Lecture 18 – Type Systems

COSE212: Programming Languages

Jihyeok Park

PLRG

2023 Fall

Recall

- We learned about **continuations** with the following topics:
  - **Continuations** (Lecture 14 & 15)
  - **First-Class Continuations** (Lecture 16)
  - **Compiling with continuations** (Lecture 17)

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- In this lecture, we will learn about **type systems**.

- We will focus on its **motivation** and the **basic concepts**.

Contents

1. Safe Language Systems
   Detecting Run-Time Errors
   Dynamic vs Static Analysis
   Soundness vs Completeness

2. Type Systems
   Types
   Types
   Type Checking
   Type Soundness

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Run-Time Errors

So far, we have designed diverse programming languages with:

- **Syntax**: a grammar that defines the structure of programs
- **Semantics**: a set of rules that defines the meaning of programs

and implemented their **interpreters** in Scala:

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However, we don't have any automatic system to **check** whether a program is evaluated without any **run-time errors**.

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- **Syntax**: a grammar that defines the structure of programs
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and implemented their **interpreters** in Scala:

However, we don't have any automatic system to **check** whether a program is evaluated without any **run-time errors**.

For example, following FAE expressions are syntactically correct, but they throw **run-time errors**:

```
/* FAE */
x * 42                  // error: free identifier
0 + (x => x)            // error: cannot add a function
1(2)                    // error: cannot apply a number
```

Errors in Saftety-Critical Software

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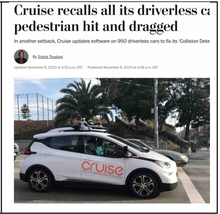
Unexpected errors in **safety-critical software** cause serious problems:



Rocket	**Financial**	**Airport**	**Auto. Vehicle**
(1996)	(2012)	(2020)	(2023)

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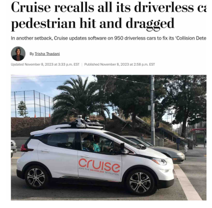

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Then, how can we **prevent** such errors?

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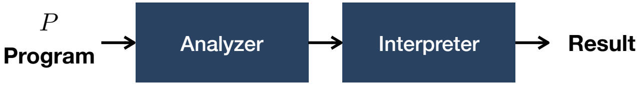
Then, how can we **prevent** such errors?

Can we **automatically** check whether a program does not have any **run-time errors**?

Detecting Run-Time Errors

We can use various **analysis** techniques to detect run-time errors:



An **analyzer** is a program that takes a program as an input and determines whether the program has a certain property. In this case, the property is **run-time errors**.

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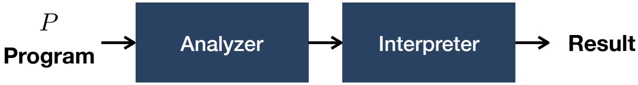

An **analyzer** is a program that takes a program as an input and determines whether the program has a certain property. In this case, the property is **run-time errors**.

We can categorize them into two groups:

- **Dynamic Analysis**: analyze programs by **executing** them
- **Static Analysis**: analyze programs **without executing** them

Dynamic Analysis

Dynamic analysis is a program analysis technique by **executing** them.

Let's perform **dynamic analysis** for the following Scala program:

```
def abs(x: Int): Int = { /* L1 */
  if (x < 0)             /* L2 */
    -x                   /* L3 */
  else                   /* L4 */
    x                    /* L5 */
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L1	-5
L2	-5
L3	5
L4	
L5	
L6	5

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L1	-5	42
L2	-5	
L3	5	
L4		42
L5		42
L6	5	42

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L1	-5	42	-7	99	0	...
L2	-5		-7			...
L3	5		7			...
L4		42		99	0	...
L5		42		99	0	...
L6	5	42	7	99	0	...

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We can **easily get** the result of the program for each **single input**.

However, it is **difficult** to get information about **all the inputs**.

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Static analysis is a program analysis technique **without executing** them.

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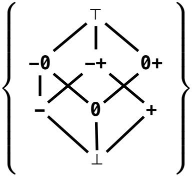

Let's define an **abstract domain** $\mathbb{D}$ for integers to analyze the program.

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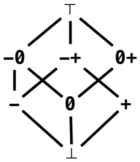

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$$
\begin{array}{lll}
\bot = \varnothing & \top = \mathbb{Z} & \\
0 = \{0\} & - = \{x \in \mathbb{Z} \mid x < 0\} & + = \{x \in \mathbb{Z} \mid x \geq 0\} \\
-0 = - \cup 0 & -+ = - \cup + & 0+ = 0 \cup +
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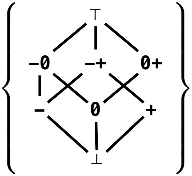

L1	
L2	
L3	
L4	
L5	
L6	

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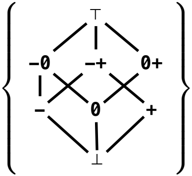
$\mathbb{D} =$ 

L1	$\top$
L2	-
L3	+
L4	0+
L5	0+
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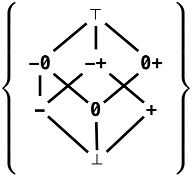

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We can prove that `abs` always returns a **non-negative** integer (i.e., 0+).

Soundness vs Completeness

- $\vDash \psi$ denotes that a statement ψ is **provable**.
- $\vdash \psi$ denotes that a statement ψ is **true**.

In a **sound** proof system, all **provable** statements are **true**.

$$\vDash \psi \qquad \Longrightarrow \qquad \vdash \psi$$

In a **complete** proof system, all **true** statements are **provable**.

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Analysis techniques can be used to prove that a program is **error-free**.

- $\vDash P$ denotes that a program P is **analyzed** as error-free.
- $\vdash P$ denotes that a program P is truly **error-free**.

Then, is dynamic/static analysis **sound** or **complete**?

Soundness vs Completeness

- **Dynamic analysis** is **complete** but **unsound**
  - All the detected errors are **true alarms** (**true positive (TP)**).
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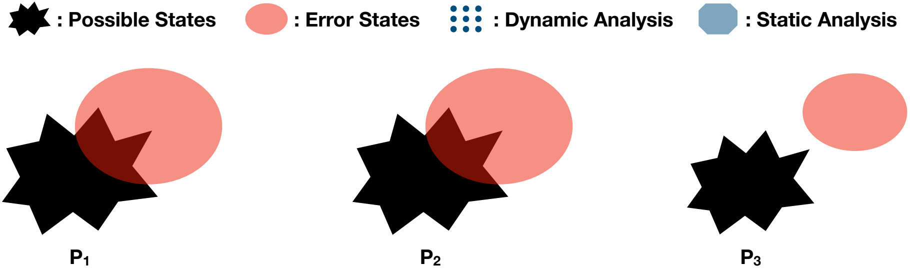

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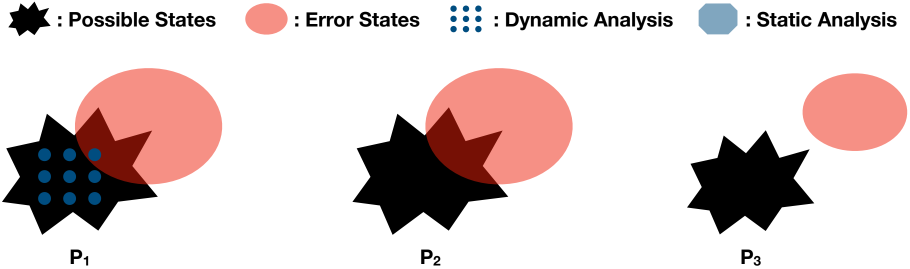

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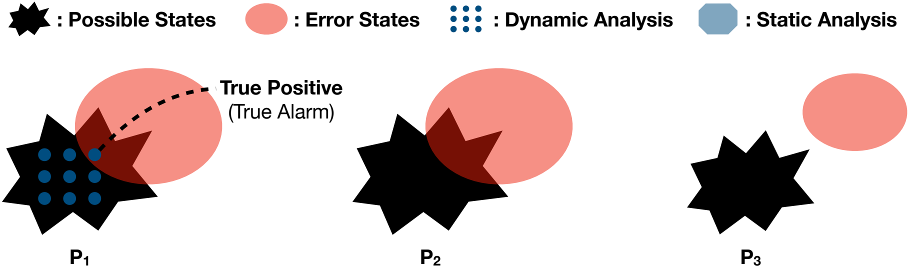

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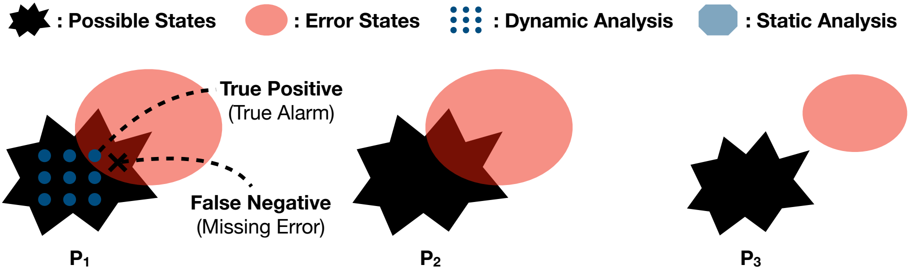

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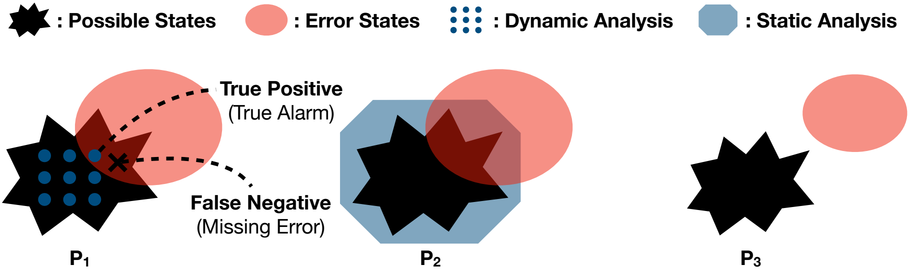

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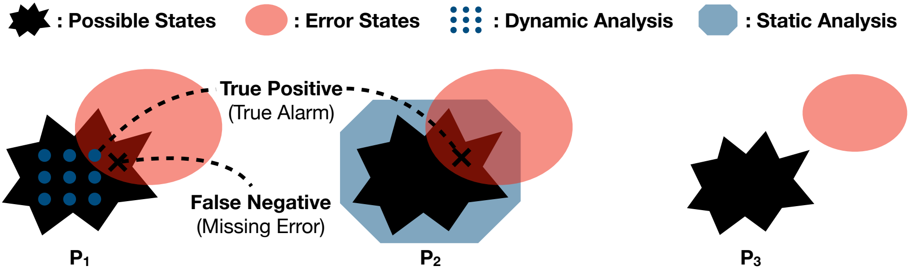

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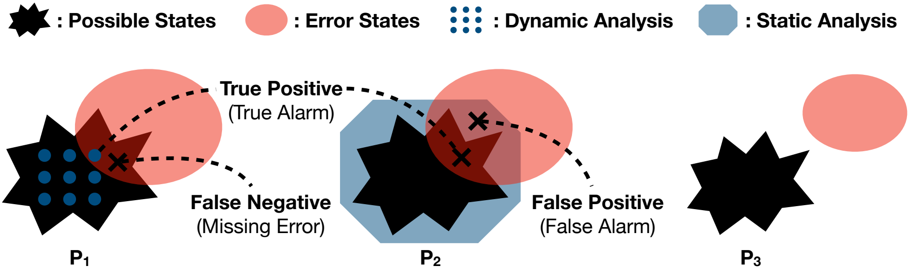

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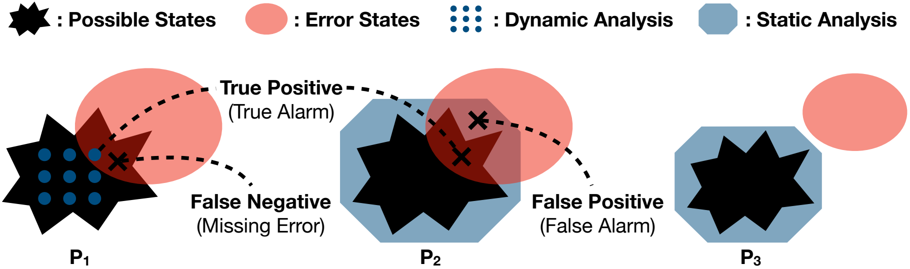

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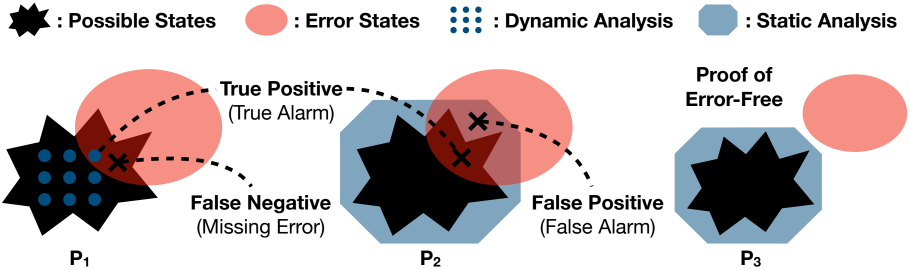

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Definition (Types)

A **type** is a set of values.

For example, the `Int` and `Boolean` types are defined in Scala:

$$
\begin{aligned}
\texttt{Int} &= \{n \in \mathbb{Z} \mid -2^{31} \leq n < 2^{31}\} \\
\texttt{Boolean} &= \{\texttt{true}, \texttt{false}\} \\
\texttt{Int => Int} &= \{f \mid f \text{ is a function from } \texttt{Int} \text{ to } \texttt{Int}\}
\end{aligned}
$$

```scala
val n: Int = 42              // 42   : Int
n + 1                        // 43   : Int
val b: Boolean = n > 10      // true : Boolean
def f(x: Int): Int = x + 1   // f    : Int => Int
f(42)                        // 43   : Int
```

Types and Type Errors

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Definition (Type Errors)

A **type error** occurs when a program tries to use a value having a type that is **incompatible** with the expected type.

For example, the following program has following type errors:

```
42 + true             // `Int` expected for `+`, but `Boolean` found
if (1) 2 else 3       // `Boolean` expected for `if`, but `Int` found
def f(x: Int): Int = x + 1
f(false)              // `Int` expected for `f`, but `Boolean` found
```

However, not all **run-time errors** are **type errors**:

```
42 / 0                // `ArithmeticException` at run-time
case class A(k: Int)
val x: A = null
x.k                   // `NullPointerException` at run-time
```

Type Checking

If the following conditions hold, we say **"the expression e has type τ"**:

- e does not cause any type error, and
- e evaluates to a value of type τ or does not terminate.

Type Checking

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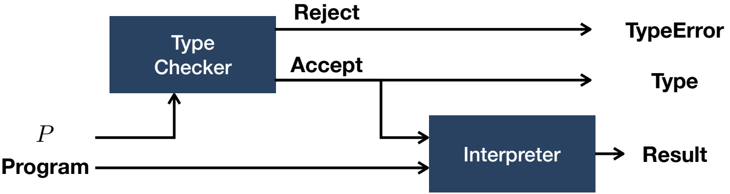
Definition (Type Checking)

Type checking is a kind of static analysis checking whether a given expression e is **well-typed**. A **type checker** returns the **type** of e if it is well-typed, or rejects it and reports the detected **type error** otherwise.



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Most of statically-typed languages has **sound** type systems.

Summary

1. Safe Language Systems
   - Detecting Run-Time Errors
   - Dynamic vs Static Analysis
   - Soundness vs Completeness

2. Type Systems
   - Types
   - Types
   - Type Checking
   - Type Soundness

Next Lecture

- Typed Languages

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