FPGA SQUARE ROOT CALCULATOR

Digital Systems Design Course Project • Academic Year 2025-2026

"From mathematical abstraction to hardware realization — bridging algorithms and architecture."

TEAM MEMBERS

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Computer Science & Engineering 5

Algorithm Design and Development & Testing

Mathematical modeling, Python/C prototyping, Algorithm verification, Testing procedures

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SystemVerilog Implementation & Hardware Integration

FPGA implementation, Pin assignment, Hardware testing, SystemVerilog optimization

KEY FEATURES

⚡ Fast Computation

Single-cycle operation using pure combinational logic, no pipelining required

🎯 High Precision

Three decimal places (0.001 resolution) achieved through fixed-point arithmetic scaling

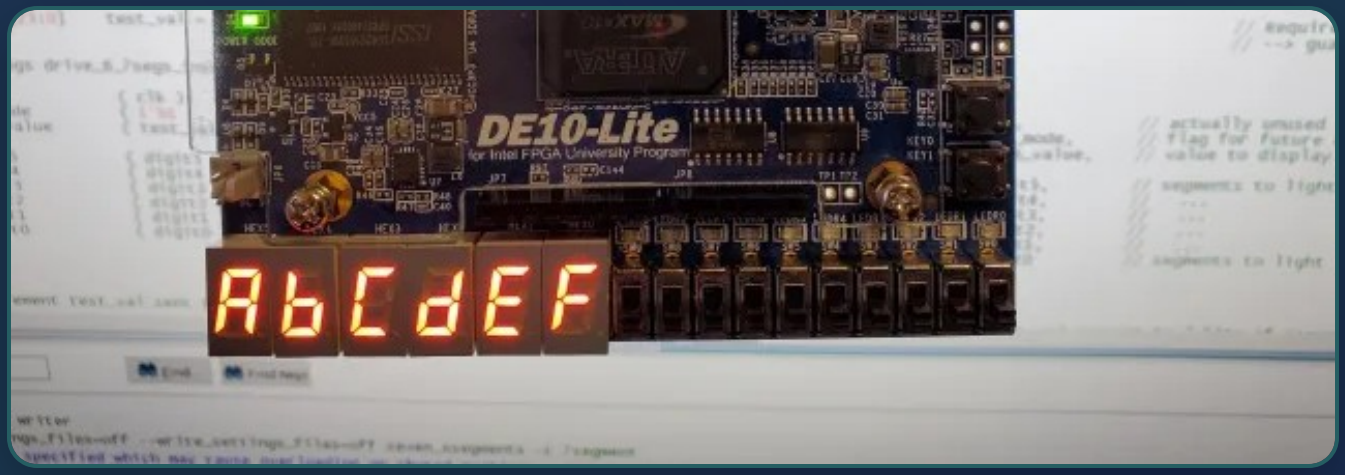
💻 Real-time Display

Five seven-segment digits with precisely positioned decimal point

Hardware Optimized

16-iteration binary search algorithm with minimal FPGA resource usage

FPGA HARDWARE IMPLEMENTATION



FPGA Development Board with Seven-Segment Displays

Input System

10-bit binary switches providing input range 0-1024

🧠 Processing Core

Binary Search Unit with exactly 16 iterations for optimal precision

📟 Output Display

5× seven-segment displays with controlled decimal point

⏱️ Performance

Pure combinational logic delivering results in single clock cycle

CORE ALGORITHM

$$\sqrt{n} \approx \sqrt{(n \times 1{,}000{,}000)} \div 1{,}000$$

Python Implementation (Algorithm Validation)

File Edit Format Run Options Window Help

```python
def sqrt_(n):
    #mashtabiryem
    scaled_n = n * 1000000

    #binary search
    low, high = 0, 40000
    for _ in range(16):
        mid = (low + high) // 2
        if mid * mid <= scaled_n:
            low = mid
        else:
            high = mid

    #find celya chact and drobnya chast
    int_part = low // 1000      #celya chast
    frac = low % 1000           #drobnya chast

    #find every digin
    tens = int_part // 10           #desatki
    ones = int_part % 10            #edinytsy
    tenths = frac // 100            #desatye
    hundredths = (frac % 100) // 10 #sotye
    thousandths = frac % 10         #tisyachnie

    return tens, ones, tenths, hundredths, thousandths

#input and output
n= int(input())
result = sqrt_(n)
print(f"√{n} ≈ {result[0]}{result[1]}.{result[2]}{result[3]}{result[4]}")
```

Python Binary Search Implementation for Algorithm Validation

Python Test Output

```
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
25
√25 ≈ 05.000
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
1
√1 ≈ 01.000
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
1023
√1023 ≈ 31.984
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
512
√512 ≈ 22.627
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
256
√256 ≈ 16.000
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
128
√128 ≈ 11.313
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
64
√64 ≈ 08.000
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
32
√32 ≈ 05.656
>>>
===== RESTART: C:/Users/kalka/OneDrive/Рабочий стол/питончик/sqrt_final.py =====
511
√511 ≈ 22.605
>>>
========================================================= RESTART: C:/Users/kalka/OneDrive/Раб
5
√5 ≈ 02.236
>>>
========================================================= RESTART: C:/Users/kalka/OneDrive/Раб
731
√731 ≈ 27.037
>>>
========================================================= RESTART: C:/Users/kalka/OneDrive/Раб
333
√333 ≈ 18.248
>>>
========================================================= RESTART: C:/Users/kalka/OneDrive/Раб
999
√999 ≈ 31.606
>>>
```

Python Algorithm Test Results

C Implementation (Batch Testing)

.vscode > .сишка > C amit.c

```c
#include <stdio.h>
int main() {
    int n;
    //vvod chisla cherez probel
    while(scanf("%d", &n) == 1) {
        //mashtabiryem
        long long scaled_n = n * 1000000LL;
        //binary search
        long long low = 0;
        long long high = 40000;
        for (int i = 0; i < 16; i++) {
            long long mid = (low + high) / 2;
            long long mid_sq = mid * mid;

            if (mid_sq <= scaled_n) {
                low = mid;
            } else {
                high = mid;
            }
        }
        //find celya chact and drobnya chast
        int int_part = low / 1000;      //celya chast
        int frac = low % 1000;          //drobnya chast
        //find every digin
        int tens = int_part / 10;           //desatki
        int ones = int_part % 10;           //edinytsy
        int tenths = frac / 100;            //desatye
        int hundredths = (frac % 100) / 10;  //sotye
        int thousandths = frac % 10;        //tisyachnie
        //input and output
        printf("%d%d.%d%d%d ", tens, ones, tenths, hundredths, thousandths);
    }
    return 0;
}
```

C Program for Batch Processing and Testing

C Test Output

```
                                                                > cd "c:\Users\kalka\OneD
mit.c -o amit } ; if ($?) { .\amit }
25 1 1023 512 256 128 64 32 511 5 731 333 999
05.000 01.000 31.984 22.627 16.000 11.313 08.000 05.656 22.605 02.236 27.037 18.248 31.606
```

C Program Test Results

ALTERNATIVE SYSTEMVERILOG IMPLEMENTATION

Optimized Root Calculation with 4-bit Approximation

Integer Part Calculation (Binary Search)

```
// Binary search for integer part
always @(*) begin
    n = sw;

    if ((8'd8 * 8'd8) <= n) begin
        root = 8;
    end else begin
        root = 0;
```

```
    end

    if (((root+4)*(root+4)) <= n) begin
        root = root + 4;
    end

    if (((root+2)*(root+2)) <= n) begin
        root = root + 2;
    end

    if (((root+1)*(root+1)) <= n) begin
        root = root + 1;
    end
end
```

Fractional Part Calculation (Minimal Difference)

```
// Calculate all possible fractional values
wire [15:0] t0 = (root*10+0)*(root*10+0);
wire [15:0] t1 = (root*10+1)*(root*10+1);
// ... t2 through t9
wire [15:0] target = n * 100;

// Find minimal difference
function [15:0] diff;
```

```
    input [15:0] a, b;
    begin
        diff = (a>b) ? (a-b) : (b-a);
    end
endfunction
```

</>

```
// Complete SystemVerilog module
module sqrtt(
    input  [7:0] sw,          // 8-bit input
    output [6:0] hex1,        // Integer part display
    output [7:0] hex0         // Fractional part display
);

    reg [4:0] root;           // Integer root (0-15)
    reg [3:0] frac;           // Fractional digit (0-9)
    reg [7:0] n;

    // Integer root calculation (binary search)
    always @(*) begin
        n = sw;
        // ... binary search logic ...
    end

    // Fractional part calculation
```

```
    wire [15:0] t0  = (root*10+0)*(root*10+0);
    wire [15:0] t1  = (root*10+1)*(root*10+1);
    // ... t2 through t9 calculations
    wire [15:0] target = n * 100;

    // Find minimal difference for fractional digit
    always @(*) begin
        reg [15:0] best;
        best = diff(t0, target);
        frac = 0;

        if (diff(t1,target) < best) begin best =
diff(t1,target); frac = 1; end
        // ... compare t2 through t9 ...
    end

    // Seven-segment display function
    function [6:0] seg7;
        input [3:0] d;
        begin
            case (d)
                4'd0: seg7 = 7'b1000000;  // "0"
                4'd1: seg7 = 7'b1111001;  // "1"
                // ... digits 2-9 ...
                default: seg7 = 7'b1111111; // Blank
            endcase
        end
```

```
    endfunction

    assign hex1 = seg7(root[3:0]);      // Integer part
    assign hex0[6:0] = seg7(frac);      // Fractional part
    assign hex0[7] = 1'b0;              // Decimal point control

endmodule
```

MAIN SYSTEMVERLOG IMPLEMENTATION

Binary Search Core

```
// Fixed-point binary search algorithm for FPGA
always_comb begin
    // Scale input for 3 decimal places precision
    scaled_n = sw * 32'd1000000;

    low = 0;
    high = 40000;    // sqrt(1024*1e6) upper boundary

    // Unrolled 16-iteration binary search
```

```
    repeat (16) begin
        mid = (low + high) >> 1;      // Efficient
division by 2
        mid_sq = mid * mid;

        if (mid_sq <= scaled_n)
            low = mid;
        else
            high = mid;
    end

    // Extract integer and fractional parts
    int_part = low / 1000;
    frac = low % 1000;
end
```

Display Output Logic

```
// Digit extraction for display output
assign frac_h = frac / 100;          // Hundredths digit
assign frac_t = (frac % 100) / 10; // Tenths digit
assign frac_o = frac % 10;           // Thousandths digit
```

```
assign int_t = int_part / 10;        // Tens digit
assign int_o = int_part % 10;        // Ones digit

// Seven-segment display drivers
assign HEX4 = seg7(int_t);           // Display tens
assign HEX3 = seg7(int_o);           // Display ones with
decimal point
assign HEX2 = seg7(frac_h);          // Display tenths
assign HEX1 = seg7(frac_t);          // Display hundredths
assign HEX0 = seg7(frac_o);          // Display
thousandths
```

TEST RESULTS

Sample Input/Output Tests

Input: 25
√25 = 05.000

Input: 100
√100 = 10.000

Input: 500
√500 = 22.360

Input: 1024
√1024 = 32.000

Performance Metrics Achieved

Accuracy
Maximum error < 0.001

Speed
1 result per clock cycle

Input Range
0-1024 fully verified

Resources
Minimal LUT utilization

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