Coordinate frame definitions and transformations for GRB polarisation analysis using `polpy`

Sujay Mate, Hancheng Li, Merlin Kole, Varun Bhalerao
Nicolas Produit, Nicolas De Angelis, Utkarsh Pathak and others

1 Introduction

The purpose of this document is to define the standard coordinate frames (and transformations between them) needed to carry out GRB polarisation analysis within the 3ML data analysis framework. The document is arranged in the following format: Section 2 describes the global and the coordinate frame definitions and Section 3 describes the transformations between them.

2 Definition of frames and the polarisation angle

There are three frames, the celestial frame (J2000), the instrument frame, and the local tangent plane frame. The following subsections define the frames along with Figures 1 and 2.

2.1 The celestial frame (J2000)

The celestial frame is defined in the standard IAU convention where the Z-axis points to the Celestial North pole and the X-axis points to the Vernal Equinox at a given Epoch, in this J2000, which is fixed at noon on January 1, 2000. A direction in the frame is defined by Right Ascension (RA/α) and Declination (Dec/δ)

2.2 The Instrument Reference Frame (IRF)

The definition of instrument reference frame varies for each instrument but typically the Z-axis (or also called the pointing axis) of the frame points towards the Zenith of the detector plane and the X and Y axis are in the plane of the detector plane. Typically, the Z-axis points towards a certain direction (α, δ) in the J2000 frame.

2.3 The Local Tangent Plane frame (LTP)

The local tangent plane frame is defined in the plane perpendicular to the source direction[1]. The Z-axis points to the direction opposite to the source direction, the X-axis points to the "local" North with respect to the IRF and the Y-axis points to the "local" East with respect to the IRF

2.4 Definition of Polarisation Angle (PA)

The IAU Convension (Commision 40, page 21) states that polarisation angle should be measured from local North increasing positively towards the local East in the celestial frame.

3 Frame transformations

3.1 LTP to IRF

Given the θ and ϕ of the source, the direction vectors of X,Y and Z axes of LTP in IRF are given as:

$$X_{LTP} = \begin{bmatrix} -\cos\theta\cos\phi \\ -\cos\theta\sin\phi \\ \sin\theta \end{bmatrix} \quad Y_{LTP} = \begin{bmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{bmatrix} \quad Z_{LTP} = \begin{bmatrix} -\sin\theta\cos\phi \\ -\sin\theta\sin\phi \\ -\cos\theta \end{bmatrix} \tag{1}$$

Hence, the transformation matrix to go from LTP to IRF can be written as:

$$\mathrm{R}_{\mathrm{LTP}}^{\mathrm{IRF}} = \begin{bmatrix} X_{LTP} & Y_{LTP} & Z_{LTP} \end{bmatrix} \tag{2}$$

$$= \begin{bmatrix} -\cos\theta\cos\phi & -\sin\phi & -\sin\theta\cos\phi \\ -\cos\theta\sin\phi & \cos\phi & -\sin\theta\sin\phi \\ \sin\theta & 0 & -\cos\theta \end{bmatrix} \tag{3}$$

[1]The LTP is same to the standard North-East-Down (NED) frame defined in aviation see https://en.wikipedia.org/wiki/Local_tangent_plane_coordinates for more details. To avoid confusion with celestial Noth and East, LTP convention is used here

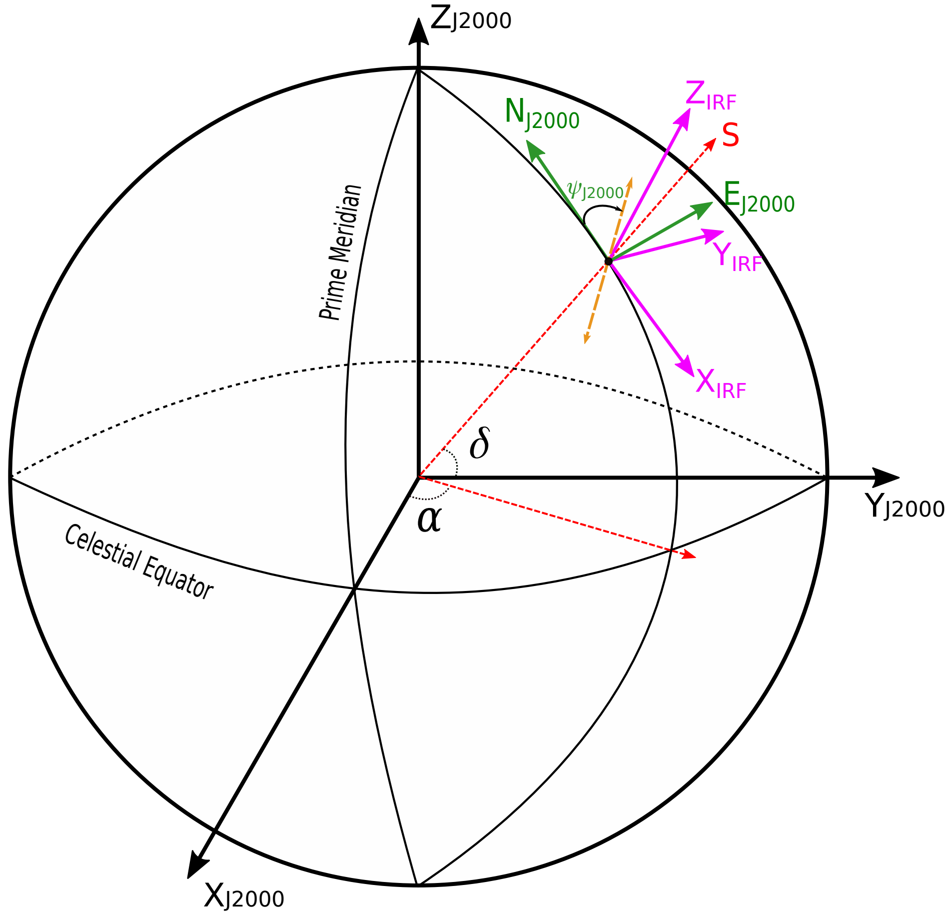

Figure 1: A diagram depicting the relation between J2000 and IRF and the definition of Polarisation Angle in the celestial frame. The three basis vectors of J2000 are shown in black, the source direction (α, δ, and its projection into the XY plane) is shown by the dotted red lines. The IRF basis vectors are shown in magenta, the Z-axis here marks the instrument pointing direction. The green vectors show the local (with respect to the celestial North pole) North and East in the tangent plane perpendicular to the source direction. The orange dotted line marks the projection of the electric field vector and the polarisation angle is defined as ψ_{J2000}).

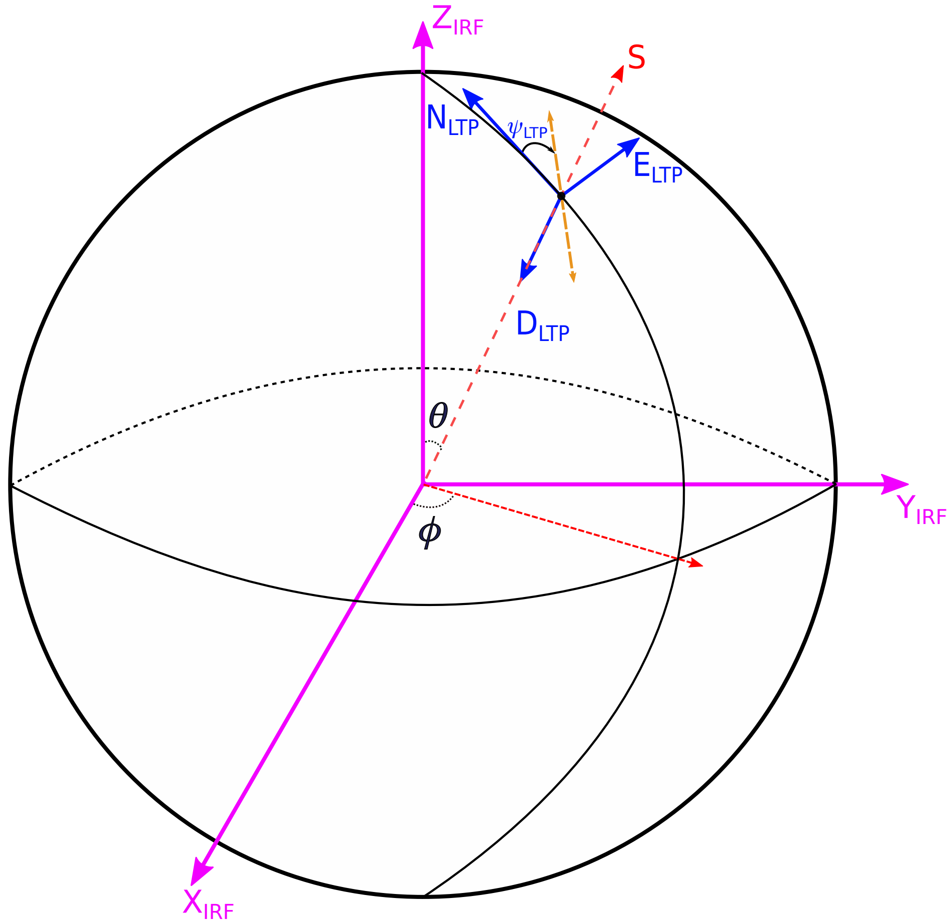

Figure 2: A diagram showing the relation between IRF and LTP and definition of Polarisation Angle in the LTP. The IRF basis vectors are shown in magenta and the source direction in IRF (θ, ϕ, and its projection into the XY plane) is shown by the dotted red lines. The LTP basis vectors are shown in blue. As before, IRF Z-axis direction corresponds the pointing direction of the instrument. The Z-axis of LTP points in the direction opposite to the source (i.e. the direction of incoming photons. The X and Y axis of LTP are the local (with respect to instrument North, i.e. the pointing direction) North and East. The orange dotted lines marks the projection of electric field vector and the Polarisation Angle in LTP is defined by ψ_{LTP})

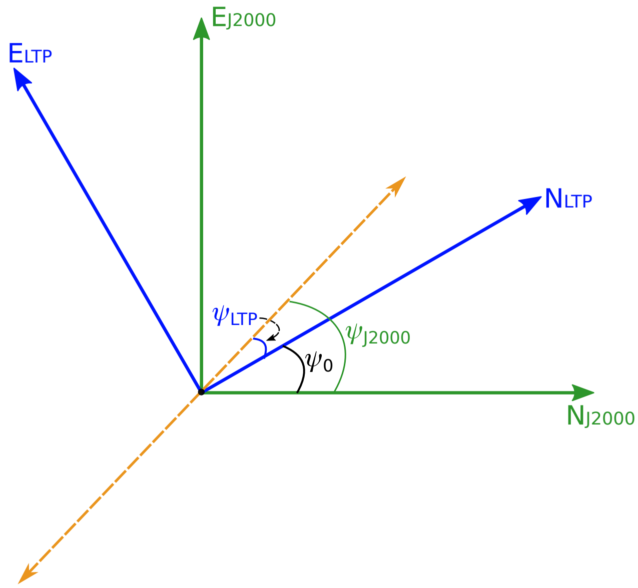

Figure 3: Projected view of the tangent plane perpendicular to the source direction as seen from the source. The orange dotted line is the projected electric field vector. As defined by the IAU convention the ψ_{J2000} is measured from the local North increasing positively towards the East. The ψ_{LTP} is defined in a similar way but with respect to the LTP North and East.

3.2 IRF to J2000

Given the RA (α) and Dec (δ) of the three instrument axes X, Y and Z, the transformation matrix to go from IRF to J2000 can be written as:

$$\mathrm{R}_{\mathrm{IRF}}^{\mathrm{J2000}} = [\vec{X} \quad \vec{Y} \quad \vec{Z}] = \begin{bmatrix} \cos\alpha_{\mathrm{X}}\cos\delta_{\mathrm{X}} & \cos\alpha_{\mathrm{Y}}\cos\delta_{\mathrm{Y}} & \cos\alpha_{\mathrm{Z}}\cos\delta_{\mathrm{Z}} \\ \sin\alpha_{\mathrm{X}}\cos\delta_{\mathrm{X}} & \sin\alpha_{\mathrm{Y}}\cos\delta_{\mathrm{Y}} & \sin\alpha_{\mathrm{Z}}\cos\delta_{\mathrm{Z}} \\ \sin\delta_{\mathrm{X}} & \sin\delta_{\mathrm{Y}} & \sin\delta_{\mathrm{Z}} \end{bmatrix} \tag{4}$$

Note that in real case, to find $\mathrm{R}_{\mathrm{LTP}}^{\mathrm{IRF}}$ knowing α and δ of $\vec{X}$ and $\vec{Z}$ are enough and $\vec{Y}$ can be estimated as:

$$\vec{Y} = \vec{Z} \times \vec{X} \tag{5}$$

3.3 PA transformation

The two PA are related as follows:

$$\psi_{\mathrm{J2000}} = \psi_{\mathrm{LTP}} + \psi_0 \tag{6}$$

where the ψ_0 is azimuth of LTP North wrt J2000 North. The azimuth is computed by transforming the Z-axis to J2000 as:

$$\mathrm{Z}_{\mathrm{LTP}}^{\mathrm{J2000}} = \mathrm{R}_{\mathrm{IRF}}^{\mathrm{J2000}} \times \mathrm{R}_{\mathrm{LTP}}^{IRF} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

$$\text{and} \quad \psi_0 = \mathrm{arctan2}\left(\frac{Z_{\mathrm{LTP}}^{\mathrm{J2000}}(y)}{Z_{\mathrm{LTP}}^{\mathrm{J2000}}(x)}\right) \tag{8}$$