Collaborative Filtering



Credits

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Agenda

Collaborative Filtering (CF)

- Pure CF approaches
- User-based nearest-neighbor
- The Pearson Correlation similarity measure
- Memory-based and model-based approaches
- Item-based nearest-neighbor
- The cosine similarity measure
- Data sparsity problems
- Recent methods (SVD, Association Rule Mining, Slope One, RF-Rec, ...)
- The Google News personalization engine
- Discussion and summary
- Literature

Recommender Systems

- In everyday life we rely on recommendations from other people either by word of mouth, recommendation letters, movie and book reviews printed in newspapers ...
- In a typical recommender system people provide recommendations as inputs, which the system then aggregates and directs to appropriate recipients
 - Aggregation of recommendations
 - Match the recommendations with those searching for recommendations.



[Resnick and Varian, 1997]

Collaborative Filtering (CF)

The most prominent approach to generate recommendations

- used by large, commercial e-commerce sites
- well-understood, various algorithms and variations exist
- applicable in many domains (book, movies, DVDs, ..)

Approach

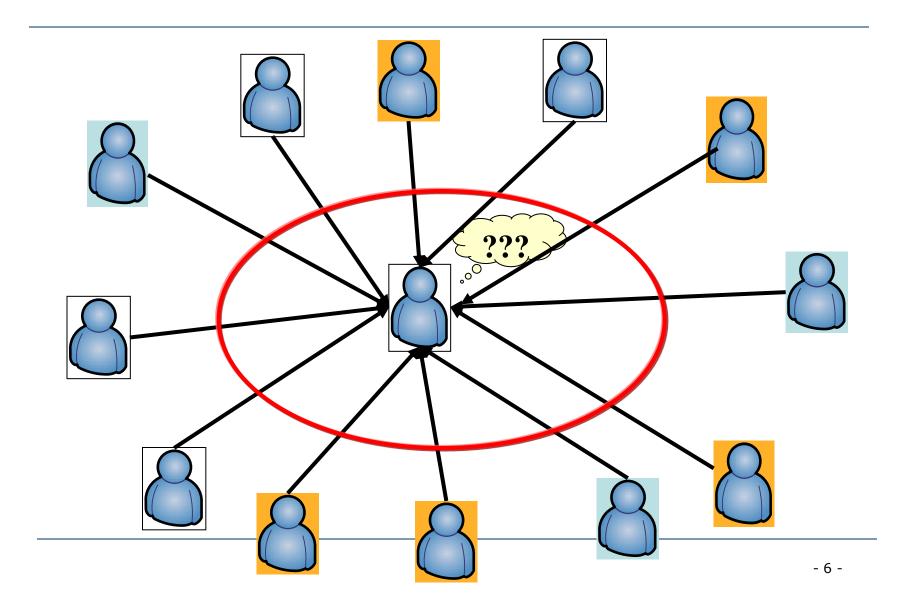
use the "wisdom of the crowd" to recommend items



Basic assumption and idea

- Users give ratings to catalog items (implicitly or explicitly)
- Customers who had similar tastes in the past, will have similar tastes in the future

Collaborative or Social Filtering



Collaborative Filtering Idea

- Trying to predict the opinion the user will have on the different items and be able to recommend the "best" items to each user
- It is based on: the user's previous likings and the opinions of other like minded users
- CF is a typical Internet application it must be supported by a networking infrastructure
 - At least many users and one server
 - But also a distributed model with many servers
- There is no stand alone CF application.

MovieLens

movielens

helping you find the right movies

Welcome to MovieLens!

Free, personalized, non-commercial, ad-free, great movie recommendations.

Have questions? Take the MovieLens Tour for answers.

Not a member? Join MovieLens now.

Need a gift idea? Try MovieLens QuickPick!

http://www.movielens.org/

New to MovieLens?

Join today!

You get **great recommendations** for movies while **helping us do research**. Learn more:

- Try out QuickPick: Our Movie Gift Recommender
- Take the MovieLens Tour
- Read our Privacy Policy
- See our Browser Requirements
- Learn about Our Research

Hello MovieLens Users!

Please log in:

Username:

Password:

Save login: 🔲

Log into MovieLens

Forgot your password?

New member? Join now

MovieLens is a free service provided by **GroupLens Research** at the **University of Minnesota**. We sometimes study how our members use MovieLens in order to learn how to build better recommendation systems. We promise to never give your personal information to anyone; see our **privacy policy** for more information.

Welcome to the new MovieLens!

Existing MovieLens users: We'd like to welcome you back to MovieLens, and let you know we have a new MovieLens FAQ you might want to read. We hope you like what you will see!

Take me to MovieLens!

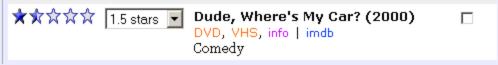
New MovieLens users: Thank you for joining MovieLens! In order to generate personalized movie recommendations, we need to know a little about what movies you have already seen. MovieLens will now display several lists of movies. If you have seen any of the listed movies, please rate them using the rating scale shown below.

Ratings are on a scale of 1 to 5:

```
★★★★ = Must See
★★★☆ = Will Enjoy
★★☆☆ = It's OK
★★☆☆ = Fairly Bad
★☆☆☆ = Awful
```

Remember: the more movies you rate, the more accurate MovieLens' predictions will be.

To rate a movie, just click on the pulldown next to the title of a movie you have seen. Blue stars will appear to indicate that your rating has been received.



This image shows that the movie 'Dude, Where's My Car?' was rated 1.5 stars.

I'm ready to start rating!

So far you have rated **0** movies. MovieLens needs at least **15** ratings from you to generate predictions for you. Please rate as many movies as you can from the list below.

		next >
	Your Rating	Movie Information
***	3.0 stars 💌	Austin Powers: International Man of Mystery (1997) Action, Adventure, Comedy
****	4.0 stars 💌	Contact (1997) Drama, Sci-Fi
???	Not seen 💌	Crouching Tiger, Hidden Dragon (Wu Hu Zang Long) (2000) Action, Adventure, Drama, Fantasy, Romance
???	Not seen 💌	Demolition Man (1993) Action, Comedy, Sci-Fi
???	Not seen 💌	Eraser (1996) Action, Drama, Thriller
???	Not seen 💌	Maverick (1994) Action, Comedy, Western
****	4.5 stars 💌	Philadelphia (1993) Drama
****	3.5 stars 💌	Piano, The (1993) Drama, Romance
???	Not seen 💌	Toy Story 2 (1999) Adventure, Animation, Children, Comedy, Fantasy
****	3.5 stars 💌	X-Men (2000) Action, Adventure, Sci-Fi

next >

Congratulations!

MovieLens can now generate personalized movie recommendations for you.

Start Using MovieLens

Remember, you can always keep rating movies you have seen. The more movies you rate, the better your predictions will be. We'd also like to tell you about some other features of MovieLens you might be interested in:

Getting recommendations. MovieLens has shortcuts
like Top Picks For You that provide you with quick
access to common searches. You can use the Search tab
to perform more advanced searches that filter by genre,
date, and more, and save your favorite searches as
personal shortcuts.

• Top Picks For You • Your Ratings • Your Wishlist • Newest Additions

 Your Wishlist. Here you can keep track of movies you haven't yet seen. You can even print this list out and take it with you to your video store.



Movie buddies. It can be a pain trying to decide what
movie a group of people should see. Let MovieLens
choose the right movie for you! You can add MovieLens
users to be your buddies and be able to generate group
movie recommendations

Prediction 3	You	Istvan
****	4.0	4.0

We will keep adding more great features as time goes on, so look for them!

Start Using MovieLens

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helping you find the right movies

Welcome fricci@unibz.it (Log Out)

You've rated **70** movies. You're the 18th visitor in the past hour. ★★★★ = Must See ★★★★☆ = Will Enjoy ★★☆☆ = It's OK ★★☆☆ = Fairly Bad ★☆☆☆ = Awful

Home | Find Movies | Q&A (new) | Preferences | Help

Shortcuts Search		There are 12019 movies matching your search: Movies without a prediction are Not Shown Movies you've rated are Not Shown					
Basic Search	Show Printer-F	You've sorted by: Prediction Show Printer-Friendly Page Download Results Permalink Suggest a Title					
Title: All Genres All Dates	Tags Related to Your Search: based on a book (1689), sci-fi (1567), comedy (1382), Nudity (Topless) (1265) action (1225), (about tags)						
Domain: All movies 💠	Page 1 of 802	1 2 3 4 802 next Skip to pa	age #:				
Use selected buddies!✓ Exclude your ratings	Prediction Your or Rating Rating	Movie Information	Wish List				
Exclude movies without predictions Search!	★★★★ Not seen ♦	Work of Director Michel Gondry, The (2003) DVD info imdb flag Movie Tuner					
	[add tag] Popular tags:	quirky ■ 🖒 🗘 notable soundtrack ■ 🖒 🛱 surreal ■ 🖒 🛱					
Select Buddies		Hearts and Minds (1974) DVD info imdb flag Movie Tuner III. Documentary, War - English, French, Vietnamese					
☐ Test Buddy	[add tag] Popular tags:	Vietnam War ುದ್ದರ್ I racism ುದ್ದರ military ುದ್ದರ					
What are buddies?	★★★★ Not seen ♦	Mad Love (1935) DVD info imdb flag Movie Tuner III Horror, Romance					
	[add tag] Popular tags:	DVD-R 🛮 ರೆ 🌣 not available from Netflix 🖺 ರೆ 🗘 DVD-RAM 🗉 ರೆ 🗘					
Advanced Search	★★★★ Not seen ♦	Double Indemnity (1944) DVD VHS info imdb flag Movie Tuner Illi Crime, Drama, Film-Noir					
	-	•					

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Shortcuts

Search

Rate and Find Movies

- Top Picks For You
- Newest Additions
- Most Often Rated
- Rate Random Movies
- Browse Movies by Tags

Your Movies

- Your Ratings
- About Your Ratings
- Your Wishlist
- Your Tags

Your Account

- Your Profile (edit)
- Preferences
- Manage Buddies
- Manage RSS Feeds

Help MovieLens

- Volunteer Center
- Vote for Titles

Work of Director Michel Gondry, The (2003)

Your Prediction: ****

Rate This Movie: Not seen 💠 Wish List: 🖂



Starring: Michel Gondry, Björk, Beck, David Grohl, David Cross, Jack White,

Meg White, Cibo Matto

Directed By: Michel Gondry, Lance Bangs, Olivier Gondry

Genres: Comedy, Documentary

Languages: English French

Average rating: **** (4.11 stars)

Your Prediction: ***** (5.0 stars)

Rated by: 71 users

Links: IMDb, Rotten Tomatoes

Movie Tags (more about tags)

Add and edit tags here or update all of your tags

Community Tags (?)

Tags represent how MovieLens users feel about this movie

01/11 02/11 03/11 bjork

creative David Cross



nel Gondr

The Work of Director Michel Gondry

The tireless creativity of director Michel Gondry is on vivid display in this collection of 27 music videos and other whimsical oddities. Released the year before Gondry's feature

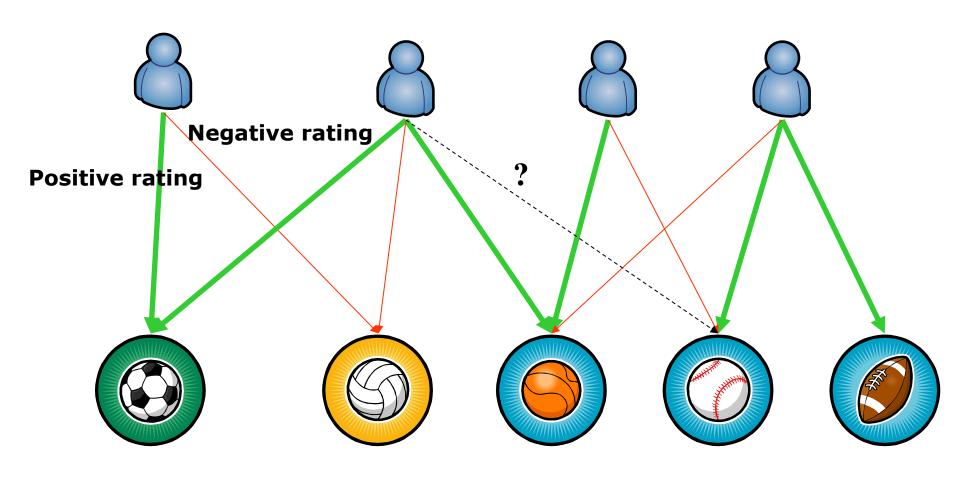
breakthrough Eternal Sunshine of the Spotless Mind, the compilation includes Kylie Minogue's "Come into My World," Bjork's "Human Behavior," Massive Attack's "Protection" and the White Stripes' Lego-centric stunner "Fell in Love with a Girl."

Report Wrong Movi@elivered by Netflix (add to queue)

Movie Tuner (?)

Find similar movies with less or more of particular qualities. The movie list below will update as you indicate your preferences

Collaborative Filtering



Collaborative Filtering Ingredients

- List of m Users and a list of n Items
- Each user has a list of items he/she expressed their opinion about (can be a null set)
- Explicit opinion a rating score (numerical scale)
 - Sometime the rating is implicitly purchase records
- Active user for whom the CF prediction task is performed
- A metric for measuring similarity between users
- A method for selecting a subset of neighbors for prediction
- A method for predicting a rating for items not currently rated by the active user.

Pure CF Approaches

Input

Only a matrix of given user—item ratings

Output types

- A (numerical) prediction indicating to what degree the current user will like or dislike a certain item
- A top-N list of recommended items

User-based nearest-neighbor collaborative filtering (1)

The basic technique

- Given an "active user" (Alice) and an item i not yet seen by Alice
 - find a set of users (peers/nearest neighbors) who liked the same items as Alice in the past and who have rated item i
 - use, e.g. the average of their ratings to predict, if Alice will like item i
 - do this for all items Alice has not seen and recommend the best-rated

Basic assumption and idea

- If users had similar tastes in the past they will have similar tastes in the future
- User preferences remain stable and consistent over time

User-based nearest-neighbor collaborative filtering (2)

Example

A database of ratings of the current user, Alice, and some other users is given:

	ltem1	Item2	Item3	Item4	ltem5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Determine whether Alice will like or dislike *Item5*, which Alice has not yet rated or seen

User-based nearest-neighbor collaborative filtering (3)

Some first questions

- How do we measure similarity?
- How many neighbors should we consider?
- How do we generate a prediction from the neighbors' ratings?



	ltem1	Item2	Item3	Item4	ltem5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Measuring user similarity (1)

A popular similarity measure in user-based CF: Pearson correlation

a, b: users

 $r_{a,p}$: rating of user a for item p

P: set of items, rated both by a and b

- Possible similarity values between -1 and 1

$$sim(a,b) = \frac{\sum_{p \in P} (r_{a,p} - \bar{r}_a)(r_{b,p} - \bar{r}_b)}{\sqrt{\sum_{p \in P} (r_{a,p} - \bar{r}_a)^2} \sqrt{\sum_{p \in P} (r_{b,p} - \bar{r}_b)^2}}$$

Measuring user similarity (2)

A popular similarity measure in user-based CF: Pearson correlation

a, b: users

 $r_{a,p}$: rating of user a for item p

P: set of items, rated both by a and b

- Possible similarity values between -1 and 1

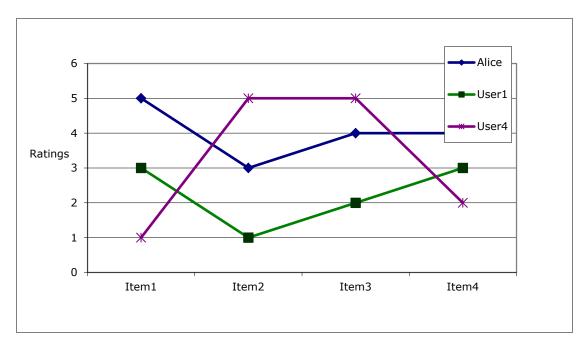
	ltem1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1



sim = 0.85 sim = 0.70 sim = 0.00sim = -0.79

Pearson correlation

Takes differences in rating behavior into account



- Works well in usual domains, compared with alternative measures
 - such as cosine similarity

Proximity Measure: Cosine

- ☐ Correlation can be replaced with a typical Information Retrieval (IR) similarity measure: **cosine**
- ☐ This has been shown to provide worse results by someone [Breese et al., 1998]
- But many uses cosine [Sarwar et al., 2000] and somebody reports that it performs better [Anand and Mobasher, 2005]

Comparison: Pearson vs. Cosine

	user 1	user 2	user 3
p1	1	2	5
p2	3	4	3
р3	4	5	2
p4	2	3	4
p5	1	2	5
p6	2	3	4
p7	2	3	4
p8	1	2	5

Pearson

	user 1	user 2	user 3
user 1	1	1	-1
user 2	1	1	-1
user 3	-1	-1	1

- User 2 ratings are those of user 1 incremented by 1
- User 3 has "opposite" preferences of user 1

Cosine

	user 1	user 2	user 3
user 1	1,00	0,99	0,76
user 2	0,99	1,00	0,84
user 3	0,76	0,84	1,00

Making predictions

A common prediction function:

$$pred(a,p) = \overline{r_a} + \frac{\sum_{b \in N} sim(a,b) * (r_{b,p} - \overline{r_b})}{\sum_{b \in N} sim(a,b)}$$



- Calculate, whether the neighbors' ratings for the unseen item i are higher or lower than their average
- Combine the rating differences use the similarity with α as a weight
- Add/subtract the neighbors' bias from the active user's average and use this as a prediction

Improving the metrics / prediction function

Value of number of co-rated items

 Use "significance weighting", by e.g., linearly reducing the weight when the number of co-rated items is low

Not all neighbor ratings might be equally "valuable"

- Agreement on commonly liked items is not so informative as agreement on controversial items
- Possible solution: Give more weight to items that have a higher variance

Case amplification

 Intuition: Give more weight to "very similar" neighbors, i.e., where the similarity value is close to 1.

Neighborhood selection

Use similarity threshold or fixed number of neighbors

Improving the metrics / prediction function

Value of number of co-rated items

- Use "significance weighting", by e.g., linearly reducing the weight when the number of co-rated items is low
- Reduce the magnitude of a similarity weight when this weight is computed using only a few ratings
- For example user similarity weight penalized by a factor proportional to the number of commonly rated items, if this number is less than a given parameter $\gamma > 0$

$$w'_{uv} = \frac{\min\{|\mathcal{I}_{uv}|, \gamma\}}{\gamma} \times w_{uv}$$

Likewise, an item similarity wi j, obtained from a few ratings, can be adjusted
 as

$$w'_{ij} = \frac{\min\{|\mathcal{U}_{ij}|, \gamma\}}{\gamma} \times w_{ij}$$

Improving the metrics / prediction function

Not all neighbor ratings might be equally "valuable"

- Agreement on commonly liked items is not so informative as agreement on controversial items
- Possible solution: Give more weight to items that have a higher variance
- A recommendation approach that addresses this problem is the *Inverse User Frequency*. Based on the information retrieval notion of IDF, a weight λ_i is given to each item i, in proportion to the log-ratio of users that have rated i:

$$\lambda_i = \log \frac{|\mathcal{U}|}{|\mathcal{U}_i|}$$

– While computing the Frequency-Weighted Pearson Correlation (FWPC) between users u and v, the correlation between the ratings given to an item i is weighted by λ_i :

$$\text{FWPC}(u,v) = \frac{\sum\limits_{i \in \mathcal{I}_{uv}} \lambda_i (r_{ui} - \overline{r}_u) (r_{vi} - \overline{r}_v)}{\sqrt{\sum\limits_{i \in \mathcal{I}_{uv}} \lambda_i (r_{ui} - \overline{r}_u)^2 \sum\limits_{i \in \mathcal{I}_{uv}} \lambda_i (r_{vi} - \overline{r}_v)^2}}$$

Memory-based and model-based approaches

User-based CF is said to be "memory-based"

- the rating matrix is directly used to find neighbors / make predictions
- does not scale for most real-world scenarios
- large e-commerce sites have tens of millions of customers and millions of items

Model-based approaches

- based on an offline pre-processing or "model-learning" phase
- at run-time, only the learned model is used to make predictions
- models are updated / re-trained periodically
- large variety of techniques used
- model-building and updating can be computationally expensive
- item-based CF is an example for model-based approaches

Item-based collaborative filtering

Basic idea:

Use the similarity between items (and not users) to make predictions

Example:

- Look for items that are similar to Item5
- Take Alice's ratings for these items to predict the rating for Item5

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

The cosine similarity measure

- Produces better results in item-to-item filtering
- Ratings are seen as vector in n-dimensional space
- Similarity is calculated based on the angle between the vectors

$$sim(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| * |\vec{b}|}$$



- Adjusted cosine similarity
 - take average user ratings into account, transform the original ratings
 - U: set of users who have rated both items a and b

$$sim(\vec{a}, \vec{b}) = \frac{\sum_{u \in U} (r_{u,a} - \overline{r_u}) (r_{u,b} - \overline{r_u})}{\sqrt{\sum_{u \in U} (r_{u,a} - \overline{r_u})^2} \sqrt{\sum_{u \in U} (r_{u,b} - \overline{r_u})^2}}$$



Making predictions

A common prediction function:

$$pred(u, p) = \frac{\sum_{i \in ratedItem(u)} sim(i, p) * r_{u,i}}{\sum_{i \in ratedItem(u)} sim(i, p)}$$



- Neighborhood size is typically also limited to a specific size
- Not all neighbors are taken into account for the prediction
- An analysis of the MovieLens dataset indicates that "in most real-world situations, a neighborhood of 20 to 50 neighbors seems reasonable" (Herlocker et al. 2002)

Pre-processing for item-based filtering

- Item-based filtering does not solve the scalability problem itself
- Pre-processing approach by Amazon.com (in 2003)
 - Calculate all pair-wise item similarities in advance
 - The neighborhood to be used at run-time is typically rather small, because only items are taken into account which the user has rated
 - Item similarities are supposed to be more stable than user similarities

Memory requirements

- Up to N² pair-wise similarities to be memorized (N = number of items) in theory
- In practice, this is significantly lower (items with no co-ratings)
- Further reductions possible
 - Minimum threshold for co-ratings
 - Limit the neighborhood size (might affect recommendation accuracy)

More on ratings – Explicit ratings

- Probably the most precise ratings
- Most commonly used (1 to 5, 1 to 7 Likert response scales)
- Research topics
 - Optimal granularity of scale; indication that 10-point scale is better accepted in movie dom.
 - An even more fine-grained scale was chosen in the joke recommender discussed by Goldberg et al. (2001), where a continuous scale (from −10 to +10) and a graphical input bar were used
 - No precision loss from the discretization
 - User preferences can be captured at a finer granularity
 - Users actually "like" the graphical interaction method
 - Multidimensional ratings (multiple ratings per movie such as ratings for actors and sound)
- Main problems
 - Users not always willing to rate many items
 - number of available ratings could be too small → sparse rating matrices → poor recommendation quality
 - How to stimulate users to rate more items?

More on ratings – Implicit ratings

- Typically collected by the web shop or application in which the recommender system is embedded
- When a customer buys an item, for instance, many recommender systems interpret this behavior as a positive rating
- Clicks, page views, time spent on some page, demo downloads ...
- Implicit ratings can be collected constantly and do not require additional efforts from the side of the user
- Main problem
 - One cannot be sure whether the user behavior is correctly interpreted
 - For example, a user might not like all the books he or she has bought; the user also might have bought a book for someone else
- Implicit ratings can be used in addition to explicit ones; question of correctness of interpretation

Data sparsity problems

Cold start problem

– How to recommend new items? What to recommend to new users?

Straightforward approaches

- Ask/force users to rate a set of items
- Use another method (e.g., content-based, demographic or simply nonpersonalized) in the initial phase
- Default voting: assign default values to items that only one of the two users to be compared has rated (Breese et al. 1998)

Alternatives

- Use better algorithms (beyond nearest-neighbor approaches)
- Example:
 - In nearest-neighbor approaches, the set of sufficiently similar neighbors might be too small to make good predictions
 - Assume "transitivity" of neighborhoods

2000: Application of Dimensionality Reduction in Recommender System, B. Sarwar et al., WebKDD Workshop

- Basic idea: Trade more complex offline model building for faster online prediction generation
- Singular Value Decomposition for dimensionality reduction of rating matrices
 - Captures important factors/aspects and their weights in the data
 - factors can be genre, actors but also non-understandable ones
 - Assumption that k dimensions capture the signals and filter out noise (K = 20 to 100)
- Constant time to make recommendations
- Approach also popular in IR (Latent Semantic Indexing), data compression,...

2008: Factorization meets the neighborhood: a multifaceted collaborative filtering model, Y. Koren, ACM SIGKDD

- Stimulated by work on Netflix competition
 - Prize of \$1,000,000 for accuracy improvement of 10% RMSE compared to own Cinematch system
 - Very large dataset (~100M ratings, ~480K users , ~18K movies)
 - Last ratings/user withheld (set K)





 Mean Absolute Error (MAE) computes the deviation between predicted ratings and actual ratings

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |p_i - r_i|$$

Root Mean Square Error (RMSE) is similar to MAE,
 but places more emphasis on larger deviation

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (p_i - r_i)^2}$$



Collaborative Filtering Issues

Pros:



well-understood, works well in some domains, no knowledge engineering required

Cons:



 requires user community, sparsity problems, no integration of other knowledge sources, no explanation of results

What is the best CF method?

 In which situation and which domain? Inconsistent findings; always the same domains and data sets; differences between methods are often very small (1/100)

How to evaluate the prediction quality?

- MAE / RMSE: What does an MAE of 0.7 actually mean?
- Serendipity (novelty and surprising effect of recommendations)
 - Not yet fully understood

What about multi-dimensional ratings?

Literature (1)

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- **[Herlocker et al. 2004]** Evaluating collaborative filtering recommender systems, ACM Transactions on Information Systems (TOIS) **22** (2004), no. 1, 5–53

Literature (2)

- [Hofmann 2004] Latent semantic models for collaborative filtering, ACM Transactions on Information Systems 22 (2004),
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- **[Huang et al. 2004]** Applying associative retrieval techniques to alleviate the sparsity problem in collaborative filtering, ACM Transactions on Information Systems 22 (2004), no. 1, 116–142
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- **[Lemire and Maclachlan 2005]** Slope one predictors for online rating-based collaborative filtering, Proceedings of the 5th SIAM International Conference on Data Mining (SDM '05) (Newport Beach, CA), 2005, pp. 471–480
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