

# Lecture 2 – Syntax and Semantics (1)

## COSE212: Programming Languages

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2023 Fall

We learn language features of **Scala**:

- **Basic Features**
  - Built-in Data Types
  - Variables
  - Functions
  - Conditionals
- **Object-Oriented Programming (OOP)**
  - Case Classes
- **Algebraic Data Types (ADTs)**
  - Pattern Matching
- **Functional Programming (FP)**
  - First-class Functions
  - Recursion
- **Immutable Collections**
  - Lists
  - Options and Pairs
  - Maps and Sets
  - For Comprehensions

## Definition (Programming Language)

A **programming language** is defined by

- **Syntax**: a grammar that defines the structure of programs
- **Semantics**: a set of rules that defines the meaning of programs

We will learn how to define the **syntax** and **semantics** of a programming language.

We define a programming language for **arithmetic expressions** (AE) as the running example.

Let's consider the arithmetic expressions (AE) supporting **addition** and **multiplication** of integers:

- $4 + 2$
- $1 * 24$
- $-42 + 4 * 10$
- $(1 + 2) * (2 + 3)$
- ...

There are **infinitely many** AEs.

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How to define all the valid AEs (**syntax**)?

How to define the expected result of each AE (**semantics**)?

## 1. Syntax

- Backus-Naur Form (BNF)

- Concrete Syntax

- Abstract Syntax

- Concrete vs. Abstract Syntax

## 2. Operational Semantics

- Inference Rules

- Big-Step Operational (Natural) Semantics

- Small-Step Operational (Reduction) Semantics

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**Backus-Naur Form (BNF)** is a notation for **context-free grammar**:

- A **nonterminal** has a name and a set of **production rules** consisting of sequences of terminals and nonterminals.
- A **terminal** is a symbol that appears in the final output.

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- A **terminal** is a symbol that appears in the final output.

For example, a nonterminal `<number>` produces all strings representing integers (allowing leading zeros) as follows:

```
<digit> ::= "0" | "1" | "2" | "3" | "4"  
          | "5" | "6" | "7" | "8" | "9"  
  
<nat>    ::= <digit> | <digit> <nat>  
  
<number> ::= <nat> | "-" <nat>
```

Let's define the **concrete syntax** of AE in BNF:

```
<expr> ::= <number>  
         | <expr> "+" <expr>  
         | <expr> "*" <expr>  
         | "(" <expr> ")"
```

It is the **surface-level** representation of programs with all the syntactic details to decide whether a given string is a valid AE or not.

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It is the **surface-level** representation of programs with all the syntactic details to decide whether a given string is a valid AE or not.

For example,  $(1+2)*3$  is a valid AE:

$\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle * \langle \text{expr} \rangle$	$\Rightarrow (\langle \text{expr} \rangle) * \langle \text{expr} \rangle$
$\Rightarrow (\langle \text{expr} \rangle + \langle \text{expr} \rangle) * \langle \text{expr} \rangle$	$\Rightarrow (\langle \text{number} \rangle + \langle \text{expr} \rangle) * \langle \text{expr} \rangle$
$\Rightarrow (1 + \langle \text{expr} \rangle) * \langle \text{expr} \rangle$	$\Rightarrow (1 + \langle \text{number} \rangle) * \langle \text{expr} \rangle$
$\Rightarrow (1+2) * \langle \text{expr} \rangle$	$\Rightarrow (1+2) * \langle \text{number} \rangle$
$\Rightarrow (1+2) * 3$	

Let's define the **concrete syntax** of AE in BNF:

```
<expr> ::= <number>
         | <expr> "+" <expr>
         | <expr> "*" <expr>
         | "(" <expr> ")"
```

We need **associativity** and **precedence** rules to disambiguate.

- "+" and "\*" are **left-associative**.

```
"1 + 2 + 3 + 4 + 5" == "((((1 + 2) + 3) + 4) + 5)"
"1 * 2 * 3 * 4 * 5" == "((((1 * 2) * 3) * 4) * 5)"
```

- "\*" has higher **precedence** than "+".

```
"1 + 2 * 3 + 4 * 5" == "((1 + (2 * 3)) + (4 * 5))"
```

Let's define the **abstract syntax** of AE in BNF:

$$\begin{array}{lcl} e & ::= & n \quad (\text{Num}) \\ & | & e + e \quad (\text{Add}) \\ & | & e \times e \quad (\text{Mul}) \end{array}$$

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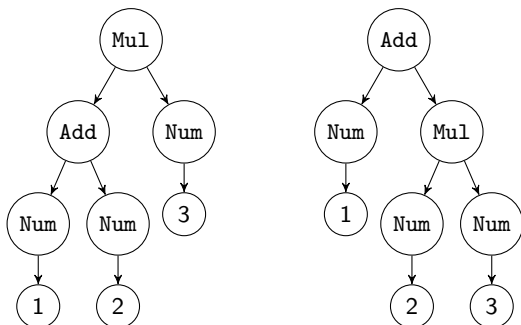
It captures only the essential structure of AE rather than the details.

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It captures only the essential structure of AE rather than the details.

The **abstract syntax trees (ASTs)** of  $(1+2)*3$  and  $1+2*3$  are as follows:





While **concrete syntax** is the **surface-level** representation of programs, **abstract syntax** is the **essential** representation of programs.

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There might be **multiple** concrete syntax for the **same** abstract syntax:

```
<expr> ::= <number>
        | <expr> "+" <expr>
        | <expr> "*" <expr>
        | "(" <expr> ")"
```

```
<expr> ::= <number>
        | "(" "+" <expr> <expr> ")"
        | "(" "*" <expr> <expr> ")"
```

```
<expr> ::= <number>
        | "ADD[" <expr> ";" <expr> "]"
        | "MUL[" <expr> ";" <expr> "]"
```

$$\begin{array}{lll} e & ::= & n \quad (\text{Num}) \\ & | & e + e \quad (\text{Add}) \\ & | & e \times e \quad (\text{Mul}) \end{array}$$

While **concrete syntax** is the **surface-level** representation of programs, **abstract syntax** is the **essential** representation of programs.

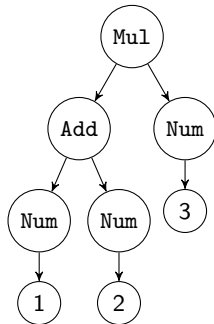
There might be **multiple** concrete syntax for the **same** abstract syntax:

$(1 + 2) * 3$

$(* (+ 1 2) 3)$

$\Rightarrow$

MUL[ADD[1; 2]; 3]



## 1. Syntax

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- Small-Step Operational (Reduction) Semantics

There exist diverse ways to define **semantics** of programming languages.

- **Axiomatic semantics** defines the meaning of a program by specifying the properties that hold after its execution.

$$\{x = n \wedge y = m\} \quad z := x + y \quad \{z = n + m\}$$

- **Denotational semantics** defines the meaning of a program by mapping it to a mathematical object that represents its meaning.

$$\llbracket e + e \rrbracket = \llbracket e \rrbracket + \llbracket e \rrbracket$$

- **Operational semantics** defines the meaning of a program by specifying how it executes on a machine.

$$\frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2}$$

• ...

In this course, we will focus on **operational semantics**, and there are two different representative styles:

- **Big-Step Operational (Natural) Semantics** defines the meaning of a program by specifying how it executes on a machine in one big step.

$$\frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2}$$

- **Small-Step Operational (Reduction) Semantics** defines the meaning of a program by specifying how it executes on a machine step-by-step.

$$\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2}$$

Operational semantics is defined by **inference rules**.

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An **inference rule** consists of multiple **premises** and one **conclusion**:

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meaning that “if all the premises are true, then the conclusion is true”:

$$premise_1 \wedge premise_2 \wedge \cdots \wedge premise_n \implies conclusion$$

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An **inference rule** consists of multiple **premises** and one **conclusion**:

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meaning that “if all the premises are true, then the conclusion is true”:

$$\text{premise}_1 \wedge \text{premise}_2 \wedge \cdots \wedge \text{premise}_n \implies \text{conclusion}$$

For example,

$$\frac{A \implies B \quad B \implies C}{A \implies C}$$

means that “if  $A$  implies  $B$ , and  $B$  implies  $C$ , then  $A$  implies  $C$ ”.

$$\boxed{\vdash e \Rightarrow n}$$

It means that *“the expression  $e$  evaluates to the number  $n$ ”*.

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It means that “the expression  $e$  evaluates to the number  $n$ ”.

Let's define the **big-step operational (natural) semantics** of AE:

$$\begin{array}{lcl}
 e ::= & n & \text{(Num)} \\
 & | & e + e \quad \text{(Add)} \\
 & | & e \times e \quad \text{(Mul)}
 \end{array}
 \quad \Rightarrow \quad
 \begin{array}{c}
 \text{NUM} \frac{}{\vdash n \Rightarrow n} \\
 \\
 \text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \\
 \\
 \text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 \times e_2 \Rightarrow n_1 \times n_2}
 \end{array}$$

$$\begin{array}{l} \text{NUM} \frac{}{\vdash n \Rightarrow n} \qquad \text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2} \qquad \text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 \times e_2 \Rightarrow n_1 \times n_2} \end{array}$$

Let's prove  $\vdash (1 + 2) \times 3 \Rightarrow 9$  by drawing a **derivation tree**:

$$\text{NUM} \frac{}{\vdash n \Rightarrow n}$$

$$\text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2}$$

$$\text{MUL} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 \times e_2 \Rightarrow n_1 \times n_2}$$

Let's prove  $\vdash (1 + 2) \times 3 \Rightarrow 9$  by drawing a **derivation tree**:

$$\begin{array}{c} \text{NUM} \frac{}{\vdash 1 \Rightarrow 1} \quad \text{NUM} \frac{}{\vdash 2 \Rightarrow 2} \\ \text{ADD} \frac{}{\vdash 1 + 2 \Rightarrow 3} \quad \text{NUM} \frac{}{\vdash 3 \Rightarrow 3} \\ \text{MUL} \frac{}{\vdash (1 + 2) \times 3 \Rightarrow 9} \end{array}$$

$$\text{NUM} \frac{}{\vdash n \Rightarrow n}$$

$$\text{ADD} \frac{\vdash e_1 \Rightarrow n_1 \quad \vdash e_2 \Rightarrow n_2}{\vdash e_1 + e_2 \Rightarrow n_1 + n_2}$$

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Let's prove  $\vdash 1 + 2 \times 3 \Rightarrow 7$  by drawing a **derivation tree**:

---


$$\vdash 1 + 2 \times 3 \Rightarrow$$

$$e_0 \rightarrow e_1$$

It means that “ $e_0$  is reduced to  $e_1$  as the result of one-step evaluation”.



$$\boxed{e_0 \rightarrow e_1}$$

It means that “ $e_0$  is reduced to  $e_1$  as the result of one-step evaluation”.

Let's define the **small-step operational (reduction) semantics** of AE:

$$\begin{array}{lcl}
 e ::= & n & \text{(Num)} \\
 & | & e + e \quad \text{(Add)} \\
 & | & e \times e \quad \text{(Mul)}
 \end{array}
 \implies
 \begin{array}{c}
 \frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2} \qquad \frac{e_1 \rightarrow e'_1}{e_1 \times e_2 \rightarrow e'_1 \times e_2} \\
 \\
 \frac{e_2 \rightarrow e'_2}{n_1 + e_2 \rightarrow n_1 + e'_2} \qquad \frac{e_2 \rightarrow e'_2}{n_1 \times e_2 \rightarrow n_1 \times e'_2} \\
 \\
 \frac{}{n_1 + n_2 \rightarrow n_1 + n_2} \qquad \frac{}{n_1 \times n_2 \rightarrow n_1 \times n_2}
 \end{array}$$

$$\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2}$$

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$$\frac{}{n_1 \times n_2 \rightarrow n_1 \times n_2}$$

Let's prove  $(1 + 2) \times 3 \rightarrow^* 9$  by showing a **reduction sequence**:

(Note that  $\rightarrow^*$  denotes the reflexive-transitive closure of  $\rightarrow$ .)

$$\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2}$$

$$\frac{e_2 \rightarrow e'_2}{n_1 + e_2 \rightarrow n_1 + e'_2}$$

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$$\frac{e_1 \rightarrow e'_1}{e_1 \times e_2 \rightarrow e'_1 \times e_2}$$

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Let's prove  $(1 + 2) \times 3 \rightarrow^* 9$  by showing a **reduction sequence**:

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$$(1 + 2) \times 3 \quad \rightarrow \quad 3 \times 3 \quad \rightarrow \quad 9$$

$$\frac{e_1 \rightarrow e'_1}{e_1 + e_2 \rightarrow e'_1 + e_2}$$

$$\frac{e_2 \rightarrow e'_2}{n_1 + e_2 \rightarrow n_1 + e'_2}$$

$$\frac{}{n_1 + n_2 \rightarrow n_1 + n_2}$$

$$\frac{e_1 \rightarrow e'_1}{e_1 \times e_2 \rightarrow e'_1 \times e_2}$$

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$$1 + 2 \times 3 \quad \rightarrow$$

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- Syntax and Semantics (2)

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