

WS 23/24 Numerics Notes

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Preface

Notes for the lecture “[WS 23/24 Numerics 0](#)” at Uni Heidelberg.

1 Floating Point Numbers

1.1 ANSI/IEEE 64 Bit

Let \tilde{a} be a 64 bit IEEE floating point number. \tilde{a} is represented as

S E ... E M ... M

Where S is the sign bit, 11 E's are the exponent bits and 52 M's are mantissa bits. Interpretation (Case analysis on value of E):

1. $E = 0$, i.e. $\tilde{a} = S \mid 0 \dots 0 \mid M$:
 1. $M = 0 \Rightarrow \tilde{a} = (-1)^S 0$
 2. $M \neq 0 \Rightarrow \tilde{a} = (-1)^S \times 2^{-1022} \times 0.M$ (**subnormal range**)
2. $1 \leq E \leq 2046 \Rightarrow \tilde{a} = (-1)^S \times 2^{E-1023} \times 1.M$ (**normal range**)
3. $E = 2047$, i.e. $\tilde{a} = S \mid 1 \dots 1 \mid M$:
 1. $M = 0 \Rightarrow \tilde{a} = (-1)^S \text{inf}$
 2. $M \neq 0 \Rightarrow \tilde{a} = \text{NaN(Not a Number)}$ (**exceptions**)

See Figure 1.1 for a visual summary.

Examples:

- **realmin** is the smallest normalized positive machine number in FP64:

$$\llbracket 0 \mid 0 \dots 01 \mid 0 \dots 0 \rrbracket_{FP64} = 2^{1-1023} \times 1.0 = 2^{-1022}$$

FP64 stands for IEEE **F**loating **P**oint **64** bit number representation. Whereas $[\cdot]_{FP64}$ is the FP64 evaluation/interpration of the machine number

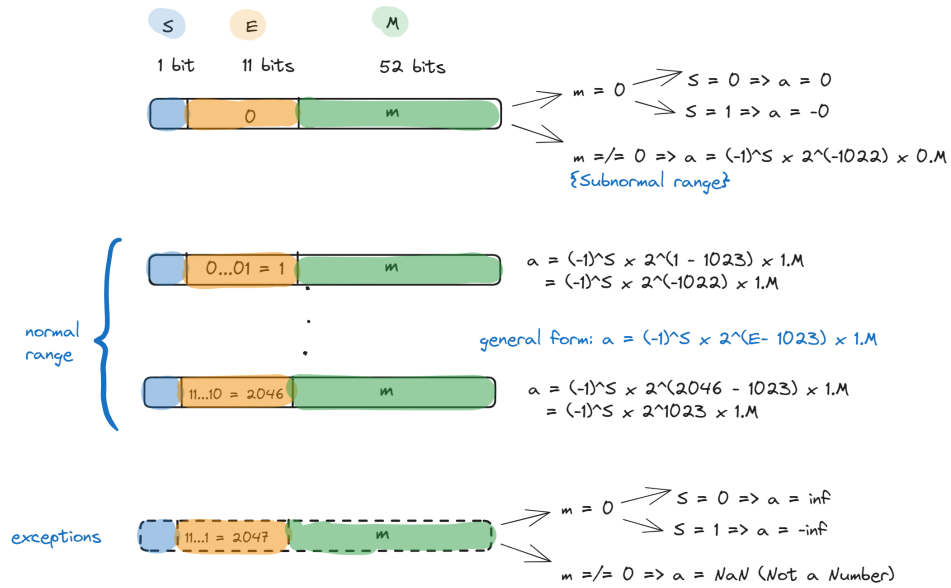


Figure 1.1: IEEE 64 bit floating point numbers interpretation

- **realmax** is the greatest normalized machine number in FP64:

$$[0|1\dots10|1\dots1]_{\text{FP64}} \approx 1.7977\text{E}308$$

- $1 = 2^0 \times 1.0 = 2^{1023-1023} \times 1.0 = [0|01\dots1|0\dots0]_{\text{FP64}}$
- **eps** is defined as the spacing in the interval $(1, 2)$. Note that the spacing is constant for each such interval, but grows as we go further down the number line. That is, the spacing in $(1000, 1001)$ is also constant, but larger.
- number right after 1 is $[0|01\dots1|0\dots1]_{\text{FP64}}$. Then the spacing, i.e. **eps** in the above definition is 2^{-52}