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Lecture 16.ns11

Exercises

Course: **Complex Networks Analysis and Visualization**
Sub-Module: **NetSci**

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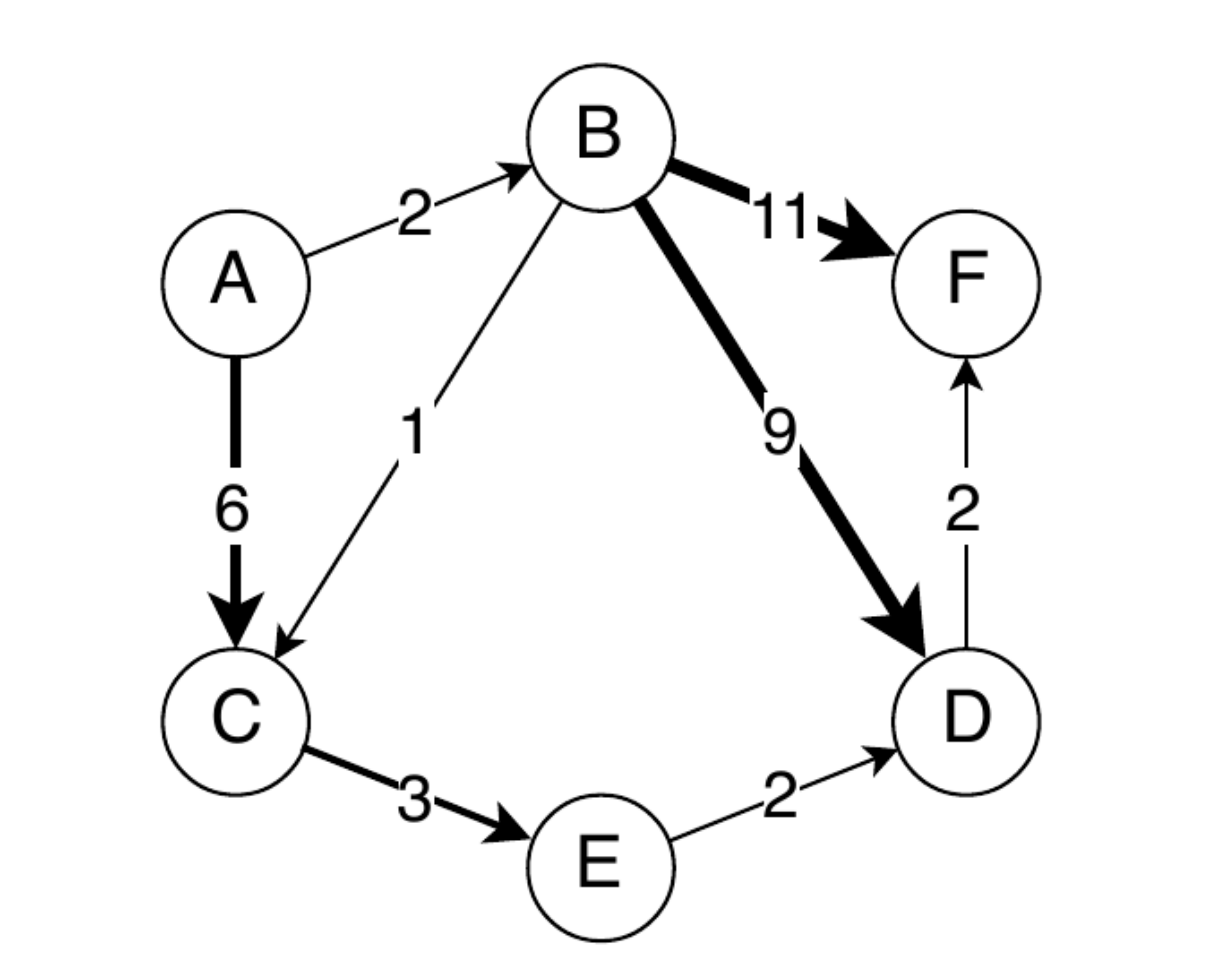
Theory

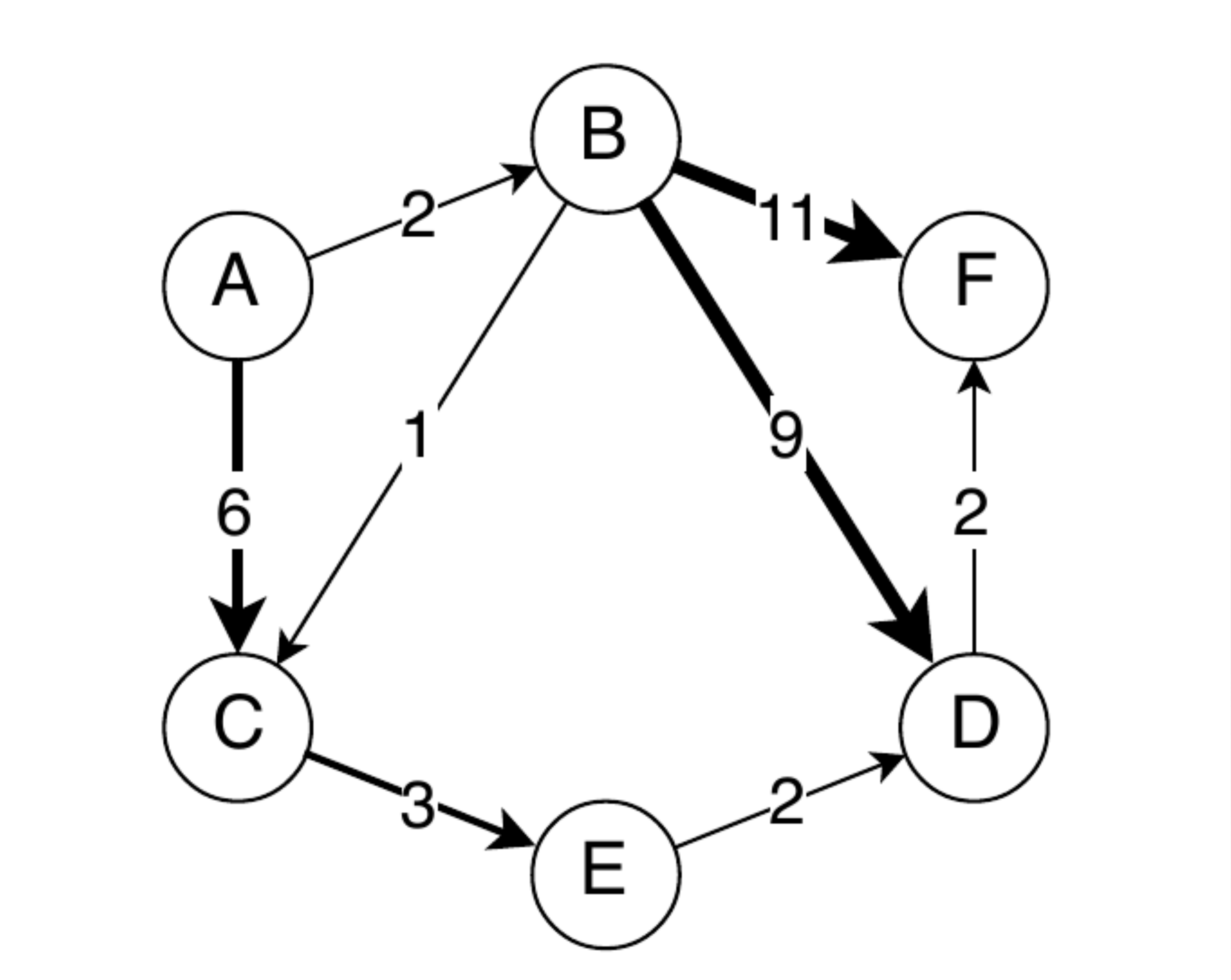
Ex 1

- Consider the weighted directed network in Figure 2.13. Which of the following most accurately describes the connectedness of this network?
- a. Strongly connected
- b. Weakly connected
- c. Disconnected
- d. None of the above



Ex 2

- What is the out strength of node C?
- What is the in strength of node D?



Ex 3, 4, 5, 6

- [3] Consider any arbitrary directed graph D along with its undirected version G. **True or False**: If the average shortest path length and diameter of the directed graph exist, they can be smaller than those of the undirected version.
- [4] Assume you have a graph with 100 nodes and 200 links. What is the average degree of nodes in this network?
- [5] What is the maximum clustering coefficient for a node in an arbitrary undirected graph?
- [6] A tree is a connected graph without cycles. What is the maximum clustering coefficient for a node? Prove that L (the number of edges) and n (the number of nodes) are related by the relation L = n-1. *Hint: use a recursive approach.*

Ex 7

- Which of the following seemingly conflicting properties are true of social networks?
- a. Social networks have short paths, yet large diameter.
- b. Social networks have small diameter, yet large average path length.
- c. Social networks have many high-degree nodes, yet are disconnected.
- d. Social networks are highly clustered, yet are not dense

Ex 8

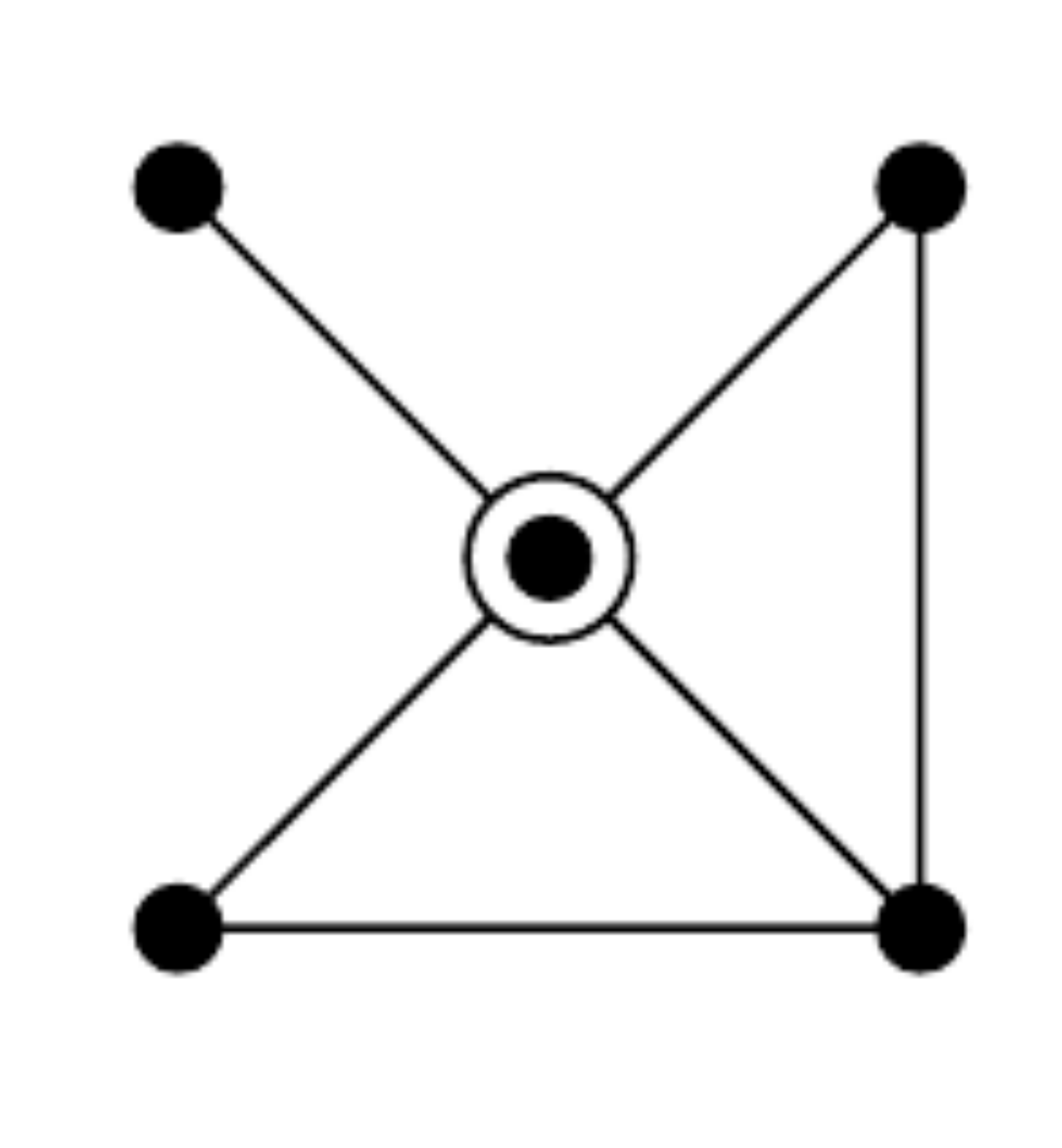
- Let A be the adjacency matrix of an undirected network and $\mathbf{1}$ be the column vector whose elements are all 1. In terms of these quantities write expressions for:
- a) The vector k whose elements are the degrees k_i of the nodes;
- b) The number m of edges in the network;
- c) The matrix N whose element N_{ij} is equal to the number of common neighbors of i and j;
- d) The total number of triangles in the network, where a triangle means three nodes, each connected by edges to both of the others.

Ex 9

- Consider a bipartite network, with its two types of nodes, and suppose that there are n1 nodes of type 1 and n2 nodes of type 2. Show that the mean degrees c1 and c2 of the two types are related by
- $$n_1 c_1 = n_2 c_2$$

Ex 10

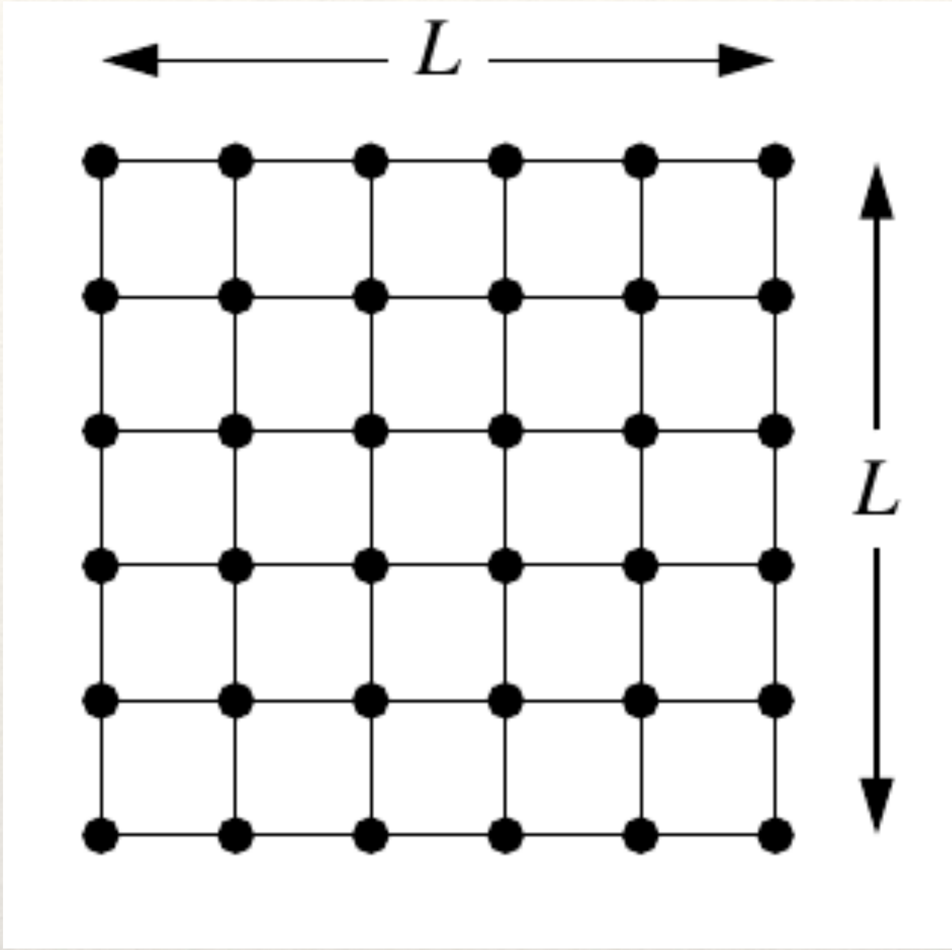
- Calculate the local clustering coefficient of each node in this network



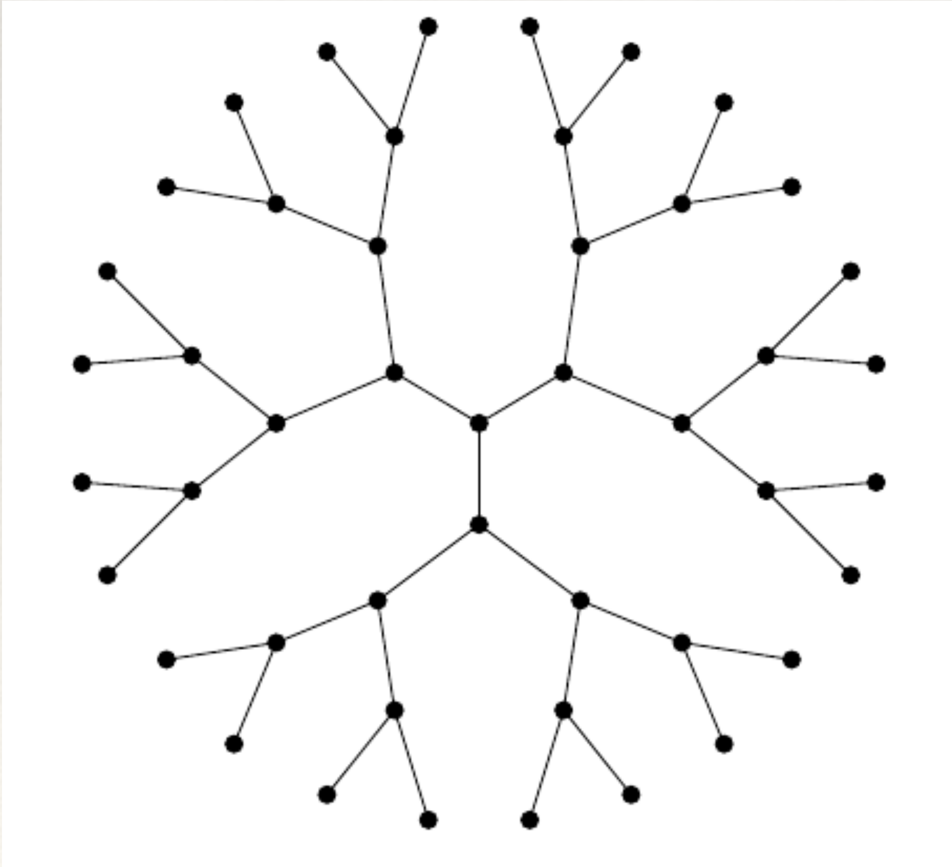
Ex 11

- One can calculate the diameter of certain types of networks exactly.
- a) What is the diameter of a clique?
- b) What is the diameter of a square portion of square lattice, with L edges (or equivalently L + 1 nodes) along each side, like this (b)
- c) A Cayley tree is a symmetric regular tree in which each node is connected to the same number k of others, until we get out to the leaves, like this (c). (We have k 3 in this picture.) Show that the number of nodes reachable in d steps from the central node is $k(k - 1)^{d-1}$ for $d \geq 1$. Hence find an expression for the diameter of the network in terms of k and the number of nodes n.
- d) Which of the networks in parts (b), and (c) displays the small-world effect, defined as having a diameter that increases as log n or slower?

b



c



Exercises

Task 1

- Define a function to generate an Erdos-Renyi (or Gilbert) random graph taking (n, L) or (n, p) as inputs. Make the following checks and test them against theory or against the NetworkX implementation
- Generate the adjacency matrix of this graph and check that it is symmetric
- Check that the average degree is correct
- Using the result of exercise 8, find the node i that is part of the largest number of triangles
- Build the matrix N as in exercise 8. What can you tell about the diagonal elements of this matrix?
- Given N, find the node j different from i that shares the largest number of neighbors with i and check with NetworkX that effectively they share N[i,j] common neighbors.
- Is i the node with the largest clustering coefficient? Why?
- What is the size of the largest connected component as a function of n?
- Plot the degree distribution, does it match the theoretical prediction?

Task 2

- Implement the Barabasi-Albert model taking n (the number of nodes) and m (the number of edges to be added at each step) as inputs. Make the following checks and compare your implementation to the NetworkX one
- Generate the adjacency matrix of this graph and check that it is symmetric
- What is the (approximate) relation between the average degree and m? Why?
- Plot the degree distribution and verify that it decays as k^{-3}. To get better results, you might want to average over multiple realizations.
- How does the average degree and the square of the average degree depend on n?
- Very the friendship paradox on this network
- Make a non-linear preferential attachment model and see how it impact the degree distribution

Task 3

- Repeat the steps of task 1 and 2 for a model we did in class or on a real world graph taken from the SNAP repository (http://snap.stanford.edu/)

Solutions

- Ex 1: weakly connected
- Ex 2: s_out(C) = 3; s_in(D) = 11
- Ex 3: False
- Ex 4: the average degree is 4
- Ex 5: 1
- Ex 6: 0, since it does not contain cycles, hence triangles
- Ex 7: d
- Ex 8: $a) A1; b) \frac{1^T A1}{2}; c) A^2; d) \frac{\mathrm{tr}(A^3)}{6}$

Solutions

- Ex 10: 0: 1; 2/3; 1; 1/3
- Ex 11: a) 1; b) L; d) c
- Ex 3: False
- Ex 4: the average degree is 4
- Ex 5: 1
- Ex 6: 0, since it does not contain cycles, hence triangles
- Ex 7: d
- Ex 8: $a) A1; b) \frac{1^T A1}{2}; c) A^2; d) \frac{\mathrm{tr}(A^3)}{6}$