CSE 6730,Final Report

Project 2: Complex Simulation

1 Project Title

Simulation of Predator-Prey Population Dynamics

2 Team Members

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3 Github

Currently, the github is at https://github.gatech.edu/ser8/complex-sys. The final submission for checkpoint and final project will be at the Folder Submission with each subfolder.
For the ABS model, we create one model with interaction but it is conflicting with macbook pro touch bar for one of our member laptop and the laptop shut down everytime running. Running on other member laptops is completely fine.
We create one static version just in case.

4 Problem Description and Purpose

The predator and prey relationship is an important ecological system. Their populations rise and fall over time as they interact and impact one another. These interactions are the prime movers of energy through food chains. Both prey and predators are affecting each other. [5]In simplest interaction, predators depend on the prey as the food source. However, any abuse of the food source may result in decease in population of the prey, and subsequently decrease the number of the predators due to lack of food. Because of such interaction, the population of the predators and the prey may oscillate, and inversely proportional to each others.[7]

Predator prey relationship is important for us to understand the impact of the relationship on the ecological system in one area. Such relationship is always complicated. Without predators, prey (normally herbivores) will cause detrimental impact on the plants in that area. However, overkill by the predators may also impact the balance of the nature. Besides, there are effects from human intervention on such relationship (eg: hunting and destroy of the habitat). Furthermore, predator-prey model can be used to describe many fundamental characteristics of ecological systems and can even be extended to other ideas like military response [2].

One of the mathematical models that simulates predator and prey interactions is the Lotka-Volterra model proposed by Alfred Lotka and Vito Volterra. Lotka helped develop the logistic equation to explain auto-catalytic chemical reactions. Volterra interconnected the logistic equation to two separate populations in competition to explain predator and prey relationships. We hope to use this intuitive model in our complex system simulation, so that we could gain more understanding on the relationship, as well as the impact of our activities on such relationship. Furthermore, we would like to investigate the equilibria and stability of our ecological system, and to facilitate in understanding the general behavior of the system by finding the equilibrium solutions of a coupled set of nonlinear ordinary differential equations.[9][1]

5 General Assumptions

Important refinements such as environmental heterogeneities, more complex food web networks, and different functional responses will be selectively added to future models in iterative process. The following assumptions are made with current model:

- The environment is assumed homogeneous with no natural disasters or variations in temperature
- All species are of the same size, produce the same amount of resources when consume
- No limited lifetime for preys and predators
- Each prey or predator has a fixed probability of reproducing at each time step.
- Unlimited food available to the prey is assumes, and so the prey(and predator)growth rates are limited by corresponding capacity and growth coefficients
- Each predator eats a constant proportion of the prey population per year; In other words, doubling the prey population will double the number eaten per predator, regardless of how big the prey population is.
- Predator reproduction is directly proportional to prey consumed; another way of expressing this is that a certain number of prey consumed results in one new predator; or that one prey consumed produces some fraction of a new predator.
- A constant proportion of the predator population dies per year. In other words, the predator death rate is independent of the amount of food available.

6 Literature Review

Simple predator and prey interaction could be model using Lotka-Volterra model. In our model, we borrow idea from [8] to build firstly our simple interaction model. Firstly, our rabbit model is following the

$$R_t = R_{t-1} + growth_R \times (\frac{capacity_R - R_{t-1}}{capacity_R})R_{t-1} \tag{1}$$

For the coyote,

$$\begin{aligned} C_t &\sim (1 - death_C) \times C_{t-1} \\ &= C_{t-1} - death_C \times C_{t-1} \end{aligned} \tag{2}$$

With the simple interaction from the first two parts, now we can combine both interaction and come out with simple interaction between the two species.

$$R_t = R_{t-1} + growth_R \times (\frac{capacity_R - R_{t-1}}{capacity_R})R_{t-1} - death_R(C_{t-1}) \times R_{t-1} \tag{3}$$

$$C_t = C_{t-1} - death_C \times C_{t-1} + growth_C(R_{t-1}) \times C_{t-1} \tag{4}$$

In equations above, death rate of rabbit is a function parameterized by the amount of coyote. Similarly, the growth rate of coyotes is a function parameterized by the amount of the rabbit. The death rate of the rabbit should be 0 if there are no coyotes, while it should approach 1 if there are many coyotes. One of the formula fulfilling this characteristics is hyperbolic function.

$$death_R(C) = 1 - \frac{1}{xC + 1} \tag{5}$$

where x determines how quickly $death_R$ increases as the number of coyotes (C) increases. Similarly, the growth rate of the coyotes should be 0 if there are no rabbits, while it should approach infinity if there are many rabbits. One of the formula fulfilling this characteristics is a linear function.

$$growth_C(R) = yR \tag{6}$$

where y determines how quickly $growth_C$ increases as number of rabbit (R) increases.

Putting all together, the final equtions are

$$R_t = R_{t-1} + growth_R \times \Big(\frac{capacity_R - R_{t-1}}{capacity_R}\Big)R_{t-1} - \Big(1 - \frac{1}{xC_{t-1}+1}\Big) \times R_{t-1} \tag{7}$$

$$C_t = C_{t-1} - death_C \times C_{t-1} + yR_{t-1}C_{t-1} \tag{8}$$

The previous relationship could be extended to multiple predators and preys relationship. In the multiple predators and preys model, we will be focusing on a specific 2+2 model with two preys: rabbits and deers ,and two predators: coyotes and wolfs. We assume that each predator preys on each prey but not on each other. And the preys population capacity are affecting each other:

$$capacity_D = capacity_{prey} - capacity_R \tag{9}$$

What difference from the simple predator and prey interaction is that, now the death rate of preys is a function parameterized by both the amount of coyotes and the amount of wolfs.

$$death_R(C, W) = (1 - \frac{1}{xC+1}) + (1 - \frac{1}{vW+1}) \tag{10}$$

$$death_D(C, W) = (1 - \frac{1}{xC+1}) + (1 - \frac{1}{vW+1}) \tag{11}$$

where v determines how quickly $death_R$ increases as the number of wolfs (W) increases. Similarly, the growth rate of predators is a function parameterized by the amount of both rabbits and deer. Where u determines how quickly $growth_W$ increases as number of rabbit (R) increases, and D_t represents the number of deer at time t.

$$growth_C(R) = yR + uD \tag{12}$$

$$growth_W(R) = yR + uD \tag{13}$$

Multiple Prey Multiple Predator model is described in the following equations:

$$R_t = R_{t-1} + growth_R \times (\frac{capacity_R - R_{t-1}}{capacity_R})R_{t-1} - (1 - \frac{1}{xC_{t-1}+1}) \times R_{t-1} - (1 - \frac{1}{vW_{t-1}+1}) \times R_{t-1} \tag{14}$$

$$D_t = D_{t-1} + growth_D \times (\frac{capacity_D - D_{t-1}}{capacity_D})D_{t-1} - (1 - \frac{1}{xC_{t-1}+1}) \times D_{t-1} - (1 - \frac{1}{vW_{t-1}+1}) \times D_{t-1} \tag{15}$$

$$C_t = C_{t-1} - death_C \times C_{t-1} + yR_{t-1}C_{t-1} + uD_{t-1}C_{t-1} \tag{16}$$

$$W_t = W_{t-1} - death_W \times W_{t-1} + yR_{t-1}W_{t-1} + uD_{t-1}W_{t-1} \tag{17}$$

7 Data Source

For this project, we do some simple simulation between rabbit and coyote. We obtained the idea from the Wikipedia for rabbits and coyotes growth and reduction rate.

8 Methodology

Our simulation will first simulate predators and prey entering and exiting a predefined area. Then through interactions, their population may affecting each others.

Traditionally, there is the nonlinear Lotka-Volterra Model of the predator-prey dynamic system [4, 6]. LVM approach is a simplified model and suitable for detailed stability analysis. However, it is also very limited model and lack of flexibility for complex interaction. Hence, we also hope to incorporate the Agent-Based Model [3] in this project to increase the completeness of our analysis. We gained most of insight of writing our simulation based on [8].

In our project, some of the ideas that we wish to investigate include:

1. Long-term population interaction among predators and prey.
2. Introduction of the uncertainties like diseases.
3. Introduction of the third parties interaction: human activity, natural disasters etc.

9 Development Platform

The programming language is Python 3. We will provide a Jupyter notebook for user interaction. In the Jupyter notebook, we will allow the user to change some of the probability and the simulation parameters to see different result of the simulation.

9.1 Lotka-Volterra Model

Currently, we have successfully model the world, rabbits and coyotes. Since this is a step by step tutorial based project, we first create the simulation with only the rabbits growth rate, and coyotes death rate. Finally, we allow some interaction between rabbits and coyotes. This is the first phase of our Lotka-Volterra Model.

In the program, simply run python notebook for the current main.ipynb. The tutorial should be self contained. Some of the current parameters that could be play with is as per table below

Parameter	Description
Single Rabbit Model	
Initial population	Initial rabbit population
Capacity	Capacity of the environment
Growth rate	How fast rabbit could grow
Single Coyote Model	
Initial population	Initial coyote population
Death rate	How fast coyote could decrease
Coyote Rabbit Interaction Model	
Initial population	Initial rabbit population
Capacity	Capacity of the environment
Growth rate	How fast rabbit could grow
Initial population	Initial coyote population
Death rate	How fast coyote could decrease
x	How fast rabbit decrease due to the coyote population
y	How fast coyote increase due to the rabbit population
Multiple Preys and Predators Model	
Initial prey population	Initial rabbit and deer population
Capacity	Capacity of the environment and capacity of rabbit
Growth rate of each prey	How fast rabbit and deer could grow
Initial predator population	Initial coyote and wolf population
Death rate of each predator	How fast coyote and wolf could decrease
Death rate ratio due to coyote	How fast rabbit and deer decrease due to the coyote population
Death rate ratio due to wolf	How fast rabbit and deer decrease due to the wolf population
Growth rate ratio due to rabbit	How fast coyote and wolf increase due to the rabbit population
Growth rate ratio due to deer	How fast coyote and wolf increase due to the deer population

9.2 Agent Based Simulation

We improve our model to Agent Based Model for the same relationship. We have one gui interaction model, and another static model populate the outcome of interaction.

10 Tutorial

The tutorial is divided into two portion. One is the basic Lotka-Volterra Model. Another is the Agent Based Simulation Model. Both the tutorials are run on Jupyter Notebook.

10.1 Lotka-Volterra Model

10.1.1 Introduction

Project topic: **Simulation of Predator-Prey Population Dynamics**
our model is single relationship, multiple relationship Our prey are rabbits and deers, while the predators are coyote and wolves. Pending for screenshot

10.1.2 Part 1: Rabbits without predators

In part 1 of the tutorial, first we introduce a relationship for a prey (rabbits) without predators. The default capcity of 605 is chosen by making some assumption. Here, we introduce the formula used.

Part 1: Rabbits without predators

According to Mother Earth News, a rabbit eats six square feet of pasture per day. Let's assume that our rabbits live in a five acre clearing in a forest: 217,800 square feet/6 square feet = 36,300 rabbit-days worth of food. For simplicity, let's assume the grass grows back in two months. Thus, the carrying capacity of five acres is 36,300/60 = 605 rabbits.

Female rabbits reproduce about six to seven times per year. They have six to ten children in a litter. According to Wikipedia, a wild rabbit reaches sexual maturity when it is about six months old and typically lives one to two years. For simplicity, let's assume that in the presence of unlimited food, a rabbit lives forever, is immediately sexually mature, and has 1.5 children every month.

For our purposes, then, let x_t be the number of rabbits in our five acre clearing on month t.

$$R_t = R_{t-1} + 1.5\frac{605 - R_{t-1}}{605} R_{t-1}$$

The formula could be put into general form

$$R_t = R_{t-1} + growth_R \times \left(\frac{capacity_R - R_{t-1}}{capacity_R}\right) R_{t-1}$$

By doing this, we allow users to interact with growth rate and the capacity value visualize different interaction

Figure 1: Introduce rabbit population without predators formula

Then, we allow users to change the default value of initial rabbit population, growth rate and capacity of the rabbit. Once users key in the value, they could visualize the dynamic of the population of the rabbits.

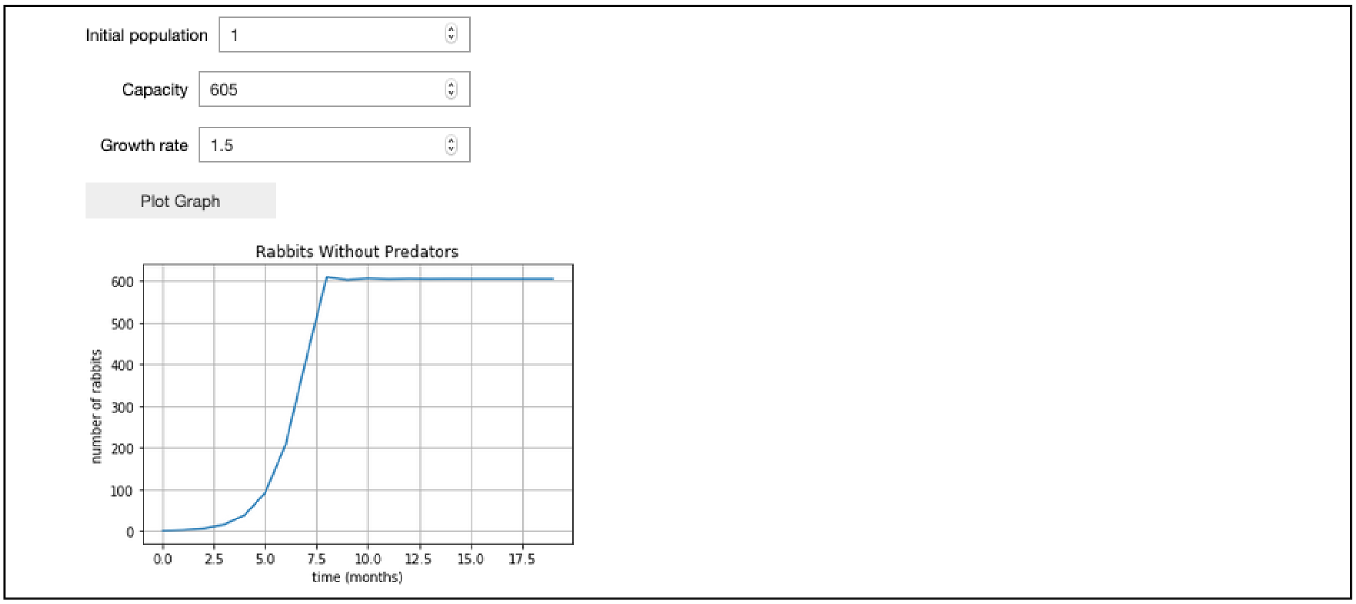


Figure 2: Interaction for different initial rabbit population, growth rate and population capacity

One important interaction is the growth rate, as we change from 1.5 to 3, the population dynamics of the rabbits become no longer stable.

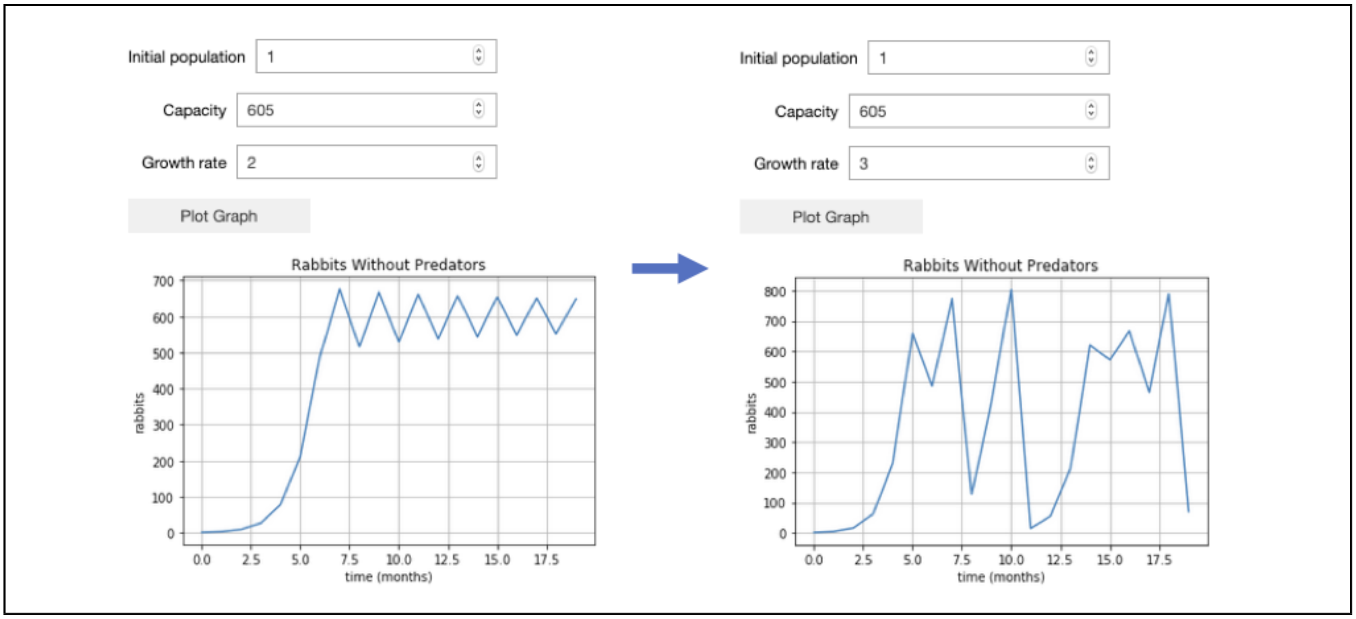


Figure 3: Changing of growth rate lead to unstable population

We could review our growth rate formula of the rabbit by changing the original formula. In current formula, there is a shaping factor. This shaping factor could be generalized to situation such as mass destruction due to destruction of fores, or more natural condition such as different in food source for different seasons.

Tweaking the Growth Function

The growth is regulated by this part of the formula:

$$\frac{capacity_R - R_{t-1}}{capacity_R}$$

That is, this fraction (and thus growth) goes to zero when the land is at capacity. As the number of rabbits goes to zero, this fraction goes to 1.0, so growth is at its highest speed. We could substitute in another function that has the same values at zero and at capacity, but has a different shape. For example,

$$\left(\frac{capacity_R - R_{t-1}}{capacity_R}\right)^{\beta}$$

where β is a positive number. For example, if β is 1.3, it indicates that the rabbits can sense that food supplies are dwindling and pre-emptively slow their reproduction.

Figure 4: Changing of growth formula

With such changes, now we have more simulate that rabbit could sense how much the food supplies are dwinding by changing their reproductive condition. With such interaction, even with the same growth rate of 3. Once we throttled their overall growth rate by this shaping condition, we could get a more realistic population dynamic.

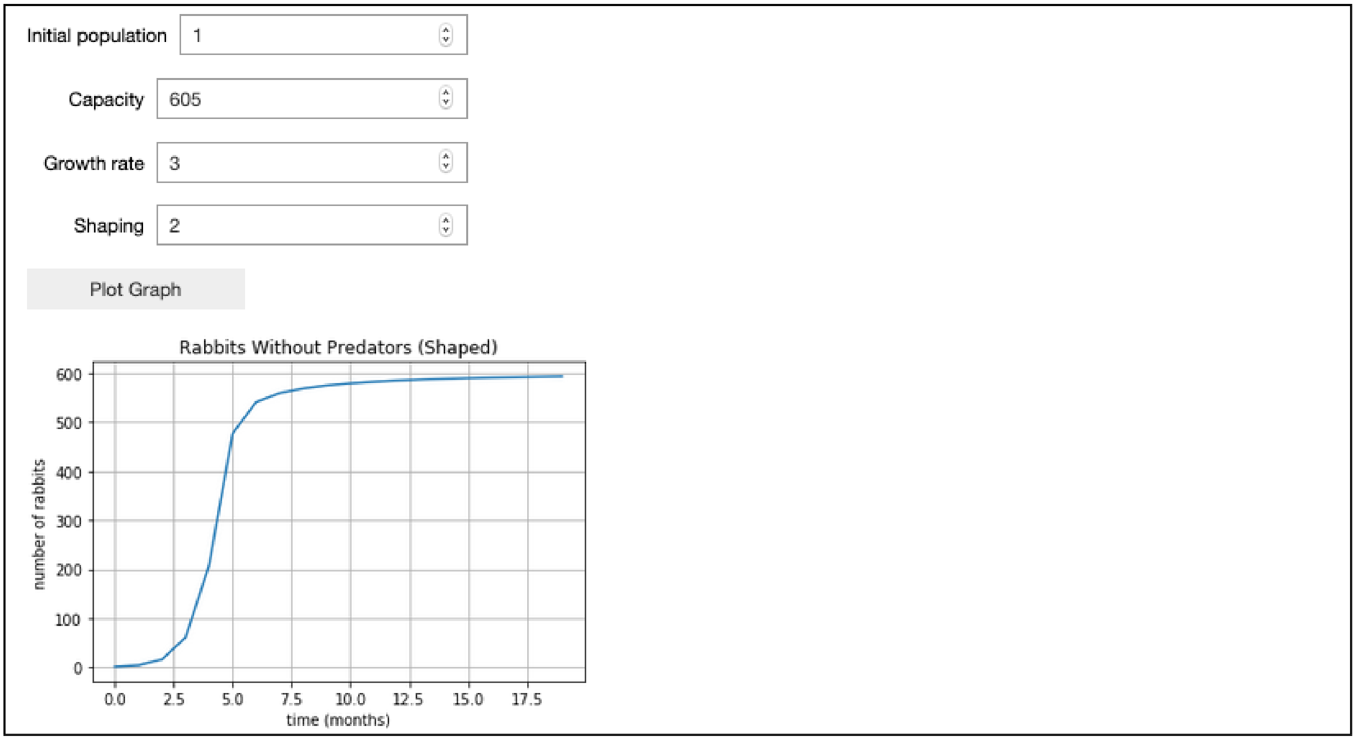
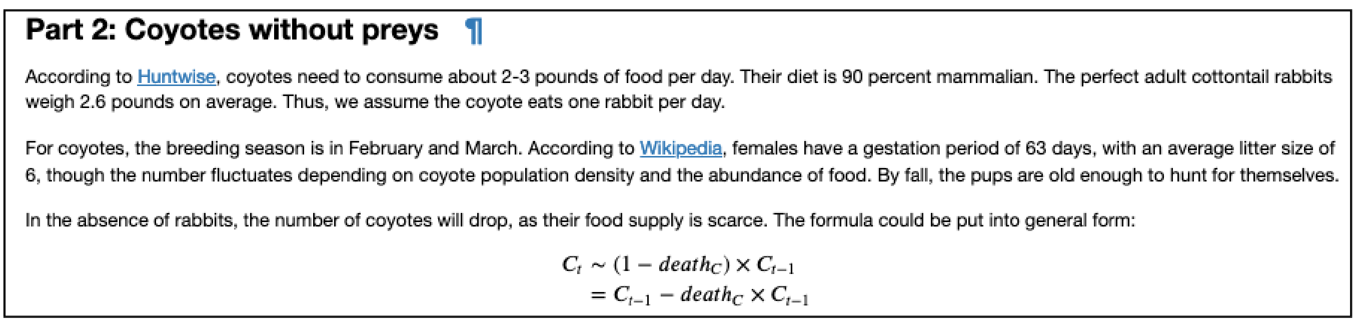


Figure 5: With shaping force, the rabbit population become stable

10.1.3 Part 2: Coyote without preys

In part 2, we introduce basic predator (coyotes) without preys. It is obvious that the formula will be a decreasing function. Since without preys, coyotes will die due to lack of food source.



Part 2: Coyotes without preys ¶

According to Huntwise, coyotes need to consume about 2-3 pounds of food per day. Their diet is 90 percent mammalian. The perfect adult cottontail rabbits weigh 2.6 pounds on average. Thus, we assume the coyote eats one rabbit per day.

For coyotes, the breeding season is in February and March. According to Wikipedia, females have a gestation period of 63 days, with an average litter size of 6, though the number fluctuates depending on coyote population density and the abundance of food. By fall, the pups are old enough to hunt for themselves.

In the absence of rabbits, the number of coyotes will drop, as their food supply is scarce. The formula could be put into general form:

$$\begin{aligned} C_t &\sim (1 - death_C) \times C_{t-1} \\ &= C_{t-1} - death_C \times C_{t-1} \end{aligned}$$

Figure 6: Introduce coyote population without preys formula

We allows users to change how fast coyotes die, as well as their initial population.

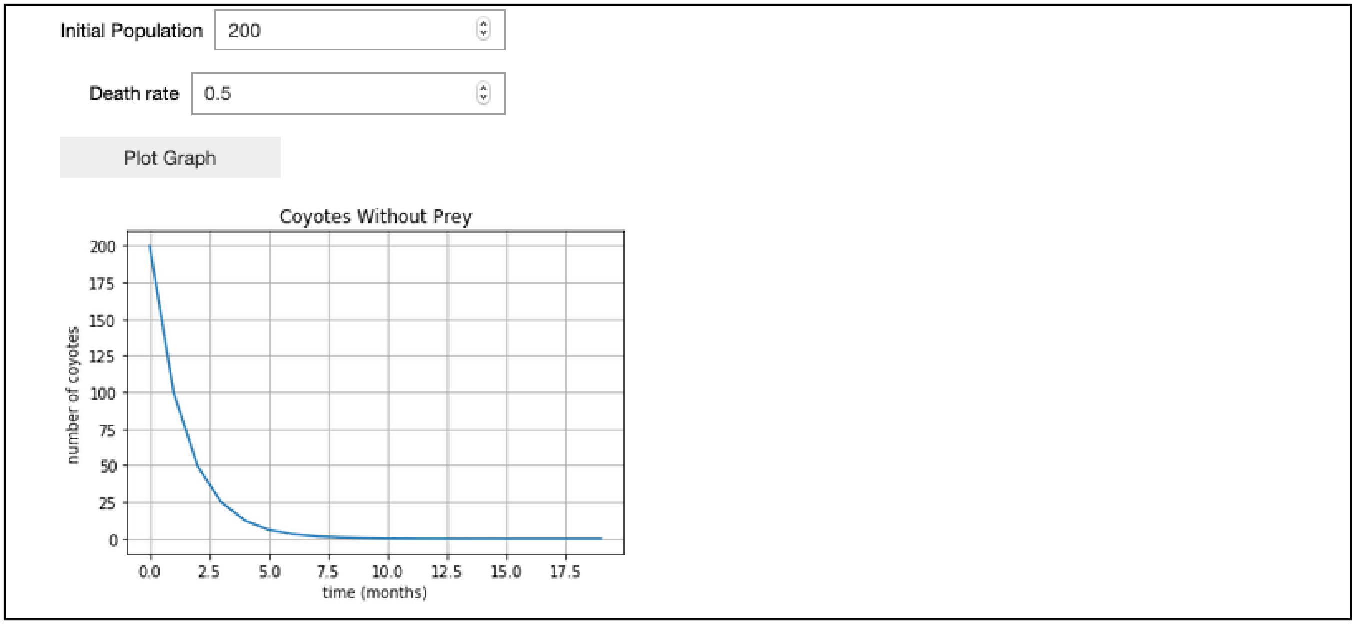


Figure 7: Interaction for different initial coyote pupulation and dead rate

10.1.4 Part 3: Interaction between coyotes and rabbits

In part 3, we start to allow interaction between coyotes and rabbits.

Part 3: Interaction Between Coyotes and Rabbit

With the simple interaction from the first two parts, now we can combine both interaction and come out with simple interaction.

$$R_t = R_{t-1} + growth_R \times \left(\frac{capacity_R - R_{t-1}}{capacity_R}\right) R_{t-1} - death_R(C_{t-1}) \times R_{t-1}$$

$$C_t = C_{t-1} - death_C \times C_{t-1} + growth_C(R_{t-1}) \times C_{t-1}$$

In equations above, death rate of rabbit is a function parameterized by the amount of coyote. Similarly, the growth rate of coyotes is a function parameterized by the amount of the rabbit.

The death rate of the rabbit should be 0 if there are no coyotes, while it should approach 1 if there are many coyotes. One of the formula fulfilling this characteristics is hyperbolic function.

$$death_R(C) = 1 - \frac{1}{xC + 1}$$

where x determines how quickly $death_R$ increases as the number of coyotes (C) increases. Similarly, the growth rate of the coyotes should be 0 if there are no rabbits, while it should approach infinity if there are many rabbits. One of the formula fulfilling this characteristics is a linear function.

$$growth_C(R) = yC$$

where y determines how quickly $growth_C$ increases as number of rabbit (R) increases.

Putting all together, the final equtions are

$$R_t = R_{t-1} + growth_R \times \left(\frac{capacity_R - R_{t-1}}{capacity_R}\right) R_{t-1} - \left(1 - \frac{1}{xC_{t-1} + 1}\right) \times R_{t-1}$$

$$C_t = C_{t-1} - death_C \times C_{t-1} + yR_{t-1}C_{t-1}$$

Figure 8: Interaction between coyotes and rabbits

Users have control for rabbit and coyote initial population, capacity for the rabbit, growth rate for rabbit and death rate for coyote. From the formula, we know the x and y are two new parameters introduced in current interactions.

Specifically, x determine how quickly rabbits are decreasing when the coyotes increases; whereas y decides how quickly coyotes are increasing when more rabbits are available.

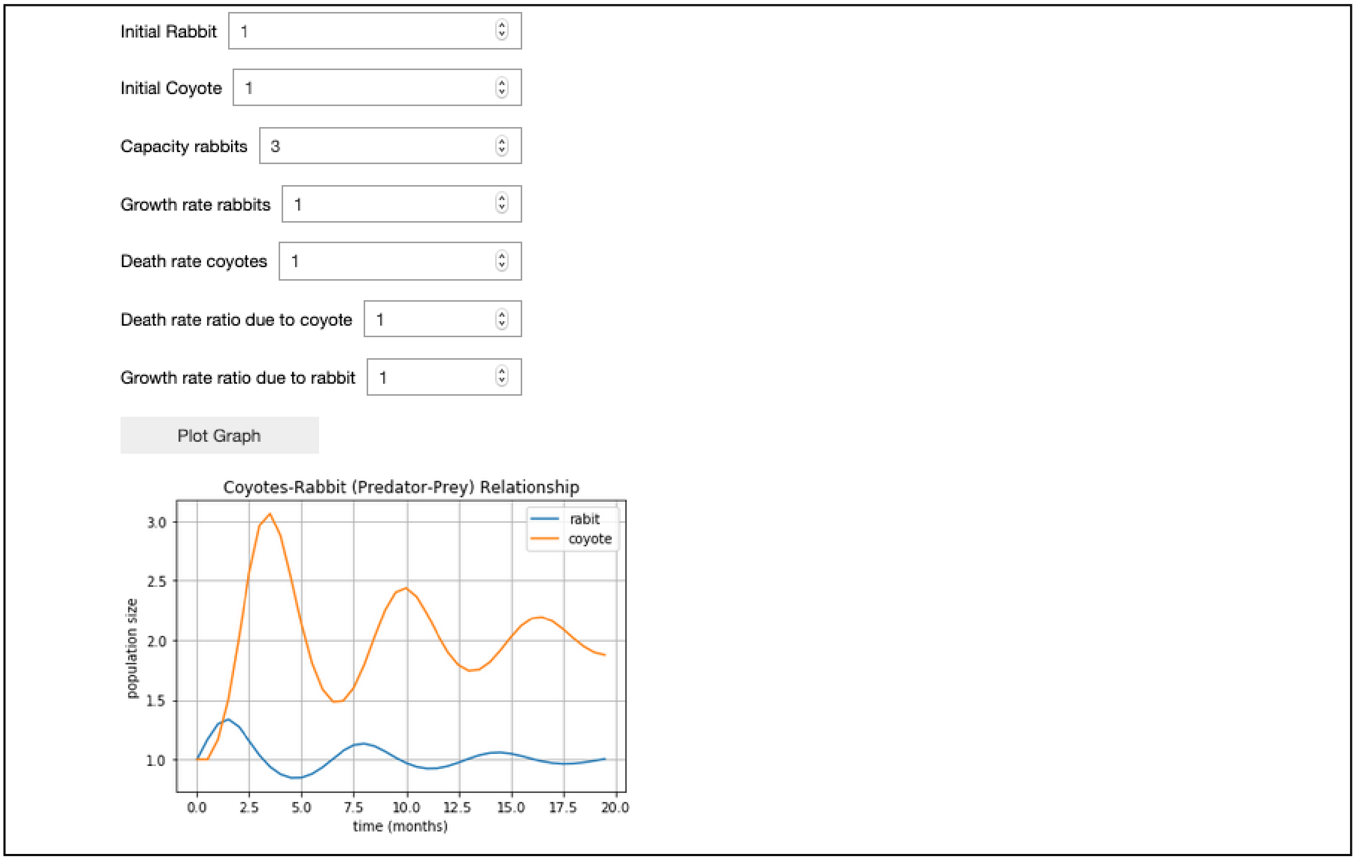
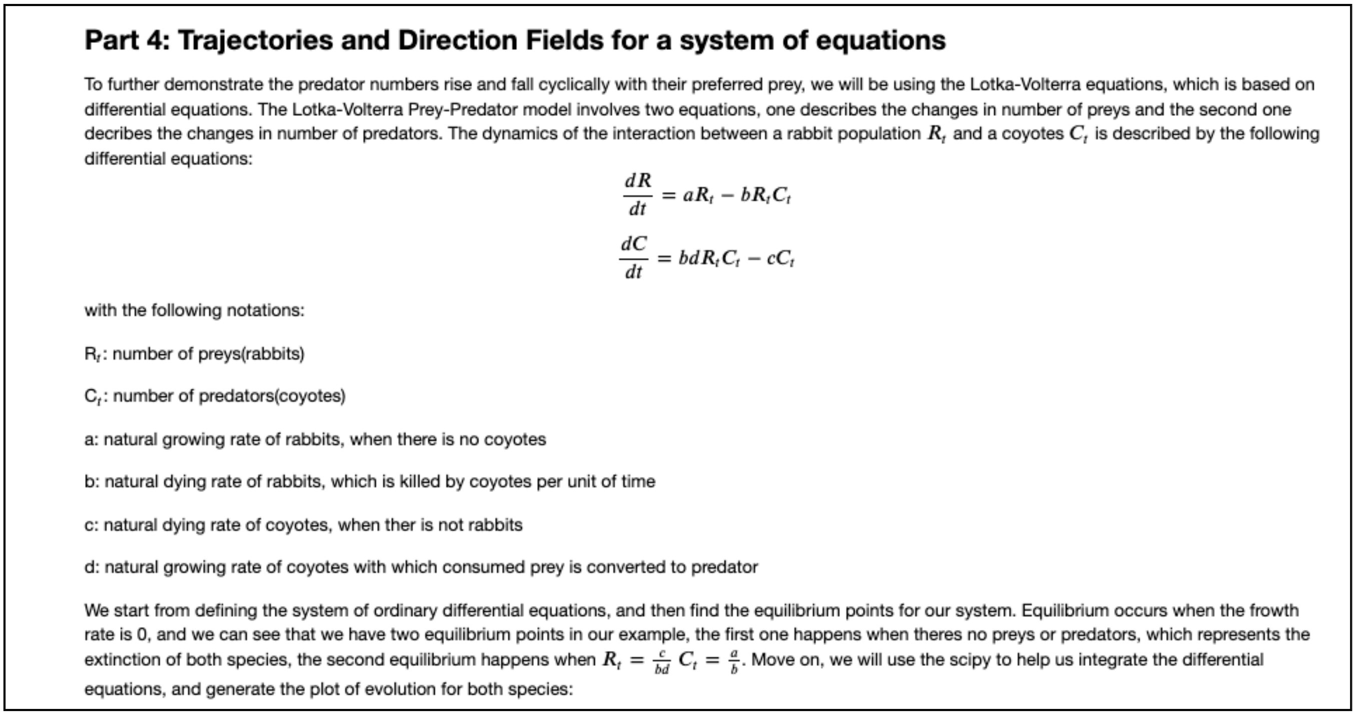


Figure 9: Users able to control interaction between coyotes and rabbits

In our model, it is obvious that the capacity of the rabbit decides the most of the population dynamic, since the coyotes are always hungry and always want to eat and reproduce in this case.

10.1.5 Part 4: Trajectories and Direction Fields for a system of equations

By looking at the trajectories and direction fields of a system of the predator and prey population, we could gain insight about their relationship.



Part 4: Trajectories and Direction Fields for a system of equations

To further demonstrate the predator numbers rise and fall cyclically with their preferred prey, we will be using the Lotka-Volterra equations, which is based on differential equations. The Lotka-Volterra Prey-Predator model involves two equations, one describes the changes in number of preys and the second one decribes the changes in number of predators. The dynamics of the interaction between a rabbit population R_t and a coyotes C_t is described by the following differential equations:

$$\frac{dR}{dt} = aR_t - bR_tC_t$$

$$\frac{dC}{dt} = bdR_tC_t - cC_t$$

with the following notations:

R_t: number of preys(rabbits)

C_t: number of predators(coyotes)

a: natural growing rate of rabbits, when there is no coyotes

b: natural dying rate of rabbits, which is killed by coyotes per unit of time

c: natural dying rate of coyotes, when ther is not rabbits

d: natural growing rate of coyotes with which consumed prey is converted to predator

We start from defining the system of ordinary differential equations, and then find the equilibrium points for our system. Equilibrium occurs when the frowth rate is 0, and we can see that we have two equilibrium points in our example, the first one happens when theres no preys or predators, which represents the extinction of both species, the second equilibrium happens when $R_t = \frac{c}{bd}$ $C_t = \frac{a}{b}$. Move on, we will use the scipy to help us integrate the differential equations, and generate the plot of evolution for both species:

Figure 10: Introduce trajectories and direction fields for interaction between coyotes and rabbits

We allows users to tune the a, b, c and d parameters to see the interaction.

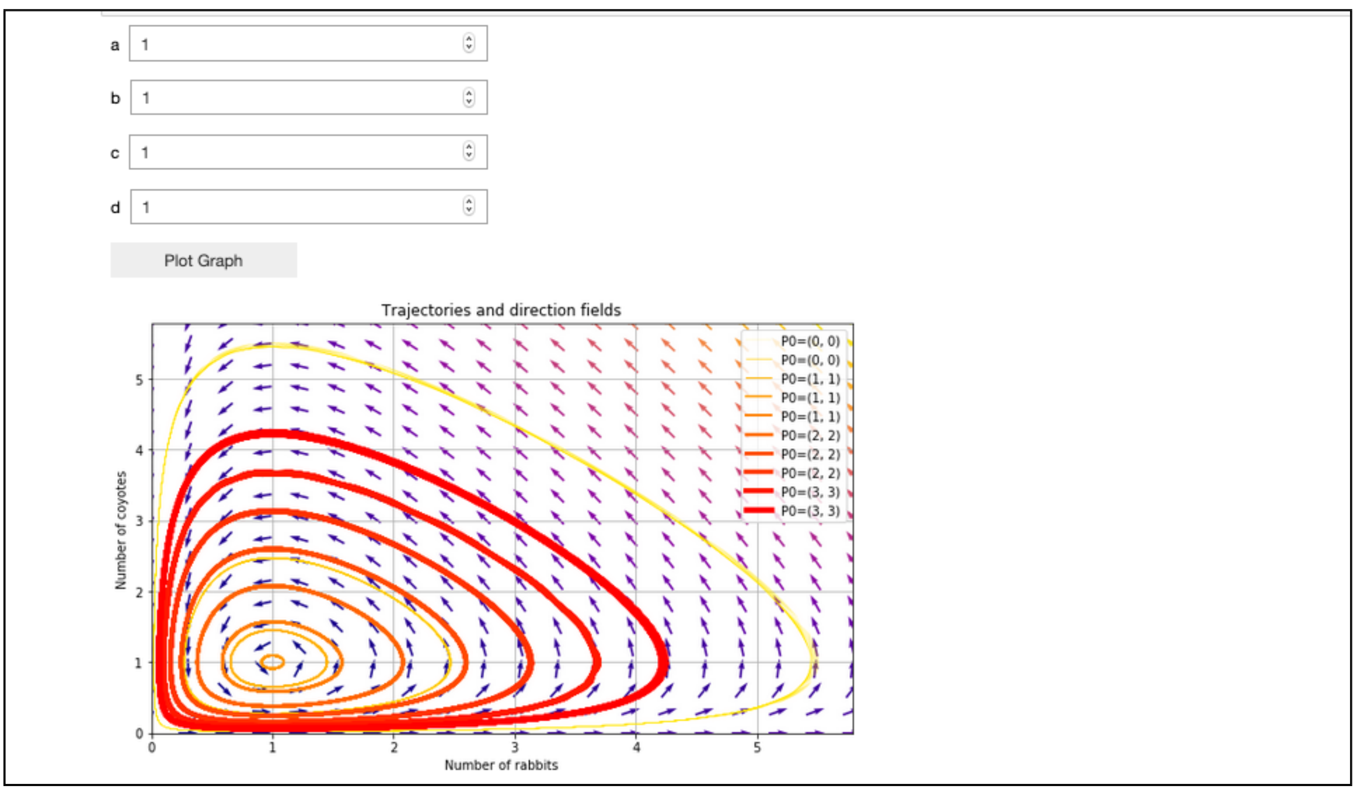


Figure 11: Trajectories and direction fields for interaction between coyotes and rabbits

10.1.6 Part 5: Multiple predators and preys relationship

We could extends our current model to multiple predators and preys relationship. The relationship is similar to the one predator one prey relationship. We assume each preys will give the same growth rate to the predators, while the predators will cause same death rate to the preys. While we could change such relationship to one to one condition, such changes may not further improve our understanding on the relationship. We could assume our case as special case where the growth rate are the same. We allows users to tune the different initial population, growth rate and death rate of different species to see the interaction.

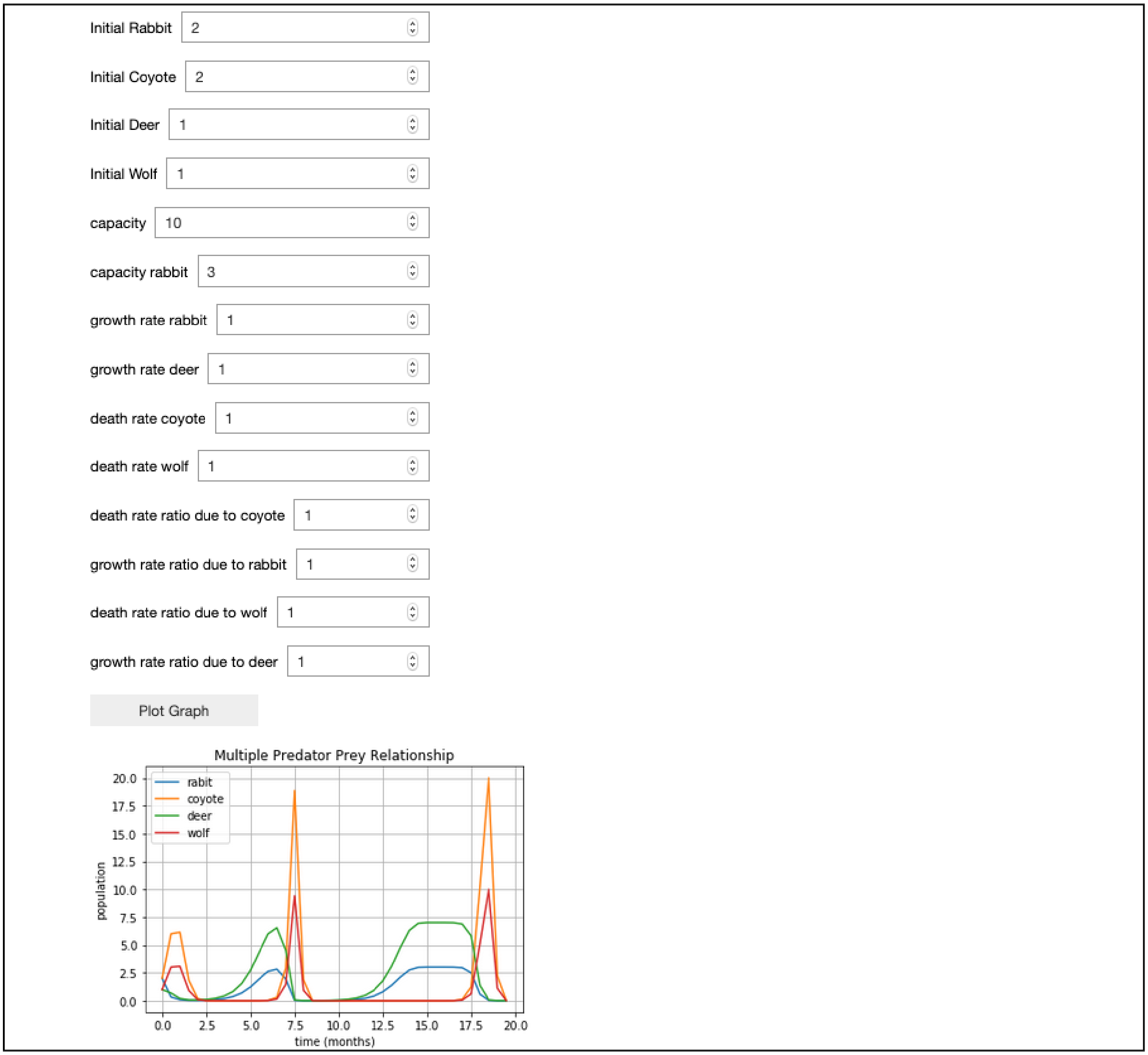


Figure 12: Interaction between multiple predators (wolves and coyotes) and preys (rabbits and deers)

10.2 Agent Based Interaction

11 Division of Labor

As we move forward on our project, we plan to work concurrently. The timeline is as below:

Task	Duration
Literature review	2 weeks
Modeling design and implementaion	4 weeks
Modeling revised	4 weeks

Task	Member
Literature review	All members
Single rabbit model	D. Aaron Hillegass
Single predator model, trajectories and direction field	Xiaotong Mu
UI interaction, predator prey interaction (single and multiple)	Siawpeng Er
Agent Based Simulation	All members
Final Report	All members

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