Surface Code Quantum Computation

Progress and Challenges

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1. Quantum Error Correction: Where We Are?

1.1 Why do we need QEC?

- Today's quantum computers are noisy: $\sim$ 1 error per 100 operations.
- Practical advantage of quantum computer needs *MegaQuOp* scale.
- Practical Application of quantum computer needs *GigaQuOp*/*TeraQuop* scale.

Application (One shot of QPE)	Logical qubits N	Depth D	QuOps (lower bound) N.D
Hubbard model (8x8) Campbell Quantum Sci. Technol., 7 015007 (2021)	162	5.8 million	0.9 GigaQuOp
Jellium / Free electron gas (5x5x5) Kivlichan et. al. Quantum, 4, 296 (2020).	378	89 million	34 GigaQuOp
RSA with 2024 bits Ekera and Gidney Quantum, 5, 433 (2021).	6,115	2.1 billion	13 TeraQuOp
FeMoCo (Li orbitals / THC method) Lee et. al. PRX Quantum, 2, 030305 (2021).	2,196	130 billion	285 TeraQuop

Figure 1: *QuOp* Scale of Practical Quantum Application[1]

1.2 Where We are?(Superconducting Qubits)

- We are at the point where the theory of QEC is beginning to be proven experimentally.
- We are still growing and improving single logical qubit.
- In 2024, Google demonstrated *sub-threshold* quantum error correction experimentally[2].

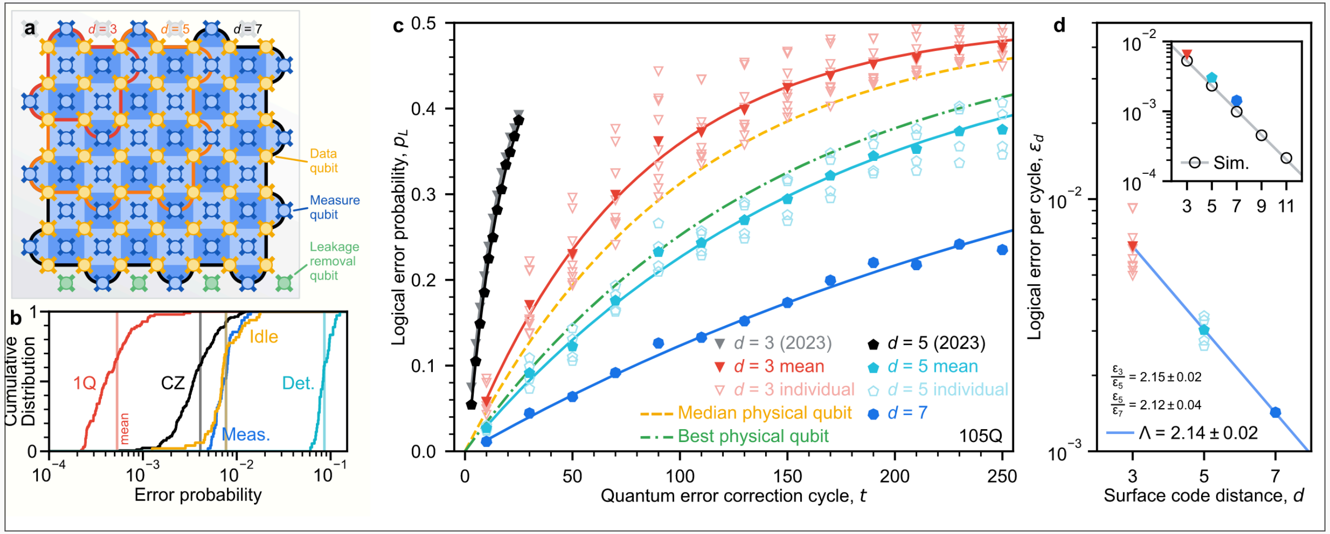

Figure 2: Google's Logical Memory Experiment(d=3,5,7)

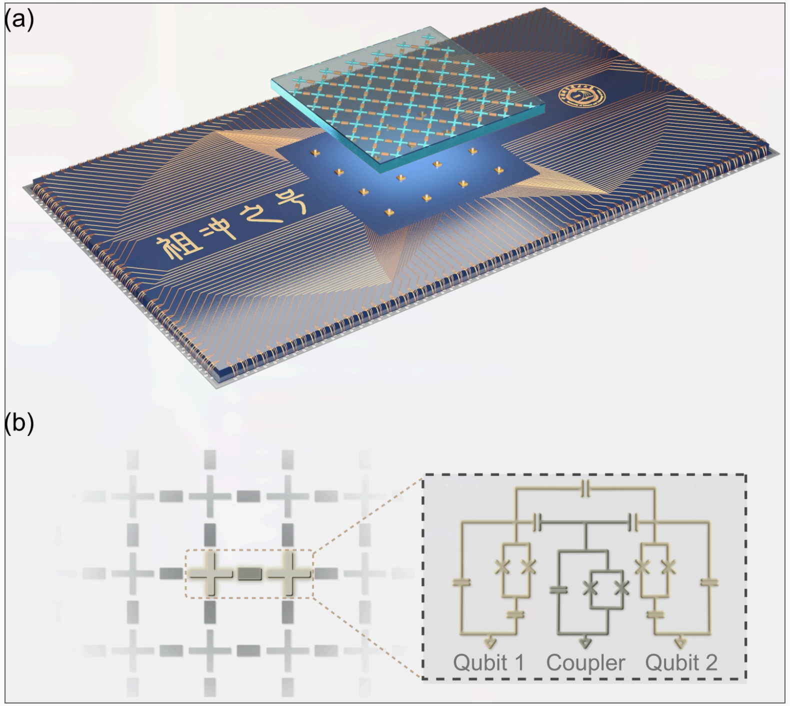

Figure 3: *Zuchongzhi2* Quantum Processor

We demonstrated $d = 3$ surface code on *Zuchongzhi2* processor in 2022[3]. The hardware has been improved since then[1]:

- 100+ qubits with tunable couplers
- average $T_1 \sim 60\mu s$
- parallel single qubit gate fidelity $\sim 99.9\%$
- parallel CZ gate fidelity $\sim 99.4\%$
- parallel readout fidelity $\sim 98.5\%$
- better support for *repeated measurements* and *reset* operations

[1] The design is optimized for targeted quantum error correction experiments.

2. Surface Code: A Practical Way towards Fault-Tolerant Quantum Computation

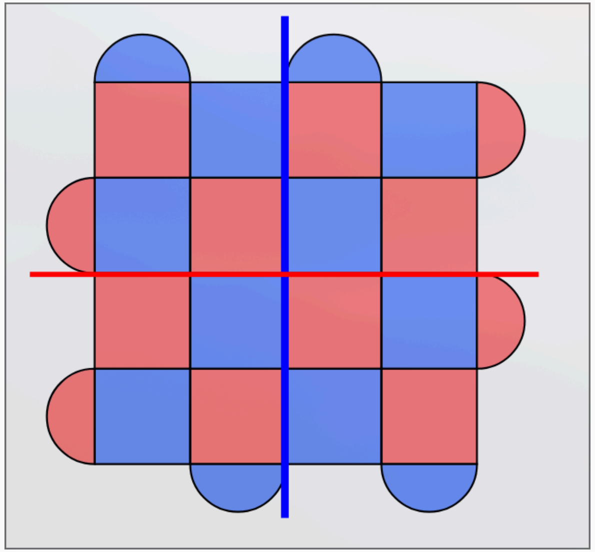

Figure 4: $d = 5$ Rotated Surface Code

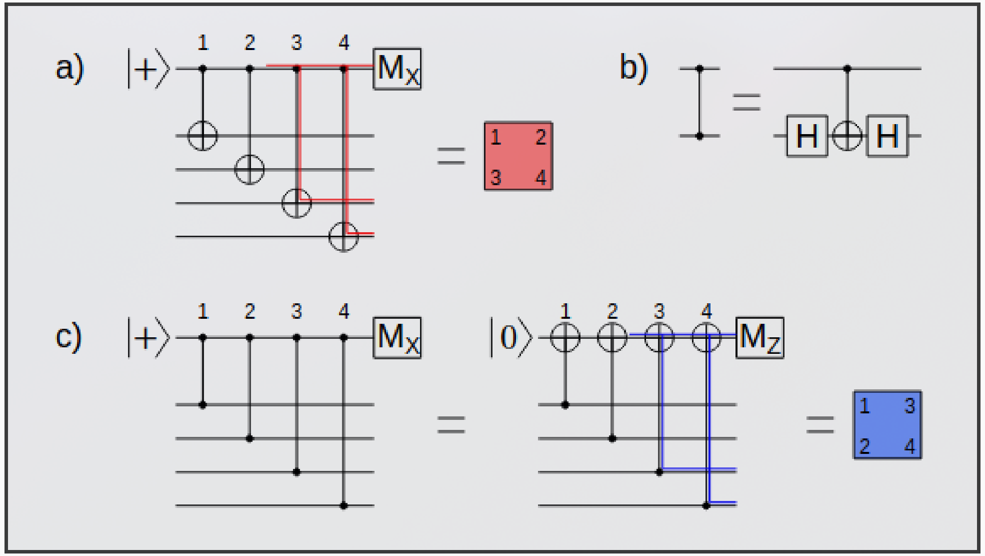

Figure 5: Syndrome Measurement Circuits

Pros

- A high error threshold: near 1%
- 2D Nearest-neighbor connectivity: suitable for superconducting qubits
- Good decoders available: MWPM, Union-Find, Tensor Network, Neural Network
- Mature protocols for logical operations: lattice surgery, transversal gates

Cons

- High resource overhead: vanishing encoding rate($\frac{1}{d^2}$)

2.2 Logical Operations: Initialization/Measurement

- Logical init in X(Z) basis: reset all data qubits in $|+\rangle(|0\rangle)$ state, takes $O(1)$ time.
- Logical meas in X(Z) basis: measure all data qubits in X(Z) basis, takes $O(1)$ time.
- Logical init/meas in Y basis requires more complex circuits, which maps between the logical Y operator and the product of specific stabilizers, takes $\frac{d}{2}$ time.[4]

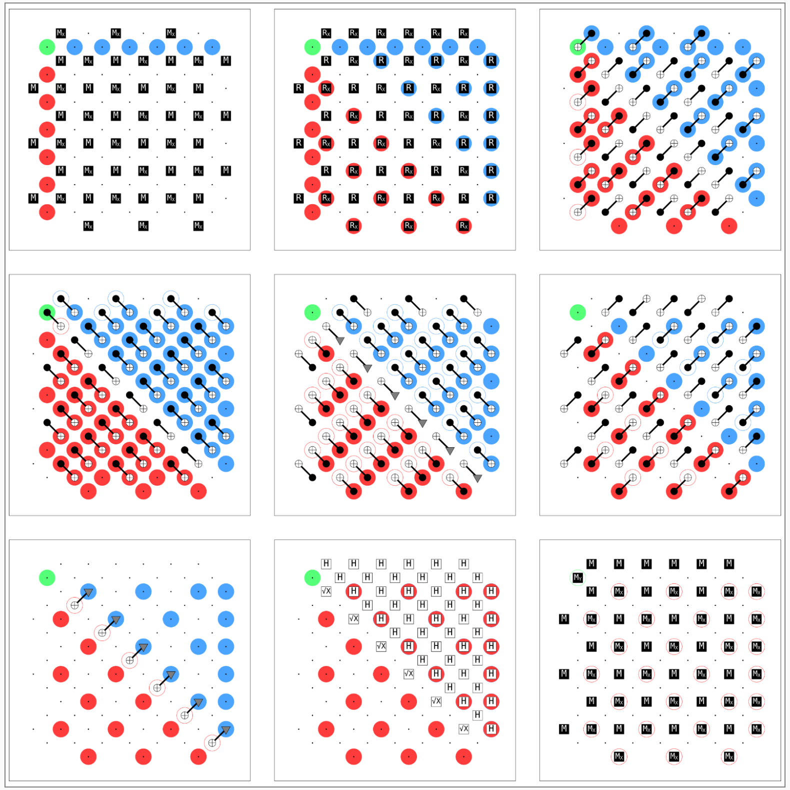

Figure 6: Y Basis Measurement

2.3 Logical Operations: Pauli Gates

Logical Pauli gates(or corrections from decoding) can be tracked in software and do not require physical operations on hardware. Takes **zero** time.

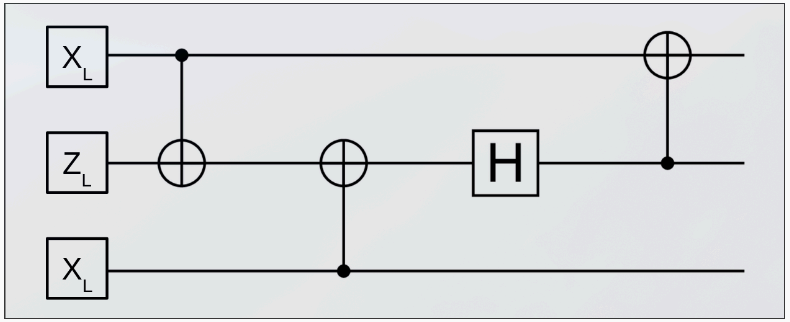

Figure 7: Logical Circuit(Raw)

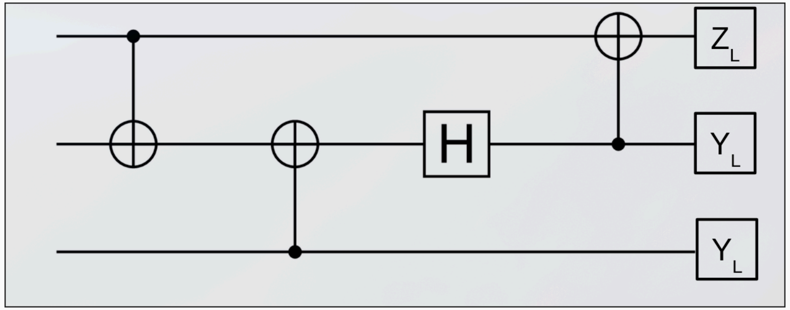

Figure 8: Equivalent Logical Circuit After Pushing Paulis

Logical Hadamard Effect can be applied with a layer of transversal H gates on data qubits, while exchanging the boundary types. Takes $O(1)$ time.

Rotating the boundary types back can be achieved in $4d^3$ spacetime volume.

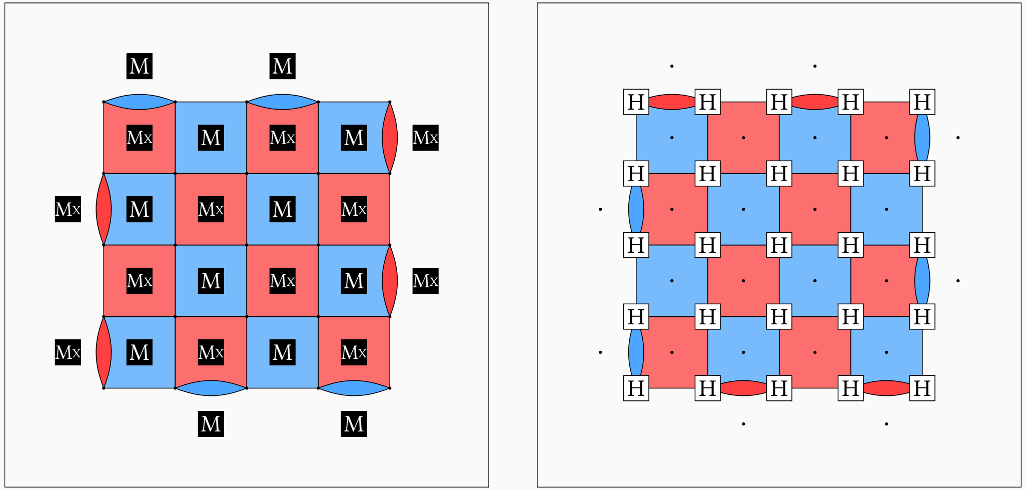

Figure 9: Logical H Effect

With space-time symmetry, the logical Hadamard effect can also be implemented in spatial direction.

Parity measurement is an elementary operation in lattice surgery based surface code quantum computation.

It can be achieved by merge and split operations between two logical qubits.

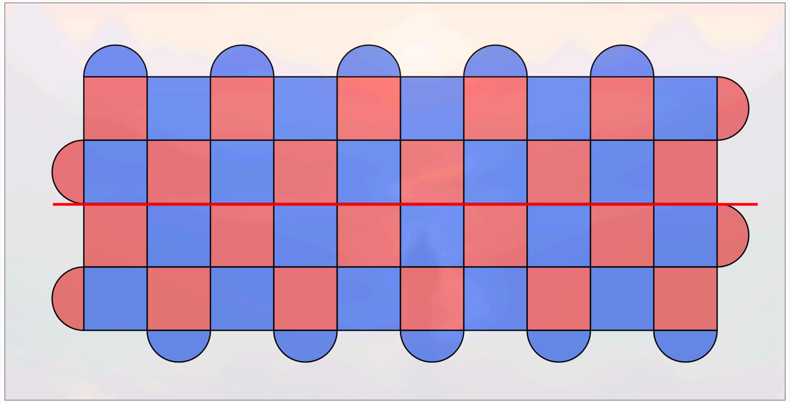

Figure 10: Product of physical qubit measurements realizes M_{XX}

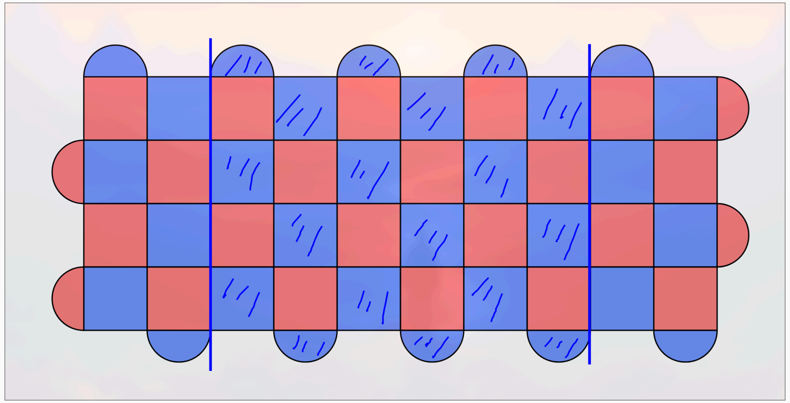

Figure 11: Product of stabilizer measurements realizes M_{ZZ}

Logical S gate can be implemented via gate teleportation[4]. Takes **$3d^3$** spacetime volume.

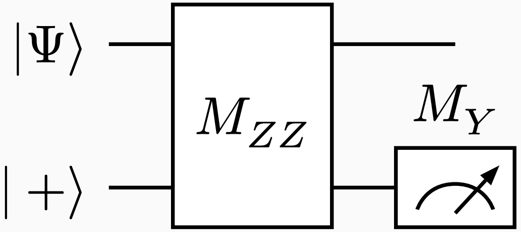

Figure 12: Logical S Gate[1]

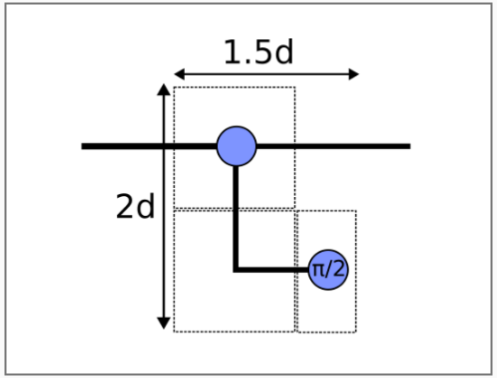

Figure 13: Logical S Gate ZX Diagram

[1] Pauli corrections conditioned on the measurement results are not plotted.

2.7 Logical Operations: CNOT

Logical CNOT gate can be implemented by two parity measurements. Takes $\mathbf{6d^3}$ spacetime volume.

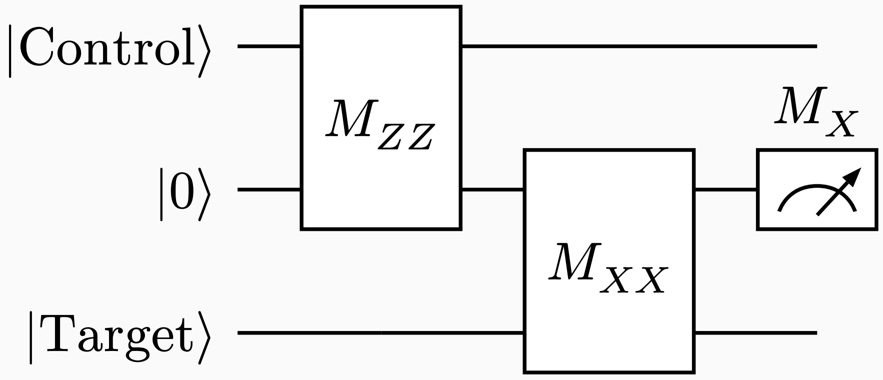

Figure 14: Logical CNOT Gate[1]

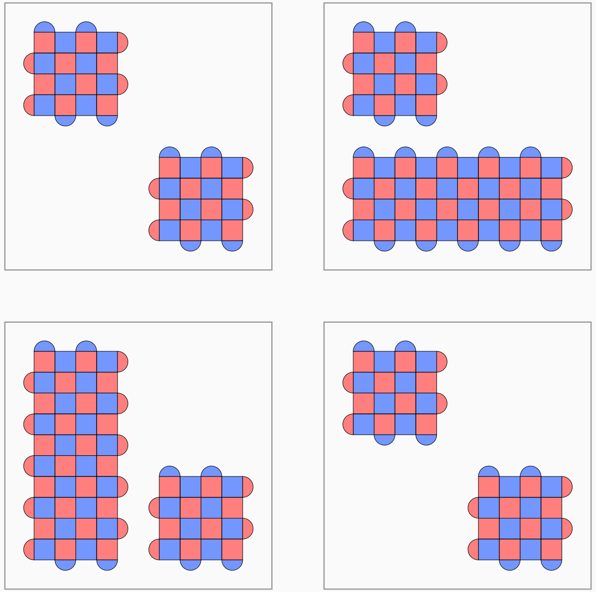

Figure 15: Logical CNOT Patches

[1] Pauli corrections conditioned on the measurement results are not plotted.

2.8 Logical Operations: T

Historically, implmenting *T gates* requires expensive magic state distillation and is the main overhead of surface code quantum computation. Now preparing a high-fidelity magic state is as cheap as a logical CNOT gate[1] by cultivation[5].

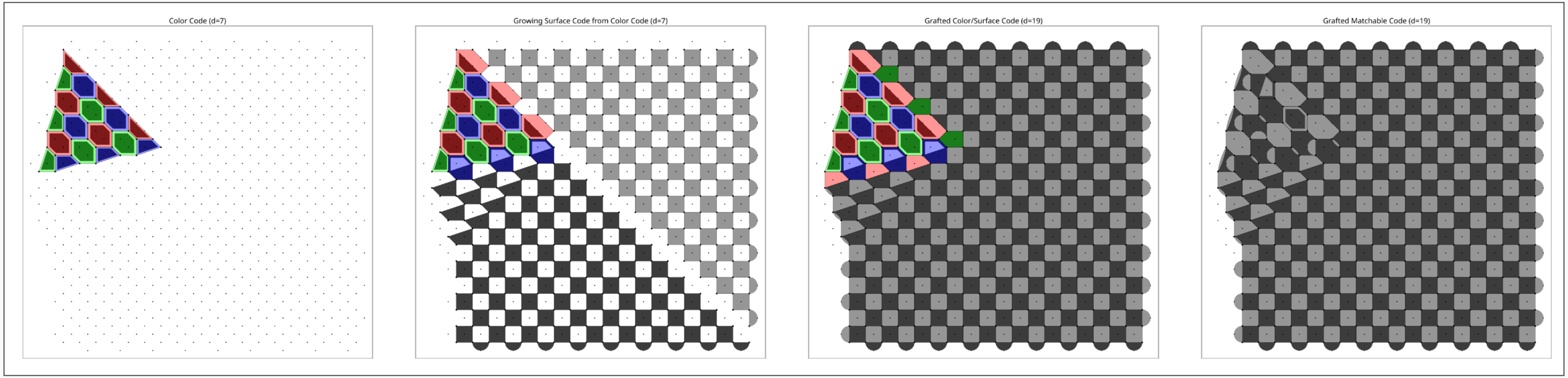

Figure 16: Magica State Cultivation

[1]for magic state fidelity achieving $1e^{-9}$

3. *TQEC*: Design Automation for Surface Code Quantum Computation

3.1 *TQEC* Community

- *TQEC* is an open-source community for design automation of Topological Quantum Error Correction, currently focusing on surface code quantum computation. It is organized by Austin G. Fowler from Google Quantum AI.
- We are building the tools to manage the complexity of scalable circuit compilation, arrangement optimization.
- It's not for good, but necessary…
- The code is open-source and available on GitHub: https://github.com/tqec/tqec. It's still in the very early stage.

3.2 High-level Workflow

The workflow for compiling a quantum algorithm or a general logical quantum circuit to the physical instructions maybe:

1. Write the quantum algorithm in a high-level quantum programming language like `Q#` or `Qualtran`.
2. Compile and optimize the quantum algorithm with *ZX-calculus*.
3. Compile the ZX-diagram representation to a 3D surface code spacetime structure with `TQEC`.
4. Compile the 3D spacetime structure to the physical instructions for the quantum hardware with `TQEC`.

- `TQEC` provides a programmable 3D spacetime representation for surface code, and its corresponding ZX diagram representation.

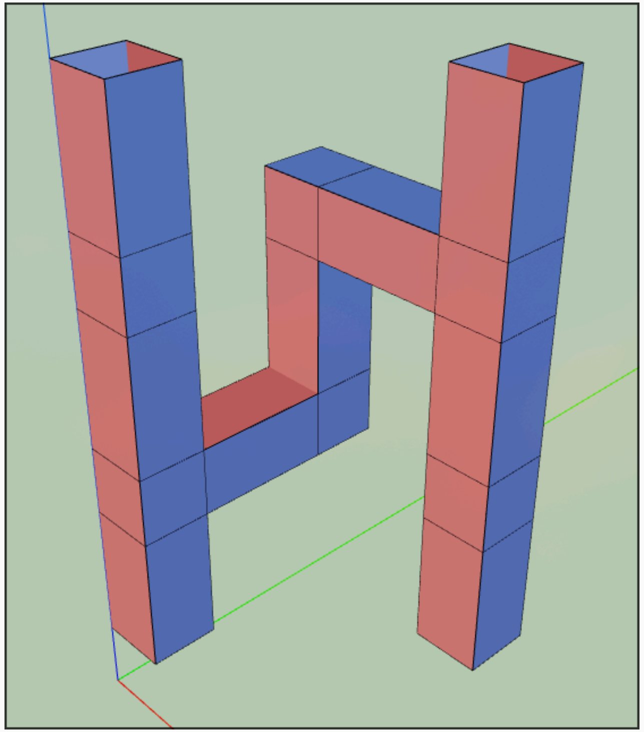

Figure 17: Logical CNOT Spacetime Diagram

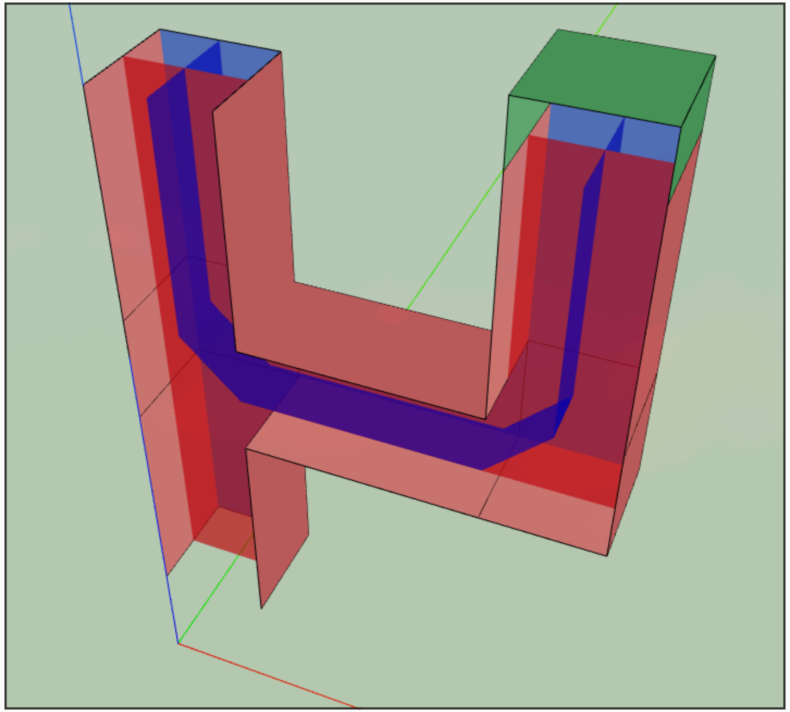

Figure 18: Logical S Spacetime Diagram

3.3 TQEC

- Constructing the detectors in the QEC circuits automatically.
- Finding the logical observables(correlation surfaces) in the surface code quantum computation automatically.
- Transpilation between the ZX-digram representation and the 3D spacetime representation.
- Compiling the 3D spacetime representation to the a simulatable `stim` circuit.
- Currently, only limited building blocks are implemented. We can construct and compile a logical CNOT gate. The compiled circuits were tested on Google's hardware.

4. Challenges and Future Directions

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- Error sources that have effects under ultra-low error rate region, e.g. cosmic rays.
- Real-time decoding: tradeoff between decoding speed and accuracy.
- The hardware/software architecture of large-scale FT quantum computer.

Thanks!

The slides are available on Github.

Bibliography

[1] Riverlane, "Quantum Error Correction Report 2024." [Online]. Available: https://www.riverlane.com/quantum-error-correction-report-2024

[2] R. Acharya *et al.*, "Quantum error correction below the surface code threshold," *arXiv preprint arXiv:2408.13687*, 2024.

[3] Y. Zhao *et al.*, "Realization of an error-correcting surface code with superconducting qubits," *Physical Review Letters*, vol. 129, no. 3, p. 30501, 2022.

[4] C. Gidney, "Inplace access to the surface code y basis," *Quantum*, vol. 8, p. 1310, 2024.

[5] C. Gidney, N. Shutty, and C. Jones, "Magic state cultivation: growing T states as cheap as CNOT gates," *arXiv preprint arXiv:2409.17595*, 2024.