

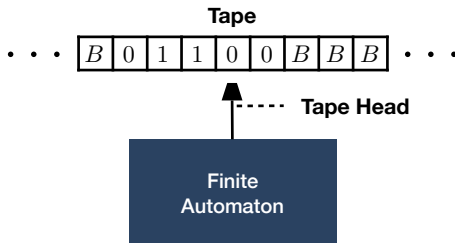
Lecture 22 – Examples of Turing Machines

COSE215: Theory of Computation

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2023 Spring



- A **Turing machine (TM)** is a finite automaton with a **tape**:
 - ① A **finite automaton** with a deterministic transition function.
 - ② A **tape** is a one-dimensional infinite array of cells.
 - Each cell contains a **tape symbol**.
 - The **blank symbol** B is a special symbol representing an empty cell.
 - ③ A **tape head** is a device that can read and write symbols on the tape.
 - It can move **left** or **right** one cell at a time.
- We can use Turing machines as **computing machines**.

1. Turing Machines as Word Recognizers

Example 1: $L = \{a^n b^n c^n \mid n \geq 0\}$

Example 2: $L = \{ww \mid w \in \{a, b\}^*\}$

Example 3: $L = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$

2. Turing Machines as Computing Machines

Example 4: Flip Bits – $f(w) = (\text{flip of } w)$

Example 5: Unary Addition – $f(a, b) = a + b$

Example 6: Data Copy – $f(w) = ww$

Example 1: $L = \{a^n b^n c^n \mid n \geq 0\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{a^n b^n c^n \mid n \geq 0\}$$

...	B	a	a	b	b	c	c	B	...
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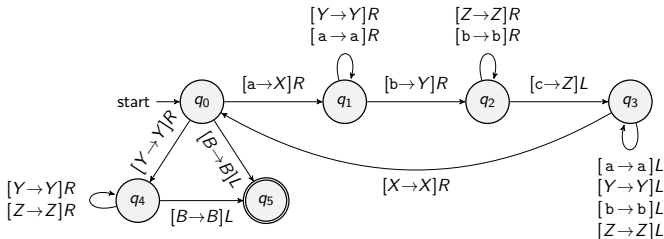
- 1: **while** there are a's **do**
- 2: Find and Replace a with X
- 3: Find and Replace b with Y
- 4: Find and Replace c with Z
- 5: **end while**
- 6: Check if only Y 's and Z 's are left

Example 1: $L = \{a^n b^n c^n \mid n \geq 0\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{a^n b^n c^n \mid n \geq 0\}$$

See the example for $aabbcc \in L(M)$.¹



¹<https://plrg.korea.ac.kr/courses/cose215/materials/tm-an-bn-cn.pdf>

Example 2: $L = \{ww \mid w \in \{a, b\}^*\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{ww \mid w \in \{a, b\}^*\}$$

...	B	a	b	b	a	b	b	B	...
-----	---	---	---	---	---	---	---	---	-----

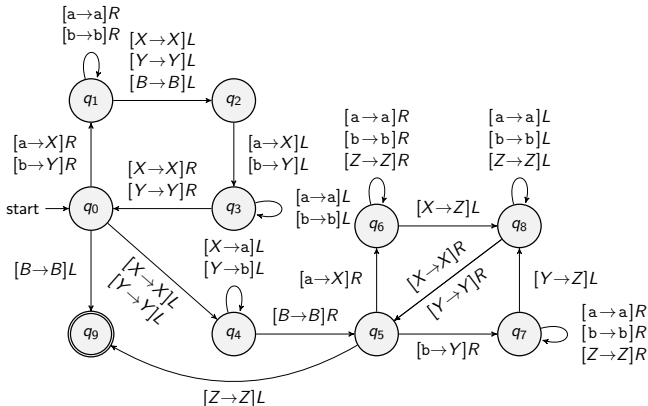
- 1: Find the middle of the input
- 2: Replace all X 's (or Y 's) with a 's (or b 's) in the first half
- 3: **while** there are input symbols in the first half **do**
- 4: Replace a (or b) with X (or Y) in the first half
- 5: Find and Replace matched X (or Y) with Z in the second half
- 6: **end while**

Example 2: $L = \{ww \mid w \in \{a, b\}^*\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{ww \mid w \in \{a, b\}^*\}$$

See the example for $abbabb \in L(M)$.²



²<https://plrg.korea.ac.kr/courses/cose215/materials/tm-w-w.pdf>

Example 3: $L = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$$

...	B	a	a	b	b	b	c	c	c	c	c	c	B	...
-----	---	---	---	---	---	---	---	---	---	---	---	---	---	-----

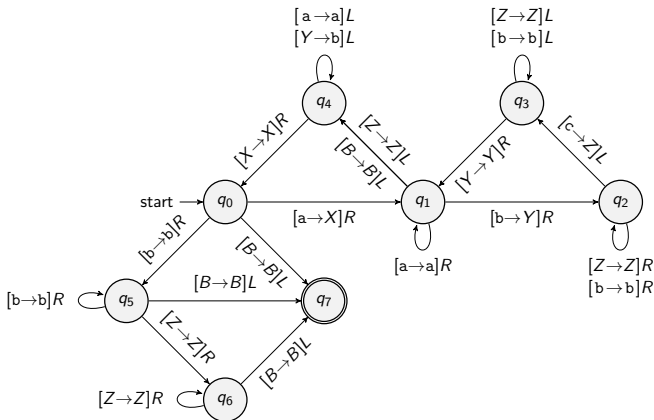
- 1: **while** there are a's **do**
- 2: Find and Replace a with X
- 3: **while** there are b's **do**
- 4: Find and Replace b with Y
- 5: Find and Replace c with Z
- 6: **end while**
- 7: Roll back all Y's to b's
- 8: **end while**
- 9: Check if only b's and Z's are left

Example 3: $L = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$

Construct a Turing machine that accepts the language:

$$L(M) = \{a^i b^j c^{i \times j} \mid i, j \geq 0\}$$

See the example for $aabbccccc \in L(M)$.³



³<https://plrg.korea.ac.kr/courses/cose215/materials/tm-ai-bj-cij.pdf>

Example 4: Flip Bits – $f(w) = (\text{flip of } w)$

Construct a Turing machine that computes the function:

$$f(w) = (\text{the flip of each bit in } w)$$

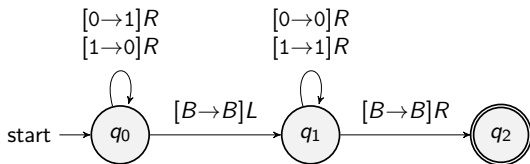
...	B	1	0	1	1	1	0	0	B	...
-----	---	---	---	---	---	---	---	---	---	-----

- 1: Flip each bit of the input: $1 \rightarrow 0$ and $0 \rightarrow 1$
- 2: Go to the first input symbol

Example 4: Flip Bits – $f(w) = (\text{flip of } w)$

$$f(w) = (\text{the flip of each bit in } w)$$

See the example for $f(1011100) = 0100011$.⁴



⁴<https://plrg.korea.ac.kr/courses/cose215/materials/tm-flip.pdf>

Example 5: Unary Addition – $f(a, b) = a + b$

Construct a Turing machine that computes the function:

$$f(a, b) = a + b \quad \text{where} \quad a \text{ and } b \text{ are unary numbers}$$

...	B	1	1	1	+	1	1	B	...
-----	---	---	---	---	---	---	---	---	-----

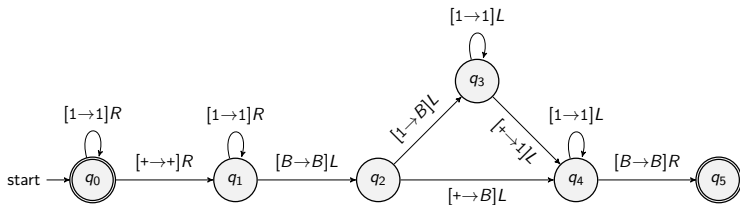
- 1: Find + after 1's
- 2: **if** the last symbol is 1 **then**
- 3: Find and Remove the last 1
- 4: Find and Replace the + with 1
- 5: **else**
- 6: Remove the +
- 7: **end if**
- 8: Go to the first input symbol

Example 5: Unary Addition – $f(a, b) = a + b$

Construct a Turing machine that computes the function:

$$f(a, b) = a + b \quad \text{where} \quad a \text{ and } b \text{ are unary numbers}$$

See the example for $f(111+11) = 11111$.⁵



⁵<https://plrg.korea.ac.kr/courses/cose215/materials/tm-unary-add.pdf>

Example 6: Data Copy – $f(w) = ww$

Construct a Turing machine that computes the function:

$$f(w) = ww \quad \text{where} \quad w \in \{0, 1\}^*$$

...	B	0	1	1	B	B	B	B	...
-----	---	---	---	---	---	---	---	---	-----

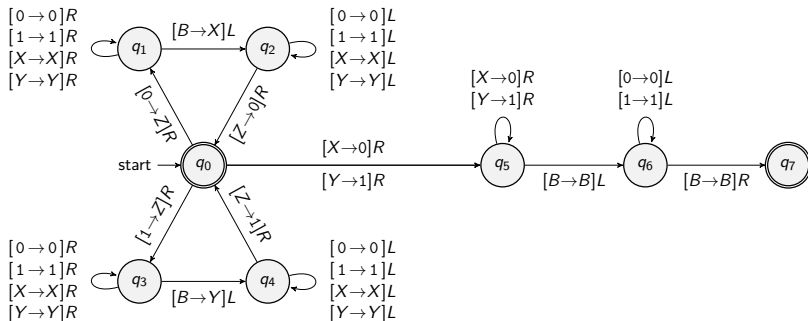
- 1: **while** there are input symbols **do**
- 2: Find and Replace 0 (or 1) with Z
- 3: Find and Fill the first blank with X (or Y) for 0 (or 1)
- 4: Roll back Z to the original 0 (or 1)
- 5: **end while**
- 6: Replace X 's and Y 's with 0's and 1's
- 7: Go to the first input symbol

Example 6: Data Copy – $f(w) = ww$

Construct a Turing machine that computes the function:

$$f(w) = ww \quad \text{where} \quad w \in \{0, 1\}^*$$

See the example for $f(011) = 011011$.⁶



⁶<https://plrg.korea.ac.kr/courses/cose215/materials/tm-data-copy.pdf>

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Example 6: Data Copy – $f(w) = ww$

- Extensions of Turing Machines

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