

# Lecture 11 – Mutable Variables

## COSE212: Programming Languages

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2023 Fall

- Mutation makes it possible to update the **contents** of a data structure or a variable after its creation.
  - **Mutable data structures**
  - **Mutable variables**
- **Mutable Data Structures** – Mutable Boxes
- BFAE – FAE with Mutable Boxes
  - Evaluation with Memories
- In this lecture, we will learn **Mutable Variables**
- **MFAE – FAE with Mutable Variables**
  - Concrete and Abstract Syntax
  - Interpreter and Natural Semantics

1. Mutable Variables
2. MFAE – FAE with Mutable Variables
  - Concrete Syntax
  - Abstract Syntax
3. Interpreter and Natural Semantics for MFAE
  - Evaluation with Memories
  - Interpreter and Natural Semantics
  - Mutable Variable
  - Identifier Lookup
  - Function Application
  - Assignment
4. Call-by-Value vs. Call-by-Reference

## 1. Mutable Variables

## 2. MFAE – FAE with Mutable Variables

Concrete Syntax

Abstract Syntax

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Function Application

Assignment

## 4. Call-by-Value vs. Call-by-Reference

A **mutable variable** is a variable whose value can be changed after its initialization.

Let's define mutable variables in Scala:

```
// A mutable variable `x` of type `Int` with 1
var x: Int = 1
x + 2          // 1 + 2 == 3 : Int

// We can reassign a mutable variable `x`
x = 2          // x == 2
x + 2          // 2 + 2 == 4 : Int

// The function `f` is impure because it uses a mutable variable `y`
var y: Int = 1
def f(x: Int): Int = x + y
f(5)           // 5 + 1 == 6 : Int
y = 3
f(5)           // 5 + 3 == 8 : Int
```

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## 4. Call-by-Value vs. Call-by-Reference

Now, let's extend FAE into MFAE to support **mutable variables**.

```
/* MFAE */  
var x = 5;  
x;           // 5  
x = 8;  
x           // 8
```

```
/* MFAE */  
var y = 1;  
var f = x => { x = x + y; x };  
f(5);        // 5 + 1 = 6  
y = 3;  
f(5);        // 5 + 3 = 8
```

For MFAE, we need to extend **expressions** of FAE with

- 1 **mutable variables** (`var`) rather than immutable variables (`val`)
- 2 **assignment** (`=`)
- 3 **sequence** of expressions

```
// expressions  
<expr> ::= ...  
          | "var" <id> "=" <expr> ";" <expr>  
          | <id> "=" <expr>  
          | <expr> ";" <expr>
```

For MFAE, we need to extend **expressions** of FAE with

- ① **mutable variables** (`var`) rather than immutable variables (`val`)
- ② **assignment** (`=`)
- ③ **sequence** of expressions



Let's define the **abstract syntax** of MFAE in BNF:

Expressions  $\mathbb{E} \ni e ::= \dots$

var x=e; e	(Var)
x=e	(Assign)
e; e	(Seq)

```
enum Expr:
    ...
    // mutable variable definition
    case Var(name: String, init: Expr, body: Expr)
    // variable assignment
    case Assign(name: String, expr: Expr)
    // sequence
    case Seq(left: Expr, right: Expr)
```

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## 4. Call-by-Value vs. Call-by-Reference

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */  
var y = 1;  
var f = x => {  
  x = x + y;  
  x * x  
};  
f(5);  
y = 3;  
f(5);
```

\*

$\sigma = [$

$]$

$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$

$M =$ 

				...
--	--	--	--	-----

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var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|c|} \hline 1 & & & & \dots \\ \hline \end{array} \end{array}$$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
    
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	...
$M$	=	1	$v$			...

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);
y = 3;
f(5);
```

\*

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$x \mapsto a_2$$

$$]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	...
$M$	=	1	$v$	5		...

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */ *
  x * x
};
f(5);
y = 3;
f(5);
    
```

$$\sigma = \begin{bmatrix} y \mapsto a_0 \\ f \mapsto a_1 \\ x \mapsto a_2 \end{bmatrix}$$

$$\begin{array}{l} \mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array} \end{array}$$

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 1 */
  x * x      /* 6 * 6 */ *
};
f(5);
y = 3;
f(5);
    
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$x \mapsto a_2$$

$$]$$

$$\begin{array}{lcl} \mathbb{A} & : & a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M & = & \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array} \end{array}$$

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$



We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);           /* 36 */
y = 3;
f(5);
    
```

$$\sigma = \left[ \begin{array}{l} y \mapsto a_0 \\ f \mapsto a_1 \end{array} \right]$$

$$\begin{array}{l} \mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots \\ M = \begin{array}{|c|c|c|c|c|} \hline 1 & v & 6 & & \dots \\ \hline \end{array} \end{array}$$

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```
/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);           /* 36 */
y = 3;
f(5);           *
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	...
$M$	=	3	$v$	6		...

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
    x = x + y;
    x * x
};
f(5);           /* 36 */
y = 3;
f(5);
    
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$x \mapsto a_3$$

$$]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	...
$M$	=	3	$v$	6	5	...

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */ *
  x * x
};
f(5);      /* 36 */
y = 3;
f(5);
    
```

$$\sigma = \left[ \begin{array}{l} y \mapsto a_0 \\ f \mapsto a_1 \\ x \mapsto a_3 \end{array} \right]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	...
$M$	=	3	$v$	6	8	...

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y; /* 5 + 3 */
  x * x      /* 8 * 8 */ *
};
f(5);        /* 36 */
y = 3;
f(5);
    
```

$$\sigma = [$$

$$y \mapsto a_0$$

$$f \mapsto a_1$$

$$x \mapsto a_3$$

$$]$$

$$\mathbb{A} : a_0 \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$M = \begin{array}{|c|c|c|c|c|} \hline 3 & v & 6 & 8 & \dots \\ \hline \end{array}$$

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

We can evaluate MFAE expressions with **memories** similar to BFAE.

Let's see how to evaluate the following MFAE expression:

```

/* MFAE */
var y = 1;
var f = x => {
  x = x + y;
  x * x
};
f(5);           /* 36 */
y = 3;
f(5);           /* 64 */
    
```

$$\sigma = \left[ \begin{array}{l} y \mapsto a_0 \\ f \mapsto a_1 \end{array} \right]$$

$\mathbb{A}$	:	$a_0$	$a_1$	$a_2$	$a_3$	$\dots$
$M$	=	3	$v$	6	8	$\dots$

where  $v = \langle \lambda x. x = x + y; x, [y \mapsto a_0] \rangle$

For MFAE, we need to 1) implement the **interpreter** with environments and **memories** by passing the updated memory in the result:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = ???
```

```
type Env = Map[Var, Addr]
type Addr = Int
type Mem = Map[Addr, Value]
```

and 2) define the **natural semantics** with environments and **memories** by passing the updated memory in the result:

$$\sigma, M \vdash e \Rightarrow v, M$$

Environments  $\sigma \in \mathbb{X} \xrightarrow{\text{fin}} \mathbb{A} \quad (\text{Env})$

Addresses  $a \in \mathbb{A} \quad (\text{Addr})$

Memories  $M \in \mathbb{A} \xrightarrow{\text{fin}} \mathbb{V} \quad (\text{Mem})$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case Var(name, init, body) =>
  val (iv, imem) = interp(init, env, mem)
  val addr = malloc(imem)
  interp(body, env + (name -> addr), imem + (addr -> iv))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Var} \frac{\sigma, M \vdash e_1 \Rightarrow v_1, M_1 \quad a \notin \text{Domain}(M_1) \quad \sigma[x \mapsto a], M_1[a \mapsto v_1] \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash \text{val } x=e_1; e_2 \Rightarrow v_2, M_2}$$

We learned one way to implement malloc in the previous lecture:

```
def malloc(mem: Mem): Addr = mem.keySet.maxOption.fold(0)(_ + 1)
```



```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
  ...
  case Id(name) => (mem(lookupId(env, name)), mem)

def lookupId(env: Env, name: String): Addr =
  env.getOrElse(name, error(s"free identifier: $name"))
```

$$\sigma, M \vdash e \Rightarrow v, M$$

$$\text{Id} \frac{x \in \text{Domain}(\sigma)}{\sigma, M \vdash x \Rightarrow M(\sigma(x)), M}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case App(fun, arg) =>
  val (fv, fmem) = interp(fun, env, mem)
  fv match
    case CloV(param, body, fenv) =>
      val (av, amem) = interp(arg, env, fmem)
      val addr = malloc(amem)
      interp(body, fenv + (param -> addr), amem + (addr -> av))
    case _ =>
      error(s"not a function: ${fv.str}")
```

$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{App} \frac{\begin{array}{l} \sigma, M \vdash e_1 \Rightarrow \langle \lambda x. e_3, \sigma' \rangle, M_1 \quad \sigma, M_1 \vdash e_2 \Rightarrow v_2, M_2 \\ a \notin \text{Domain}(M_2) \quad \sigma'[x \mapsto a], M_2[a \mapsto v_2] \vdash e_3 \Rightarrow v_3, M_3 \end{array}}{\sigma, M \vdash e_1(e_2) \Rightarrow v_3, M_3}$$

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case Assign(name, expr) =>
  val (ev, emem) = interp(expr, env, mem)
  (ev, emem + (lookupId(env, name) -> ev))
```

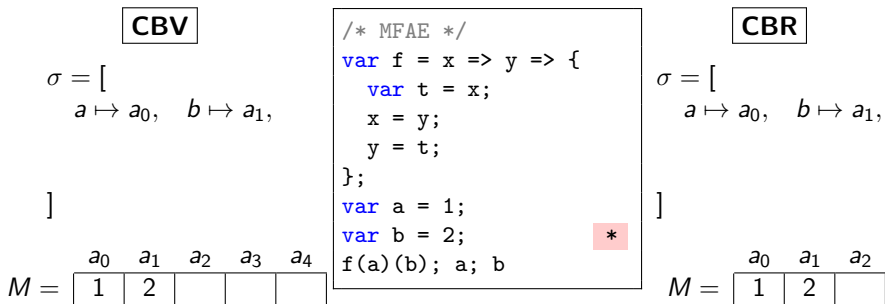
$$\boxed{\sigma, M \vdash e \Rightarrow v, M}$$

$$\text{Assign} \frac{\sigma, M \vdash e \Rightarrow v, M' \quad x \in \text{Domain}(\sigma)}{\sigma, M \vdash x=e \Rightarrow v, M'[\sigma(x) \mapsto v]}$$

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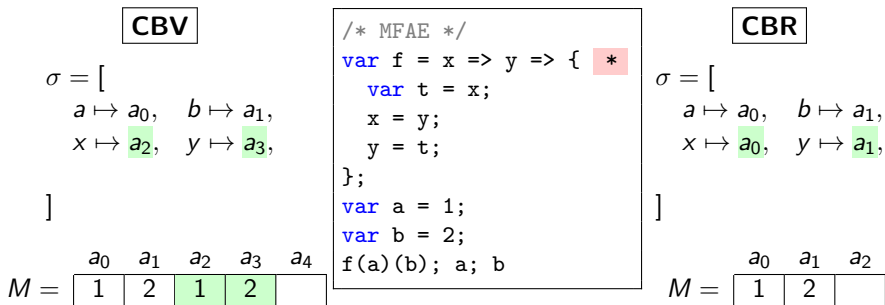
The current semantics of MFAE is based on the **call-by-value (CBV)** strategy, because the argument expression is always evaluated and the result value is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** strategy instead; if the argument expression is an identifier, the parameter points to its address.



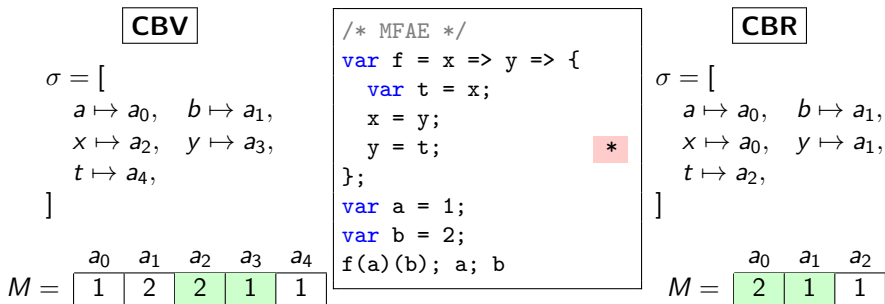
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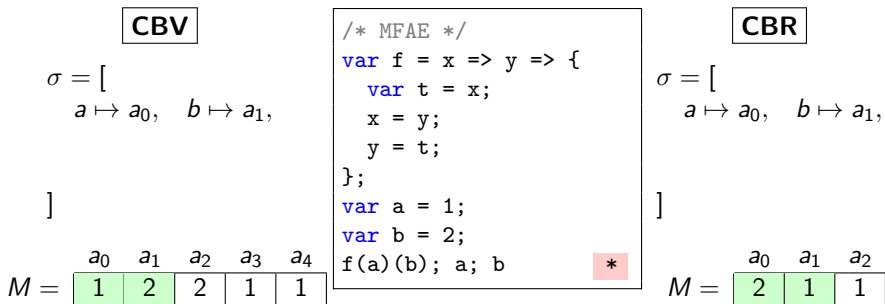
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The current semantics of MFAE is based on the **call-by-value (CBV)** strategy because the argument expression is always evaluated, and the result value is passed to the parameter.

However, we can define the semantics of MFAE in another way by using the **call-by-reference (CBR)** strategy instead; if the argument expression is an identifier, the parameter points to its address.





We can design and implement the semantics of MFAE with the **call-by-reference** strategy by adding the following cases:

```
def interp(expr: Expr, env: Env, mem: Mem): (Value, Mem) = expr match
...
case App(fun, arg) =>
  val (fv, fmem) = interpCBR(fun, env, mem)
  fv match
    case CloV(param, body, fenv) => arg match
      case Id(name) =>
        val addr = lookupId(env, name)
        interpCBR(body, fenv + (param -> addr), fmem)
      case _ => ...
    case _ => error(s"not a function: ${fv.str}")
...

```

$$\text{App}_x \frac{\sigma, M \vdash e_1 \Rightarrow \langle \lambda x'. e_2, \sigma' \rangle, M_1 \quad x \in \text{Domain}(\sigma) \quad \sigma'[x' \mapsto \sigma(x)], M_1 \vdash e_2 \Rightarrow v_2, M_2}{\sigma, M \vdash e_1(x) \Rightarrow v_2, M_2}$$

- Please see this document<sup>1</sup> on GitHub.
  - Implement `interp` function.
  - Implement `interpCBR` function.
- It is just an exercise, and you **don't need to submit** anything.
- However, some exam questions might be related to this exercise.

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<sup>1</sup><https://github.com/ku-plrg-classroom/docs/tree/main/cose212/mfae>.

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- Garbage Collection

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