)</code> | Median | 11.5 |
| <code>np.max(x)</code> | Maximum value | 13 |
| <code>np.min(x)</code> | Minimum value | 10 |
| <code>np.std(x)</code> | Standard deviation | 1.11803... |

Mathematical reductions (array → number)

```
x = np.array([ [11, 22, 33, 44], [5, 4, 3, 2] ])
```

Function:

Evaluates to:

```
[[11 22 33 44]
 [ 5  4  3  2]]
```

```
np.sum(x, axis=0)
```

```
[16 26 36 46]
```

```
np.mean(x, axis=1)
```

```
[27.5  3.5]
```

```
np.median(x, axis=0)
```

```
[ 8. 13. 18. 23.]
```

```
np.max(x)
```

```
44
```

```
np.min(x, axis=1)
```

```
[11  2]
```

```
np.std(x)
```

```
14.84082...
```


Mathematical reductions (array → number)

```
x = np.array([[11, 22, 33, 44], [5, 4, 3, 2]])
```

| | | | | | | |
|----------|------|----|----|-----|--|----------|
| [| [11 | 22 | 33 | 44] | | axis = 1 |
| | [5 | 4 | 3 | 2]] | | |
| | | | | | | |
| axis = 0 | | | | | | |

Function:

Evaluates to:

```
np.sum(x, axis=0)
```

```
[16 26 36 46]
```

```
np.mean(x, axis=1)
```

```
[27.5  3.5]
```

```
np.median(x, axis=0)
```

```
[ 8. 13. 18. 23.]
```

```
np.max(x)
```

```
44
```

```
np.min(x, axis=1)
```

```
[11  2]
```

```
np.std(x)
```

```
14.84082...
```

Functions to create new arrays

Function:

`np.zeros(4)`

`np.ones(4)`

`np.full(4, 2)`

`np.arange(4)`

`np.arange(0, 1, 0.25)`

`np.linspace(0, 1, 5)`

Purpose:

Array of given length
filled with zeros

Array of given length
filled with ones

Array of given length
filled with given value

Same as `range()`...

...except floats and fractional
increments are allowed

Returns the given number of
evenly spaced values from
start to end (both are inclusive)

Evaluates to arrays:

`[0., 0., 0., 0.]`

`[1., 1., 1., 1.]`

`[2, 2, 2, 2]`

`[0, 1, 2, 3]`

`[0., 0.25, 0.5, 0.75]`

`[0., 0.25, 0.5, 0.75, 1.]`

Functions to create new arrays

Function:

`np.zeros ((4 , 3))`

Purpose:

Array of given length
filled with zeros

Evaluates to arrays:

```
[ [ 0.  0.  0. ]  
  [ 0.  0.  0. ]  
  [ 0.  0.  0. ]  
  [ 0.  0.  0. ] ]
```

`np.ones ((4 ,))`

Array of given length
filled with ones

```
[ 1.  1.  1.  1. ]
```

`np.full ((2 , 3 , 4) , 2)`

Array of given length
filled with given value

```
[ [ [ 2  2  2  2 ]  
    [ 2  2  2  2 ]  
    [ 2  2  2  2 ] ]
```

```
[ [ 2  2  2  2 ]  
  [ 2  2  2  2 ]  
  [ 2  2  2  2 ] ] ]
```