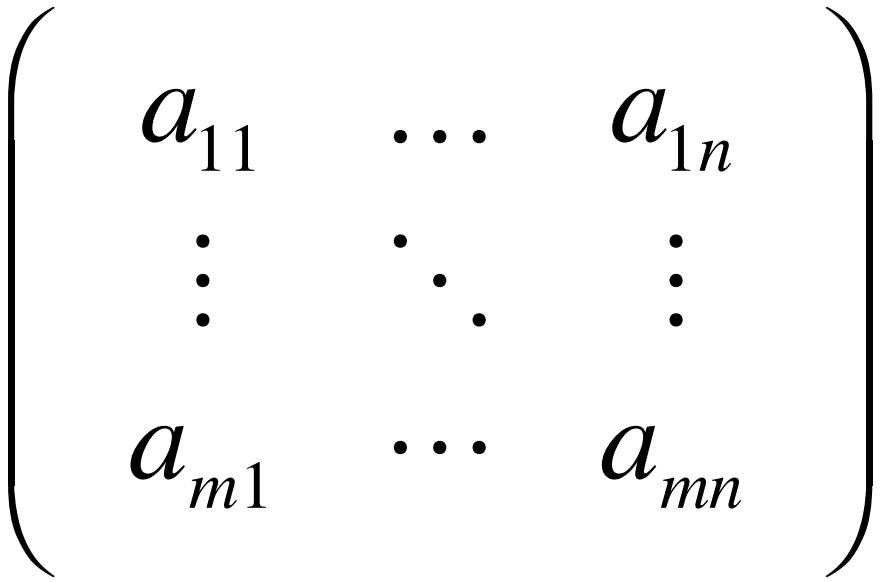
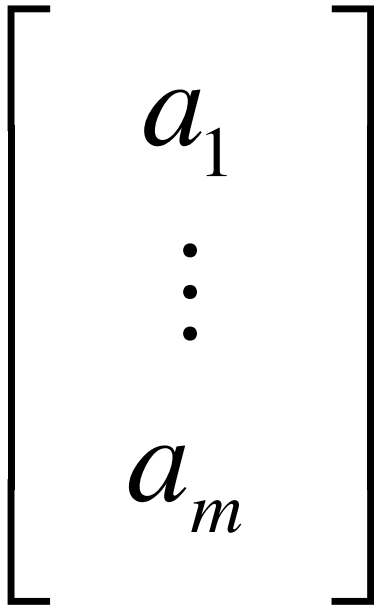
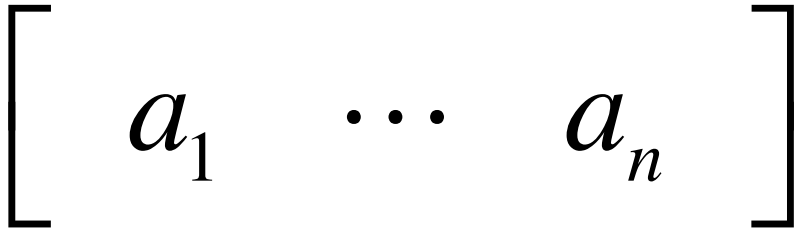
Linear algebra

Matrices



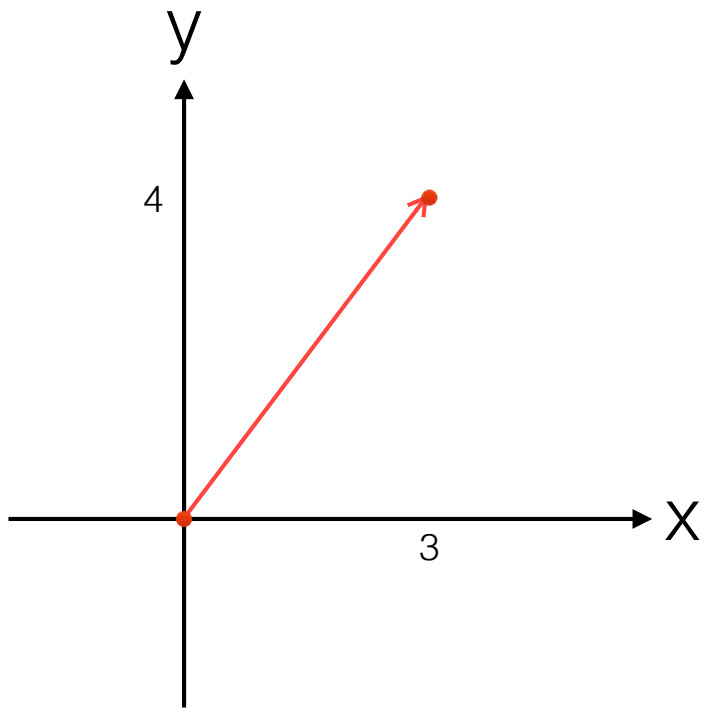
Vectors


a_1
a_m

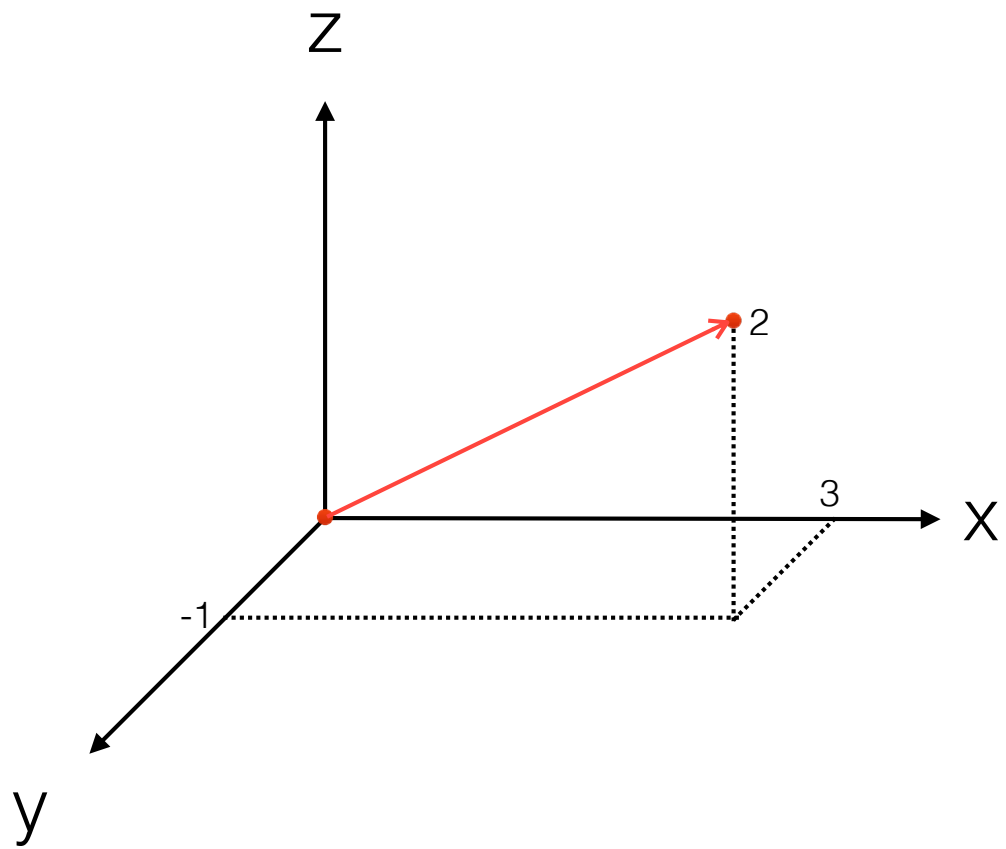


a sequence of numbers

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

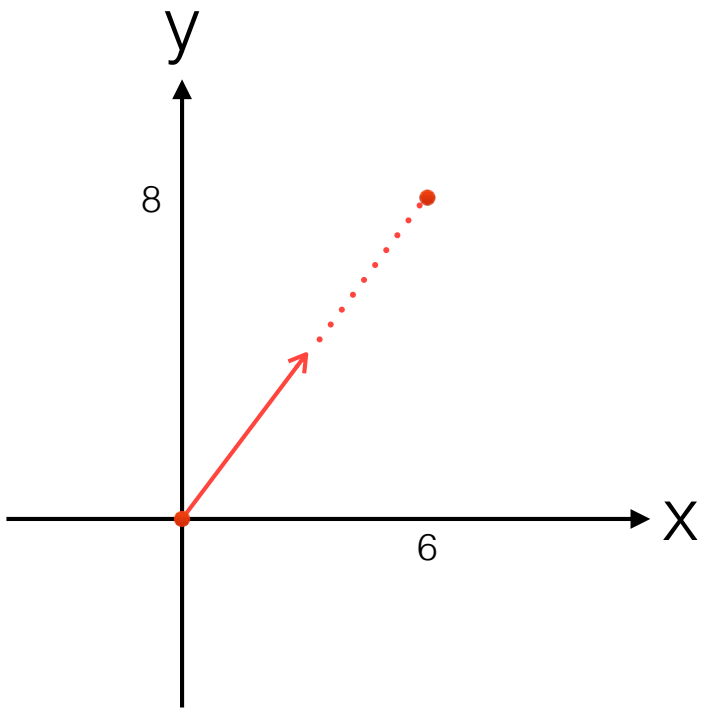


$$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

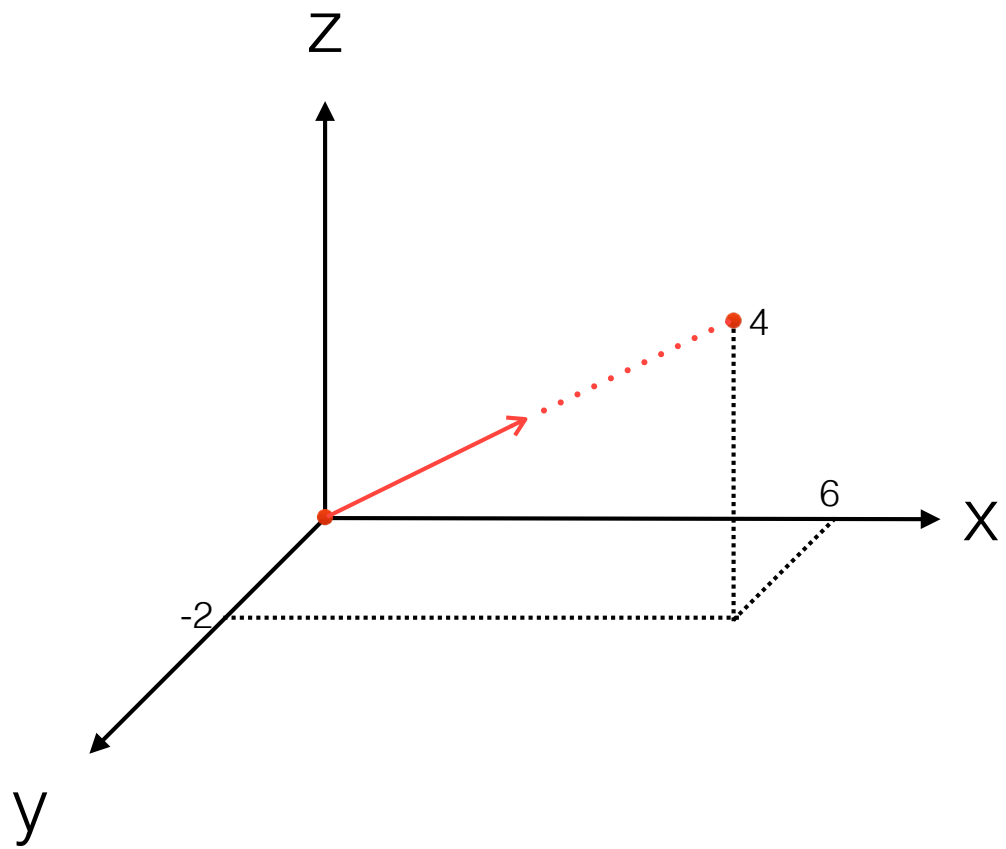


vector multiplication

$$0.5\begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

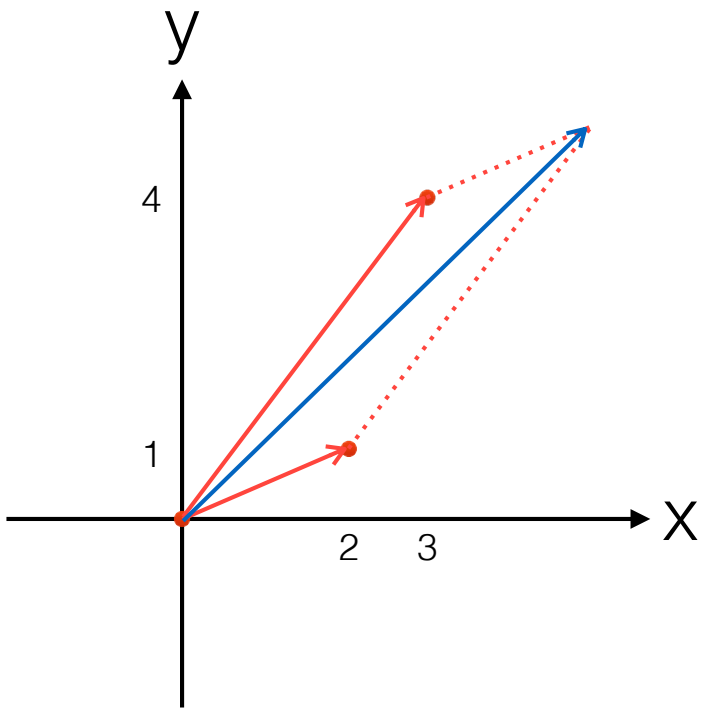


$$0.5\begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix}$$

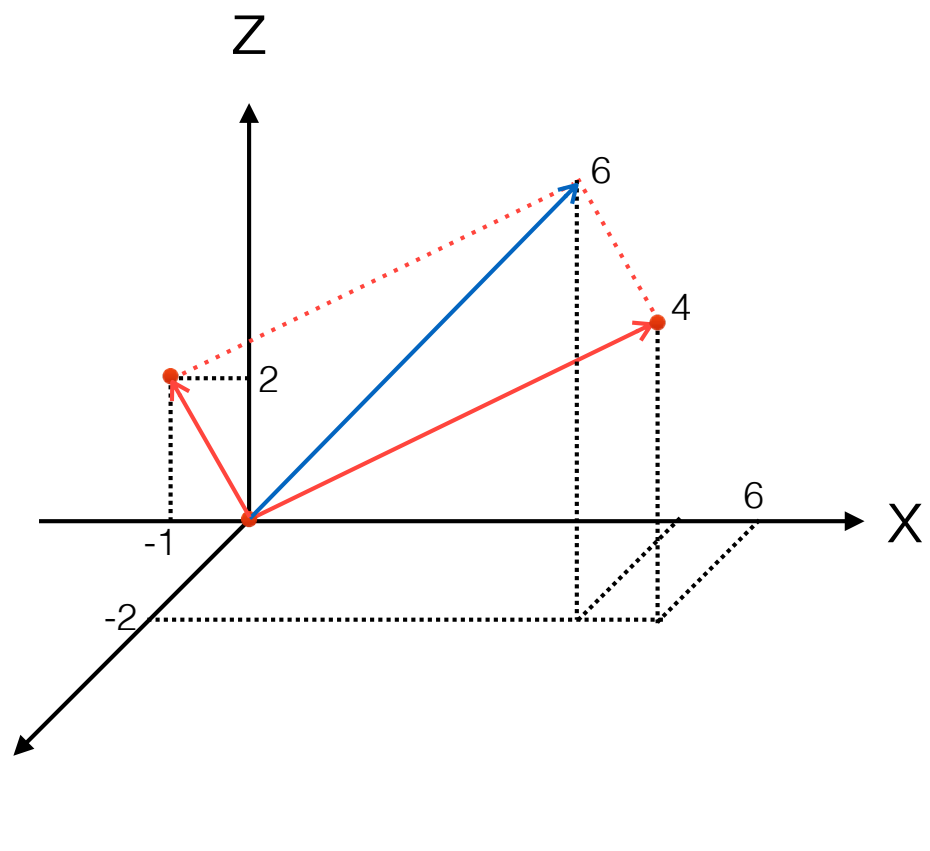


vector addition

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} 6 \\ -2 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

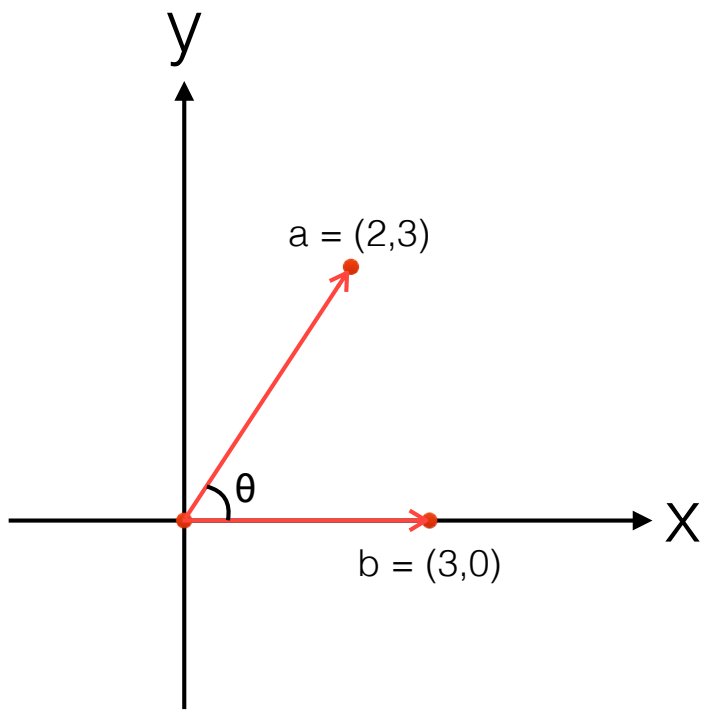


vector product

inner product
outer product
tensor product

inner product

a = [2, 3]
b = [3, 0]



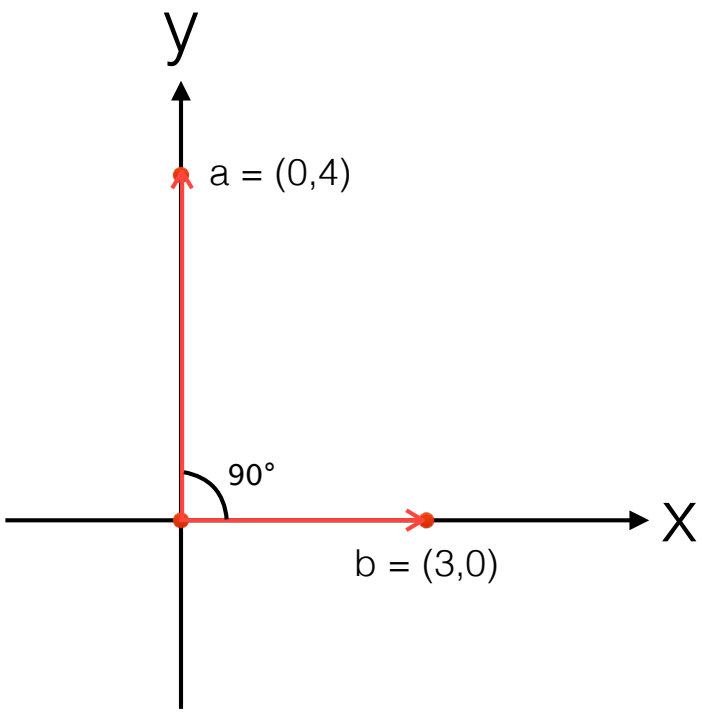
algebraic: $a \cdot b = 2x3 + 3x0$

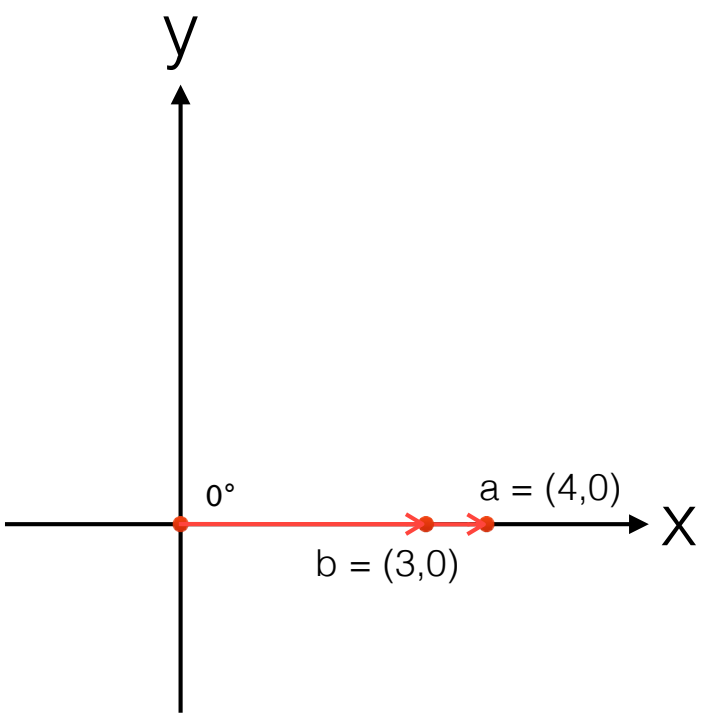
geometric: $a \cdot b = |a||b|\cos(\theta)$

How to measure informational similarity between a and b?

- cosine(θ)
- a-b distance

inner product



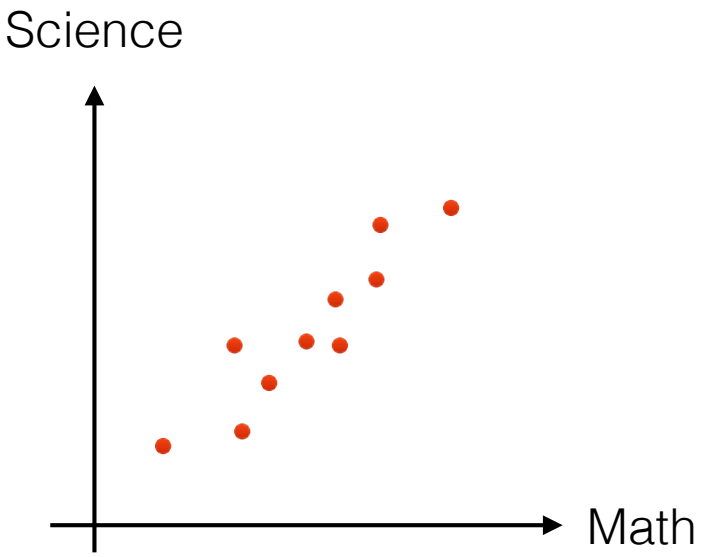


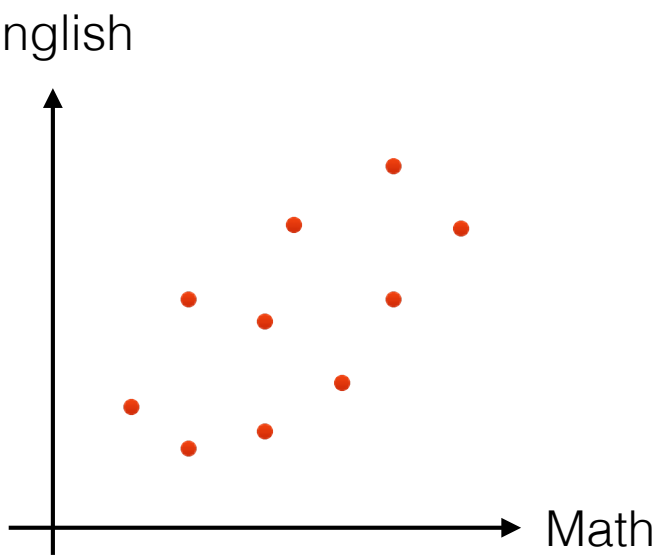
$a \cdot b = 4x3x\cos(\pi/2)$

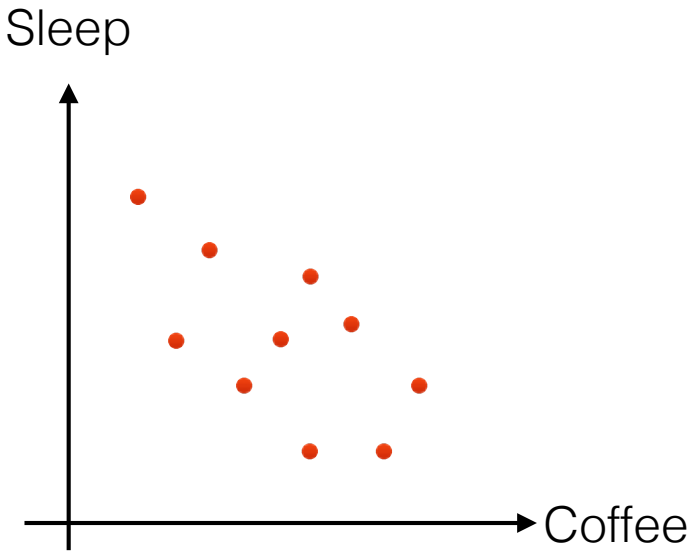
$a \cdot b = 4x3x\cos(0)$

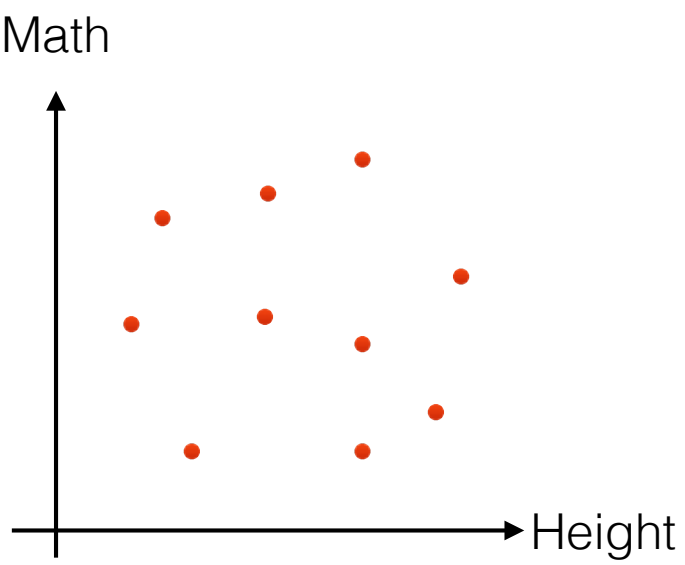
correlation

$-1 \leq r \leq 1$

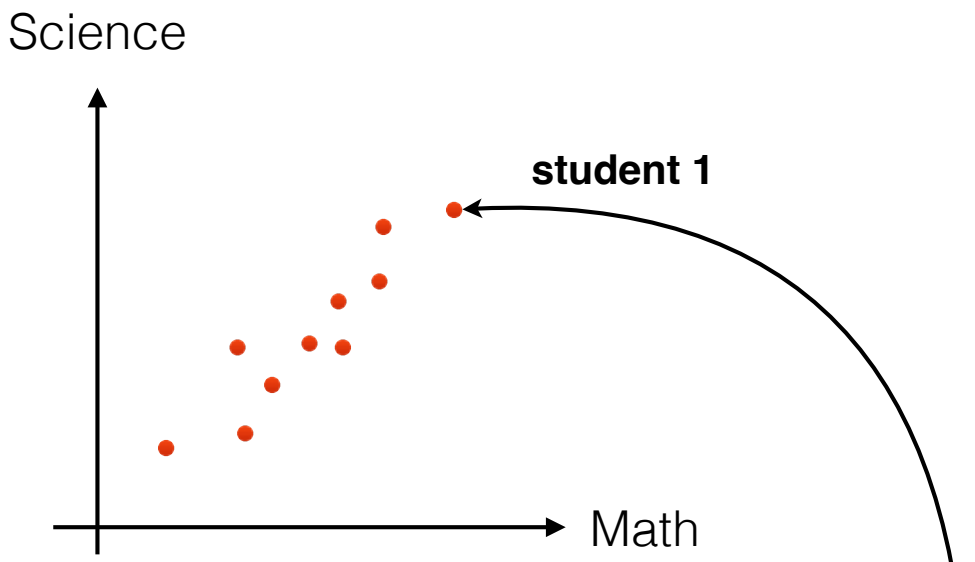








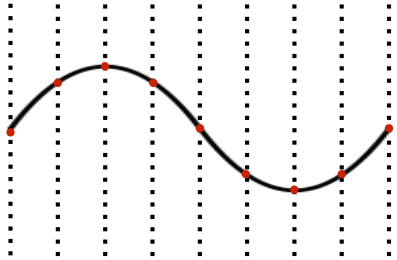
correlation = cosine



science = [90, 48, 60, 89, 25, 48, 55, 72, 36, 51]
math = [92, 54, 55, 92, 30, 41, 56, 74, 39, 48]

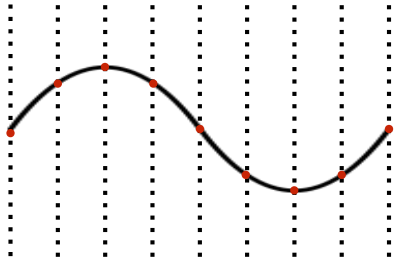
$$a \cdot b = |a||b|\cos(\theta)$$
$$\rightarrow \cos(\theta) = a \cdot b \,/\, |a||b|$$

inner product
signal vectors



a = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

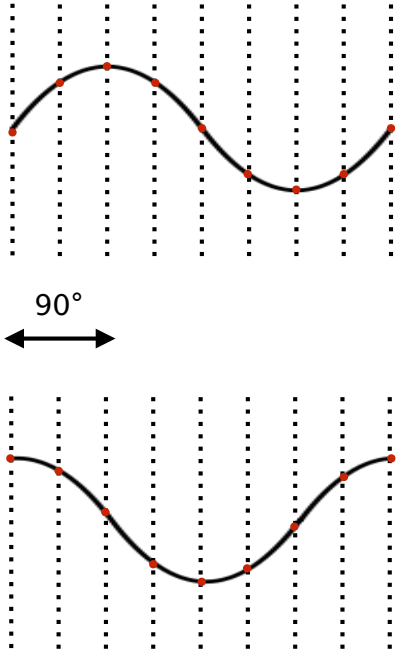
0°



b = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

$a \cdot b = ?$

inner product
signal vectors



a = [0, 0.85, 1, 0.85, 0, -0.85, -1, -0.85, 0]

b = [1, 0.85, 0, -0.85, -1, -0.85, 0, 0.85, 1]

$a \cdot b = ?$

inner product
signal vectors