hw2

3.31

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1、 I. 解析解

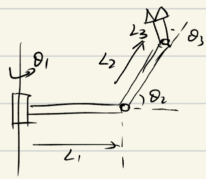
$$L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3) = P_x$$

$$L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3) = P_y$$

旋转矩阵：等价于平面旋转 $\theta_1+\theta_2+\theta_3$, i.e. $\sin(\theta_1+\theta_2+\theta_3) = r_{21}$, $\cos(\theta_1+\theta_2+\theta_3) = r_{11}$

故有：
$$\begin{cases} L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) = P_x - L_3\cdot r_{11} \\ L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) = P_y - L_3\cdot r_{21} \\ \theta_3 = \arctan\frac{r_{21}}{r_{11}} - \theta_1 - \theta_2 \end{cases}$$

II 见附件 hw2.ipynb 代码.



2、 由运动学传递关系：

$$^{i+1}W_{i+1} = {}^{i+1}_{i}R\,{}^{i}W_i + {}^{i}\dot{\theta}_{i+1}\,{}^{i+1}\hat{z}_{i+1}$$

$$^{i+1}V_{i+1} = {}^{i+1}_{i}R\,({}^{i}V_i + {}^{i}W_i \times {}^{i}P_{i+1}).$$

$$^{1}W_1 = (0,0,\dot{\theta}_1)^T \qquad {}^{1}V_1 = (0,0,0)^T$$

$$\rightarrow {}^{2}W_2 = {}^{2}_{1}R\,{}^{1}W_1 + (0,0,\dot{\theta}_2)^T = (S_2\dot{\theta}_1\ \ C_2\dot{\theta}_1\ \ \dot{\theta}_2)^T \qquad {}^{2}V_2 = (0,0,-L_1\dot{\theta}_1)^T$$

$$\rightarrow {}^{3}W_3 = {}^{3}_{2}R\,{}^{2}W_2 + (0,0,\dot{\theta}_3)^T = (\dot{\theta}_1(S_2S_3+C_2C_3),\ \dot{\theta}_1(C_2C_3-S_2S_3),\ \dot{\theta}_2+\dot{\theta}_3)^T$$

$$^{3}V_3 = {}^{3}_{2}R\,({}^{2}V_2 + {}^{2}W_2\times{}^{2}P_3) = \begin{pmatrix} C_3 & S_3 & \\ -S_3 & C_3 & \\ & & 1\end{pmatrix}\cdot\left[\begin{pmatrix}0\\0\\-L_1\dot{\theta}_1\end{pmatrix} + \begin{pmatrix}S_2\dot{\theta}_1\\C_2\dot{\theta}_1\\\dot{\theta}_2\end{pmatrix}\times\begin{pmatrix}L_2\\0\\0\end{pmatrix}\right] = \begin{pmatrix}L_2\dot{\theta}_2S_3\\L_2\dot{\theta}_2C_3\\-L_2C_2\dot{\theta}_1-L_1\dot{\theta}_1\end{pmatrix}$$

$$\rightarrow {}^{4}W_4 = {}^{3}W_3, \qquad {}^{4}V_4 = {}^{4}_{3}R\,({}^{3}V_3 + {}^{3}W_3\times{}^{3}P_4) \quad 而\ {}^{4}_{3}R = 1$$

$$^{4}V_4 = {}^{3}V_3 + {}^{3}W_3\times{}^{3}P_4 = (L_2\dot{\theta}_2S_3,\ L_2\dot{\theta}_2C_3 + L_3(\dot{\theta}_2+\dot{\theta}_3),\ -L_1\dot{\theta}_1 - L_2C_2\dot{\theta}_1 + L_3(S_2S_3-C_2C_3)\dot{\theta}_1)^T$$

其中
$$^{3}W_3\times{}^{3}P_4 = \begin{pmatrix}\dot{\theta}_1(S_2S_3+C_2C_3)\\\dot{\theta}_1(C_2C_3-S_2S_3)\\\dot{\theta}_2+\dot{\theta}_3\end{pmatrix}\times\begin{pmatrix}L_3\\0\\0\end{pmatrix} = \begin{pmatrix}0\\L_3(\dot{\theta}_2+\dot{\theta}_3)\\-L_3\dot{\theta}_1(C_2C_3-S_2S_3)\end{pmatrix}$$

故
$$^{4}J(\Theta) = \frac{d\,{}^{4}V_4}{d\,\dot{\Theta}} = \begin{pmatrix}0 & L_2S_3 & 0\\0 & L_2C_3 & L_3\\-L_1-L_2C_2+L_3(S_2S_3-C_2C_3) & 0 & 0\end{pmatrix}$$

3.
$$^{0}V_3 = J(\Theta)\,\dot{\Theta} \qquad 由2.\ {}^{3}V_3 = (L_2\dot{\theta}_2S_3\ \ L_2\dot{\theta}_2C_3\ \ -(L_1+L_2C_2)\dot{\theta}_1)^T$$

$$^{0}V_3 = {}^{0}_{3}R\,{}^{3}V_3 = \begin{pmatrix}C_1C_{23} & -C_1S_{23} & S_1\\S_1C_{23} & -S_1S_{23} & -C_1\\S_{23} & C_{23} & 0\end{pmatrix}\cdot\begin{pmatrix}L_2\dot{\theta}_2S_3\\L_2\dot{\theta}_2C_3\\-(L_2C_2+L_1)\dot{\theta}_1\end{pmatrix} = \begin{pmatrix}-L_2\dot{\theta}_2C_1S_2-(L_1+L_2C_2)S_1\dot{\theta}_1\\-L_2\dot{\theta}_2S_1S_2+(L_1+L_2C_2)C_1\dot{\theta}_1\\L_2\dot{\theta}_2C_2\end{pmatrix}$$

故
$$^{0}J(\Theta)_3 = \begin{pmatrix}-(L_1+L_2C_2)S_1 & -L_2C_1S_2 & 0\\(L_1+L_2C_2)C_1 & -L_2S_1S_2 & 0\\0 & L_2C_2 & 0\end{pmatrix}$$